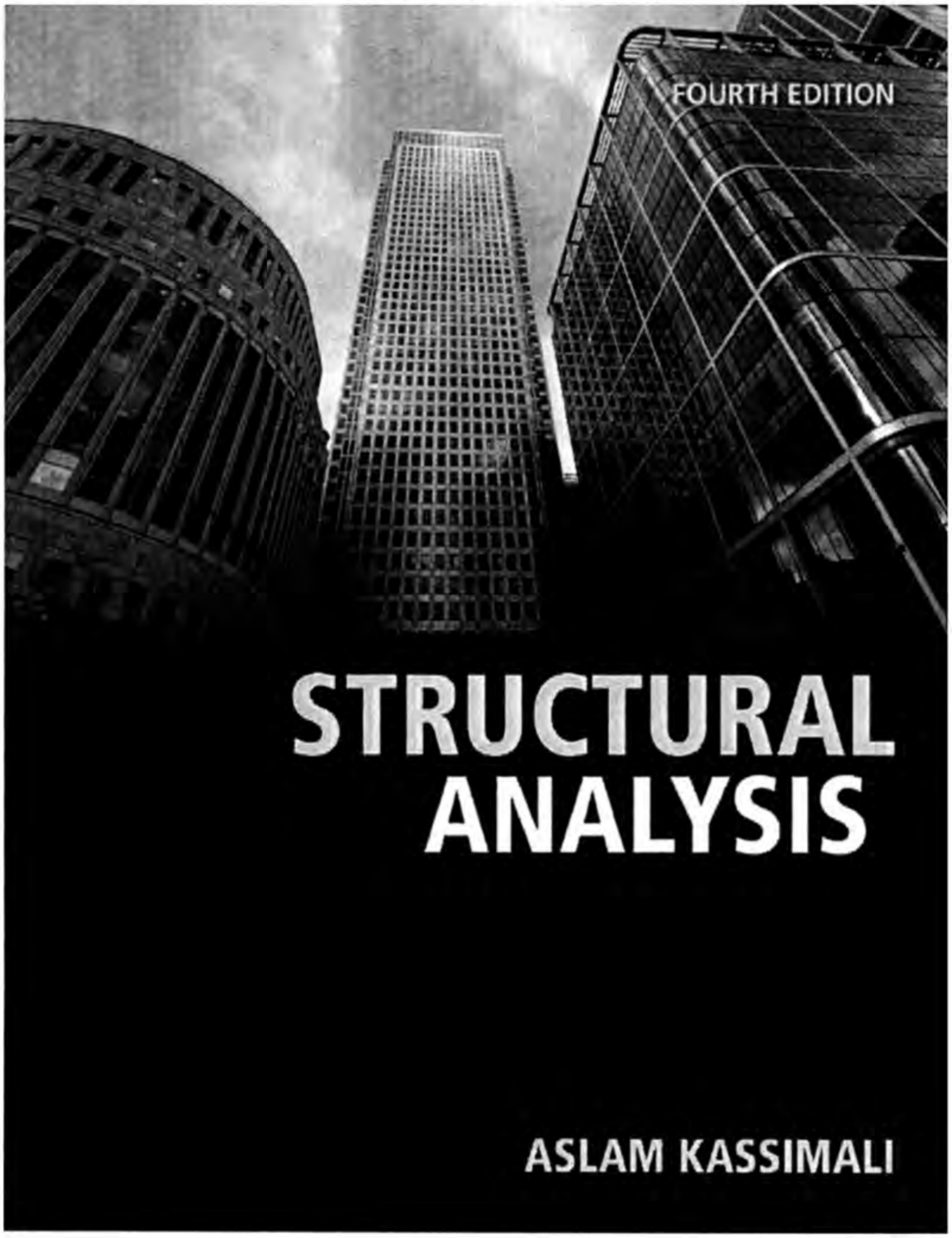
An Instructor's Solutions Manual to Accompany

Structural Analysis, 4th Edition

Aslam Kassimali



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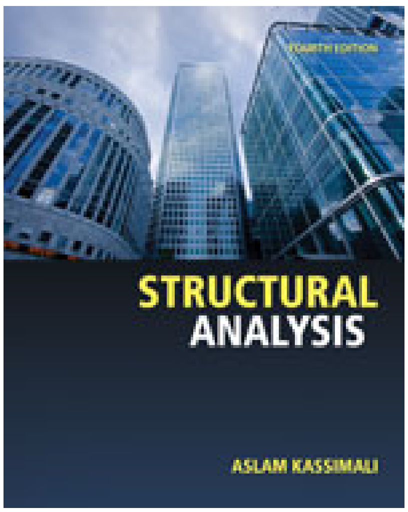
An Instructor's Solutions Manual

For

Structural Analysis
Fourth Edition

Aslam Kassimali
Southern Illinois University Carbondale



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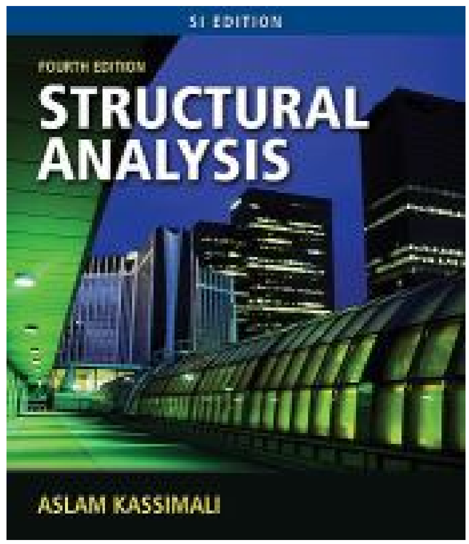

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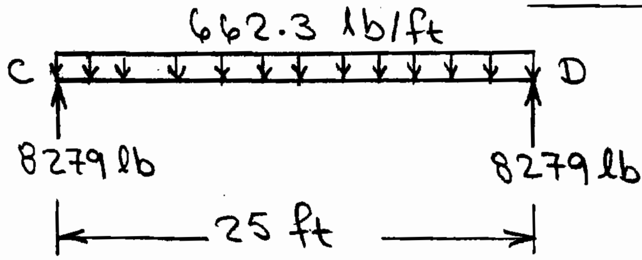
Chapter Two
Loads on Structures

CHAPTER 2

2.1 Beam CD

Uniformly distributed load $= 150(12)\left(\frac{4}{12}\right) + 490\left(\frac{18.3}{144}\right)$

$= \underline{662.3 \text{ lb/ft}}$



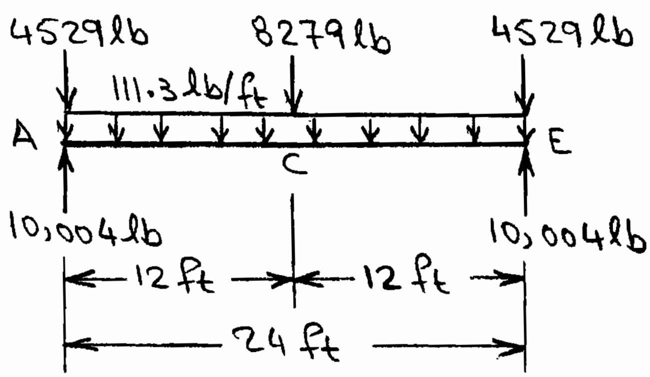
Girder AE

Uniformly distributed load $= 490\left(\frac{32.7}{144}\right) = \underline{111.3 \text{ lb/ft}}$

Concentrated load at C $= \underline{8279 \text{ lb}}$

Concentrated loads at A and E

$= \left[150(6)\left(\frac{4}{12}\right) + 490\left(\frac{18.3}{144}\right)\right]\left(\frac{25}{2}\right) = \underline{4529 \text{ lb}}$



$\boxed{2.2}$ See solution of Problem 2.1

<u>Beam CD</u> Uniformly distributed load

$= 662.3 + 120\left(\frac{6}{12}\right)(7) = 662.3 + 420 = \underline{1082.3\ \text{lb/ft}}$

<u>Girder AE</u> Uniformly distributed load $= \underline{111.3\ \text{lb/ft}}$

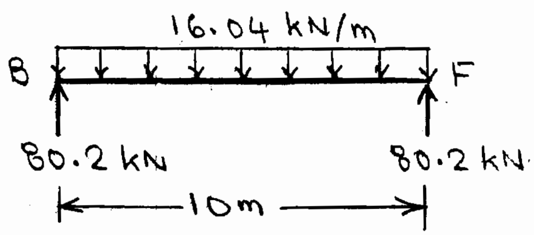
Concentrated load at C $= 8279 + 420\left(\frac{25}{2}\right) = \underline{13{,}529\ \text{lb}}$

Concentrated loads at A and E $= \underline{4529\ \text{lb}}$

$\boxed{2.3}$ <u>Beam BF</u>

Uniformly distributed load

$= 23.6(5)\left(\frac{130}{1000}\right) + 77\left(\frac{9100}{10^6}\right) = \underline{16.04\ \text{kN/m}}$



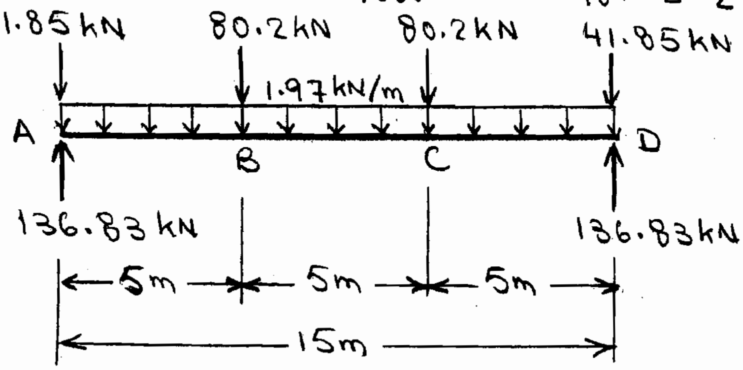
<u>Girder AD</u>

Uniformly distributed load $= 77\left(\frac{25600}{10^6}\right) = \underline{1.97\ \text{kN/m}}$

Concentrated loads at B and C $= \underline{80.2\ \text{kN}}$

Concentrated loads at A and D

$= \left[23.6(2.5)\left(\frac{130}{1000}\right) + 77\left(\frac{9100}{10^6}\right)\right]\frac{10}{2} = \underline{41.85\ \text{kN}}$



$\boxed{2.4}$

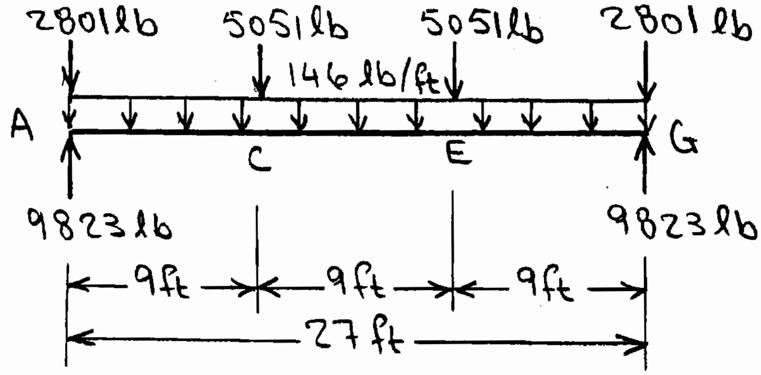
Uniformly distributed load $= 490\left(\frac{42.9}{144}\right) = \underline{146\ lb/ft}$

Concentrated loads at A and G

$$= \left[150(4.5)\left(\frac{4}{12}\right) + 490\left(\frac{16.2}{144}\right)\right]\left(\frac{20}{2}\right) = \underline{2801\ lb}$$

Concentrated loads at C and E

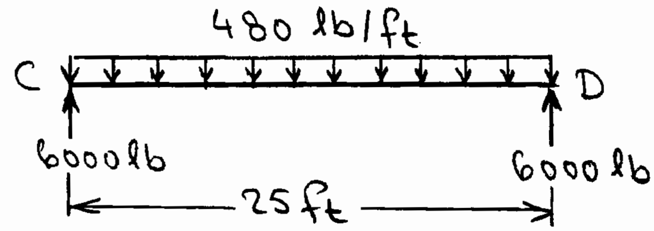
$$= \left[150(9)\left(\frac{4}{12}\right) + 490\left(\frac{16.2}{144}\right)\right]\left(\frac{20}{2}\right) = \underline{5051\ lb}$$



$\boxed{2.5}$ Live load = 40 psf

Beam CD

Uniformly distributed load = 40(12) = 480 lb/ft

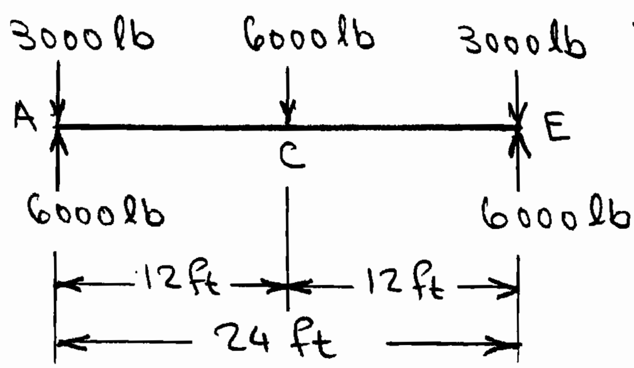


Girder AE

Concentrated load at C = 6000 lb

Concentrated loads at A and E = $[40(6)]\left(\frac{25}{2}\right)$

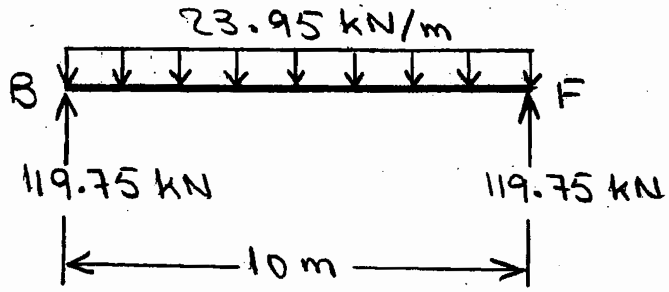
= 3000 lb



$\boxed{2.6}$ Live load = 4.79 kPa = 4.79 kN/m^2

Beam BF

Uniformly distributed load = 4.79(5) = $\underline{23.95 \text{ kN/m}}$

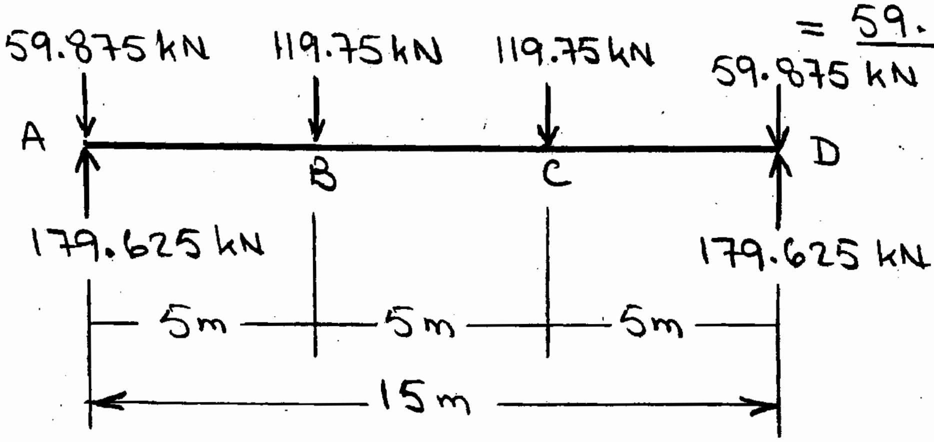


Girder AD

Concentrated loads at B and C = $\underline{119.75 \text{ kN}}$

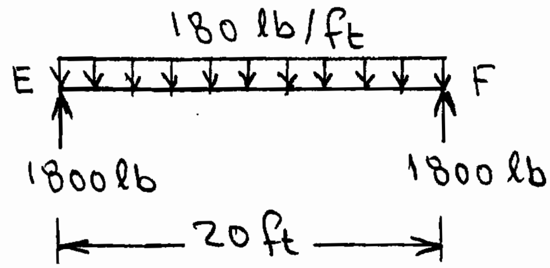
Concentrated loads at A and D = $[4.79(2.5)]\frac{10}{2}$

$= \underline{59.875 \text{ kN}}$



2.7 Beam EF

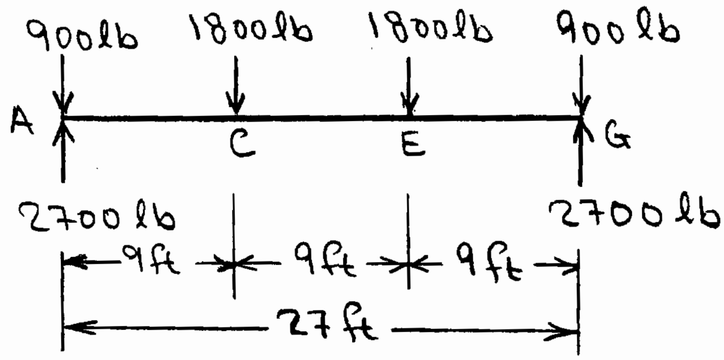
Uniformly distributed load = 20(9) = 180 lb/ft



Girder AG

Concentrated loads at C and E = 1800 lb

Concentrated loads at A and G = 1800/2 = 900 lb



Column A   Concentrated load = 2700 lb

$\boxed{2.8}$ $V = 85$ mph, $h = 40 + (15/2) = 47.5$ ft,

$I = 1.0$, $z_g = 1200$ ft, $\alpha = 7.0$, $K_{zt} = 1$

and $K_d = 1$

$$K_h = 2.01\left(\frac{47.5}{1200}\right)^{2/7} = 0.8$$

$$q_h = 0.00256(0.8)(1)(1)(85)^2(1) = 14.8 \text{ psf}$$

$$G = 0.85$$

For $\theta = 45°$ and $h/L = 47.5/30 = 1.58$:

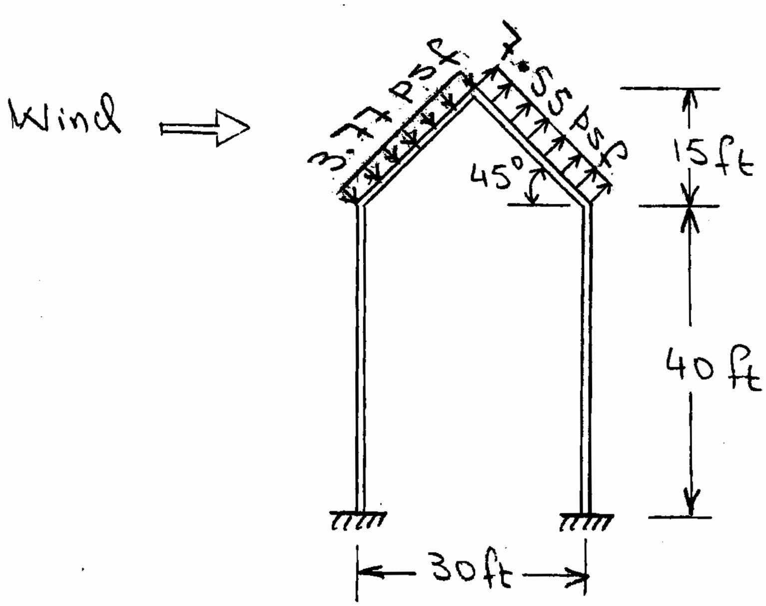
$C_p = 0.3$ for windward side

$C_p = -0.6$ for leeward side

Thus, the wind pressures are:

$p_h = 14.8(0.85)(0.3) = \underline{3.77 \text{ psf}}$ for windward side

$p_h = 14.8(0.85)(-0.6) = \underline{-7.55 \text{ psf}}$ for leeward side



$\boxed{2.9}$ $V = 40\ m/s$, $h = 12 + \frac{5}{2} = 14.5\ m$

$I = 1.15$, $z_g = 366\ m$, $\alpha = 7.0$, $K_{zt} = 1$

and $K_d = 1$

$$K_h = 2.01\left(\frac{14.5}{366}\right)^{2/7} = 0.8$$

$$q_h = 0.613(0.8)(1)(1)(40)^2(1.15) = 902.34\ N/m^2$$

$G = 0.85$

Roof slope: $\theta = \tan^{-1}(5/6) = 39.8^\circ$

$$\frac{h}{L} = \frac{14.5}{12} = 1.21$$

$C_p = -0.1$ and 0.25 for windward side

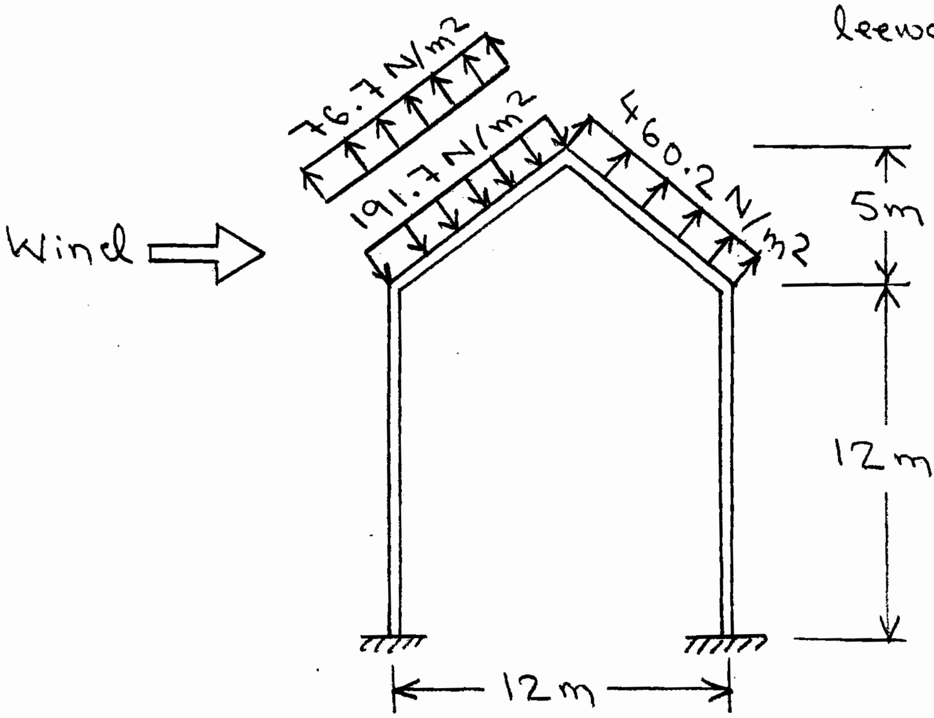
$C_p = -0.6$ for leeward side

Thus, the wind pressures are:

$p_h = (902.34)(0.85)(-0.1) = \underline{-76.7\ N/m^2}$

$p_h = (902.34)(0.85)(0.25) = \underline{191.7\ N/m^2}$ for windward side

$p_h = (902.34)(0.85)(-0.6) = \underline{-460.2\ N/m^2}$ for leeward side



$\boxed{2.10}$ $V = 90$ mph, $h = 30 + \frac{11}{2} = 35.5$ ft

$I = 1.15$, $z_g = 900$ ft, $\alpha = 9.5$, $K_{zt} = 1$

and $K_d = 1$

$$K_h = 2.01\left(\frac{35.5}{900}\right)^{2/9.5} = 1.02$$

$$q_h = 0.00256(1.02)(1)(1)(90)^2(1.15) = 24.32 \text{ psf}$$

$G = 0.85$

Roof slope: $\theta = \tan^{-1}(11/20) = 28.8^\circ$

$$\frac{h}{L} = \frac{35.5}{40} = 0.89$$

$C_p = -0.3$ and 0.2 for windward side

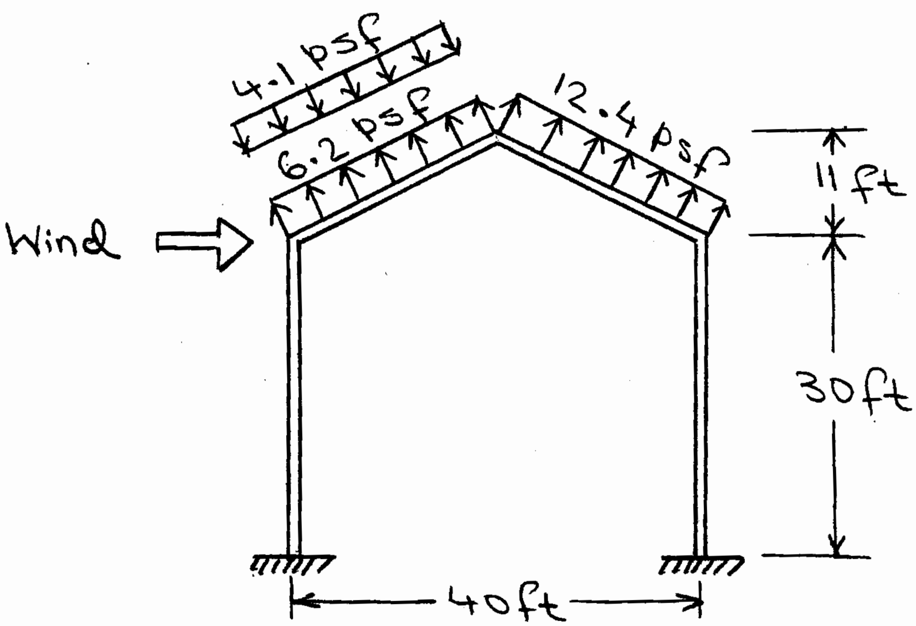
$C_p = -0.6$ for leeward side

Thus, the wind pressures are:

$p_h = 24.32(0.85)(-0.3) = \underline{-6.2 \text{ psf}}$ for windward side

$p_h = 24.32(0.85)(0.2) = \underline{4.1 \text{ psf}}$ for windward side

$p_h = 24.32(0.85)(-0.6) = \underline{-12.4 \text{ psf}}$ for leeward side



2.11 $V = 90$ mph, $I = 1.15$, $z_g = 900$ ft, $\alpha = 9.5$

From the solution of Problem 2.10:

$q_h = 24.32$ psf and $G = 0.85$

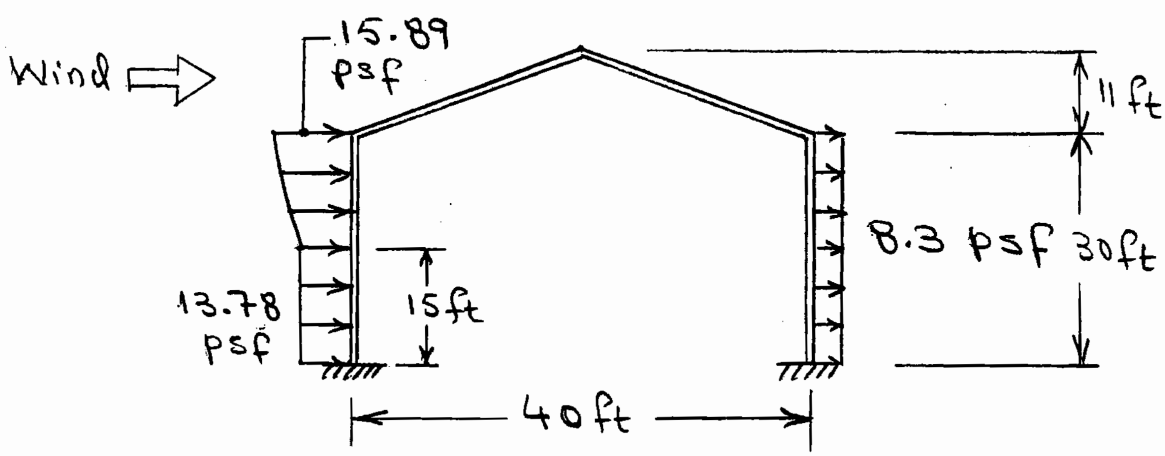
Leeward wall: For $L/B = 40/30 = 1.33$, $C_p = -0.4$

Thus, the wind pressure, $p_h = 24.32(0.85)(-0.4)$

$= \underline{-8.3 \text{ psf}}$

Windward wall: $C_p = 0.8$

z (ft)	K_z	q_z (psf)	p_z (psf)
30	0.98	23.37	15.89
25	0.95	22.65	15.4
20	0.90	21.46	14.59
15	0.85	20.27	13.78



$\boxed{2.12}$ $p_g = 20\,psf$, $C_e = 1$, $C_t = 1$, $I = 1.2$

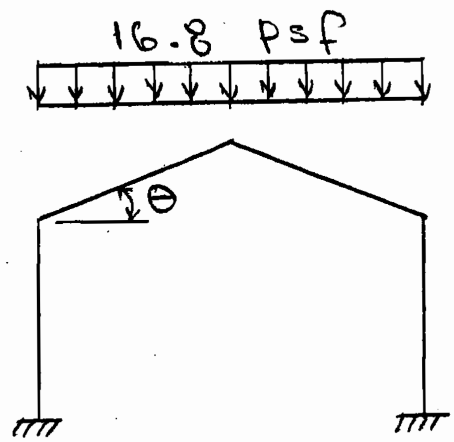
$p_f = 0.7\,C_e\,C_t\,I\,p_g = 0.7\,(1)(1)(1.2)(20) = 16.8\,psf$

$\theta = \tan^{-1}(11/20) = 28.8°$, $\frac{70}{W} + 0.5 = \frac{70}{20} + 0.5 = 4°$

Therefore, the minimum values of p_f need not be considered.

$C_s = 1$

Balanced load $= p_s = C_s\,p_f = 1\,(16.8) = \underline{16.8\ psf}$



Balanced
Snow Load

$\boxed{2.13}$ $p_g = 1.2\ kN/m^2$, $C_e = 1$, $C_t = 1$, $I = 1.1$

$p_f = 0.7 C_e C_t I p_g = 0.7(1)(1)(1.1)(1.2) = 0.92\ kN/m^2$

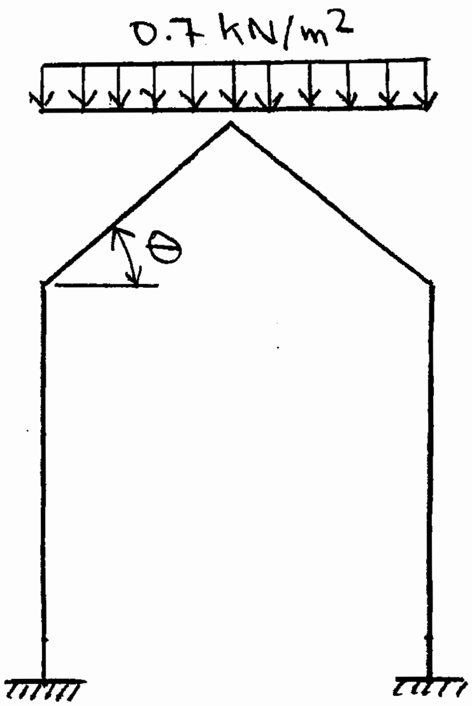
$\theta = \tan^{-1}(5/6) = 39.8°$, $W = 6m = 19.7\ ft$

$\frac{70}{W} + 0.5 = \frac{70}{19.7} + 0.5 = 4.1°$

Therefore, the minimum values of p_f need not be considered.

$C_s = 1 - \frac{\theta - 30°}{40°} = 0.76$

Balanced Load $= p_s = C_s p_f = 0.76(0.92) = \underline{0.7\ kN/m^2}$



Balanced Snow Load

Chapter Three
Equilibrium and Support Reactions

CHAPTER 3

3.1 (a) Internally stable, with $r = 3$. It is statically determinate

(b) Internally stable, with $r = 5$. It is statically indeterminate. $i_e = 5 - 3 = 2$

(c) Internally unstable, with $r = 6$ and $e_c = 2$. As $r > 3 + e_c$, the structure is statically indeterminate. $i_e = 6 - (3 + 2) = 1$

(d) Internally unstable with $r = 3$ and $e_c = 1$. As $r < 3 + e_c$, the beam is statically unstable.

3.2 (a) Internally stable, with $r = 5$. It is statically indeterminate. $i_e = 5 - 3 = 2$.

(b) Internally unstable, with $r = 5$ and $e_c = 2$. As $r = 3 + e_c$, the beam is statically determinate.

(c) Internally stable, with $r = 6$. It is statically indeterminate. $i_e = 6 - 3 = 3$

(d) Internally unstable, with $r = 6$ and $e_c = 1$. As $r > 3 + e_c$, the arch is statically indeterminate. $i_e = 6 - (3 + 1) = 2$.

(e) Internally unstable, with $r = 5$ and $e_c = 2$. As $r = 3 + e_c$, the frame is statically determinate.

$\boxed{3.3}$ (a) Internally unstable, with $r = 5$ and $e_c = 3$.

As $r < 3 + e_c$, the beam is <u>statically unstable</u>.

(b) Internally unstable, with $r = 4$ and $e_c = 2$.

As $r < 3 + e_c$, the beam is <u>statically unstable</u>.

(c) Internally stable, with $r = 4$. It is <u>statically indeterminate</u>. $i_e = 4 - 3 = \underline{1}$.

(d) Internally unstable, with $r = 4$ and $e_c = 2$.

As $r < 3 + e_c$, the frame is <u>statically unstable</u>.

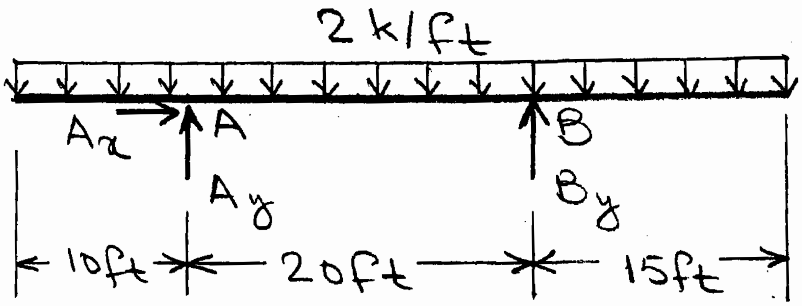
$\boxed{3.4}$ (a) Internally stable, with $r = 4$. It is <u>statically indeterminate.</u> $i_e = 4 - 3 = \underline{1}$.

(b) Internally unstable, with $r = 4$ and $e_c = 2$. As $r < 3 + e_c$, the frame is <u>statically unstable.</u>

(c) Internally stable, with $r = 6$. It is <u>statically indeterminate</u>. $i_e = 6 - 3 = \underline{3}$.

(d) Internally unstable, with $r = 6$ and $e_c = 3$. As $r = 3 + e_c$, the frame is <u>statically determinate.</u>

3.5



$\Sigma F_x = 0$ $\qquad\qquad$ $\underline{A_x = 0}$

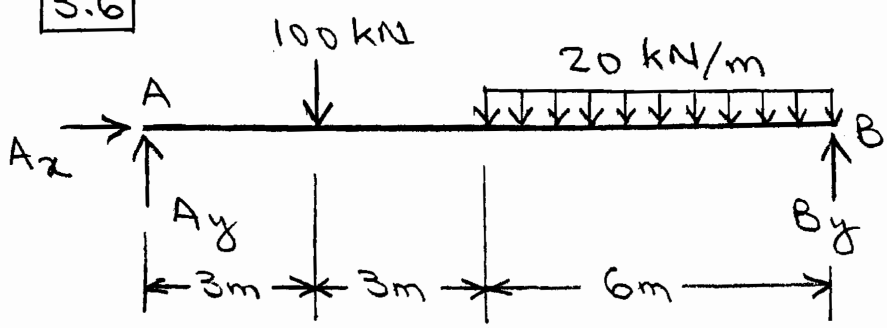
$+\circlearrowright \Sigma M_B = 0 \qquad -A_y(20) + 2(45)(7.5) = 0$

$\underline{A_y = 33.75 \text{ k} \uparrow}$

$+\uparrow \Sigma F_y = 0 \qquad 33.75 - 2(45) + B_y = 0$

$\underline{B_y = 56.25 \text{ k} \uparrow}$

3.6



$\Sigma F_x = 0$ $\qquad\qquad$ $\underline{A_x = 0}$

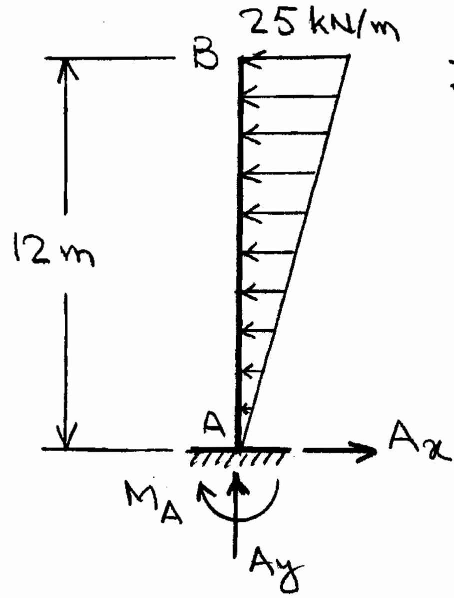
$+\circlearrowright \Sigma M_B = 0 \qquad -A_y(12) + 100(9) + 20(6)3 = 0$

$\underline{A_y = 105 \text{ kN} \uparrow}$

$+\uparrow \Sigma F_y = 0 \qquad 105 - 100 - 20(6) + B_y = 0$

$\underline{B_y = 115 \text{ kN} \uparrow}$

3.7



$\xrightarrow{+} \Sigma F_x = 0$

$-\frac{1}{2}(25)(12) + A_x = 0$

$\underline{A_x = 150 \text{ kN} \rightarrow}$

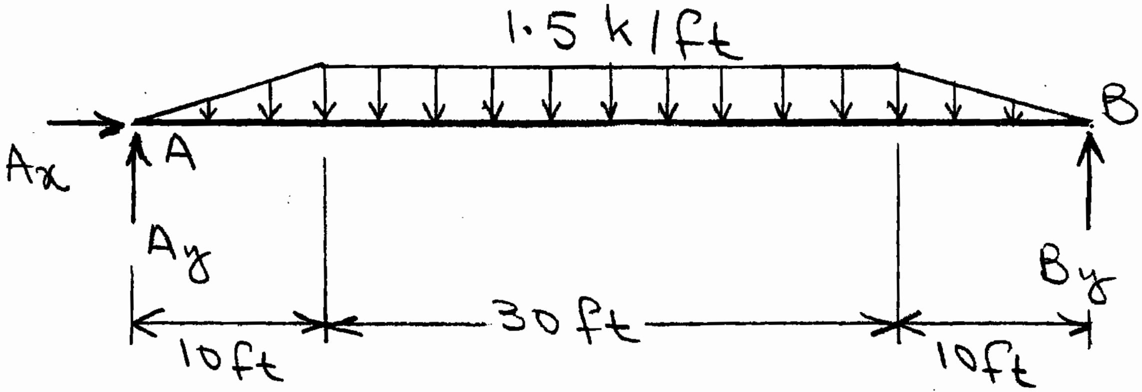
$\Sigma F_y = 0 \qquad \underline{A_y = 0}$

$+\circlearrowright \Sigma M_A = 0$

$-M_A + \frac{1}{2}(25)(12)(8) = 0$

$\underline{M_A = 1200 \text{ kN.m} \circlearrowright}$

$\boxed{3.8}$



$\Sigma F_x = 0$ $\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ $\underline{A_x = 0}$

$+\circlearrowleft \Sigma M_B = 0$

$$-A_y(50) + \frac{1}{2}(1.5)10\left(\frac{10}{3}+40\right) + 1.5(30)(25) + \frac{1}{2}(1.5)10\left(\frac{2}{3}\right)(10) = 0$$

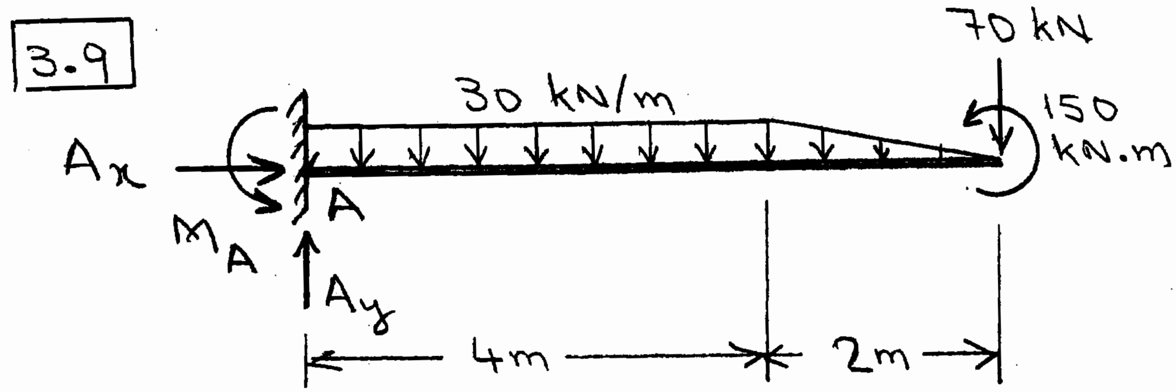
$$\underline{A_y = 30\ \text{k}\uparrow}$$

$+\uparrow \Sigma F_y = 0$

$$30 - \frac{1}{2}(1.5)(10)(2) - 1.5(30) + B_y = 0$$

$$\underline{B_y = 30\ \text{k}\uparrow}$$

3.9



$\Sigma F_x = 0$ $\qquad$ $\underline{A_x = 0}$

$+\uparrow \Sigma F_y = 0 \qquad A_y - 30(4) - \frac{1}{2}(30)(2) - 70 = 0$

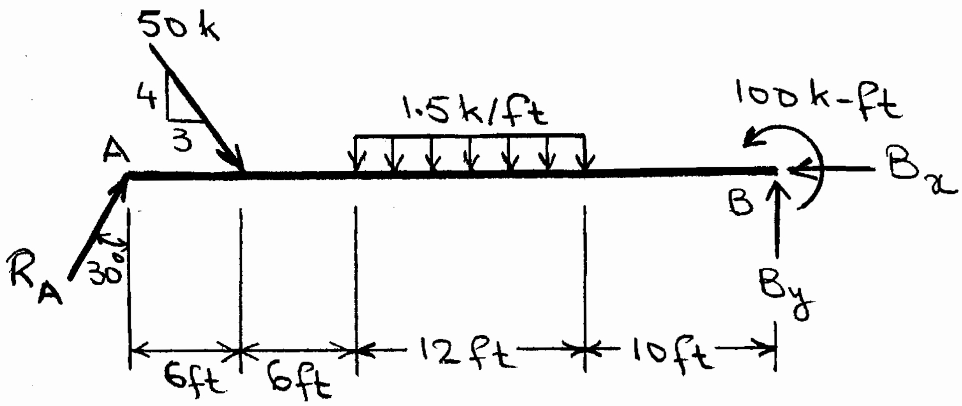
$\underline{A_y = 220 \text{ kN} \uparrow}$

$+\circlearrowleft \Sigma M_A = 0$

$M_A - 30(4)(2) - \frac{1}{2}(30)(2)\left(4 + \frac{2}{3}\right) - 70(6) + 150 = 0$

$\underline{M_A = 650 \text{ kN.m} \circlearrowleft}$

$\boxed{3.10}$



$+\circlearrowleft \Sigma M_B = 0$

$$-R_A(\cos 30°)(34) + \frac{4}{5}(50)(28) + 1.5(12)(16) + 100 = 0$$

$\underline{R_A = 51.2\text{ k} \nearrow}$

$\overset{+}{\rightarrow} \Sigma F_x = 0$

$$51.2(\sin 30°) + \frac{3}{5}(50) - B_x = 0$$

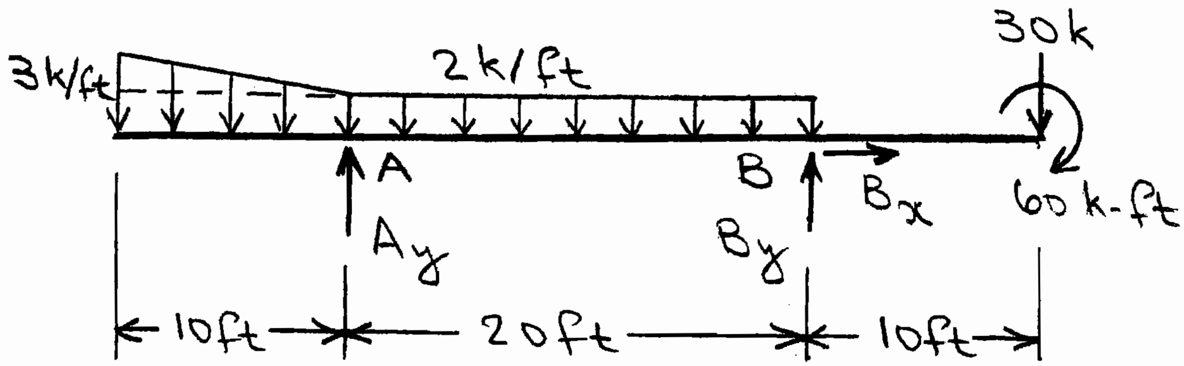
$\underline{B_x = 55.6\text{ k} \leftarrow}$

$+\uparrow \Sigma F_y = 0$

$$51.2(\cos 30°) - \frac{4}{5}(50) - 1.5(12) + B_y = 0$$

$\underline{B_y = 13.65\text{ k} \uparrow}$

$\boxed{3.11}$



$$\Sigma F_x = 0 \qquad \underline{B_x = 0}$$

$$+\circlearrowleft \Sigma M_A = 0$$

$$\frac{1}{2}(1)(10)\left(\frac{20}{3}\right) - 2(30)(5) + B_y(20) - 30(30) - 60 = 0$$

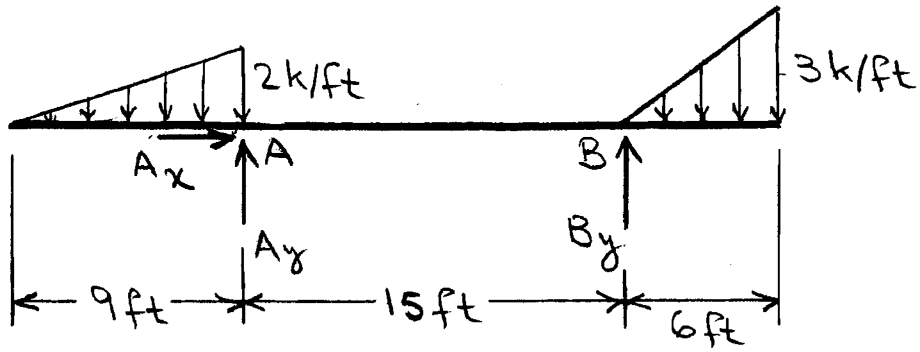
$$\underline{B_y = 61.33\text{ k}\uparrow}$$

$$+\uparrow \Sigma F_y = 0$$

$$-\frac{1}{2}(1)(10) - 2(30) + A_y + 61.33 - 30 = 0$$

$$\underline{A_y = 33.67\text{ k}\uparrow}$$

$\boxed{3.12}$



$\Sigma F_x = 0$ $\qquad$ $\underline{A_x = 0}$

$+\circlearrowright \Sigma M_B = 0$

$\frac{1}{2}(2)(9)\left[\frac{1}{3}(9)+15\right] - A_y(15) - \frac{1}{2}(3)(6)(4) = 0$

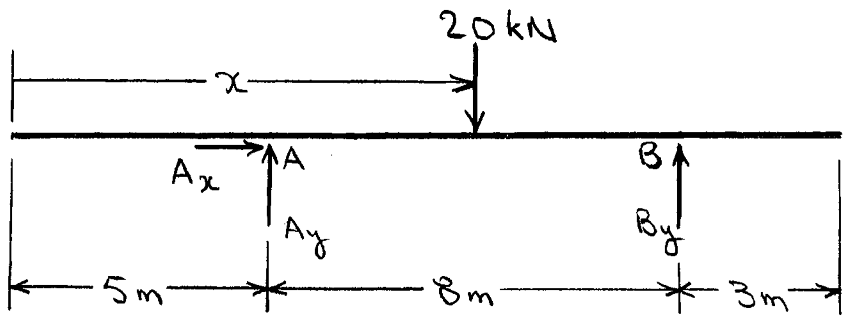
$\underline{A_y = 8.4\,k \uparrow}$

$+\uparrow \Sigma F_y = 0$

$-\frac{1}{2}(2)(9) + 8.4 + B_y - \frac{1}{2}(3)(6) = 0$

$\underline{B_y = 9.6\,k \uparrow}$

3.14



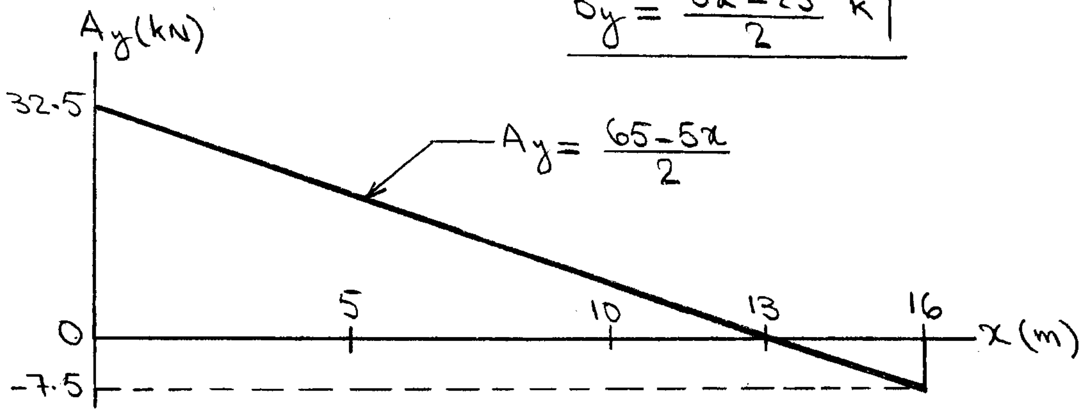
$\Sigma F_x = 0$ $\qquad\qquad$ $\underline{A_x = 0}$

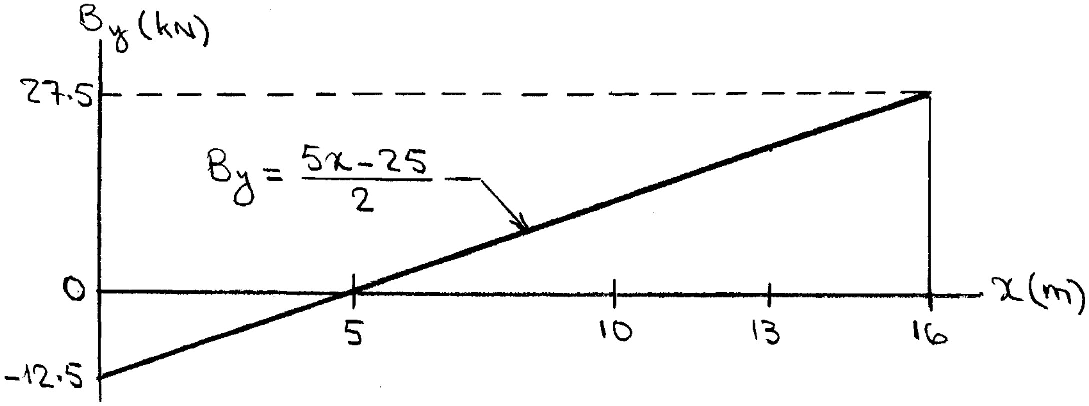
$+\circlearrowleft \Sigma M_B = 0$ $\qquad$ $-A_y(8) + 20(13 - x) = 0$

$$\underline{A_y = \frac{65 - 5x}{2} \text{ k}\uparrow}$$

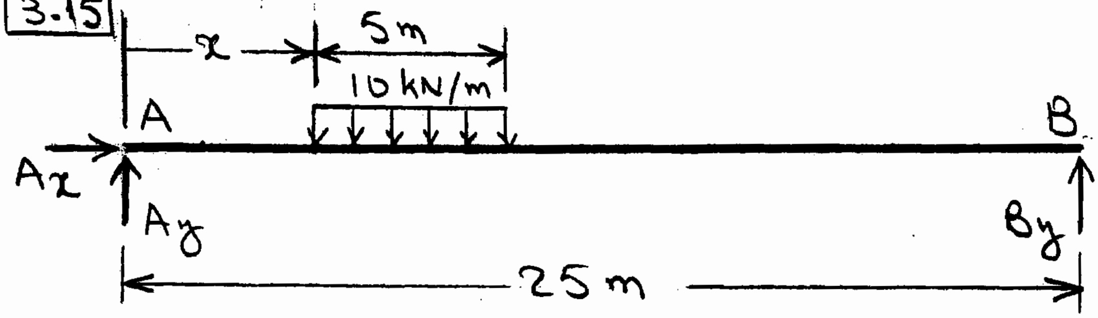
$+\uparrow \Sigma F_y = 0$ $\qquad$ $\frac{65 - 5x}{2} - 20 + B_y = 0$

$$\underline{B_y = \frac{5x - 25}{2} \text{ k}\uparrow}$$





3.15



$\Sigma F_x = 0 \qquad\qquad A_x = 0$

<u>$0 \leqslant x \leqslant 20$ m:</u>

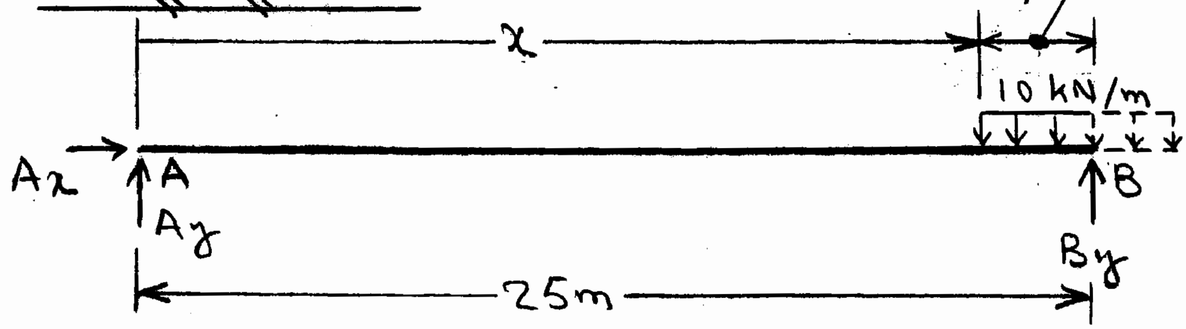
$+\circlearrowleft \Sigma M_B = 0 \qquad -A_y(25) + 10(5)[25 - (x+25)] = 0$

<u>$A_y = 45 - 2x$ kN ↑</u>

$+\uparrow \Sigma F_y = 0 \qquad (45 - 2x) - 10(5) + B_y = 0$

<u>$B_y = 5 + 2x$ kN ↑</u>

<u>$20\text{ m} \leqslant x \leqslant 25$ m:</u>



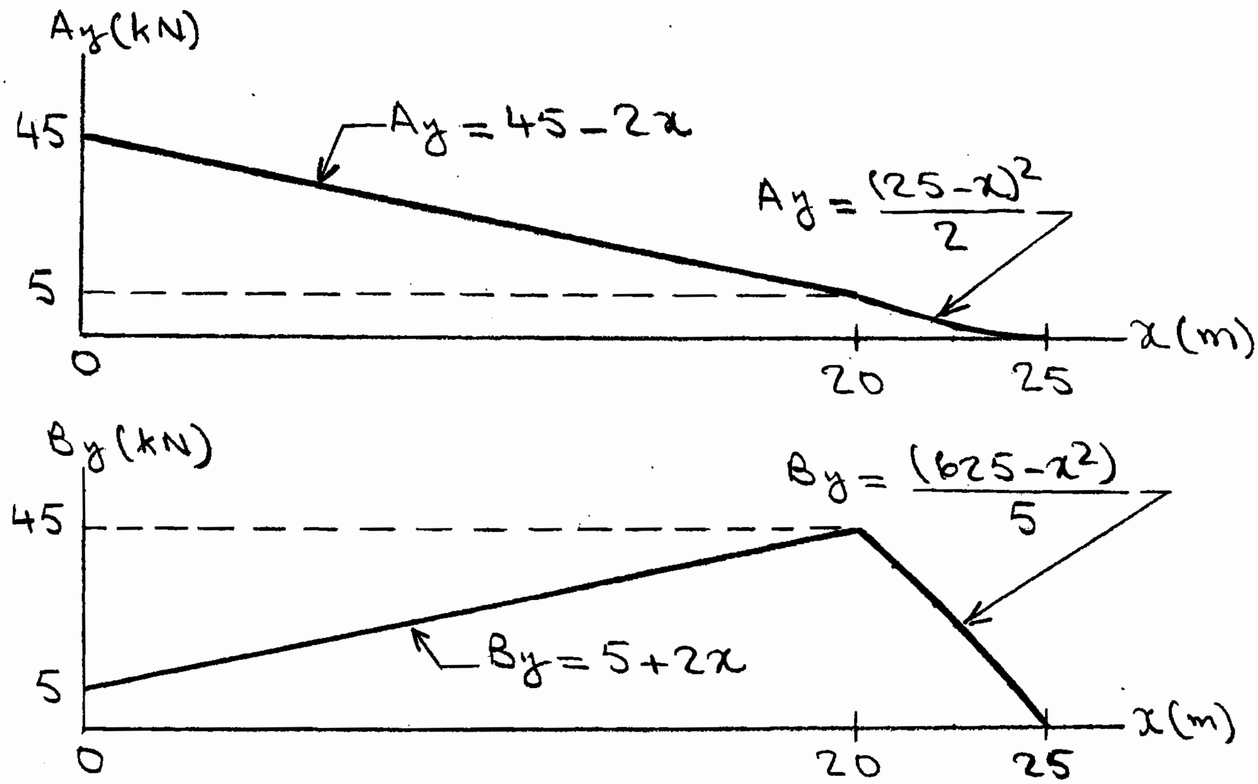
$+\circlearrowleft \Sigma M_B = 0$

$-A_y(25) + 10(25-x)\left(\frac{25-x}{2}\right) = 0 \qquad$ <u>$A_y = \frac{(25-x)^2}{5}$ kN ↑</u>

$+\uparrow \Sigma F_y = 0 \qquad \frac{(25-x)^2}{5} - 10(25-x) + B_y = 0$

<u>$B_y = \frac{(625 - x^2)}{5}$ kN ↑</u>

13.15 (contd.)

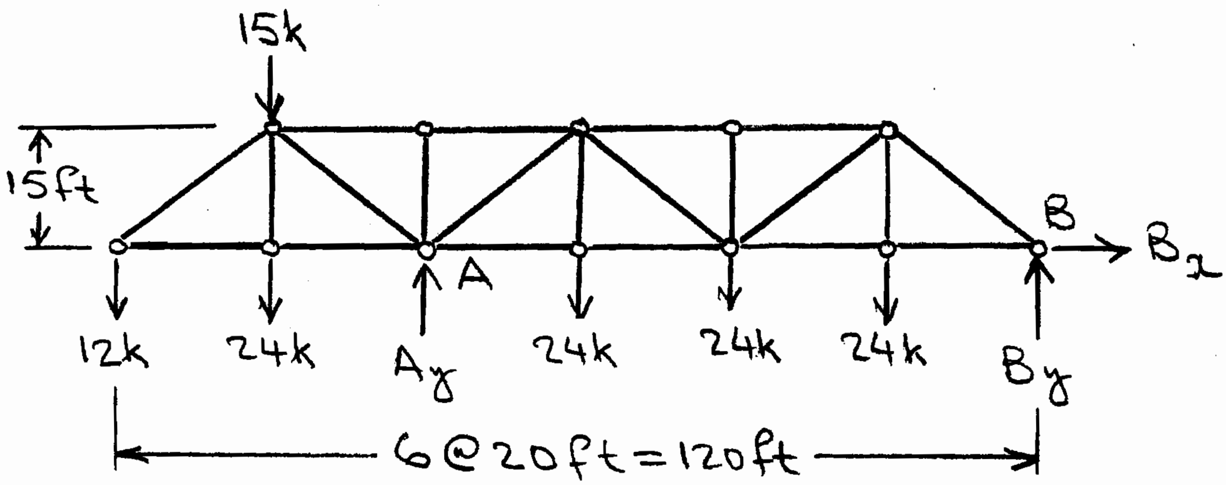


Ay(kN)
45
5
0
Ay = 45 − 2x
Ay = (25 − x)^2 / 2
20
25
x(m)
By(kN)
45
5
0
By = (625 − x^2) / 5
By = 5 + 2x
20
25
x(m)

$\boxed{3.16}$ $\underline{B_x = 0}$

$$A_y = B_y = \frac{2(50)+70}{2} = \underline{85\ \text{kN}\uparrow}$$

3.17



$\Sigma F_x = 0$ $\qquad$ $\underline{B_x = 0}$

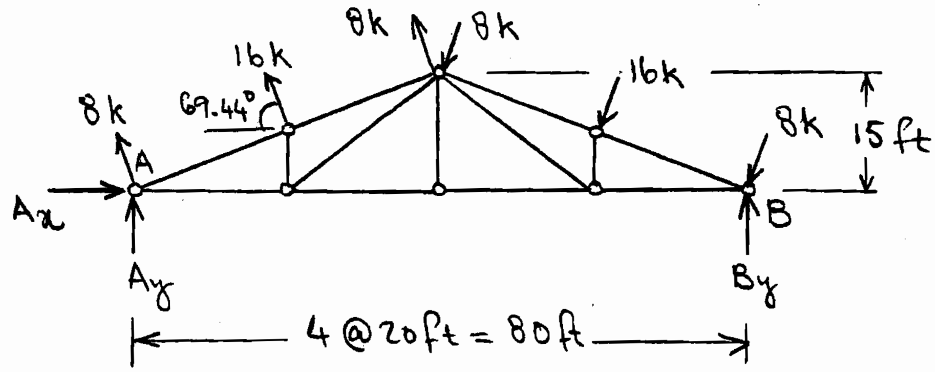
$$A_y = 12\left(\frac{6}{4}\right) + (24+15)\left(\frac{5}{4}\right) + 24\left(\frac{3}{4} + \frac{1}{2} + \frac{1}{4}\right)$$

$$\underline{A_y = 102.75\text{ k}\uparrow}$$

$$B_y = 12 + 15 + 4(24) - 102.75$$

$$\underline{B_y = 20.25\text{ k}\uparrow}$$

3.18



$\xrightarrow{+} \Sigma F_x = 0 \qquad A_x - (8+16+8+8+16+8)\cos 69.44° = 0$

$\underline{A_x = 22.48\text{ k} \rightarrow}$

$+\circlearrowleft \Sigma M_B = 0 \qquad -A_y(80) + 2[16(7.5)+8(15)]\cos 69.44°$

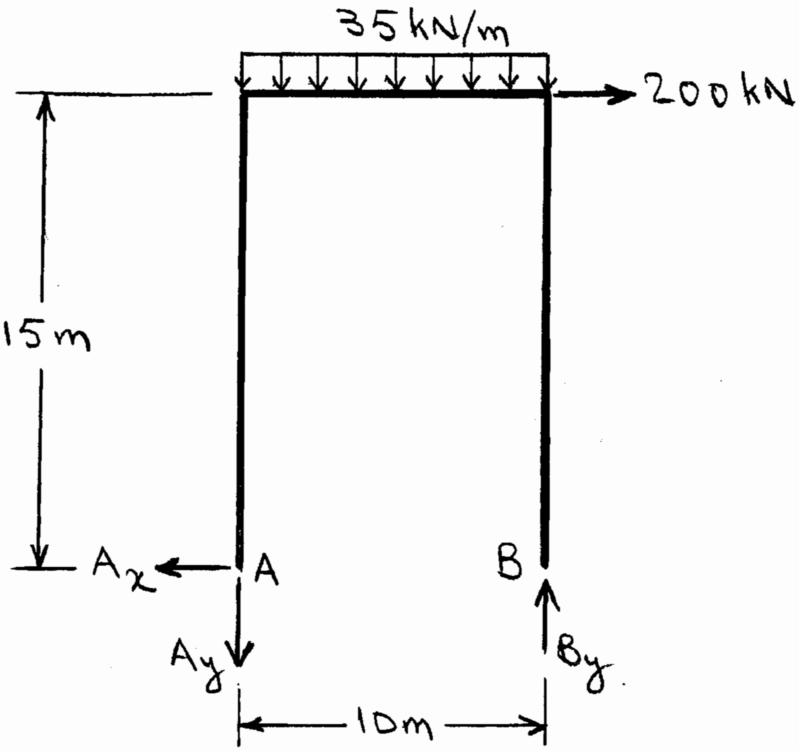
$+ [-8(80) - 16(60) + 16(20)]\sin 69.44° = 0$

$A_y = -12.87\text{ k} \qquad \underline{A_y = 12.87\text{ k} \downarrow}$

$+\uparrow \Sigma F_y = 0 \qquad -12.87 + (8+16+8-8-16-8)\sin 69.44° + B_y = 0$

$\underline{B_y = 12.87\text{ k} \uparrow}$

$\boxed{3.19}$



$\overset{+}{\rightarrow} \Sigma F_x = 0$ $\qquad\qquad$ $\underline{A_x = 200\text{ kN} \leftarrow}$

$+\circlearrowright \Sigma M_B = 0$

$$A_y(10) + 35(10)(5) - 200(15) = 0$$

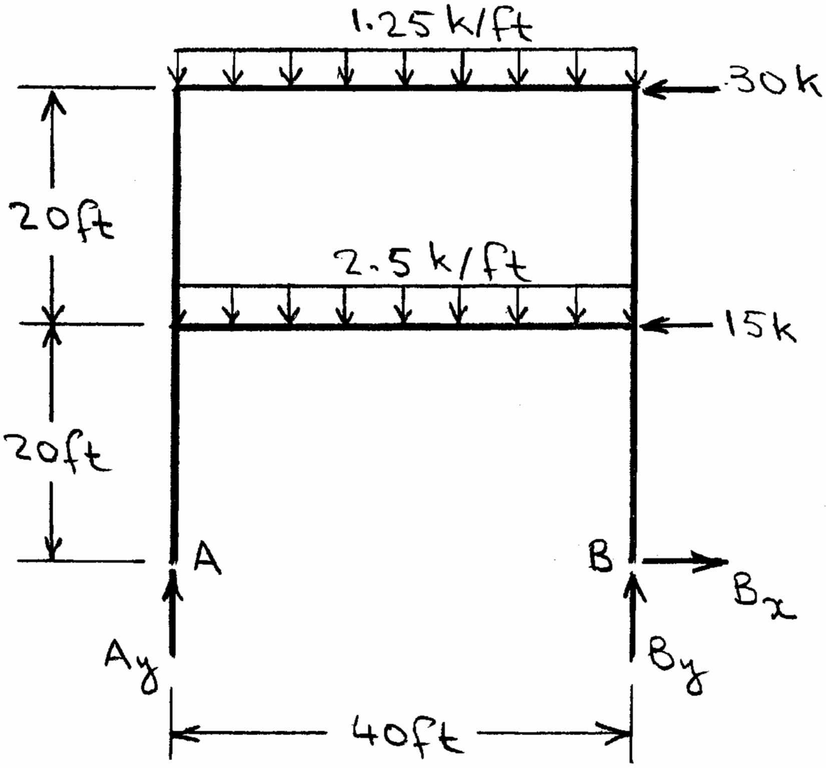
$\underline{A_y = 125\text{ kN}\downarrow}$

$+\uparrow \Sigma F_y = 0$

$$-125 - 35(10) + B_y = 0$$

$\underline{B_y = 475\text{ kN}\uparrow}$

3.20



$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad B_x - 15 - 30 = 0 \qquad \underline{B_x = 45\text{k} \rightarrow}$

$+\circlearrowleft \Sigma M_B = 0$

$$-A_y(40) + 2.5(40)(20) + 15(20) + 1.25(40)(20) + 30(40) = 0$$

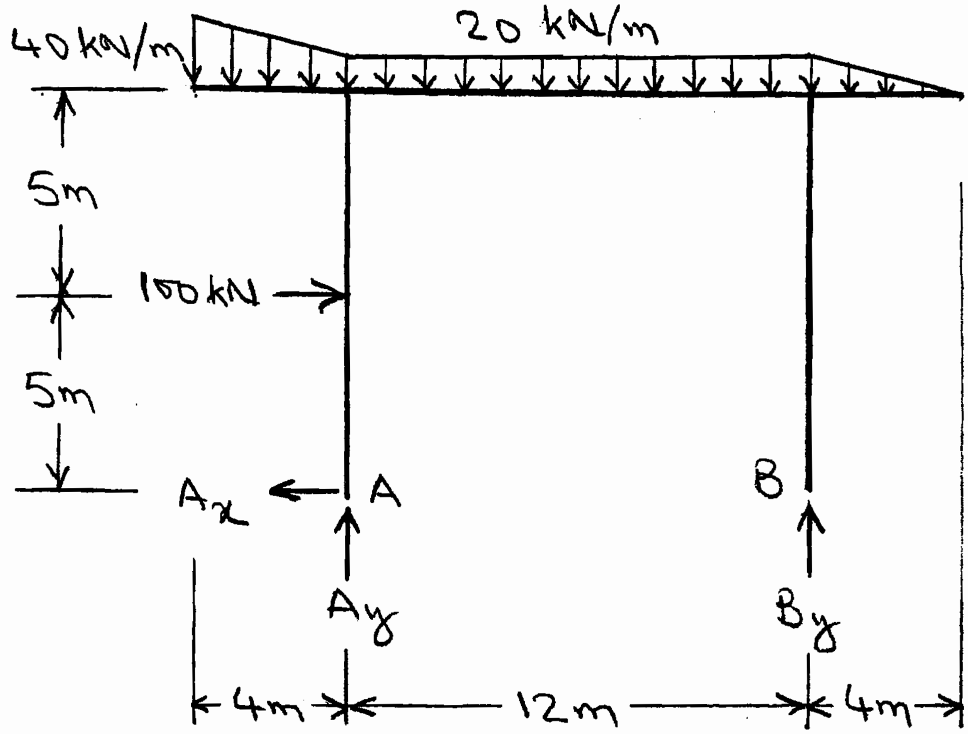
$$\underline{A_y = 112.5\text{ k} \uparrow}$$

$+\uparrow \Sigma F_y = 0$

$$112.5 - 2.5(40) - 1.25(40) + B_y = 0$$

$$\underline{B_y = 37.5\text{ k} \uparrow}$$

3.21



$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad 100 - A_x = 0$

$$\underline{A_x = 100 \text{ kN} \leftarrow}$$

$+\circlearrowleft \Sigma M_A = 0$

$$-100(5) + \tfrac{1}{2}(20)(4)\left(\tfrac{2}{3}\right)4 - 20(16)4 + B_y(12)$$
$$- \tfrac{1}{2}(20)4\left(12 + \tfrac{4}{3}\right) = 0$$

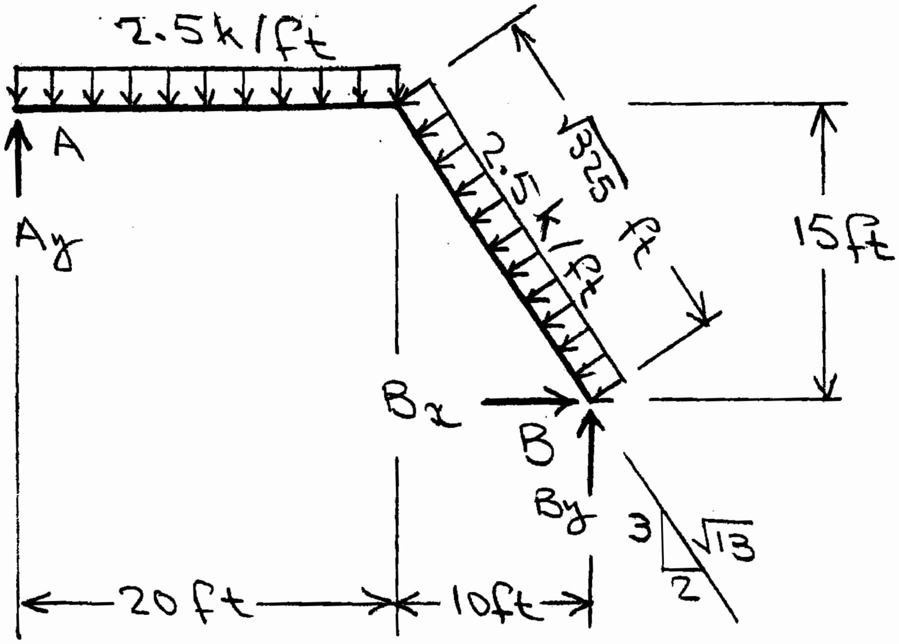
$$\underline{B_y = 183.89 \text{ kN} \uparrow}$$

$+\uparrow \Sigma F_y = 0$

$$A_y - \tfrac{1}{2}(20)4 - 20(16) - \tfrac{1}{2}(20)4 + 183.89 = 0$$

$$\underline{A_y = 216.11 \text{ kN} \uparrow}$$

3.22



$$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad B_x - 2.5(\sqrt{325})\left(\frac{3}{\sqrt{13}}\right) = 0$$

$$\underline{B_x = 37.5 \text{ k} \rightarrow}$$

$$+\circlearrowright \Sigma M_B = 0$$

$$-A_y(30) + 2.5(20)(20) + 2.5(\sqrt{325})\left(\frac{\sqrt{325}}{2}\right) = 0$$

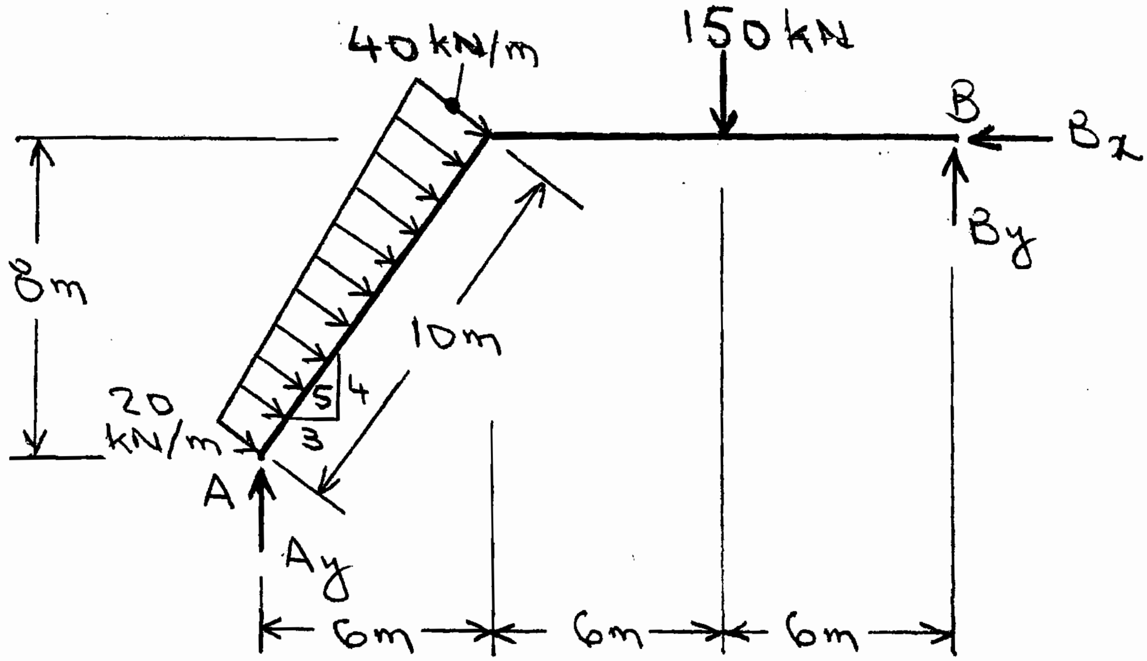
$$\underline{A_y = 46.875 \text{ k} \uparrow}$$

$$+\uparrow \Sigma F_y = 0$$

$$46.875 - 2.5(20) - 2.5(\sqrt{325})\left(\frac{2}{\sqrt{13}}\right) + B_y = 0$$

$$\underline{B_y = 28.125 \text{ k} \uparrow}$$

3.23



$$\xrightarrow{+} \Sigma F_x = 0 \qquad \left(\frac{20+40}{2}\right)10\left(\frac{4}{5}\right) - B_x = 0$$

$$\underline{B_x = 240 \text{ kN} \leftarrow}$$

$$+\circlearrowright \Sigma M_A = 0$$

$$-20(10)5 - \frac{1}{2}(20)10\left(\frac{20}{3}\right) - 150(12) + 240(8) + B_y(18) = 0$$

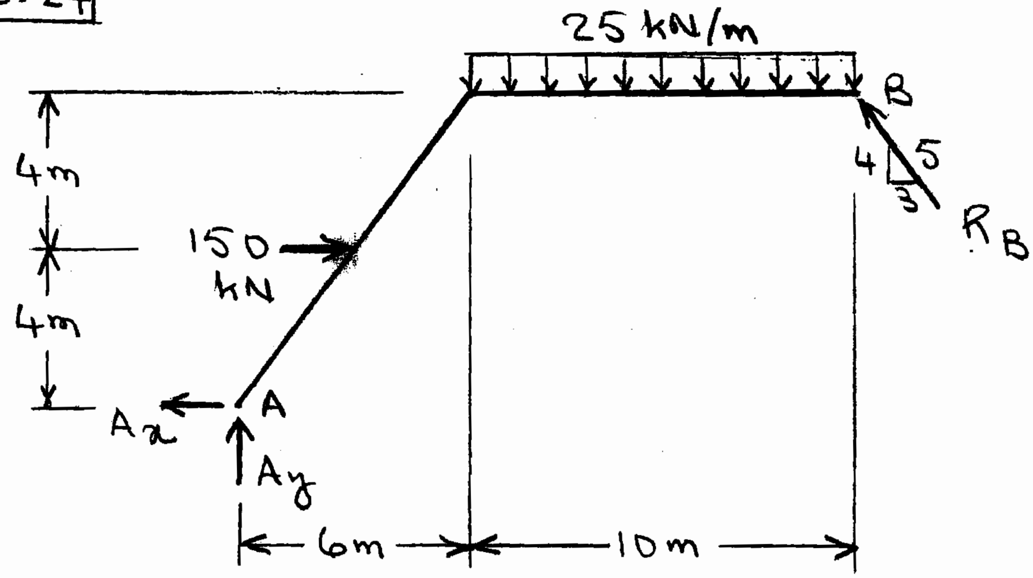
$$\underline{B_y = 85.93 \text{ kN} \uparrow}$$

$$+\uparrow \Sigma F_y = 0$$

$$A_y - \left(\frac{20+40}{2}\right)10\left(\frac{3}{5}\right) - 150 + 85.93 = 0$$

$$\underline{A_y = 244.07 \text{ kN} \uparrow}$$

3.24



$+\circlearrowright \sum M_A = 0$

$$-150(4) - 25(10)(11) + \frac{3}{5}R_B(8) + \frac{4}{5}R_B(16) = 0$$

$$\underline{R_B = 190.3 \text{ kN} \nwarrow}$$

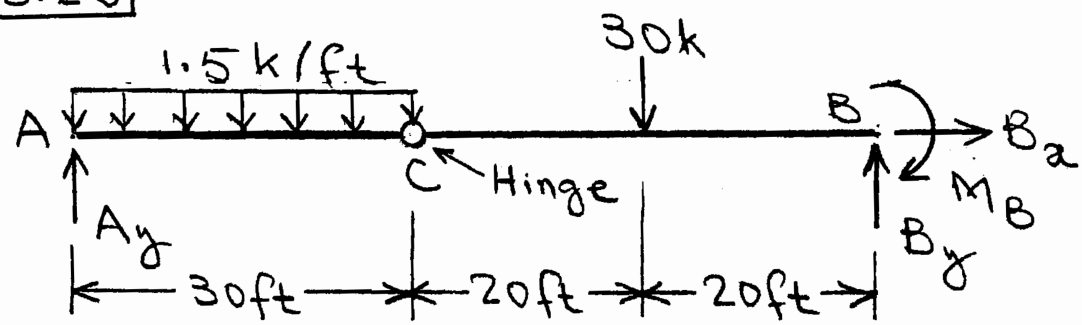
$\xrightarrow{+} \sum F_x = 0 \qquad -A_x + 150 - \frac{3}{5}(190.3) = 0$

$$\underline{A_x = 35.8 \text{ kN} \leftarrow}$$

$+\uparrow \sum F_y = 0 \qquad A_y - 25(10) + \frac{4}{5}(190.3) = 0$

$$\underline{A_y = 97.7 \text{ kN} \uparrow}$$

3.25



$\Sigma F_x = 0$ $\qquad$ $\underline{B_x = 0}$

$+\circlearrowleft \Sigma M_C^{AC} = 0$ $\qquad$ $-A_y(30) + 1.5(30)15 = 0$

$\underline{A_y = 22.5\text{ k}\uparrow}$

$+\uparrow \Sigma F_y = 0$ $\qquad$ $22.5 - 1.5(30) - 30 + B_y = 0$

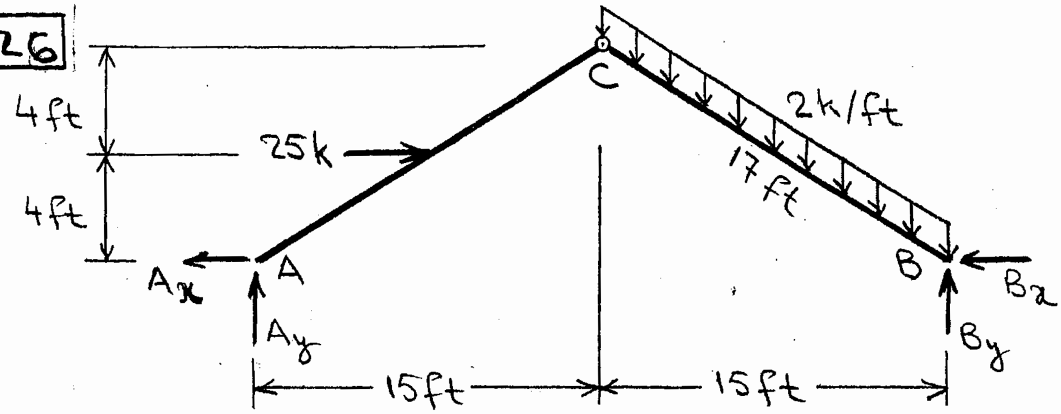
$\underline{B_y = 52.5\text{ k}\uparrow}$

$+\circlearrowleft \Sigma M_A = 0$

$-1.5(30)15 - 30(50) + 52.5(70) - M_B = 0$

$\underline{M_B = 1500\text{ k-ft} \circlearrowright}$

3.26



$+\circlearrowright \Sigma M_B = 0$

$-A_y(30) - 25(4) + 2(17)\left(\frac{15}{2}\right) = 0$ $\qquad$ $\underline{A_y = 5.17\,k\uparrow}$

$+\circlearrowright \Sigma M_C^{AC} = 0$

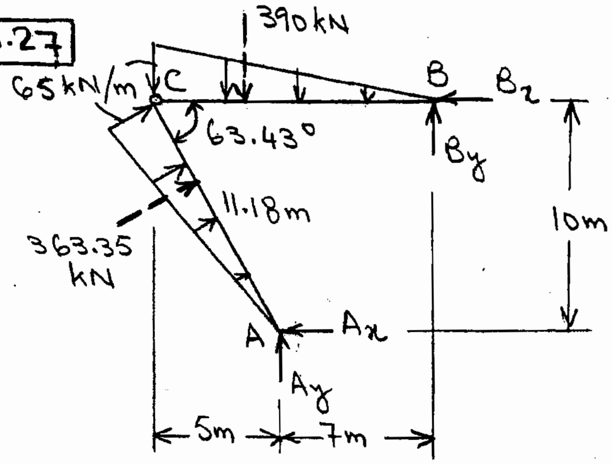
$-5.17(15) - A_x(8) + 25(4) = 0$ $\qquad$ $\underline{A_x = 2.81\,k\leftarrow}$

$\overset{+}{\rightarrow} \Sigma F_x = 0$

$-2.81 + 25 - B_x = 0$ $\qquad$ $\underline{B_x = 22.19\,k\leftarrow}$

$+\uparrow \Sigma F_y = 0 \qquad 5.17 - 2(17) + B_y = 0$ $\qquad$ $\underline{B_y = 28.83\,k\uparrow}$

3.27



$+\circlearrowright \Sigma M_C^{BC} = 0$

$-390\left(\frac{12}{3}\right) + B_y(12) = 0$

$\underline{B_y = 130\,kN\uparrow}$

$+\circlearrowright \Sigma M_A = 0$

$B_x(10) + 130(7) + 390(1)$

$-363.35\left(\frac{2}{3}\right)11.18 = 0$

$\underline{B_x = 140.82\,kN\leftarrow}$

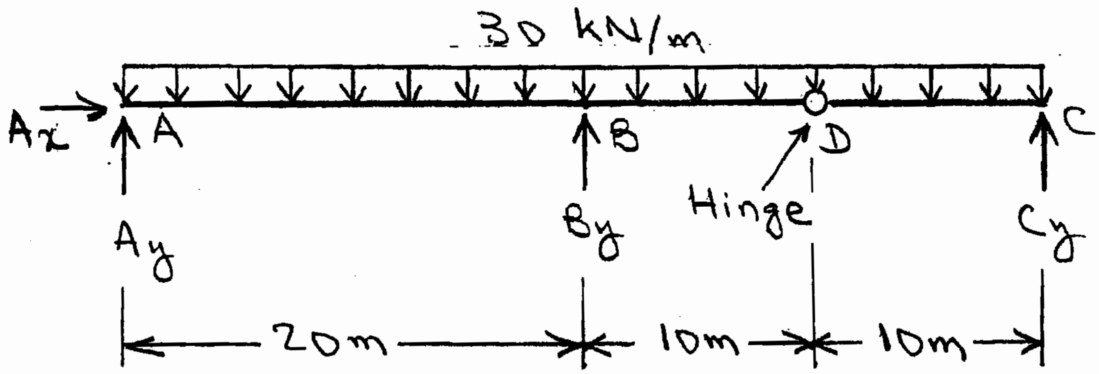
$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad 363.35 \sin 63.43^\circ - A_x - 140.82 = 0$

$\underline{A_x = 184.16\,kN\leftarrow}$

$+\uparrow \Sigma F_y = 0 \qquad A_y + 363.35 \cos 63.43^\circ - 390 + 130 = 0$

$\underline{A_y = 97.48\,kN\uparrow}$

$\boxed{3.28}$



$\Sigma F_x = 0$ $\qquad$ $\underline{A_x = 0}$

$+\circlearrowright \Sigma M_D^{CD} = 0 \qquad C_y(10) - 30(10)5 = 0$

$\underline{C_y = 150\ \text{kN} \uparrow}$

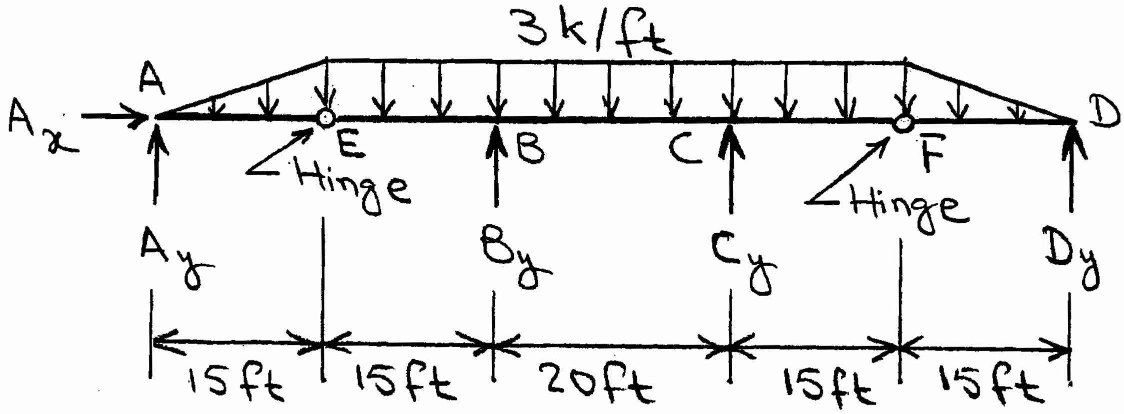
$+\circlearrowright \Sigma M_A = 0 \qquad -30(40)20 + B_y(20) + 150(40) = 0$

$\underline{B_y = 900\ \text{kN} \uparrow}$

$+\uparrow \Sigma F_y = 0 \qquad A_y - 30(40) + 900 + 150 = 0$

$\underline{A_y = 150\ \text{kN} \uparrow}$

3.29



$\Sigma F_x = 0$ $\qquad$ $\underline{A_x = 0}$

$+\circlearrowleft \Sigma M_E^{AE} = 0 \qquad -A_y(15) + \frac{1}{2}(3)15(5) = 0$

$\underline{A_y = 7.5\text{ k}\uparrow}$

$+\circlearrowleft \Sigma M_F^{DF} = 0 \qquad D_y(15) - \frac{1}{2}(3)15(5) = 0$

$\underline{D_y = 7.5\text{ k}\uparrow}$

$+\circlearrowleft \Sigma M_C = 0$

$$-7.5(50) + \frac{1}{2}(3)15(40) + 3(50)10 - B_y(20)$$
$$-\frac{1}{2}(3)15(20) + 7.5(30) = 0$$

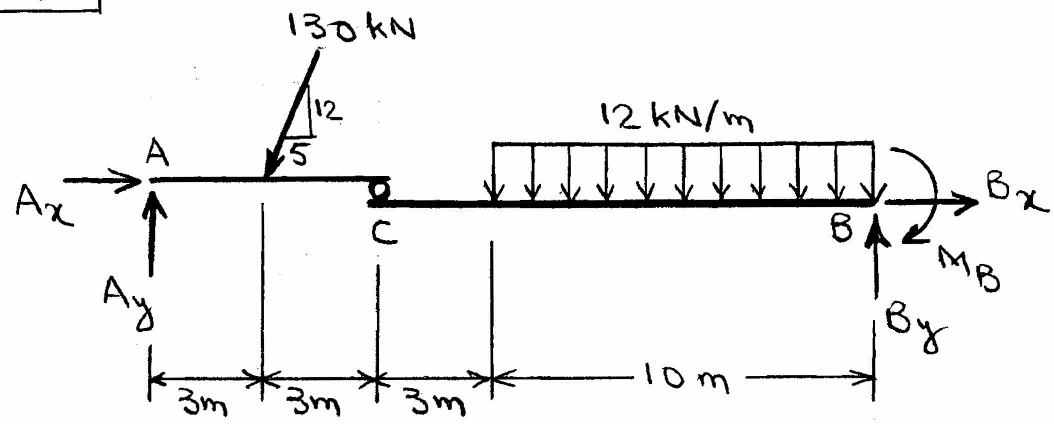
$\underline{B_y = 90\text{ k}\uparrow}$

$+\uparrow \Sigma F_y = 0$

$$7.5 - \frac{1}{2}(3)15 - 3(50) - \frac{1}{2}(3)15 + 90 + C_y + 7.5 = 0$$

$\underline{C_y = 90\text{ k}\uparrow}$

3.30



$$\xrightarrow{+} \Sigma F_x^{AC} = 0 \qquad A_x - \frac{5}{13}(130) = 0 \qquad \underline{A_x = 50 \text{ kN} \rightarrow}$$

$$+\circlearrowleft \Sigma M_C^{AC} = 0 \qquad -A_y(6) + \frac{12}{13}(130)(3) = 0 \qquad \underline{A_y = 60 \text{ kN} \uparrow}$$

$$\xrightarrow{+} \Sigma F_x = 0 \qquad 50 - \frac{5}{13}(130) + B_x = 0 \qquad \underline{B_x = 0}$$

$$+\uparrow \Sigma F_y = 0 \qquad 60 - \frac{12}{13}(130) - 12(10) + B_y = 0$$

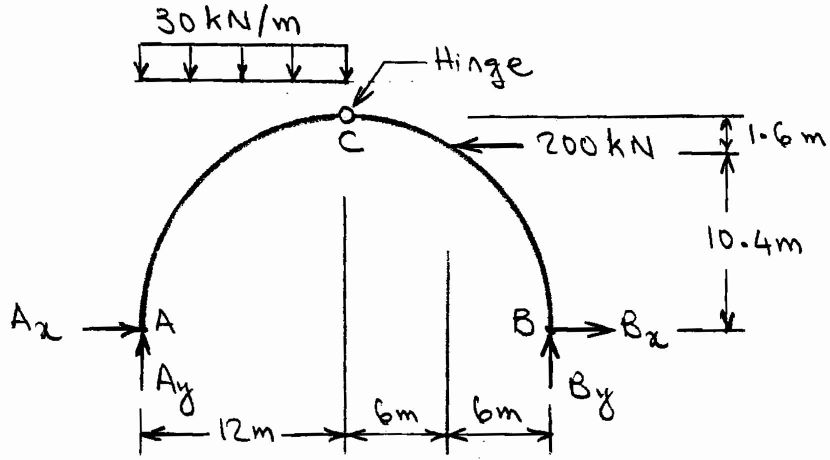
$$\underline{B_y = 180 \text{ kN} \uparrow}$$

$$+\circlearrowleft \Sigma M_A = 0$$

$$-\frac{12}{13}(130)(3) - 12(10)(9+5) + 180(19) - M_B = 0$$

$$\underline{M_B = 1380 \text{ kN}\cdot\text{m} \circlearrowright}$$

3.31



$+\circlearrowleft \Sigma M_B = 0 \qquad -A_y(24) + 30(12)(18) + 200(10.4) = 0$

$\underline{A_y = 356.67 \text{ kN} \uparrow}$

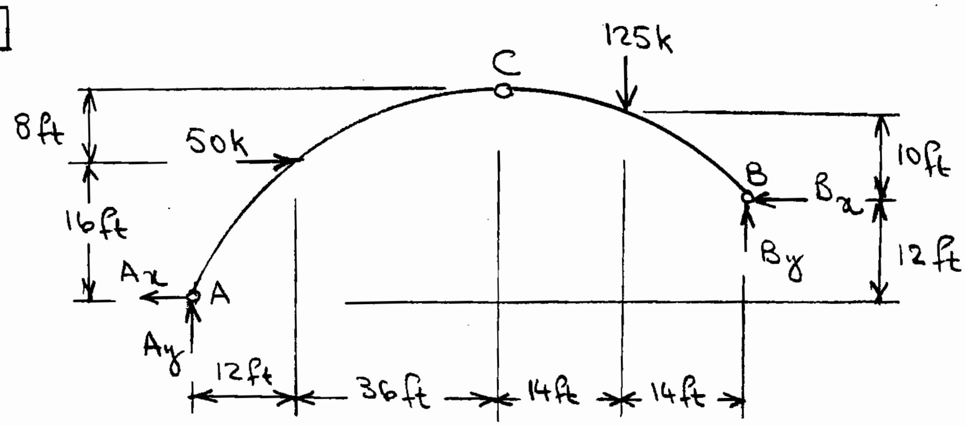
$+\circlearrowleft \Sigma M_C^{AC} = 0 \qquad A_x(12) - 356.67(12) + 30(12)(6) = 0$

$\underline{A_x = 176.67 \text{ kN} \rightarrow}$

$\Sigma F_x = 0 \qquad 176.67 - 200 + B_x = 0 \qquad \underline{B_x = 23.33 \text{ kN} \rightarrow}$

$\Sigma F_y = 0 \qquad 356.67 - 30(12) + B_y = 0 \qquad \underline{B_y = 3.33 \text{ kN} \uparrow}$

3.32



$+\circlearrowleft \Sigma M_C^{AC} = 0 \qquad -A_x(24) - A_y(48) + 50(8) = 0$

$3A_x + 6A_y = 50 \quad \text{—— (1)}$

$+\circlearrowleft \Sigma M_B = 0 \qquad -A_x(12) - A_y(76) - 50(4) + 125(14) = 0$

$6A_x + 38A_y = 775 \quad \text{—— (2)}$

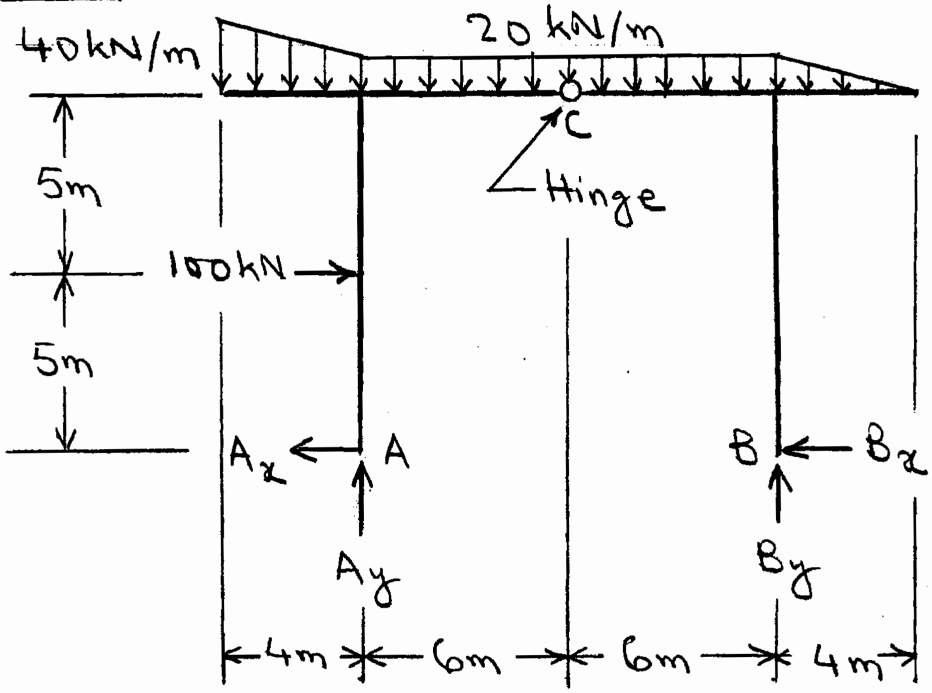
Solving (1) and (2) simultaneously, we obtain

$A_x = -35.25\text{k} = \underline{35.25\text{k} \rightarrow} \qquad \underline{A_y = 25.96 \text{ k} \uparrow}$

$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad 35.25 + 50 - B_x = 0 \qquad \underline{B_x = 85.25 \text{ k} \leftarrow}$

$+\uparrow \Sigma F_y = 0 \qquad 25.96 - 125 + B_y = 0 \qquad \underline{B_y = 99.04 \text{ k} \uparrow}$

3.33



$+\circlearrowright \Sigma M_A = 0$

$$-100(5) + \frac{1}{2}(20)(4)\left(\frac{8}{3}\right) - 20(16)4 - \frac{1}{2}(20)4\left(12+\frac{4}{3}\right) + B_y(12) = 0$$

$$\underline{B_y = 183.89 \text{ kN} \uparrow}$$

$+\circlearrowright \Sigma M_C^{BC} = 0$

$$-20(6)3 - \frac{1}{2}(20)4\left(6+\frac{4}{3}\right) + 183.89(6) - B_x(10) = 0$$

$$\underline{B_x = 45 \text{ kN} \leftarrow}$$

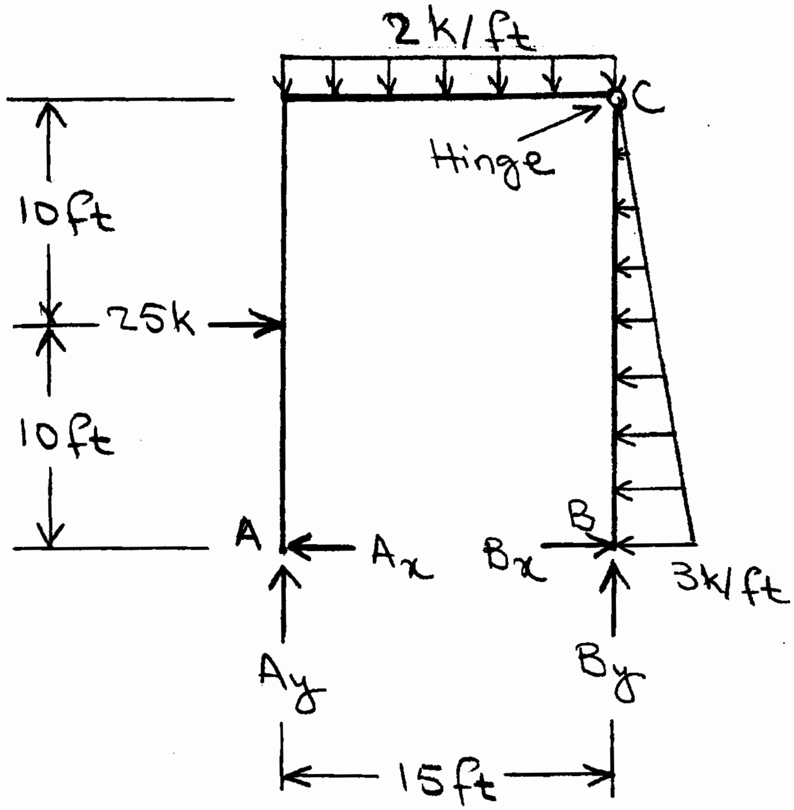
$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad 100 - A_x - 45 = 0$

$$\underline{A_x = 55 \text{ kN} \leftarrow}$$

$+\uparrow \Sigma F_y = 0 \qquad A_y - \frac{1}{2}(20)4 - 20(16) - \frac{1}{2}(20)4 + 183.89 = 0$

$$\underline{A_y = 216.11 \text{ kN} \uparrow}$$

$\boxed{3.34}$



$+\circlearrowleft \Sigma M_A = 0$

$$-25(10) - 2(15)7.5 + \tfrac{1}{2}(3)20\left(\tfrac{20}{3}\right) + B_y(15) = 0$$

$$\underline{B_y = 18.33\text{ k}\uparrow}$$

$+\Sigma M_C^{BC} = 0 \qquad -\tfrac{1}{2}(3)20\left(\tfrac{40}{3}\right) + B_x(20) = 0$

$$\underline{B_x = 20\text{ k}\rightarrow}$$

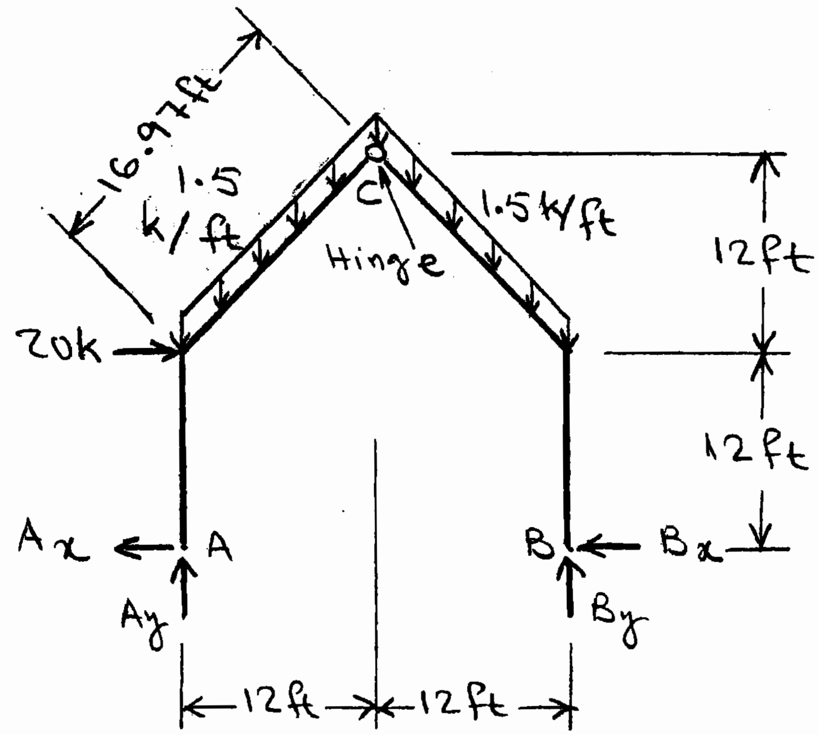
$\overset{+}{\rightarrow}\Sigma F_x = 0 \qquad -A_x + 25 - \tfrac{1}{2}(3)20 + 20 = 0$

$$\underline{A_x = 15\text{ k}\leftarrow}$$

$+\uparrow \Sigma F_y = 0 \qquad A_y - 2(15) + 18.33 = 0$

$$\underline{A_y = 11.67\text{ k}\uparrow}$$

$\boxed{3.35}$



$+\circlearrowright \Sigma M_B = 0$

$-A_y(24) - 20(12) + 1.5(16.97)(18) + 1.5(16.97)(6) = 0$

$\underline{A_y = 15.46 \text{ k} \uparrow}$

$+\circlearrowright \Sigma M_C^{AC} = 0$

$-A_x(24) - 15.46(12) + 20(12) + 1.5(16.97)(6) = 0$

$\underline{A_x = 8.63 \text{ k} \leftarrow}$

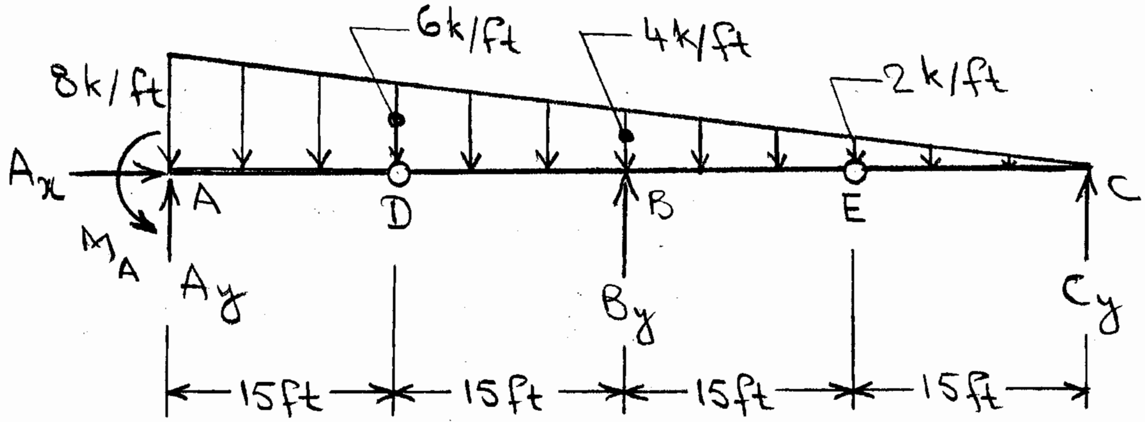
$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad -8.63 + 20 - B_x = 0$

$\underline{B_x = 11.37 \text{ k} \leftarrow}$

$+\uparrow \Sigma F_y = 0 \qquad 15.46 - 2(1.5)(16.97) + B_y = 0$

$\underline{B_y = 35.45 \text{ k} \uparrow}$

3.36



$\Sigma F_x = 0$ $\quad\quad$ $\underline{A_x = 0}$

$+\circlearrowright \Sigma M_E^{CE} = 0 \quad C_y(15) - \frac{1}{2}(2)(15)(5) = 0$

$\underline{C_y = 5\text{ k}\uparrow}$

$+\circlearrowright \Sigma M_D^{CD} = 0 \quad 5(45) - \frac{1}{2}(6)(45)(15) + B_y(15) = 0$

$\underline{B_y = 120\text{ k}\uparrow}$

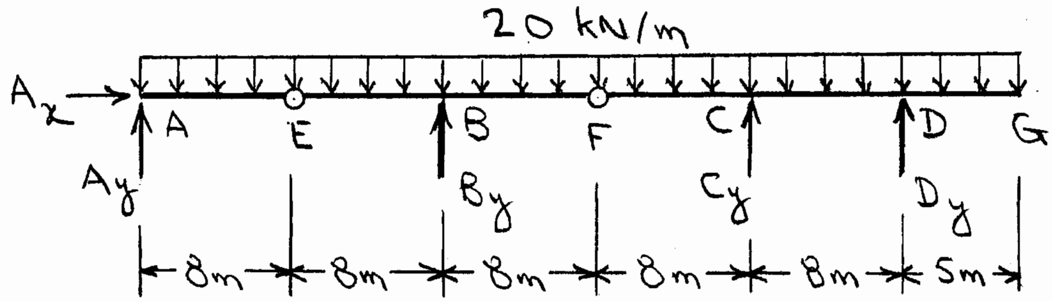
$+\circlearrowright \Sigma M_A = 0 \quad M_A - \frac{1}{2}(8)(60)(20) + 120(30) + 5(60) = 0$

$\underline{M_A = 900\text{ k-ft}\circlearrowleft}$

$+\uparrow \Sigma F_y = 0 \quad A_y - \frac{1}{2}(8)(60) + 120 + 5 = 0$

$\underline{A_y = 115\text{ k}\uparrow}$

3.37



$\Sigma F_x = 0$ $\qquad\qquad$ $\underline{A_x = 0}$

$+\circlearrowright \Sigma M_E^{AE} = 0 \qquad -A_y(8) + 20(8)(4) = 0 \qquad \underline{A_y = 80 \text{ kN}\uparrow}$

$+\circlearrowright \Sigma M_F^{AF} = 0 \qquad -80(24) + 20(24)(12) - B_y(8) = 0$

$\underline{B_y = 480 \text{ kN}\uparrow}$

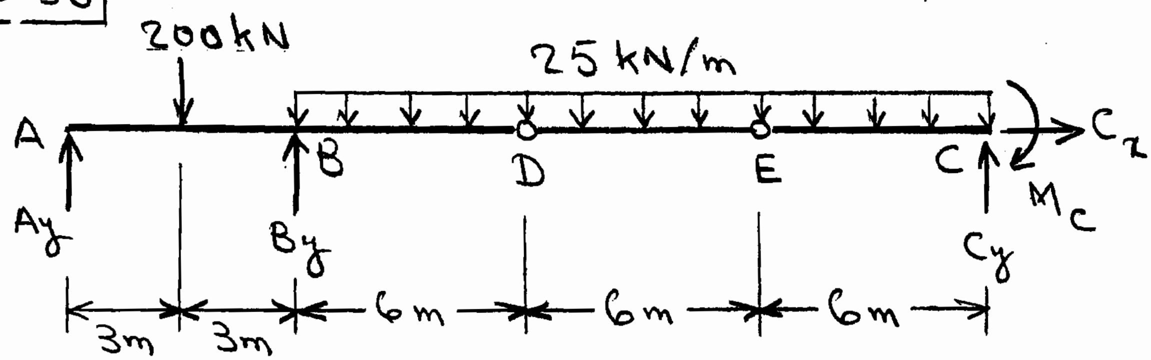
$+\circlearrowright \Sigma M_D = 0 \qquad -80(40) - 480(24) - C_y(8) + 20(45)(17.5) = 0$

$\underline{C_y = 128.75 \text{ kN}\uparrow}$

$+\uparrow \Sigma F_y = 0 \qquad 80 + 480 + 128.75 + D_y - 20(45) = 0$

$\underline{D_y = 211.25 \text{ kN}\uparrow}$

$\boxed{3.38}$



$\sum F_x = 0$ $\qquad\qquad$ $\underline{C_x = 0}$

$+\circlearrowright \sum M_D^{AD} = 0 \quad -A_y(12) + 200(9) - B_y(6) + 25(6)(3) = 0$

$$2A_y + B_y = 375 \qquad (1)$$

$+\circlearrowright \sum M_E^{AE} = 0 \quad -A_y(18) + 200(15) - B_y(12) + 25(12)(6) = 0$

$$1.5A_y + B_y = 400 \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously, we obtain

$$A_y = -50 \text{ kN} = \underline{50 \text{ kN} \downarrow}$$

$$\underline{B_y = 475 \text{ kN} \uparrow}$$

$+\uparrow \sum F_y = 0 \quad -50 - 200 + 475 - 25(18) + C_y = 0$

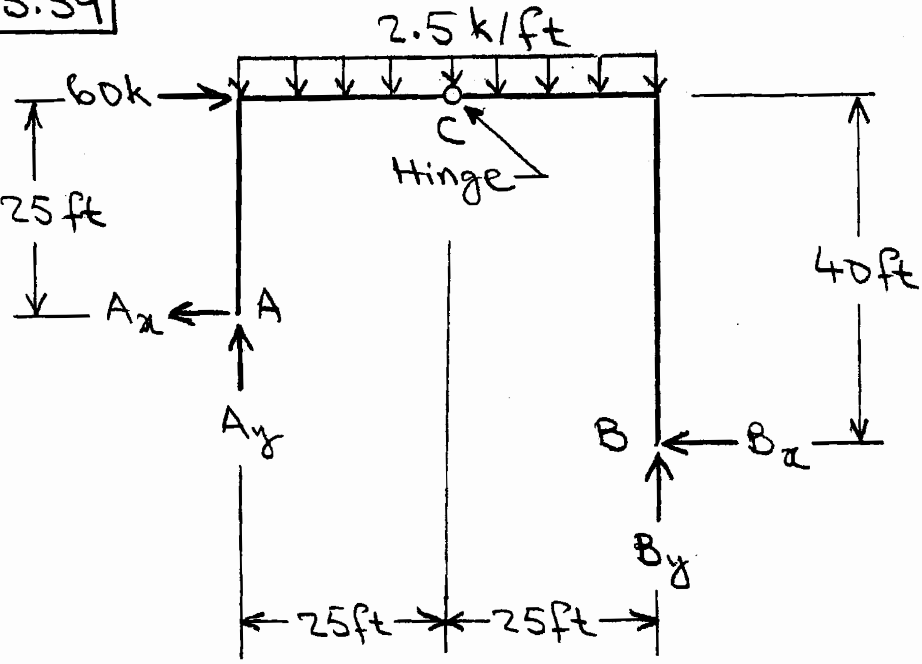
$$\underline{C_y = 225 \text{ kN} \uparrow}$$

$+\circlearrowright \sum M_C = 0$

$$50(24) + 200(21) - 475(18) + 25(18)(9) - M_C = 0$$

$$\underline{M_C = 900 \text{ kN.m} \circlearrowright}$$

$\boxed{3.39}$



$$+\circlearrowleft \sum M_A = 0$$

$$-60(25) - 2.5(50)25 - B_x(15) + B_y(50) = 0$$

$$3B_x - 10B_y = -925 \qquad (1)$$

$$+\circlearrowleft \sum M_C^{BC} = 0$$

$$-2.5(25)(12.5) - B_x(40) + B_y(25) = 0$$

$$8B_x - 5B_y = -156.25 \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously, we obtain

$$\underline{B_x = 47.12 \text{ k} \leftarrow}$$

$$\underline{B_y = 106.63 \text{ k} \uparrow}$$

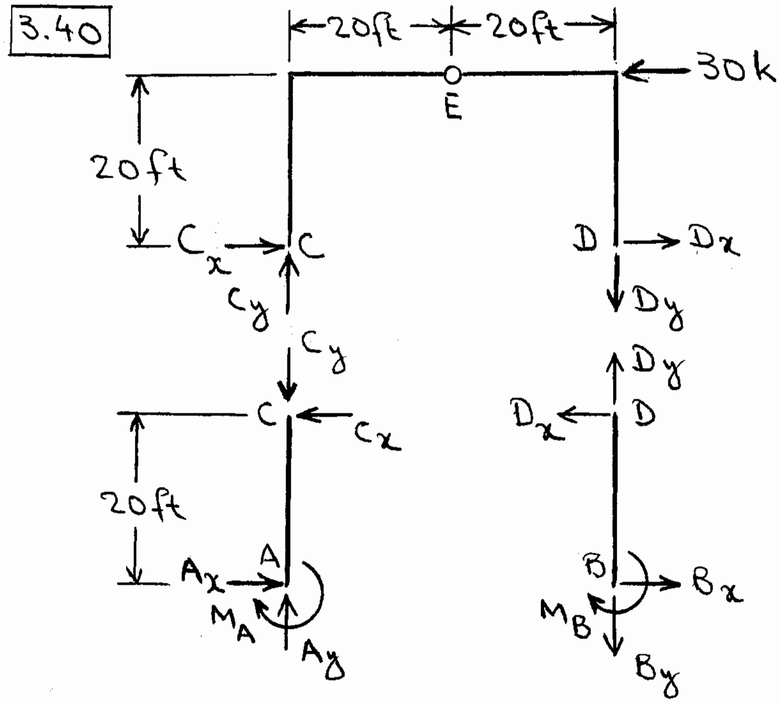
$$\overset{+}{\rightarrow} \sum F_x = 0 \qquad -A_x + 60 - 47.12 = 0$$

$$\underline{A_x = 12.88 \text{ k} \leftarrow}$$

$$+\uparrow \sum F_y = 0 \qquad A_y - 2.5(50) + 106.63 = 0$$

$$\underline{A_y = 18.37 \text{ k} \uparrow}$$

3.40



Considering the free body of portion CED, we determine the forces at the internal hinges C and D to be

$+\circlearrowleft \Sigma M_D = 0 \qquad -C_y(40) + 30(20) = 0 \qquad C_y = 15\text{ k}$

$+\circlearrowleft \Sigma M_E^{CE} = 0 \qquad C_x(20) - 15(20) = 0 \qquad C_x = 15\text{ k}$

$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad 15 - 30 + D_x = 0 \qquad D_x = 15\text{ k}$

$+\uparrow \Sigma F_y = 0 \qquad 15 - D_y = 0 \qquad D_y = 15\text{ k}$

Considering the equilibrium of portion AC, we compute the reactions at support A to be

$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad A_x - 15 = 0 \qquad \underline{A_x = 15\text{ k} \rightarrow}$

$+\uparrow \Sigma F_y = 0 \qquad A_y - 15 = 0 \qquad \underline{A_y = 15\text{ k} \uparrow}$

$+\circlearrowleft \Sigma M_A = 0 \qquad 15(20) - M_A = 0 \qquad \underline{M_A = 300\text{ k-ft} \circlearrowright}$

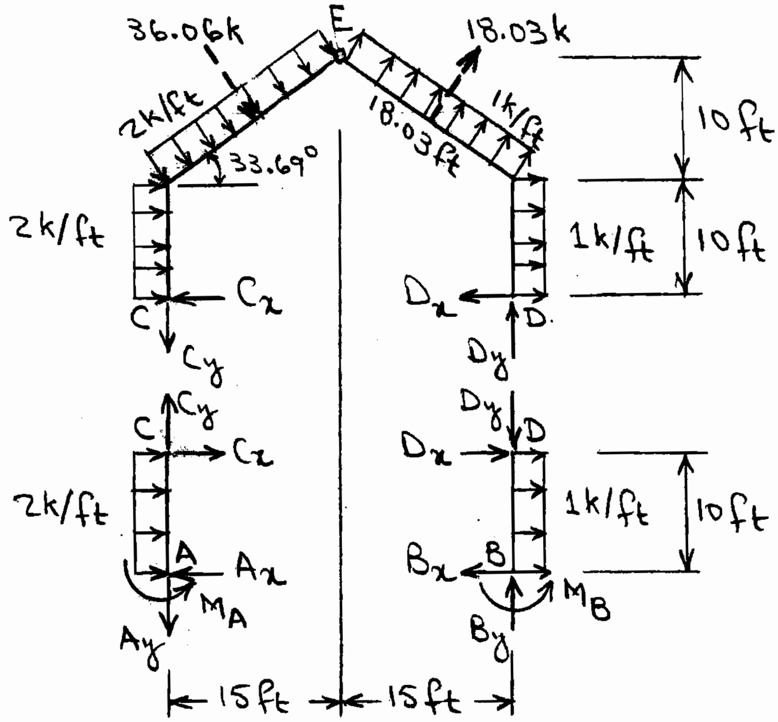
Similarly, considering the equilibrium of portion BD:

$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad B_x - 15 = 0 \qquad \underline{B_x = 15\text{ k} \rightarrow}$

$+\uparrow \Sigma F_y = 0 \qquad -B_y + 15 = 0 \qquad \underline{B_y = 15\text{ k} \downarrow}$

$+\circlearrowleft \Sigma M_B = 0 \qquad 15(20) - M_B = 0 \qquad \underline{M_B = 300\text{ k-ft} \circlearrowright}$

3.41



Considering the free body of portion CED:

$+\circlearrowleft \Sigma M_D = 0 \quad C_y(30) - (2+1)(10)5 - (36.06+18.03)\sin 33.69°(15) + 36.06\cos 33.69°(22.5) - 18.03\cos 33.69°(7.5) = 0$

$C_y = 1.25\text{k}$

$+\circlearrowleft \Sigma M_E^{CE} = 0 \quad -C_x(20) + 1.25(15) + 2(10)(15) + 36.06\left(\frac{18.03}{2}\right) = 0$

$C_x = 32.19\text{ k}$

$\overset{+}{\rightarrow}\Sigma F_x = 0 \quad -32.19 + (2+1)10 + (36.06+18.03)\sin 33.69° - D_x = 0$

$D_x = 27.81\text{ k}$

$+\uparrow \Sigma F_y = 0 \quad -1.25 - 36.06\cos 33.69° + 18.03\cos 33.69° + D_y = 0$

$D_y = 16.25\text{ k}$

Free body of portion AC:

$\overset{+}{\rightarrow}\Sigma F_x = 0 \quad 32.19 + 2(10) - A_x = 0 \qquad \underline{A_x = 52.19\text{k} \leftarrow}$

$+\uparrow \Sigma F_y = 0 \quad -A_y + 1.25 = 0 \qquad \underline{A_y = 1.25\text{k}\downarrow}$

$+\circlearrowleft \Sigma M_A = 0 \quad M_A - 32.19(10) - 2(10)5 = 0 \qquad \underline{M_A = 421.9\text{ k-ft} \circlearrowleft}$

Free body of portion BD:

$\overset{+}{\rightarrow}\Sigma F_x = 0 \quad 27.81 + 1(10) - B_x = 0 \qquad \underline{B_x = 37.81\text{k} \leftarrow}$

$+\uparrow \Sigma F_y = 0 \quad B_y - 16.25 = 0 \qquad \underline{B_y = 16.25\text{k}\uparrow}$

$+\circlearrowleft \Sigma M_B = 0 \quad M_B - 27.81(10) - 1(10)5 = 0 \qquad \underline{M_B = 328.1\text{ k-ft} \circlearrowleft}$

Chapter Four
Plane and Space Trusses

CHAPTER 4

4.1 (a) $m = 2$, $r = 3$, $j = 3$; $m + r < 2j$; Unstable

(b) $m = 3$, $r = 3$, $j = 3$; $m + r = 2j$

Statically determinate

(c) $m = 2$, $r = 4$, $j = 3$; $m + r = 2j$

Statically determinate

(d) $m = 3$, $r = 4$, $j = 4$; $m + r < 2j$; Unstable

4.2 (a) $m = 6$, $r = 3$, $j = 4$; $m + r > 2j$

Statically indeterminate; $i = (6 + 3) - 2(4) = 1$

(b) $m = 9$, $r = 3$, $j = 6$; $m + r = 2j$

Statically determinate

(c) $m = 9$, $r = 3$, $j = 6$; $m + r = 2j$

Unstable, because two rigid portions are connected by an internal hinge and three reactions are not sufficient to prevent relative rotation of one rigid part with respect to the other.

(d) $m = 7$, $r = 4$, $j = 5$; $m + r > 2j$

Statically indeterminate; $i = (7 + 4) - 2(5) = 1$

$\boxed{4.3}$ (a) $m = 17,\ r = 3,\ j = 9;\ \ m + r > 2j$

Statically indeterminate; $i = (17+3)+2(9) = \underline{2}$

(b) $m = 12,\ r = 3,\ j = 7;\ \ m + r > 2j$

Statically indeterminate; $i = (12+3) - 2(7) = \underline{1}$

(c) $m = 25,\ r = 4,\ j = 14;\ \ m + r > 2j$

Statically indeterminate; $i = (25+4) - 2(14) = \underline{1}$

(d) $m = 24,\ r = 6,\ j = 15;\ \ m + r = 2j$

Statically determinate

$\boxed{4.4}$ (a) $m = 22,\ r = 4,\ j = 13;\ m + r = 2j$

Statically determinate

(b) $m = 14,\ r = 3,\ j = 9;\ m + r < 2j$; Unstable

(c) $m = 11,\ r = 3,\ j = 7;\ \ m + r = 2j$

Statically determinate

(d) $m = 23,\ r = 4,\ j = 13;\ m + r > 2j$

Statically indeterminate; $i = (23+4) - 2(13) = \underline{1}$

$\boxed{4.5}$ (a) $m = 14,\ r = 4,\ j = 9;\ m + r = 2j$

Unstable, because two rigid portions are connected by a hinge.

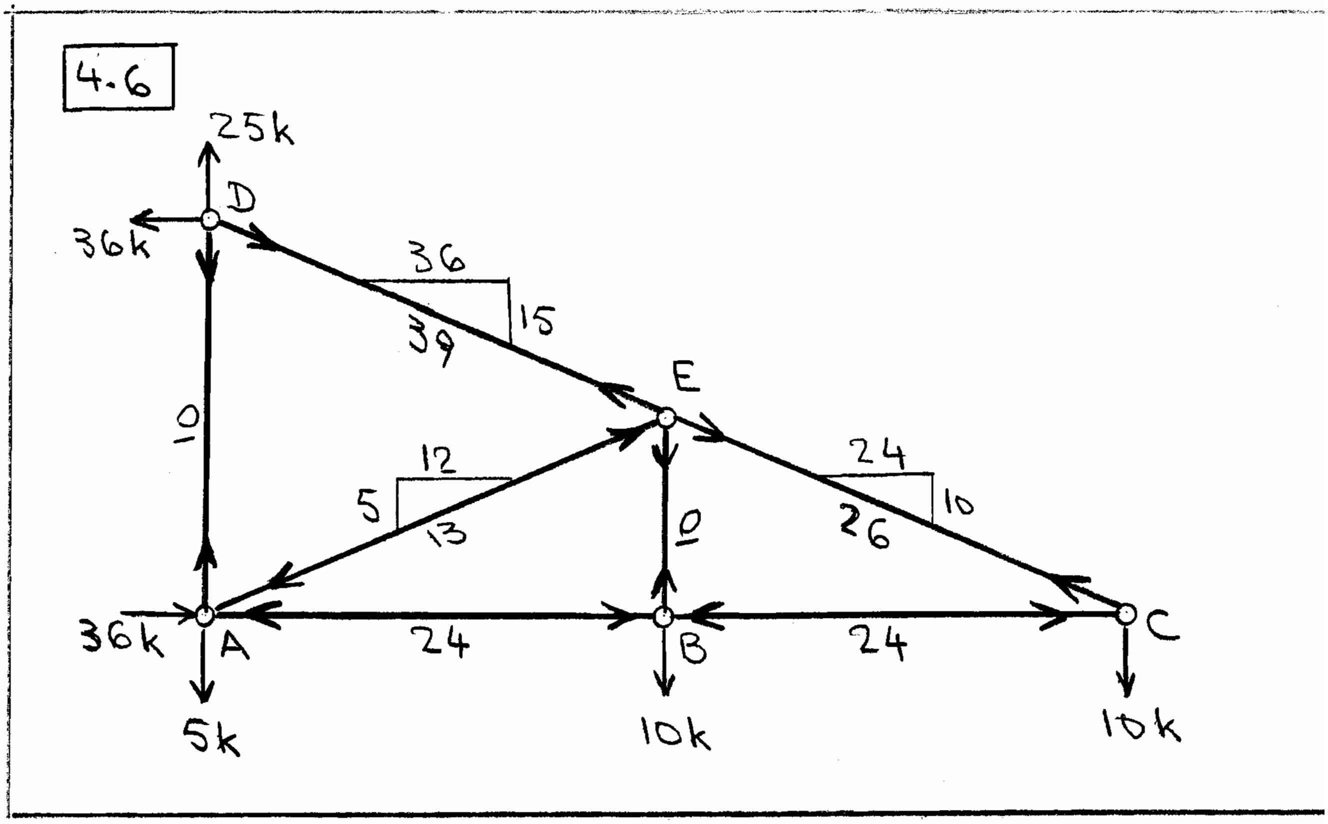
(b) $m = 24,\ r = 3,\ j = 14;\ m + r < 2j$; Unstable

(c) $m = 9,\ r = 3,\ j = 6;\ m + r = 2j$

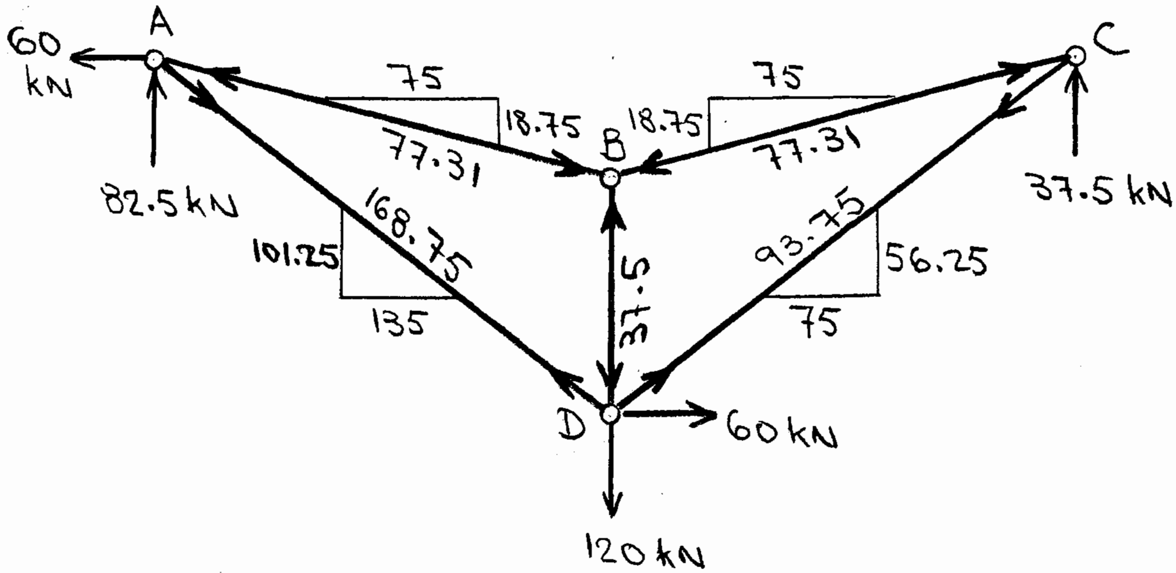
Statically determinate

(d) $m = 25,\ r = 3,\ j = 14;\ m + r = 2j$

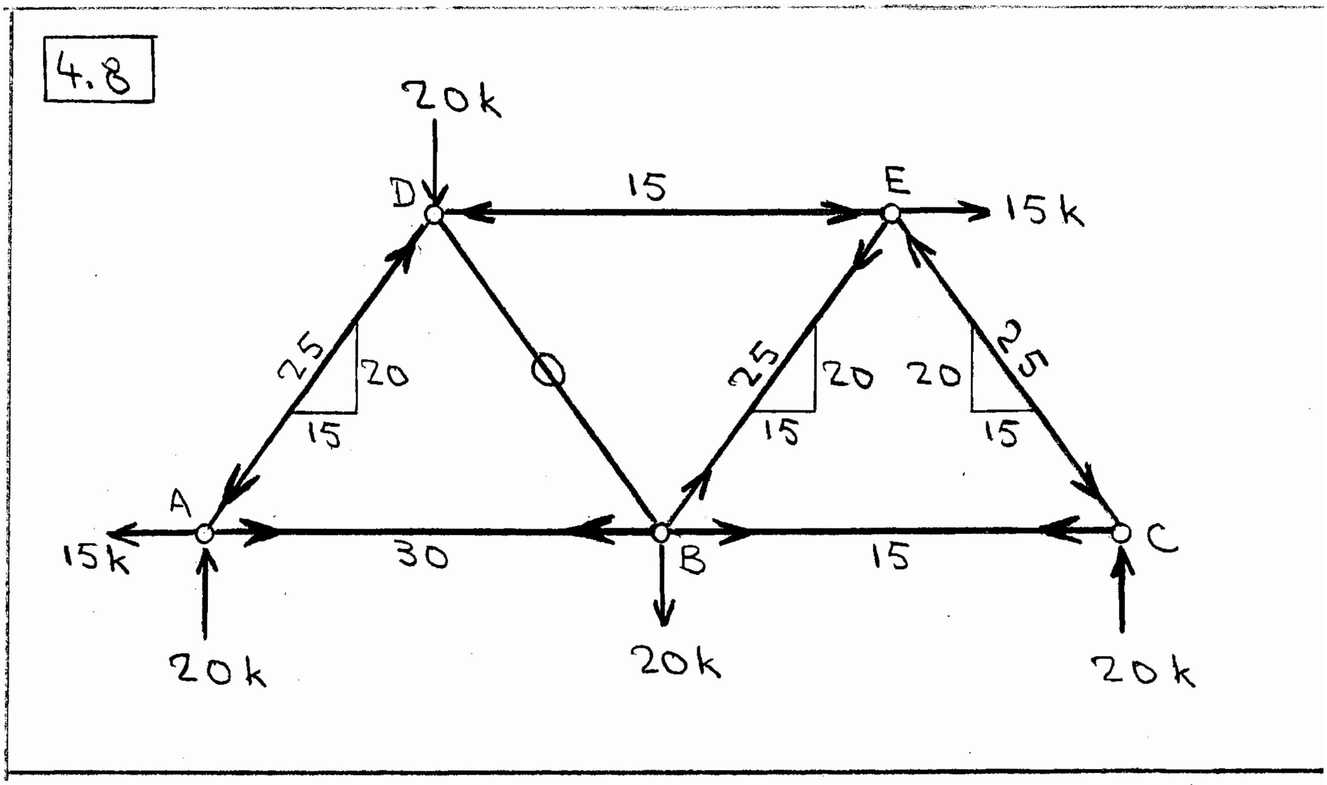
Unstable, because two rigid portions are connected by three parallel members


4.6
25k
D
36k
36
15
39
E
0
12
5
13
0
24
10
26
36k
A
24
B
24
C
5k
10k
10k

4.7

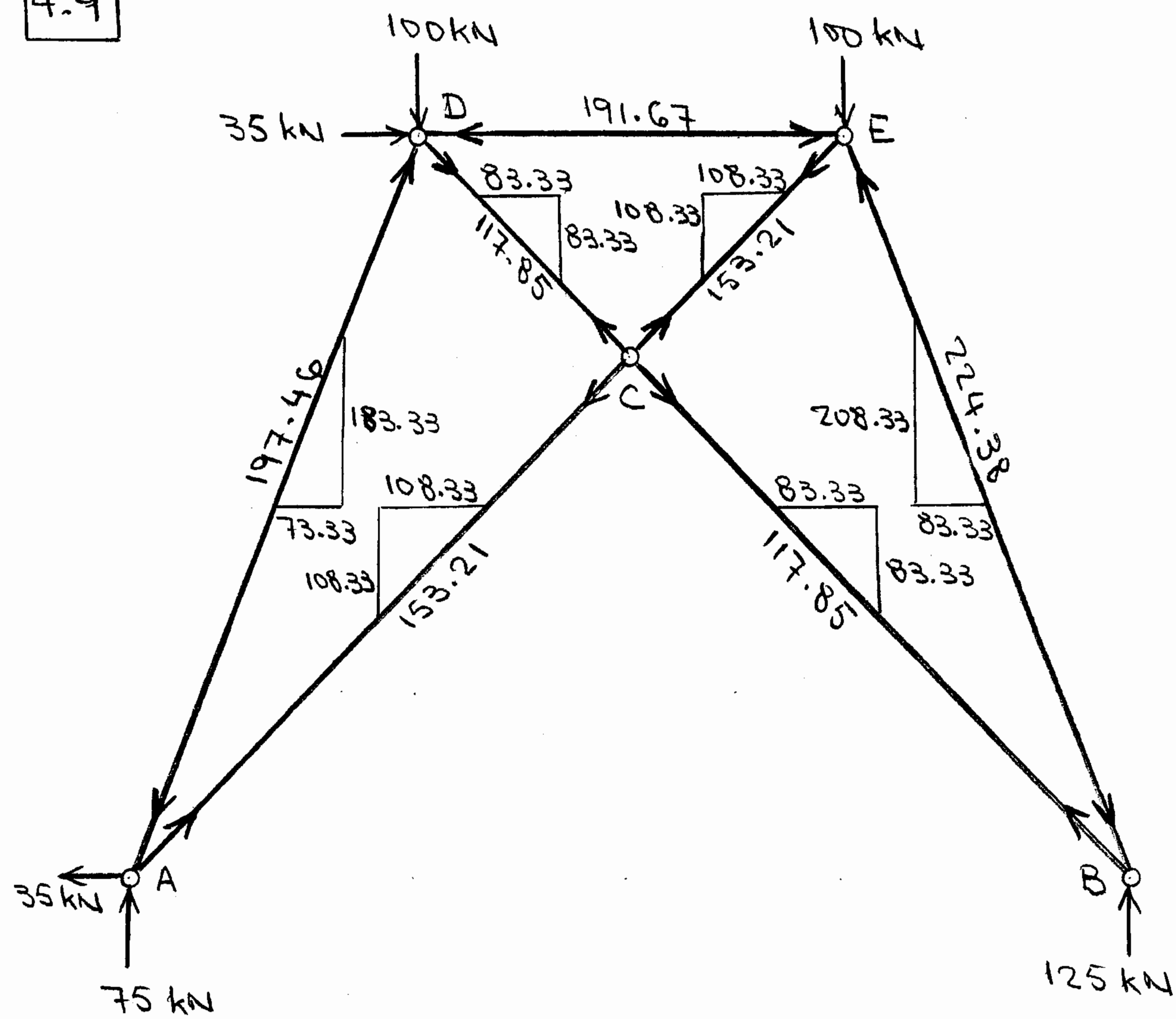


A
B
C
D
60 kN
82.5 kN
75
18.75
77.31
75
18.75
77.31
37.5 kN
168.75
101.25
135
37.5
93.75
56.25
75
60 kN
120 kN

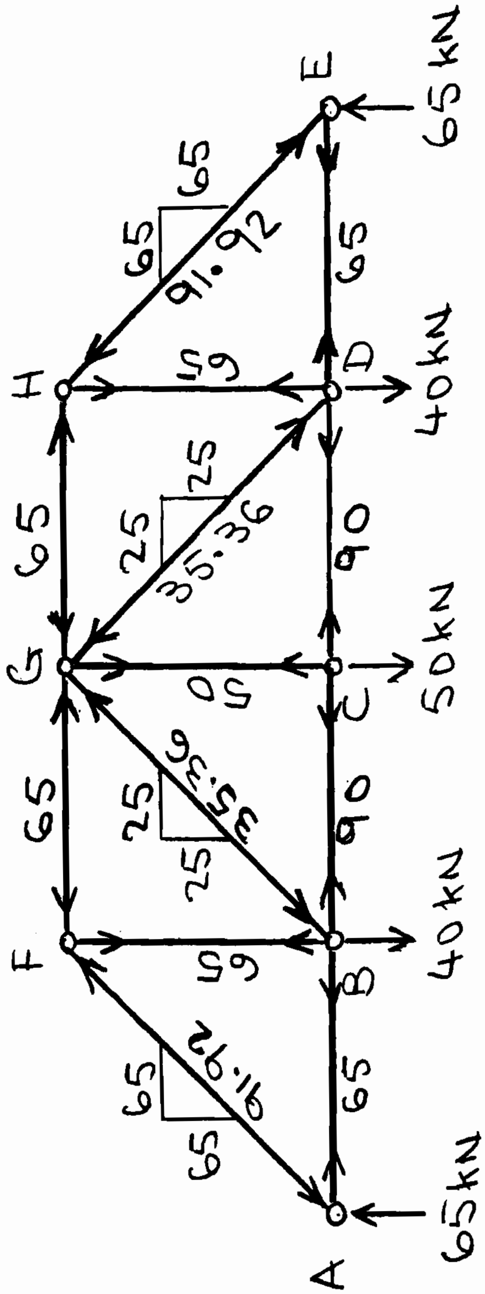


4.8
20k
D
15
E
15k
25
20
15
0
25
20
20
25
15
15
A
15k
30
B
15
C
20k
20k
20k

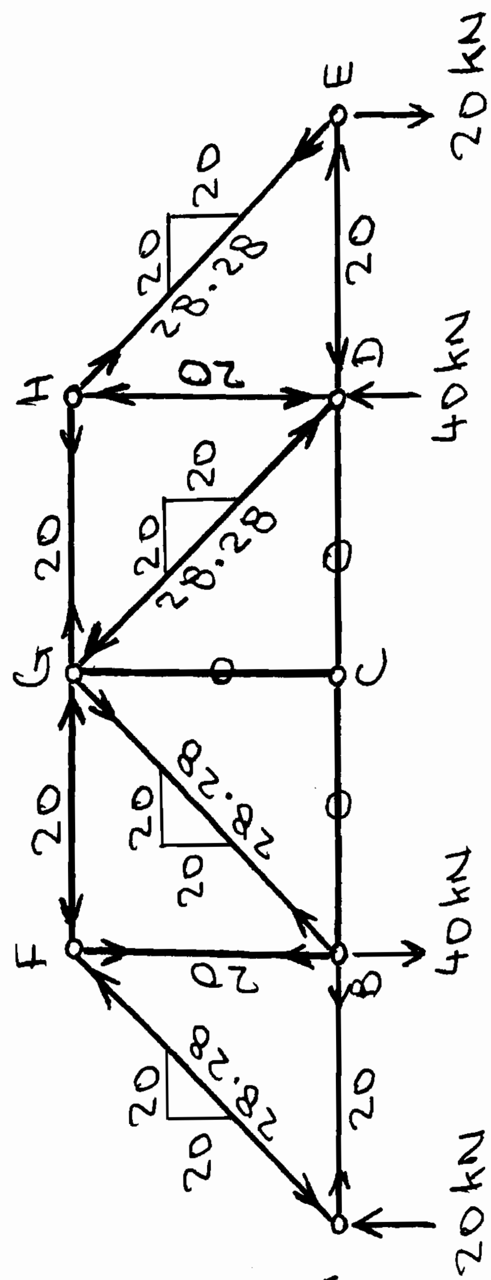
4.9


100kN
100 kN
35 kN
D
191.67
E
83.33
108.33
108.33
83.33
117.85
153.21
197.46
183.33
C
208.33
224.38
108.33
83.33
73.33
83.33
108.33
153.21
117.85
83.33
35 kN
A
B
75 kN
125 kN

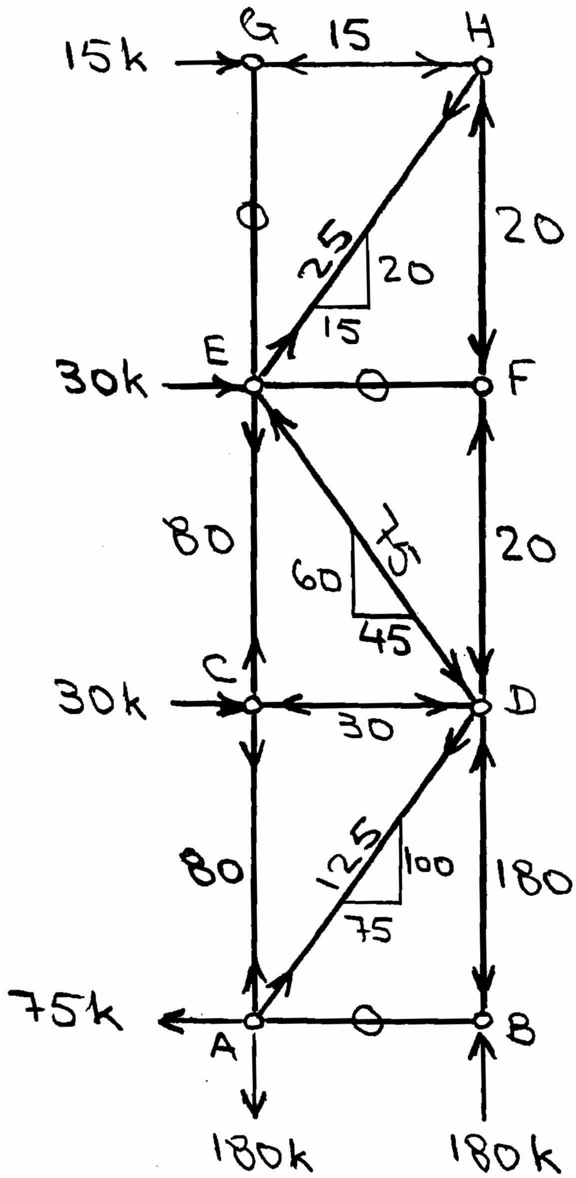
4.10



4.11

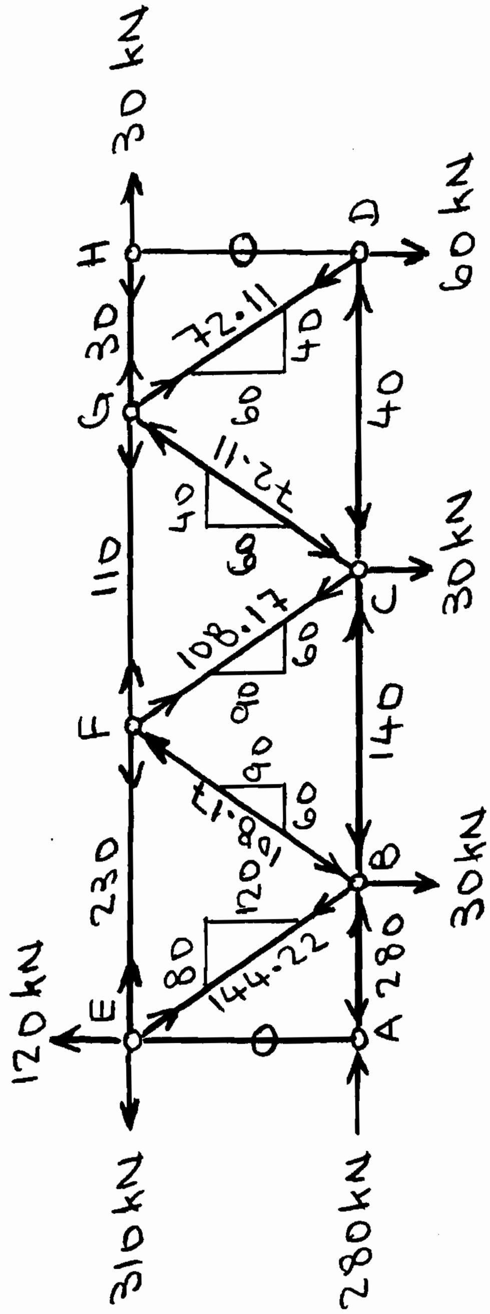


4.12

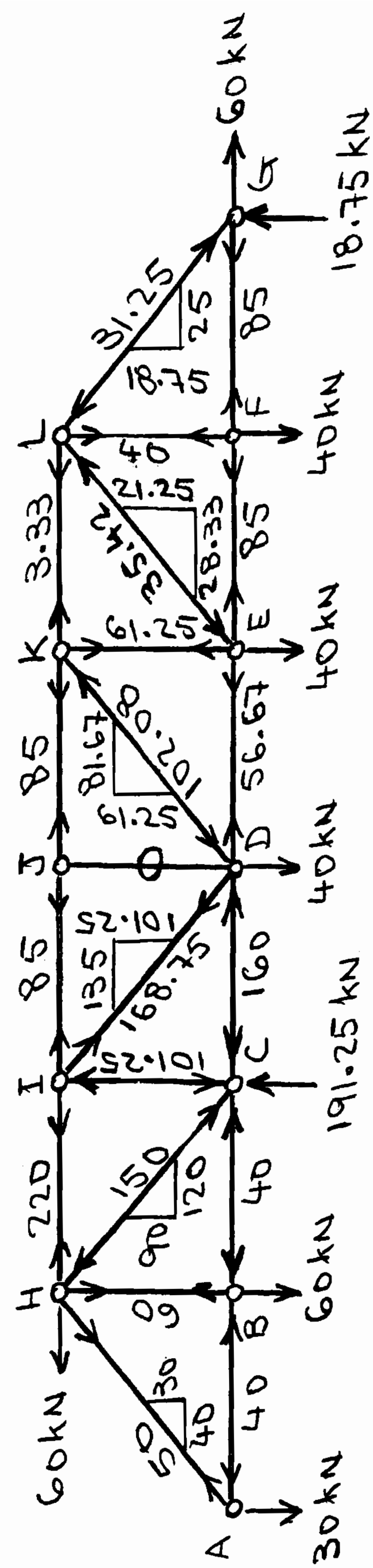


G
15
H
15k
0
20
25
20
15
E
30k
0
F
80
75
60
45
20
C
30k
30
D
80
125
100
75
180
75k
A
0
B
180k
180k

4.13

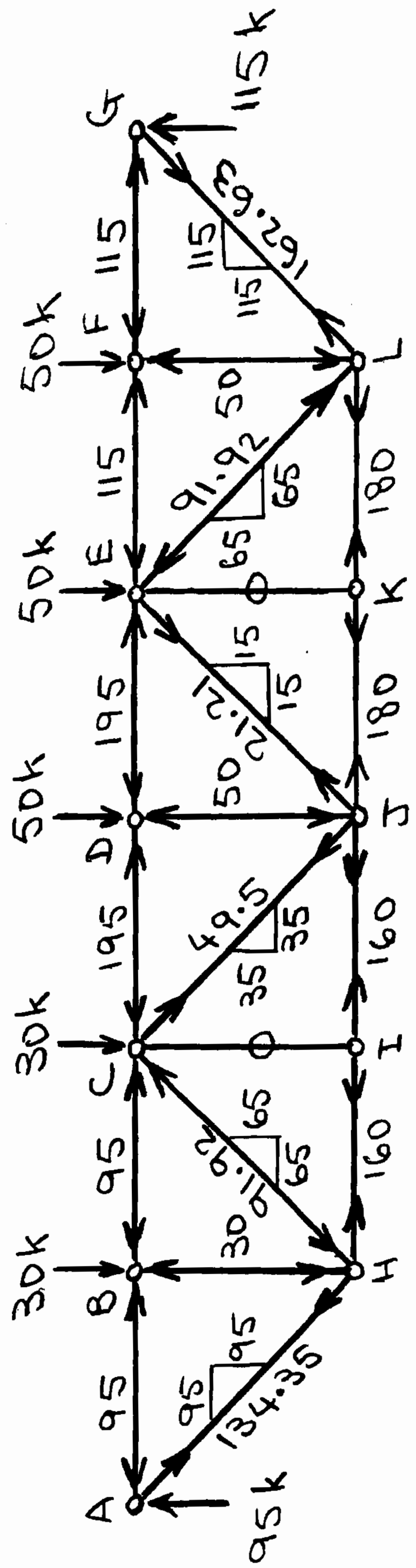

30 kN
H
D
60 kN
30
72.11
40
60
G
40
72.11
40
60
110
30 kN
C
108.17
60
90
F
140
90
60
108.17
230
30 kN
B
120
80
144.22
280
120 kN
E
A
0
0
310 kN
280 kN

4.14

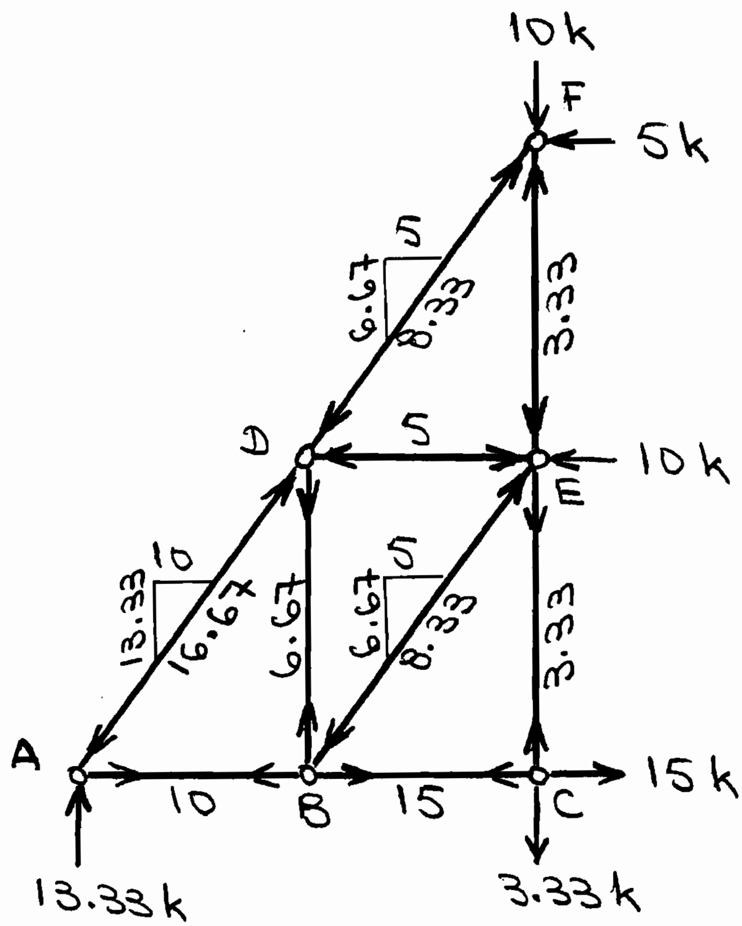


A
B
C
D
E
F
G
H
I
J
K
L
30 kN
60 kN
60 kN
191.25 kN
40 kN
40 kN
40 kN
18.75 kN
60 kN
50
30
40
40
60
150
90
120
40
220
101.25
135
168.75
101.25
160
85
0
85
81.67
61.25
102.08
56.67
61.25
3.33
35.42
21.25
28.33
85
40
31.25
18.75
25
85

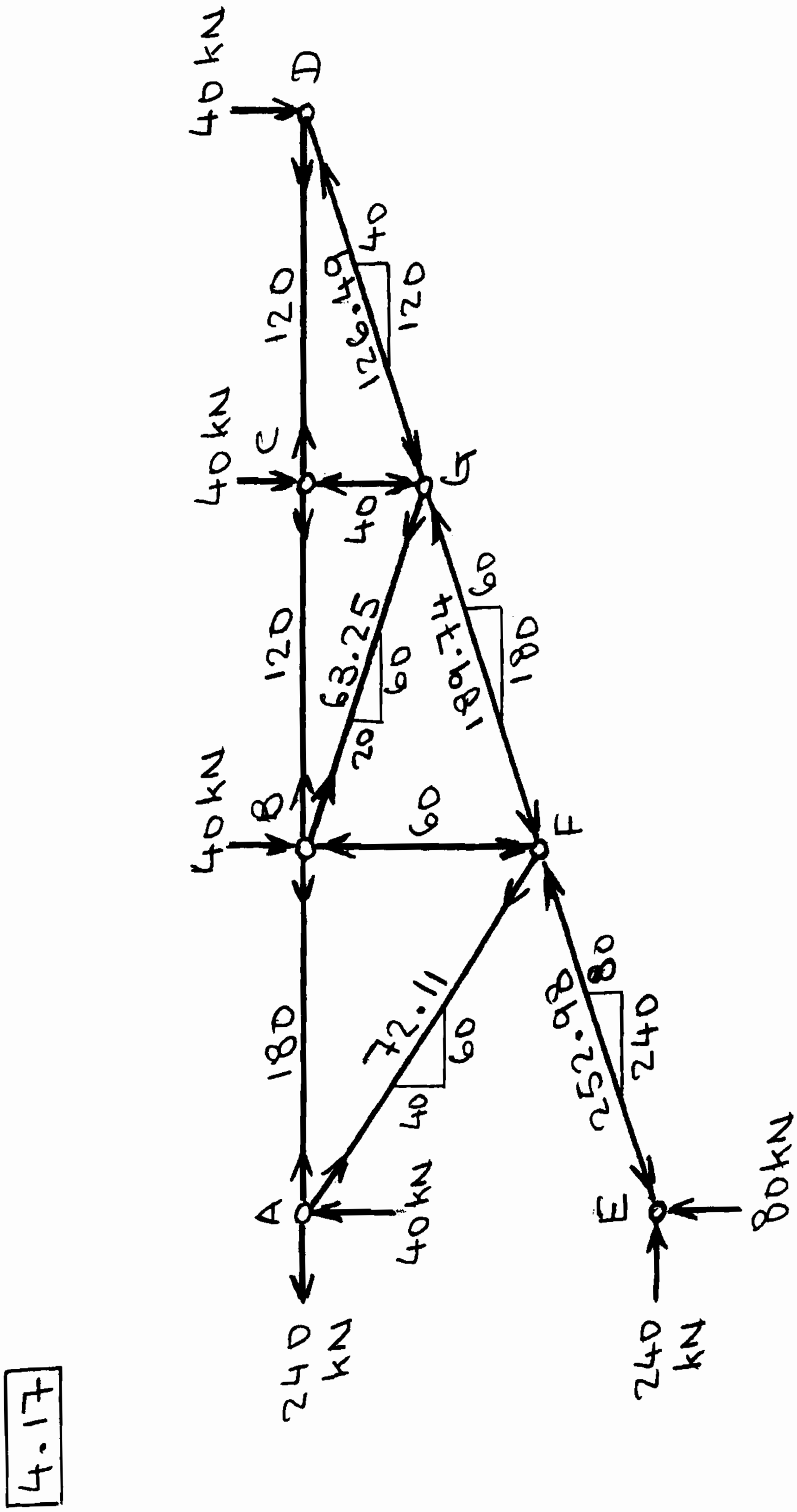
4.15


A
B
C
D
E
F
G
H
I
J
K
L
95 k
30k
30k
50k
50k
50k
115 k
95
95
195
195
115
115
134.35
30
91.92
0
49.5
50
21.21
0
91.92
50
162.63
160
160
180
180

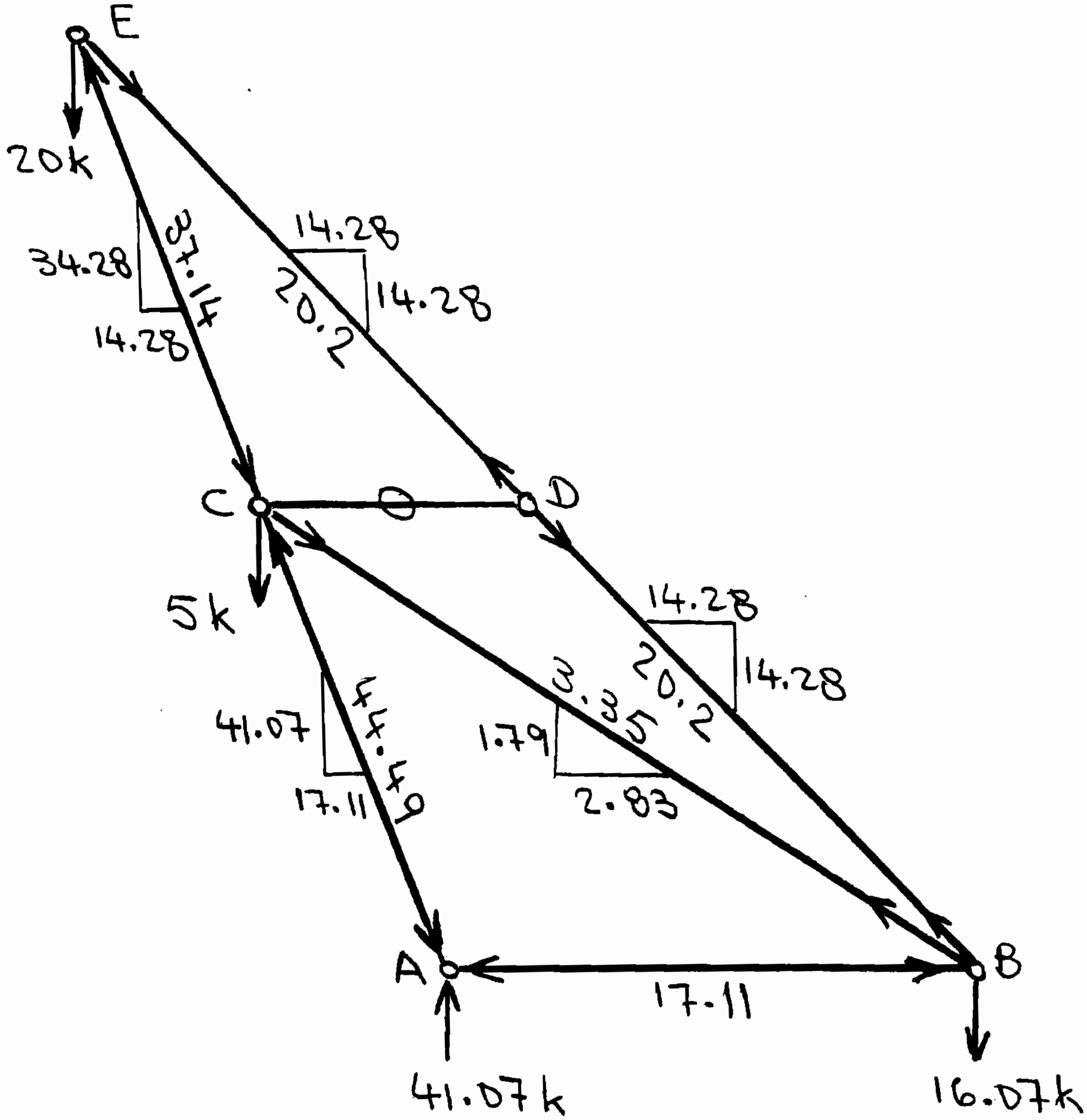
4.16



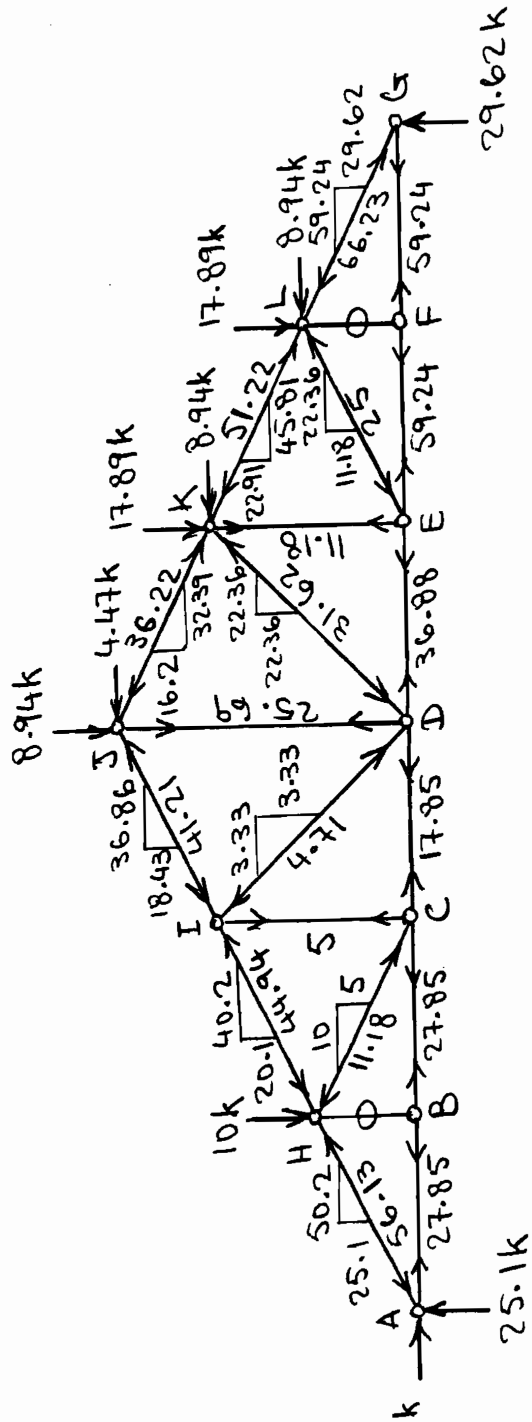
10k
F
5k
5
6.67
8.33
3.33
D
5
10k
E
10
13.33
16.67
6.67
5
6.67
8.33
3.33
A
10
B
15
C
15k
13.33k
3.33k


4.17
40 kN
D
40 kN
C
40 kN
B
A
40 kN
240 kN
E
80 kN
240 kN
F
G
120
126.49
40
120
40
120
63.25
60
20
189.74
60
180
60
180
72.11
60
40
252.98
80
240

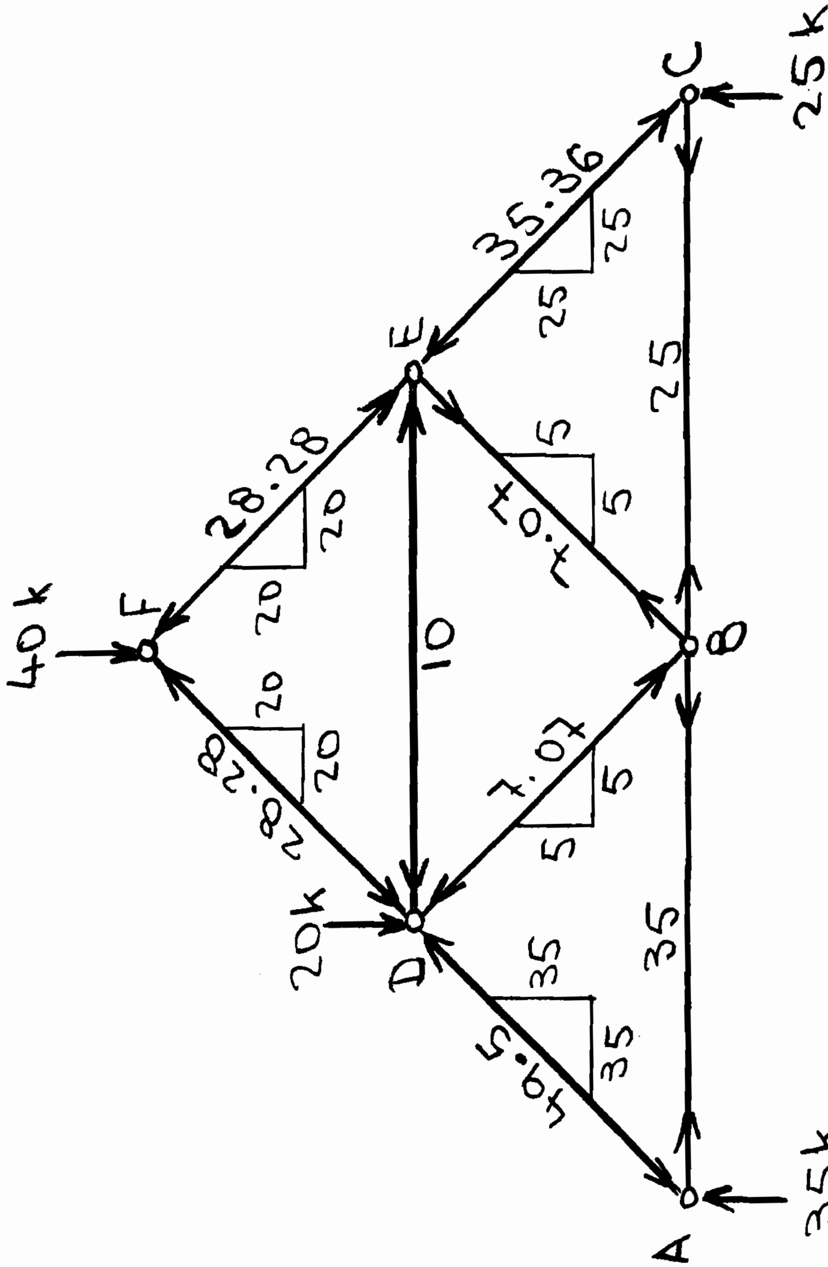
4.18


E
20k
34.28
37.14
14.28
14.28
20.2
14.28
C
0
D
5k
14.28
20.2
14.28
41.07
44.49
17.11
3.35
1.79
2.83
A
17.11
B
41.07k
16.07k

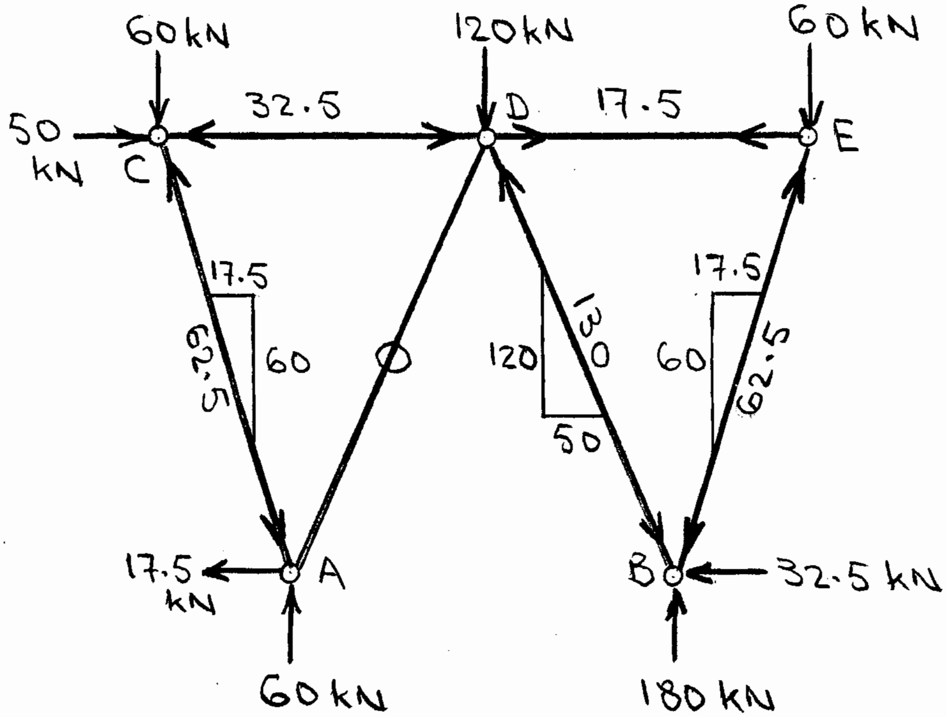
4.19


A
B
C
D
E
F
G
H
I
J
K
L
22.35k
25.1k
10k
8.94k
4.47k
17.89k
8.94k
17.89k
8.94k
29.62k
25.1
50.2
56.13
27.85
27.85
0
20.1
40.2
44.94
10
5
11.18
27.85
5
18.43
36.86
41.21
3.33
3.33
4.71
17.85
16.2
36.22
32.39
25.69
22.36
22.36
31.62
36.88
11.18
22.91
51.22
45.81
22.36
11.18
25
59.24
0
59.24
59.24
66.23
29.62
59.24

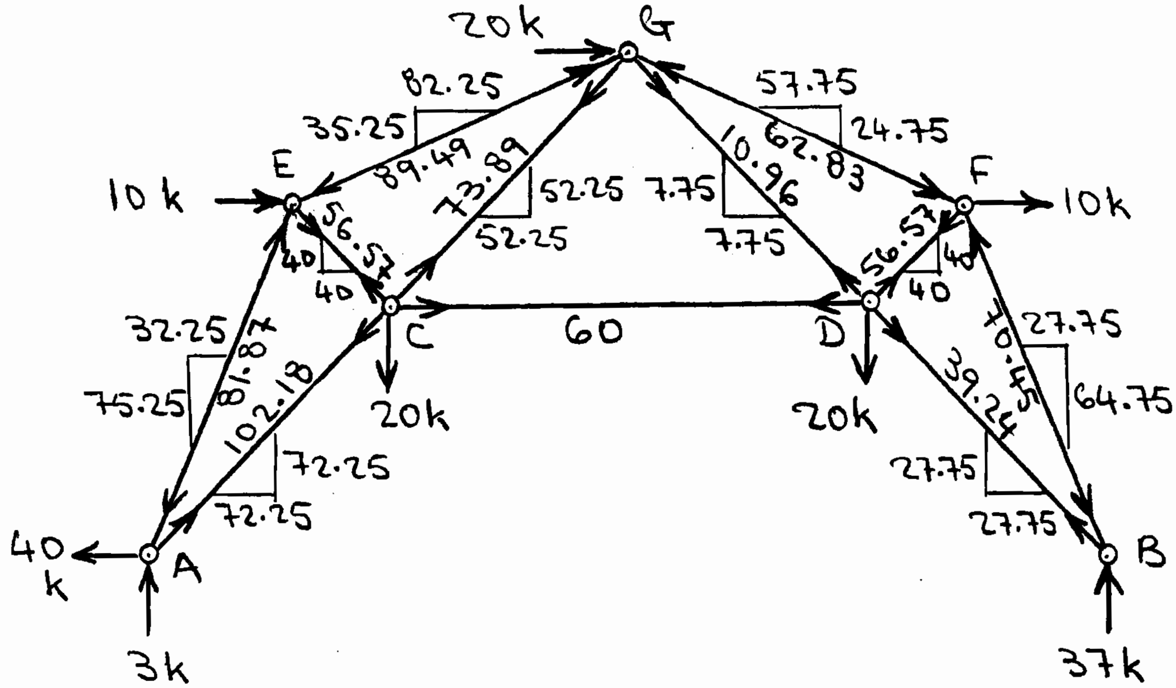
4.20



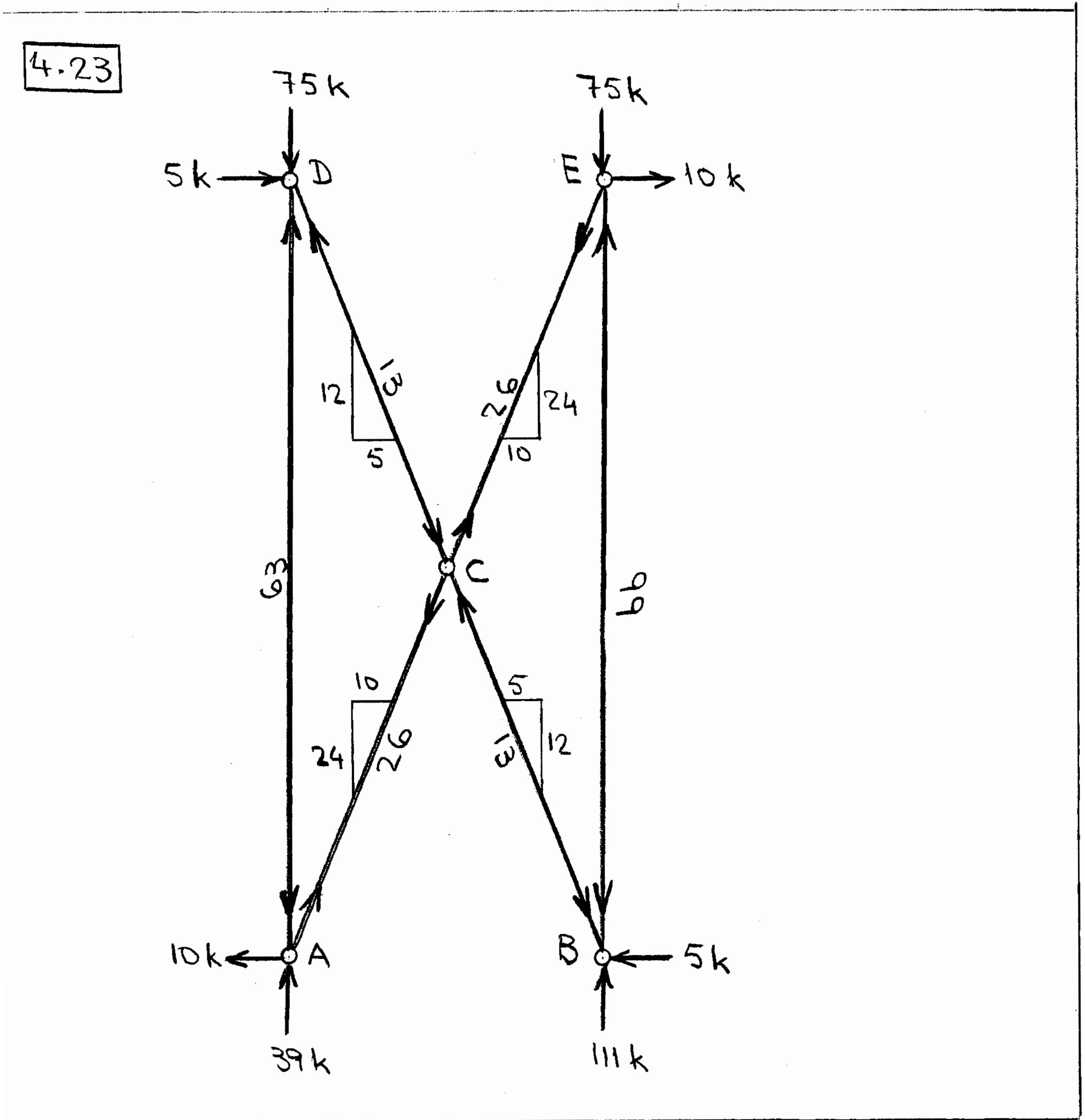
4.21


60kN
120kN
60kN
50 kN
C
32.5
D
17.5
E
17.5
62.5
60
0
120
130
50
60
17.5
62.5
17.5 kN
A
B
32.5 kN
60kN
180kN

4.22

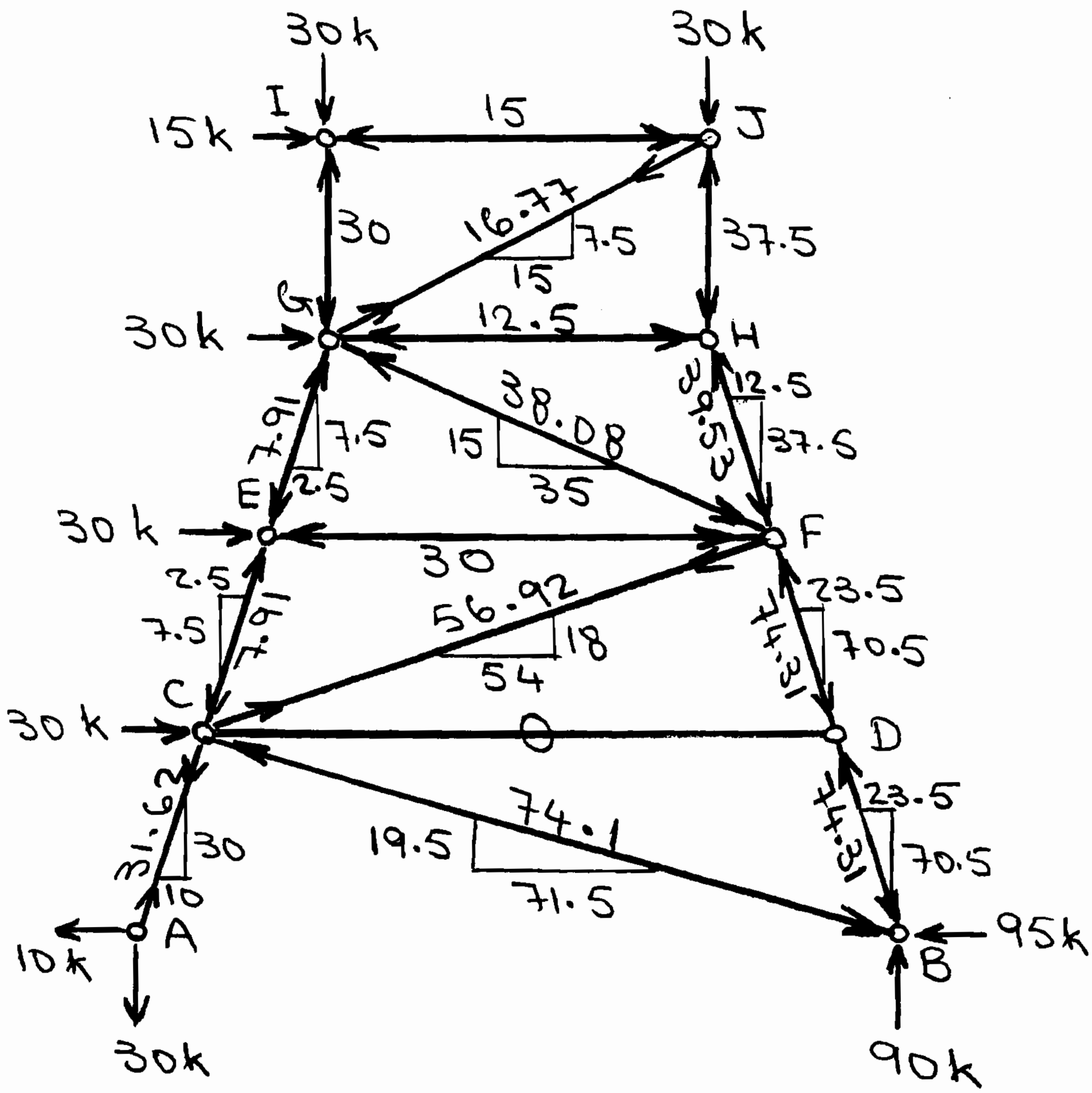


20k
G
82.25
35.25
89.49
73.89
57.75
62.83
24.75
10.96
E
F
10 k
10k
52.25
7.75
52.25
7.75
56.57
56.57
40
40
40
40
C
D
60
32.25
81.87
75.25
102.18
72.25
72.25
20k
20k
27.75
70.45
64.75
39.24
27.75
27.75
40
k
A
B
3k
37k

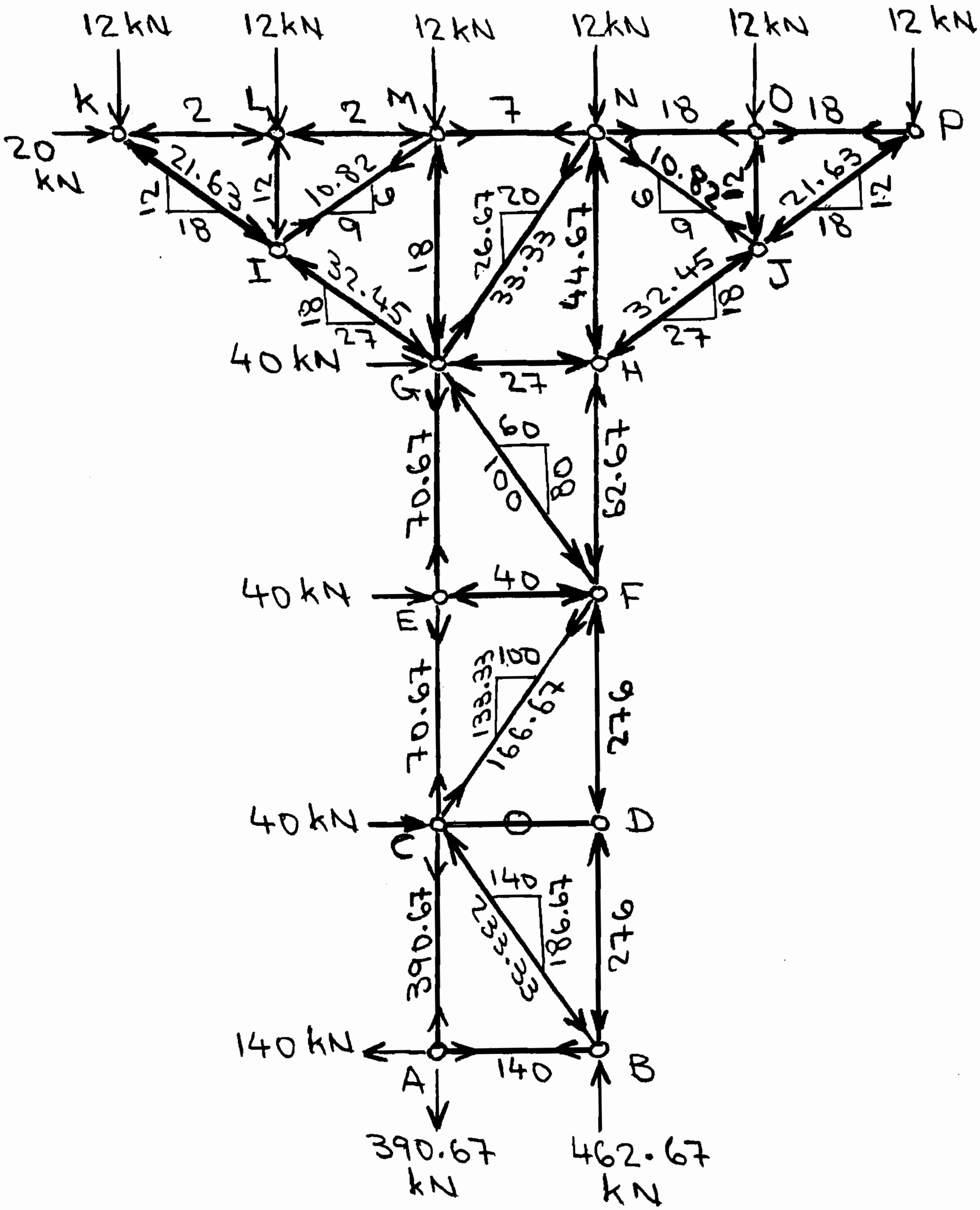


4.23
75 k
75 k
5 k
D
E
10 k
12
13
5
26
24
10
63
C
99
10
5
24
26
13
12
10 k
A
B
5 k
39 k
111 k

4.24

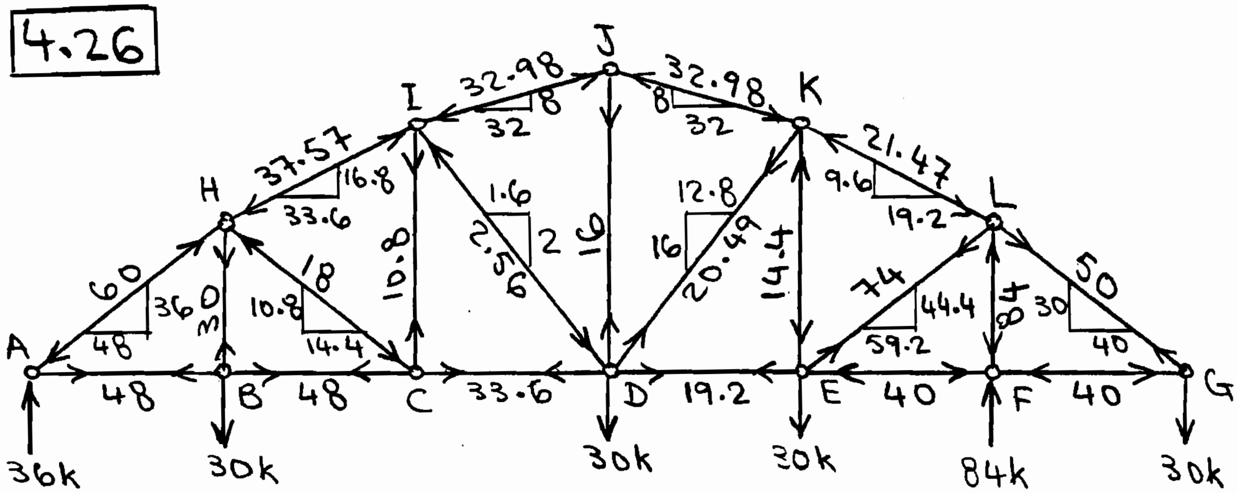

30k
30k
I
J
15k
15
30
16.77
7.5
15
37.5
30k
G
12.5
H
12.5
7.91
7.5
38.08
15
35
2.5
39.53
37.5
E
30k
30
F
2.5
7.91
7.5
56.92
18
54
74.31
23.5
70.5
C
30k
0
D
31.62
30
10
19.5
74.1
71.5
74.31
23.5
70.5
A
10k
30k
B
95k
90k

4.25



12 kN
12 kN
12 kN
12 kN
12 kN
12 kN
K
L
M
N
O
P
20 kN
2
2
7
18
18
21.63
12
18
12
10.82
6
9
26.67
20
33.33
18
44.67
10.82
6
9
12
21.63
18
12
I
J
32.45
18
27
32.45
18
27
40 kN
G
H
27
70.67
60
100
80
62.67
40 kN
E
40
F
70.67
133.33
100
166.67
276
40 kN
C
0
D
390.67
140
186.67
233.33
276
140 kN
A
140
B
390.67 kN
462.67 kN

4.26



32.98
J
32.98
K
I
8
32
8
32
37.57
21.47
H
16.8
9.6
L
33.6
19.2
1.6
12.8
16.8
60
18
10.8
2.56
2
16
16
20.49
14.4
74
50
36
30
10.8
14.4
44.4
8
30
A
48
59.2
40
48
B
48
C
33.6
D
19.2
E
40
F
40
G
36k
30k
30k
30k
84k
30k

4.27 Reactions:

$\xrightarrow{+} \Sigma F_x = 0 \quad A_x + 50 = 0 \quad A_x = -50 \quad \underline{A_x = 50\text{ kN} \leftarrow}$

$+\circlearrowleft \Sigma M_C^{AC} = 0 \quad -A_y(8) + 120(4) = 0 \quad \underline{A_y = 60\text{ kN} \uparrow}$

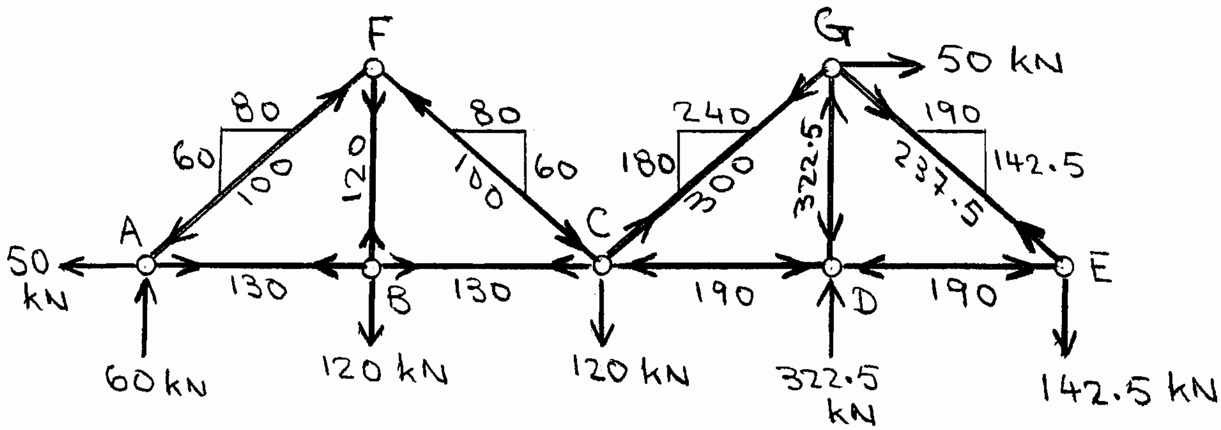
$+\circlearrowleft \Sigma M_E = 0 \quad -60(16) + 120(12) + 120(8) - 50(3) - D_y(4) = 0$

$\underline{D_y = 322.5\text{ kN} \uparrow}$

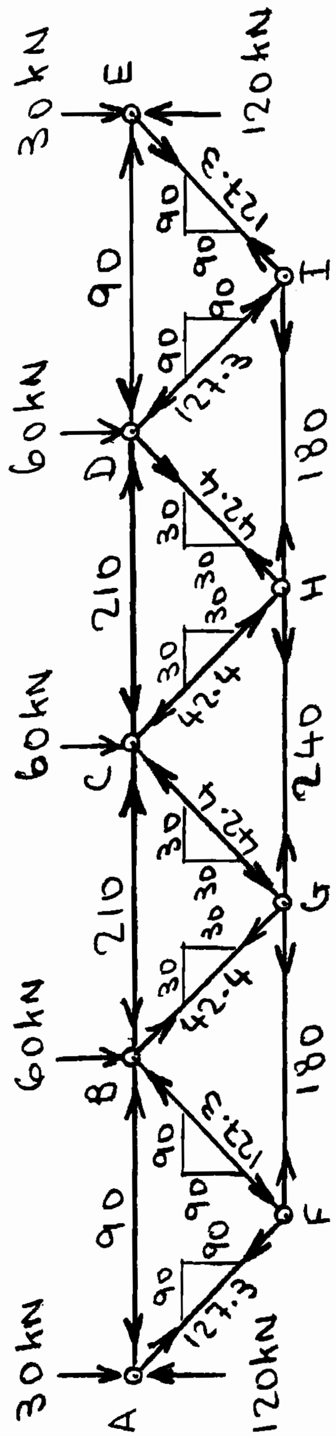
$+\uparrow \Sigma F_y = 0 \quad 60 - 120 - 120 + 322.5 + E_y = 0$

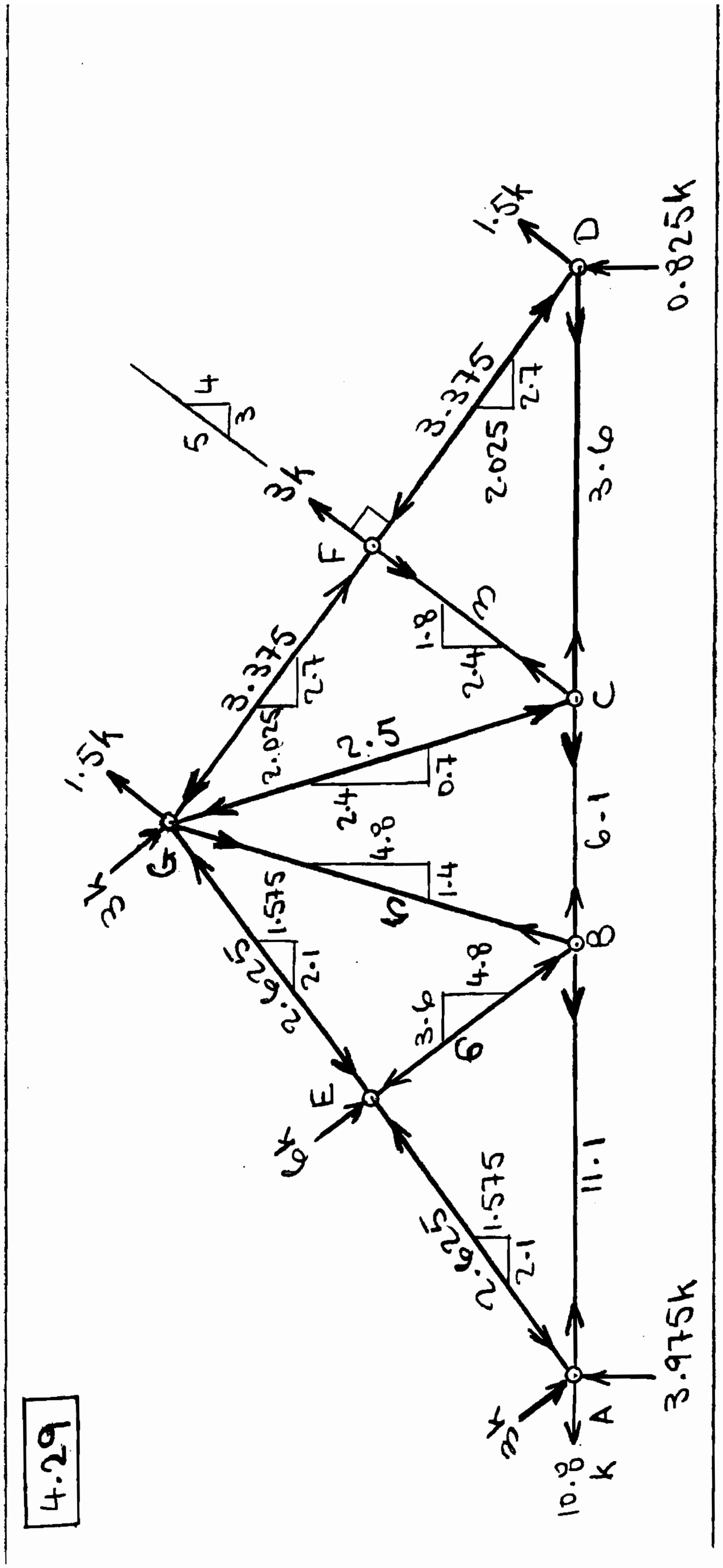
$E_y = -142.5\text{ kN} \quad \underline{E_y = 142.5\text{ kN} \downarrow}$

Member Forces:

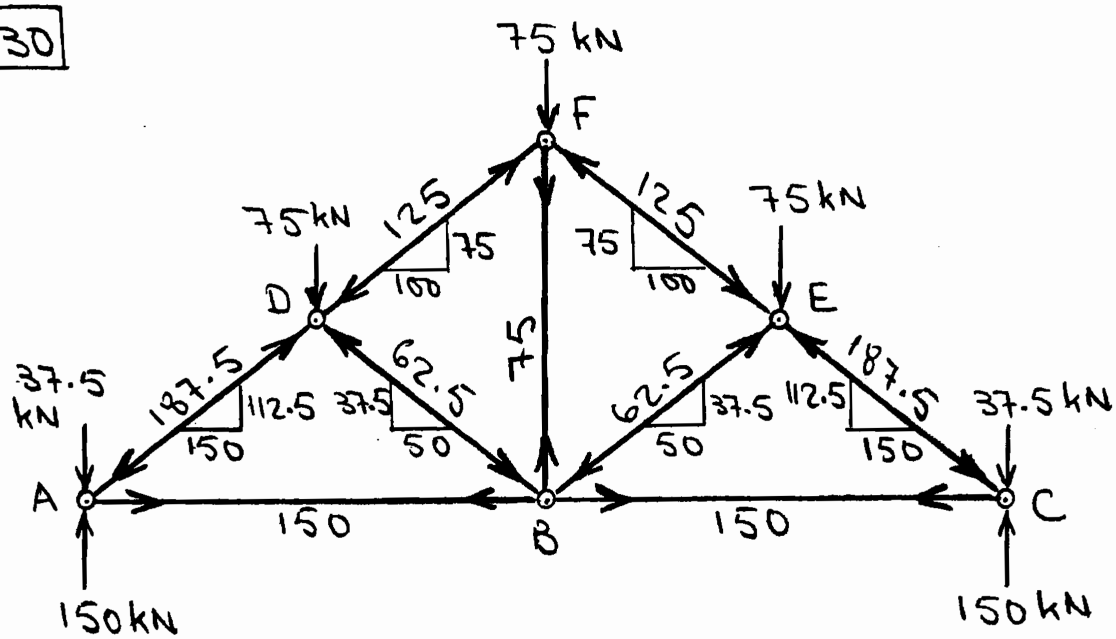


4.28

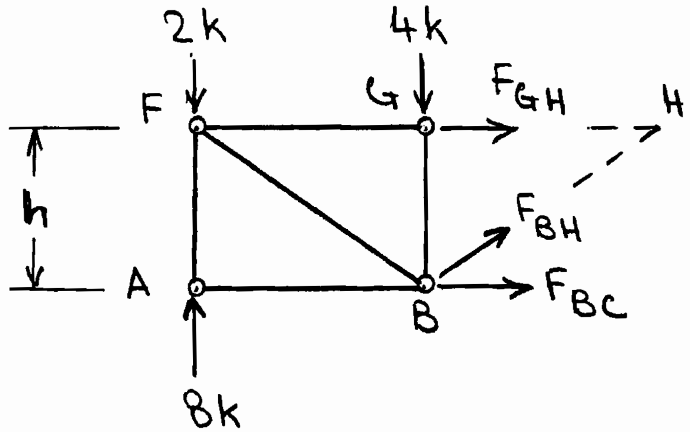

30 kN
120 kN
60 kN
60 kN
60 kN
30 kN
120 kN
A
B
C
D
E
F
G
H
I
90
210
210
90
127.3
42.4
42.4
42.4
42.4
127.3
127.3
127.3
180
240
180


4.29
A
B
C
D
E
F
G
1.5k
0.825k
3.375
2.025
2.7
3.6
3k
5
4
3
1.8
2.4
2.5
0.7
4.8
1.4
6.1
5
1.575
2.1
2.625
3.6
6
6k
11.1
3.975k
10.8 k
3k

4.30


75 kN
F
125
75
100
125
75
100
75kN
75kN
D
E
75
37.5
kN
187.5
112.5
150
37.5
62.5
50
62.5
50
37.5
112.5
150
187.5
37.5kN
A
150
B
150
C
150kN
150kN

4.31 Section through members BC, BH and GH:



$+\circlearrowleft \Sigma M_B = 0 \qquad -8(6) + 2(6) - F_{GH}(h) = 0$

$$F_{GH} = -\frac{36}{h} \qquad (1)$$

$+\circlearrowleft \Sigma M_H = 0 \qquad -8(12) + 2(12) + 4(6) + F_{BC}(h) = 0$

$$F_{BC} = \frac{48}{h} \qquad (2)$$

Equations (1) and (2) indicate that the magnitudes of F_{GH} and F_{BC} are inversely proportional to the truss height h.

For h = 3 ft: $F_{GH} = -\frac{36}{3} = -12\text{ k} = \underline{12\text{ k (C)}}$

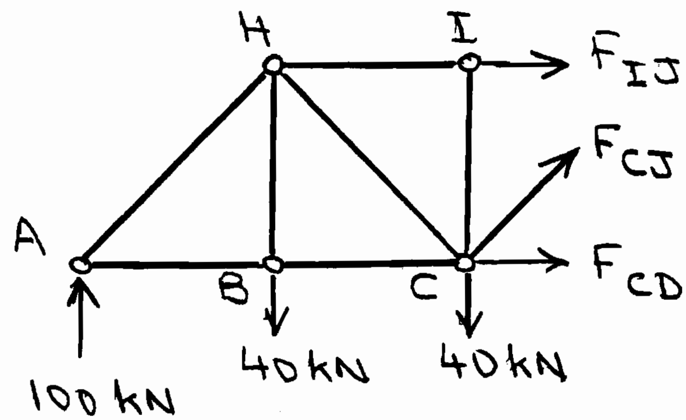
$F_{BC} = \frac{48}{3} = \underline{16\text{k (T)}}$

For h = 6 ft: $F_{GH} = -\frac{36}{6} = -6\text{k} = \underline{6\text{k (C)}}$

$F_{BC} = \frac{48}{6} = \underline{8\text{ k (T)}}$

4.32

Section through members CD, CJ and IJ:



$+\uparrow \Sigma F_y = 0$

$$100 - 2(40) + \left(\frac{1}{\sqrt{2}}\right) F_{CJ} = 0$$

$$F_{CJ} = -28.28 \text{ kN}$$

$$\underline{F_{CJ} = 28.28 \text{ kN (C)}}$$

$+\circlearrowleft \Sigma M_C = 0$

$$-100(10) + 40(5) - F_{IJ}(5) = 0$$

$$F_{IJ} = -160 \text{ kN}$$

$$\underline{F_{IJ} = 160 \text{ kN (C)}}$$

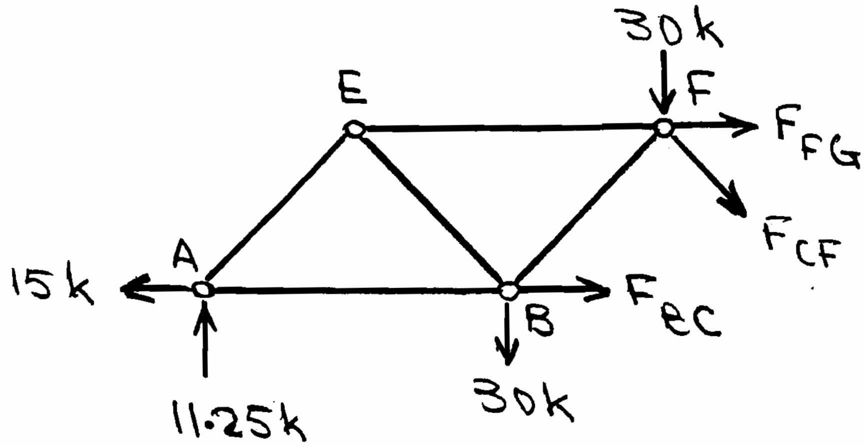
$\xrightarrow{+} \Sigma F_x = 0$

$$F_{CD} - \left(\frac{1}{\sqrt{2}}\right) 28.28 - 160 = 0$$

$$\underline{F_{CD} = 180 \text{ kN (T)}}$$

$\boxed{4.33}$

Section through members BC, CF and FG:



$+\uparrow \Sigma F_y = 0$

$$11.25 - 30 - 30 - \left(\frac{1}{\sqrt{2}}\right) F_{CF} = 0$$

$$F_{CF} = -68.94\ k$$

$$\underline{F_{CF} = 68.94\ k\ (C)}$$

$+\circlearrowleft \Sigma M_F = 0$

$$-15(10) - 11.25(30) + 30(10) + F_{BC}(10) = 0$$

$$\underline{F_{BC} = 18.75\ k\ (T)}$$

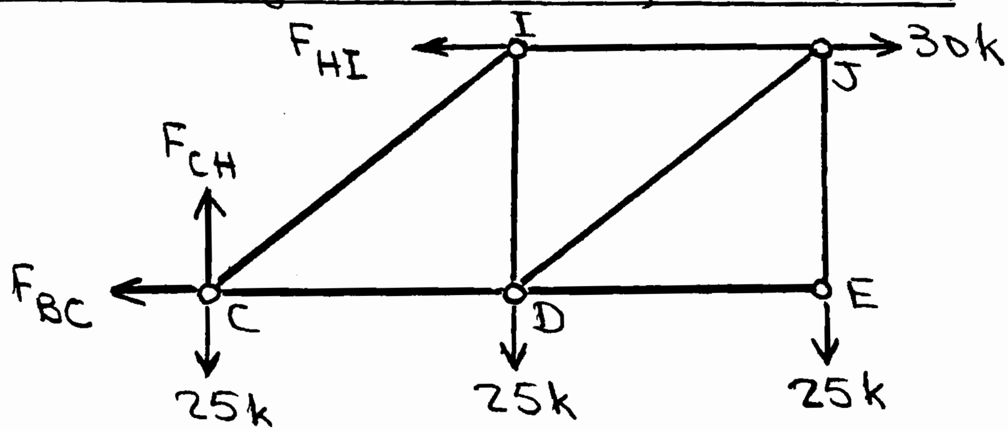
$\xrightarrow{+} \Sigma F_x = 0$

$$-15 + 18.75 - \left(\frac{1}{\sqrt{2}}\right) 68.94 + F_{FG} = 0$$

$$\underline{F_{FG} = 45\ k\ (T)}$$

4.34

Section through members BC, CH and HI:



$+\uparrow \Sigma F_y = 0 \qquad F_{CH} - 3(25) = 0$

$$\underline{F_{CH} = 75\text{ k (T)}}$$

$+\circlearrowleft \Sigma M_C = 0$

$F_{HI}(15) - 30(15) - 25(20) - 25(40) = 0$

$$\underline{F_{HI} = 130\text{ k (T)}}$$

$\overset{+}{\rightarrow} \Sigma F_x = 0$

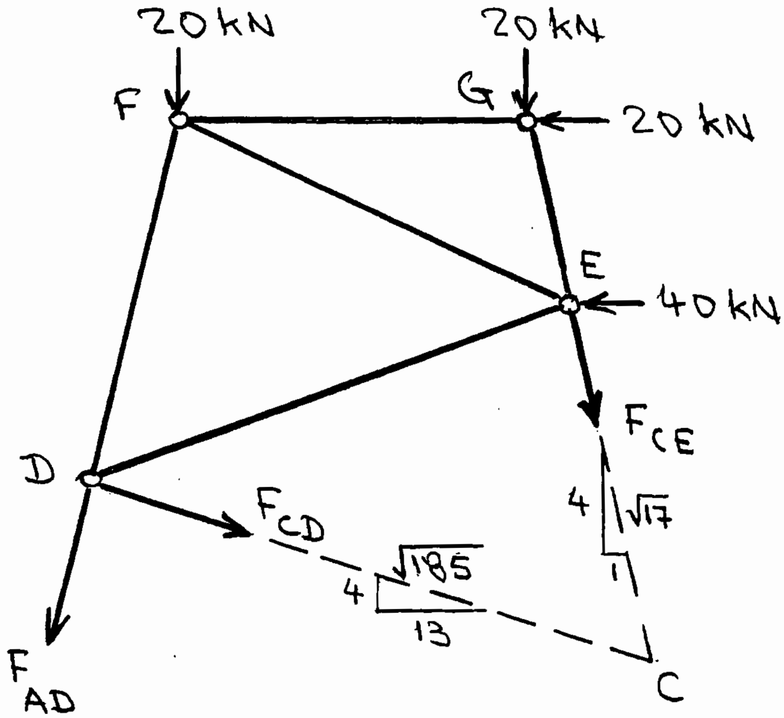
$-F_{BC} - 130 + 30 = 0$

$F_{BC} = -100\text{ k}$

$$\underline{F_{BC} = 100\text{ k (C)}}$$

$\boxed{4.35}$

Section through members AD, CD and CE:



$$+\circlearrowleft \Sigma M_D = 0$$

$$-20(1.5) - 20(7.5) + 20(6) + 40(3) - \frac{1}{\sqrt{17}} F_{CE}(3) - \frac{4}{\sqrt{17}} F_{CE}(8.25) = 0$$

$$\underline{F_{CE} = 6.87 \text{ kN (T)}}$$

$$+\circlearrowleft \Sigma M_C = 0$$

$$20(8.25) + 20(2.25) + 20(9) + 40(6) + \frac{1}{\sqrt{17}} F_{AD}(3) + \frac{4}{\sqrt{17}} F_{AD}(9.75) = 0$$

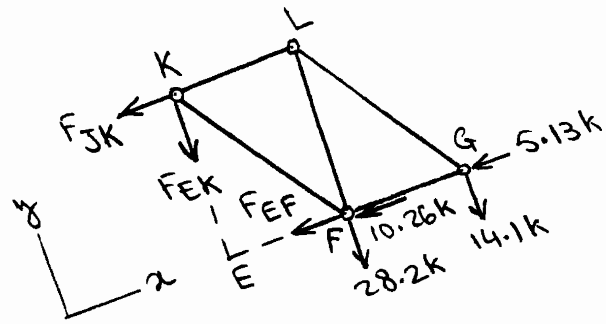
$$F_{AD} = -61.85 \text{ kN} \qquad \underline{F_{AD} = 61.85 \text{ kN (C)}}$$

$$+\uparrow \Sigma F_y = 0$$

$$\frac{4}{\sqrt{17}}(61.85) - 20 - 20 - \frac{4}{\sqrt{17}}(6.87) - \frac{4}{\sqrt{185}} F_{CD} = 0$$

$$F_{CD} = 45.34 \text{ kN (T)}$$

4.36 Section through members EF, EK and JK:



$\Sigma F_y = 0 \quad -F_{EK} - 28.2 - 14.1 = 0$

$F_{EK} = -42.3\,k = \underline{42.3\,k\,(C)}$

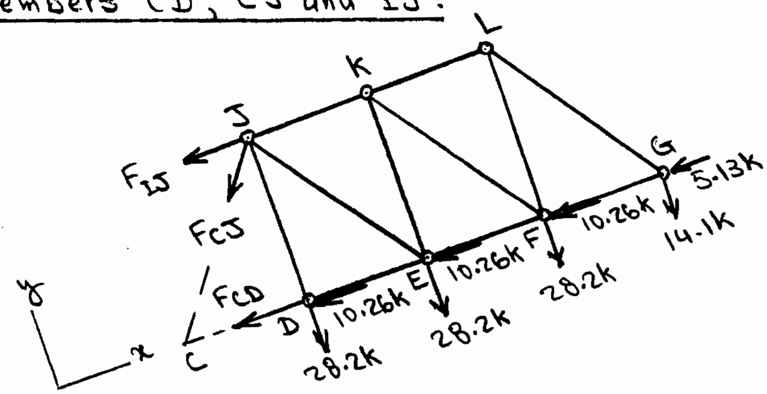
$+\circlearrowleft \Sigma M_K = 0 \quad -F_{EF}(20) - 28.2(15) - 14.1(30) - (5.13 + 10.26)(20) = 0$

$F_{EF} = -57.69\,k = \underline{57.69\,k\,(C)}$

$+\circlearrowleft \Sigma M_E = 0 \quad F_{JK}(20) - 28.2(15) - 14.1(30) = 0$

$F_{JK} = 42.3\,k\,(T)$

Section through members CD, CJ and IJ:



$\Sigma F_y = 0 \quad -\left(\frac{4}{5}\right)F_{CJ} - 3(28.2) - 14.1 = 0$

$F_{CJ} = -123.38\,k = \underline{123.38\,k\,(C)}$

$+\circlearrowleft \Sigma M_J = 0 \quad -F_{CD}(20) - [3(10.26) + 5.13](20) - 28.2(15+30) - 14.1(45) = 0$

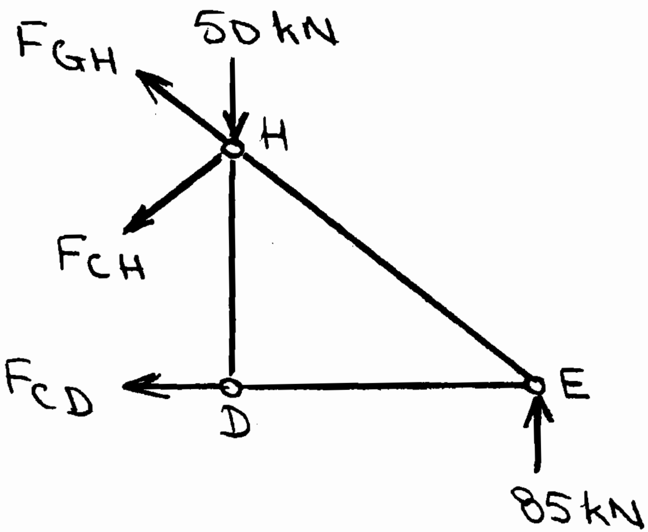
$F_{CD} = -131.09\,k = \underline{131.09\,k\,(C)}$

$+\circlearrowleft \Sigma M_C = 0 \quad F_{IJ}(20) - 28.2(15+30+45) - 14.1(60) = 0$

$\underline{F_{IJ} = 169.2\,k\,(T)}$

$\boxed{4.37}$

Section through members CD, CH and GH:



$+\circlearrowright \Sigma M_E = 0$

$$50(4) + \frac{4}{5} F_{CH}(3) + \frac{3}{5} F_{CH}(4) = 0$$

$$F_{CH} = -41.67 \text{ kN} \qquad \underline{F_{CH} = 41.67 \text{ kN (C)}}$$

$+\uparrow \Sigma F_y = 0$

$$\frac{3}{5} F_{GH} - 50 + \frac{3}{5}(41.67) + 85 = 0$$

$$F_{GH} = -100 \text{ kN} \qquad \underline{F_{GH} = 100 \text{ kN (C)}}$$

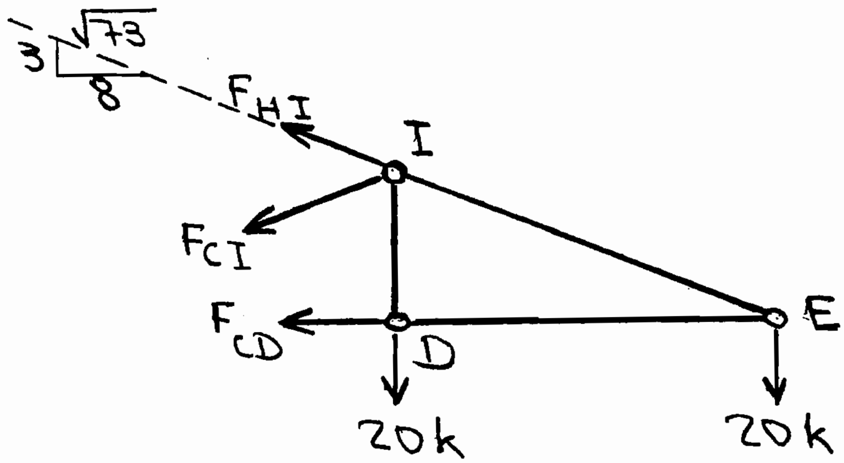
$+\circlearrowright \Sigma M_H = 0$

$$-F_{CD}(3) + 85(4) = 0$$

$$\underline{F_{CD} = 113.33 \text{ kN (T)}}$$

4.38

Section through members CD, CI and HI:



$$+\circlearrowright \Sigma M_I = 0 \qquad -F_{CD}(3.75) - 20(10) = 0$$

$$F_{CD} = -53.33 \text{ k} \qquad \underline{F_{CD} = 53.33 \text{ k (C)}}$$

$$+\circlearrowright \Sigma M_E = 0$$

$$20(10) + \frac{8}{\sqrt{73}} F_{CI}(3.75) + \frac{3}{\sqrt{73}} F_{CI}(10) = 0$$

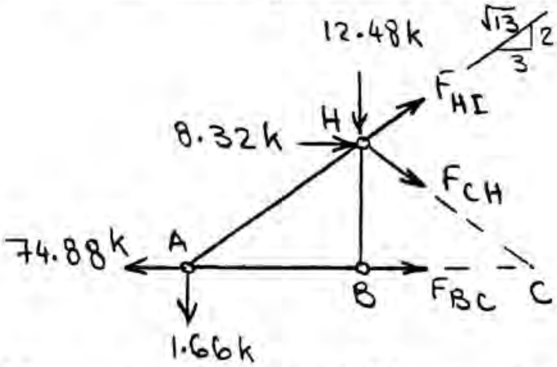
$$F_{CI} = -28.48 \text{ k} \qquad \underline{F_{CI} = 28.48 \text{ k (C)}}$$

$$+\uparrow \Sigma F_y = 0$$

$$\frac{3}{\sqrt{73}} F_{HI} + \frac{3}{\sqrt{73}}(28.48) - 20 - 20 = 0$$

$$\underline{F_{HI} = 85.44 \text{ k (T)}}$$

4.39 Section through members BC, CH and HI:



$+\circlearrowleft \Sigma M_H = 0 \quad -74.88\left(\frac{40}{3}\right) + 1.66(20) + F_{BC}\left(\frac{40}{3}\right) = 0$

$\underline{F_{BC} = 72.39\text{ k (T)}}$

$+\circlearrowleft \Sigma M_C = 0 \quad 1.66(40) + 12.48(20) - 8.32\left(\frac{40}{3}\right)$

$- \frac{3}{\sqrt{13}}F_{HI}\left(\frac{40}{3}\right) - \frac{2}{\sqrt{13}}F_{HI}(20) = 0$

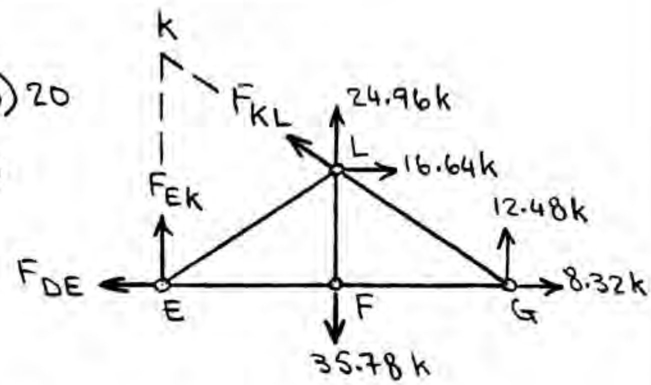
$\underline{F_{HI} = 9.24\text{ k (T)}}$

$+\circlearrowleft \Sigma M_A = 0 \quad -8.32\left(\frac{40}{3}\right) - 12.48(20) - \frac{3}{\sqrt{13}}F_{CH}\left(\frac{40}{3}\right)$

$- \frac{2}{\sqrt{13}}F_{CH}(20) = 0$

$F_{CH} = -16.25\text{ k} = \underline{16.25\text{ k (C)}}$

Section through members DE, EK and KL:



$+\circlearrowleft \Sigma M_K = 0$

$-F_{DE}\left(\frac{80}{3}\right) + (24.96 - 35.78)20$

$+ 12.48(40) + 16.64\left(\frac{40}{3}\right)$

$+ 8.32\left(\frac{80}{3}\right) = 0$

$\underline{F_{DE} = 27.25\text{ k (T)}}$

$+\circlearrowleft \Sigma M_G = 0$

$-F_{EK}(40) + (35.78 - 24.96)20 - 16.64\left(\frac{40}{3}\right) = 0$

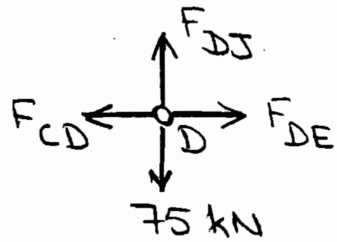
$F_{EK} = -0.14\text{ k} = \underline{0.14\text{ k (C)}}$

$+\circlearrowleft \Sigma M_E = 0 \quad (24.96 - 35.78)20 + 12.48(40) - 16.64\left(\frac{40}{3}\right)$

$+ \frac{3}{\sqrt{13}}F_{KL}\left(\frac{40}{3}\right) + \frac{2}{\sqrt{13}}F_{KL}(20) = 0$

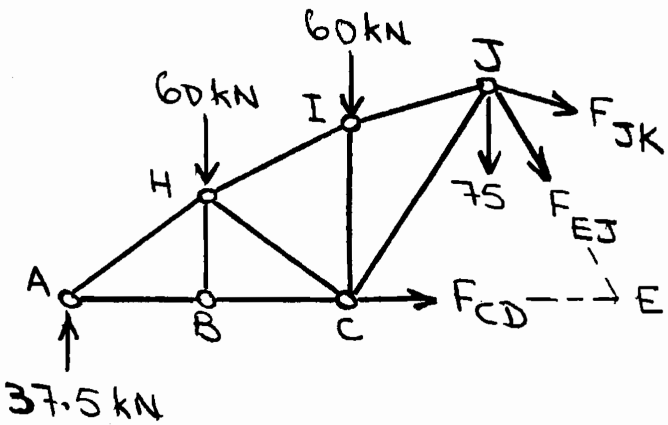
$F_{KL} = -2.75\text{ k} = \underline{2.75\text{ k (C)}}$

$\boxed{4.40}$ Joint D:



$\Sigma F_y = 0 \qquad F_{DJ} = 75 \text{ kN (T)}$

Section through members CD, DJ, EJ and JK:



$+\circlearrowright \Sigma M_J = 0$

$-37.5(12) + 60(8) + 60(4) + F_{CD}(6) = 0$

$F_{CD} = -45 \text{ kN} \qquad \underline{F_{CD} = 45 \text{ kN (C)}}$

$+\circlearrowleft \Sigma M_E = 0$

$-37.5(16) + 60(12) + 60(8) + 75(4)$

$-\frac{4}{\sqrt{17}} F_{JK}(6) + \frac{1}{\sqrt{17}} F_{JK}(4) = 0$

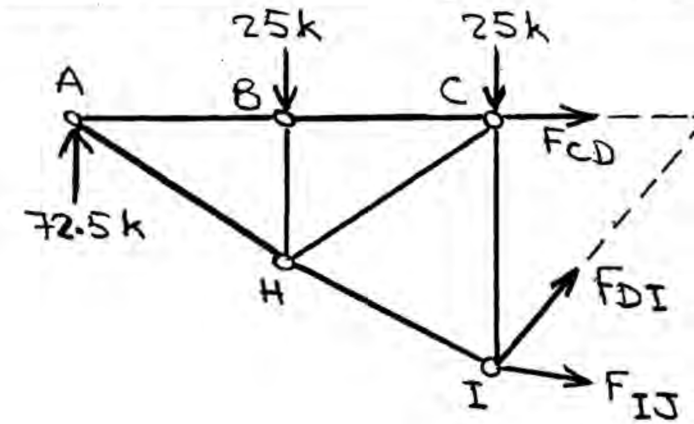
$\underline{F_{JK} = 185.54 \text{ kN (T)}}$

$+\uparrow \Sigma F_y = 0$

$37.5 - 60 - 60 - 75 - \frac{3}{\sqrt{13}} F_{EJ} - \frac{1}{\sqrt{17}}(185.54) = 0$

$F_{EJ} = -243.37 \text{ kN} \quad F_{EJ} = 243.37 \text{ kN (C)}$

4.41 Section through members CD, DI and IJ:



$+\circlearrowleft \Sigma M_I = 0 \qquad -72.5(60) + 25(30) - F_{CD}(35) = 0$

$F_{CD} = -102.86\text{ k} \qquad \underline{F_{CD} = 102.86\text{ k (C)}}$

$+\circlearrowleft \Sigma M_D = 0$

$-72.5(90) + 25(60) + 25(30) + \frac{6}{\sqrt{37}} F_{IJ}(35)$

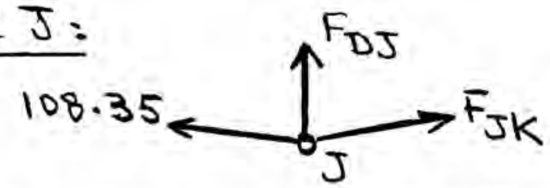
$+ \frac{1}{\sqrt{37}} F_{IJ}(30) = 0 \qquad F_{IJ} = 108.35\text{ k (T)}$

$+\uparrow \Sigma F_y = 0$

$72.5 - 25 - 25 - \frac{1}{\sqrt{37}}(108.35) + \frac{7}{\sqrt{85}} F_{DI} = 0$

$F_{DI} = -6.17\text{ k} \qquad \underline{F_{DI} = 6.17\text{ k (C)}}$

Joint J:



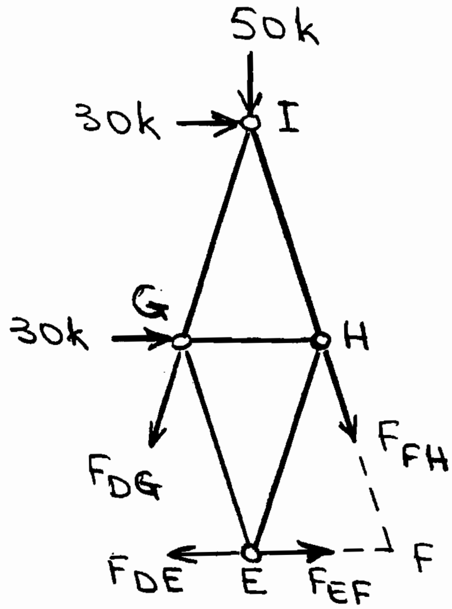
$\xrightarrow{+} \Sigma F_x = 0 \qquad -\frac{6}{\sqrt{37}}(108.35) + \frac{6}{\sqrt{37}} F_{JK} = 0$

$F_{JK} = 108.35\text{ k (T)}$

$+\uparrow \Sigma F_y = 0 \qquad 2\left(\frac{1}{\sqrt{37}}\right)108.35 + F_{DJ} = 0$

$F_{DJ} = -35.63\text{ k} \qquad \underline{F_{DJ} = 35.63\text{ k (C)}}$

4.42 Section through members DG, DE, EF and FH:



$+\circlearrowleft \Sigma M_F = 0$

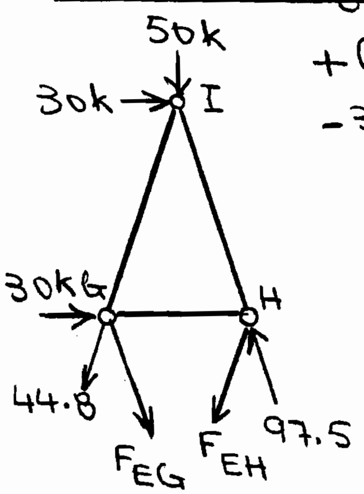
$$50(6) - 30(18) - 30(9) + \frac{1}{\sqrt{10}} F_{DG}(9) + \frac{3}{\sqrt{10}} F_{DG}(9) = 0$$

$$\underline{F_{DG} = 44.8 \text{ k (T)}}$$

$+\uparrow \Sigma F_y = 0 \qquad -\frac{3}{\sqrt{10}}(44.8) - 50 - \frac{3}{\sqrt{10}} F_{FH} = 0$

$$F_{FH} = -97.5 \text{ k} = 97.5 \text{ k (C)}$$

Section through members DG, EG, EH and FH:



$+\circlearrowleft \Sigma M_G = 0$

$$-30(9) - 50(3) + \frac{3}{\sqrt{10}}(97.5)(6) - \frac{3}{\sqrt{10}} F_{EH}(6) = 0$$

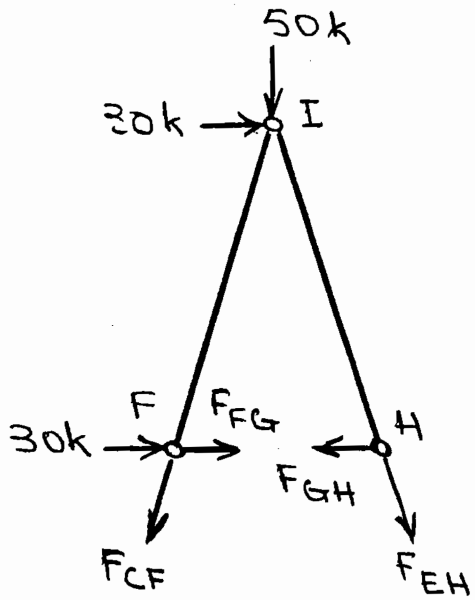
$$\underline{F_{EH} = 23.72 \text{ k (T)}}$$

$+\circlearrowleft \Sigma M_H = 0$

$$50(3) - 30(9) + \frac{3}{\sqrt{10}}(44.8)(6) + \frac{3}{\sqrt{10}} F_{EG}(6) = 0$$

$F_{EG} = -23.72 \text{ k} \qquad \underline{F_{EG} = 23.72 \text{ k (C)}}$

4.43 Section through members CF, FG, GH and EH:



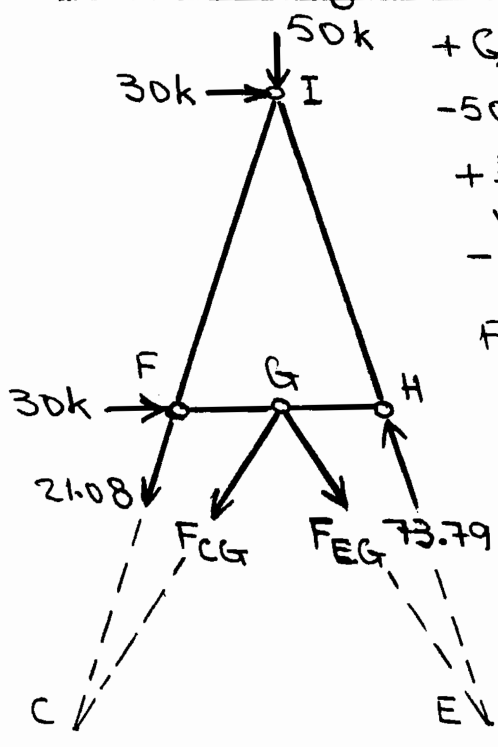
$+\circlearrowright \Sigma M_H = 0 \qquad 50(3) - 30(9) + \frac{3}{\sqrt{10}} F_{CF}(6) = 0$

$\underline{F_{CF} = 21.08 \text{ k (T)}}$

$+\circlearrowright \Sigma M_F = 0 \qquad -50(3) - 30(9) - \frac{3}{\sqrt{10}} F_{EH}(6) = 0$

$F_{EH} = -73.79 \text{ k} = 73.79 \text{ k (C)}$

Section through members CF, CG, EG and EH:



$+\circlearrowright \Sigma M_C = 0$

$-50(6) - 30(18) - 30(9) + \frac{1}{\sqrt{10}}(73.79)(9)$
$+ \frac{3}{\sqrt{10}}(73.79)(9) - \frac{2}{\sqrt{13}} F_{EG}(9)$
$- \frac{3}{\sqrt{13}} F_{EG}(6) = 0$

$F_{EG} = -27.04 \text{ k}$

$\underline{F_{EG} = 27.04 \text{ k (C)}}$

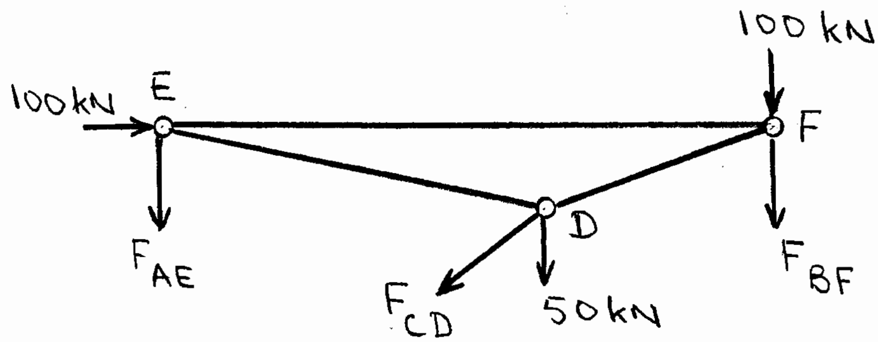
$+\circlearrowright \Sigma M_E = 0$

$50(6) - 30(18) - 30(9)$
$+ \frac{1}{\sqrt{10}}(21.08)(9) + \frac{3}{\sqrt{10}}(21.08)(9)$
$+ \frac{2}{\sqrt{13}} F_{CG}(9) + \frac{3}{\sqrt{13}} F_{CG}(6) = 0$

$\underline{F_{CG} = 27.04 \text{ k (T)}}$

4.44

Section through members AE, CD and BF:



$\xrightarrow{+} \Sigma F_x = 0 \qquad 100 - \left(\frac{4}{5}\right) F_{CD} = 0 \qquad \underline{F_{CD} = 125 \text{ kN (T)}}$

$+\circlearrowright \Sigma M_F = 0 \qquad F_{AE}(16) - \frac{4}{5}(125)(2) + \frac{3}{5}(125)(6) + 50(6) = 0$

$F_{AE} = -34.38 \text{ kN} = \underline{34.38 \text{ kN (C)}}$

$+\uparrow \Sigma F_y = 0 \qquad 34.38 - \frac{3}{5}(125) - 50 - 100 - F_{BF} = 0$

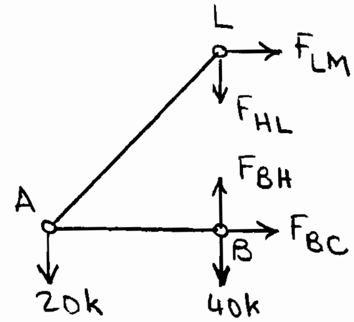
$F_{BF} = -190.62 \text{ kN} = \underline{190.62 \text{ kN (C)}}$

4.45 Section through members BC, BH, HL and LM:

$+\circlearrowleft \Sigma M_B = 0$

$20(30) - F_{LM}(30) = 0$

$\underline{F_{LM} = 20\text{ k (T)}}$



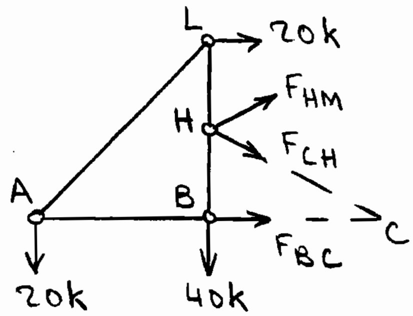
Section through members BC, CH, HM and LM:

$+\circlearrowleft \Sigma M_C = 0$

$20(60) + 40(30) - 20(30)$

$- \frac{2}{\sqrt{5}} F_{HM}(15) - \frac{1}{\sqrt{5}} F_{HM}(30) = 0$

$\underline{F_{HM} = 67.08\text{ k (T)}}$



$+\uparrow \Sigma F_y = 0 \qquad -20 - 40 + \frac{1}{\sqrt{5}}(67.08) - \frac{1}{\sqrt{5}} F_{CH} = 0$

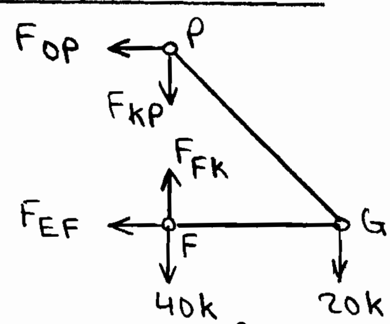
$F_{CH} = -67.08\text{k} = \underline{67.08\text{k (C)}}$

Section through members EF, FK, KP, and OP:

$+\circlearrowleft \Sigma M_P = 0$

$-F_{EF}(30) - 20(30) = 0$

$F_{EF} = -20\text{k} = \underline{20\text{k (C)}}$



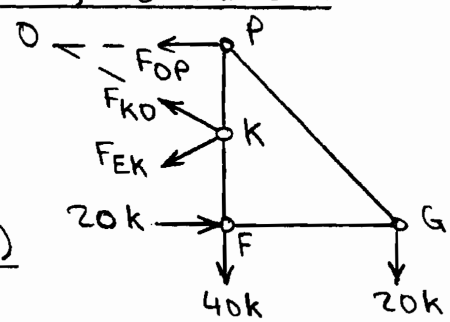
Section through members EF, EK, KO and OP:

$+\circlearrowleft \Sigma M_O = 0$

$-20(60) - 40(30) + 20(30)$

$- \frac{2}{\sqrt{5}} F_{EK}(15) - \frac{1}{\sqrt{5}} F_{EK}(30) = 0$

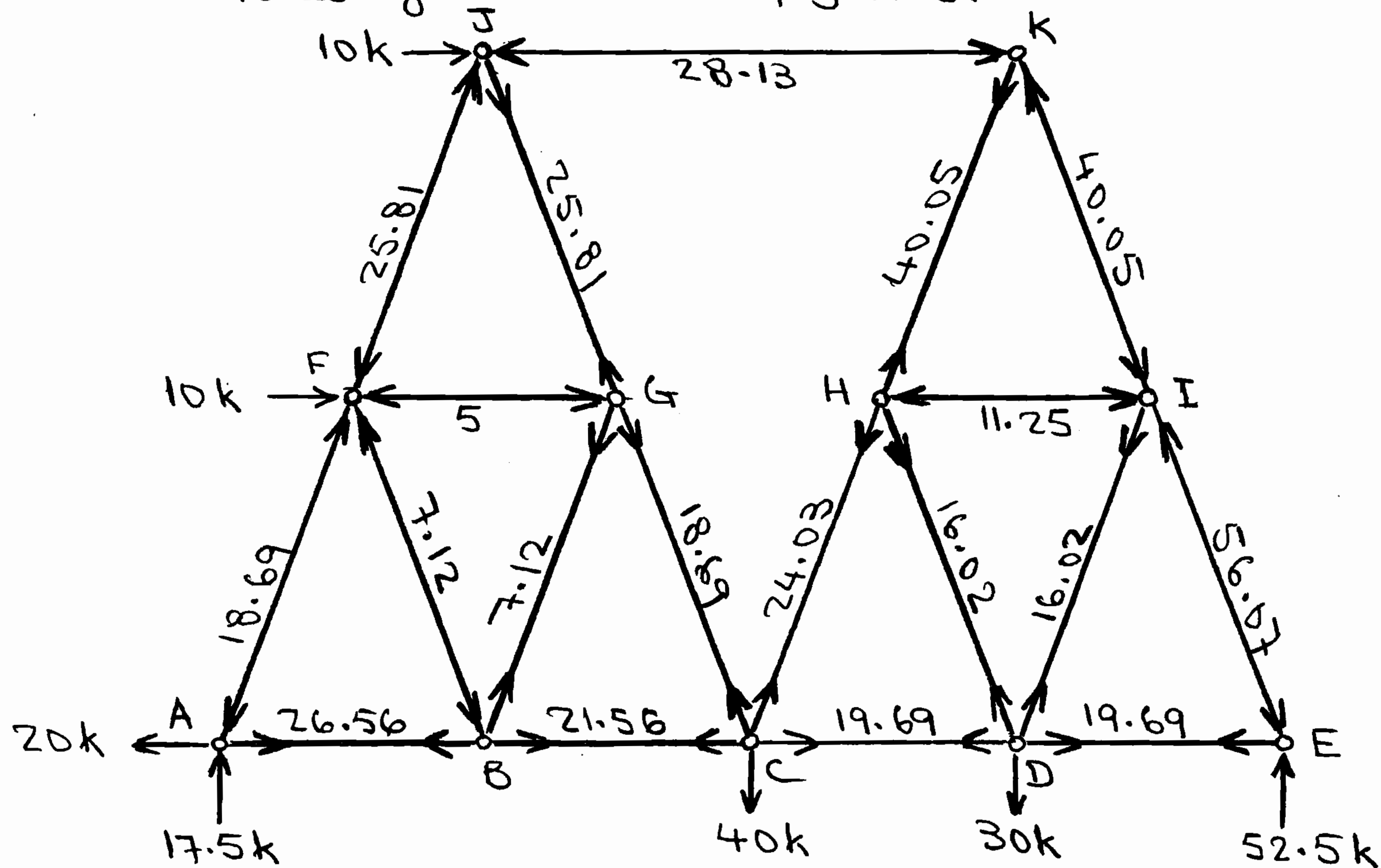
$F_{EK} = -67.08\text{ k} = \underline{67.08\text{ k (C)}}$



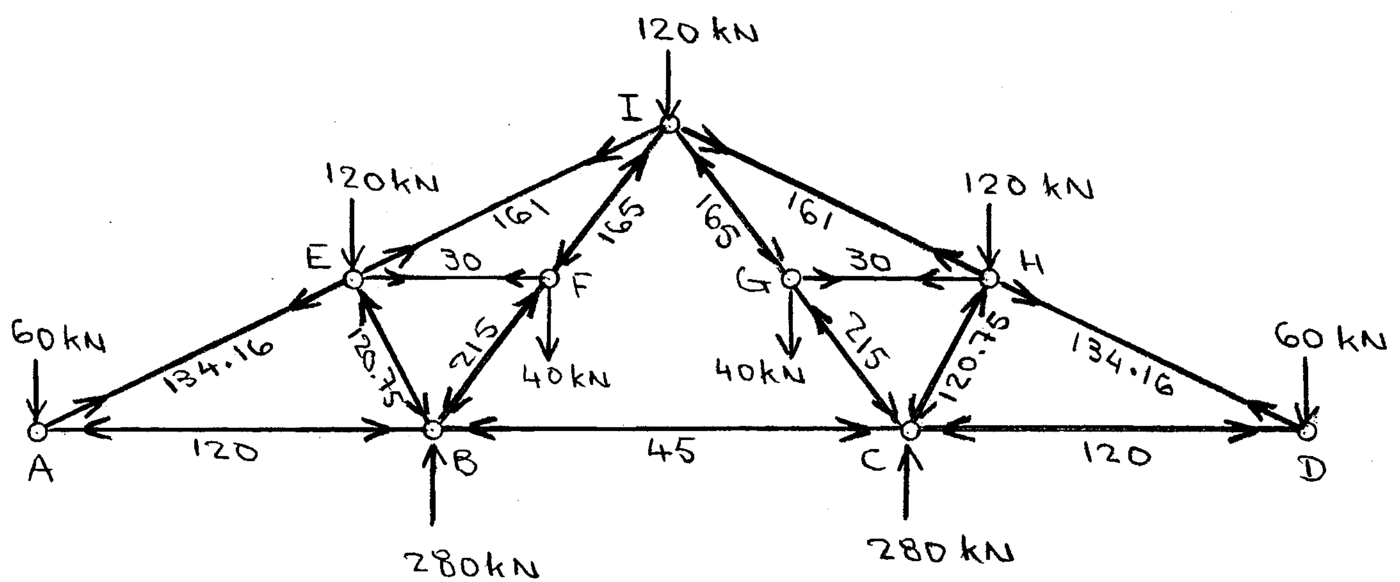
$+\uparrow \Sigma F_y = 0$

$\frac{1}{\sqrt{5}}(67.08) + \frac{1}{\sqrt{5}} F_{KO} - 40 - 20 = 0 \qquad \underline{F_{KO} = 67.08\text{ k (T)}}$

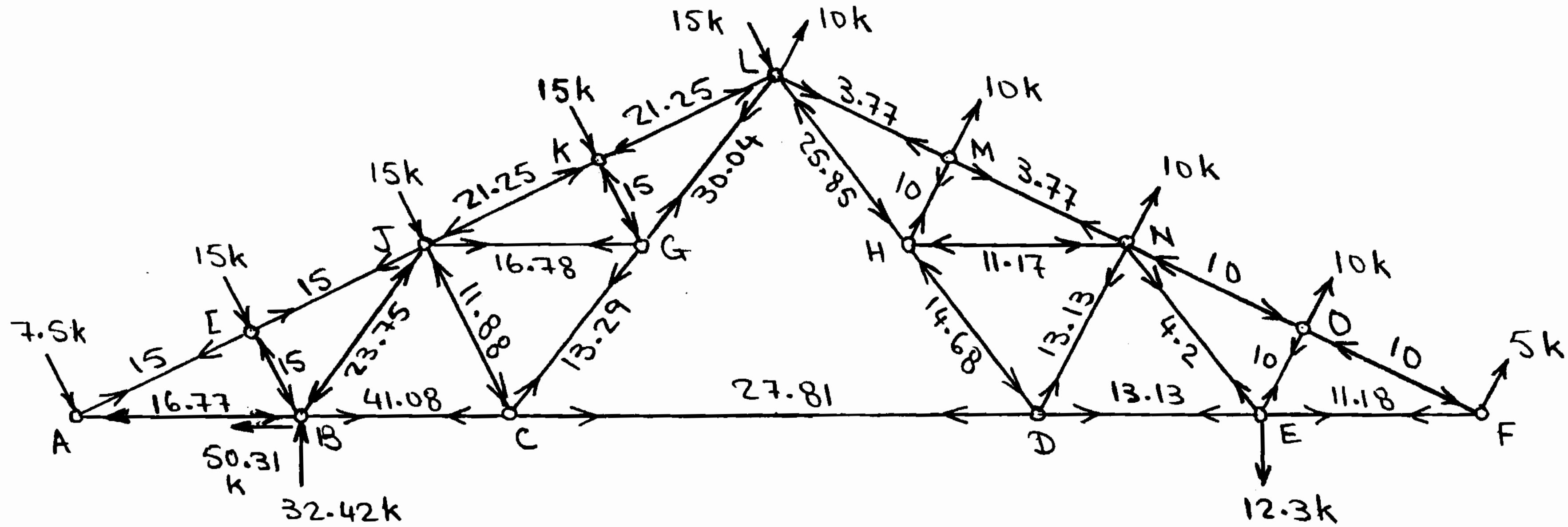
4.46 Considering a section through members BC, CG and JK, compute the force in member JK: $F_{JK} = 28.13\,k\,(C)$. Then, determine the remaining member forces by the method of joints.



4.47 Considering a section through members BC, FI and EI, compute the force in member BC: $\underline{F_{BC} = 45\text{ kN (C)}}$. Then, determine the remaining member forces by the method of joints.

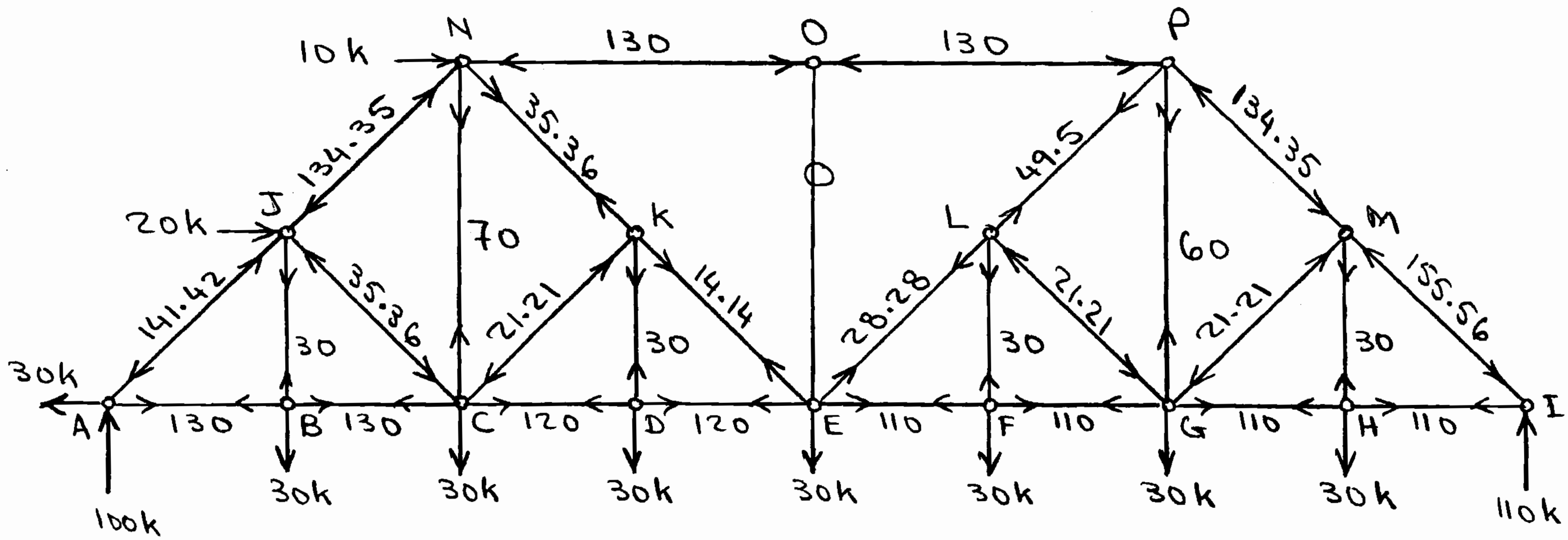


4.48 Considering a section through members CD, GL and KL, compute the force in member CD: $F_{CD} = 27.81\text{ k (T)}$. Then, determine the remaining member forces by the method of joints.

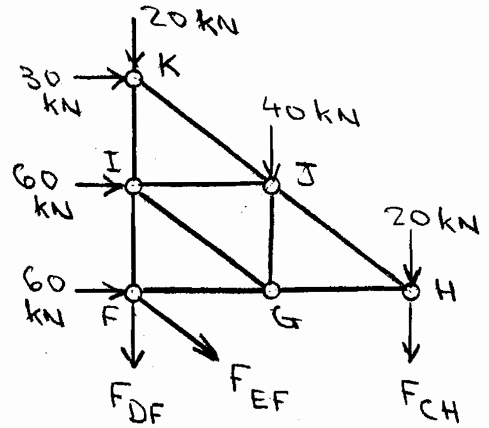


4.49 Considering a section through members DE, EK and NO, compute the force in member NO: $\underline{F_{NO} = 130\text{k (C)}}$.

Then, determine the remaining member forces by the method of joints.



4.50 Section through members DF, EF and CH:



$+\circlearrowleft \Sigma M_F = 0 \qquad -30(6)-60(3)-40(4)-20(8)-F_{CH}(8)=0$

$$F_{CH} = -85 \text{ kN} = \underline{85 \text{ kN (C)}}$$

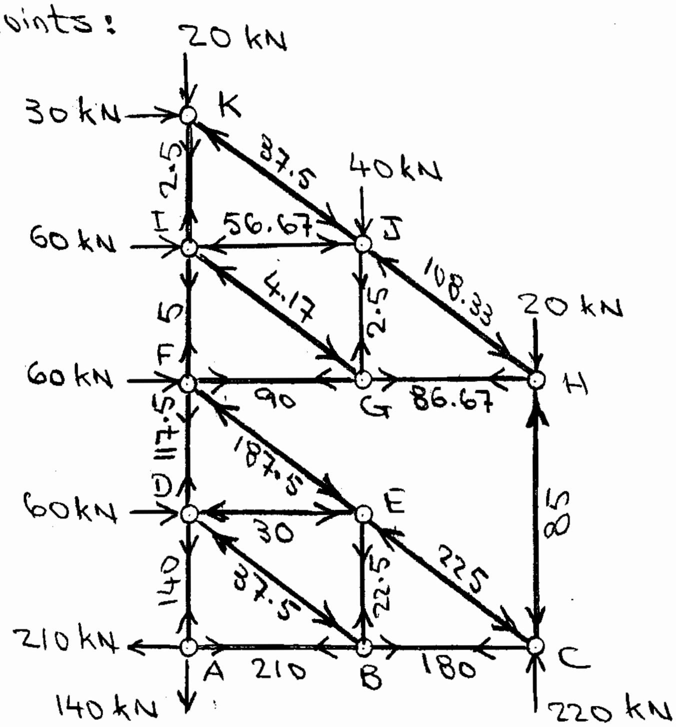
$\overset{+}{\rightarrow}\Sigma F_x = 0 \qquad 30+60+60+\left(\frac{4}{5}\right)F_{EF}=0$

$$F_{EF} = -187.5 \text{ kN} = \underline{187.5 \text{ kN (C)}}$$

$+\uparrow \Sigma F_y = 0 \qquad -20-40-20-F_{DF}+85+\left(\frac{3}{5}\right)(187.5)=0$

$$\underline{F_{DF} = 117.5 \text{ (T)}}$$

The remaining member forces can now be determined by the method of joints:



4.51

Member	Projection			Length (ft)
	x (ft)	y (ft)	z (ft)	
AD	3	10	5	11.58
BD	8	10	7	14.59
CD	2	10	4	10.95

Joint D:

$$\sum F_x = 0 \quad -\left(\frac{3}{11.58}\right)F_{AD} + \left(\frac{8}{14.59}\right)F_{BD} + \left(\frac{2}{10.95}\right)F_{CD} + 6 = 0$$

$$\sum F_y = 0 \quad -\left(\frac{10}{11.58}\right)F_{AD} - \left(\frac{10}{14.59}\right)F_{BD} - \left(\frac{10}{10.95}\right)F_{CD} - 12 = 0$$

$$\sum F_z = 0 \quad \left(\frac{5}{11.58}\right)F_{AD} + \left(\frac{7}{14.59}\right)F_{BD} - \left(\frac{4}{10.95}\right)F_{CD} = 0$$

Solving these simultaneous equations, we obtain:

$$\underline{F_{AD} = 1.12\text{ k (T)}}$$

$$F_{BD} = -7.56\text{ k} = \underline{7.56\text{ k (C)}}$$

$$F_{CD} = -8.54\text{ k} = \underline{8.54\text{ k (C)}}$$

4.52

Member	Projection			Length (m)
	x (m)	y (m)	z (m)	
AB	0	0	3	3
AC	4	0	3	5
BC	4	0	0	4
AD	0	7	3	7.62
BD	0	7	0	7
CD	4	7	0	8.06

Joint D:

$\Sigma F_x = 0 \qquad \left(\frac{4}{8.06}\right) F_{CD} = 0 \qquad \underline{F_{CD} = 0}$

$\Sigma F_z = 0 \qquad \left(\frac{3}{7.62}\right) F_{AD} + 40 = 0 \qquad F_{AD} = -101.6\text{ kN}$

$\underline{F_{AD} = 101.6\text{ kN (C)}}$

$\Sigma F_y = 0 \qquad \left(\frac{7}{7.62}\right) 101.6 - F_{BD} - 60 = 0$

$\underline{F_{BD} = 33.33\text{ kN (T)}}$

Joint B:

$\Sigma F_x = 0 \qquad \underline{F_{BC} = 0}$

$\Sigma F_z = 0 \qquad \underline{F_{AB} = 0}$

Joint C:

$\Sigma F_z = 0 \qquad \left(\frac{3}{5}\right) F_{AC} = 0 \qquad \underline{F_{AC} = 0}$

4.53

Member	Projection			Length (ft)
	x (ft)	y (ft)	z (ft)	
AB	20	0	0	20
AC	14	0	10	17.2
BC	6	0	10	11.66
AD	10	20	5	22.91
BD	10	20	5	22.91
CD	4	20	5	21

Joint D:

$$\Sigma F_x = 0 \quad -\left(\frac{10}{22.91}\right)F_{AD} + \left(\frac{10}{22.91}\right)F_{BD} + \left(\frac{4}{21}\right)F_{CD} + 25 = 0$$

$$\Sigma F_y = 0 \quad -\left(\frac{20}{22.91}\right)F_{AD} - \left(\frac{20}{22.91}\right)F_{BD} - \left(\frac{20}{21}\right)F_{CD} - 30 = 0$$

$$\Sigma F_z = 0 \quad \left(\frac{5}{22.91}\right)F_{AD} + \left(\frac{5}{22.91}\right)F_{BD} - \left(\frac{5}{21}\right)F_{CD} = 0$$

Solving these equations, we obtain:

$\underline{F_{AD} = 16.63 \text{ k (T)}}$ $\qquad F_{BD} = -33.84 \text{ k} = \underline{33.84 \text{ k (C)}}$

$F_{CD} = -15.74 \text{ k} = \underline{15.74 \text{ k (C)}}$

Joint A:

$$\Sigma F_z = 0 \quad -\left(\frac{5}{22.91}\right)16.63 - \left(\frac{10}{17.2}\right)F_{AC} = 0$$

$$F_{AC} = -6.24 \text{ k} = \underline{6.24 \text{ k (C)}}$$

$$\Sigma F_x = 0 \quad \left(\frac{10}{22.91}\right)16.63 - \left(\frac{14}{17.2}\right)6.24 + F_{AB} = 0$$

$$F_{AB} = -2.18 \text{ k} = \underline{2.18 \text{ k (C)}}$$

Joint B:

$$\Sigma F_z = 0 \quad \left(\frac{5}{22.91}\right)33.84 - \left(\frac{10}{11.66}\right)F_{BC} = 0$$

$$\underline{F_{BC} = 8.61 \text{ k (T)}}$$

4.54

Member	Projection			Length (m)
	x (m)	y (m)	z (m)	
AB	8	0	0	8
CD	8	0	0	8
BC	0	0	4	4
AD	0	0	4	4
BD	8	0	4	8.94
AE	4	8	2	9.17
BE	4	8	2	9.17
CE	4	8	2	9.17
DE	4	8	2	9.17

Joint C: $\Sigma F_y = 0 \qquad \left(\frac{8}{9.17}\right) F_{CE} = 0 \qquad \underline{F_{CE} = 0}$

$\Sigma F_z = 0 \qquad \underline{F_{BC} = 0}$

Joint E: $\Sigma F_x = 0 \qquad \frac{4}{9.17}\left(-F_{AE} + F_{BE} - F_{DE}\right) + 30 = 0$

$\Sigma F_y = 0 \qquad -\frac{8}{9.17}\left(F_{AE} + F_{BE} + F_{DE}\right) - 60 = 0$

$\Sigma F_z = 0 \qquad \frac{2}{9.17}\left(F_{AE} + F_{BE} - F_{DE}\right) + 40 = 0$

Solving these equations, we obtain:

$F_{AE} = -57.31 \text{ kN} = \underline{57.31 \text{ kN (C)}}$;

$F_{BE} = -68.78 \text{ kN} = \underline{68.78 \text{ kN (C)}}$ and $\underline{F_{DE} = 57.31 \text{ kN (T)}}$

Joint A:

$\Sigma F_x = 0 \qquad -\left(\frac{4}{9.17}\right) 57.31 + F_{AB} = 0 \qquad \underline{F_{AB} = 25 \text{ kN (T)}}$

$\Sigma F_z = 0 \qquad \left(\frac{2}{9.17}\right) 57.31 - F_{AD} = 0 \qquad \underline{F_{AD} = 12.5 \text{ kN (T)}}$

Joint D:

$\Sigma F_z = 0 \qquad \left(\frac{2}{9.17}\right) 57.31 + 12.5 + \left(\frac{4}{8.94}\right) F_{BD} = 0$

$F_{BD} = -55.87 \text{ kN} = \underline{55.87 \text{ kN (C)}}$

$\Sigma F_x = 0 \qquad \left(\frac{4}{9.17}\right) 57.31 - \left(\frac{8}{8.94}\right) 55.87 + F_{CD} = 0$

$\underline{F_{CD} = 25 \text{ kN (T)}}$

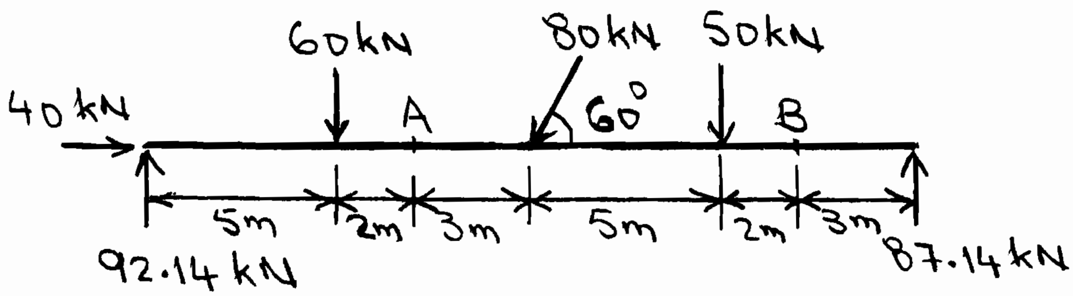
4.55

Member	Force (k)
AB	29.17 (T)
CD	15.83 (C)
BC	7.22 (T)
AD	0
AC	0.5 (T)
EF	28.33 (T)
GH	10.0 (C)
FG	5.55 (C)
EH	5.55 (C)
FH	2.0 (T)
AE	4.12 (T)
BF	8.25 (C)
CG	80.41 (C)
DH	39.17 (C)
BE	52.25 (C)
CF	8.83 (T)
DG	45.12 (T)
AH	2.21 (C)

Chapter Five
Beams and Frames: Shear and Bending Mo~~ve~~ment

CHAPTER 5

5.1



$$Q_A = \underline{-40 \text{ kN}}$$

$$S_A = 92.14 - 60 = \underline{32.14 \text{ kN}}$$

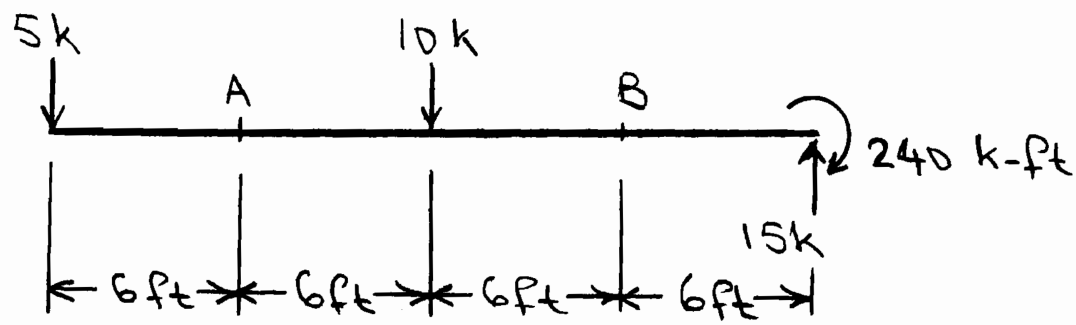
$$M_A = 92.14(7) - 60(2) = \underline{524.98 \text{ kN.m}}$$

$$Q_B = \underline{0}$$

$$S_B = \underline{-87.14 \text{ kN}}$$

$$M_B = 87.14(3) = \underline{261.42 \text{ kN.m}}$$

$\boxed{5.2}$



$Q_A = \underline{0}$

$S_A = \underline{-5\ k}$

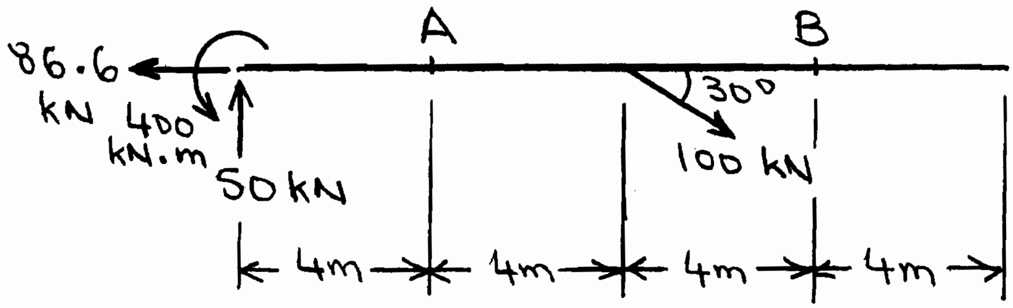
$M_A = -5(6) = \underline{-30\ k\text{-}ft}$

$Q_B = \underline{0}$

$S_B = -5 - 10 = \underline{-15\ k}$

$M_B = -5(18) - 10(6) = \underline{-150\ k\text{-}ft}$

$\boxed{5.3}$



$$Q_A = 100 \cos 30^\circ = \underline{86.6 \text{ kN}}$$

$$S_A = 100 \sin 30^\circ = \underline{50 \text{ kN}}$$

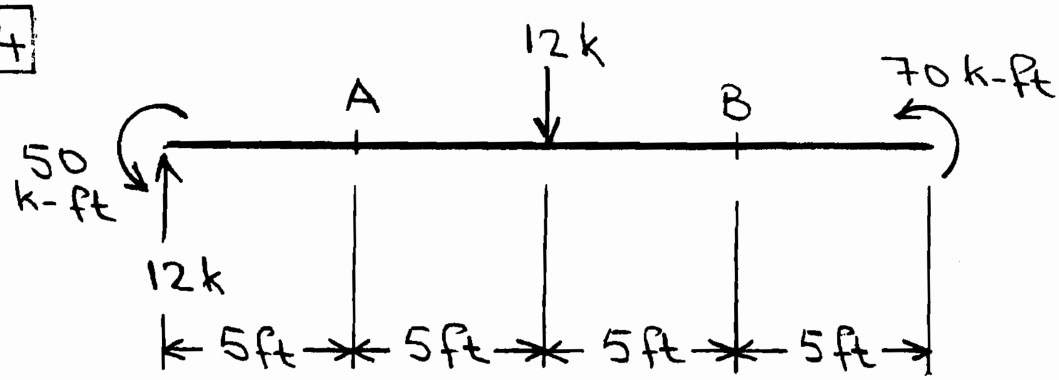
$$M_A = -100 \sin 30^\circ (4) = \underline{-200 \text{ kN.m}}$$

$$Q_B = \underline{0}$$

$$S_B = \underline{0}$$

$$M_B = \underline{0}$$

$\boxed{5.4}$



$Q_A = \underline{0}$

$S_A = \underline{12\text{ k}}$

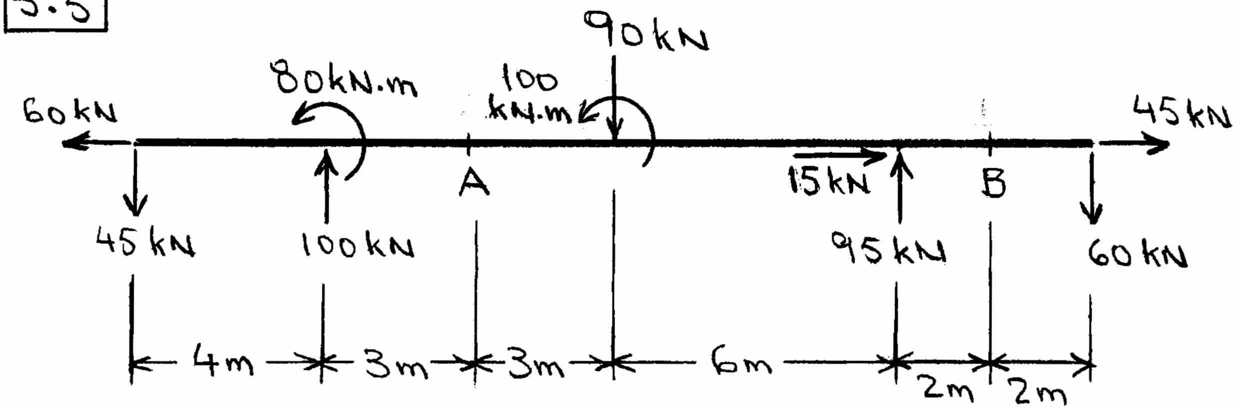
$M_A = 12(5) - 50 = \underline{10\text{ k-ft}}$

$Q_B = \underline{0}$

$S_B = \underline{0}$

$M_B = \underline{70\text{ k-ft}}$

5.5



$Q_A = \underline{60 \text{ kN}}$

$S_A = -45 + 100 = \underline{55 \text{ kN}}$

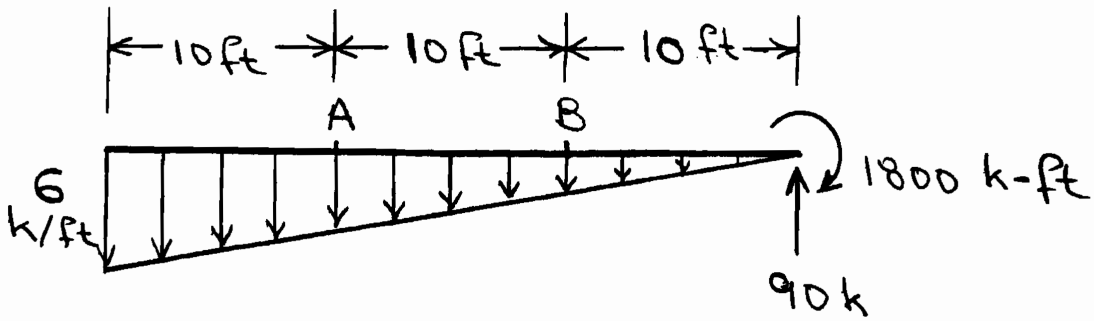
$M_A = -45(7) - 80 + 100(3) = \underline{-95 \text{ kN.m}}$

$Q_B = \underline{45 \text{ kN}}$

$S_B = \underline{60 \text{ kN}}$

$M_B = -60(2) = \underline{-120 \text{ kN.m}}$

$\boxed{5.6}$



$Q_A = \underline{0}$

$S_A = -\frac{(6+4)}{2}(10) = \underline{-50\text{ k}}$

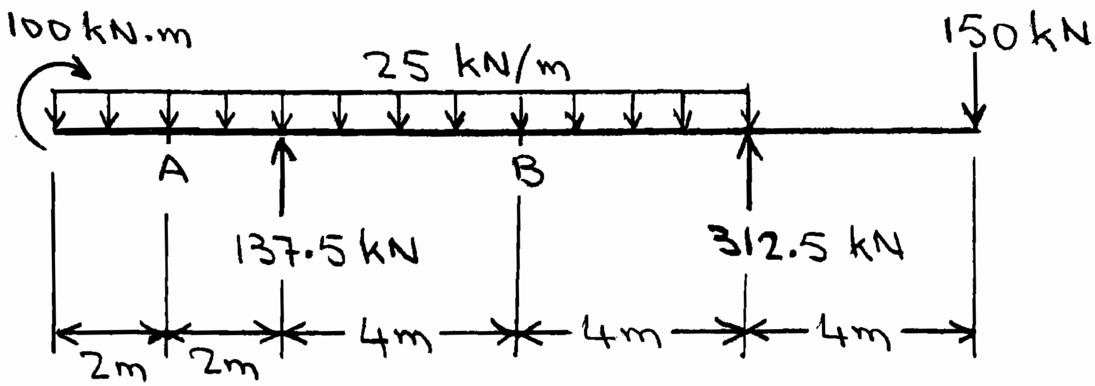
$M_A = -4(10)5 - \frac{1}{2}(2)(10)\left(\frac{20}{3}\right) = \underline{-266.67\text{ k-ft}}$

$Q_B = \underline{0}$

$S_B = -\frac{(6+2)}{2}(20) = \underline{-80\text{ k}}$

$M_B = -2(20)10 - \frac{1}{2}(4)(20)\left(\frac{40}{3}\right) = \underline{-933.33\text{ k-ft}}$

5.7



$Q_A = \underline{0}$

$S_A = -25(2) = \underline{-50 \text{ kN}}$

$M_A = 100 - 25(2)(1) = \underline{50 \text{ kN.m}}$

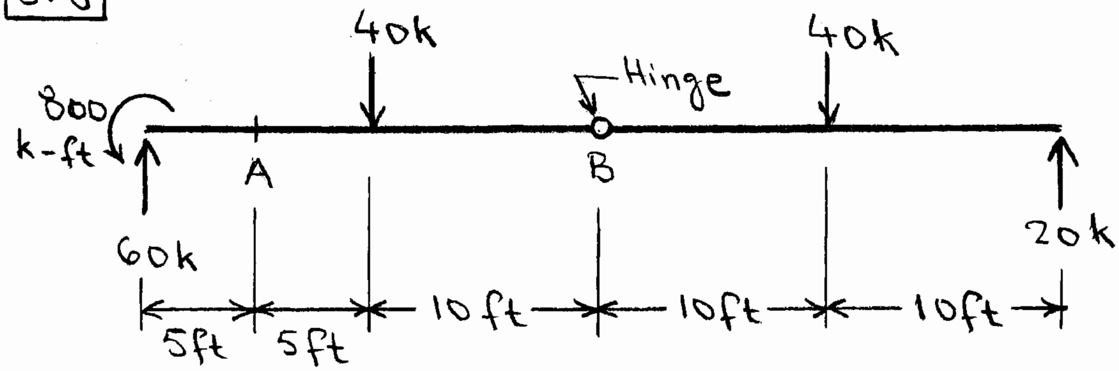
$Q_B = \underline{0}$

$S_B = 150 - 312.5 + 25(4) = \underline{-62.5 \text{ kN}}$

$M_B = -150(8) + 312.5(4) - 25(4)(2)$

$M_B = \underline{-150 \text{ kN.m}}$

$\boxed{5.8}$



$Q_A = \underline{0}$

$S_A = \underline{60\ k}$

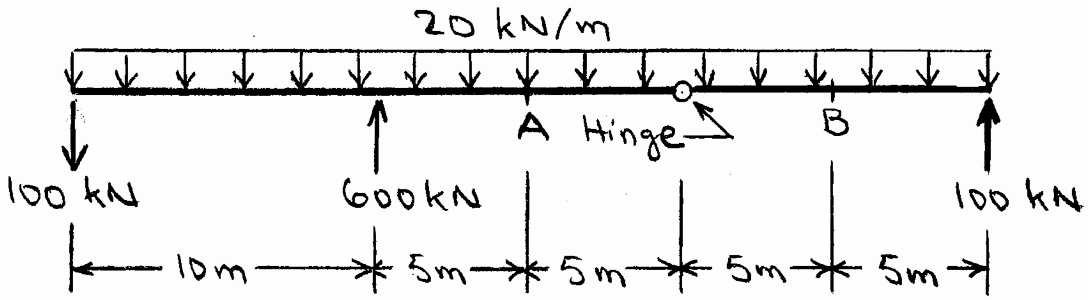
$M_A = -800 + 60(5) = \underline{-500\ k\text{-}ft}$

$Q_B = \underline{0}$

$S_B = -20 + 40 = \underline{20\ k}$

$M_B = 20(20) - 40(10) = \underline{0}$

5.9



$Q_A = \underline{0}$

$S_A = -100 - 20(15) + 600 = \underline{200 \text{ kN}}$

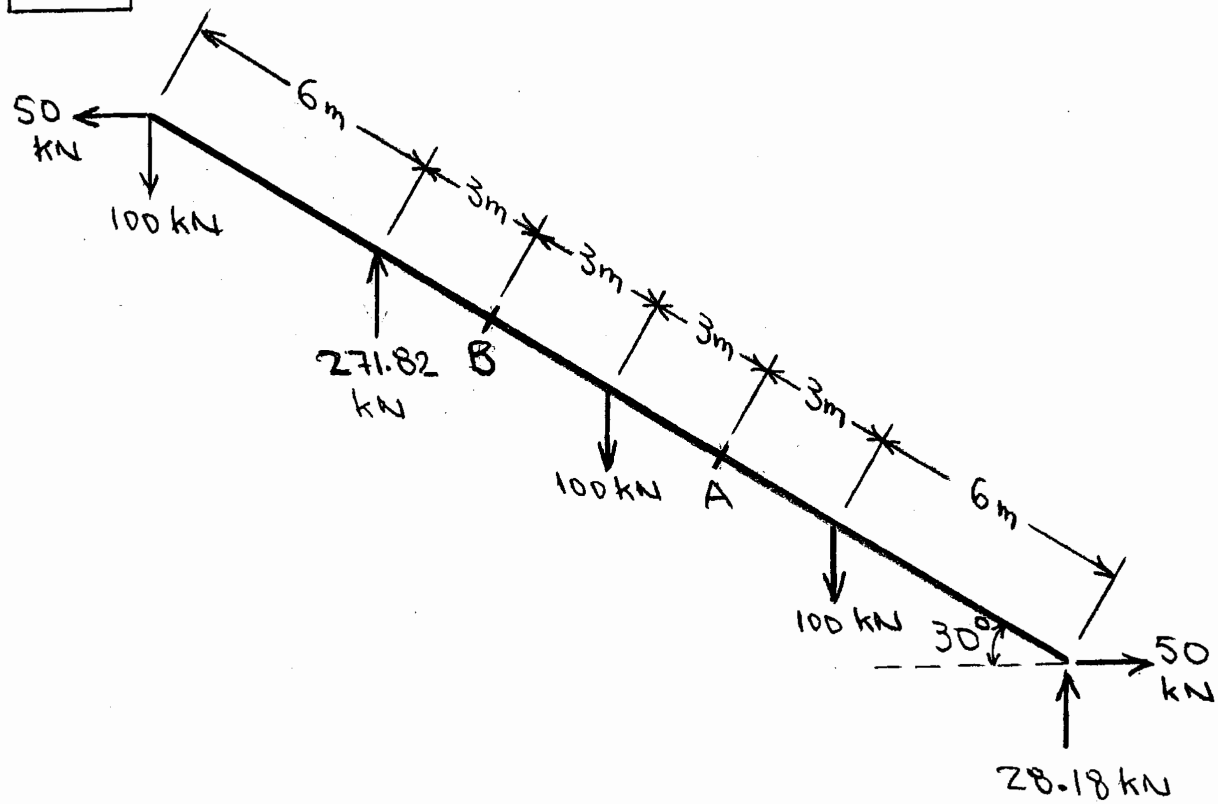
$M_A = -100(15) - 20(15)(7.5) + 600(5) = \underline{-750 \text{ kN.m}}$

$Q_B = \underline{0}$

$S_B = -100 + 20(5) = \underline{0}$

$M_B = 100(5) - 20(5)(2.5) = \underline{250 \text{ kN.m}}$

5.10



$$Q_A = 50(\cos 30°) - 28.18(\sin 30°) + 100(\sin 30°) = \underline{79.2 \text{ kN}}$$

$$S_A = -50(\sin 30°) - 28.18(\cos 30°) + 100(\cos 30°) = \underline{37.2 \text{ kN}}$$

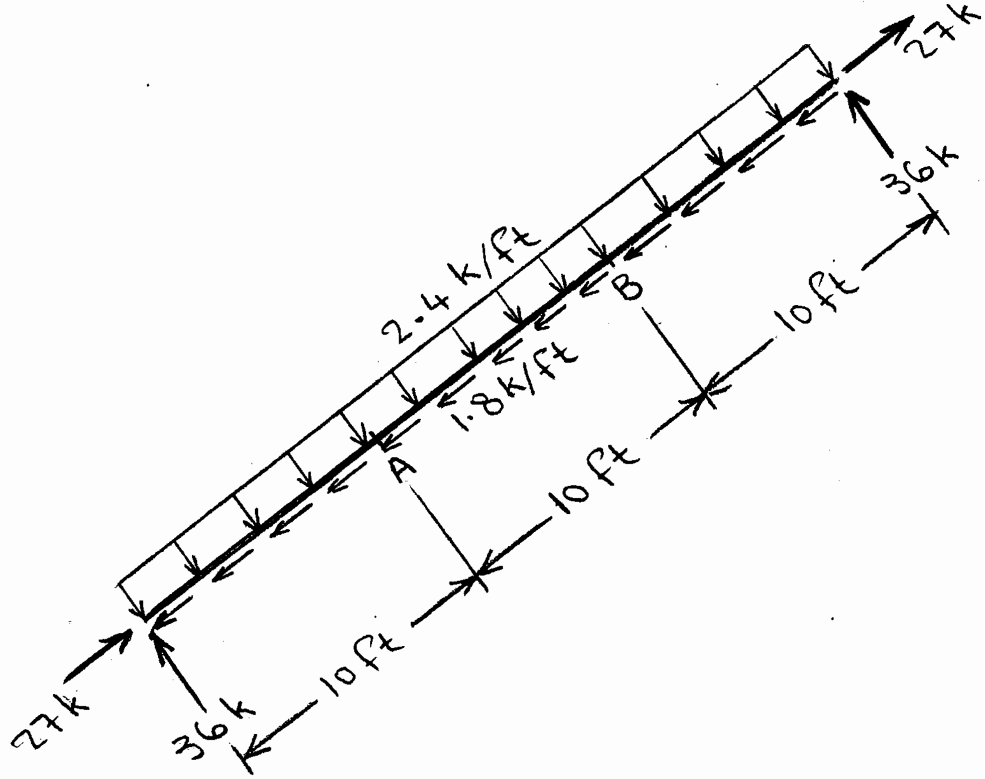
$$M_A = 50(\sin 30°)(9) + 28.18(\cos 30°)(9) - 100(\cos 30°)(3)$$
$$= \underline{184.8 \text{ kN.m}}$$

$$Q_B = 50(\cos 30°) - 100(\sin 30°) + 271.82(\sin 30°) = \underline{129.2 \text{ kN}}$$

$$S_B = -50(\sin 30°) - 100(\cos 30°) + 271.82(\cos 30°) = \underline{123.8 \text{ kN}}$$

$$M_B = -50(\sin 30°)(9) - 100(\cos 30°)(9) + 271.82(\cos 30°)(3)$$
$$= \underline{-298.2 \text{ kN.m}}$$

5.11



$Q_A = -27 + 1.8(10) = \underline{-9\,k}$

$S_A = 36 - 2.4(10) = \underline{12\,k}$

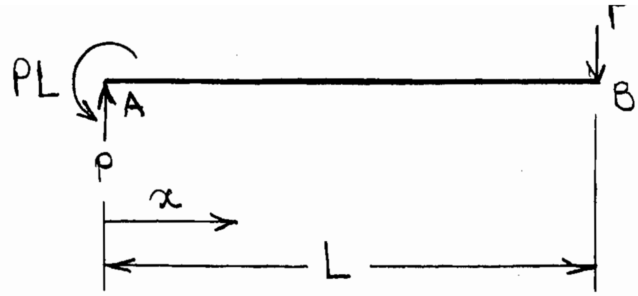
$M_A = 36(10) - 2.4(10)(5) = \underline{240\text{ k-ft}}$

$Q_B = 27 - 1.8(10) = \underline{9\,k}$

$S_B = -36 + 2.4(10) = \underline{-12\,k}$

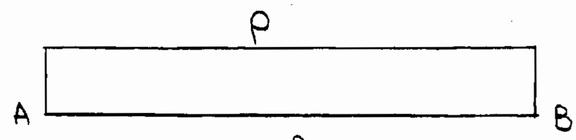
$M_B = 36(10) - 2.4(10)(5) = \underline{240\text{ k-ft}}$

$\boxed{5.12}$

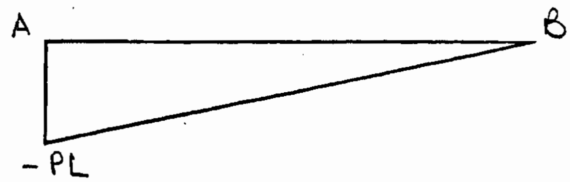


$S = \underline{P}$ for $0 < x < L$

$M = -PL + Px = \underline{P(x-L)}$ for $0 < x \leq L$

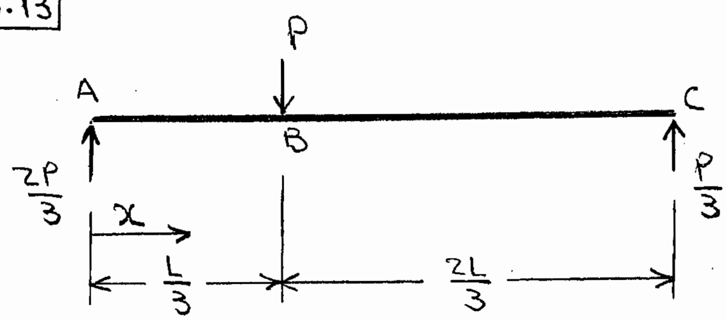


Shear Diagram



Bending Moment Diagram

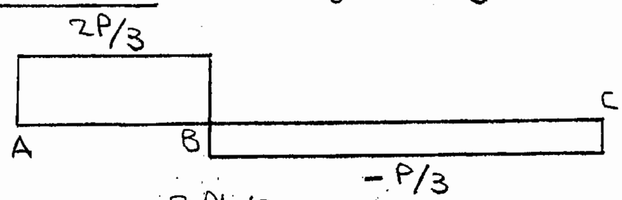
$\boxed{5.13}$



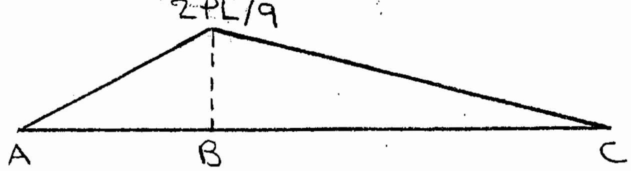
$\underline{0 < x < \frac{L}{3}:}$ $\quad S = \frac{2P}{3}$ $\quad M = \frac{2P}{3}x$

$\underline{\frac{L}{3} < x < L:}$ $\quad S = \frac{2P}{3} - P = -\frac{P}{3}$ $\quad M = \frac{2P}{3}x - P\left(x - \frac{L}{3}\right)$

$$= \frac{P}{3}(L - x)$$

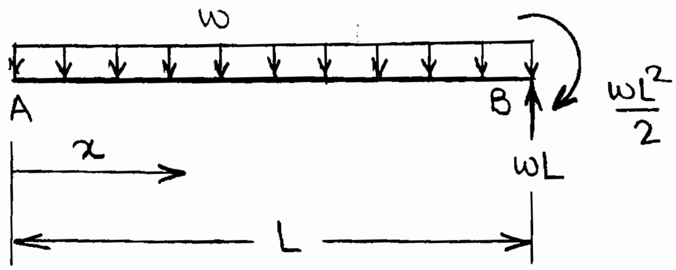


Shear Diagram



Bending Moment Diagram

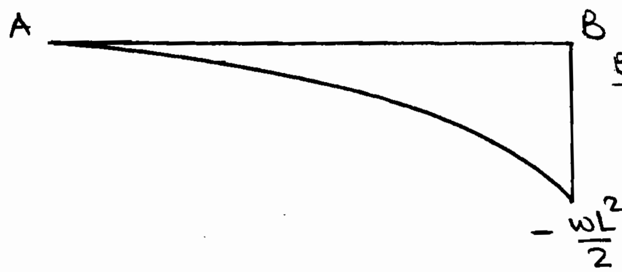
5.14



$$S = -wx \quad \text{for} \quad 0 \leqslant x < L$$

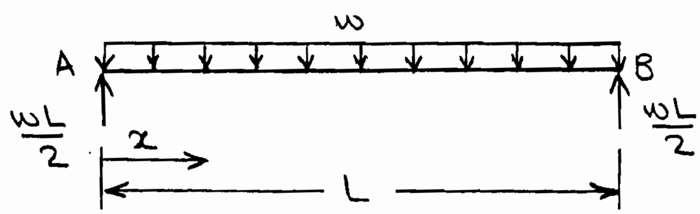
$$M = -\frac{wx^2}{2} \quad \text{for} \quad 0 \leqslant x \leqslant L$$

Shear Diagram



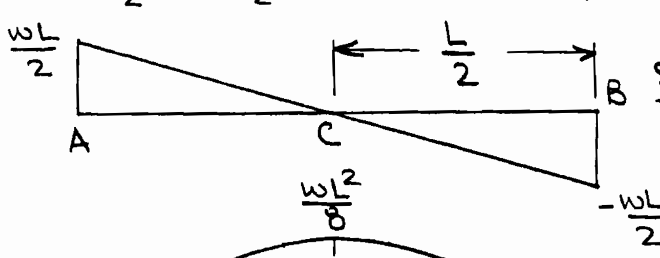
Bending Moment Diagram

5.15

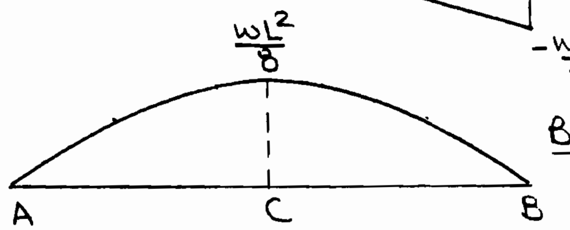


$$S = \frac{wL}{2} - wx = w\left(\frac{L}{2} - x\right) \quad \text{for} \quad 0 < x < L$$

$$M = \frac{wL}{2}x - \frac{wx^2}{2} = \frac{wx}{2}(L - x) \quad \text{for} \quad 0 \leqslant x \leqslant L$$

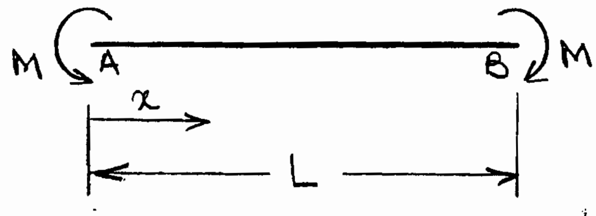


Shear Diagram



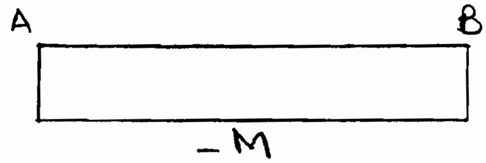
Bending Moment Diagram

5.16



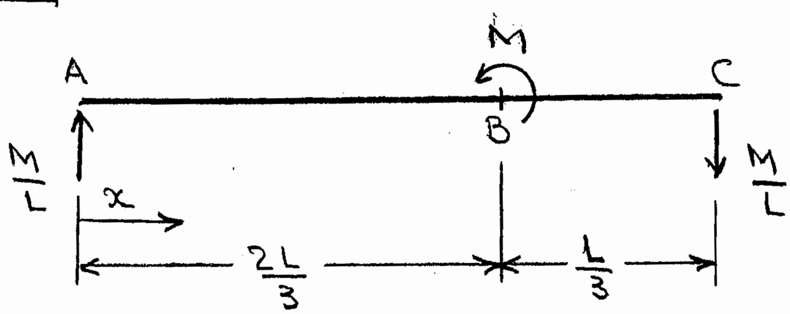
$S = \underline{0}$

$M_x = \underline{-M}$ for $0 < x < L$



Bending Moment Diagram

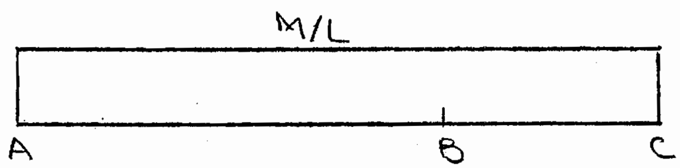
5.17



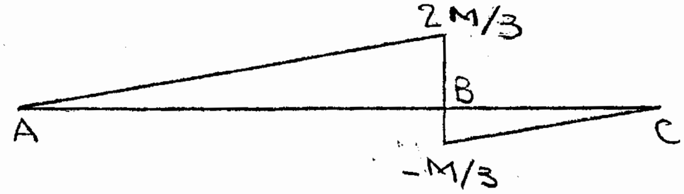
$S = \frac{M}{L}$ for $0 < x < L$

$B.M. = \frac{Mx}{L}$ for $0 \leq x < \frac{2L}{3}$

$B.M. = \frac{Mx}{L} - M = \frac{M}{L}(x - L)$ for $\frac{2L}{3} < x \leq L$

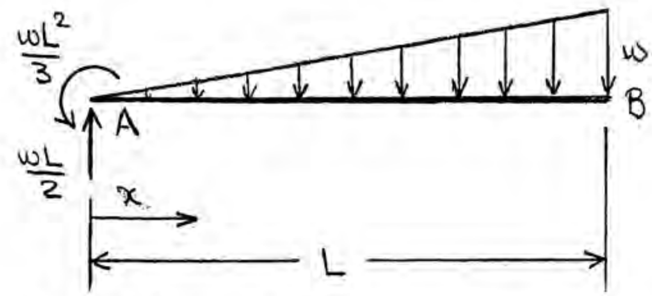


Shear Diagram



Bending Moment Diagram

5.18

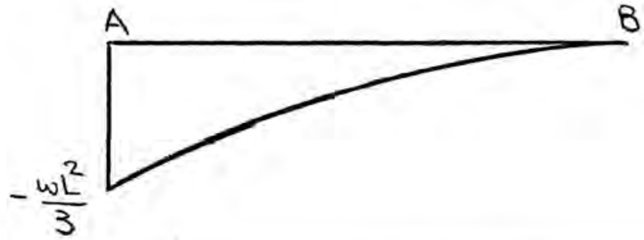


$$S = \frac{wL}{2} - \frac{1}{2}(x)\left(\frac{wx}{L}\right) = \frac{wL}{2}\left[1 - \left(\frac{x}{L}\right)^2\right] \qquad 0 < x \leq L$$

$$M = -\frac{wL^2}{3} + \left(\frac{wL}{2}\right)x - \left(\frac{wx^2}{2L}\right)\left(\frac{x}{3}\right) = -\frac{wL^2}{3}\left[1 - \frac{3}{2}\left(\frac{x}{L}\right) + \frac{1}{2}\left(\frac{x}{L}\right)^3\right]$$

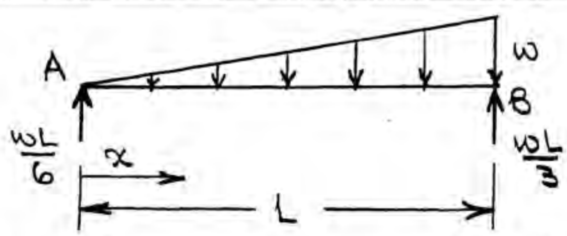
$$0 < x \leq L$$

Shear Diagram



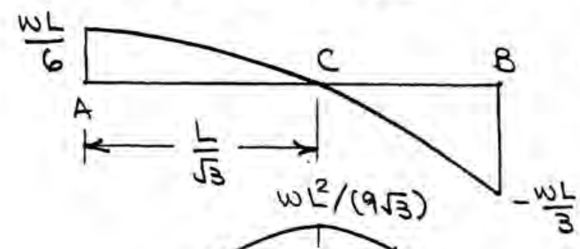
Bending Moment Diagram

5.19

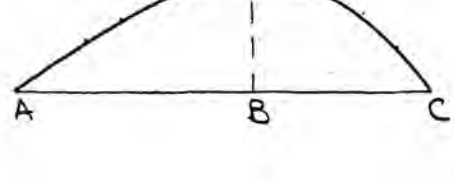


$$S = \frac{wL}{6} - \frac{1}{2}\left(\frac{wx}{L}\right)x = \frac{w}{6L}(L^2 - 3x^2) \quad \text{for } 0 < x < L$$

$$M = \frac{wL}{6}x - \frac{wx^2}{2L}\left(\frac{x}{3}\right) = \frac{wx}{6L}(L^2 - x^2) \quad \text{for } 0 \leq x \leq L$$

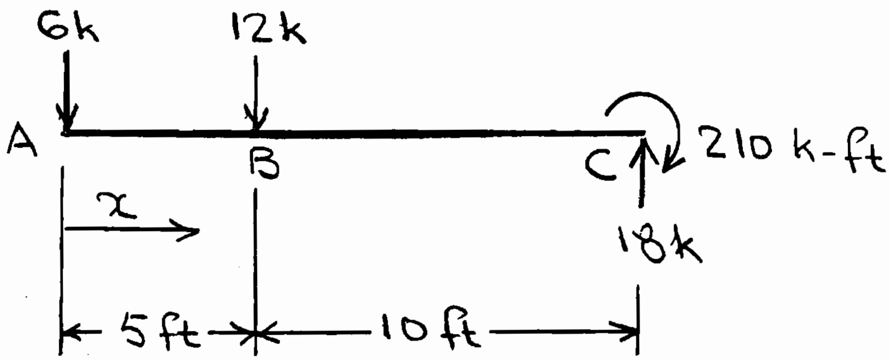


Shear Diagram



Bending Moment Diagram

$\boxed{5.20}$

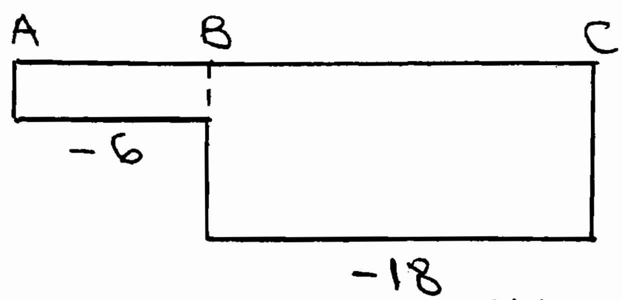


$\underline{0 < x < 5':}$ $\quad S = -6$

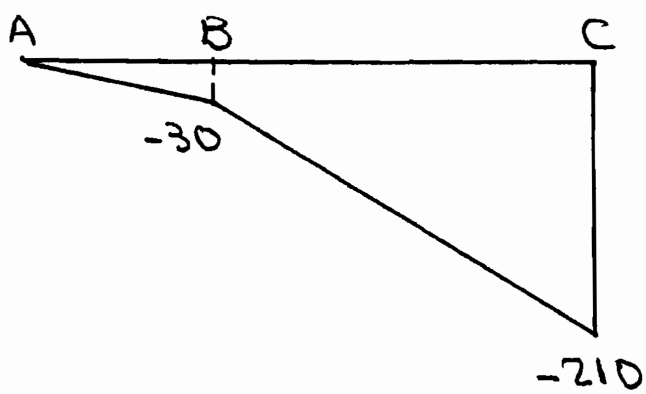
$$M = -6x$$

$\underline{5' < x < 15':}$ $\quad S = -6 - 12 = -18$

$$M = -6x - 12(x-5) = 60 - 18x$$

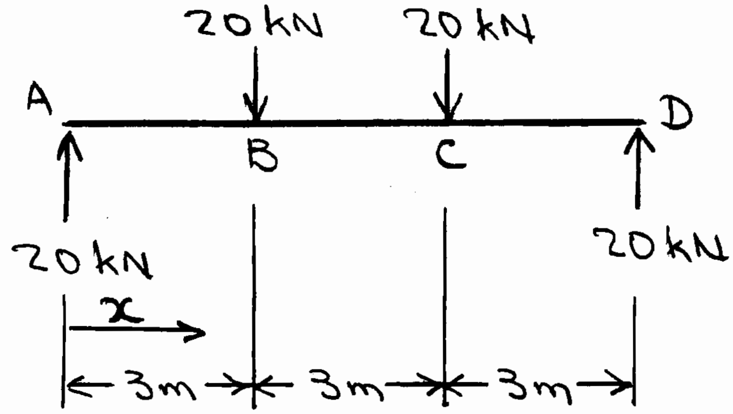


Shear Diagram (k)



Bending Moment Diagram (k-ft)

5.21



$0 < x < 3m$:

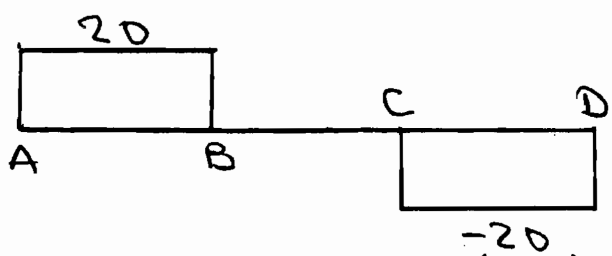
$$S = 20$$
$$M = 20x$$

$3m < x < 6m$:

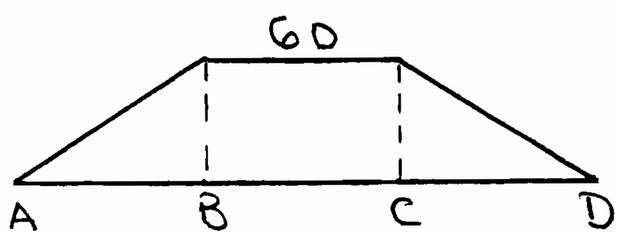
$$S = 20 - 20 = 0$$
$$M = 20x - 20(x-3) = 60$$

$6m < x < 9m$:

$$S = 20 - 20 - 20 = -20$$
$$M = 20x - 20(x-3) - 20(x-6)$$
$$= 180 - 20x$$

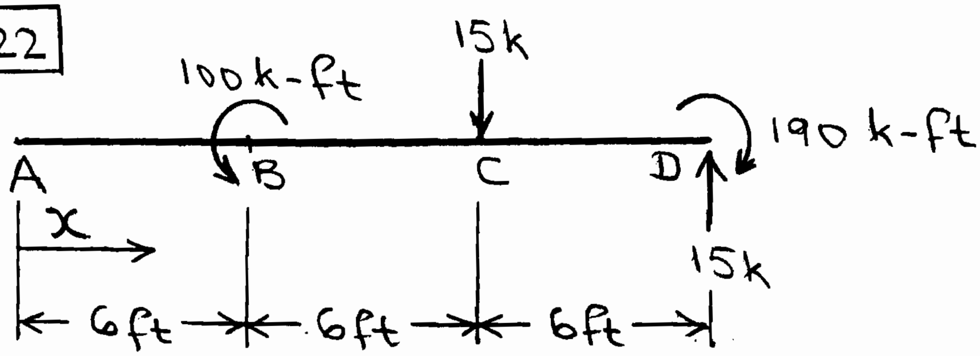


Shear Diagram (kN)



Bending Moment Diagram (kN.m)

5.22



$\underline{0 \leqslant x < 6':}$ $\quad S = 0$

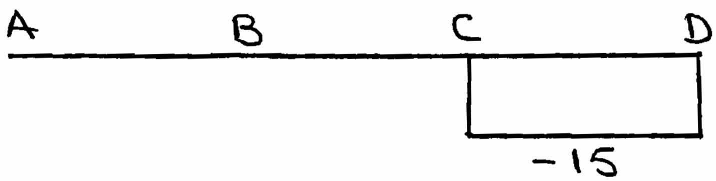
$\quad M = 0$

$\underline{6' < x < 12':}$ $\quad S = 0$

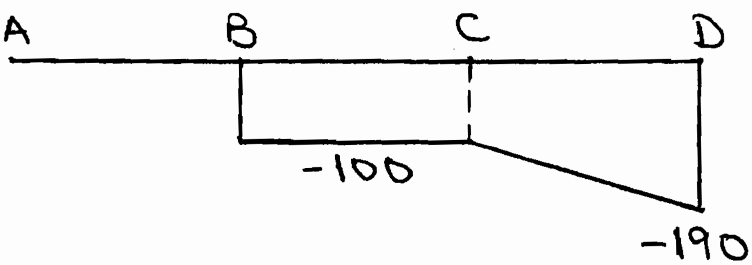
$\quad M = -100$

$\underline{12' < x < 18':}$ $\quad S = -15$

$\quad M = -100 - 15(x-12) = 80 - 15x$

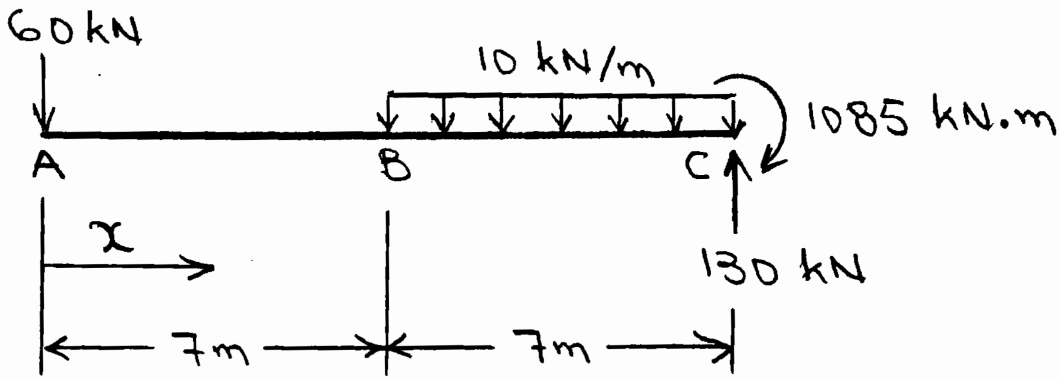


Shear Diagram (k)



Bending Moment Diagram (k-ft)

5.23

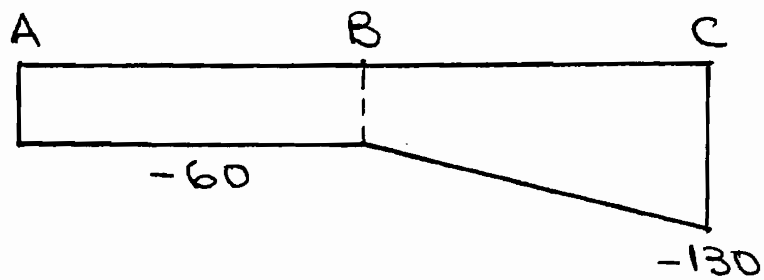


$0 < x \leqslant 7\text{m}$:

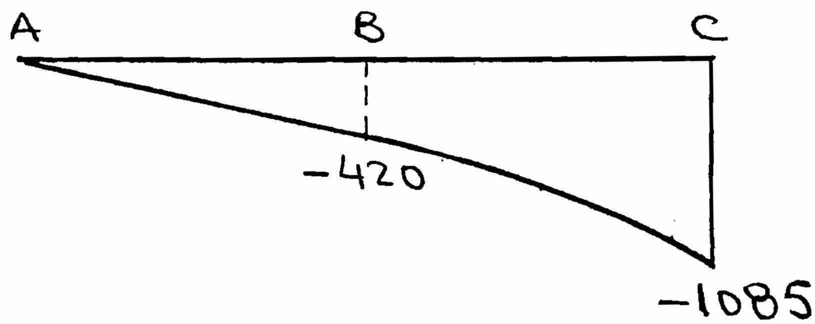
$$S = -60$$
$$M = -60x$$

$7\text{m} \leqslant x < 14\text{m}$:

$$S = -60 - 10(x-7) = 10(1-x)$$
$$M = -60x - \frac{10(x-7)^2}{2}$$
$$= -5x^2 + 10x - 245$$

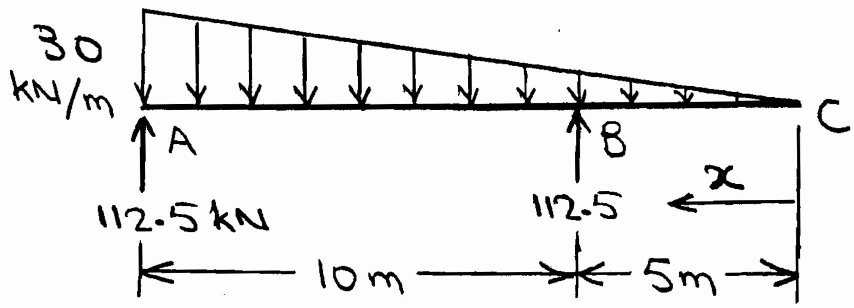


Shear Diagram (kN)



Bending Moment Diagram (kN.m)

5.24



$0 \leq x < 5\text{m}$:

$$S = \frac{1}{2}(2x)x = x^2$$

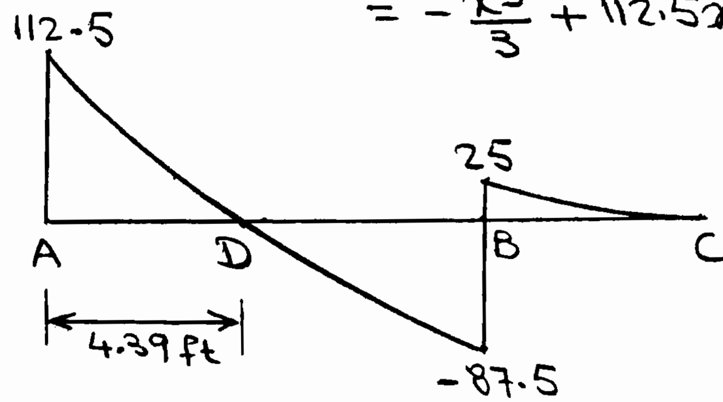
$$M = -\frac{1}{2}(2x)x\left(\frac{x}{3}\right) = -\frac{x^3}{3}$$

$5\text{m} < x < 10\text{m}$:

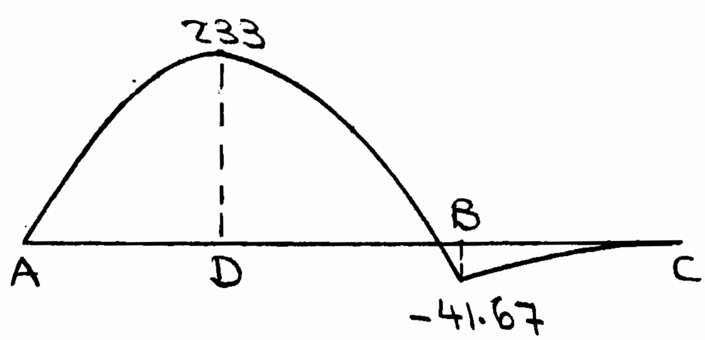
$$S = x^2 - 112.5$$

$$M = -\frac{x^3}{3} + 112.5(x-5)$$

$$= -\frac{x^3}{3} + 112.5x - 562.5$$

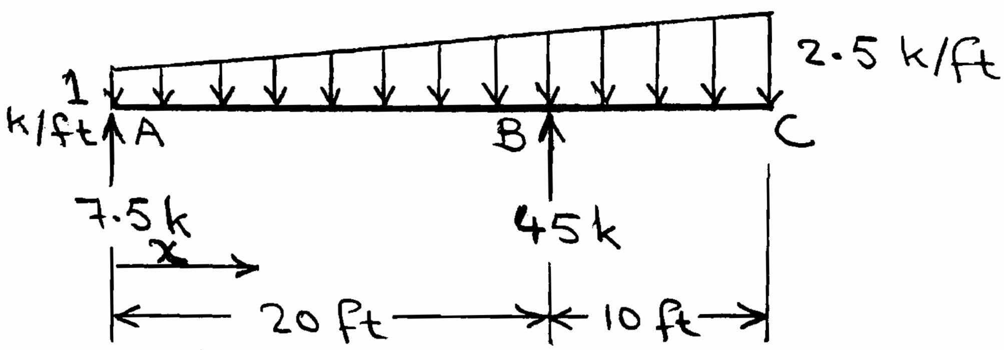


Shear Diagram (kN)



Bending Moment Diagram (kN.m)

$\boxed{5.25}$

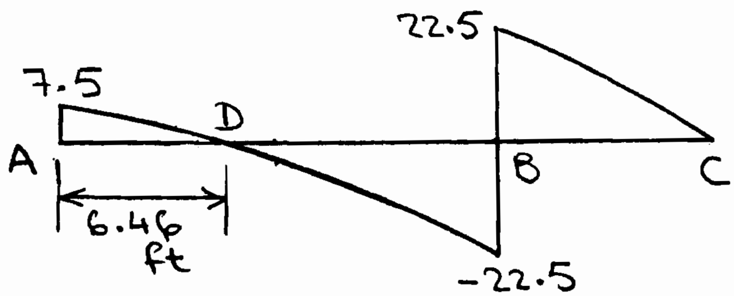


$\underline{0 < x < 20':}$

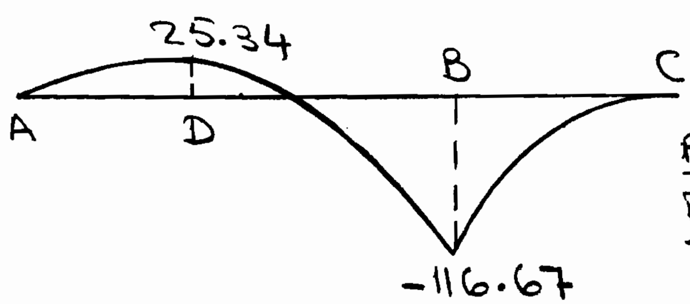
$$S = 7.5 - 1(x) - \frac{1}{2}\left(\frac{x}{20}\right)x$$
$$= -\frac{x^2}{40} - x + 7.5$$
$$M = 7.5x - 1\left(\frac{x^2}{2}\right) - \frac{1}{2}\left(\frac{x}{20}\right)x\left(\frac{x}{3}\right)$$
$$= -\frac{x^3}{120} - \frac{x^2}{2} + 7.5x$$

$\underline{20' < x < 30':}$

$$S = 7.5 - 1(x) - \frac{1}{2}\left(\frac{x}{20}\right)x + 45$$
$$= -\frac{x^2}{40} - x + 52.5$$
$$M = 7.5x - 1\left(\frac{x^2}{2}\right) - \frac{x^3}{120} + 45(x-20)$$
$$= -\frac{x^3}{120} - \frac{x^2}{2} + 52.5x - 900$$

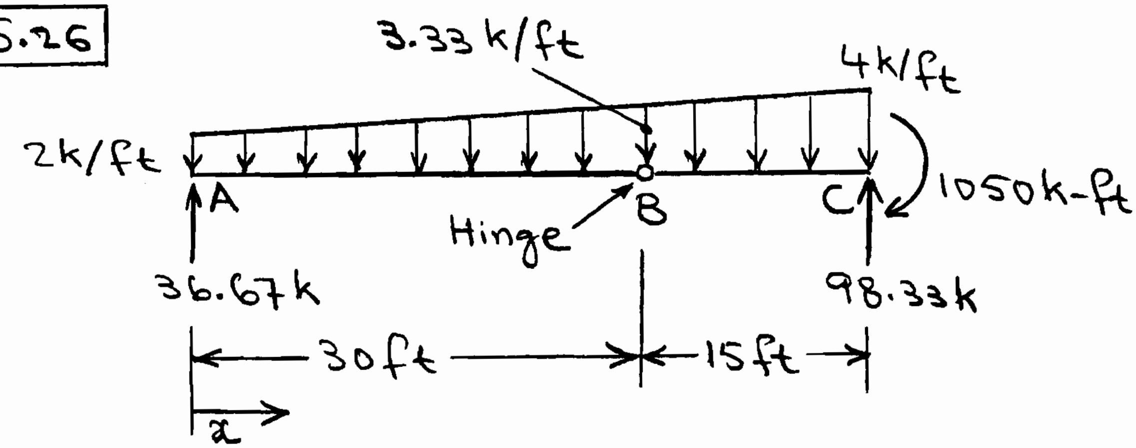


Shear Diagram (k)



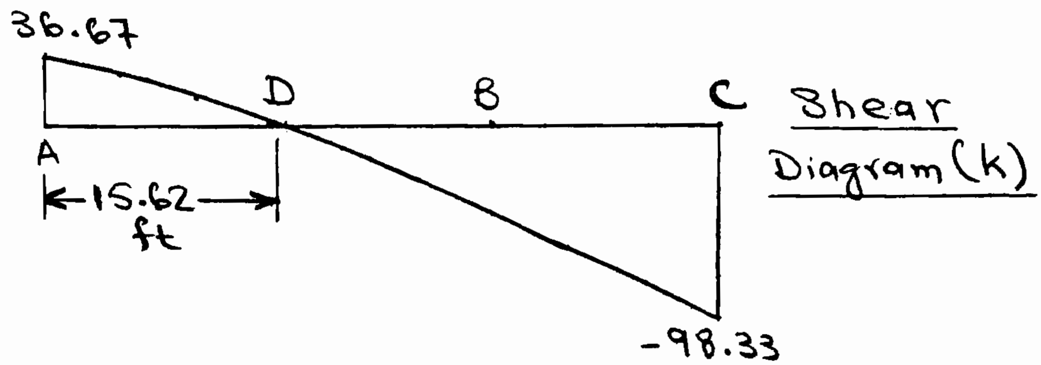
Bending Moment Diagram (k-ft)

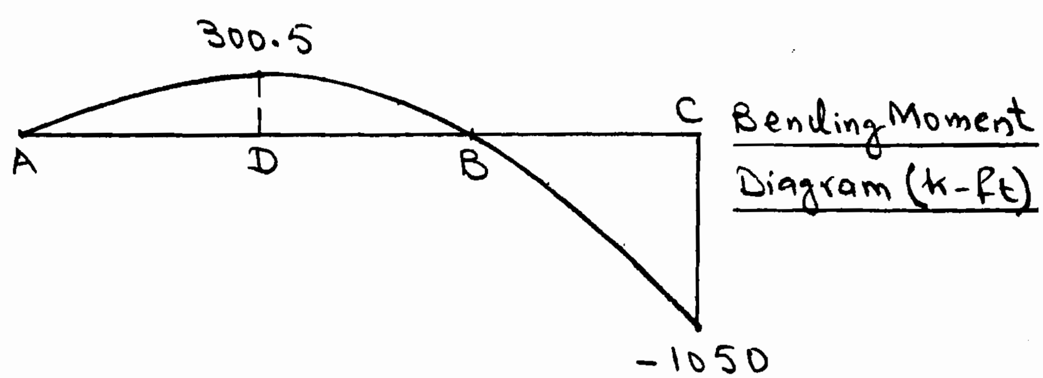
5.26



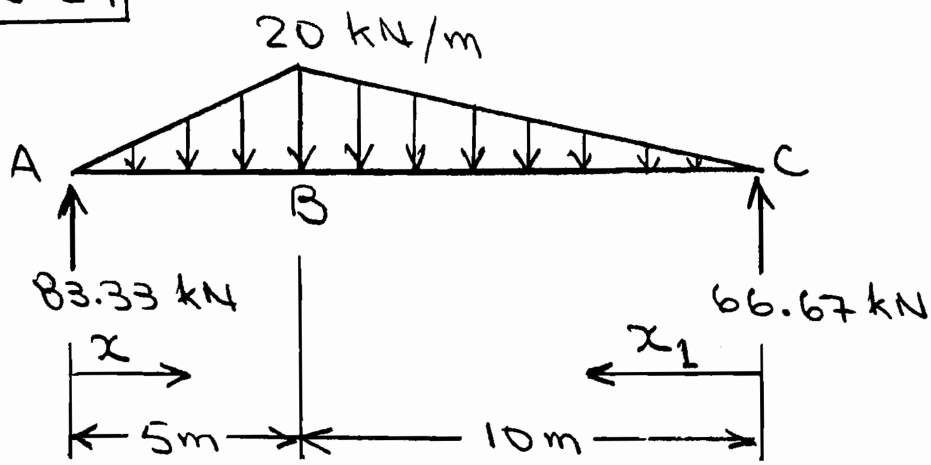
$\underline{0 \leqslant x \leqslant 45'}$:

$$S = 36.67 - 2x - \frac{1}{2}\left(\frac{2}{45}x\right)x$$
$$= 36.67 - 2x - \frac{x^2}{45}$$
$$M = 36.67x - 2(x)\frac{x}{2} - \frac{1}{2}\left(\frac{2}{45}x\right)x\left(\frac{x}{3}\right)$$
$$= 36.67x - x^2 - \frac{x^3}{135}$$





5.27



$0 < x \leq 5m$:

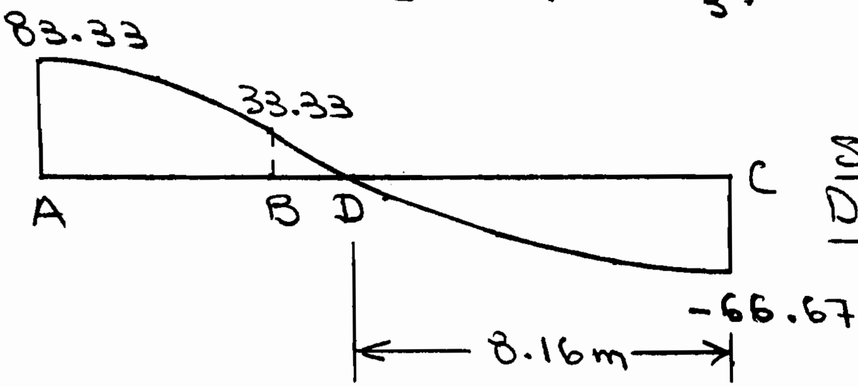
$$S = 83.33 - \frac{1}{2}\left(\frac{20x}{5}\right)x = -2x^2 + 83.33$$

$$M = 83.33x - \frac{1}{2}\left(\frac{20x}{5}\right)x\left(\frac{x}{3}\right) = -\frac{2x^3}{3} + 83.33x$$

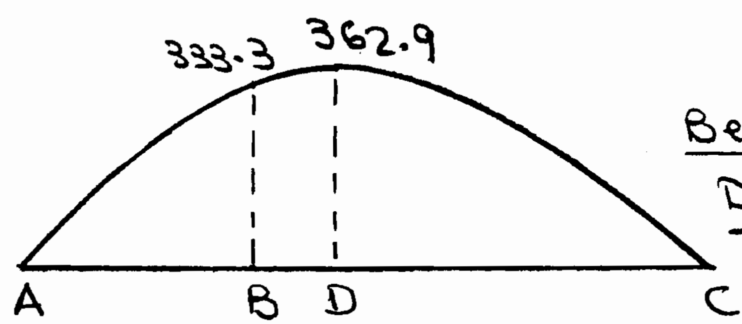
$0 < x_1 \leq 10\,m$:

$$S = -66.67 + \frac{1}{2}(2x_1)x_1 = x_1^2 - 66.67$$

$$M = 66.67x_1 - \frac{1}{2}(2x_1)x_1\left(\frac{x_1}{3}\right) = -\frac{x_1^3}{3} + 66.67x_1$$

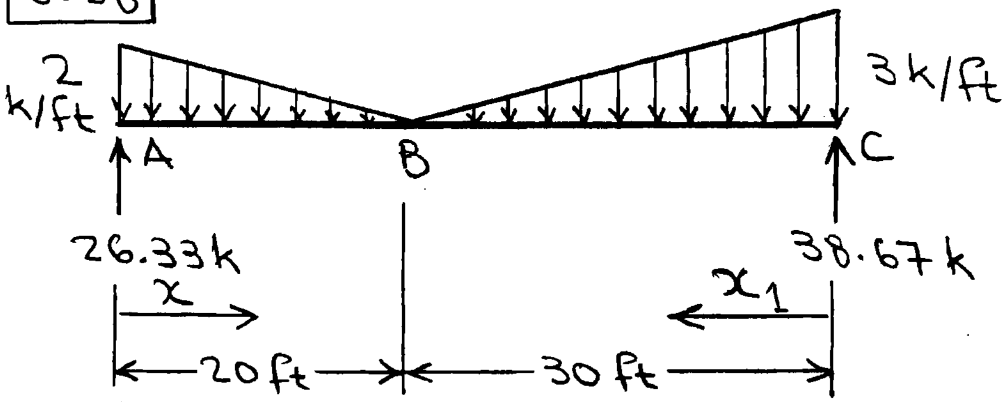


Shear Diagram (kN)



Bending Moment Diagram (kN.m)

5.28



<u>$0 < x \leqslant 20'$:</u>

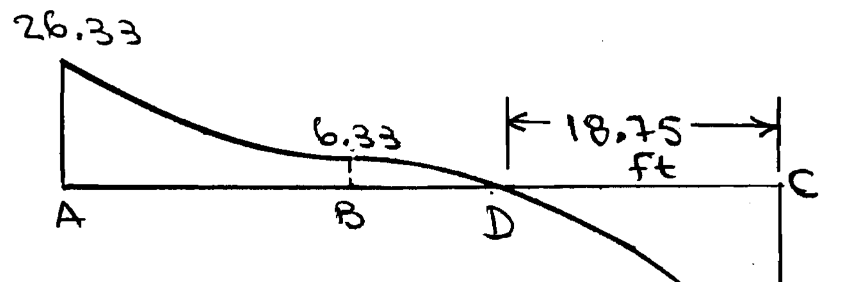
$$S = 26.33 - 2x + \frac{1}{2}\left(\frac{x}{10}\right)x = \frac{x^2}{20} - 2x + 26.33$$

$$M = 26.33x - x^2 + \frac{1}{2}\left(\frac{x}{10}\right)x\left(\frac{x}{3}\right) = \frac{x^3}{60} - x^2 + 26.33x$$

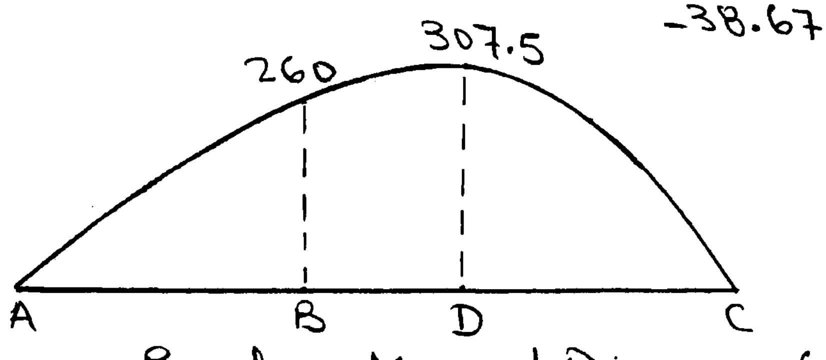
<u>$0 < x_1 \leqslant 30'$:</u>

$$S = -38.67 + 3x_1 - \frac{1}{2}\left(\frac{x_1}{10}\right)x_1 = -\frac{x_1^2}{20} + 3x_1 - 38.67$$

$$M = 38.67x_1 - 3\frac{x_1^2}{2} + \frac{1}{2}\left(\frac{x_1}{10}\right)x_1\left(\frac{x_1}{3}\right) = \frac{x_1^3}{60} - \frac{3x_1^2}{2} + 38.67x_1$$

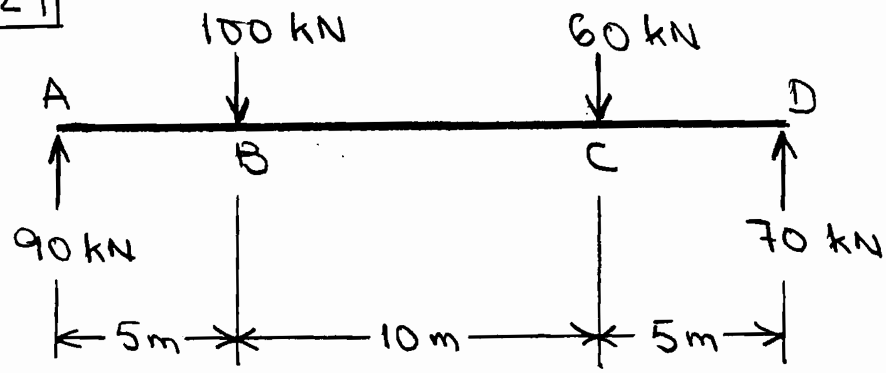


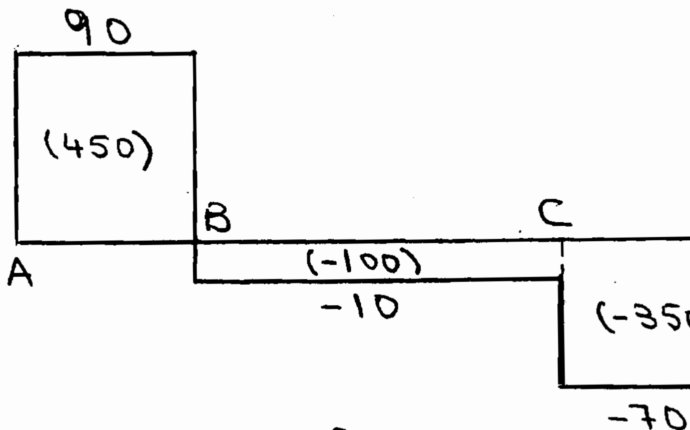
<u>Shear Diagram (k)</u>



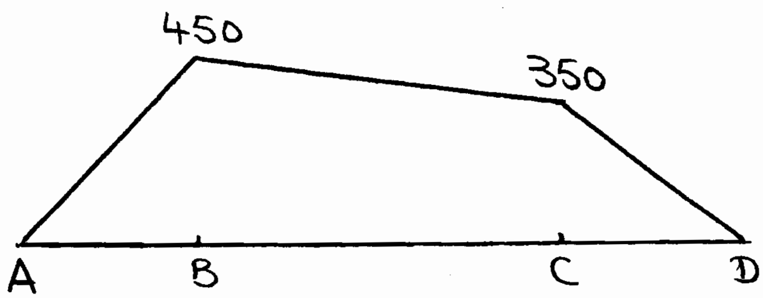
<u>Bending Moment Diagram (k-ft)</u>

5.29





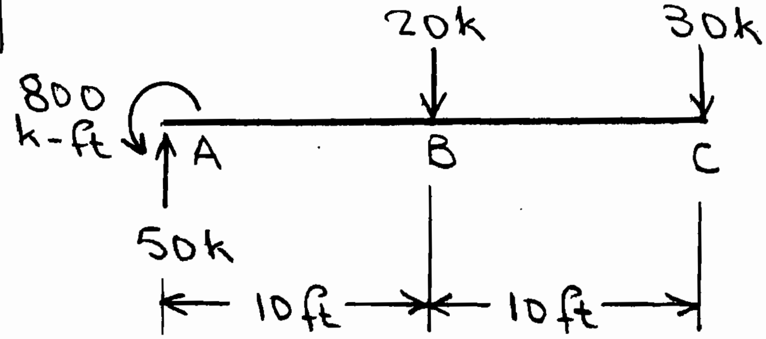
Shear Diagram (kN)

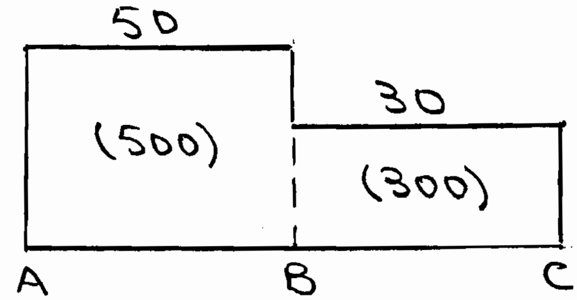


Bending Moment Diagram (kN.m)

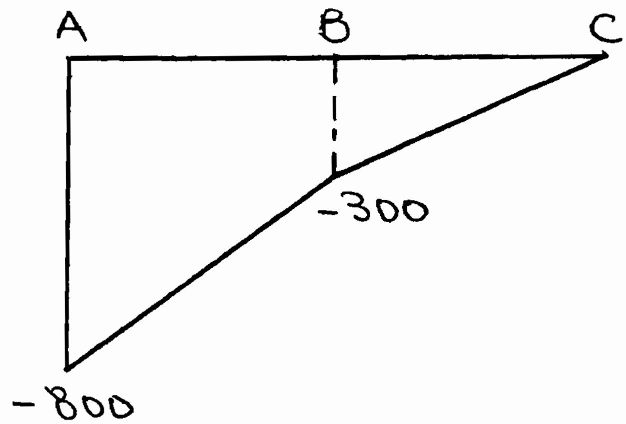
Qualitative Deflected Shape

5.30





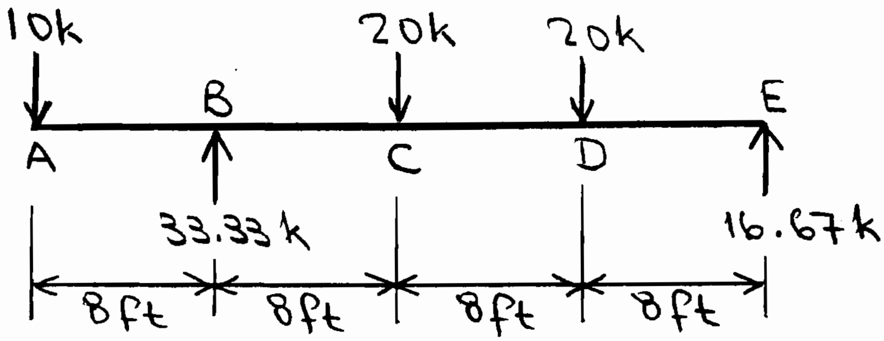
Shear Diagram (k)

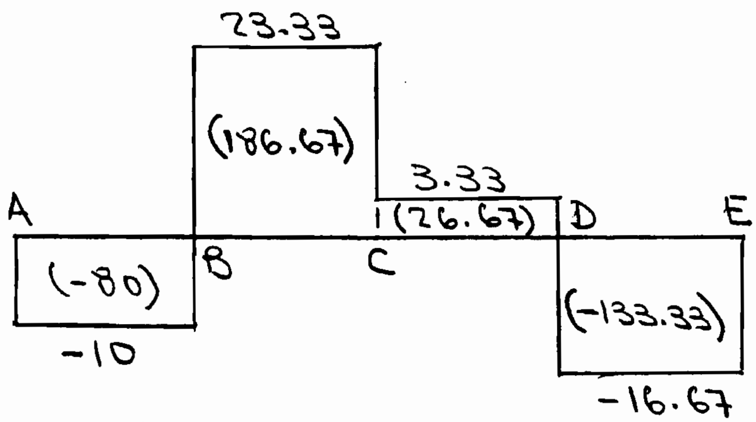


Bending Moment Diagram (k-ft)

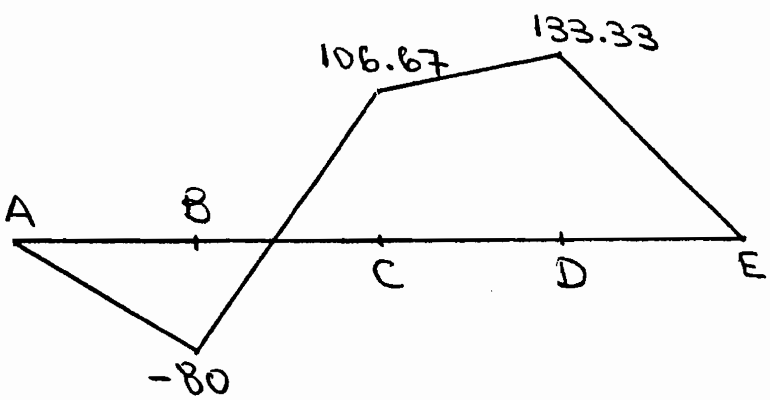
Qualitative Deflected Shape

5.31





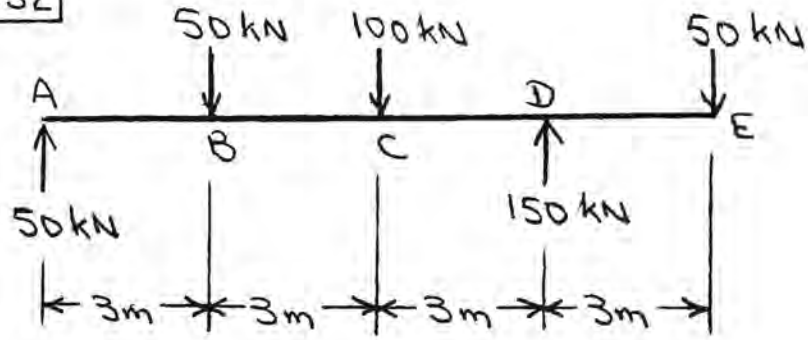
Shear Diagram (k)

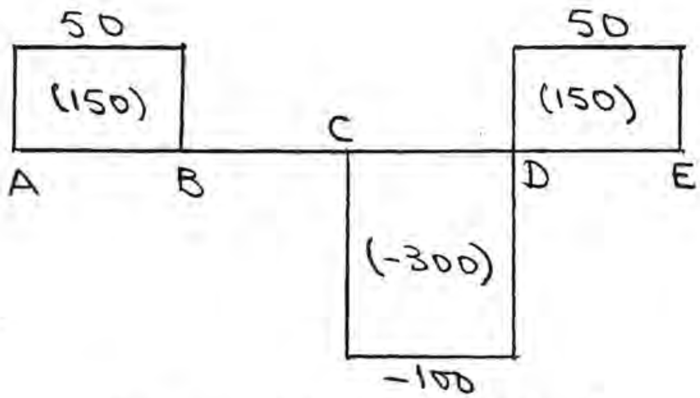


Bending Moment Diagram (k-ft)

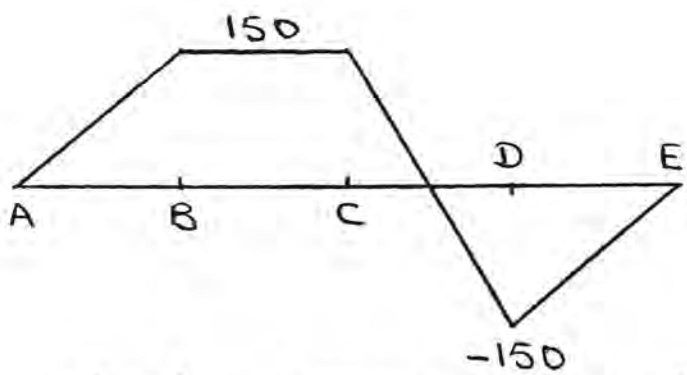
Qualitative Deflected Shape

5.32

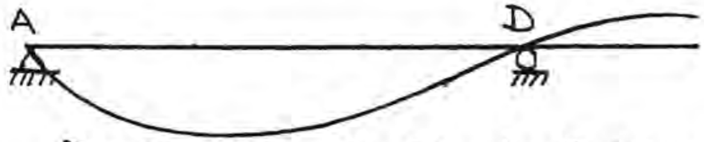




Shear Diagram (kN)

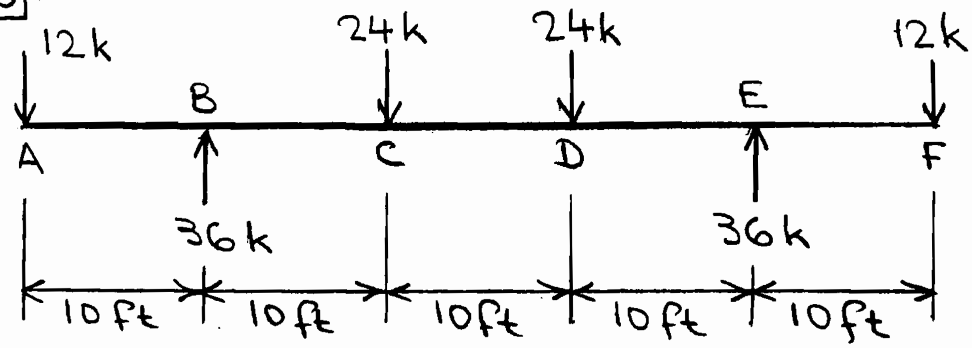


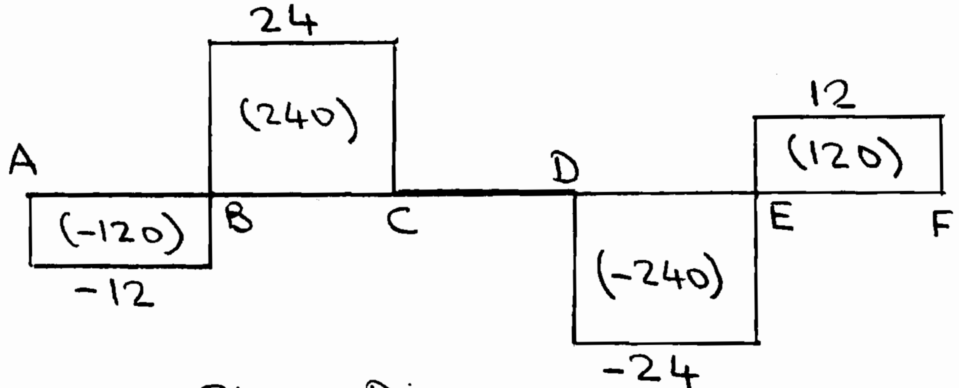
Bending Moment Diagram (kN.m)



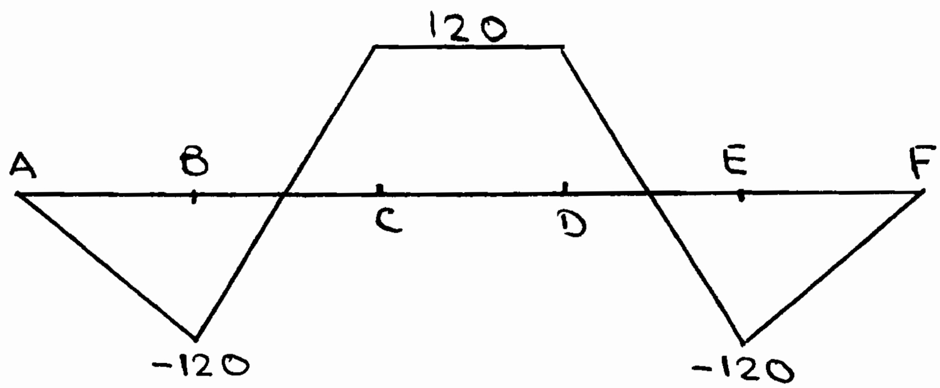
Qualitative Deflected Shape

5.33

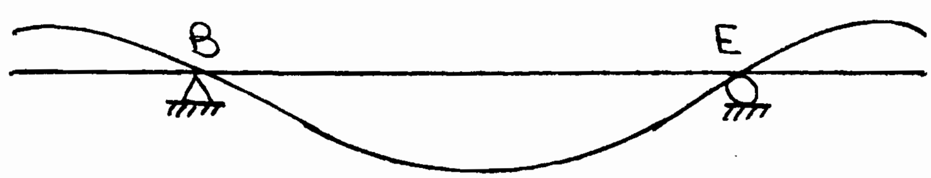




Shear Diagram (k)

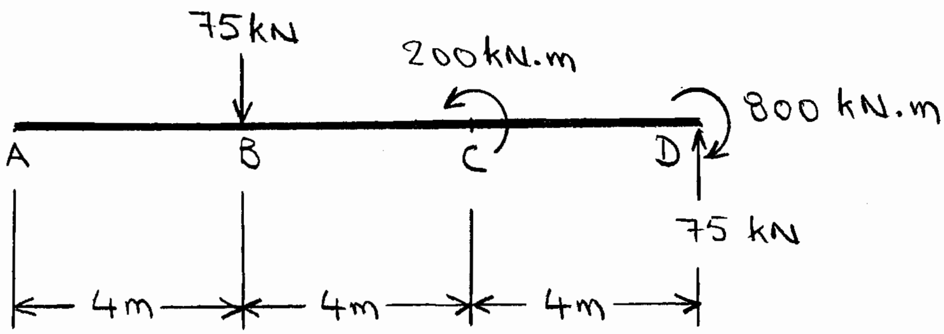


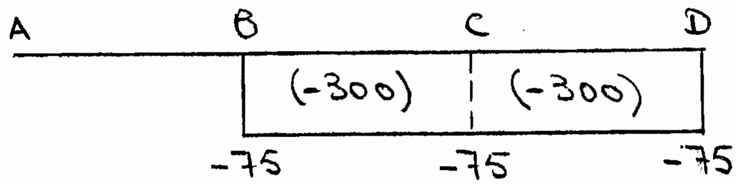
Bending Moment Diagram (k-ft)



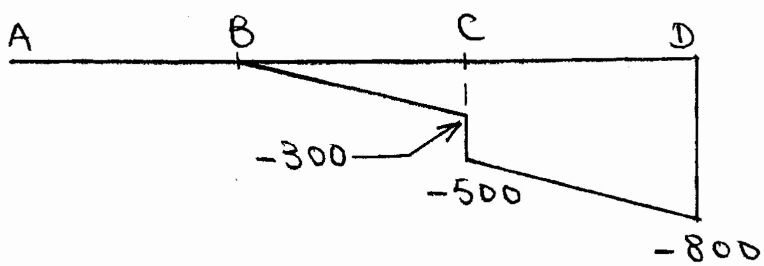
Qualitative Deflected Shape

5.34

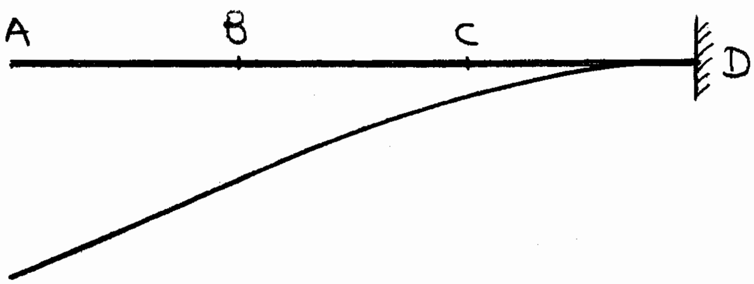




Shear Diagram (kN)

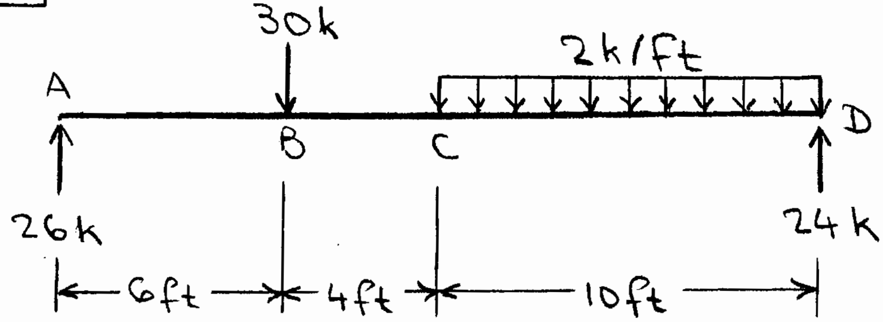


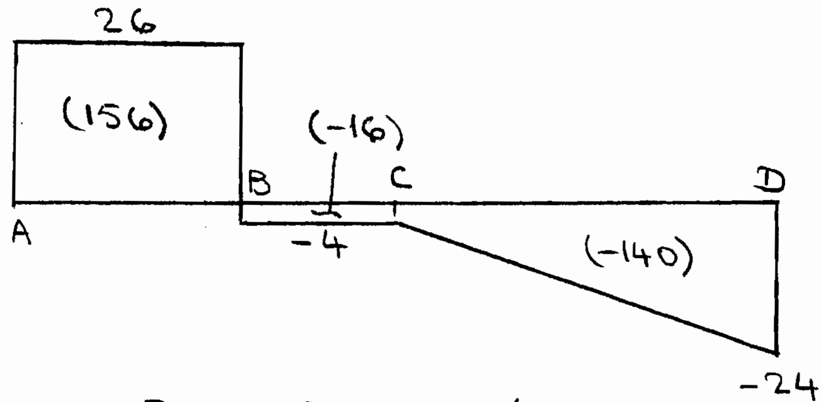
Bending Moment Diagram (kN.m)



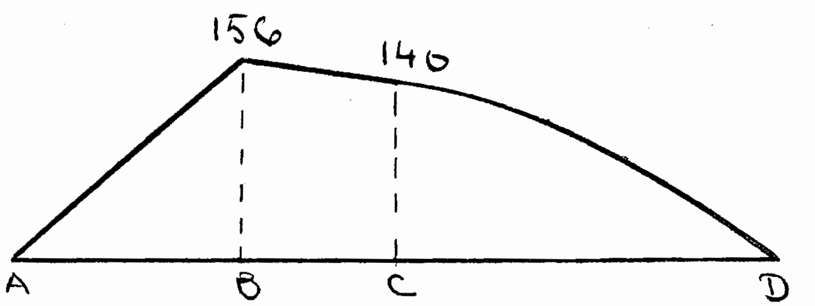
Qualitative Deflected Shape

5.35





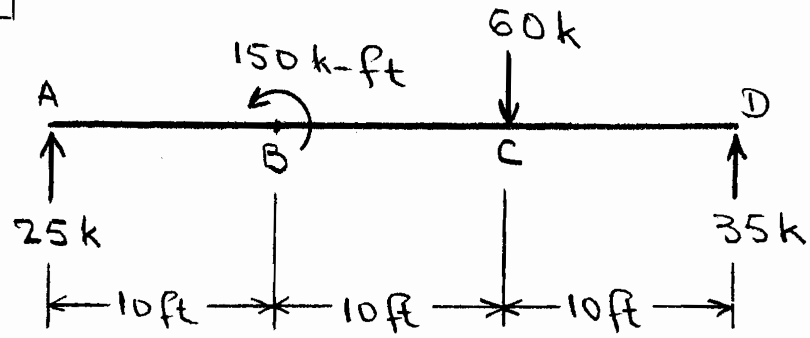
Shear Diagram (k)

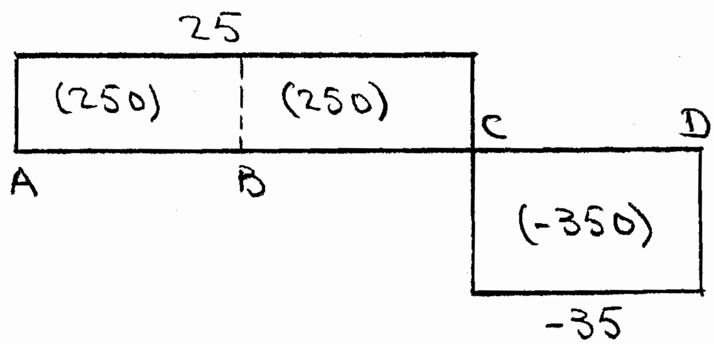


Bending Moment Diagram (k-ft)

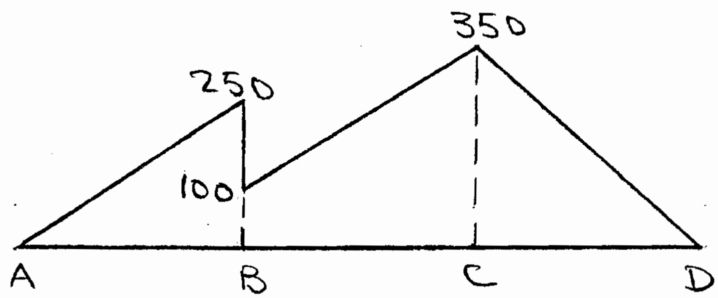
Qualitative Deflected Shape

5.36





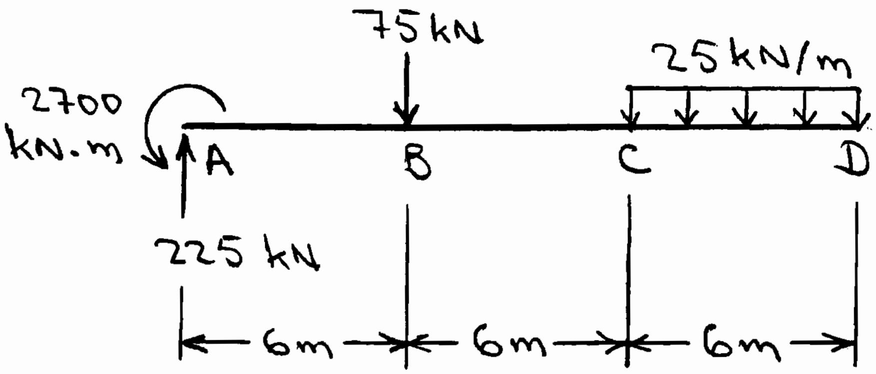
Shear Diagram (k)

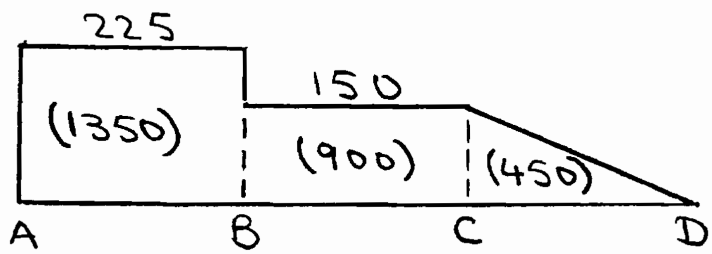


Bending Moment Diagram (k-ft)

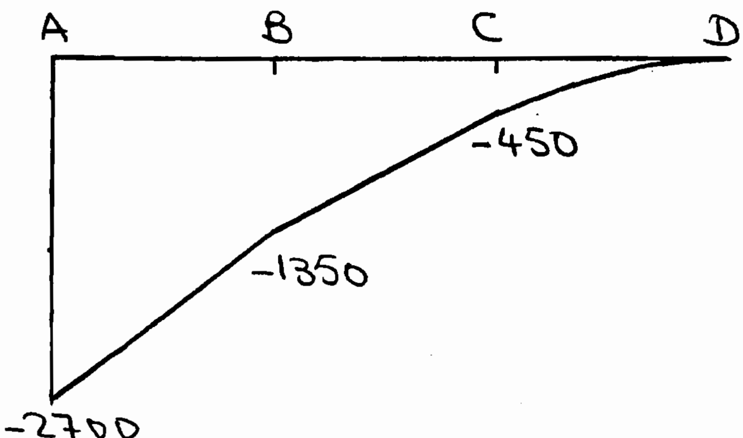
Qualitative Deflected Shape

5.37

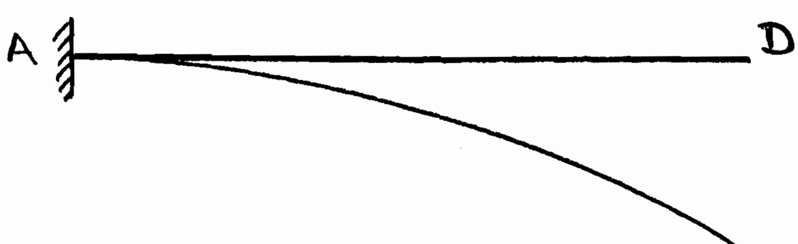




Shear Diagram (kN)

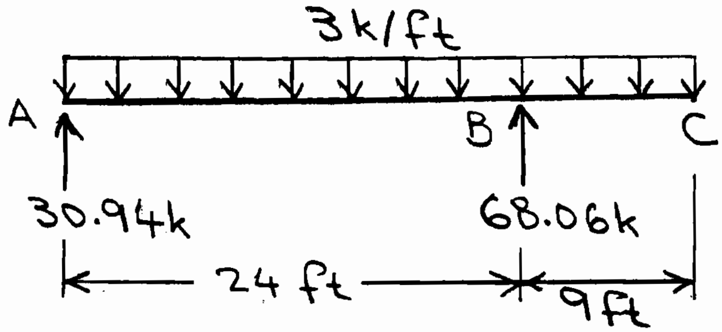


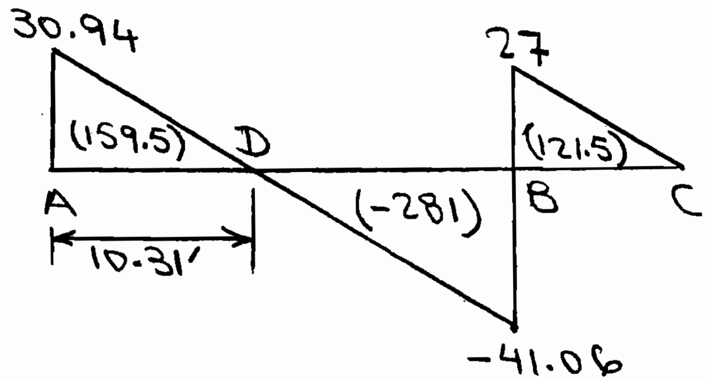
Bending Moment Diagram (kN.m)



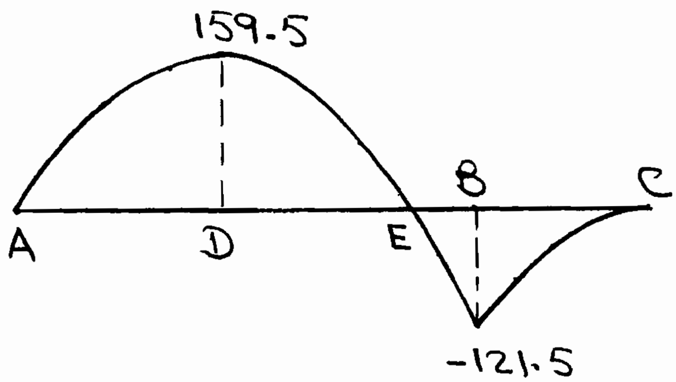
Qualitative Deflected Shape

5.38

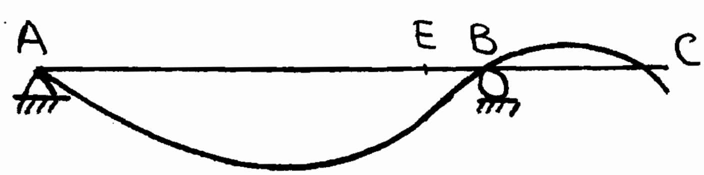




Shear Diagram (k)

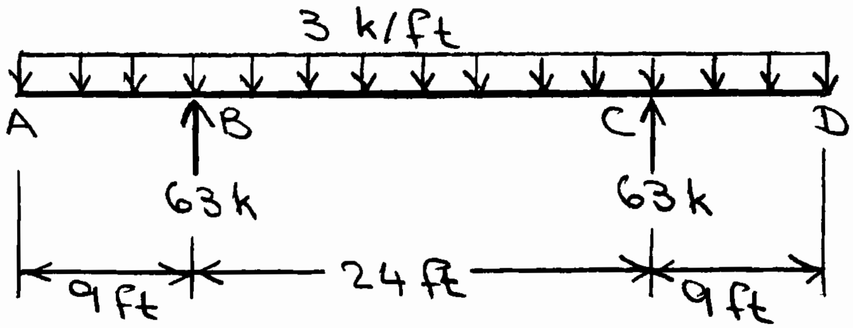


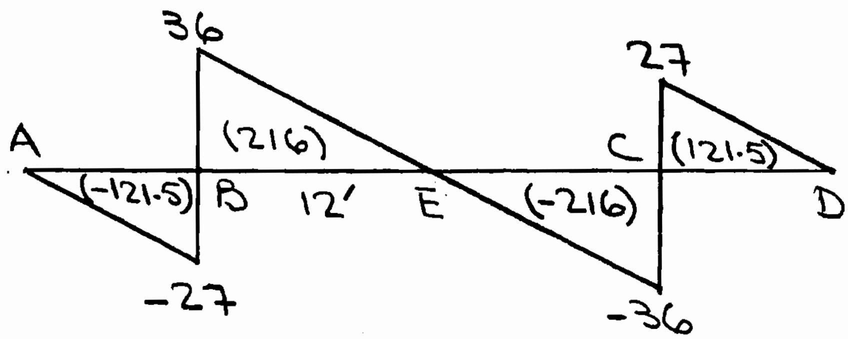
Bending Moment Diagram (k-ft)



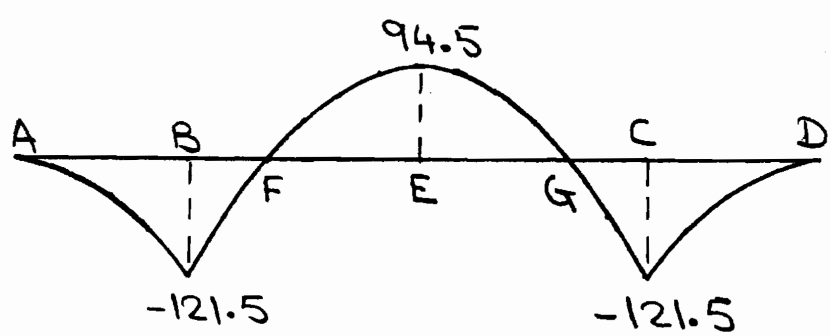
Qualitative Deflected Shape

5.39

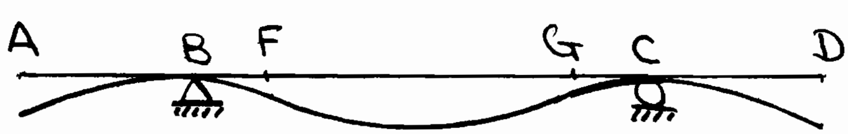




Shear Diagram (k)

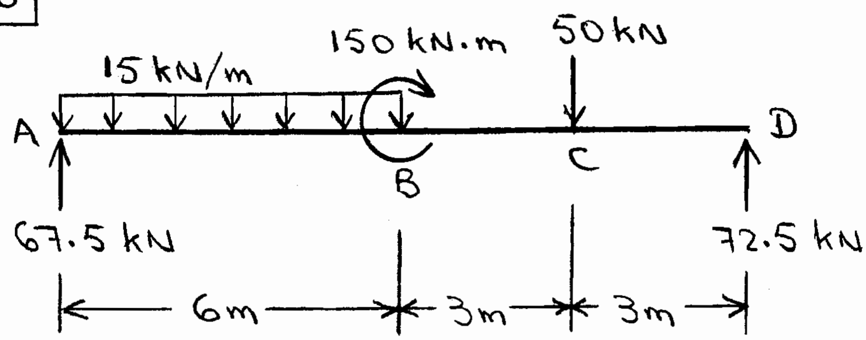


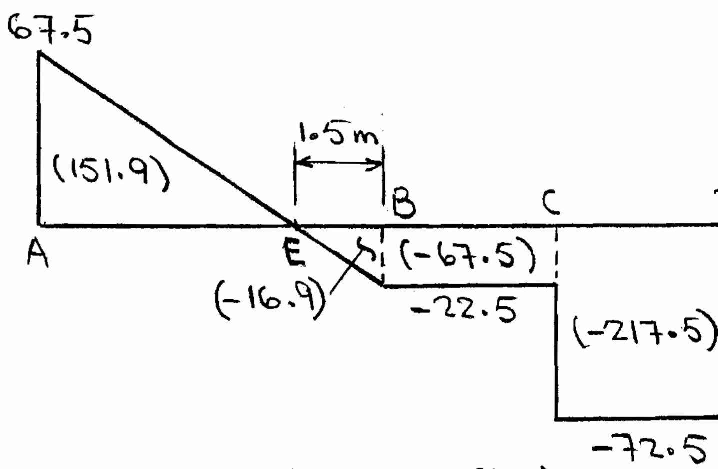
Bending Moment Diagram (k-ft)



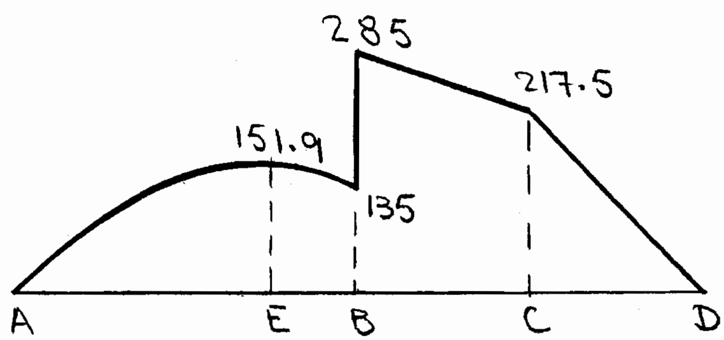
Qualitative Deflected Shape

5.40





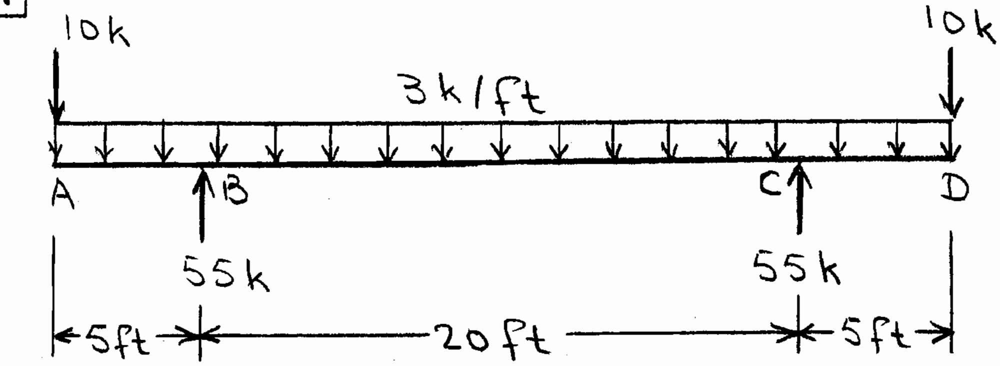
Shear Diagram (kN)

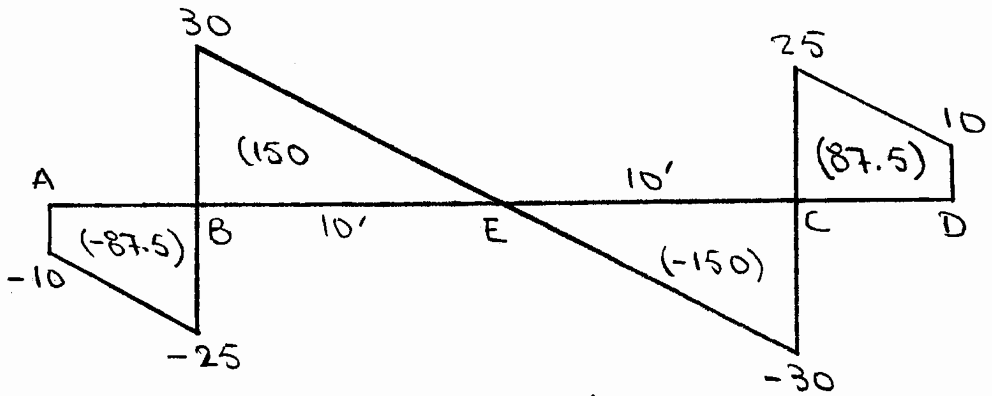


Bending Moment Diagram (kN.m)

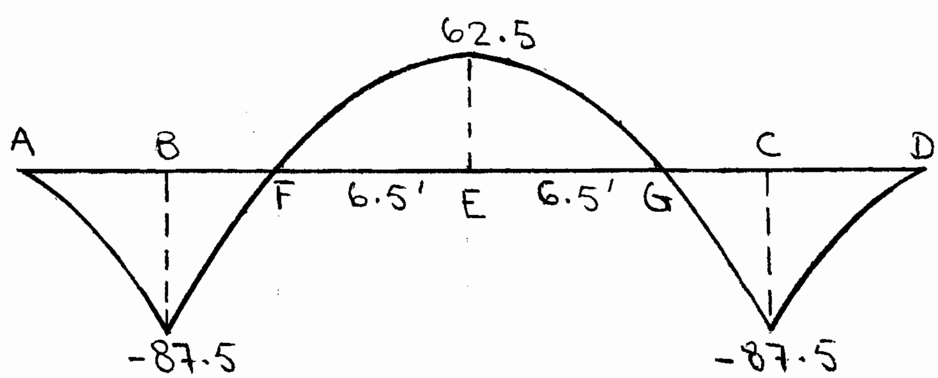
Qualitative Deflected Shape

5.41

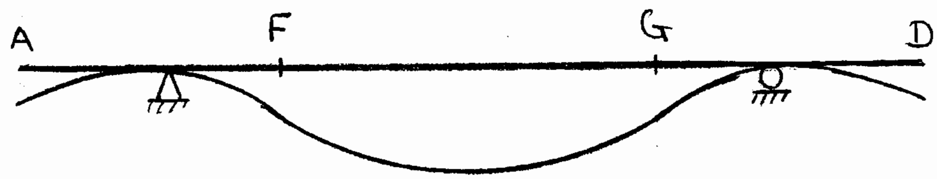




Shear Diagram (k)

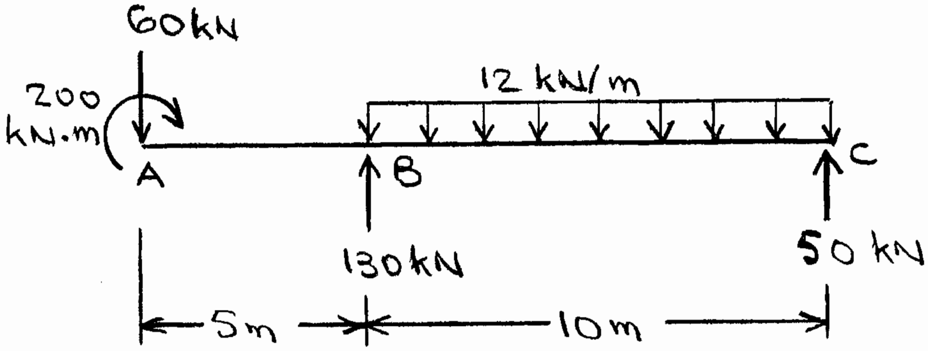


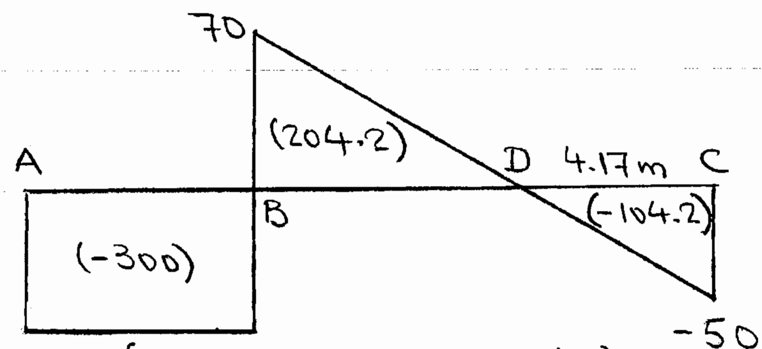
Bending Moment Diagram (k-ft)



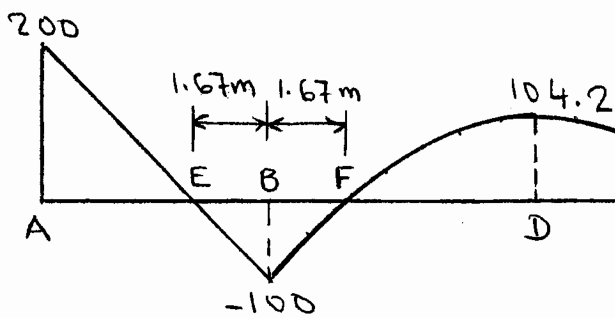
Qualitative Deflected Shape

5.42

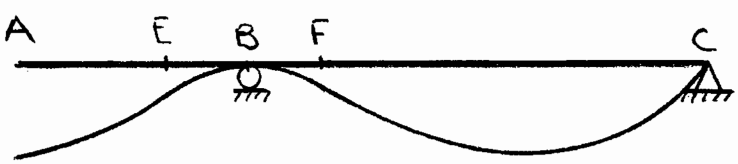




Shear Diagram (kN)

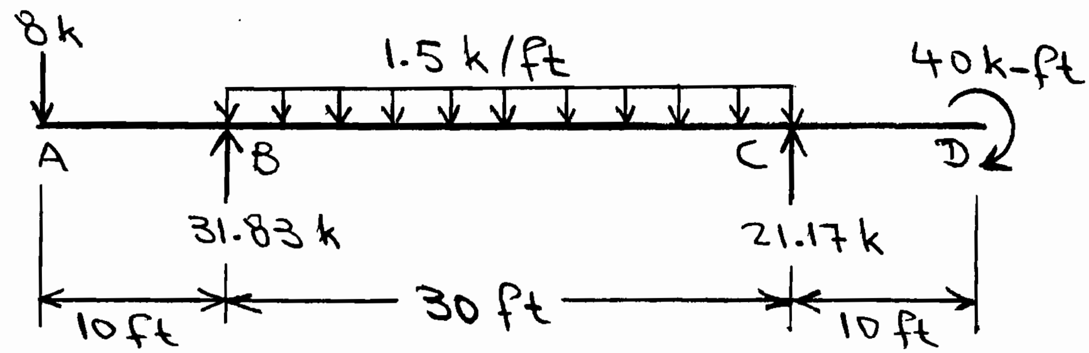


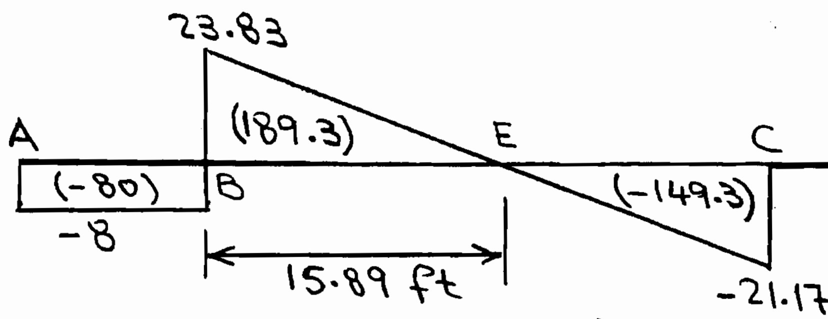
Bending Moment Diagram (kN.m)



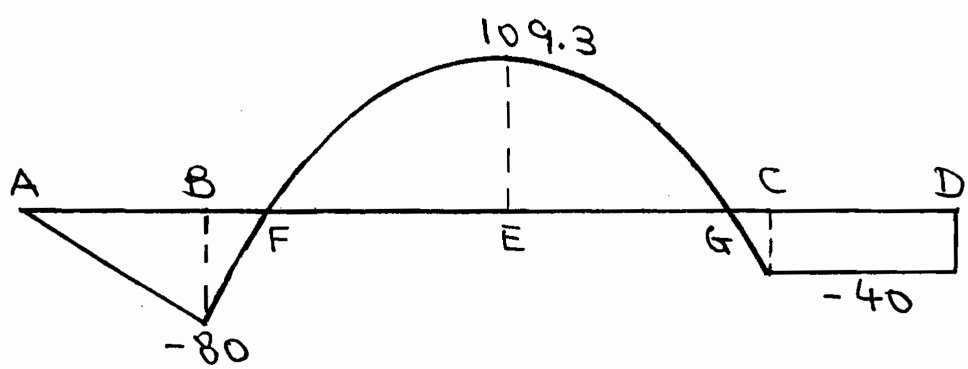
Qualitative Deflected Shape

$\boxed{5.43}$





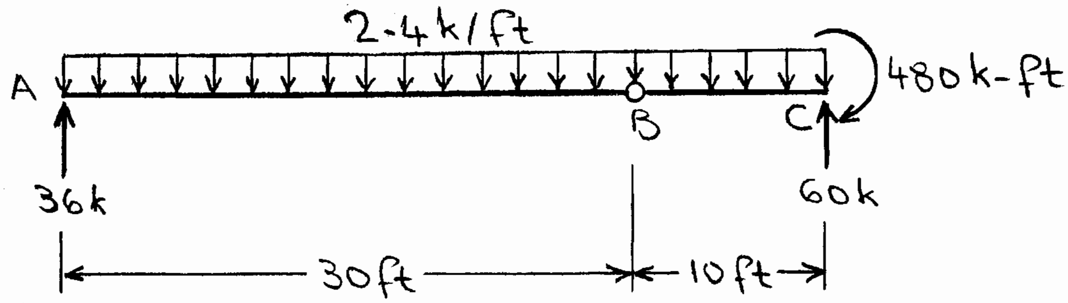
Shear Diagram (k)

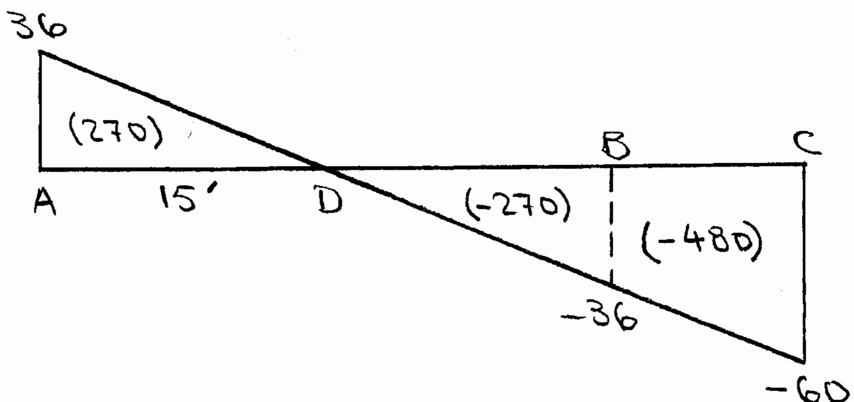


Bending Moment Diagram (k-ft)

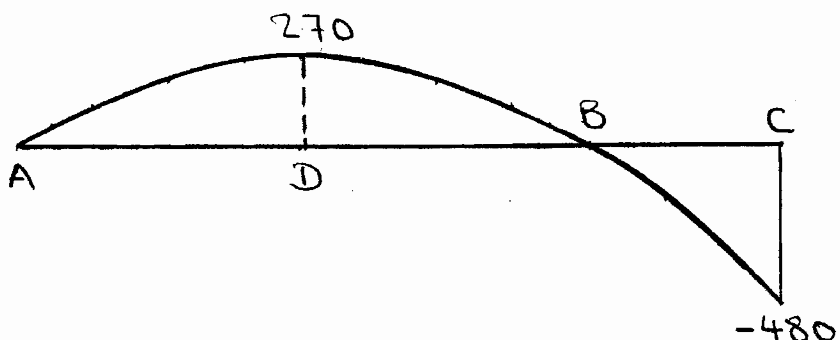
Qualitative Deflected Shape

5.44





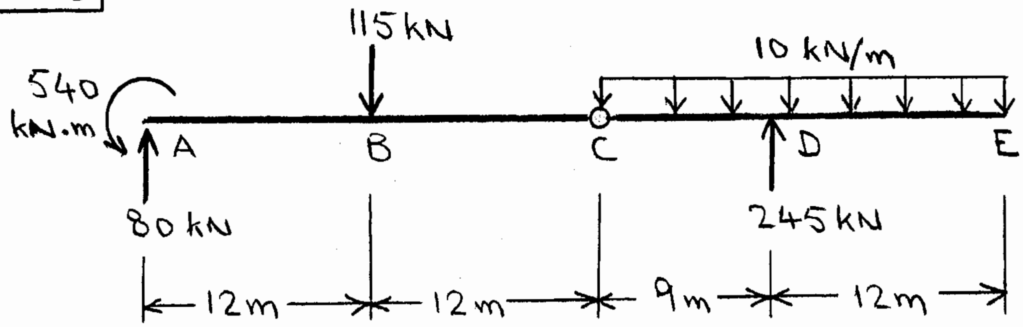
Shear Diagram (k)

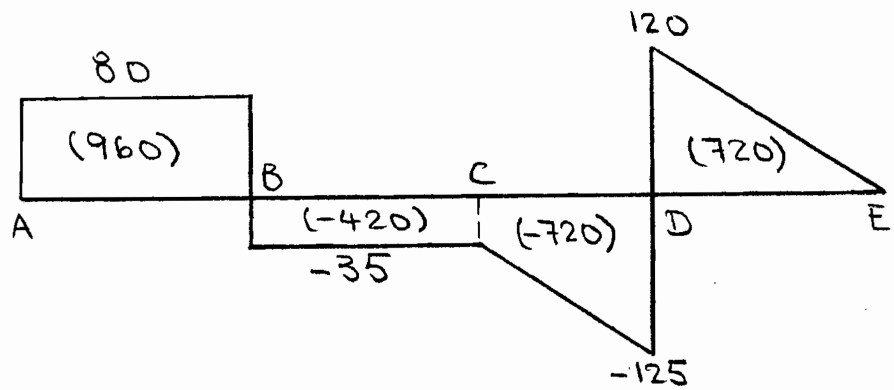


Bending Moment Diagram (k-ft)

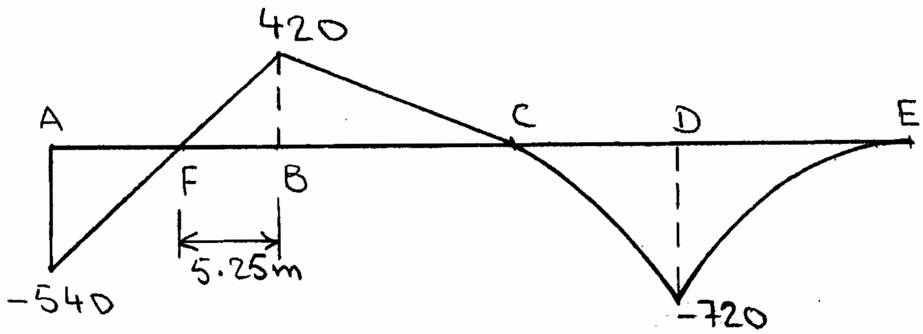
Qualitative Deflected Shape

5-45

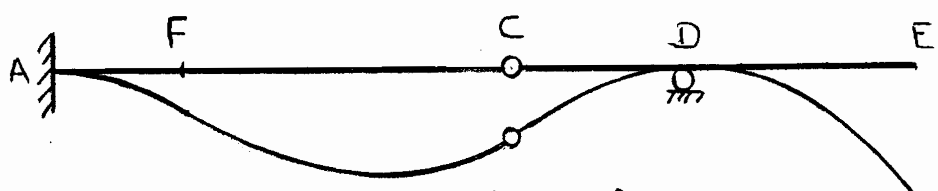




Shear Diagram (kN)

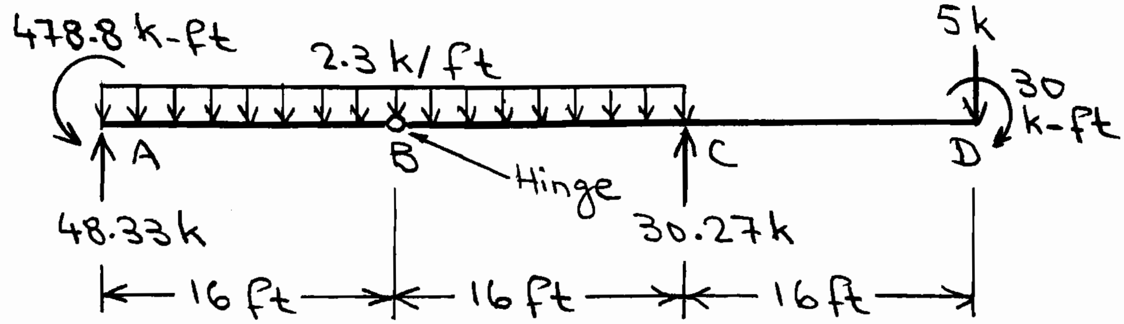


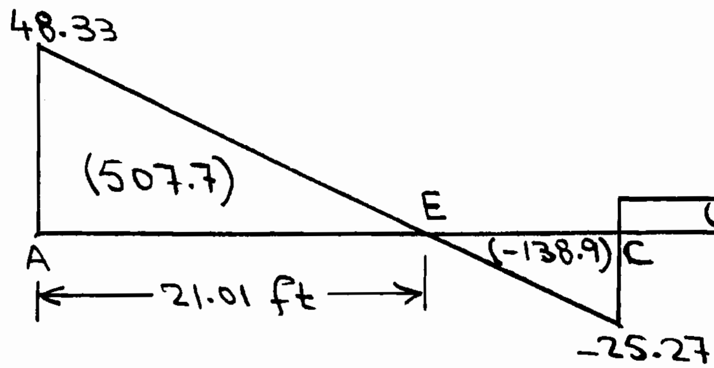
Bending Moment Diagram (kN.m)



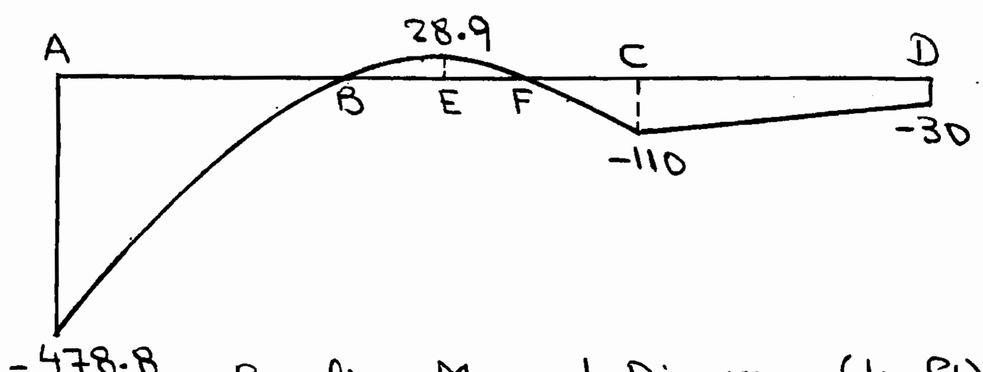
Qualitative Deflected Shape

5.46

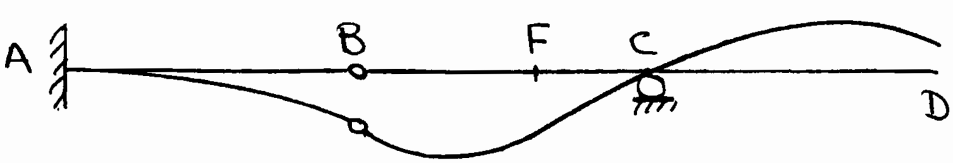




Shear Diagram (k)

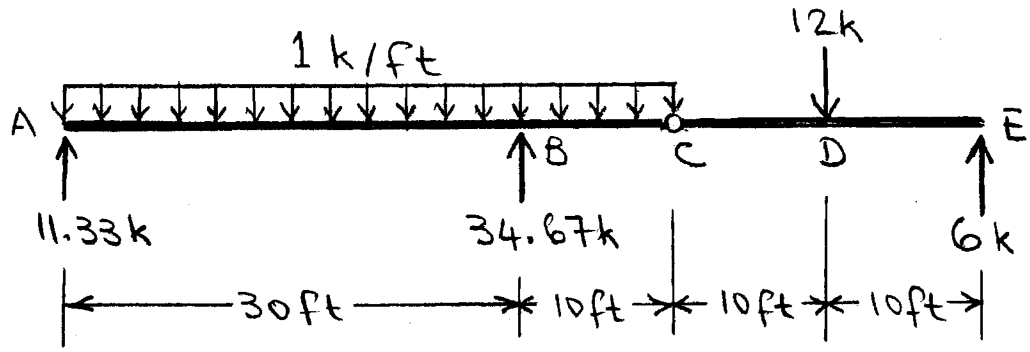


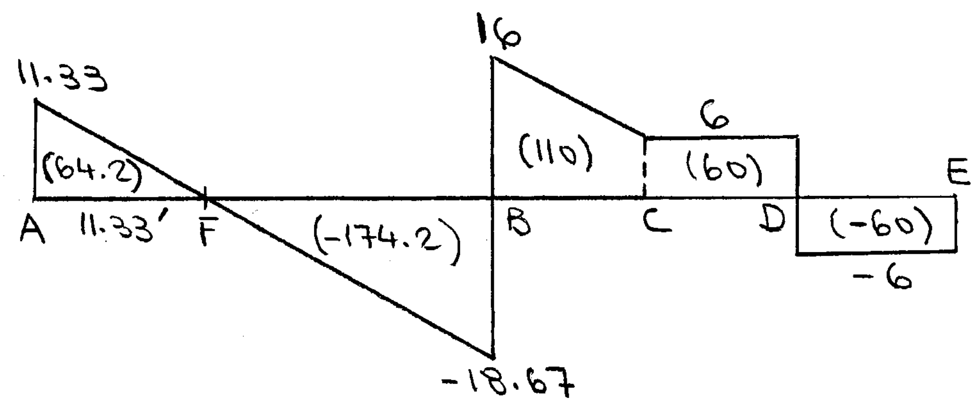
Bending Moment Diagram (k-ft)



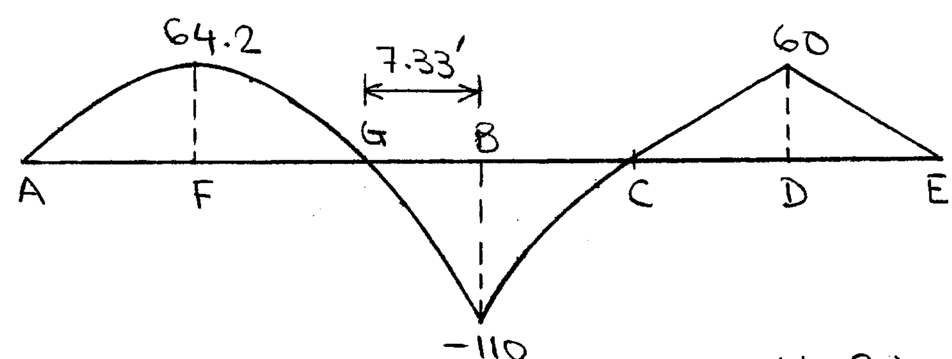
Qualitative Deflected Shape

5.47

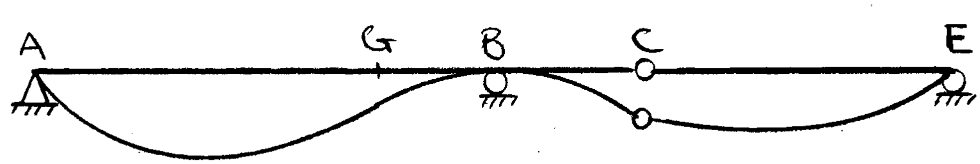




Shear Diagram (k)

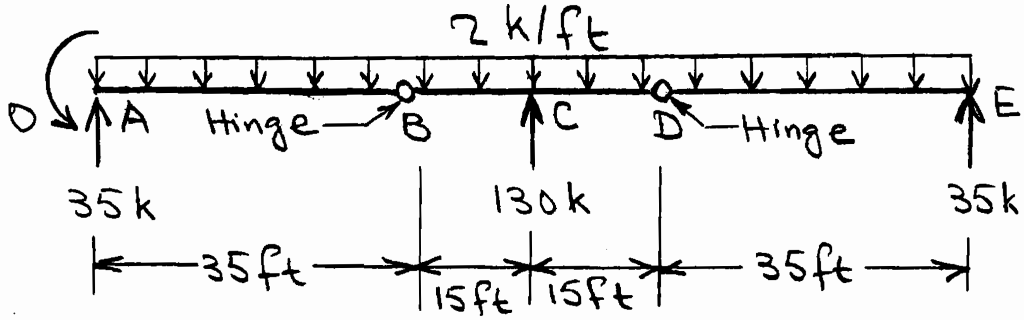


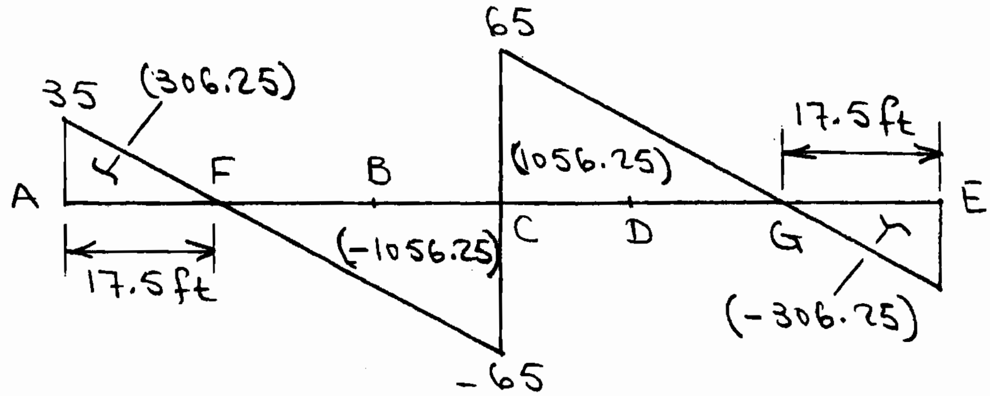
Bending Moment Diagram (k-ft)



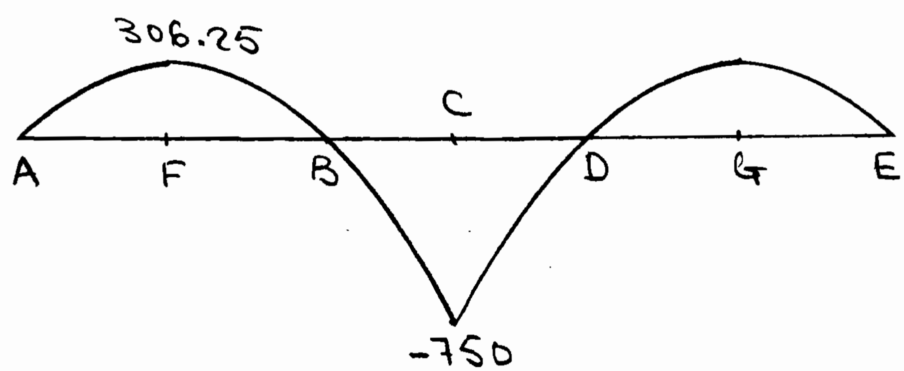
Qualitative Deflected Shape

5.48

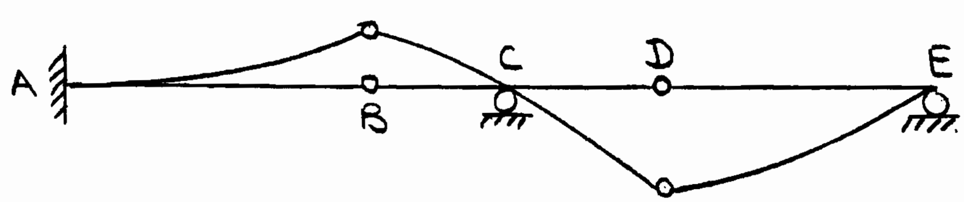




Shear Diagram (k)

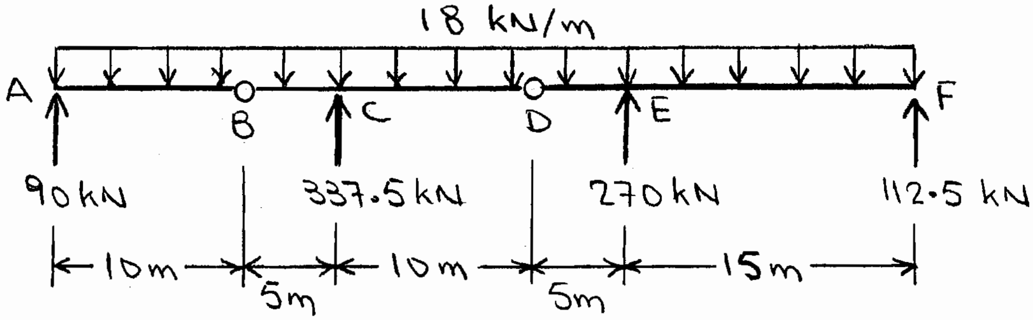


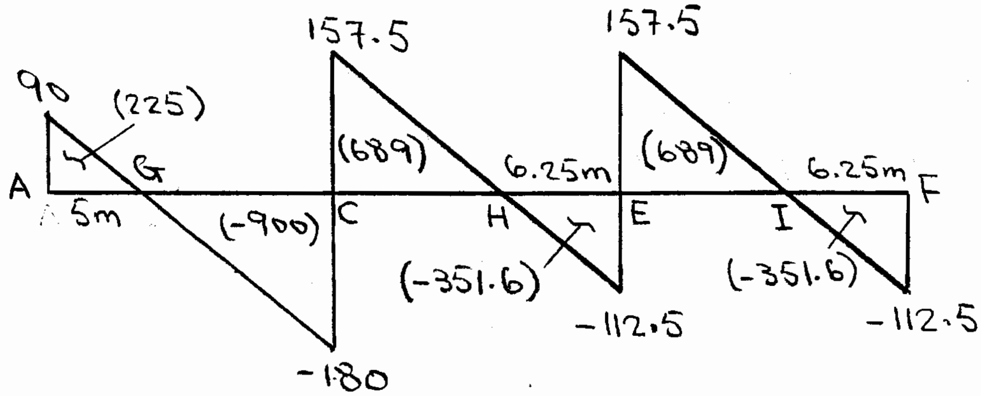
Bending Moment Diagram (k-ft)



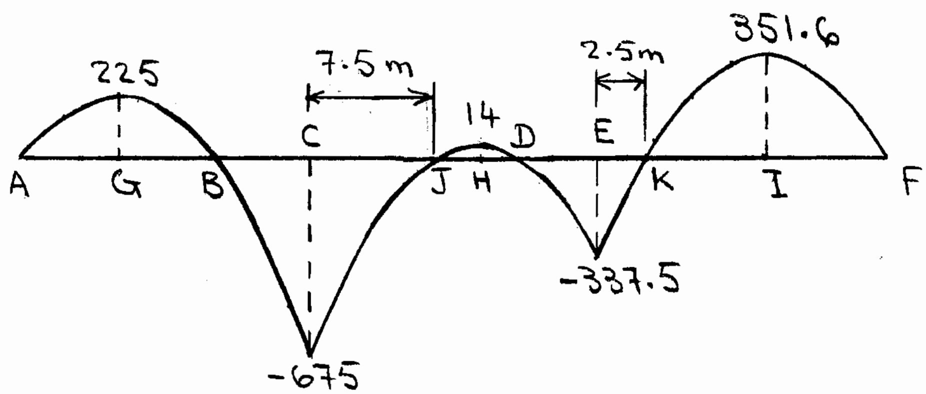
Qualitative Deflected Shape

5.49

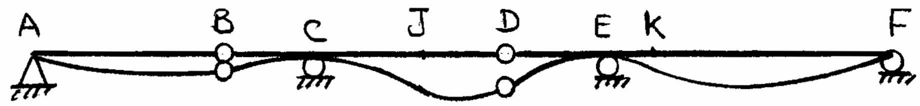




Shear Diagram (kN)

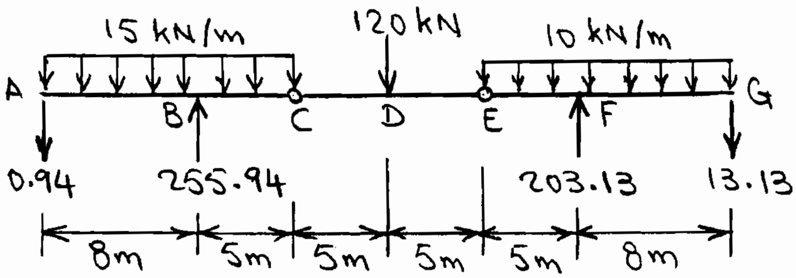


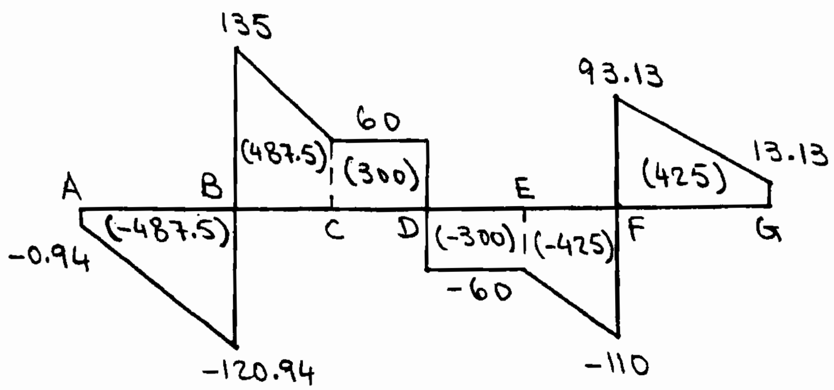
Bending Moment Diagram (kN.m)



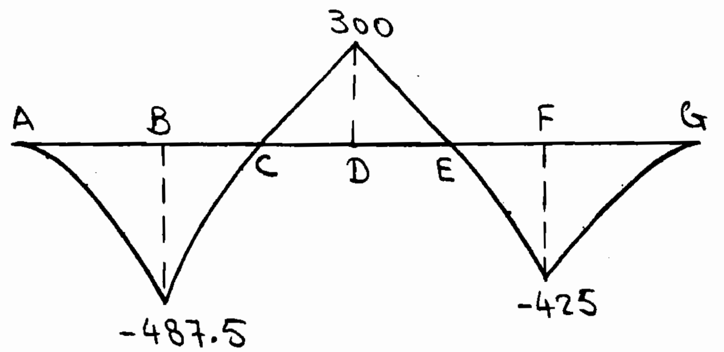
Qualitative Deflected Shape

5-50

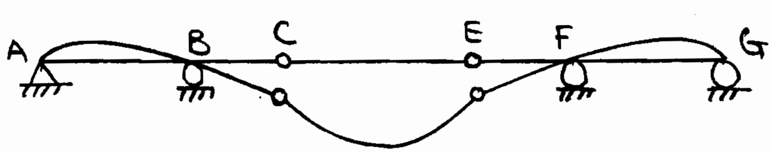




Shear Diagram (kN)

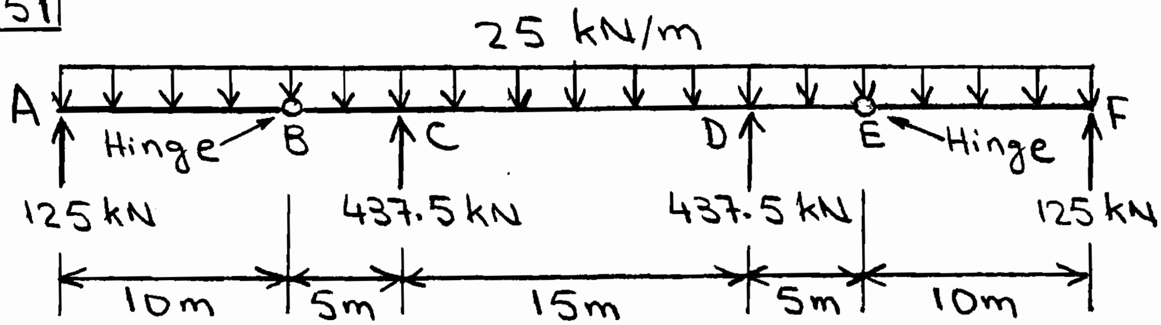


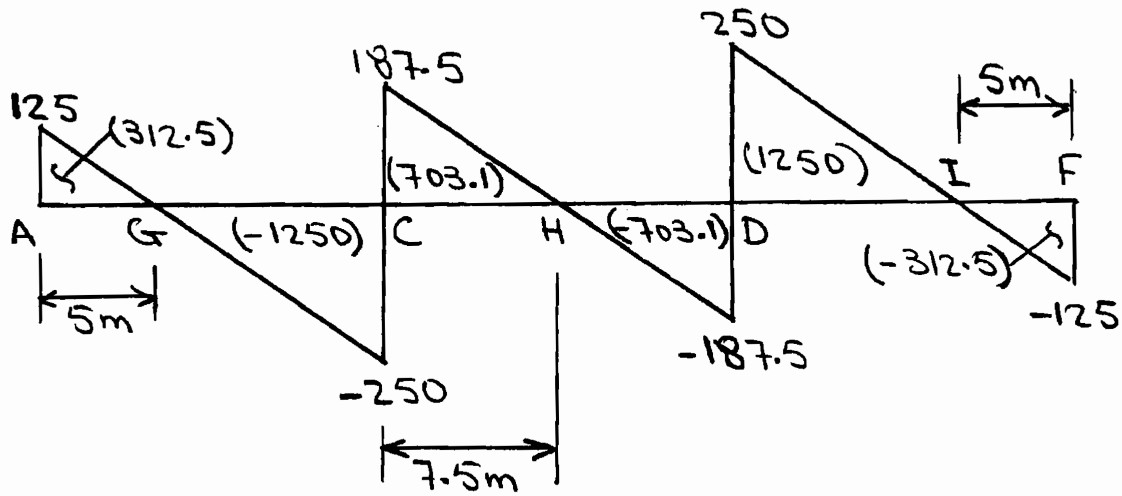
Bending Moment Diagram (kN.m)



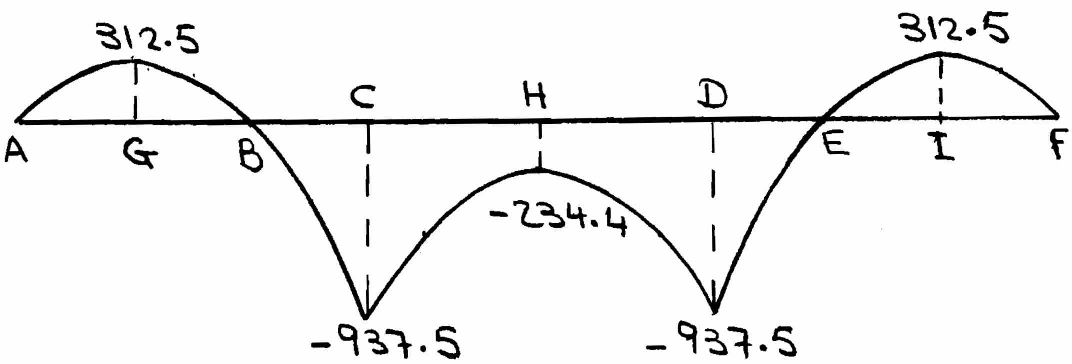
Qualitative Deflected Shape

5.51

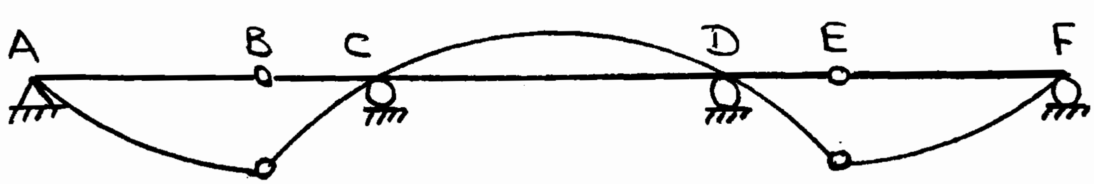




Shear Diagram (kN)

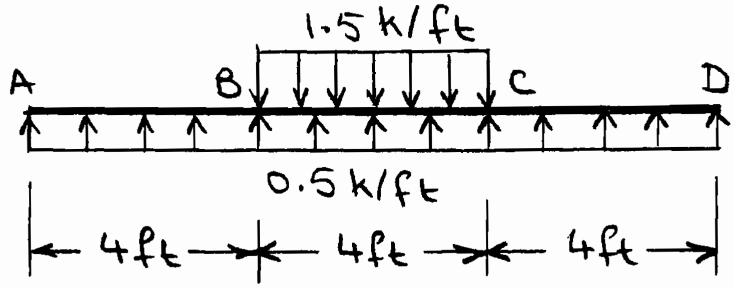


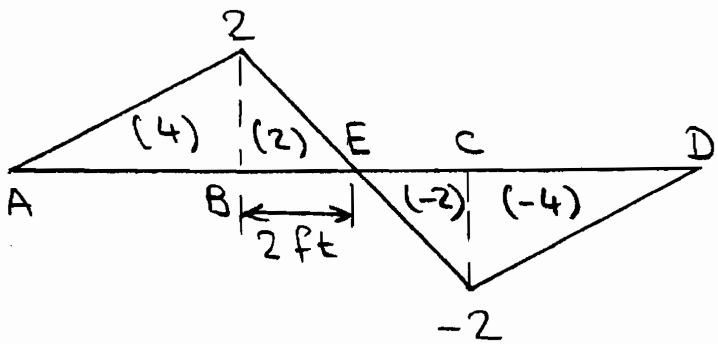
Bending Moment Diagram (kN.m)



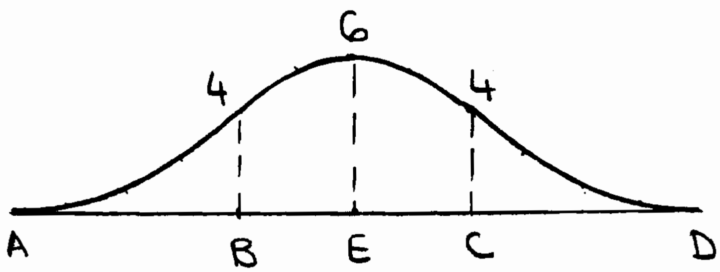
Qualitative Deflected Shape

$\boxed{5.52}$



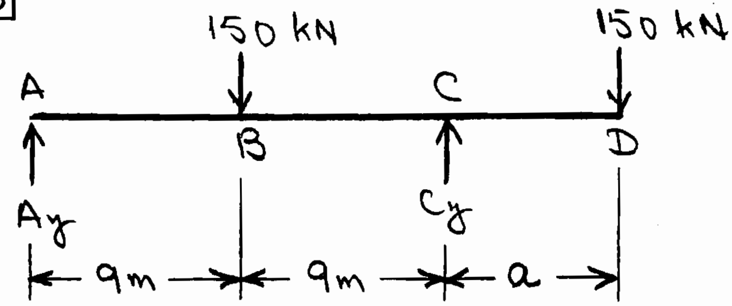


Shear Diagram (k)



Bending Moment Diagram (k-ft)

5.53



$$+\circlearrowleft \Sigma M_C = 0 \qquad -A_y(18) + 150(9) - 150a = 0$$

$$A_y = 75 - \frac{25}{3}a$$

Maximum positive bending moment occurs at B:

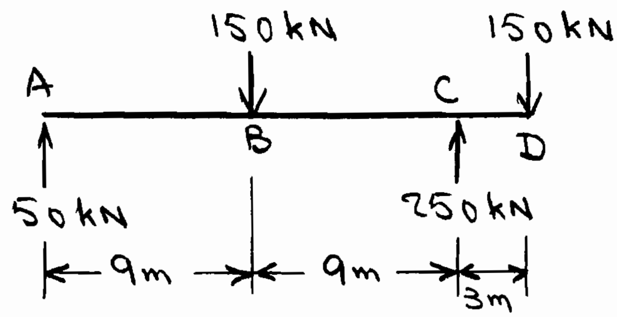
$$M_B = A_y(9) = 675 - 75a \qquad (1)$$

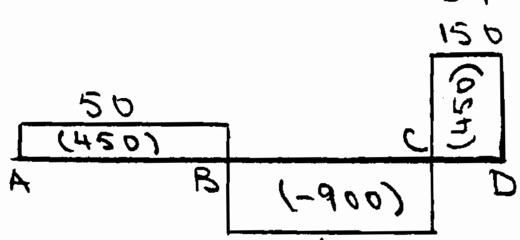
Maximum negative bending moment occurs at C:

$$M_C = 150a \qquad (2)$$

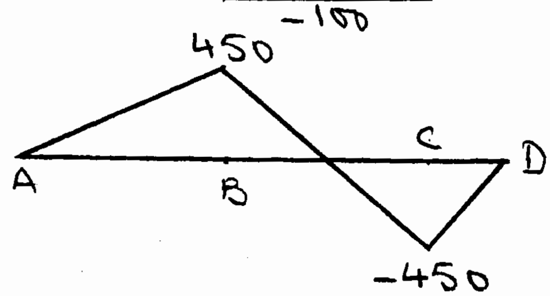
Equating Eqs. (1) and (2):

$$675 - 75a = 150a \qquad \underline{a = 3\text{ m}}$$



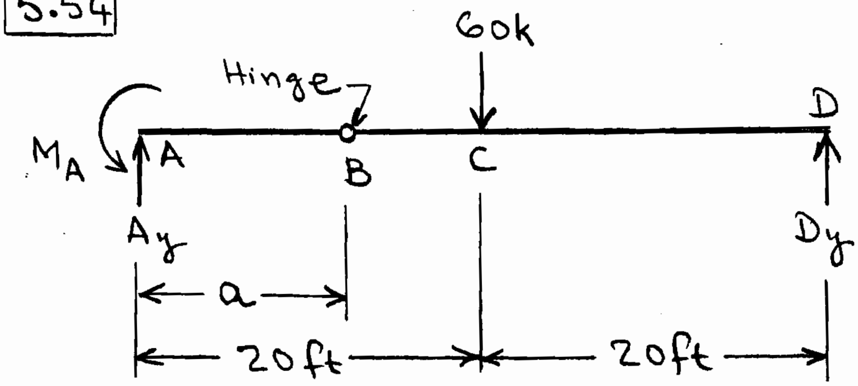


Shear Diagram (kN)



Bending Moment Diagram (kN.m)

$\boxed{5.54}$



$$+\circlearrowleft \Sigma M_B^{BD} = 0 \qquad -60(20-a) + D_y(40-a) = 0$$

$$D_y = \frac{60(20-a)}{40-a}$$

Maximum positive bending moment occurs at C:

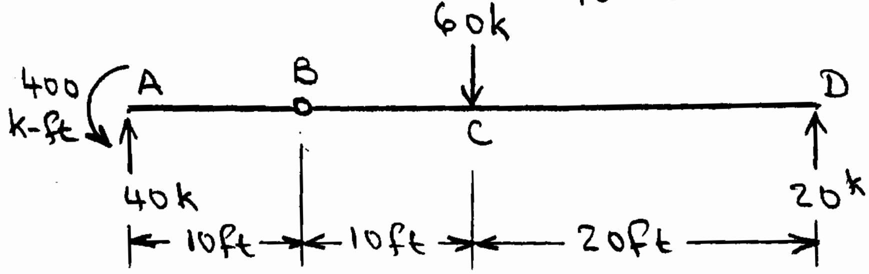
$$M_C = D_y(20) = \frac{1200(20-a)}{40-a} \qquad (1)$$

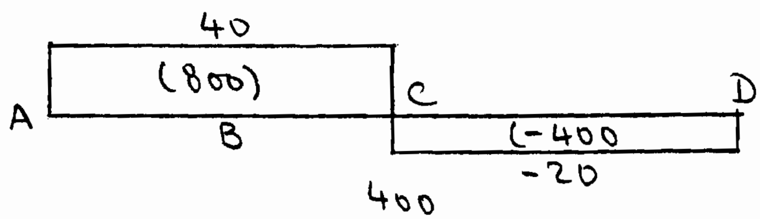
Maximum negative bending moment occurs at A:

$$M_A = 60(20) - D_y(40) = 1200 - \frac{2400(20-a)}{40-a} \qquad (2)$$

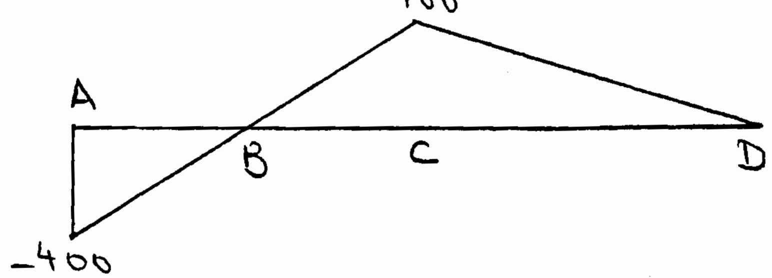
Equating Eqs. (1) and (2):

$$\frac{1200(20-a)}{40-a} = 1200 - \frac{2400(20-a)}{40-a} \qquad \underline{a = 10\text{ ft}}$$





<u>Shear Diagram (k)</u>



<u>Bending Moment Diagram (k-ft)</u>

$\boxed{5.55}$ (a) $m = 4,\ j = 5,\ r = 3,\ e_c = 0$

$3m + r = 3j + e_c$; <u>Statically determinate</u>

(b) $m = 6,\ j = 7,\ r = 3,\ e_c = 1$

$3m + r < 3j + e_c$; <u>Unstable</u>

(c) $m = 7,\ j = 6,\ r = 3,\ e_c = 0$

$3m + r > 3j + e_c$; <u>Statically indeterminate</u>

$i = (21 + 3) - 18 = \underline{6}$

(d) $m = 7,\ j = 7,\ r = 6,\ e_c = 1$

$3m + r > 3j + e_c$; <u>Statically indeterminate</u>

$i = (21 + 6) - (21 + 1) = \underline{5}$

$\boxed{5.56}$ (a) $m = 6,\ j = 6,\ r = 4,\ e_c = 2$

$3m + r > 3j + e_c$; <u>Statically indeterminate</u>

$i = (18 + 4) - (18 + 2) = \underline{2}$

(b) $m = 6,\ j = 7,\ r = 6,\ e_c = 3$

$3m + r = 3j + e_c$; <u>Statically determinate</u>

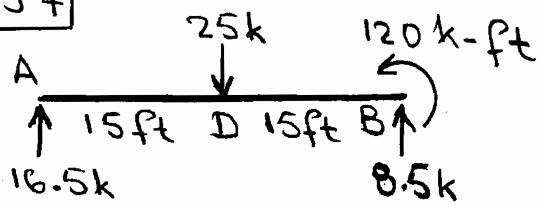
(c) $m = 5,\ j = 6,\ r = 4,\ e_c = 2$

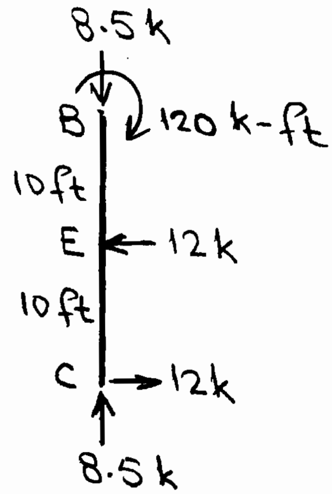
$3m + r < 3j + e_c$; <u>Unstable</u>

(d) <u>Statically indeterminate</u>

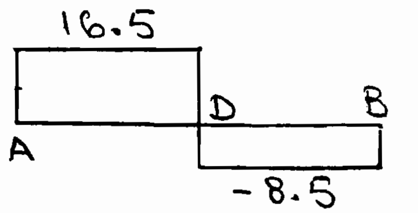
$i = 3(8) = \underline{24}$

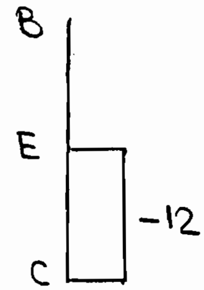
$\boxed{5.57}$



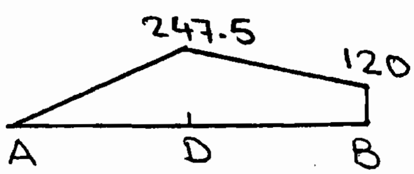


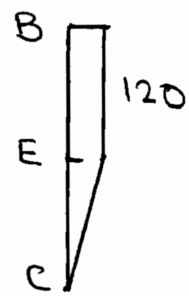
Member End Forces



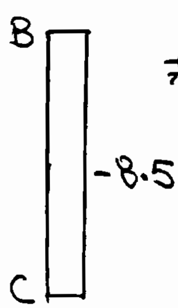


Shear Diagrams (k)

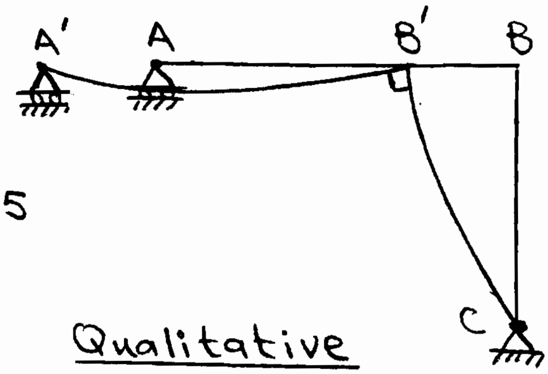




Bending Moment Diagrams (k-ft)

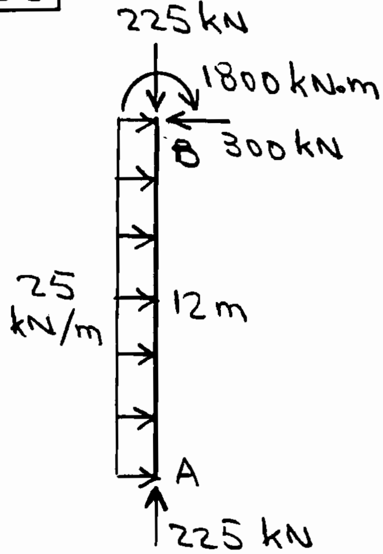


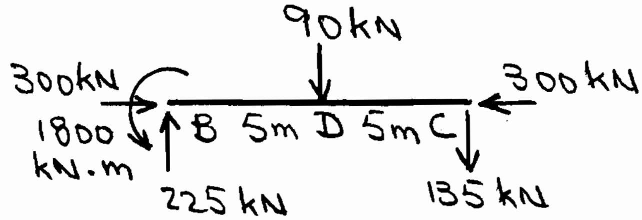
Axial Force Diagrams (k)



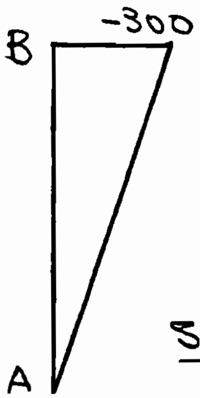
Qualitative Deflected Shape

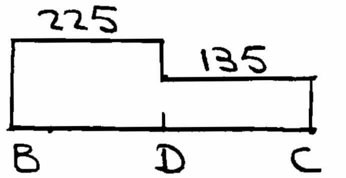
5.58



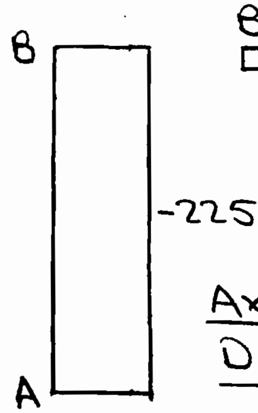


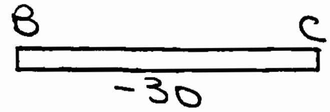
Member End Forces



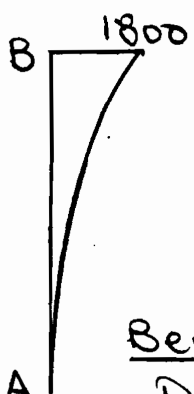


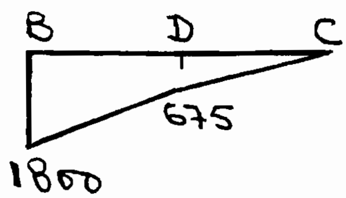
Shear Diagrams (kN)



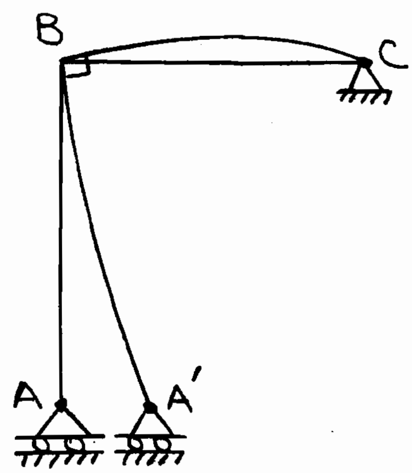


Axial Force Diagrams (kN)



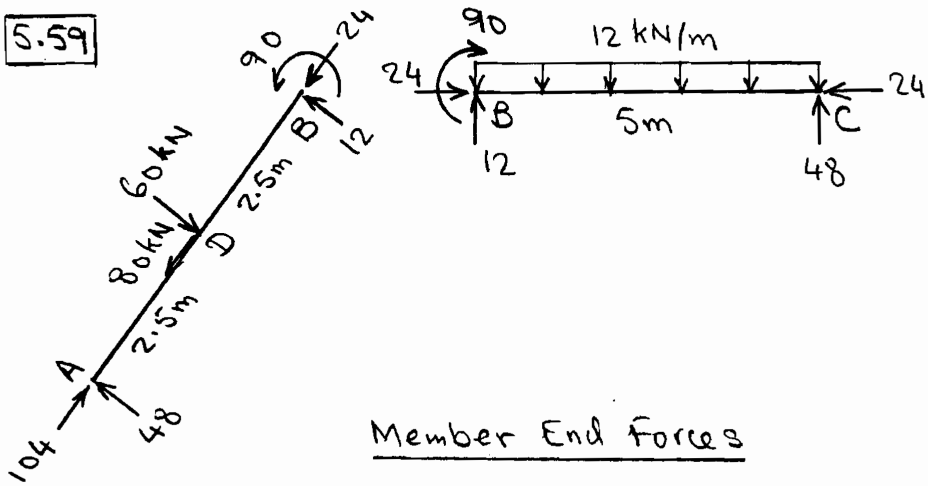


Bending Moment Diagrams (kN.m)

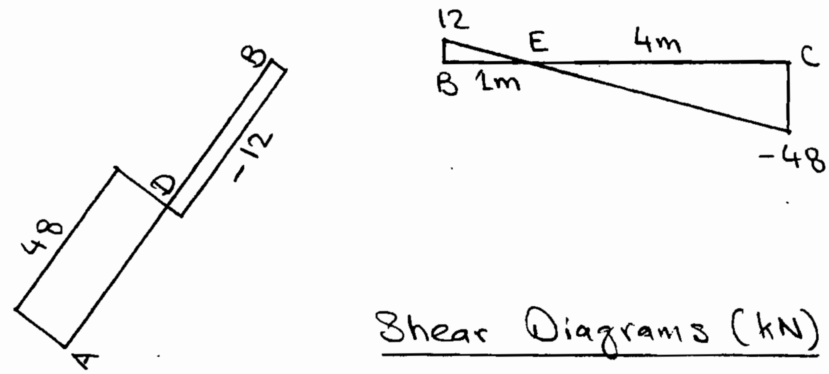


Qualitative Deflected Shape

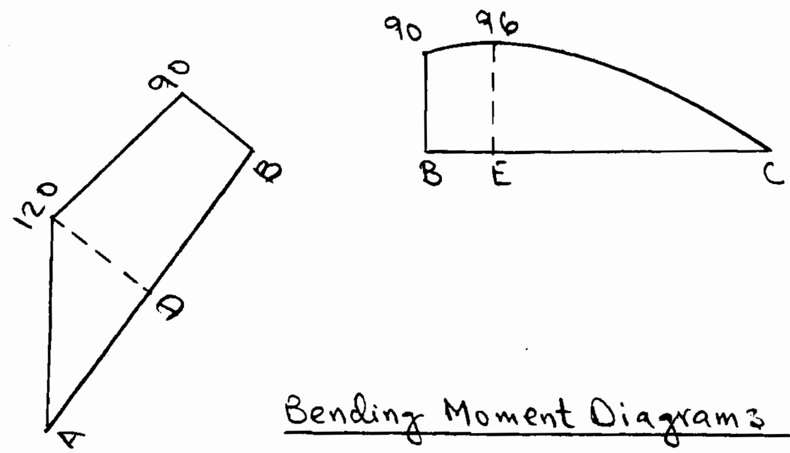
5.59



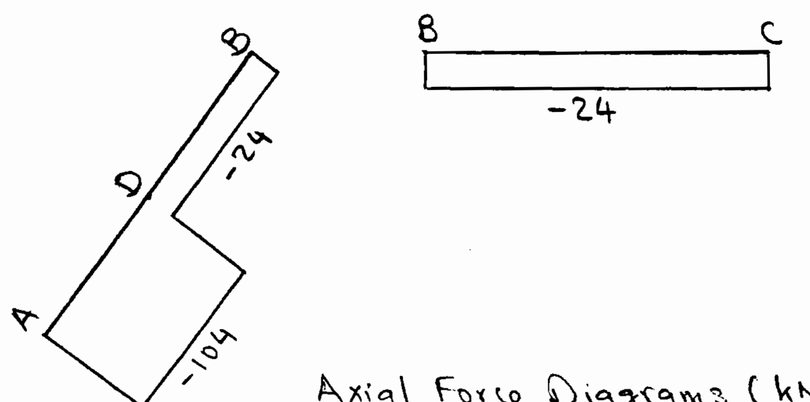
Member End Forces



Shear Diagrams (kN)

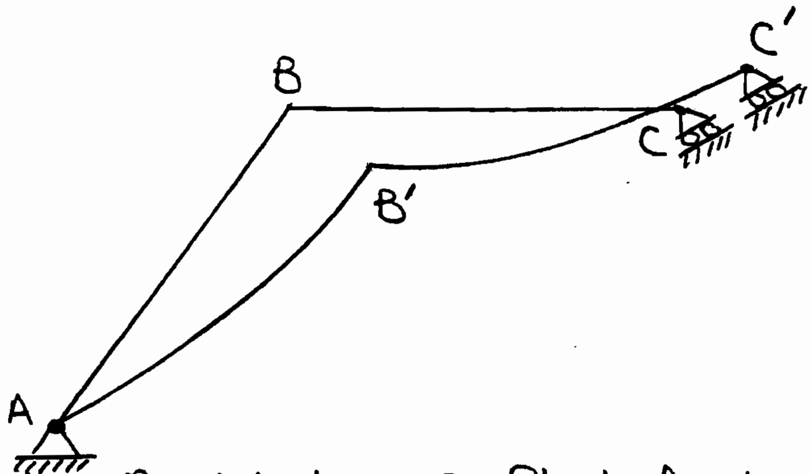


Bending Moment Diagrams (kN.m)



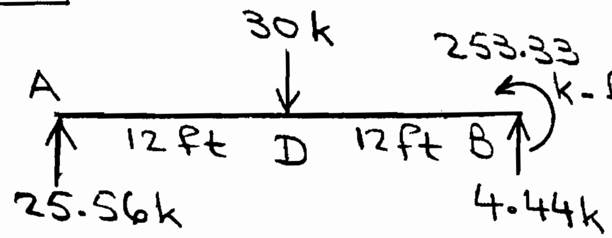
Axial Force Diagrams (kN)

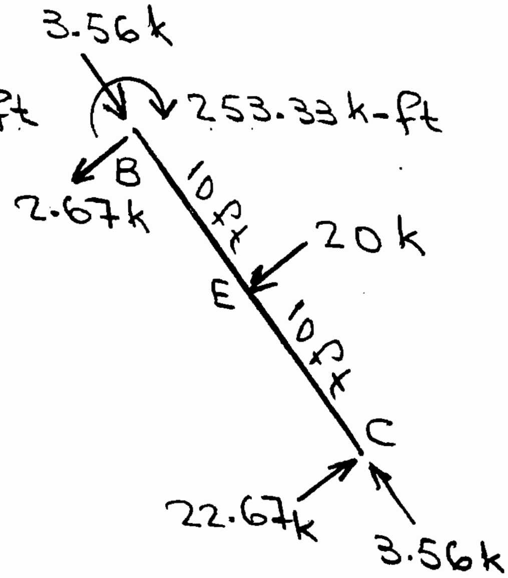
5.59 (contd.)



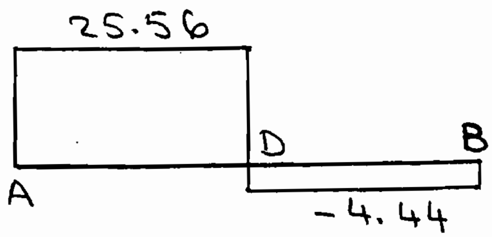
Qualitative Deflected Shape

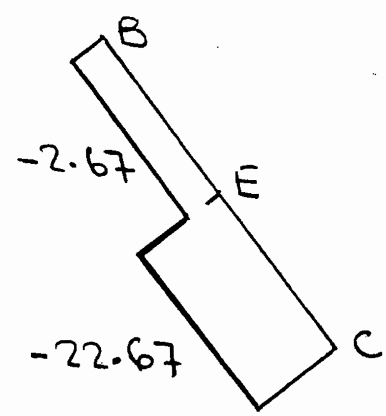
$\boxed{5.60}$



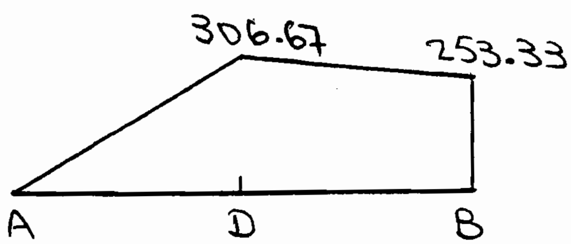


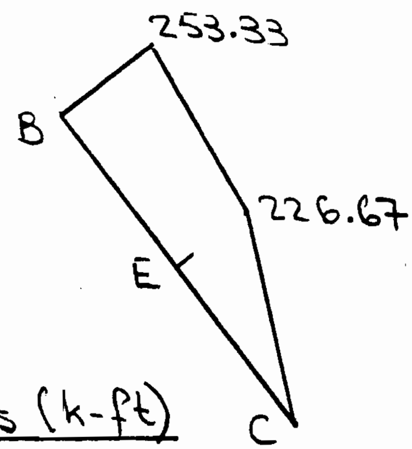
Member End Forces





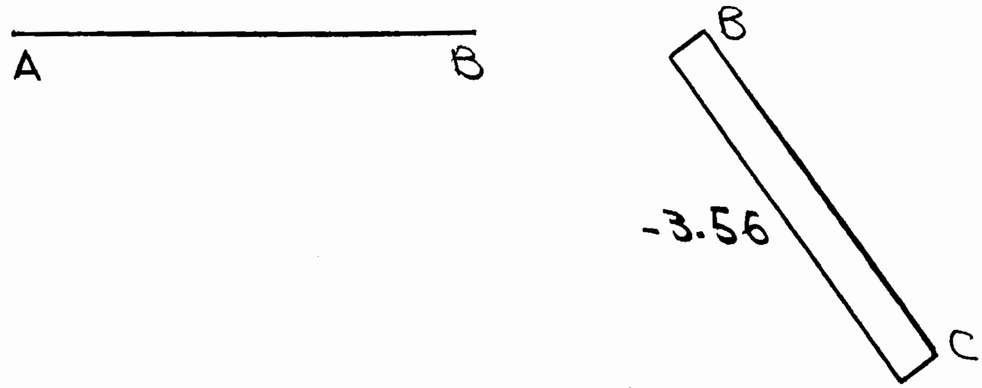
Shear Diagrams (k)



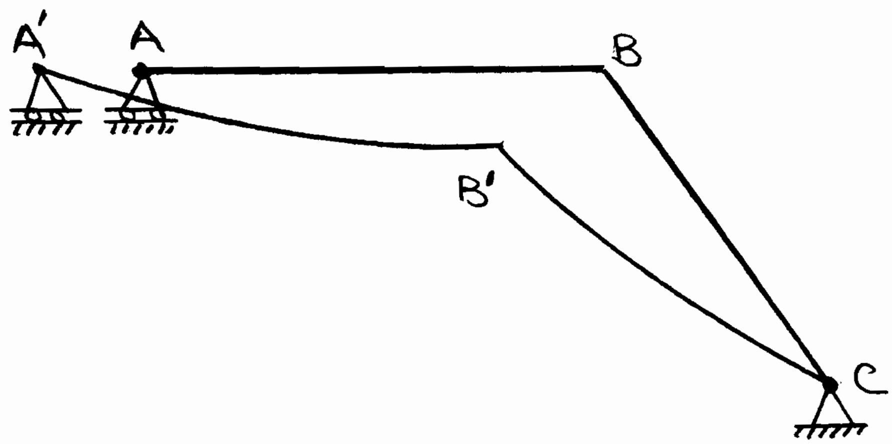


Bending Moment Diagrams (k-ft)

5.60 (contd.)

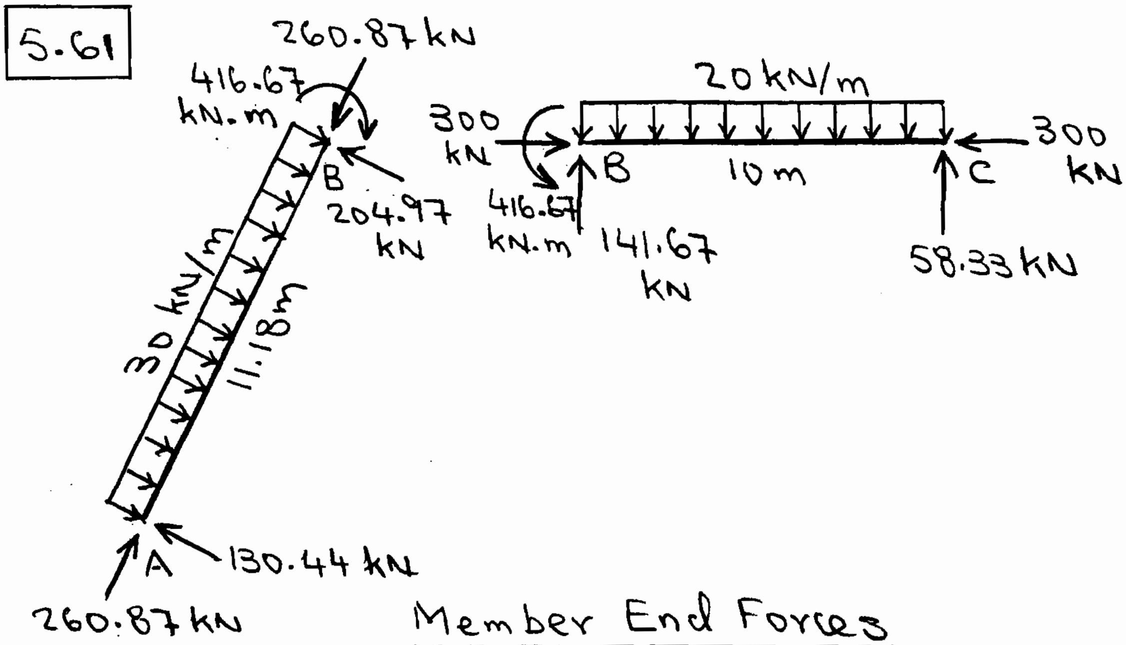


Axial Force Diagrams (k)

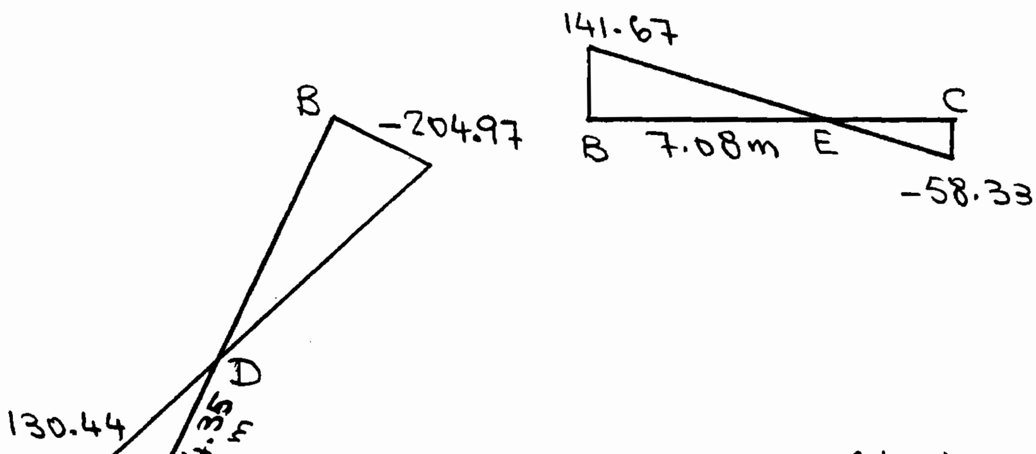


Qualitative Deflected Shape

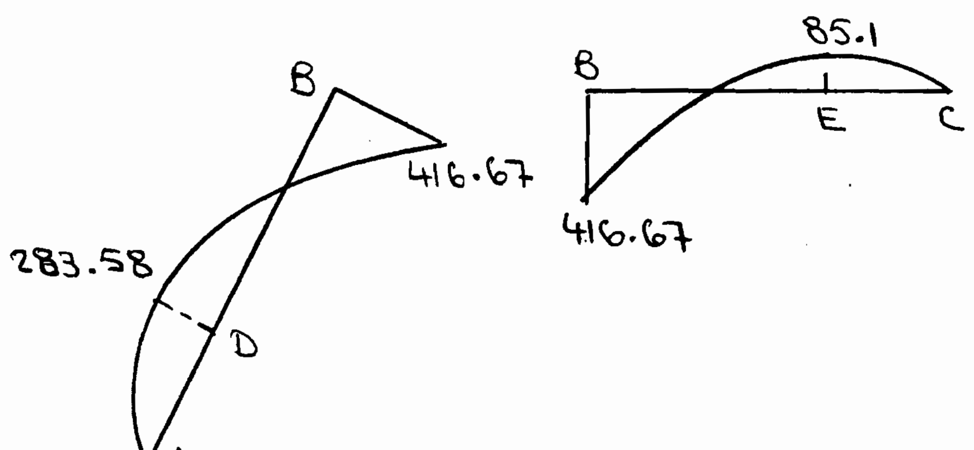
$\boxed{5.61}$



Member End Forces

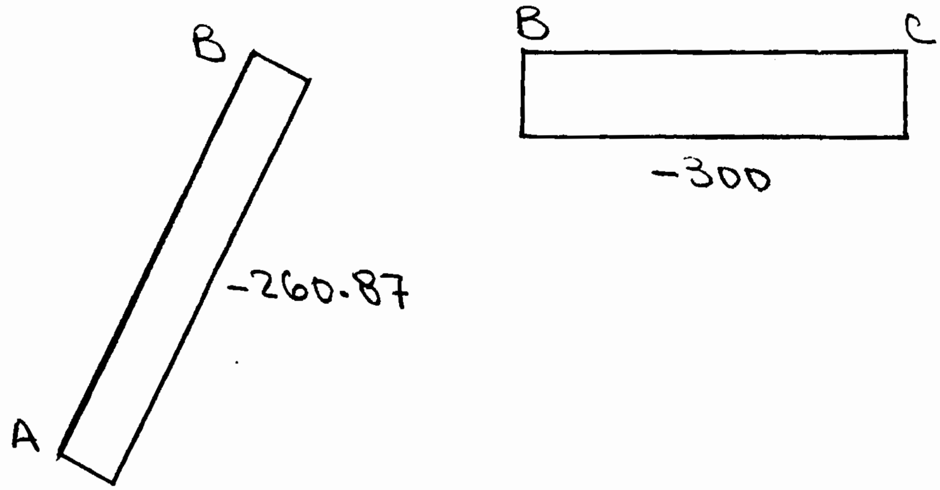


Shear Diagrams (kN)

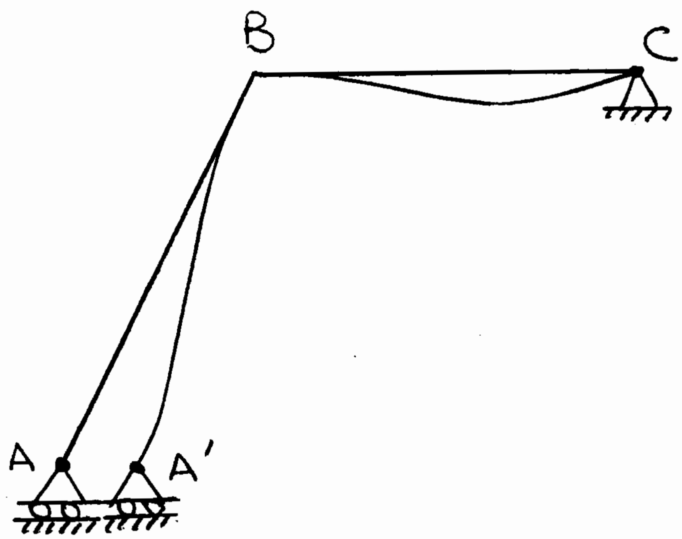


Bending Moment Diagrams (kN.m)

5.61 (contd.)

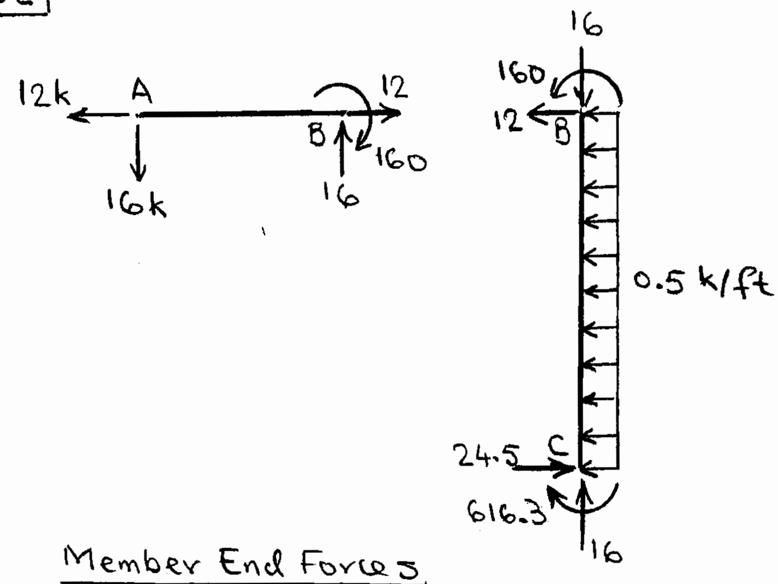


Axial Force Diagrams (kN)

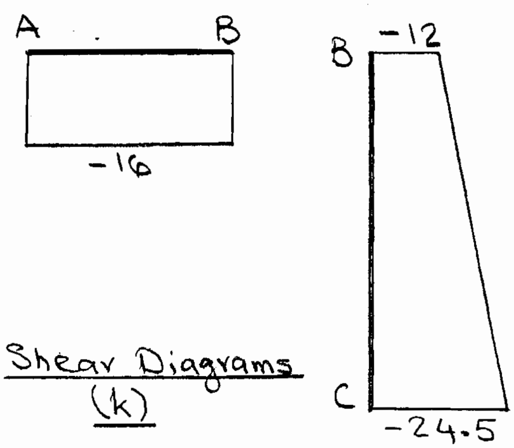


Qualitative Deflected Shape

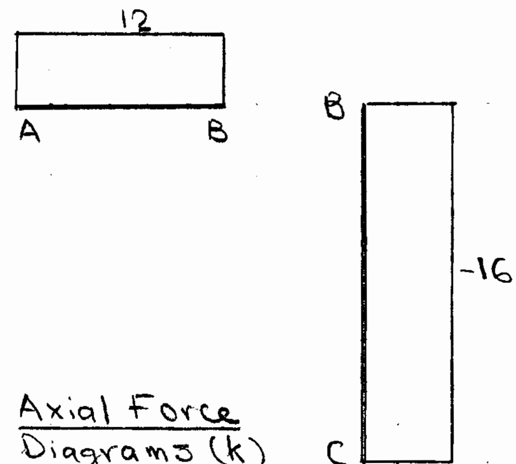
5.62



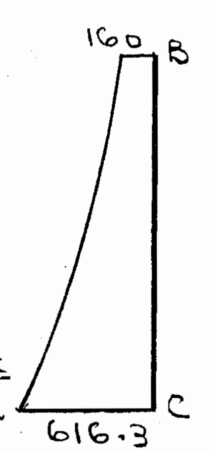
Member End Forces



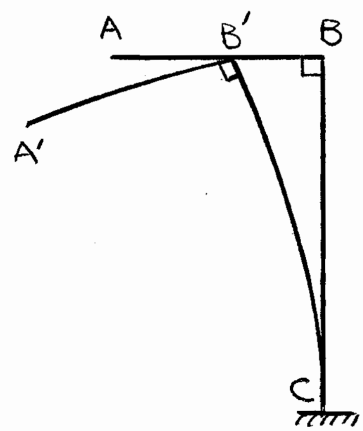
Shear Diagrams (k)



Axial Force Diagrams (k)

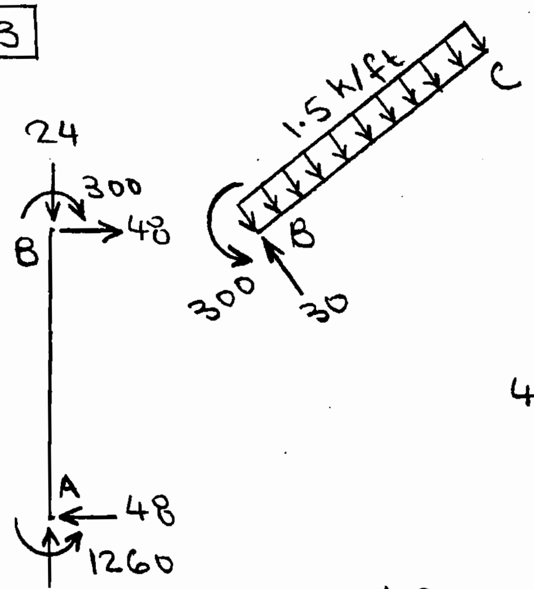


Bending Moment Diagrams (k-ft)

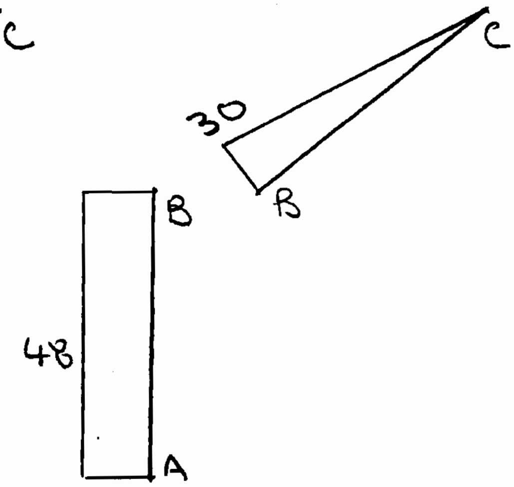


Qualitative Deflected Shape

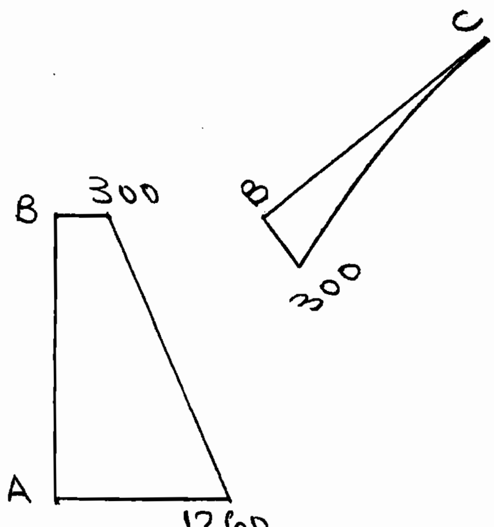
5.63



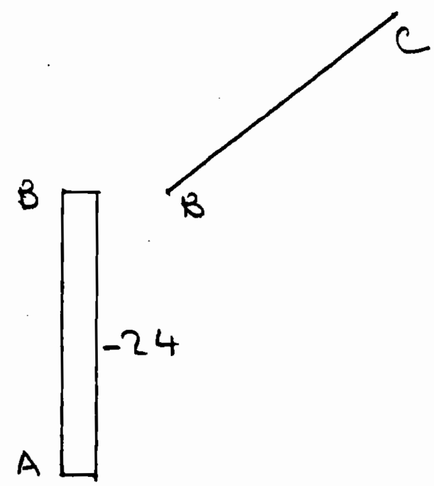
Member End Forces



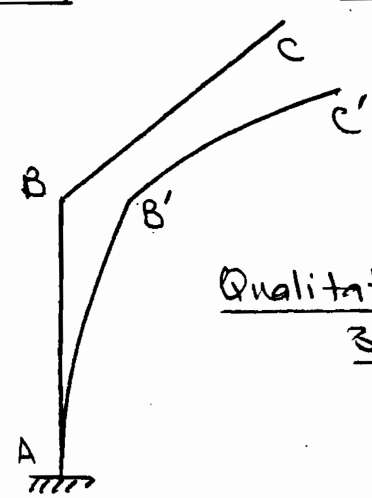
Shear Diagrams (k)



Bending Moment Diagrams (k-ft)

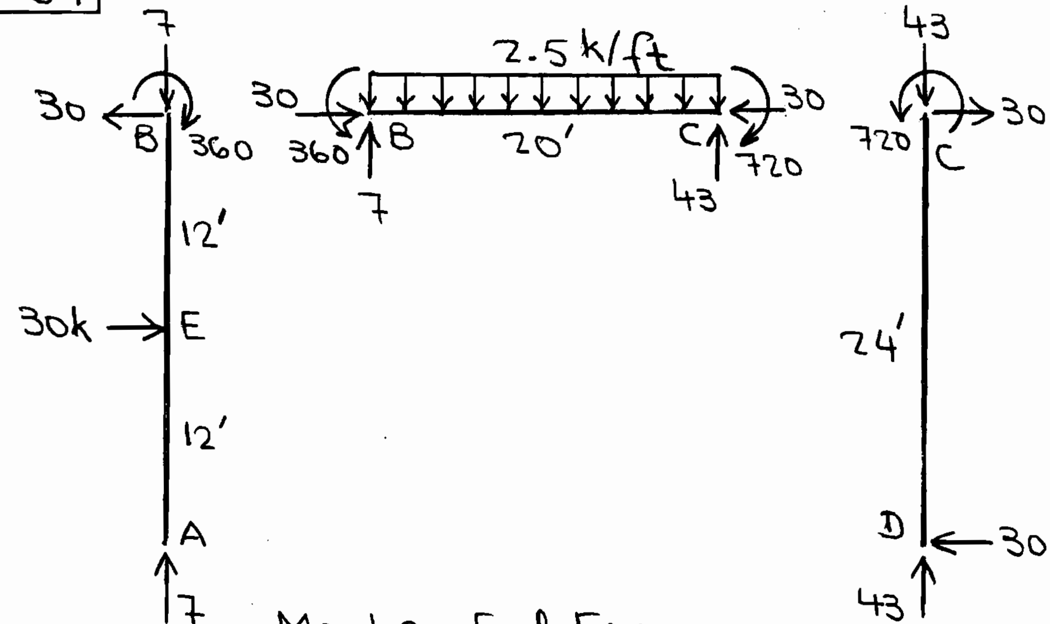


Axial Force Diagrams (kN)

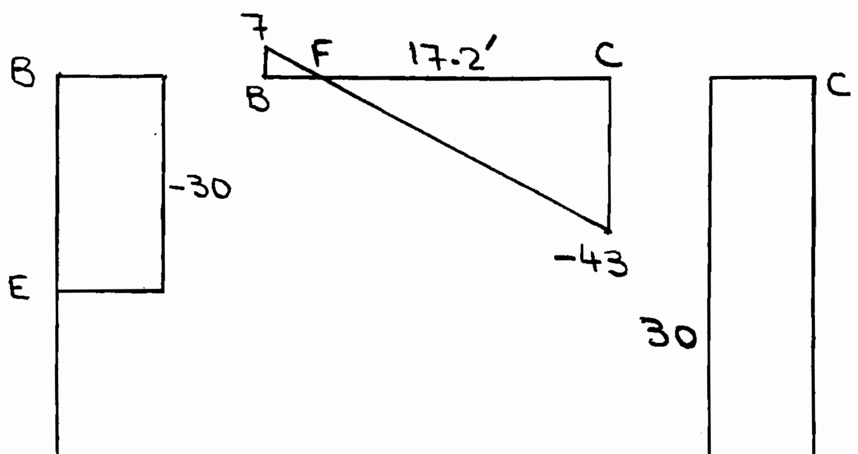


Qualitative Deflected Shape

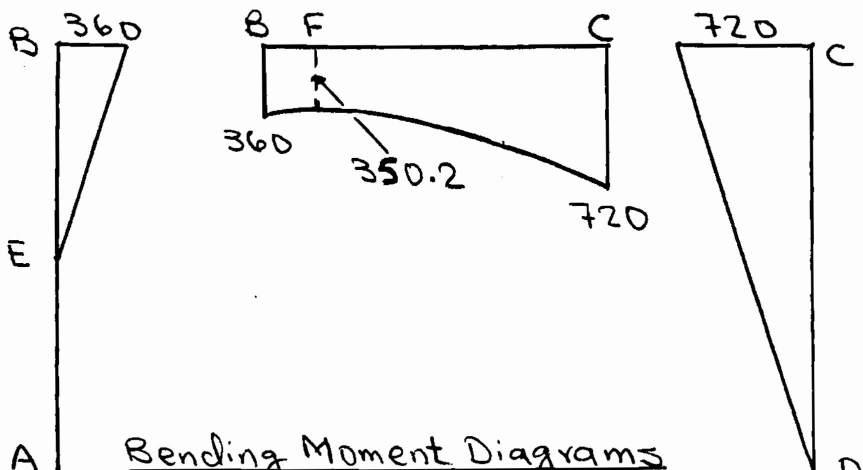
5.64



Member End Forces

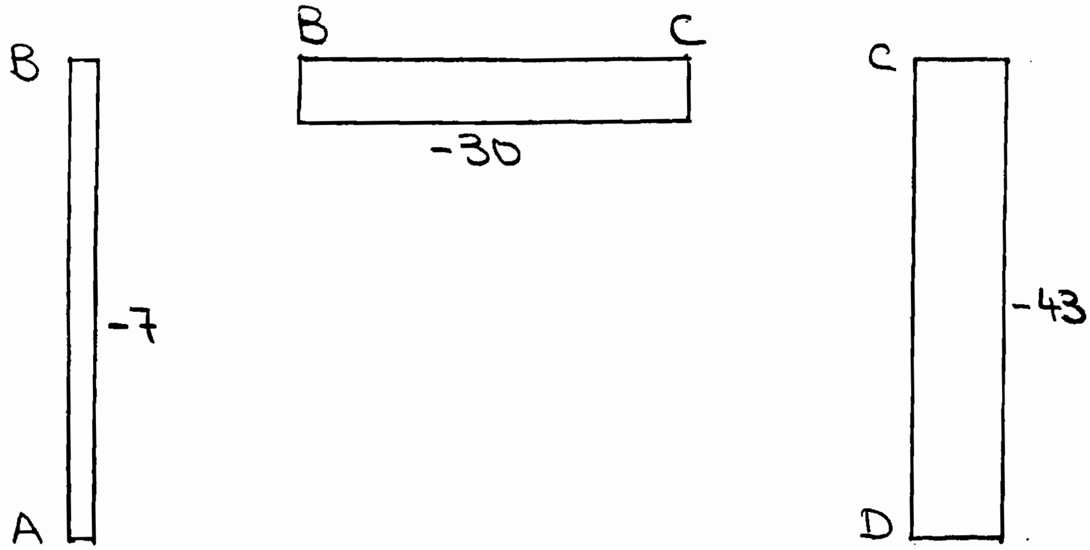


Shear Diagrams (k)

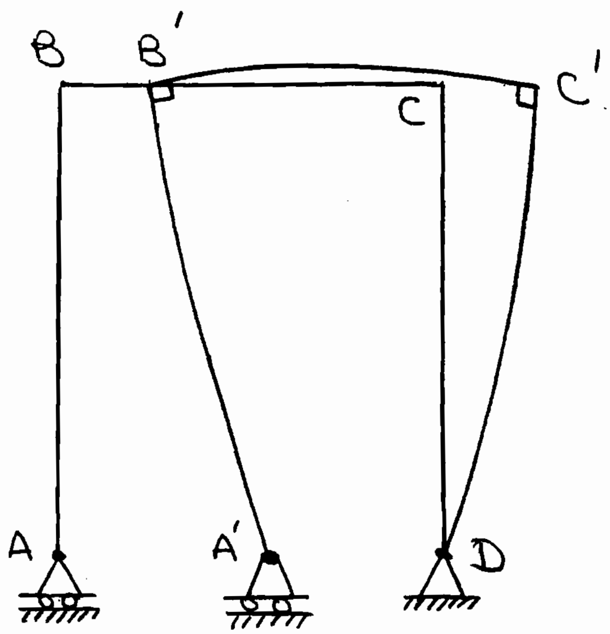


Bending Moment Diagrams (k-ft)

S-64 (contd.)

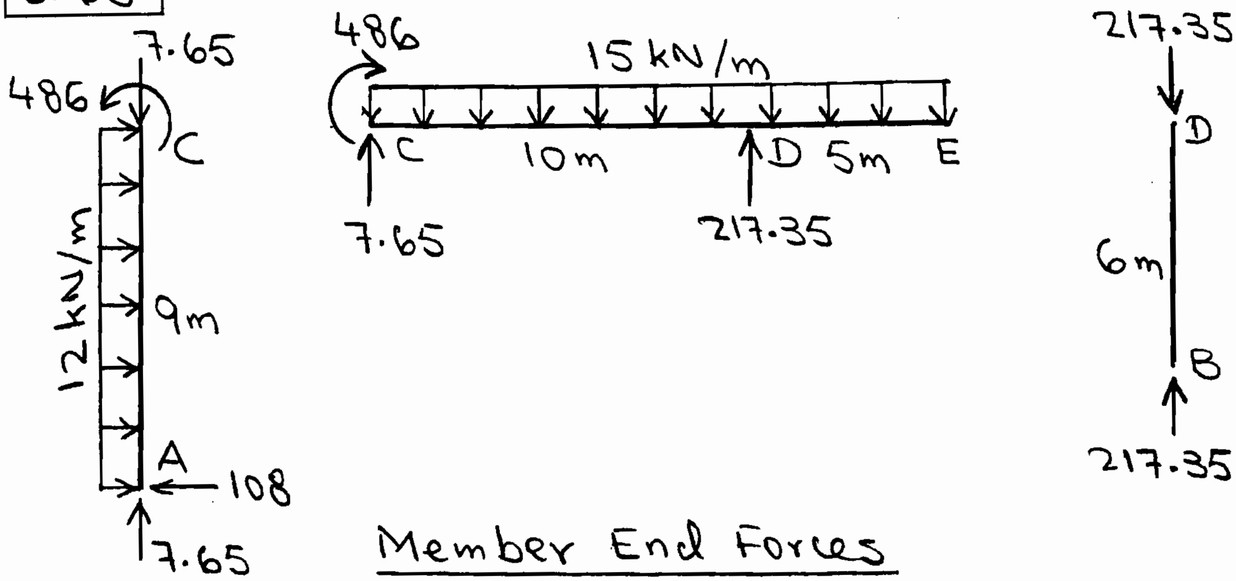


Axial Force Diagrams (k)

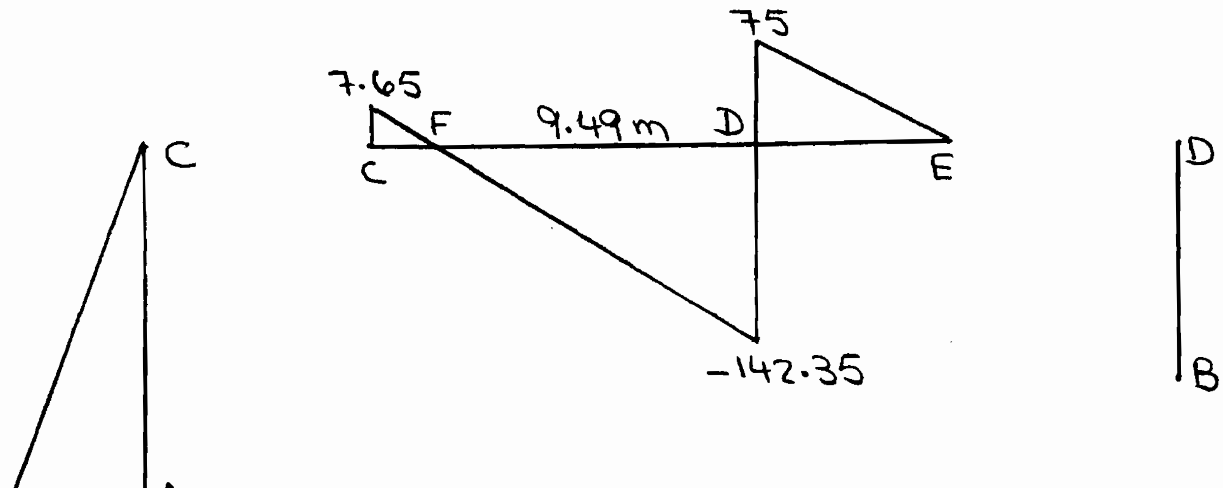


Qualitative Deflected Shape

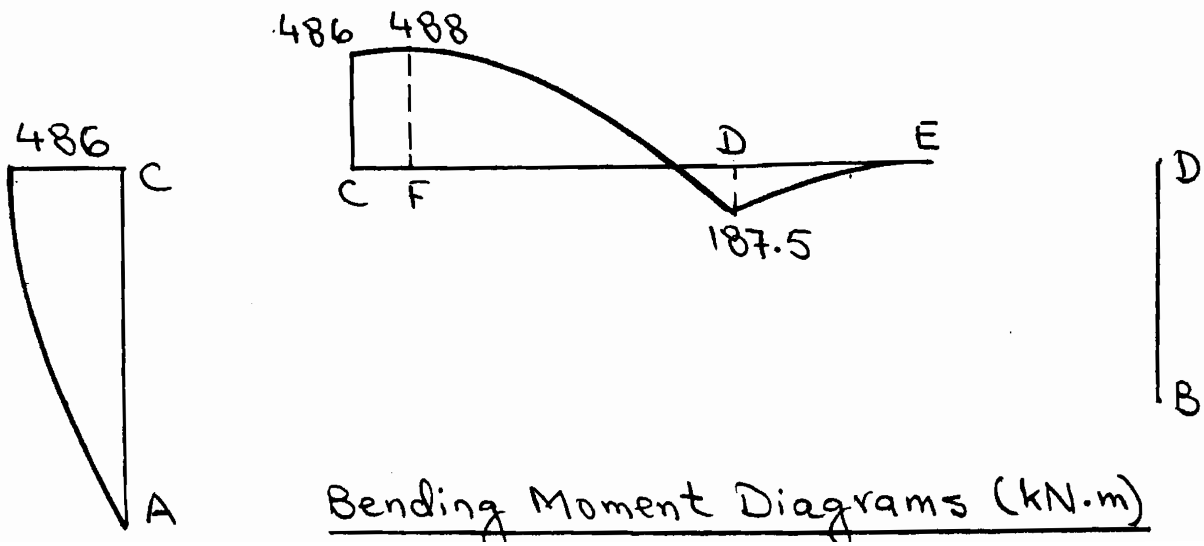
5.65



Member End Forces

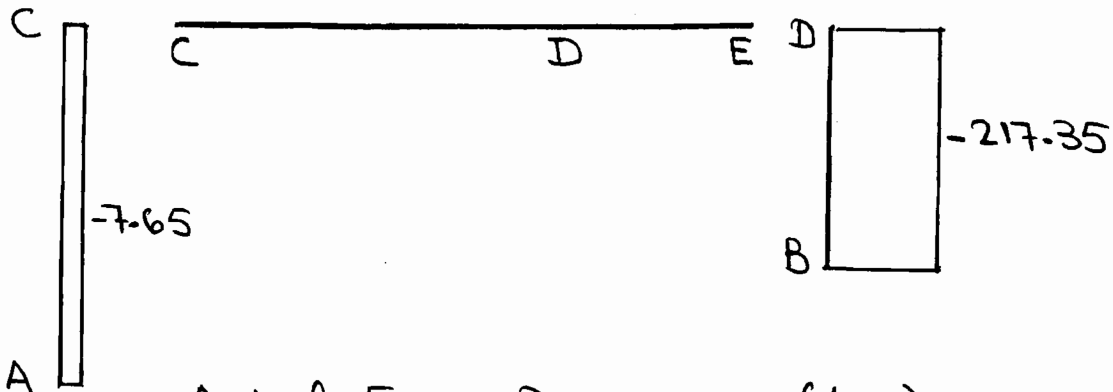


Shear Diagrams (kN)

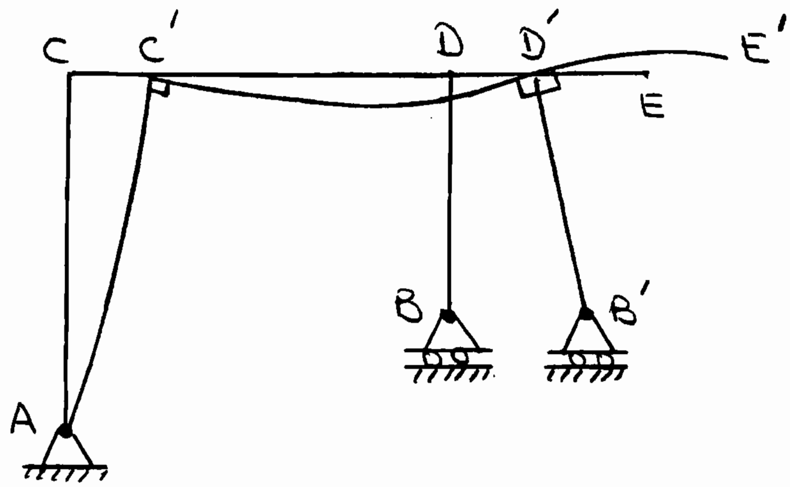


Bending Moment Diagrams (kN.m)

5.65 (contd.)

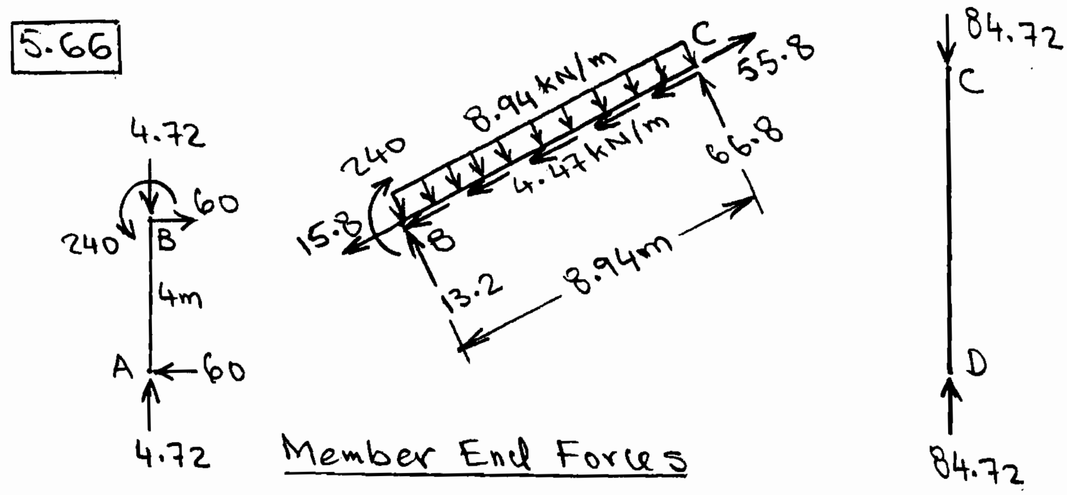


Axial Force Diagrams (kN)

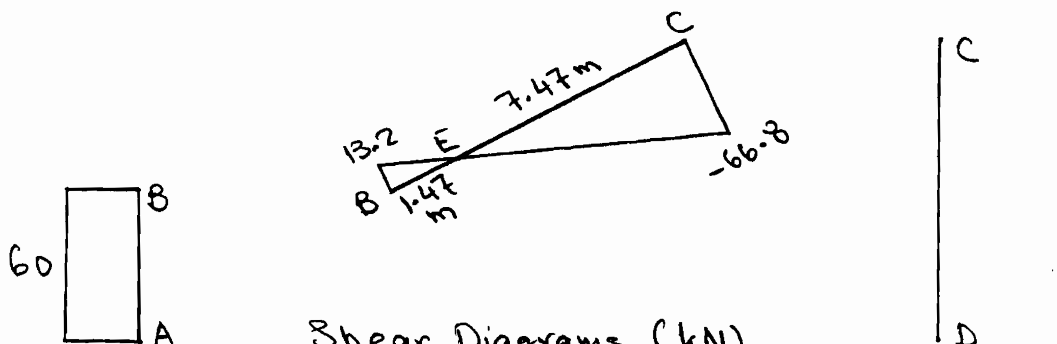


Qualitative Deflected Shape

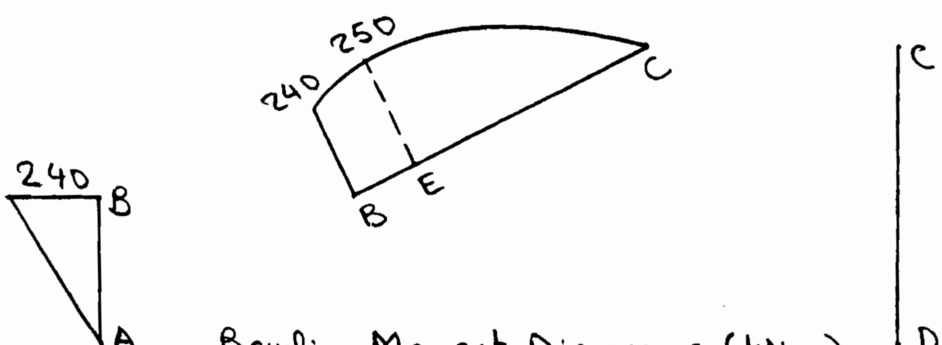
$\boxed{5.66}$



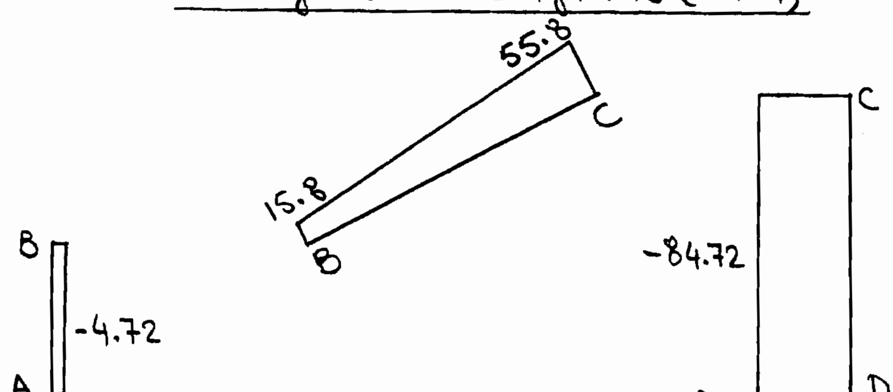
Member End Forces



Shear Diagrams (kN)

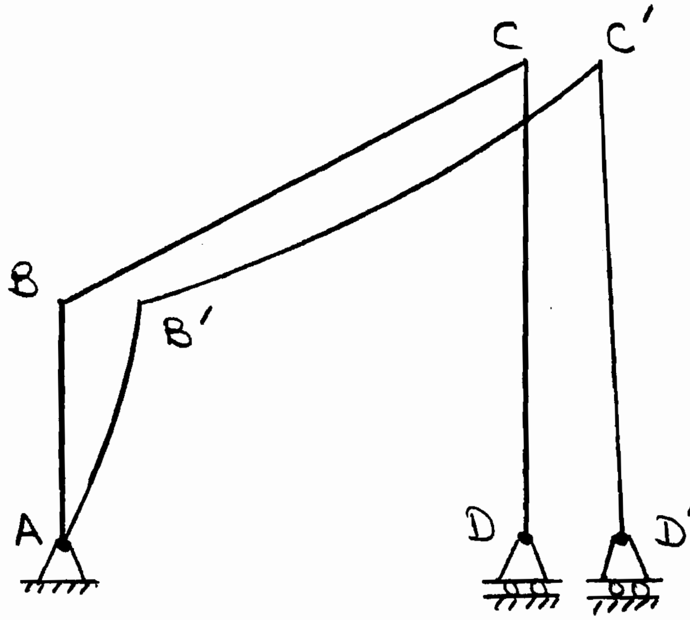


Bending Moment Diagrams (kN-m)



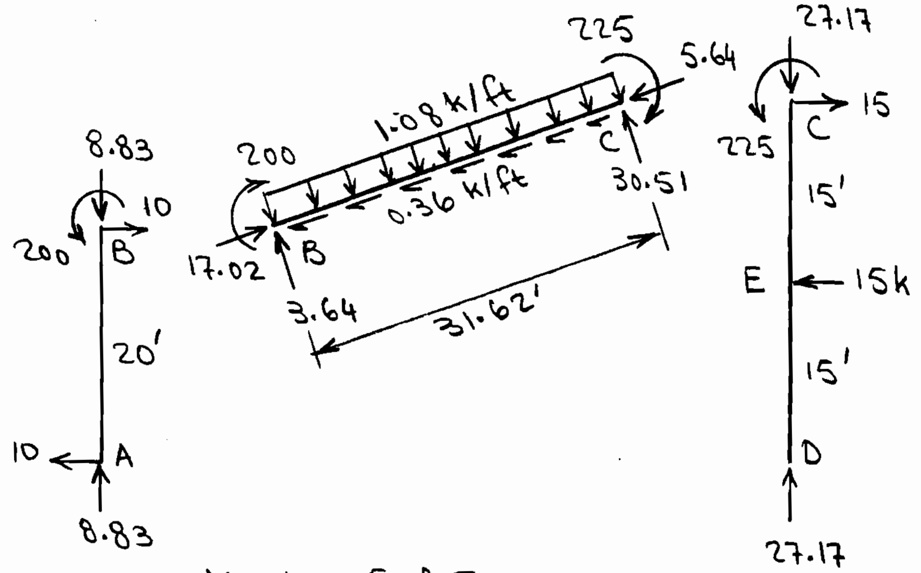
Axial Force Diagrams (kN)

5-66 (contd.)

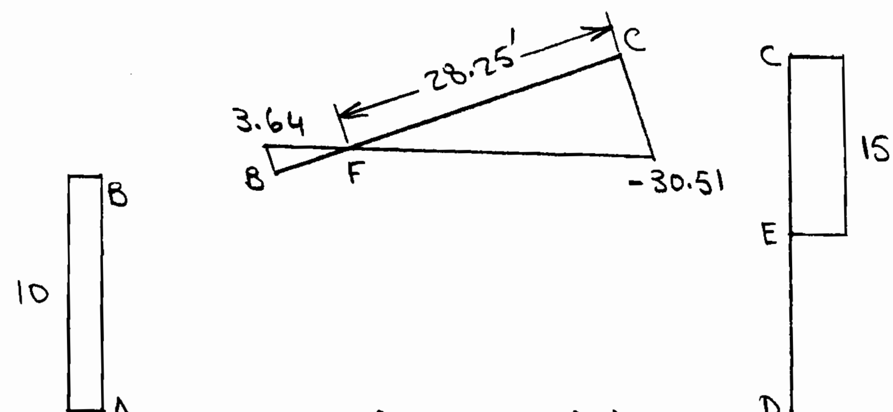


Qualitative Deflected Shape

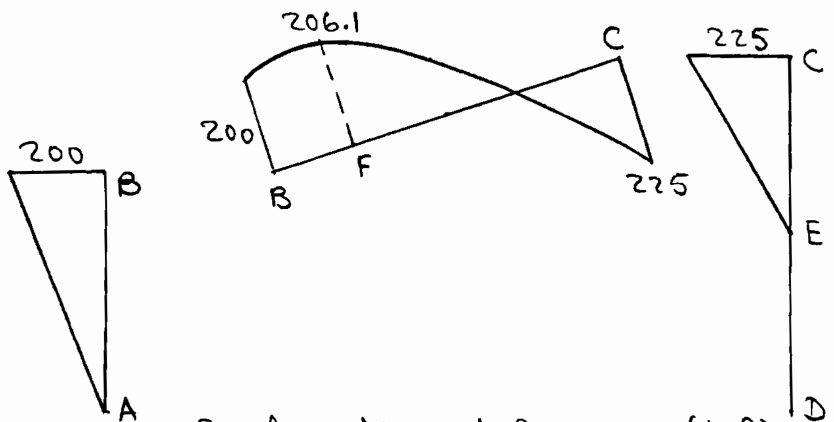
5.67



Member End Forces

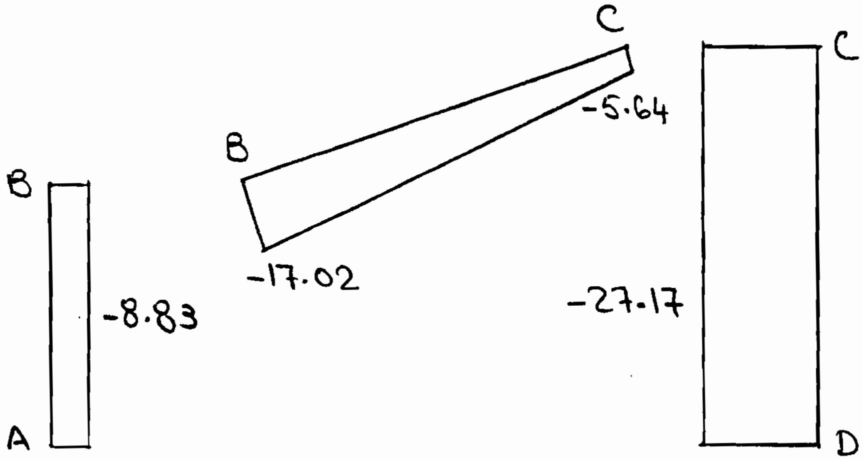


Shear Diagrams (k)

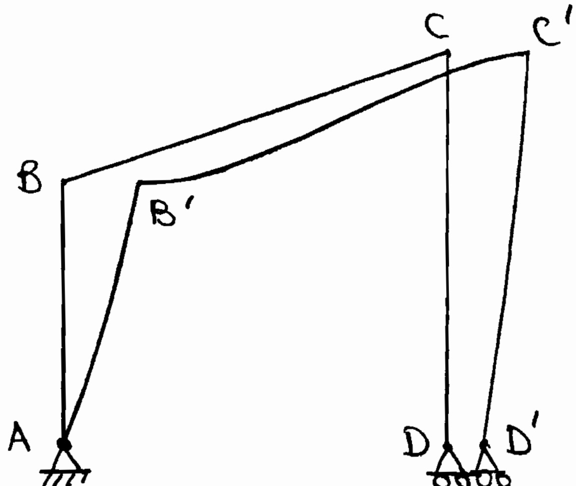


Bending Moment Diagrams (k-ft)

5.67 (Contd.)

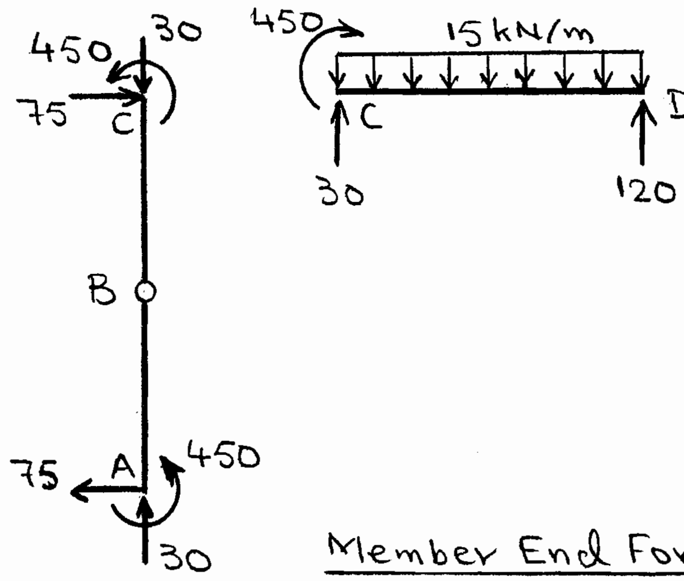


Axial Force Diagrams (k)

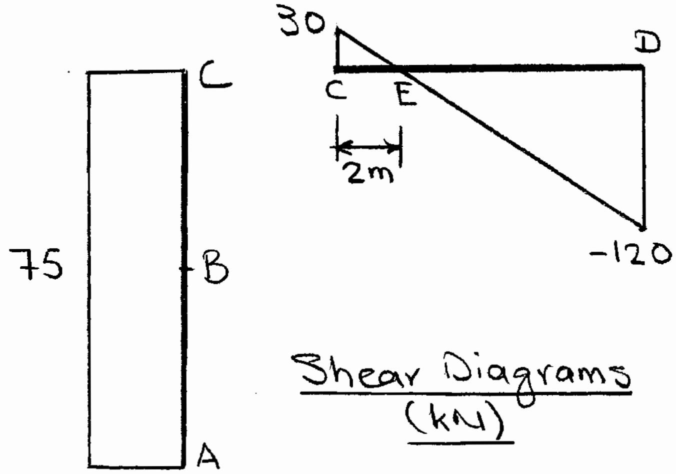


Qualitative Deflected Shape

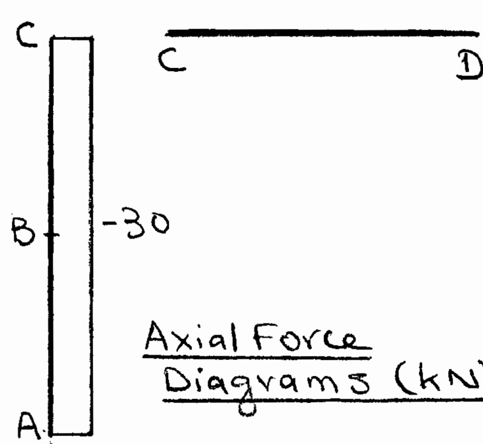
5.68



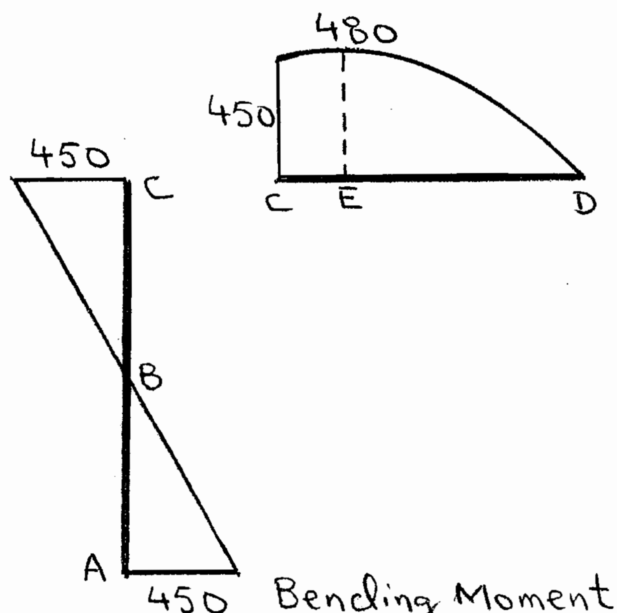
Member End Forces



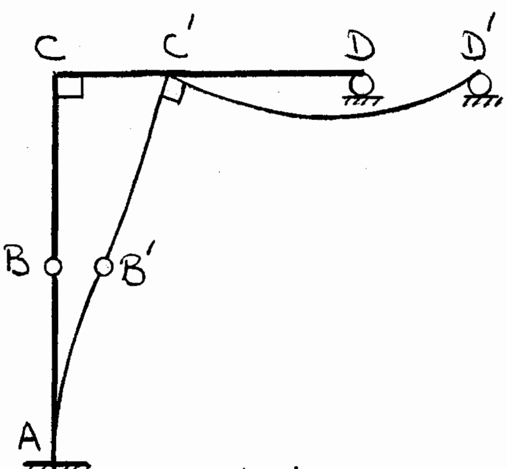
Shear Diagrams (kN)



Axial Force Diagrams (kN)

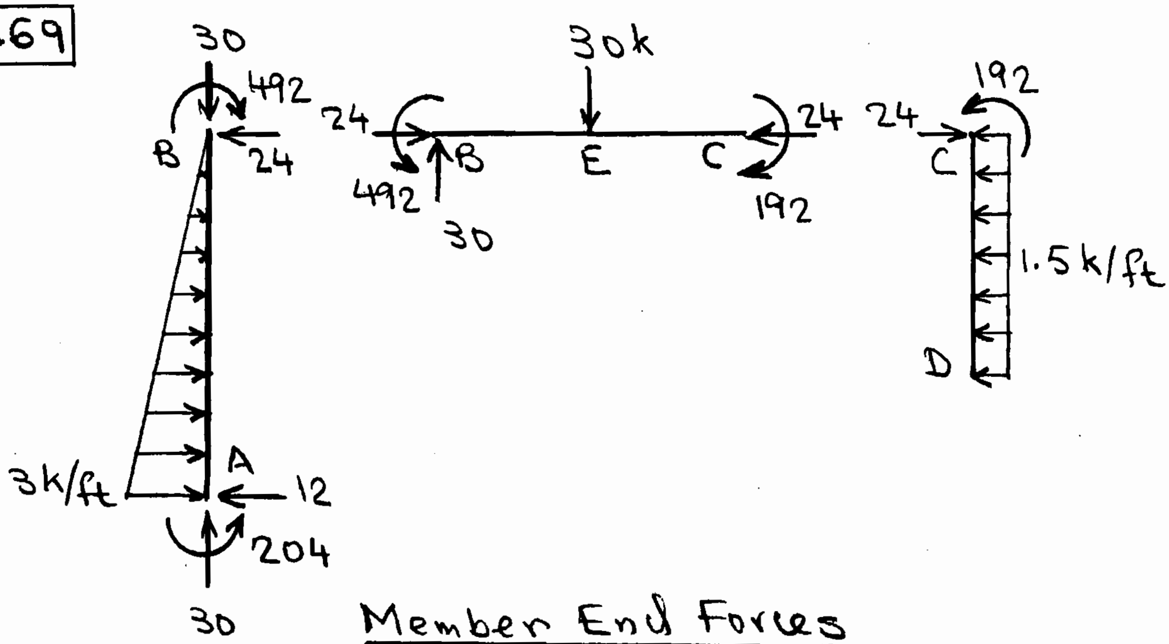


Bending Moment Diagrams (kN.m)

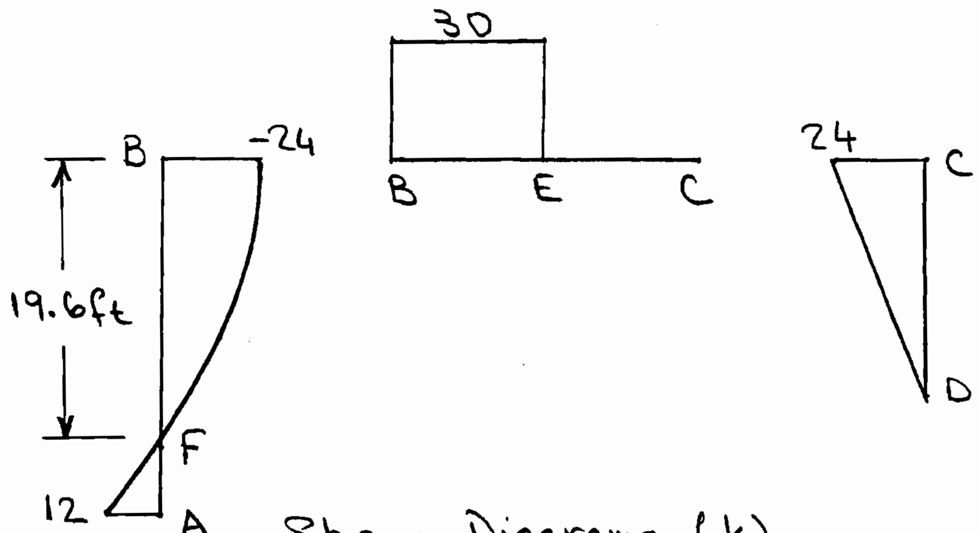


Qualitative Deflected Shape

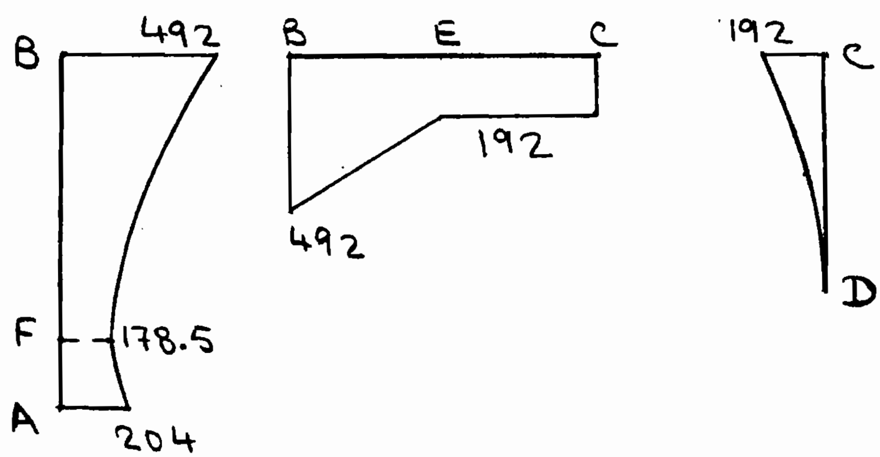
5.69



Member End Forces

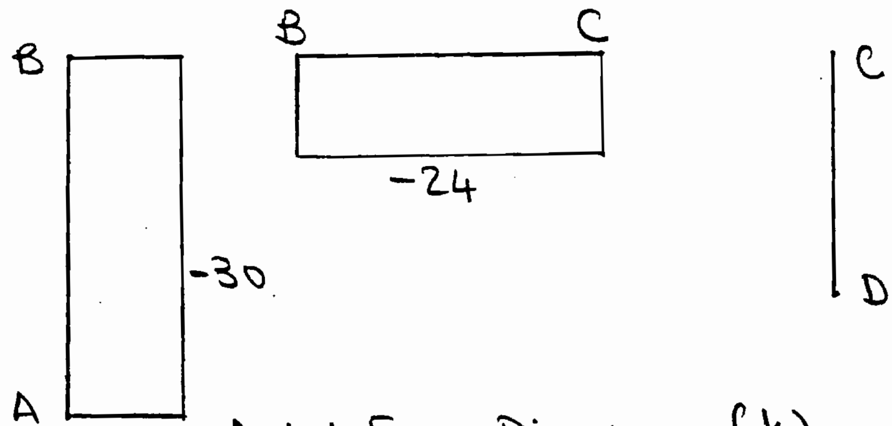


Shear Diagrams (k)

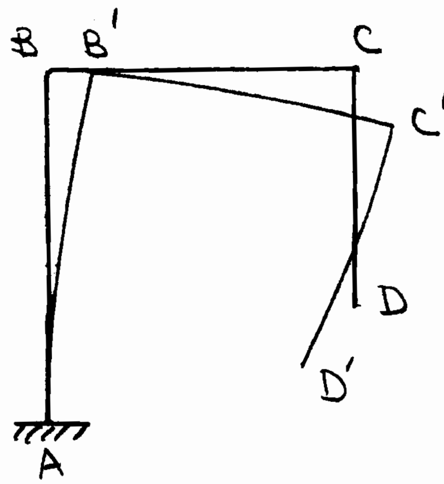


Bending Moment Diagrams (k-ft)

5.69 (Contd.)

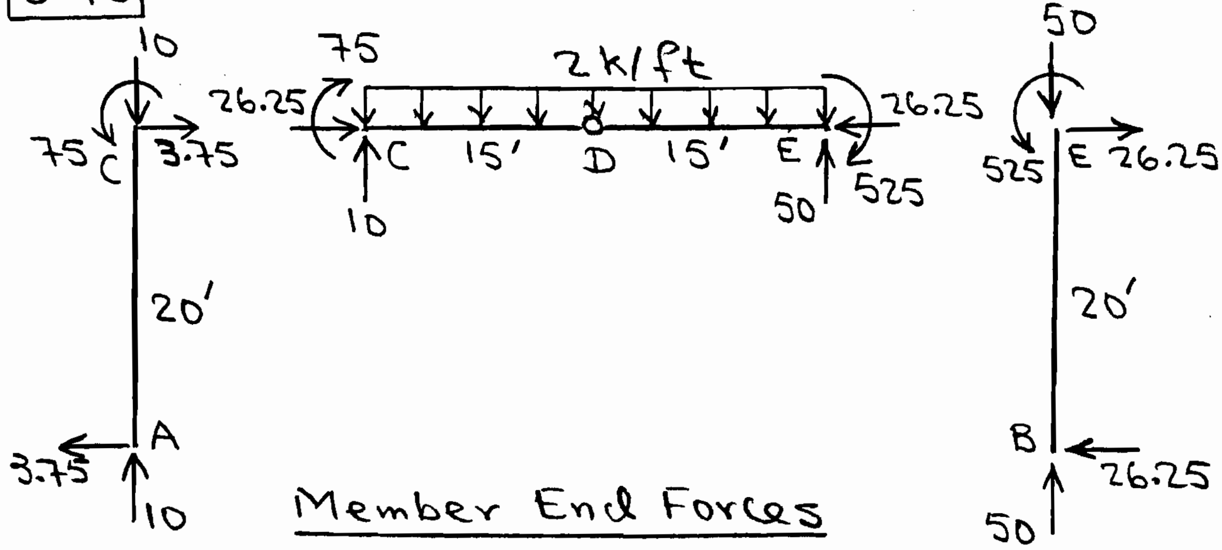


Axial Force Diagrams (k)

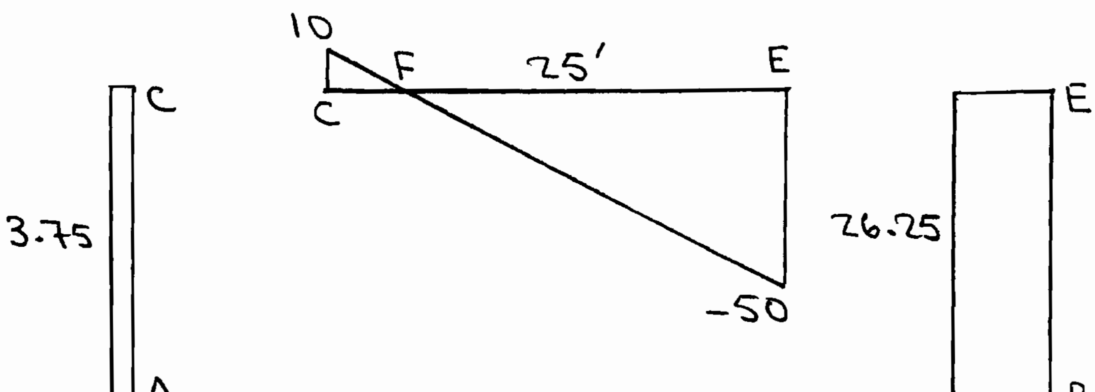


Qualitative Deflected Shape

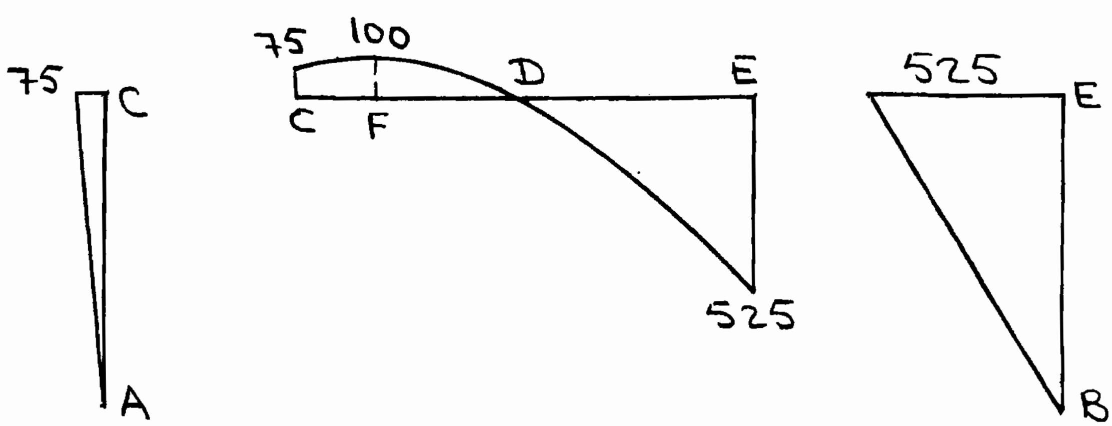
5.70



Member End Forces

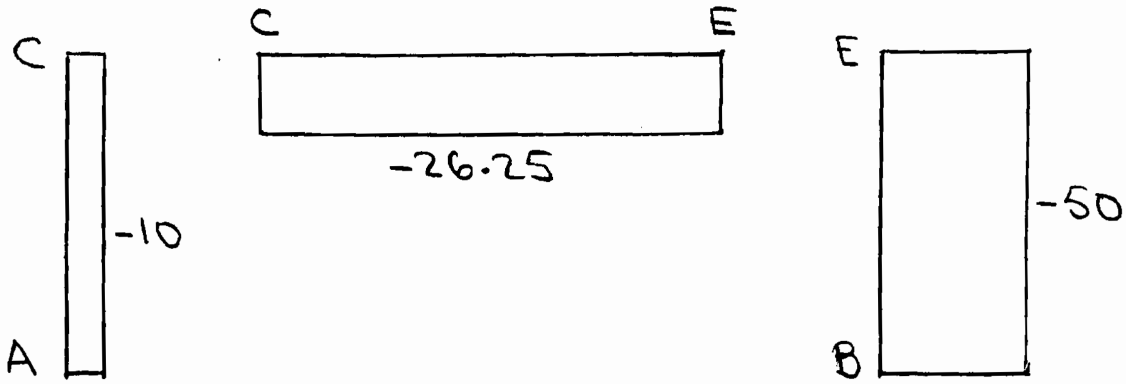


Shear Diagrams (k)

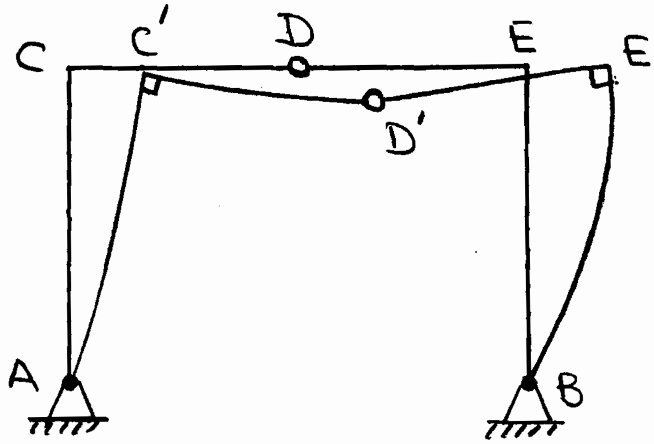


Bending Moment Diagrams (k-ft)

5.70 (contd.)

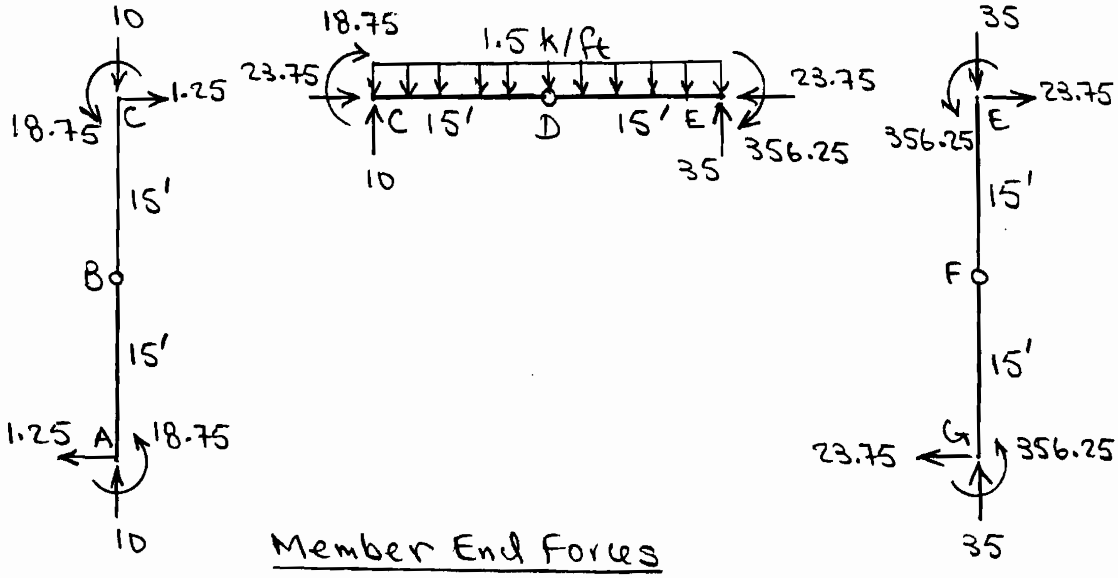


Axial Force Diagrams (k)

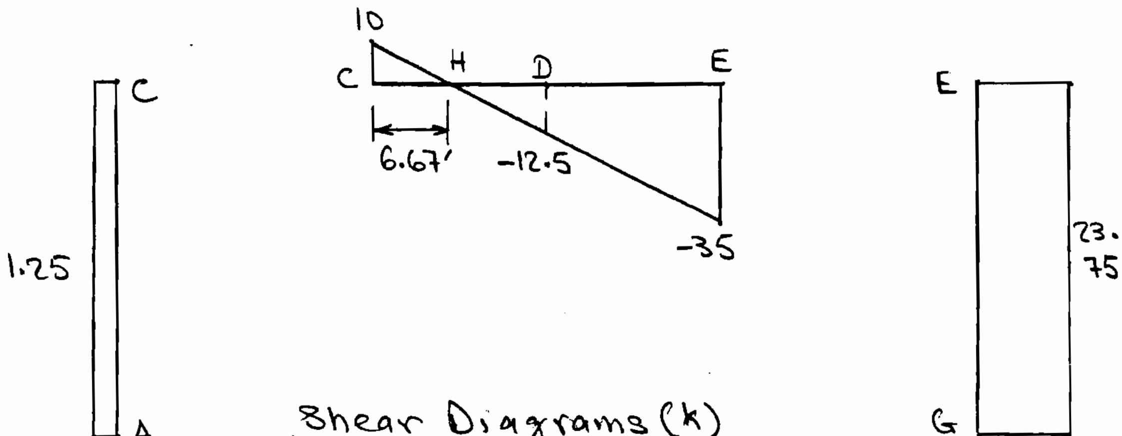


Qualitative Deflected Shape

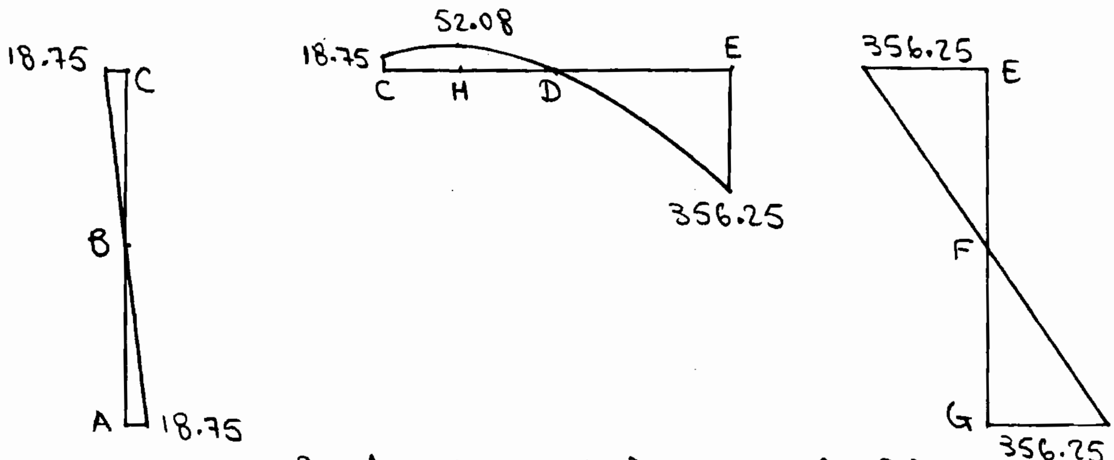
5.71



Member End Forces

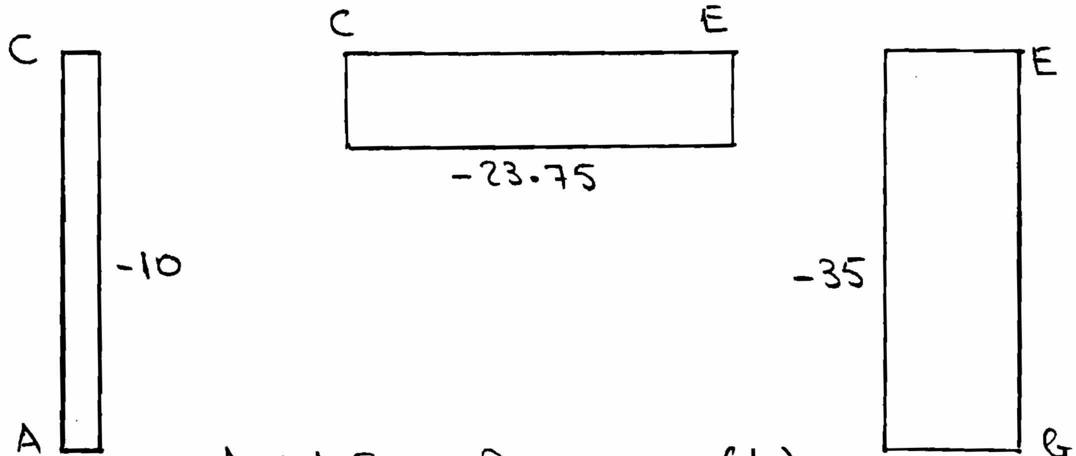


Shear Diagrams (k)

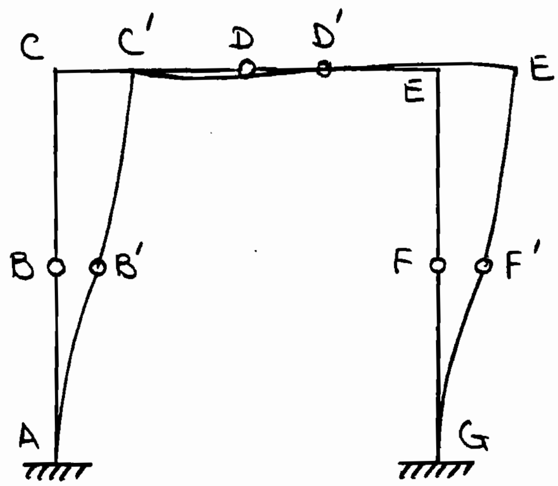


Bending Moment Diagrams (k-ft)

5.71 (Contd.)



Axial Force Diagrams (k)

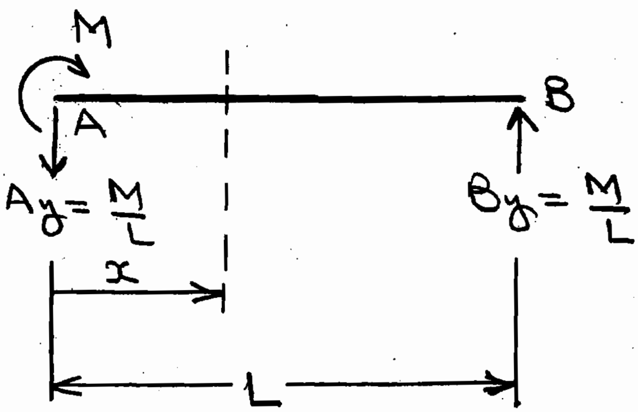


Qualitative Deflected Shape

Chapter Six
Deflections of Beams: Geometric Methods

CHAPTER 6

$\boxed{6.1}$



$$M_x = M - \frac{M}{L}(x) = \frac{M}{L}(L - x)$$

$$\frac{d^2y}{dx^2} = \frac{M}{EIL}(L-x)$$

$$\theta = \frac{dy}{dx} = \int \frac{M}{EIL}(L-x)\,dx = \frac{M}{EIL}\left(Lx - \frac{x^2}{2}\right) + C_1$$

$$y = \int \left[\frac{M}{EIL}\left(Lx - \frac{x^2}{2}\right) + C_1\right] dx$$

$$= \frac{M}{EIL}\left(\frac{Lx^2}{2} - \frac{x^3}{6}\right) + C_1 x + C_2$$

Boundary Conditions:

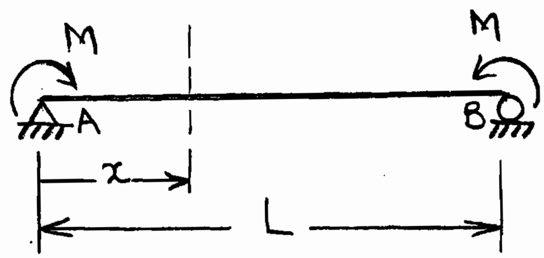
at $x = 0$, $y = 0$ $\quad\therefore C_2 = 0$

at $x = L$, $y = 0$ $\quad\therefore C_1 = -\frac{ML}{3EI}$

$$\underline{\theta = -\frac{M}{6EIL}(3x^2 - 6Lx + 2L^2)}$$

$$\underline{y = -\frac{M}{6EIL}(x^3 - 3Lx^2 + 2L^2x)}$$

$\boxed{6.2}$



$$M_x = M$$

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

$$\theta = \frac{dy}{dx} = \int \frac{M}{EI}\,dx = \frac{Mx}{EI} + C_1$$

$$y = \int \left(\frac{Mx}{EI} + C_1\right)dx = \frac{Mx^2}{2EI} + C_1x + C_2$$

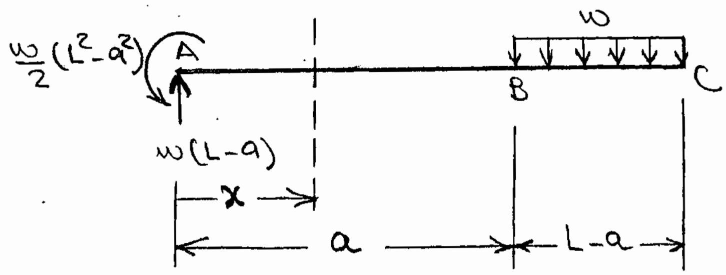
Boundary conditions:

at $x = 0$, $y = 0$ $\quad\therefore C_2 = 0$

at $x = L$, $y = 0$ $\quad\therefore C_1 = -\frac{ML}{2EI}$

$$\underline{\theta = \frac{M}{EI}\left(x - \frac{L}{2}\right)} \; ; \qquad \underline{y = \frac{Mx}{2EI}(x - L)}$$

6.3



Segment AB: $0 \leqslant x \leqslant a$

$$M = -\frac{w}{2}(L^2-a^2) + w(L-a)x$$

$$\frac{d^2y}{dx^2} = \frac{1}{EI}\left[-\frac{w}{2}(L^2-a^2) + w(L-a)x\right]$$

$$EI\theta = -\frac{w}{2}(L^2-a^2)x + w(L-a)\frac{x^2}{2} + C_1$$

$$EIy = -\frac{w}{2}(L^2-a^2)\frac{x^2}{2} + w(L-a)\frac{x^3}{6} + C_1x + C_2$$

Using the boundary conditions, $\theta = 0$ and $y = 0$ at $x = 0$, we obtain $C_1 = 0$ and $C_2 = 0$

Thus,

$$\underline{\theta = \frac{wx}{2EI}\left[a^2 - L^2 + (L-a)x\right]}$$

$$\underline{y = \frac{wx^2}{2EI}\left[\frac{a^2-L^2}{2} + \frac{(L-a)x}{3}\right]}$$

Segment BC: $a \leqslant x \leqslant L$

$$M = -\frac{w}{2}(L^2-a^2) + w(L-a)x - \frac{w}{2}(x-a)^2$$

$$EI\theta = -\frac{w}{2}(L^2-a^2)x + w(L-a)\frac{x^2}{2} - \frac{w}{2}\left(\frac{x^3}{3} + a^2x - ax^2\right) + C_3$$

$$EIy = -\frac{w}{2}(L^2-a^2)\frac{x^2}{2} + w(L-a)\frac{x^3}{6} - \frac{w}{2}\left(\frac{x^4}{12} + \frac{a x^2}{2} - \frac{a x^3}{3}\right) + C_3x + C_4$$

By using the conditions that at $x = a$, $\theta_{B,Left} = \theta_{B,Right}$ and $y_{B,Left} = y_{B,Right}$, we obtain:

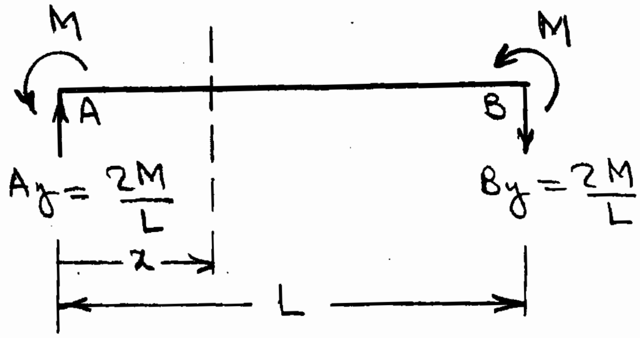
$$C_3 = \frac{wa^3}{6} \quad \text{and} \quad C_4 = -\frac{wa^4}{24}$$

Thus

$$\underline{\theta = \frac{w}{2EI}\left[xL(x-L) - \frac{x^3}{3} + \frac{a^3}{3}\right]}$$

$$y = \frac{w}{2EI}\left[x^2L\left(\frac{x}{3} - \frac{L}{2}\right) - \frac{x^4}{12} - \frac{a^4}{12} + \frac{a^3x}{3}\right]$$

$\boxed{6.4}$



$$M_x = \frac{2M}{L}x - M$$

$$\frac{d^2y}{dx^2} = \frac{M}{EI}\left(\frac{2x}{L} - 1\right)$$

$$\theta = \frac{dy}{dx} = \int \frac{M}{EI}\left(\frac{2x}{L} - 1\right)dx = \frac{Mx}{EI}\left(\frac{x}{L} - 1\right) + C_1$$

$$y = \int\left[\frac{Mx}{EI}\left(\frac{x}{L} - 1\right) + C_1\right]dx = \frac{Mx^2}{EI}\left(\frac{x}{3L} - \frac{1}{2}\right) + C_1x + C_2$$

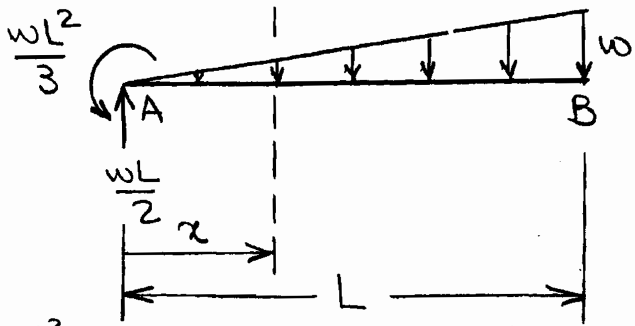
Boundary conditions:

at $x = 0$, $y = 0$ $\quad\therefore\ C_2 = 0$

at $x = L$, $y = 0$ $\quad\therefore\ C_1 = \frac{ML}{6EI}$

$$\theta = \frac{M}{6EIL}\left(6x^2 - 6Lx + L^2\right); \quad y = \frac{Mx}{6EIL}\left(2x^2 - 3Lx + L^2\right)$$

$\boxed{6.5}$



$$M = -\frac{wL^2}{3} + \frac{wL}{2}x - \frac{1}{2}x\left(\frac{wx}{L}\right)\frac{x}{3}$$

$$EI\frac{d^2y}{dx^2} = -\frac{wL^2}{3} + \frac{wL}{2}x - \frac{wx^3}{6L}$$

$$EI\theta = -\frac{wL^2}{3}x + \frac{wL}{2}\left(\frac{x^2}{2}\right) - \frac{w}{6L}\left(\frac{x^4}{4}\right) + C_1$$

$$EIy = -\frac{wL^2}{3}\left(\frac{x^2}{2}\right) + \frac{wL}{2}\left(\frac{x^3}{6}\right) + \frac{w}{6L}\left(\frac{x^5}{20}\right) + C_1x + C_2$$

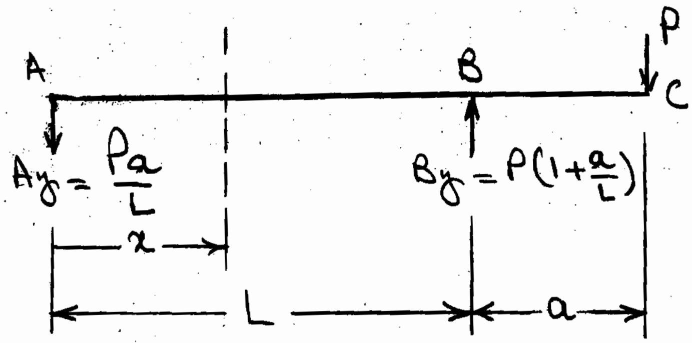
Using the boundary conditions that at $x=0$, $\theta=0$ and $y=0$, we obtain $C_1=0$ and $C_2=0$.

Thus,

$$\underline{\theta = \frac{wx}{24EIL}\left(-8L^3 + 6L^2x - x^3\right)}$$

$$\underline{y = \frac{wx^2}{120EIL}\left(-20L^3 + 10L^2x - x^3\right)}$$

$\boxed{6.6}$



Segment AB: $0 \leqslant x \leqslant L$

$$M = -\frac{Pax}{L}$$

$$\frac{d^2y}{dx^2} = -\frac{Pax}{EIL}$$

$$\theta = -\frac{Pa}{EIL}\left(\frac{x^2}{2}\right) + C_1$$

$$y = -\frac{Pa}{2EIL}\left(\frac{x^3}{3}\right) + C_1 x + C_2$$

Applying the boundary conditions, $y=0$ at $x=0$ and $y=0$ at $x=L$, we obtain

$$C_2 = 0 \qquad C_1 = \frac{PaL}{6EI}$$

Thus,

$$\underline{\theta = \frac{PaL}{6EI}\left(1 - 3\frac{x^2}{L^2}\right)}; \quad \underline{y = \frac{PaLx}{6EI}\left(1 - \frac{x^2}{L^2}\right)}$$

Segment BC: $L \leqslant x \leqslant (L+a)$

$$M = -P(L+a-x)$$

$$\frac{d^2y}{dx^2} = -\frac{P}{EI}(L+a-x)$$

$$\theta = -\frac{P}{EI}\left(Lx + ax - \frac{x^2}{2}\right) + C_3$$

$$y = -\frac{P}{EI}\left(\frac{Lx^2}{2} + \frac{ax^2}{2} - \frac{x^3}{6}\right) + C_3 x + C_4$$

6.6 (Contd.)

By using the condition that at $x = L$,
$\theta_{B,Left} = \theta_{B,Right}$, we obtain

$$C_3 = \frac{PL}{EI}\left(\frac{L}{2} + \frac{2a}{3}\right)$$

Next, by applying the condition that at $x = L$, $y = 0$, we obtain

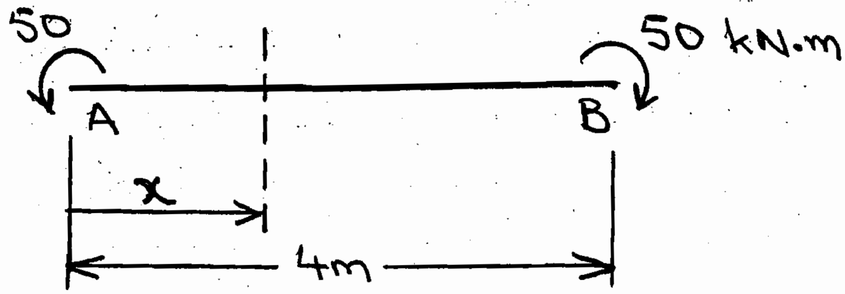
$$C_4 = -\frac{PL^2}{6EI}(L+a)$$

Thus,

$$\underline{\theta = \frac{PL}{EI}\left[\frac{x^2}{2L} - x\left(1+\frac{a}{L}\right) + \frac{L}{2} + \frac{2a}{3}\right]}$$

$$\underline{y = \frac{PL}{EI}\left[\frac{x^3}{6L} - \frac{x^2}{2}\left(1+\frac{a}{L}\right) + x\left(\frac{L}{2} + \frac{2a}{3}\right) - \frac{L}{6}(L+a)\right]}$$

6.7



$M_x = -50$

$$\frac{d^2y}{dx^2} = -\frac{50}{EI}$$

$$\theta = \frac{dy}{dx} = -\frac{50}{EI}x + C_1$$

$$y = -\frac{25}{EI}x^2 + C_1 x + C_2$$

Boundary Conditions:

at $x = 0$, $\theta = 0$ $\therefore C_1 = 0$

at $x = 0$, $y = 0$ $\therefore C_2 = 0$

Thus,

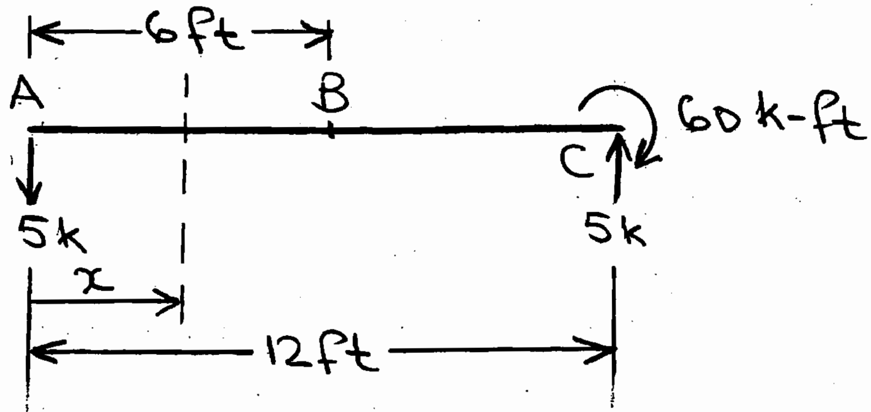
$$\theta = -\frac{50x}{EI} \quad ; \quad y = -\frac{25x^2}{EI}$$

At $x = 4m$:

$$\theta = -\frac{50(4)}{70(164)} = -0.0174\,\text{rad} = \underline{0.0174\,\text{rad}}$$

$$y = -\frac{25(4)^2}{70(164)} = -0.0348\,m = \underline{34.8\,mm\downarrow}$$

$\boxed{6.8}$



$$M_x = -5x$$

$$\frac{d^2y}{dx^2} = -\frac{5x}{EI}$$

$$\theta = \frac{dy}{dx} = -\frac{5x^2}{2EI} + C_1$$

$$y = -\frac{5x^3}{6EI} + C_1 x + C_2$$

Boundary Conditions:

at $x = 0$, $y = 0$ $\therefore C_2 = 0$

at $x = 12'$, $y = 0$ $\therefore C_1 = \frac{120}{EI}$

Thus,

$$\theta = \frac{5}{2EI}(-x^2 + 48)$$

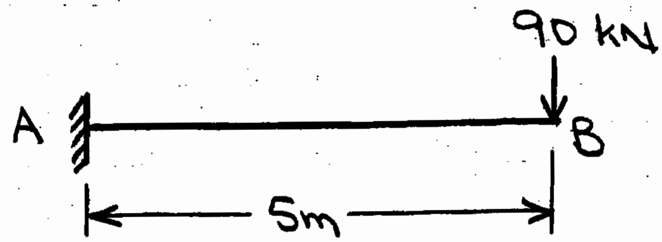
$$y = \frac{5x}{6EI}(-x^2 + 144)$$

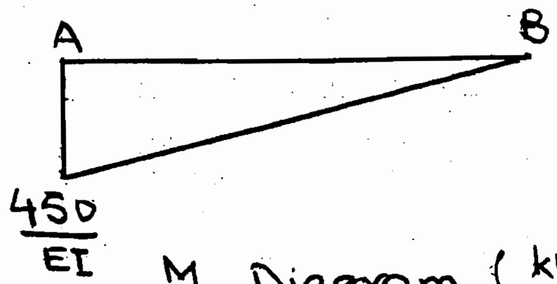
By substituting $x = 6$ ft, $E = 10000(144)$ ksf and $I = \frac{800}{(12)^4}$ ft.^{4}, we obtain

$\underline{\theta = 0.00054 \text{ rad}}$

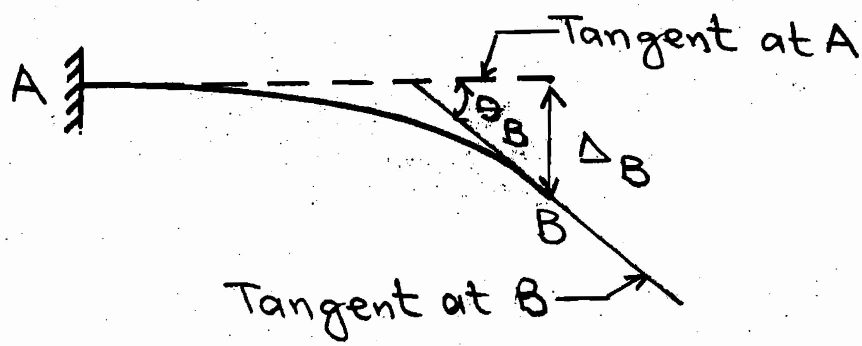
$y = 0.00972 \text{ ft} = \underline{0.117 \text{ in.} \uparrow}$

6.9





$\frac{M}{EI}$ Diagram $\left(\frac{kN.m}{EI}\right)$



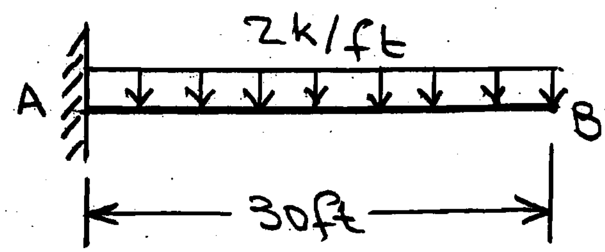
$$\theta_B = \theta_{BA} = \frac{1}{2}(5)\left(\frac{450}{EI}\right) = \frac{1125 \text{ kN.m}^2}{EI}$$

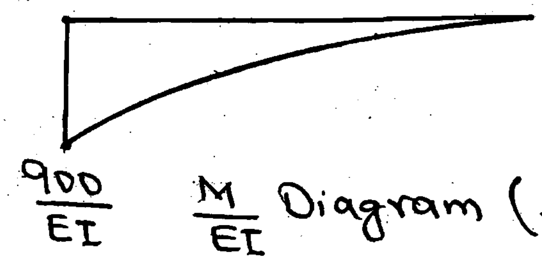
$$= \frac{1125}{200(800)} = \underline{0.00703 \text{ rad}}$$

$$\Delta_B = \Delta_{BA} = \frac{1125}{EI}\left(\frac{10}{3}\right) = \frac{3750 \text{ kN.m}^3}{EI}$$

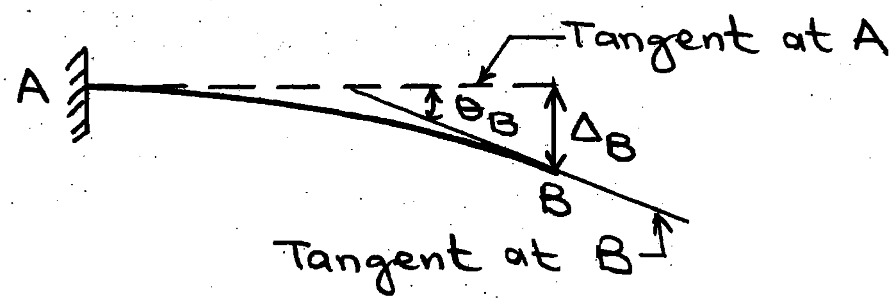
$$= \frac{3750}{200(800)} = 0.0234 \text{ m} = \underline{23.4 \text{ mm} \downarrow}$$

6.10





$\frac{M}{EI}$ Diagram $\left(\frac{k\text{-}ft}{EI}\right)$



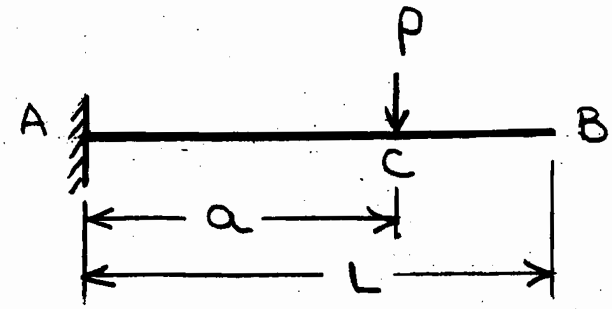
$$\theta_B = \theta_{BA} = \frac{1}{3}(30)\left(\frac{900}{EI}\right) = \frac{9000 \text{ k-ft}^2}{EI}$$

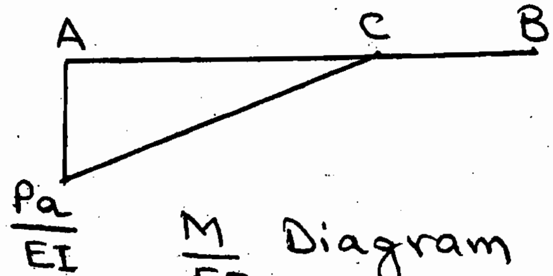
$$= \frac{9000(12)^2}{29000(3000)} = \underline{0.0149 \text{ rad}}$$

$$\Delta_B = \Delta_{BA} = \frac{9000}{EI}\left(\frac{90}{4}\right) = \frac{202500 \text{ k-ft}^3}{EI}$$

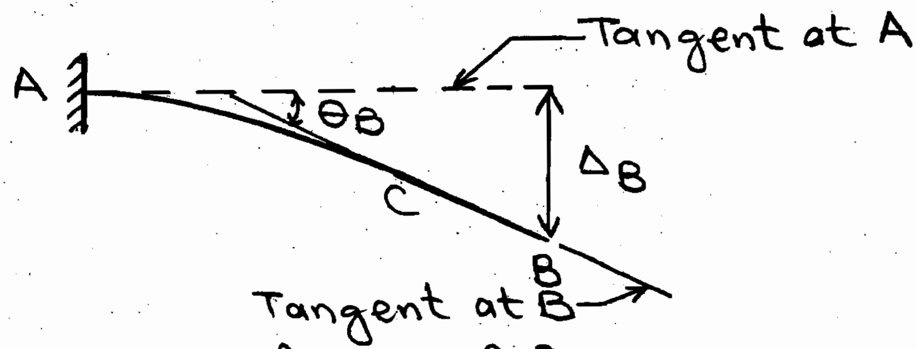
$$= \frac{202500(12)^3}{29000(3000)} = \underline{4.022 \text{ in.} \downarrow}$$

$\boxed{6.11}$





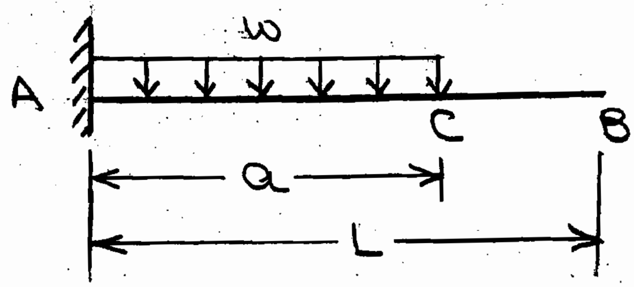
$\frac{M}{EI}$ Diagram



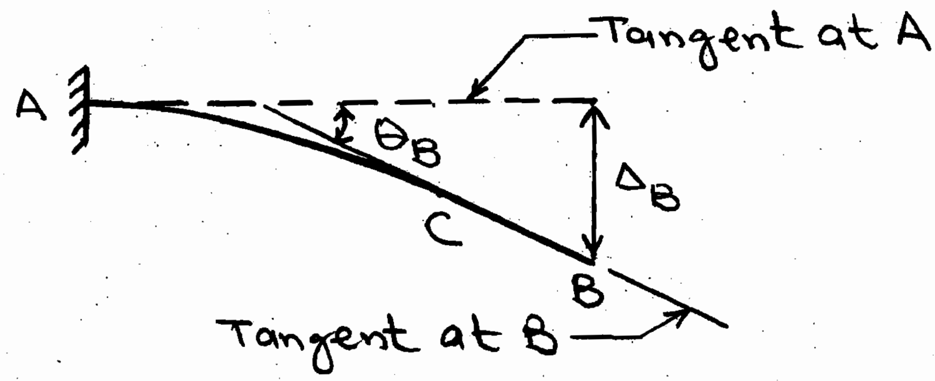
$$\theta_B = \theta_{BA} = \frac{1}{2}(a)\frac{Pa}{EI} = \frac{Pa^2}{2EI}$$

$$\Delta_B = \Delta_{BA} = \frac{Pa^2}{2EI}\left(L - \frac{a}{3}\right) = \frac{Pa^2}{6EI}(3L - a) \downarrow$$

6.12



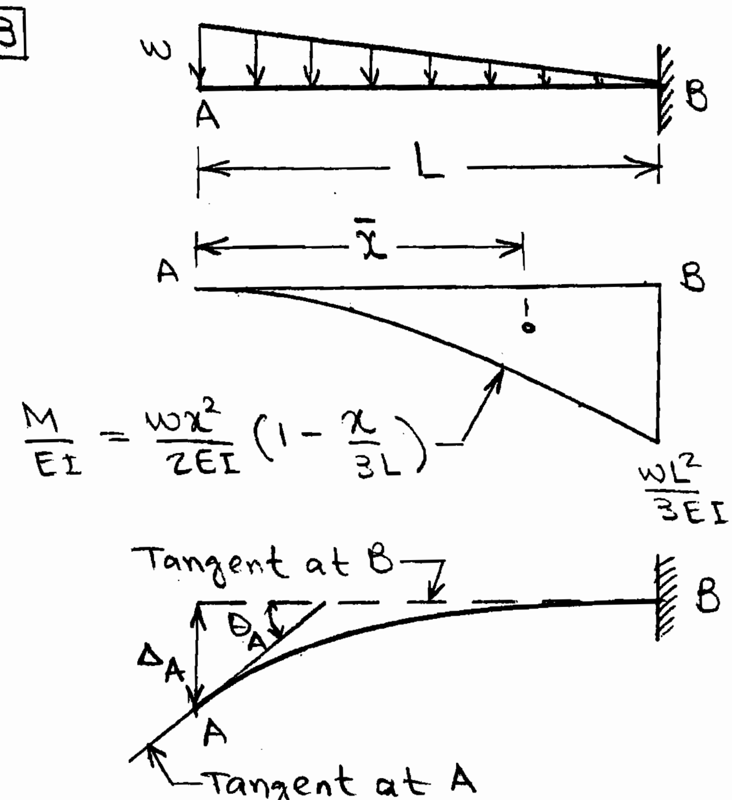
$\frac{wa^2}{2EI}$ $\quad$ $\underline{\frac{M}{EI}\text{ Diagram}}$



$$\theta_B = \theta_{BA} = \frac{1}{3}(a)\left(\frac{wa^2}{2EI}\right) = \underline{\frac{wa^3}{6EI}}$$

$$\Delta_B = \Delta_{BA} = \frac{wa^3}{6EI}\left(L - \frac{a}{4}\right) = \underline{\frac{wa^3}{24EI}(4L - a)\downarrow}$$

6.13

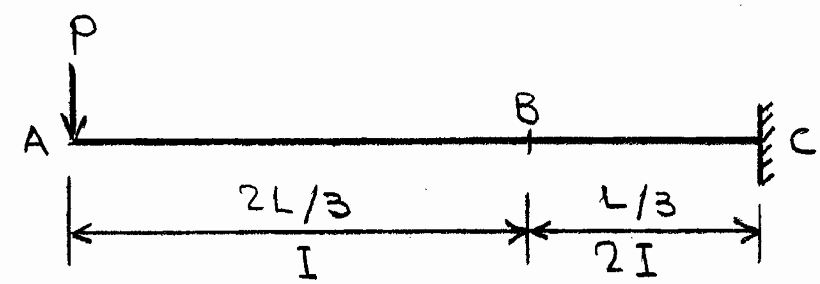


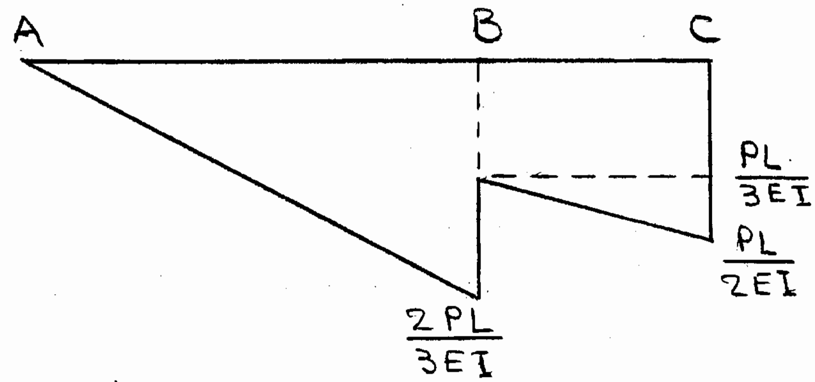
$$\theta_A = \theta_{AB} = \int_0^L \frac{M}{EI}\, dx = \int_0^L \frac{wx^2}{2EI}\left(1 - \frac{x}{3L}\right) dx = \underline{\frac{wL^3}{8EI}}$$

$$\Delta_A = \Delta_{AB} = \int_0^L \frac{M}{EI}(x)\, dx = \int_0^L \frac{wx^2}{2EI}\left(1 - \frac{x}{3L}\right) x\, dx$$

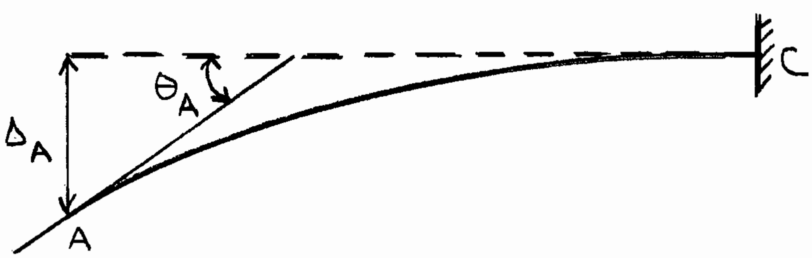
$$= \underline{\frac{11}{120}\frac{wL^4}{EI} \downarrow}$$

6.14





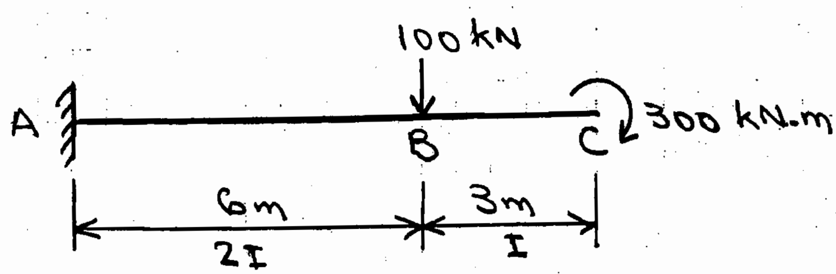
$\frac{M}{EI}$ Diagram



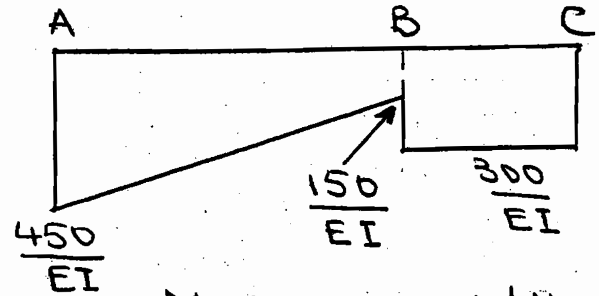
$$\theta_A = \theta_{AC} = \frac{1}{2}\left(\frac{2L}{3}\right)\left(\frac{2PL}{3EI}\right) + \frac{L}{3}\left(\frac{PL}{3EI}\right) + \frac{1}{2}\left(\frac{L}{3}\right)\left(\frac{PL}{6EI}\right) = \underline{\frac{13PL^2}{36EI}}$$

$$\Delta_A = \Delta_{AC} = \frac{2PL^2}{9EI}\left(\frac{4L}{9}\right) + \frac{PL^2}{9EI}\left(\frac{5L}{6}\right) + \frac{PL^2}{36EI}\left(\frac{8L}{9}\right) = \underline{\frac{35PL^3}{162EI}} \downarrow$$

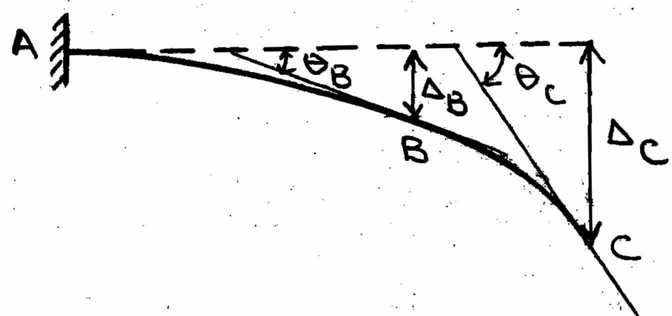
6.15



$I = 500(10^6)\ mm^4$



$\frac{M}{EI}$ Diagram $\left(\frac{kN.m}{EI}\right)$



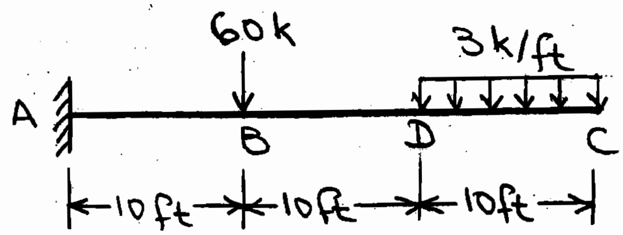
$$\theta_B = \theta_{BA} = \frac{1}{EI}\left[150(6) + \frac{1}{2}(300)6\right] = \frac{1800\ kN.m^2}{EI}$$
$$= \frac{1800}{70(500)} = \underline{0.0514\ rad}$$

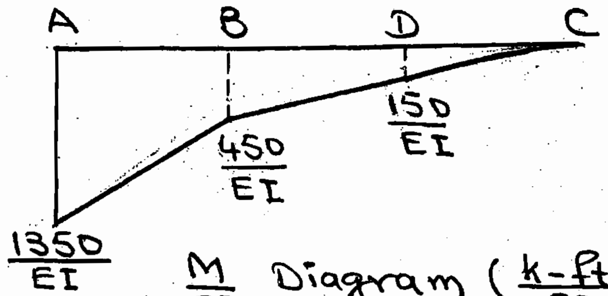
$$\Delta_B = \Delta_{BA} = \frac{1}{EI}\left[150(6)3 + \frac{1}{2}(300)(6)4\right] = \frac{6300\ kN.m^3}{EI}$$
$$= \frac{6300}{70(500)} = 0.18\ m = \underline{180\ mm \downarrow}$$

$$\theta_C = \theta_{CA} = \frac{1}{EI}\left[1800 + 300(3)\right] = \frac{2700\ kN.m^2}{EI}$$
$$= \frac{2700}{70(500)} = \underline{0.0771\ rad}$$

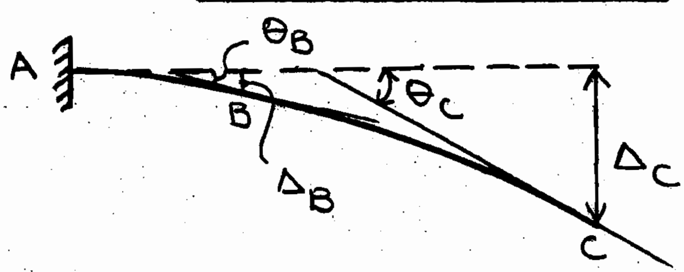
$$\Delta_C = \Delta_{CA} = \frac{1}{EI}\left[150(6)6 + \frac{1}{2}(300)(6)7 + 300(3)(1.5)\right]$$
$$= \frac{13050\ kN.m^3}{EI} = \frac{13050}{70(500)} = 0.373\ m = \underline{373\ mm \downarrow}$$

6.16





$\frac{M}{EI}$ Diagram $\left(\frac{k\text{-}ft}{EI}\right)$



$$\theta_B = \theta_{BA} = \frac{1}{EI}\left[450(10) + \frac{1}{2}(900)10\right] = \frac{9000\ k\text{-}ft^2}{EI}$$

$$= \frac{9000(12)^2}{29000(4000)} = \underline{0.0112\ \text{rad}}$$

$$\Delta_B = \Delta_{BA} = \frac{1}{EI}\left[450(10)5 + \frac{1}{2}(900)(10)\left(\frac{20}{3}\right)\right] = \frac{52500\ k\text{-}ft^3}{EI}$$

$$= \frac{52500(12)^3}{29000(4000)} = \underline{0.782\ \text{in.}\downarrow}$$

$$\theta_C = \theta_{CA} = \frac{1}{EI}\left[9000 + 150(10) + \frac{1}{2}(300)10) + \frac{1}{3}(150)10\right]$$

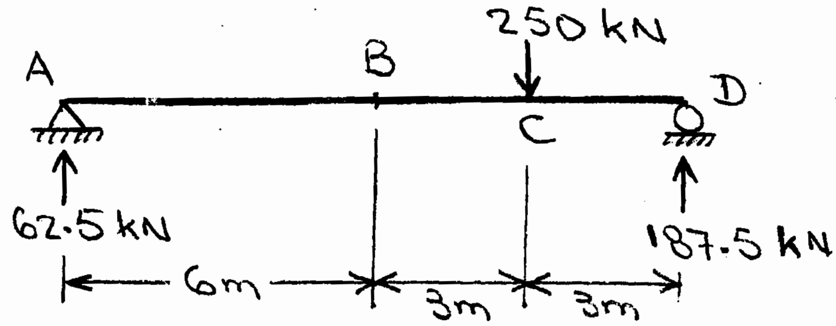
$$= \frac{12500\ k\text{-}ft^2}{EI} = \frac{12500(12)^2}{29000(4000)} = \underline{0.0155\ \text{rad}}$$

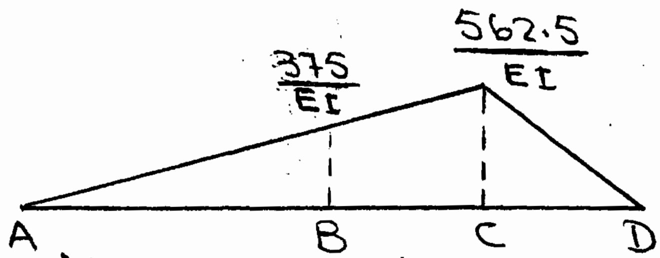
$$\Delta_C = \Delta_{CA} = \frac{1}{EI}\left[450(10)25 + \frac{1}{2}(900)10\left(\frac{20}{3}+20\right)\right.$$

$$\left. + 150(10)15 + \frac{1}{2}(300)10\left(\frac{20}{3}+10\right) + \frac{1}{3}(150)10\left(\frac{30}{4}\right)\right]$$

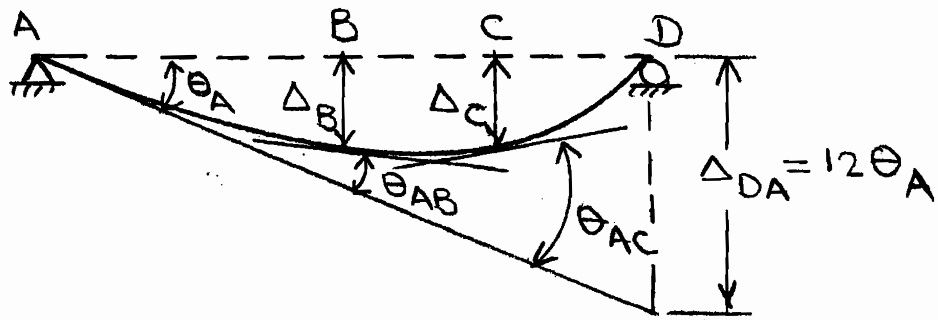
$$= \frac{283750\ k\text{-}ft^3}{EI} = \frac{283750(12)^3}{29000(4000)} = \underline{4.227\ \text{in.}\downarrow}$$

6.17





$\frac{M}{EI}$ Diagram $\left(\frac{kN.m}{EI}\right)$



$$\Delta_{DA} = \frac{1}{EI}\left[\frac{1}{2}(562.5)9(6) + \frac{1}{2}(562.5)3(2)\right] = \frac{16875}{EI}$$

$$\theta_A = \frac{\Delta_{DA}}{12} = \frac{1406.25}{EI}$$

$$\theta_B = \theta_A - \theta_{AB} = \frac{1}{EI}\left[1406.25 - \frac{1}{2}(375)6\right]$$

$$= \frac{281.25 \text{ kN.m}^2}{EI} = \frac{281.25}{200(462)} = \underline{0.00304 \text{ rad}}$$

$$\Delta_B = 6\theta_A - \Delta_{BA} = \frac{1}{EI}\left[6(1406.25) - \frac{1}{2}(375)6(2)\right]$$

$$= \frac{6187.5 \text{ kN.m}^3}{EI} = \frac{6187.5}{200(462)} = 0.067 \text{ m}$$

$$\underline{\Delta_B = 67 \text{ mm}\downarrow}$$

6.17 (contd.)

$$\theta_C = \theta_{AC} - \theta_A = \frac{1}{EI}\left[\frac{1}{2}(562.5)9 - 1406.25\right]$$

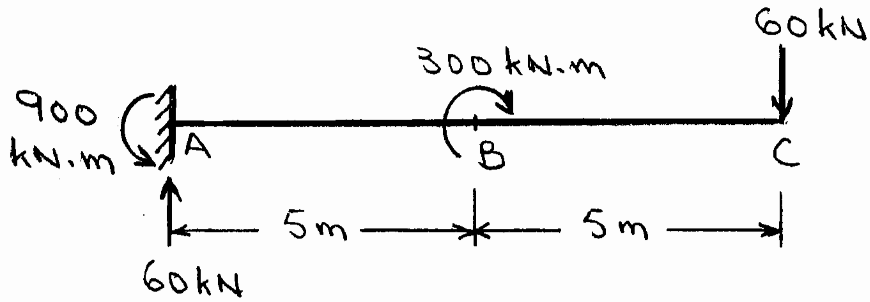
$$= \frac{1125 \text{ kN}\cdot\text{m}^2}{EI} = \frac{1125}{200(462)} = \underline{0.0122 \text{ rad}}$$

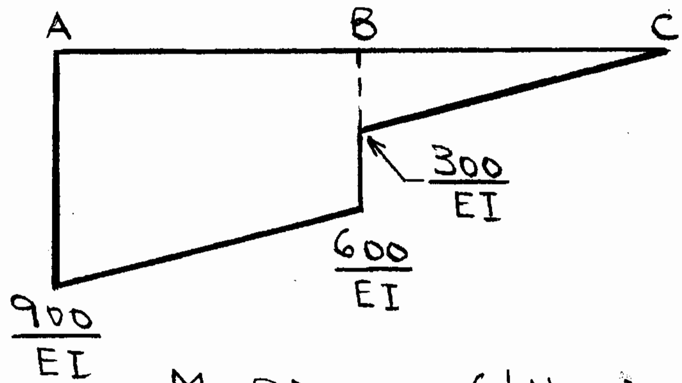
$$\Delta_C = 9\theta_A - \Delta_{CA} = \frac{1}{EI}\left[9(1406.25) - \frac{1}{2}(562.5)9(3)\right]$$

$$= \frac{5062.5 \text{ kN}\cdot\text{m}^3}{EI} = \frac{5062.5}{200(462)} = 0.0548 \text{ m}$$

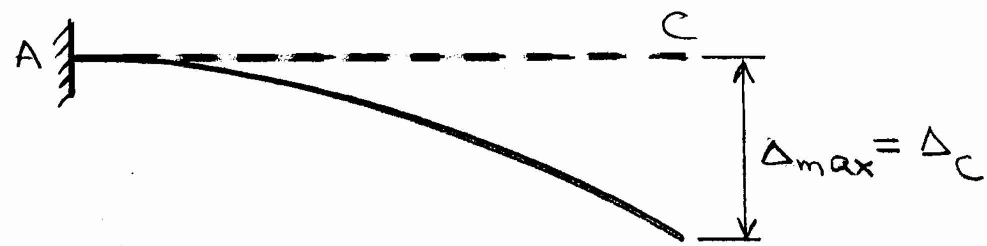
$$\underline{\Delta_C = 54.8 \text{ mm} \downarrow}$$

6.18





$\frac{M}{EI}$ Diagram $\left(\frac{kN.m}{EI}\right)$



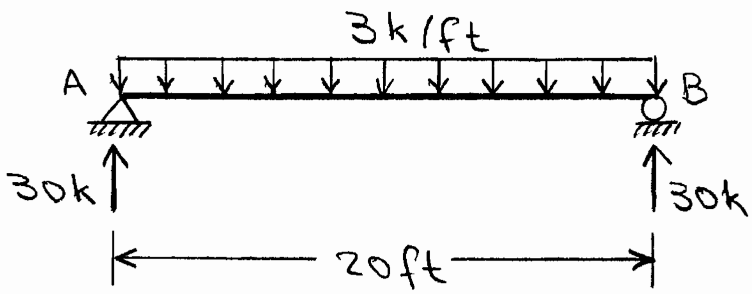
$$\Delta_{max} = \Delta_C = \Delta_{CA} = \frac{1}{EI}\left[600(5)(7.5) + \frac{1}{2}(300)(5)\left(\frac{25}{3}\right) + \frac{1}{2}(300)(5)\left(\frac{10}{3}\right)\right] = \frac{31250 \text{ kN.m}^3}{EI}$$

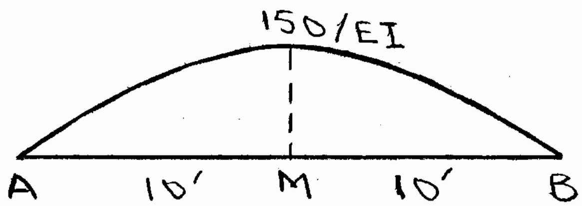
$$\Delta_{max} = \frac{L}{360}$$

$$\frac{31250}{200(10^6)I} = \frac{10}{360}$$

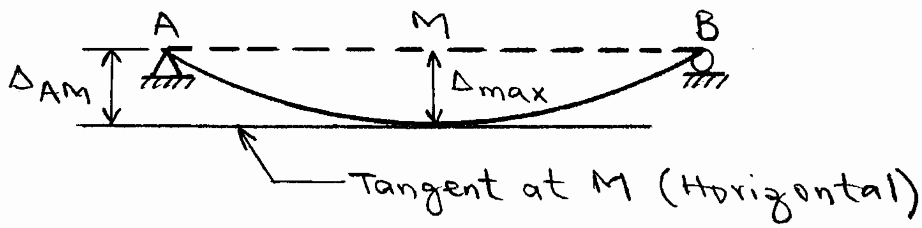
from which, $I = 5625(10^{-6})\,m^4 = \underline{5625(10^6)\,mm^4}$

6.19





$\frac{M}{EI}$ Diagram $\left(\frac{k\text{-}ft}{EI}\right)$



The maximum deflection occurs at the beam midspan, M, where the tangent is horizontal.

$$\Delta_{max} = \Delta_{AM} = \frac{1}{EI}\left[\frac{2}{3}(150)(10)\left(\frac{50}{8}\right)\right]$$

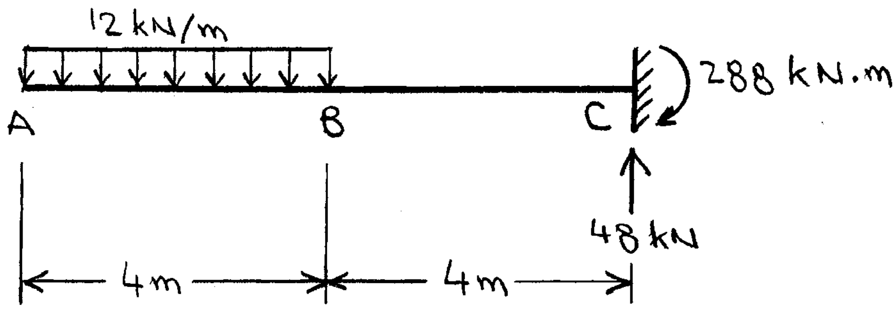
$$= \frac{6250 \text{ k-ft}^3}{EI}$$

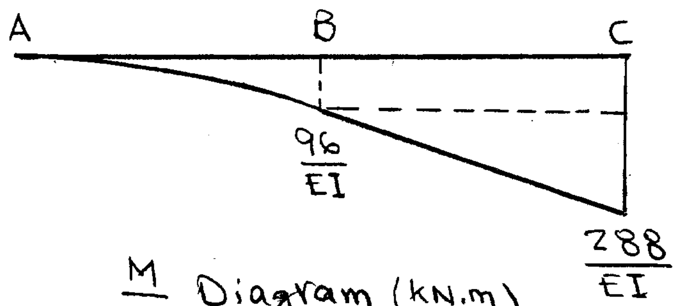
$$\Delta_{max} = \frac{L}{360}$$

$$\frac{6250(12)^3}{29000(I)} = \frac{20(12)}{360}$$

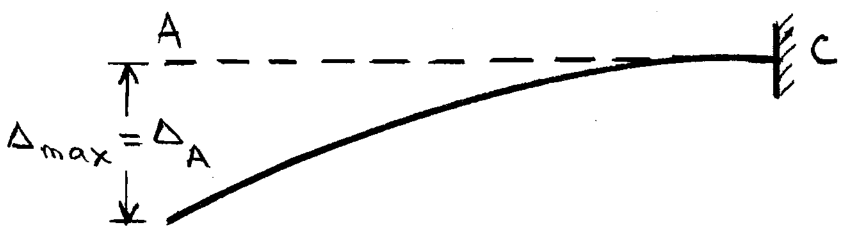
from which, $\underline{I = 559 \text{ in}^4}$

6.20





$\frac{M}{EI}$ Diagram $\left(\frac{kN.m}{EI}\right)$



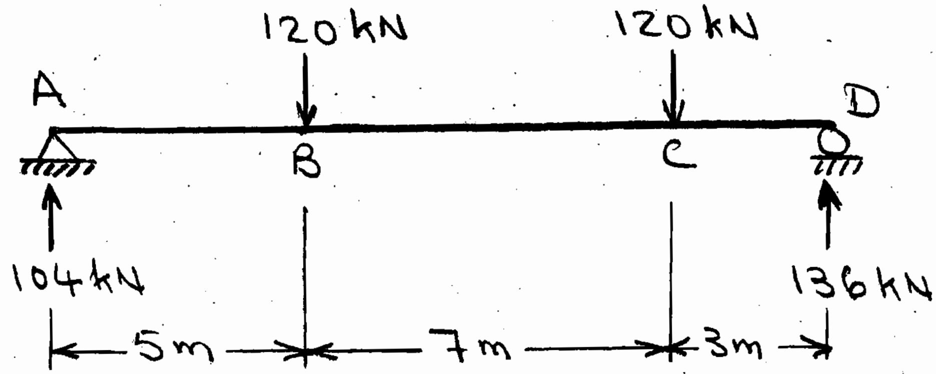
$$\Delta_{max} = \Delta_A = \Delta_{AC} = \frac{1}{EI}\left[\frac{1}{3}(96)(4)(3) + 96(4)(6) + \frac{1}{2}(192)4\left(\frac{8}{3}+4\right)\right] = \frac{5248 \text{ kN.m}^3}{EI}$$

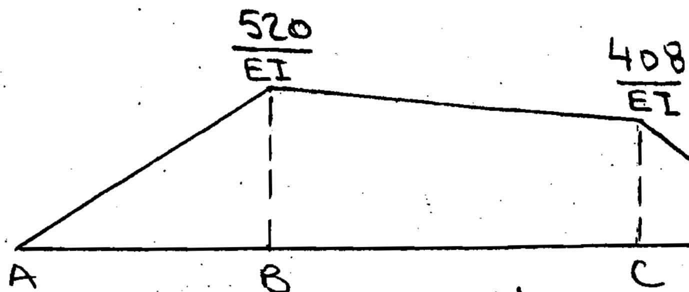
$$\Delta_{max} = \frac{L}{360}$$

$$\frac{5248}{70(10^6)(I)} = \frac{8}{360}$$

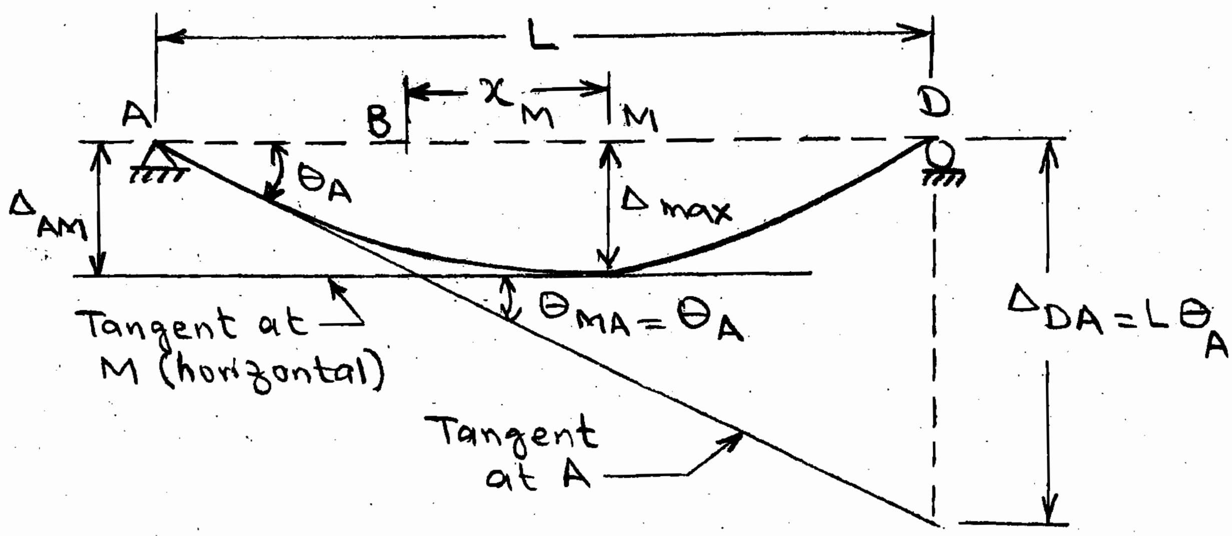
from which, $I = 3374(10^{-6})\,m^4 = \underline{3374\,(10^6)\,mm^4}$

6.21





$\frac{M}{EI}$ Diagram $\left(\frac{kN.m}{EI}\right)$



$$\Delta_{DA} = \frac{1}{EI}\left[\frac{1}{2}(520)(5)\left(\frac{5}{3}+10\right) + (408)(7)(6.5) + \frac{1}{2}(112)(7)\left(\frac{14}{3}+3\right) + \frac{1}{2}(408)(3)(2)\right]$$

$$= \frac{37960 \text{ kN.m}^3}{EI}$$

$$\theta_A = \frac{\Delta_{DA}}{L} = \frac{37960}{15(EI)} = \frac{2530.67 \text{ kN.m}^2}{EI}$$

6.21 (contd.)

If the maximum deflection occurs at M, then

$$\theta_{MA} = \theta_A$$

$$\frac{1}{EI}\left[\frac{1}{2}(520)(5) + 520x_M - \frac{1}{2}\left(\frac{112}{7}x_M\right)x_M\right] = \frac{2530.67}{EI}$$

$$8x_M^2 - 520x_M + 1230.67 = 0$$

$$x_M = 2.46 \text{ m}$$

$$\Delta_{max} = \Delta_{AM} = \frac{1}{EI}\left[\frac{1}{2}(520)(5)\left(\frac{10}{3}\right)\right.$$

$$\left. + 520(2.46)\left(\frac{2.46}{2} + 5\right) - \frac{1}{2}\left(\frac{112}{7}\right)(2.46)(2.46)(6.64)\right]$$

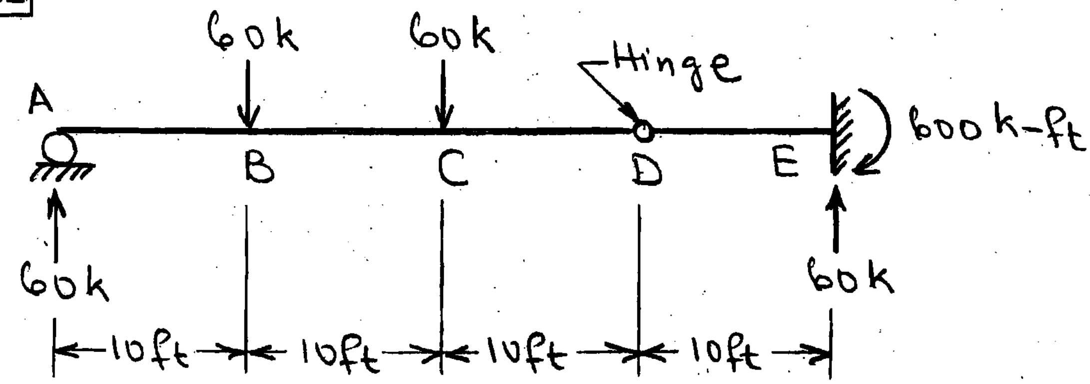
$$= \frac{11981.29 \text{ kN.m3}}{EI}$$

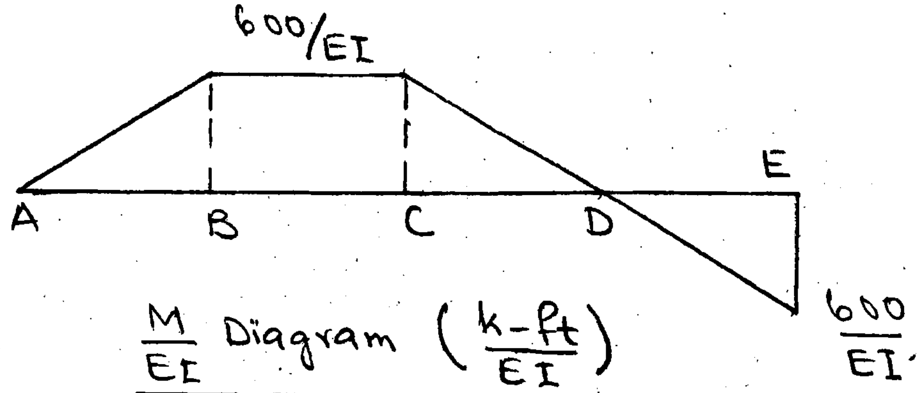
$$\Delta_{max} = \frac{L}{360}$$

$$\frac{11981.29}{30(10^6)I} = \frac{15}{360}$$

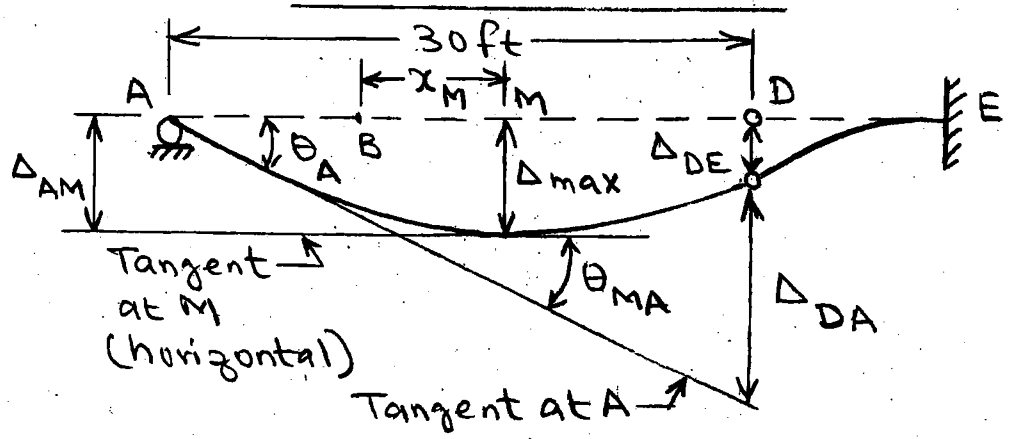
from which $I = 9585(10^{-6})\,m^4 = \underline{9585(10^6)\,mm^4}$

$\boxed{6.22}$





$\frac{M}{EI}$ Diagram $\left(\frac{k\text{-}ft}{EI}\right)$



$$\Delta_{DA} = \frac{1}{EI}\left[\frac{1}{2}(600)(10)\left(\frac{10}{3}+20\right) + 600(10)(15) + \frac{1}{2}(600)(10)\left(\frac{20}{3}\right)\right] = \frac{180000\ k\text{-}ft^3}{EI}$$

$$\Delta_{DE} = \frac{1}{EI}\left[\frac{1}{2}(600)(10)\left(\frac{20}{3}\right)\right] = \frac{20000\ k\text{-}ft^3}{EI}$$

$$\theta_A = \frac{\Delta_{DA}+\Delta_{DE}}{30} = \frac{180000+20000}{30} = \frac{6666.67}{EI}$$

6.22 (contd.)

If the maximum deflection occurs at M, then

$$\theta_{MA} = \theta_A$$

$$\frac{1}{EI}\left[\frac{1}{2}(600)(10) + 600\,x_M\right] = \frac{6666.67}{EI}$$

$$x_M = 6.11 \text{ ft}$$

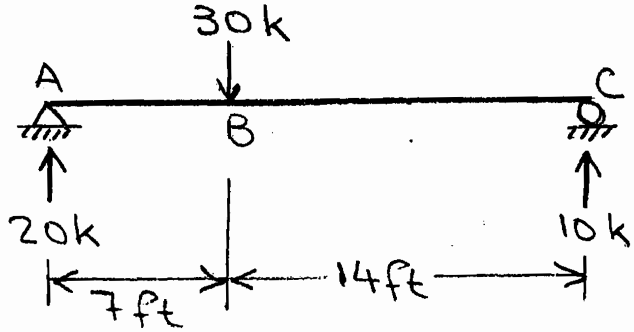
$$\Delta_{max} = \Delta_{AM} = \frac{1}{EI}\left[\frac{1}{2}(600)(10)\left(\frac{20}{3}\right) + (600)(6.11)(13.06)\right] = \frac{67878 \text{ k-ft}^3}{EI}$$

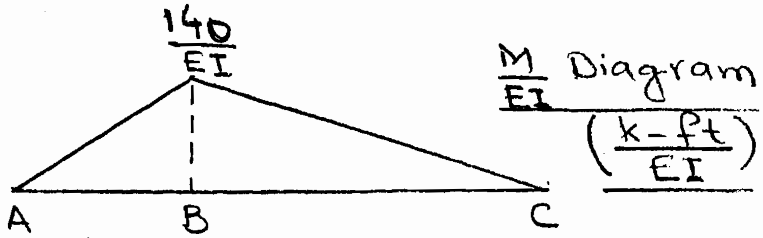
$$\Delta_{max} = \frac{L}{360}$$

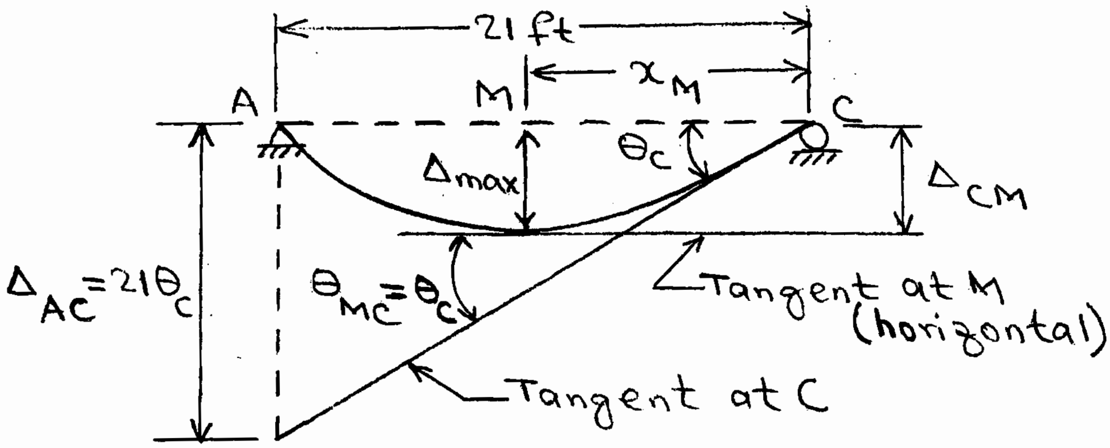
$$\frac{67878\,(12)^3}{29000\,(I)} = \frac{40(12)}{360}$$

from which $\underline{I = 3033 \text{ in}^4}$

6.23







$$\Delta_{AC} = \frac{1}{EI}\left[\frac{1}{2}(140)7\left(\frac{14}{3}\right) + \frac{1}{2}(140)14\left(7+\frac{14}{3}\right)\right] = \frac{13720}{EI}$$

$$\theta_C = \frac{\Delta_{AC}}{21} = \frac{653.33}{EI}$$

$$\theta_{MC} = \theta_C$$

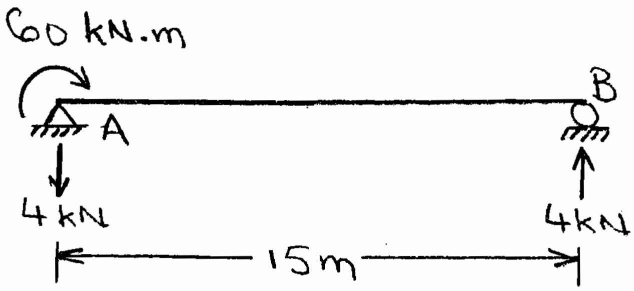
$$\frac{1}{EI}\left[\frac{1}{2}\left(\frac{140}{14}x_M\right)x_M\right] = \frac{653.33}{EI}$$

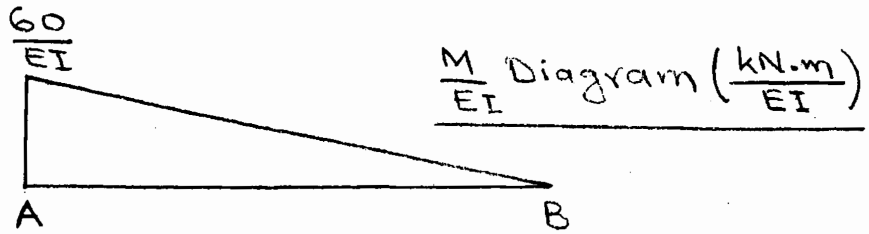
from which, $x_M = 11.43$ ft

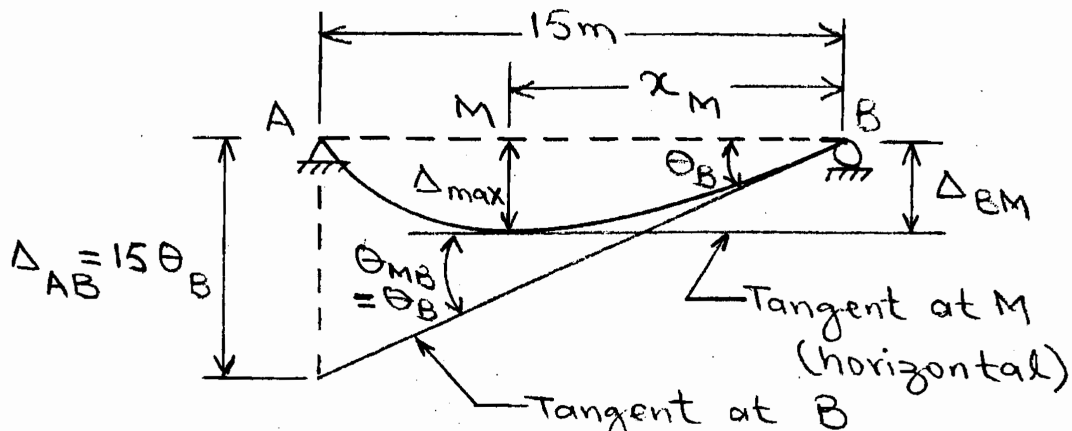
$$\Delta_{max} = \Delta_{CM} = \frac{1}{EI}\left[\frac{1}{2}(114.3)(11.43)\frac{2}{3}(11.43)\right]$$

$$= \frac{4978.81 \text{ k-ft}^3}{EI} = \frac{4978.81(12)^3}{10000(500)} = \underline{1.72 \text{ in.} \downarrow}$$

$\boxed{6.24}$







$$\Delta_{AB} = \frac{1}{EI}\left[\frac{1}{2}(60)15(5)\right] = \frac{2250}{EI}$$

$$\theta_B = \frac{\Delta_{AB}}{15} = \frac{150}{EI}$$

$$\theta_{MB} = \theta_B$$

$$\frac{1}{EI}\left[\frac{1}{2}\left(\frac{60}{15}x_M\right)x_M\right] = \frac{150}{EI}$$

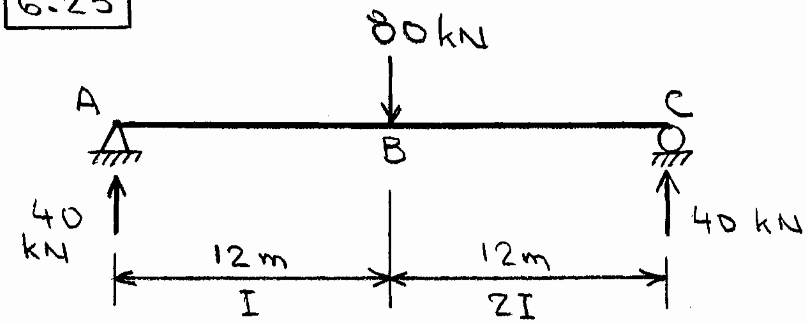
from which $x_M = 8.66$ m

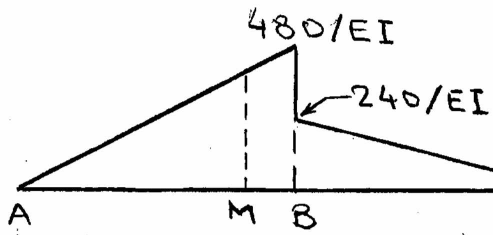
$$\Delta_{max} = \Delta_{BM} = \frac{1}{EI}\left[\frac{1}{2}(34.64)(8.66)\frac{2}{3}(8.66)\right]$$

$$= \frac{866 \text{ kN.m}^3}{EI} = \frac{866}{70(712)} = 0.0174 \text{ m}$$

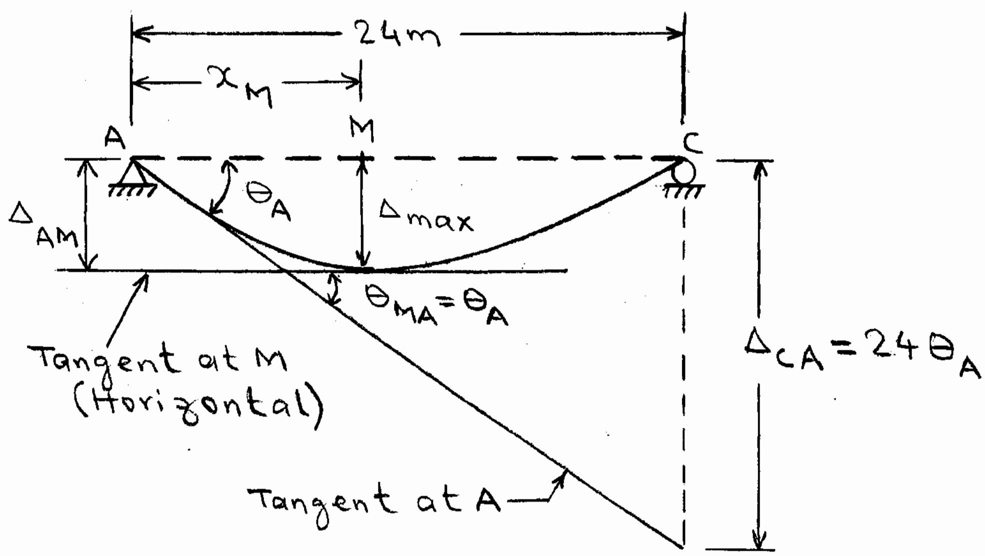
$$\Delta_{max} = 17.4 \text{ mm}\downarrow$$

6.25





$\frac{M}{EI}$ Diagram $\left(\frac{kN.m}{EI}\right)$



$$\Delta_{CA}=\frac{1}{EI}\left[\frac{1}{2}(480)(12)(16)+\frac{1}{2}(240)(12)(8)\right]=\frac{57600\ kN.m^3}{EI}$$

$$\theta_A=\frac{\Delta_{CA}}{24}=\frac{57600}{24(EI)}=\frac{2400\ kN.m^2}{EI}$$

If the maximum deflection occurs at M, then

$$\theta_{MA}=\theta_A$$

$\boxed{6.25 \text{ (contd.)}}$

$$\frac{1}{EI}\left[\frac{1}{2}(40x_M)x_M\right] = \frac{2400}{EI}$$

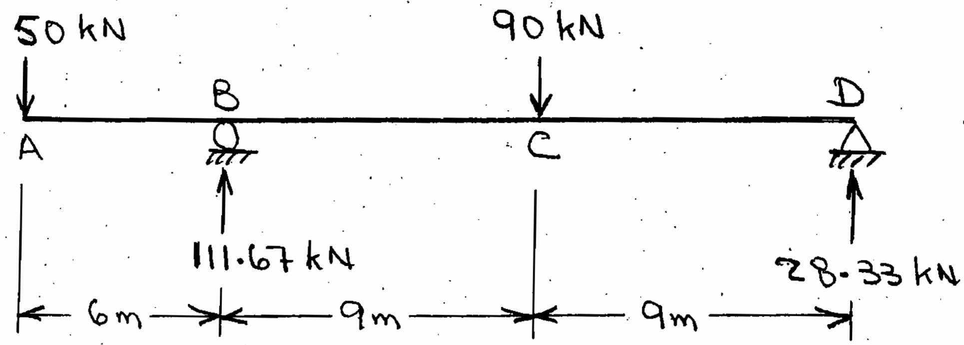
from which, $\underline{x_M = 10.95 \text{ m}}$

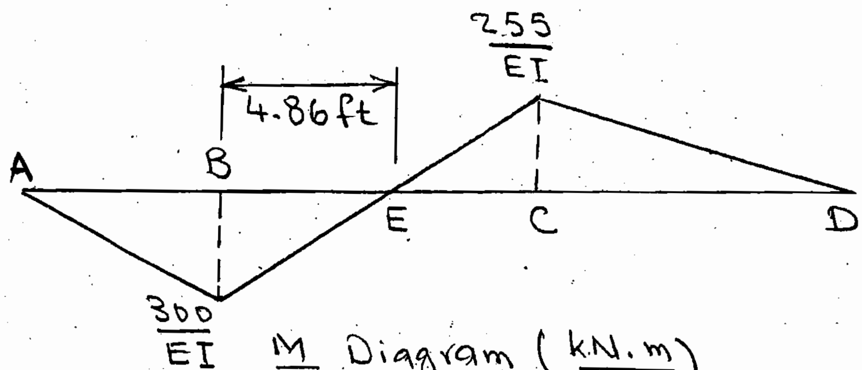
$$\Delta_{max} = \Delta_{AM} = \frac{1}{EI}\left[\frac{1}{2}(438)(10.95)\frac{2}{3}(10.95)\right]$$

$$= \frac{17506 \text{ kN}\cdot\text{m}^3}{EI} = \frac{17506}{200(600)} = 0.146 \text{ m}$$

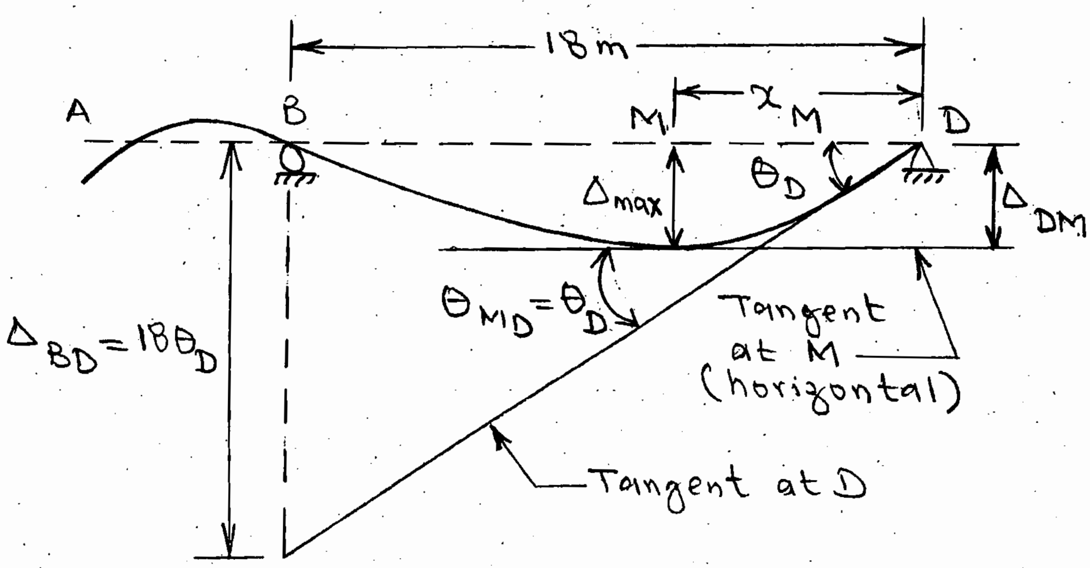
$$\underline{\Delta_{max} = 146 \text{ mm} \downarrow}$$

6.26





$\frac{M}{EI}$ Diagram $\left(\frac{kN.m}{EI}\right)$



$$\Delta_{BD} = \frac{1}{EI}\left[-\frac{1}{2}(300)(4.86)\left(\frac{4.86}{3}\right) + \frac{1}{2}(255)(4.14)(7.62) + \frac{1}{2}(255)(9)(12)\right] = \frac{16611.24\ kN.m^3}{EI}$$

$$\theta_D = \frac{\Delta_{BD}}{18} = \frac{16611.24}{18(EI)} = \frac{922.85\ kN.m^2}{EI}$$

$\boxed{6.26 \text{(contd.)}}$ $\theta_{MD} = \theta_D$

$$\frac{1}{EI}\left[\frac{1}{2}\left(\frac{255}{9}x_M\right)x_M\right] = \frac{922.85}{EI}$$

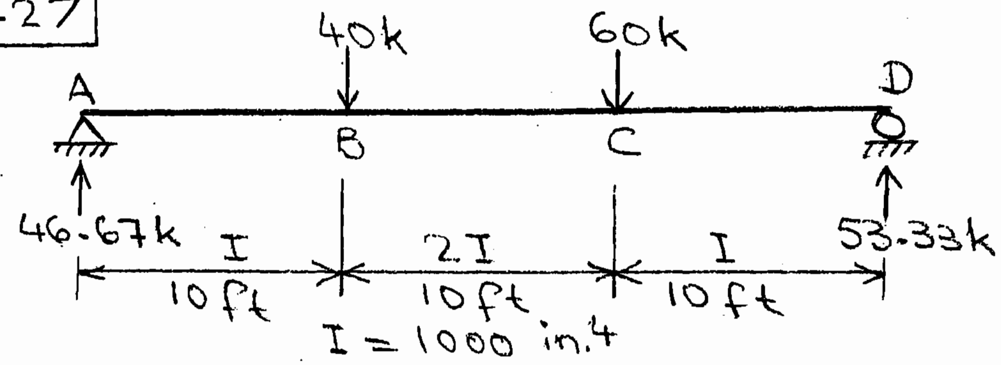
from which, $x_M = 8.07$ m

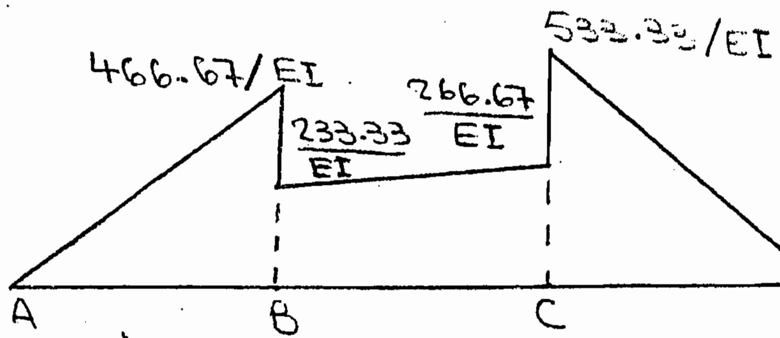
$$\Delta_{max} = \Delta_{DM} = \frac{1}{EI}\left[\frac{1}{2}(228.65)(8.07)\frac{2}{3}(8.07)\right]$$

$$= \frac{4963.6 \text{ kN.m}^3}{EI} = \frac{4963.6}{70(95)} = 0.746 \text{m}$$

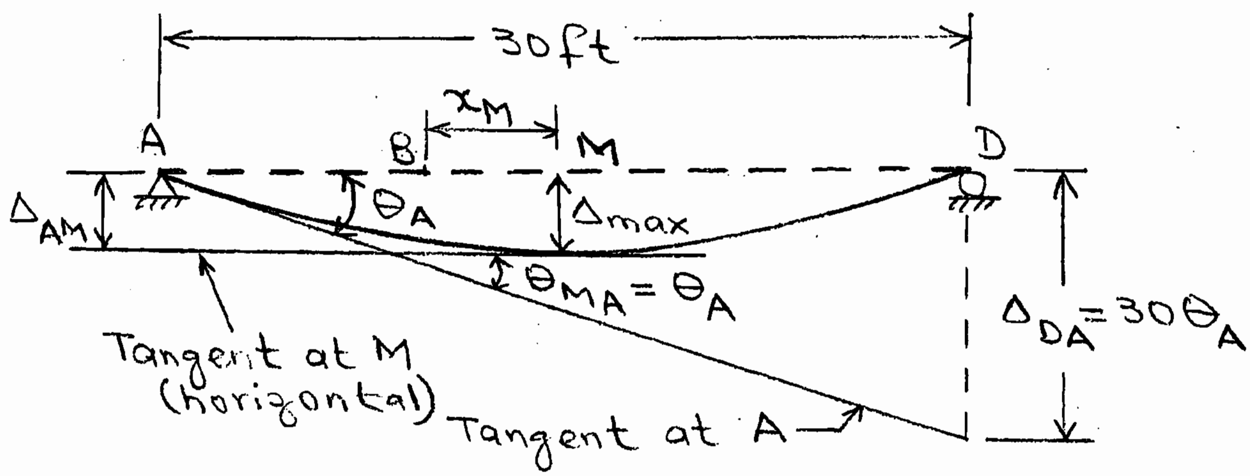
$$\underline{\Delta_{max} = 746 \text{ mm}\downarrow}$$

6.27





$\frac{M}{EI}$ Diagram $\left(\frac{k\text{-}ft}{EI}\right)$



$$\Delta_{DA} = \frac{1}{EI}\left[\frac{1}{2}(466.67)10\left(\frac{10}{3}+20\right) + \frac{1}{2}(33.33)10\left(\frac{10}{3}+10\right) + 233.33(10)15 + \frac{1}{2}(533.33)10\left(\frac{20}{3}\right)\right]$$

$$= \frac{109444.4 \text{ k-ft}^3}{EI}$$

$$\theta_A = \frac{\Delta_{DA}}{30} = \frac{3648.15 \text{ k-ft}^2}{EI}$$

$\boxed{6.27 \text{ (contd.)}}$

If the maximum deflection occurs at M, then

$$\theta_{MA} = \theta_A$$

$$\frac{1}{EI}\left[\frac{1}{2}(466.67)10 + 233.33 x_M + \frac{1}{2}\left(\frac{33.33}{10} x_M\right) x_M\right] = \frac{3648.15}{EI}$$

$$1.67 x_M^2 + 233.33 x_M - 1314.8 = 0$$

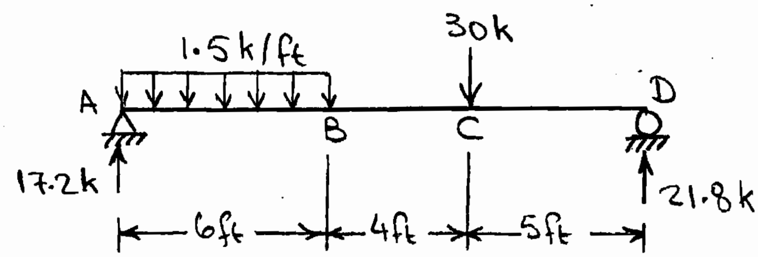
$$x_M = 5.42 \text{ ft}$$

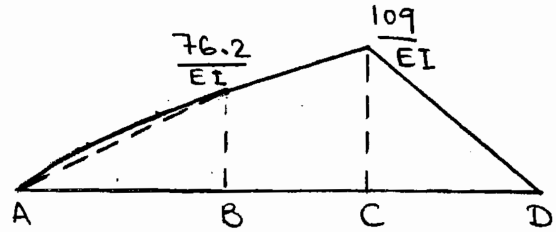
$$\Delta_{max} = \Delta_{AM} = \frac{1}{EI}\left[\frac{1}{2}(466.67)10\left(\frac{20}{3}\right) + 233.33(5.42)\left(\frac{5.42}{2} + 10\right) + \frac{1}{2}(18.06)(5.42)(3.61 + 10)\right]$$

$$= \frac{32296 \text{ k-ft}^3}{EI}$$

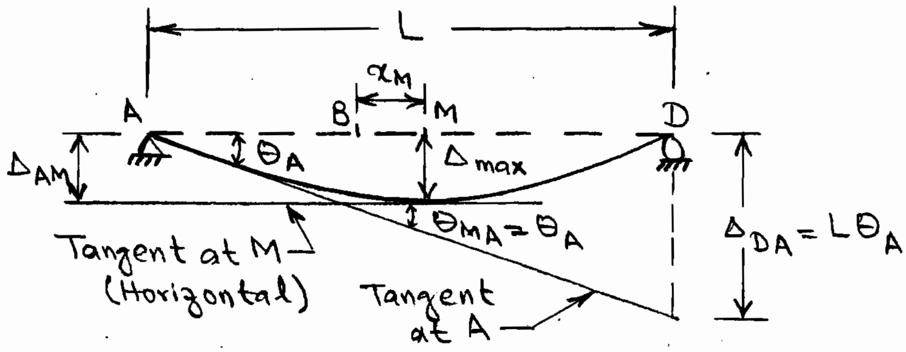
$$= \frac{32296(12)^3}{29000(1000)} = \underline{1.92 \text{ in.} \downarrow}$$

6.28





$\frac{M}{EI}$ Diagram $\left(\frac{\text{k-ft}}{EI}\right)$



$$\Delta_{DA} = \frac{1}{EI}\left[\frac{1}{2}(76.2)(6)(11) + \frac{2}{3}\frac{(1.5)(6)^2}{8}(6)(12) + (76.2)(4)(7) + \frac{1}{2}(32.8)(4)\left(\frac{4}{3}+5\right) + \frac{1}{2}(109)(5)\left(\frac{10}{3}\right)\right]$$

$$= \frac{6296 \text{ k-ft}^3}{EI}$$

$$\theta_A = \frac{6296}{15EI} = \frac{419.73 \text{ k-ft}^2}{EI}$$

By setting $\theta_{MA} = \theta_A$, we write

$$\frac{1}{EI}\left[\frac{1}{2}(76.2)(6) + \frac{2}{3}\frac{(1.5)(6)^2}{8}(6) + \frac{1}{2}(152.4 + 8.2x_M)x_M\right] = \frac{419.73}{EI}$$

$$4.1x_M^2 + 76.2x_M - 164.13 = 0$$

$$x_M = 1.95 \text{ ft}$$

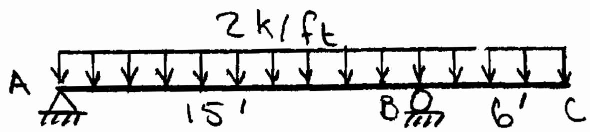
$\boxed{6.28 \text{(Contd.)}}$

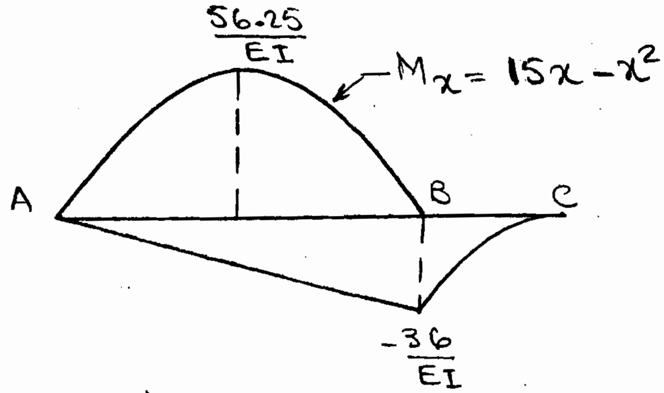
$$\Delta_{max} = \Delta_{AM} = \frac{1}{EI}\left[\frac{1}{2}(76.2)(6)(4) + \frac{2}{3}\frac{(1.5)(6)^3}{8}(3)\right.$$

$$\left. + 76.2(1.95)\left(\frac{1.95}{2}+6\right) + \frac{1}{2}(16)(1.95)(1.3+6)\right]$$

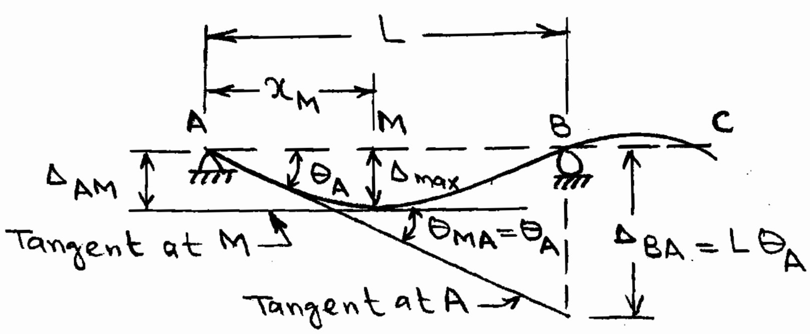
$$= \frac{2145.7 \text{ k-ft}^3}{EI} = \frac{2145.7(12)^3}{(1500)(20000)} = \underline{0.124 \text{ in.}\downarrow}$$

6.29





$\frac{M}{EI}$ Diagram $\left(\frac{k\text{-}ft}{EI}\right)$



$$\Delta_{BA} = \frac{1}{EI}\left[\frac{2}{3}(56.25)15(7.5) - \frac{1}{2}(36)(15)5\right]$$

$$= \frac{2868.75 \text{ k-ft}^3}{EI}$$

$$\theta_A = \frac{2868.75}{15\,EI} = \frac{191.25 \text{ k-ft}^2}{EI}$$

$$\theta_{MA} = \theta_A$$

$$\frac{1}{EI}\left[\frac{1}{2}(15x_M - x_M^2)x_M + \frac{2}{3}\left(\frac{2x_M^2}{8}\right)x_M - \frac{1}{2}(2.4x_M)x_M\right] = \frac{191.25}{EI}$$

$$-\frac{x_M^3}{3} + 6.3x_M^2 = 191.25$$

from which $x_M = 6.92$ ft

6.29 (contd.)

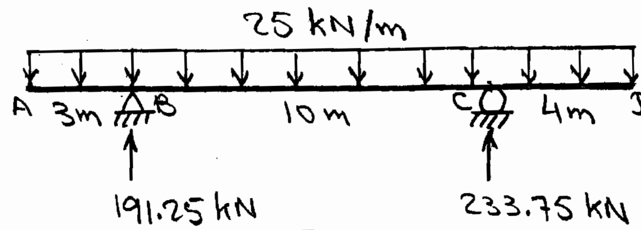
$$\Delta_{max} = \Delta_{AM} = \frac{1}{EI}\left[\frac{1}{2}(55.91)(6.92)(4.61)\right.$$

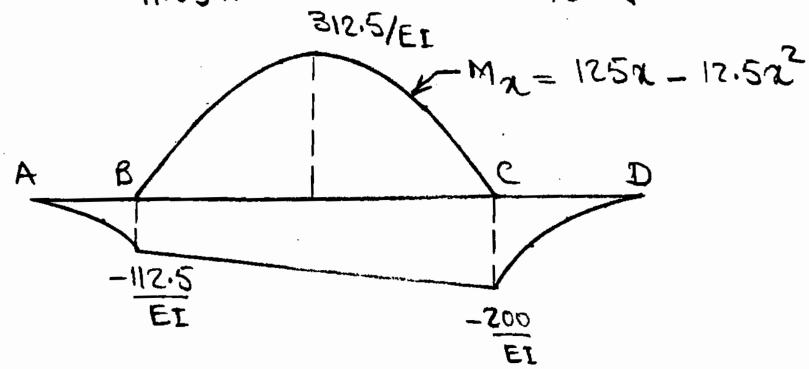
$$\left. + \frac{2}{3}(11.97)\frac{(6.92)^2}{2} - \frac{1}{2}(16.61)(6.92)\frac{2}{3}(6.92)\right]$$

$$= \frac{817.73\ \text{k-ft}^3}{EI} = \frac{817.73(12)^3}{29000(3500)}$$

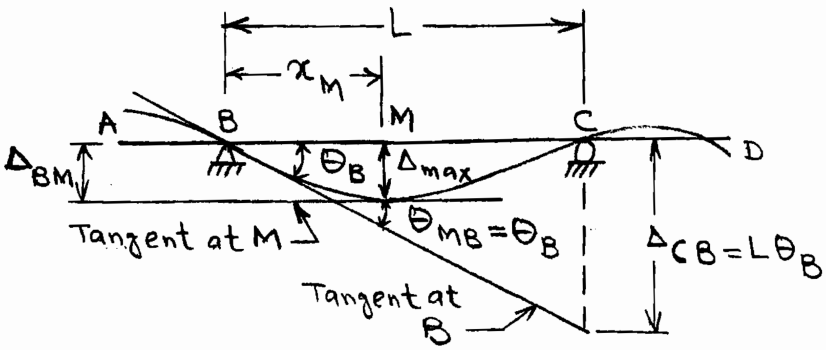
$$= \underline{0.0139\ \text{in.}\downarrow}$$

6.30





$\frac{M}{EI}$ Diagram $\left(\frac{kN\text{-}m}{EI}\right)$



$$\Delta_{CB} = \frac{1}{EI}\left[\frac{2}{3}(312.5)(10)(5) - (112.5)(10)(5) - \frac{1}{2}(87.5)(10)\left(\frac{10}{3}\right)\right]$$

$$= \frac{3333.33 \text{ kN-m}^3}{EI}$$

$$\theta_B = \frac{3333.33}{10EI} = \frac{333.33 \text{ kN-m}^2}{EI}$$

$$\theta_{MB} = \theta_A$$

$$\frac{1}{EI}\left[\frac{1}{2}(125x_M - 12.5x_M^2)x_M + \frac{2}{3}\left(\frac{25x_M^2}{8}\right)x_M - 112.5x_M - \frac{1}{2}(8.75x_M)x_M\right] = \frac{333.33}{EI}$$

$$-4.167x_M^3 + 58.125x_M^2 - 112.5x_M = 333.33$$

From which $x_M = 4.77$ m

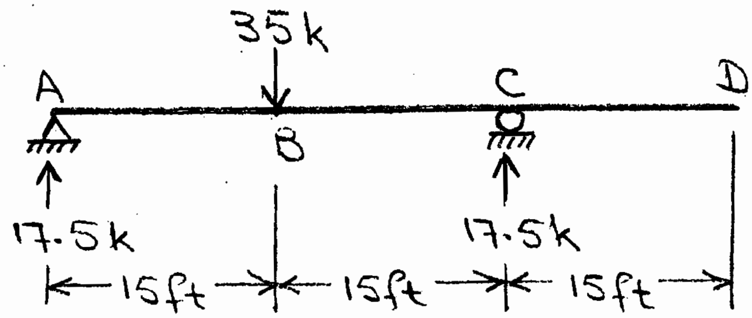
$\boxed{6.30 \text{(Contd.)}}$

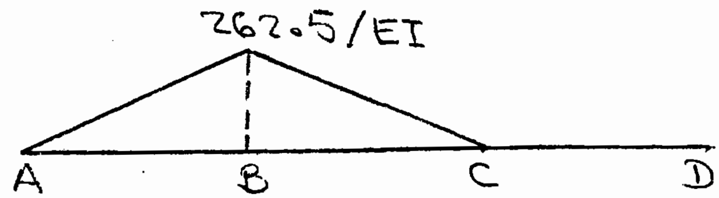
$$\Delta_{max} = \Delta_{BM} = \frac{1}{EI}\left[\frac{1}{2}(311.84)(4.77)(3.18) + \frac{2}{3}(71.1) \times \frac{(4.77)^2}{2} - (112.5)\frac{(4.77)^2}{2} - \frac{1}{2}(41.74)(4.77)(3.18)\right]$$

$$= \frac{1307.91 \text{ kN-m}^3}{EI} = \frac{1307.91}{(200)(500)} = 0.0131 \text{ m}$$

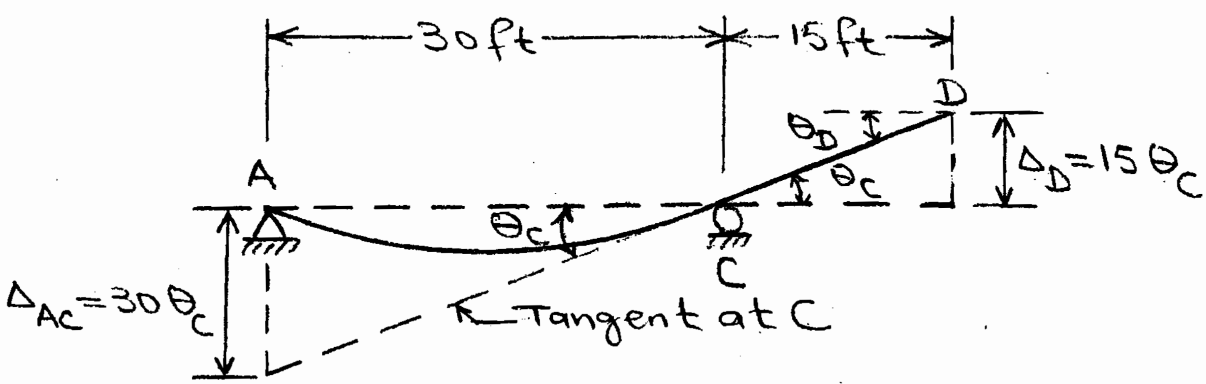
$$\underline{\Delta_{max} = 13.1 \text{ mm} \downarrow}$$

$\boxed{6.31}$





$\frac{M}{EI}$ Diagram $\left(\frac{k\text{-}ft}{EI}\right)$



$$\Delta_{AC} = \frac{1}{EI}\left[\frac{1}{2}(262.5)30(15)\right] = \frac{59062.5}{EI}$$

$$\theta_C = \frac{\Delta_{AC}}{30} = \frac{1968.75}{EI}$$

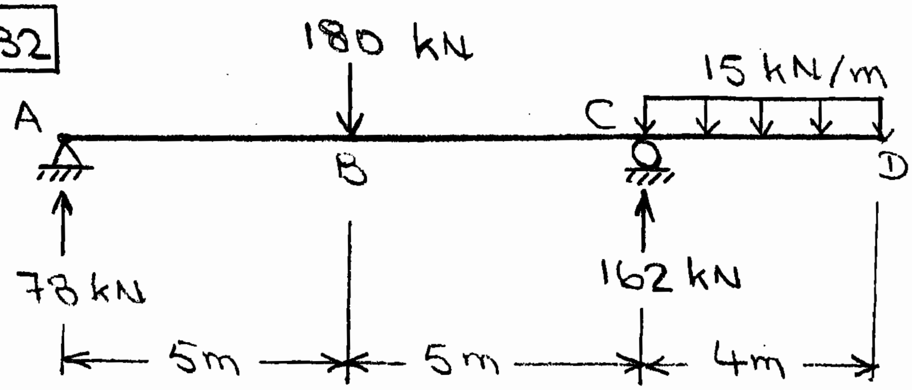
$$\theta_D = \theta_C = \frac{1968.75 \text{ k-ft}^2}{EI} = \frac{1968.75(12)^2}{10000(2500)}$$

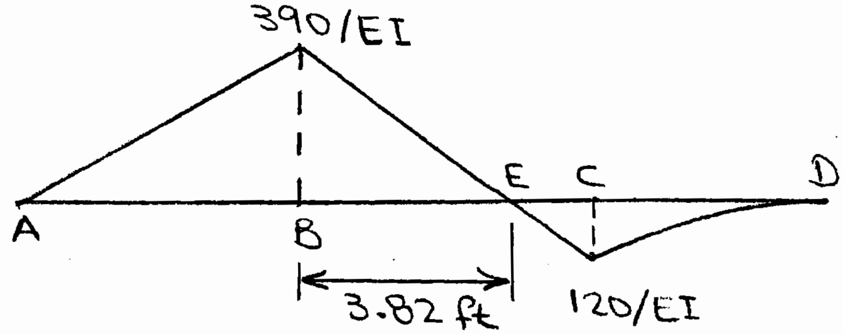
$$\underline{\theta_D = 0.01134 \text{ rad}}$$

$$\Delta_D = 15\,\theta_C = \frac{29531.25 \text{ k-ft}^3}{EI} = \frac{29531.25\,(12)^3}{10000(2500)}$$

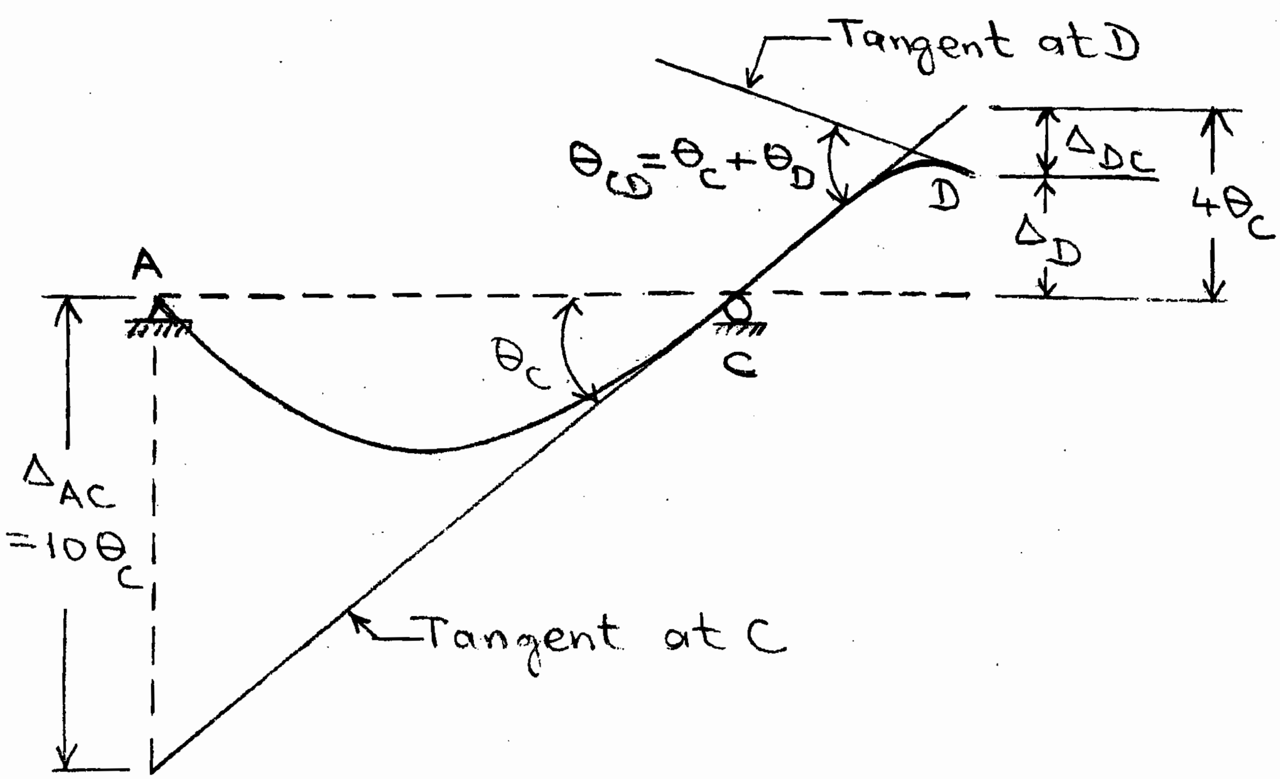
$$\underline{\Delta_D = 2.04 \text{ in.} \uparrow}$$

6.32





$\frac{M}{EI}$ Diagram $\left(\frac{kN.m}{EI}\right)$



6.32 (contd.)

$$\Delta_{AC} = \frac{1}{EI}\left[\frac{1}{2}(390)5\left(\frac{10}{3}\right) + \frac{1}{2}(390)3.82\left(\frac{3.82}{3}+5\right) - \frac{1}{2}(120)(1.18)(9.607)\right] = \frac{7243}{EI}$$

$$\theta_C = \frac{\Delta_{AC}}{10} = \frac{724.3}{EI}$$

$$\theta_{CD} = \frac{1}{EI}\left[\frac{1}{3}(120)4\right] = \frac{160}{EI}$$

$$\theta_D = \theta_{CD} - \theta_C = \frac{1}{EI}(160 - 724.3) = -\frac{564.3 \text{ kN.m}^2}{EI}$$

$$= -\frac{564.3}{70(2340)} = -0.00345 \text{ rad}$$

$$\underline{\theta_D = 0.00345 \text{ rad}}$$

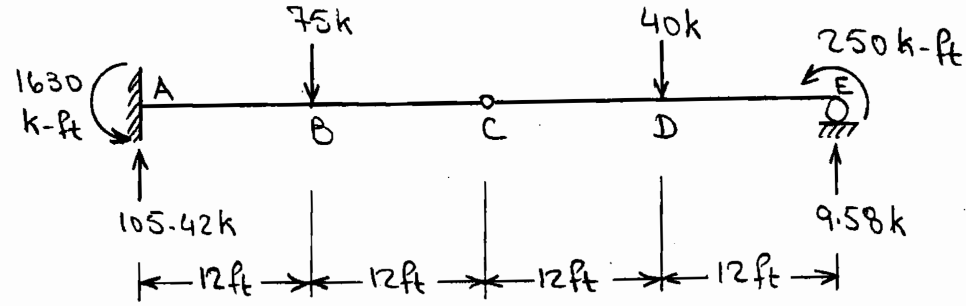
$$\Delta_{DC} = \frac{1}{EI}[160(3)] = \frac{480}{EI}$$

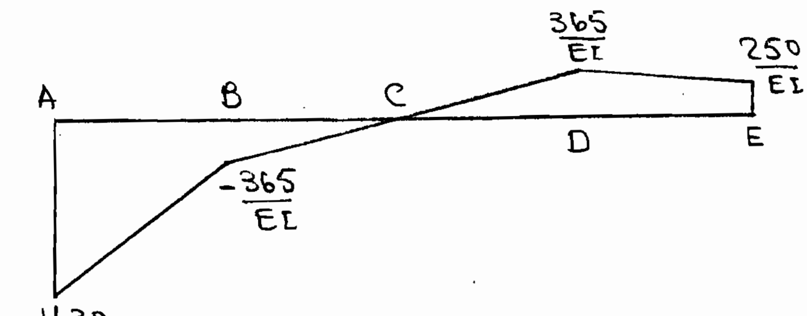
$$\Delta_D = 4\theta_C - \Delta_{DC} = \frac{1}{EI}[4(724.3) - 480]$$

$$= \frac{2417.2 \text{ kN.m}^3}{EI} = \frac{2417.2}{70(2340)} = 0.01476 \text{ m}$$

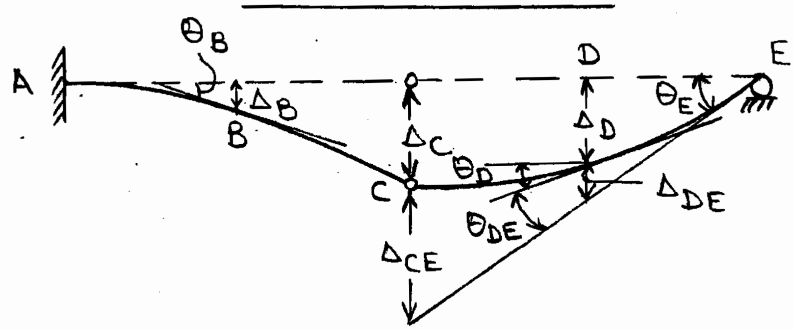
$$\underline{\Delta_D = 14.76 \text{ mm} \uparrow}$$

$\boxed{6.33}$





M/EI Diagram (k-ft/EI)



$$\theta_B = \theta_{BA} = \frac{1}{EI}\left[\left(\frac{1630+365}{2}\right)12\right] = \frac{11970 \text{ k-ft}^2}{EI} = \underline{0.0099 \text{ rad.} \triangledown}$$

$$\Delta_B = \Delta_{BA} = \frac{1}{EI}\left[(365)12(6) + \frac{1}{2}(1265)(12)(8)\right] = \frac{87000}{EI} = \underline{0.86 \text{ in.} \downarrow}$$

$$\Delta_C = \Delta_{CA} = \frac{1}{EI}\Big[365(12)(18) + \frac{1}{2}(1265)(12)(20) + \frac{1}{2}(365)$$
$$\times(12)8\Big] = \frac{248160 \text{ k-ft}^3}{EI}$$

$$\Delta_{CE} = \frac{1}{EI}\left[\frac{1}{2}(365)(12)(8) + (250)(12)(18) + \frac{1}{2}(115)(12)(16)\right]$$
$$= \frac{82560 \text{ k-ft}^3}{EI}$$

$$\theta_E = \frac{\Delta_C + \Delta_{CE}}{24} = \frac{248160 + 82560}{24EI} = \frac{13780 \text{ k-ft}^2}{EI}$$

$\boxed{6.33 \text{(contd.)}}$

$$\theta_{DE} = \frac{1}{EI}\left[\left(\frac{365+250}{2}\right)12\right] = \frac{3690 \text{ k-ft}^2}{EI}$$

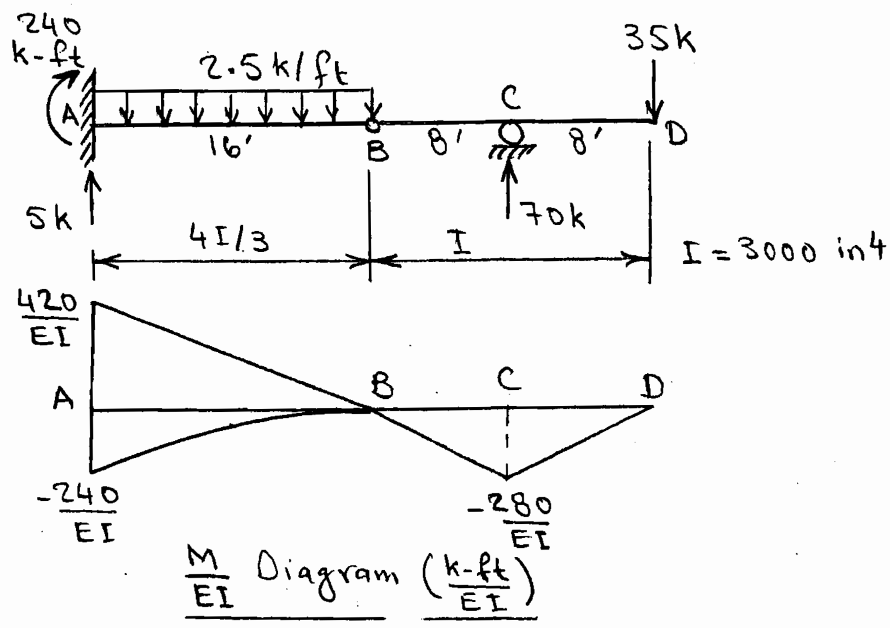
$$\theta_D = \theta_E - \theta_{DE} = \frac{13780 - 3690}{EI} = \frac{10090 \text{ k-ft}^2}{EI} = \underline{0.0084 \text{ rad.}}$$

$$\Delta_{DE} = \frac{1}{EI}\left[(250)(12)(6) + \frac{1}{2}(115)(12)(4)\right] = \frac{20760 \text{ k-ft}^3}{EI}$$

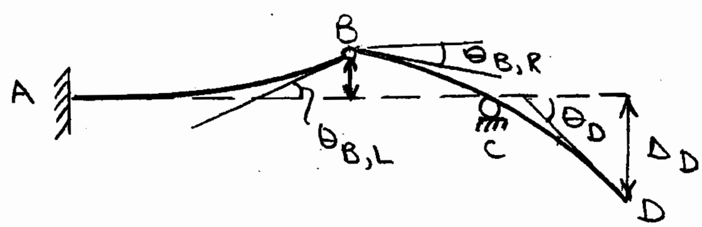
$$\Delta_D = 12\theta_E - \Delta_{DE} = \frac{1}{EI}\left[12(13780) - 20760\right] = \frac{144600 \text{ k-ft}^3}{EI}$$

$$\underline{\Delta_D = 1.44 \text{ in.} \downarrow}$$

6.34



$\frac{M}{EI}$ Diagram $\left(\frac{k\text{-}ft}{EI}\right)$



$$\theta_{B,Left} = \theta_{AB} = \frac{1}{EI}\left[\frac{1}{2}(420)16 - \frac{1}{3}(240)16\right]$$

$$= \frac{2080\ k\text{-}ft^2}{EI} = \frac{2080\,(12)^2}{30000\,(3000)} = \underline{0.0033\ rad.}$$

$$\Delta_B = \Delta_{BA} = \frac{1}{EI}\left[\frac{1}{2}(420)16\,(10.67) - \frac{1}{3}(240)16\,(12)\right]$$

$$= \frac{20480\ k\text{-}ft^3}{EI} = \frac{20480\,(12)^3}{30000\,(3000)} = \underline{0.39\ in\uparrow}$$

$$\Delta_{BC} = \frac{1}{EI}\left[\frac{1}{2}(280)8\left(\frac{16}{3}\right)\right] = \frac{5973.33}{EI}$$

$$\Delta_{BC} + \Delta_B = 8\,\theta_C$$

$$\theta_C = \frac{1}{8EI}(5973.33 + 20480) = \frac{3306.67}{EI}$$

$$\theta_{BC} = \frac{1}{EI}\left[\frac{1}{2}(280)8\right] = \frac{1120}{EI}$$

$$\theta_{B,Right} = \theta_C - \theta_{BC} = \frac{1}{EI}(3306.67 - 1120)$$

$$= \frac{2186.67\ k\text{-}ft^2}{EI} = \underline{0.0035\ rad.}$$

6.34 (contd.)

$$\theta_{CD} = \frac{1}{EI}\left[\frac{1}{2}(280)8\right] = \frac{1120}{EI}$$

$$\theta_D = \theta_C + \theta_{CD} = \frac{1}{EI}(3306.67 + 1120)$$

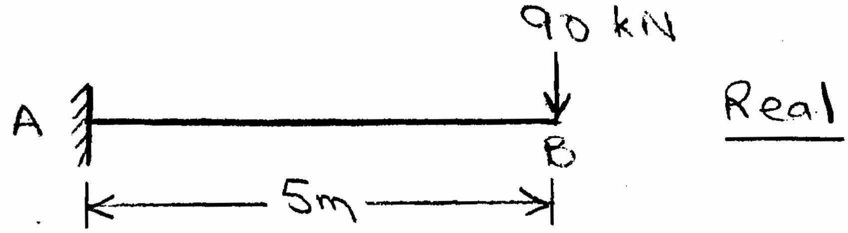
$$= \frac{4426.67 \text{ k-ft}^2}{EI} = \underline{0.0071 \text{ rad.}}$$

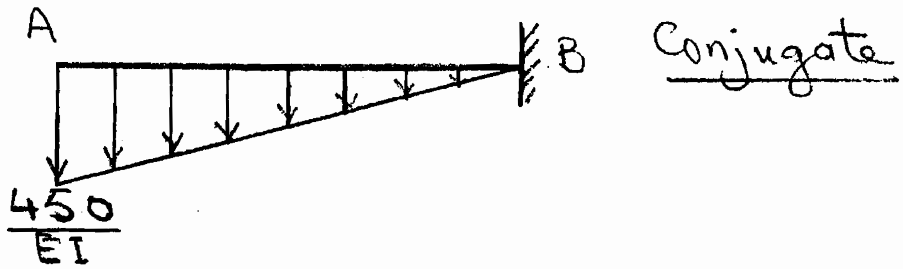
$$\Delta_{DC} = \frac{1}{EI}\left[1120\left(\frac{16}{3}\right)\right] = \frac{5973.33}{EI}$$

$$\Delta_D = 8\theta_C + \Delta_{DC} = \frac{1}{EI}\left[8(3306.67) + 5973.33\right]$$

$$= \frac{32426.67 \text{ k-ft}^3}{EI} = \underline{0.62 \text{ in} \downarrow}$$

6.35





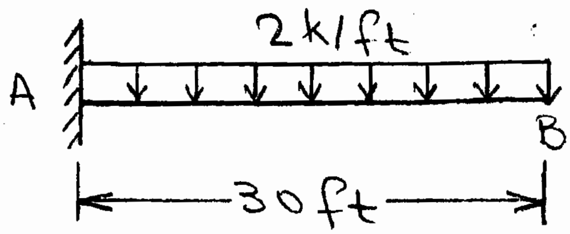
$$\theta_B = -\frac{1}{2}(5)\left(\frac{450}{EI}\right) = -\frac{1125 \text{ kN}\cdot\text{m}^2}{EI} = -\frac{1125}{200(800)}$$

$$= -0.00703 \text{ rad} = \underline{0.00703 \text{ rad}}$$

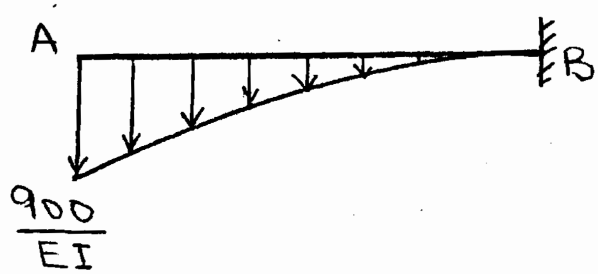
$$\Delta_B = -\frac{1}{2}(5)\left(\frac{450}{EI}\right)\left(\frac{10}{3}\right) = -\frac{3750 \text{ kN}\cdot\text{m}^3}{EI}$$

$$= -\frac{3750}{200(800)} = -0.0234 \text{m} = \underline{23.4 \text{ mm}\downarrow}$$

6.36



Real



Conjugate

$$\theta_B = -\frac{1}{3}\left(\frac{900}{EI}\right)30 = -\frac{9000 \text{ k-ft}^2}{EI}$$

$$= -\frac{9000\,(12)^2}{29000(3000)} = -0.0149 \text{ rad}$$

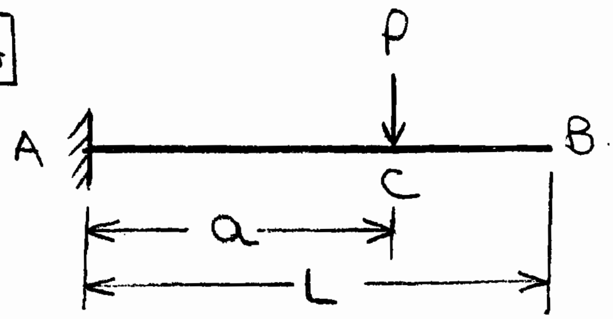
$$\underline{\theta_B = 0.0149 \text{ rad}}$$

$$\Delta_B = -\frac{9000}{EI}\left(\frac{3}{4}\right)30 = -\frac{202500 \text{ k-ft}^3}{EI}$$

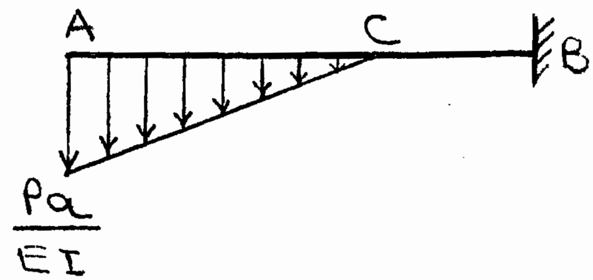
$$= -\frac{202500\,(12)^3}{29000\,(3000)} = -4.022 \text{ in.}$$

$$\underline{\Delta_B = 4.022 \text{ in.} \downarrow}$$

6.37



Real



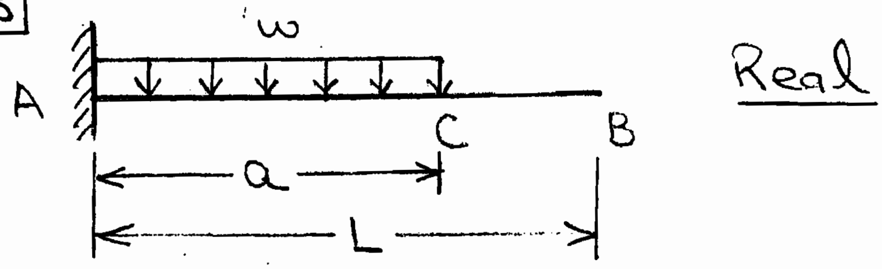
Conjugate

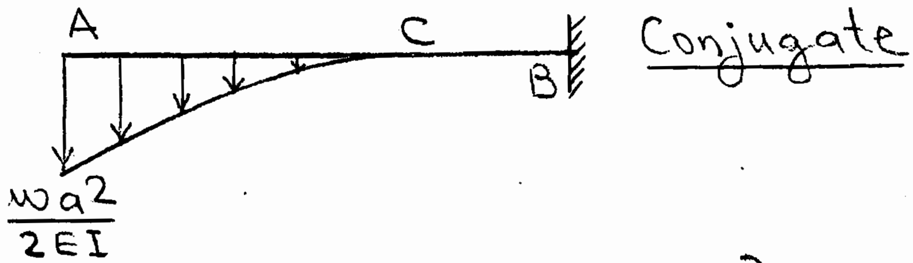
$$\theta_B = -\frac{1}{2}\left(\frac{Pa}{EI}\right)a = -\frac{Pa^2}{2EI} = \frac{Pa^2}{EI}$$

$$\Delta_B = -\frac{Pa^2}{2EI}\left(L - \frac{a}{3}\right) = -\frac{Pa^2}{6EI}(3L-a)$$

$$= \frac{Pa^2}{6EI}(3L-a) \downarrow$$

6.38



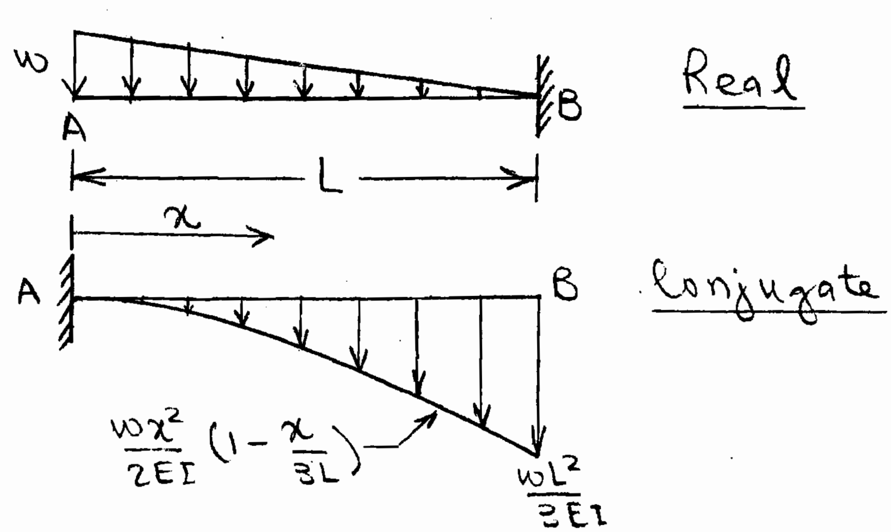


$$\theta_B = -\frac{1}{3}\left(\frac{wa^2}{2EI}\right)a = -\frac{wa^3}{6EI} = \underline{\frac{wa^3}{6EI}}$$

$$\Delta_B = -\frac{wa^3}{6EI}\left(L - \frac{a}{4}\right) = -\frac{wa^3}{24EI}(4L - a)$$

$$= \underline{\frac{wa^3}{24EI}(4L - a)\downarrow}$$

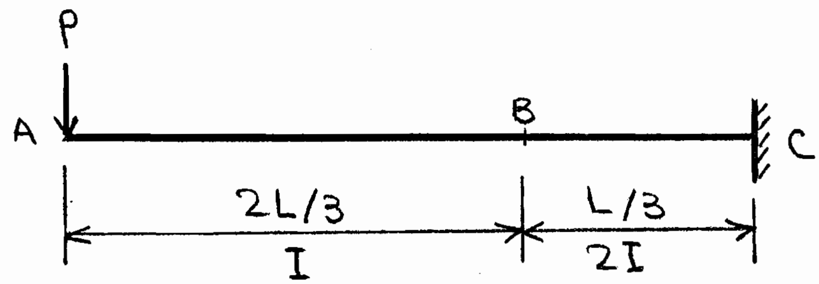
$\boxed{6.39}$



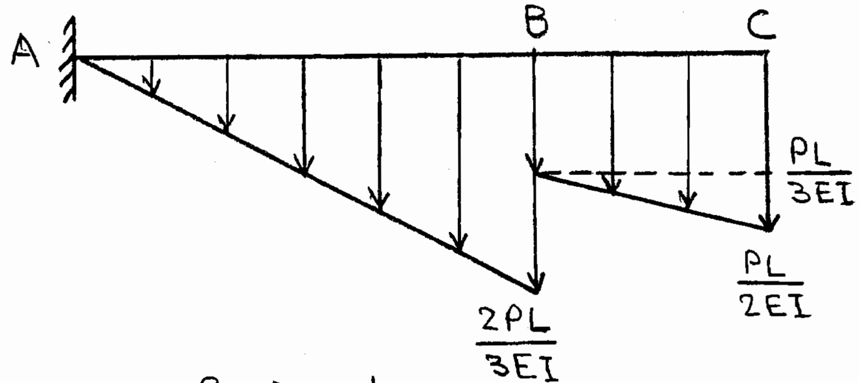
$$\theta_A = \int_0^L \frac{wx^2}{2EI}\left(1-\frac{x}{3L}\right)dx = \underline{\frac{wL^3}{8EI}}$$

$$\Delta_A = \int_0^L \frac{wx^2}{2EI}\left(1-\frac{x}{3L}\right)x\,dx = -\frac{11}{120}\frac{wL^4}{EI} = \underline{\frac{11}{120}\frac{wL^4}{EI}\downarrow}$$

6.40



Real



Conjugate

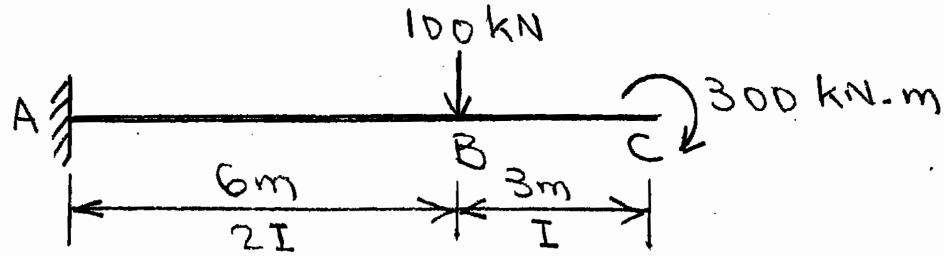
$$\theta_A = \frac{1}{2}\left(\frac{2L}{3}\right)\left(\frac{2PL}{3EI}\right) + \frac{L}{3}\left(\frac{PL}{3EI}\right) + \frac{1}{2}\left(\frac{L}{3}\right)\left(\frac{PL}{6EI}\right) = \underline{\frac{13PL^2}{36EI}}$$

$$\Delta_A = -\frac{2PL^2}{9EI}\left(\frac{4L}{9}\right) - \frac{PL^2}{9EI}\left(\frac{5L}{6}\right) - \frac{PL^2}{36EI}\left(\frac{8L}{9}\right) = -\frac{35PL^3}{162EI}$$

$$= \underline{\frac{35PL^3}{162EI}\downarrow}$$

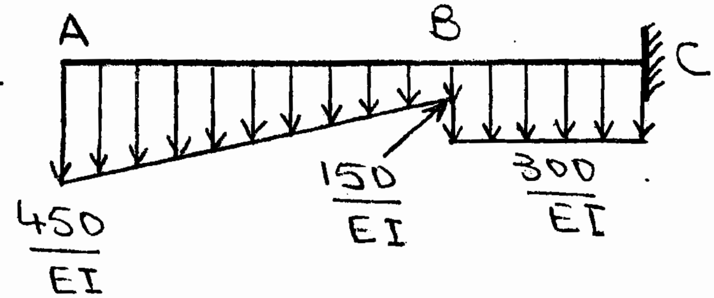
$\boxed{6.41}$

Real



$I = 500(10^6)\ mm^4$

Conjugate



$$\theta_B = \frac{1}{EI}\left[-150(6) - \frac{1}{2}(300)6\right] = -\frac{1800\ kN.m^2}{EI}$$

$$= -\frac{1800}{70(500)} = -0.0514\ rad = \underline{0.0514\ rad}$$

$$\Delta_B = \frac{1}{EI}\left[-150(6)3 - \frac{1}{2}(300)6(4)\right] = -\frac{6300\ kN.m^3}{EI}$$

$$= -\frac{6300}{70(500)} = -0.18\ m = \underline{180\ mm \downarrow}$$

$$\theta_C = \frac{1}{EI}\left[-1800 - 300(3)\right] = -\frac{2700\ kN.m^2}{EI}$$

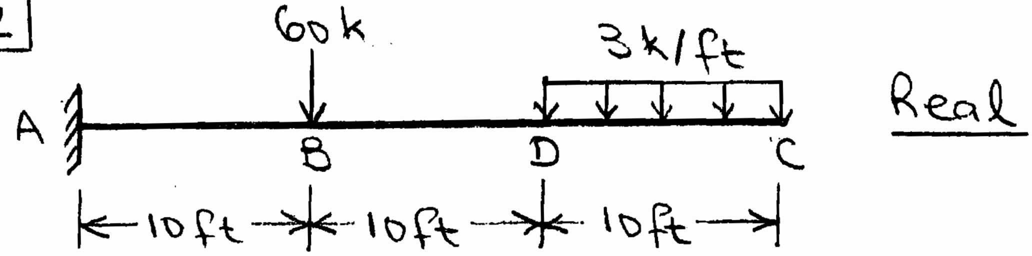
$$= -\frac{2700}{70(500)} = -0.0771\ rad = \underline{0.0771\ rad}$$

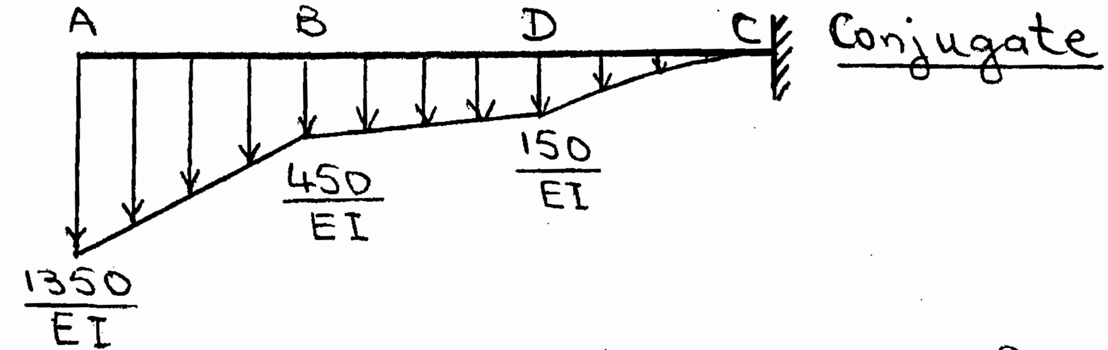
$$\Delta_C = \frac{1}{EI}\left[-150(6)(6) - \frac{1}{2}(300)6(7) - 300(3)(1.5)\right]$$

$$= -\frac{13050\ kN.m^3}{EI} = -\frac{13050}{70(500)} = -0.373\ m$$

$$= \underline{373\ mm \downarrow}$$

6.42





$$\theta_B = \frac{1}{EI}\left[-450(10) - \frac{1}{2}(900)10\right] = -\frac{9000\text{ k-ft}^2}{EI}$$

$$= -\frac{9000(12)^2}{29000(4000)} = -0.0112\text{ rad} = \underline{0.0112\text{ rad} \circlearrowright}$$

$$\Delta_B = \frac{1}{EI}\left[-450(10)5 - \frac{1}{2}(900)10\left(\frac{20}{3}\right)\right] = -\frac{52500\text{ k-ft}^3}{EI}$$

$$= -\frac{52500(12)^3}{29000(4000)} = -0.782\text{ in.} = \underline{0.782\text{ in.}\downarrow}$$

$$\theta_C = \frac{1}{EI}\left[-9000 - 150(10) - \frac{1}{2}(300)10 - \frac{1}{3}(150)10\right]$$

$$= -\frac{12500\text{ k-ft}^2}{EI} = -\frac{12500(12)^2}{29000(4000)} = -0.0155\text{ rad}$$

$$= \underline{0.0155\text{ rad} \circlearrowright}$$

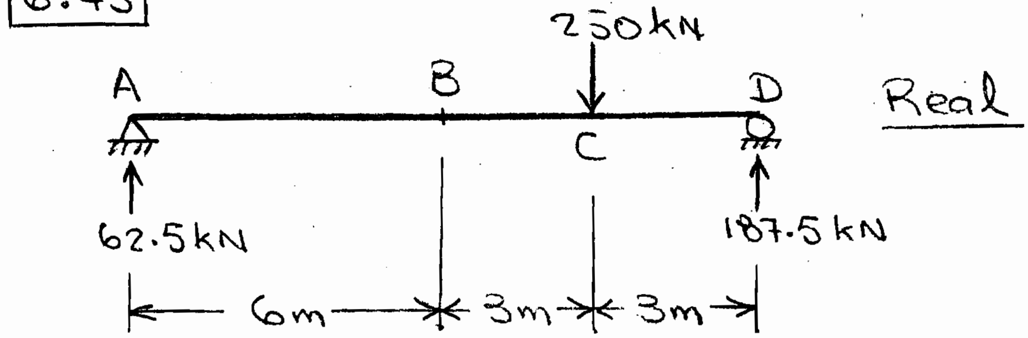
$$\Delta_C = \frac{1}{EI}\left[-450(10)25 - \frac{1}{2}(900)10\left(\frac{20}{3}+20\right)\right.$$

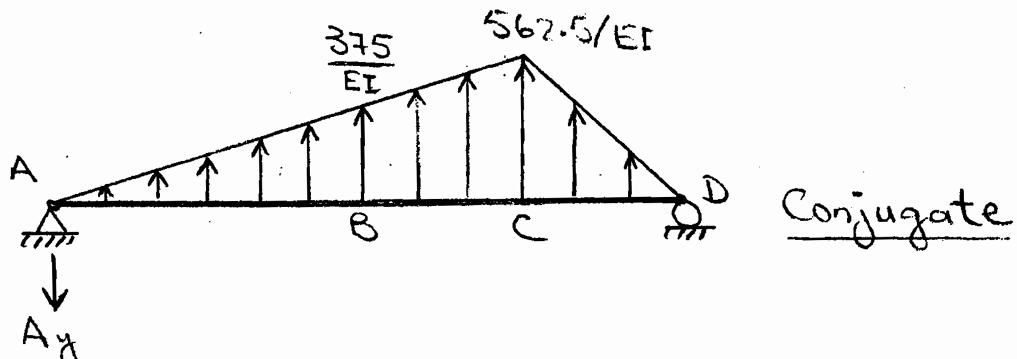
$$\left. -150(10)15 - \frac{1}{2}(300)10\left(\frac{20}{3}+10\right) - \frac{1}{3}(150)10\left(\frac{30}{4}\right)\right]$$

$$= -\frac{283750\text{ k-ft}^3}{EI} = -\frac{283750(12)^3}{29000(4000)}$$

$$= -4.227\text{ in.} = \underline{4.227\text{ in.}\downarrow}$$

6.43





Reaction for conjugate beam:

$+\circlearrowleft \Sigma M_D = 0$

$$A_y(12) - \frac{1}{2}\left(\frac{562.5}{EI}\right)9(6) - \frac{1}{2}\left(\frac{562.5}{EI}\right)3(2) = 0$$

$$A_y = \frac{1406.25 \text{ kN.m}^2}{EI}$$

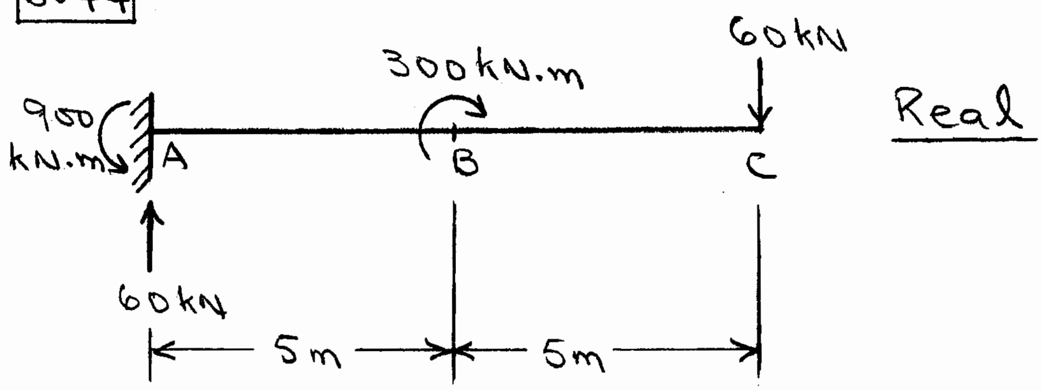
$$\theta_B = \frac{1}{EI}\left[-1406.25 + \frac{1}{2}(375)6\right] = -\frac{281.25 \text{ kN.m}^2}{EI}$$
$$= -\frac{281.25}{200(462)} = -0.00304 \text{ rad} = \underline{0.00304 \text{ rad}}$$

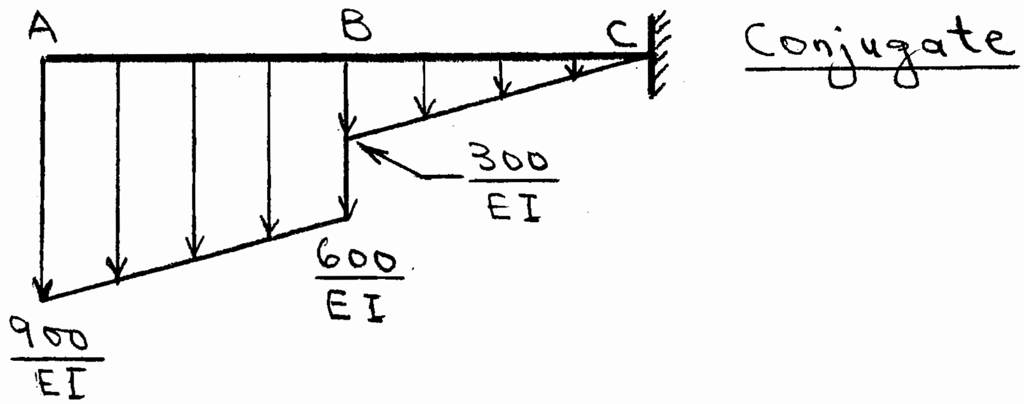
$$\Delta_B = \frac{1}{EI}\left[-1406.25(6) + \frac{1}{2}(375)6(2)\right] = -\frac{6187.5 \text{ kN.m}^3}{EI}$$
$$= -\frac{6187.5}{200(462)} = -0.067 \text{ m} = \underline{67 \text{ mm} \downarrow}$$

$$\theta_C = \frac{1}{EI}\left[-1406.25 + \frac{1}{2}(562.5)9\right] = \frac{1125 \text{ kN.m}^2}{EI}$$
$$= \frac{1125}{200(462)} = \underline{0.0122 \text{ rad}}$$

$$\Delta_C = \frac{1}{EI}\left[-1406.25(9) + \frac{1}{2}(562.5)9(3)\right] = -\frac{5062.5 \text{ kN.m}^3}{EI}$$
$$= -\frac{5062.5}{200(462)} = -0.0548 \text{ m} = \underline{54.8 \text{ mm} \downarrow}$$

6.44





$$\Delta_{max} = \Delta_C = -\frac{1}{EI}\left[600(5)(7.5) + \frac{1}{2}(300)(5)\left(\frac{25}{3}\right) + \frac{1}{2}(300)(5)\left(\frac{10}{3}\right)\right] = \frac{-31250 \text{ kN.m}^3}{EI}$$

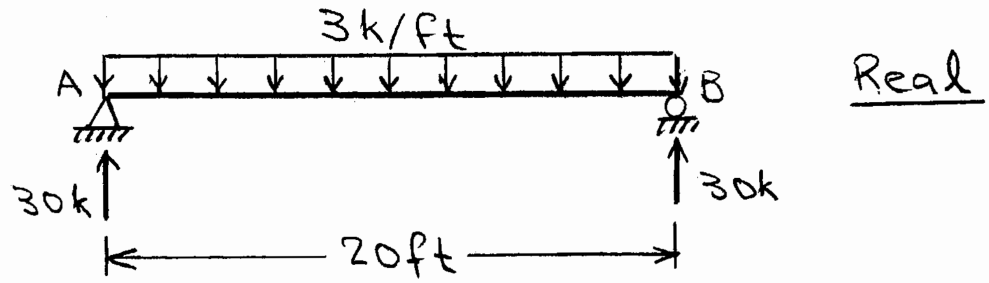
$$= \frac{31250 \text{ kN.m}^3}{EI} \downarrow$$

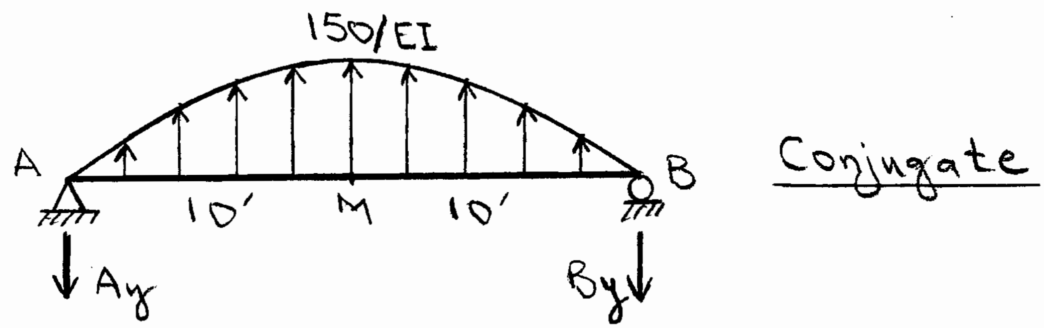
$$\Delta_{max} = \frac{L}{360}$$

$$\frac{31250}{200(10^6)I} = \frac{10}{360}$$

from which, $I = 5625(10^{-6})\,m^4 = \underline{5625(10^6)\,mm^4}$

6.45





Reactions for conjugate beam:

$$A_y = B_y = \frac{1}{2}\left(\frac{2}{3}\right)\left(\frac{150}{EI}\right)(20) = \frac{1000 \text{ k-ft}^2}{EI}$$

$$\Delta_{max} = \Delta_M = \frac{1}{EI}\left[-1000(10) + \frac{2}{3}(150)(10)\left(\frac{30}{8}\right)\right]$$

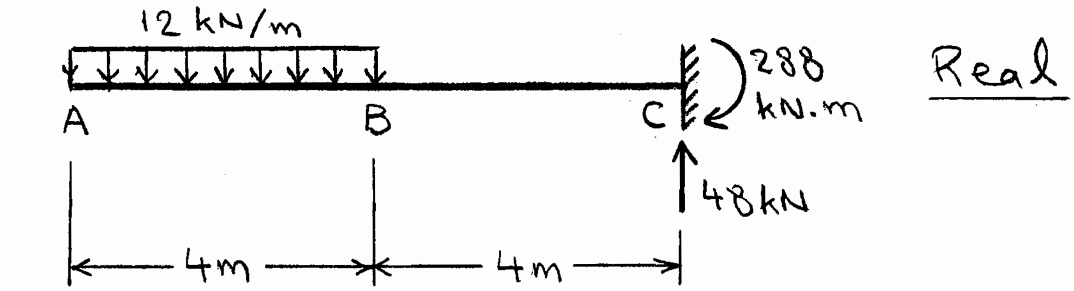
$$= -\frac{6250 \text{ k-ft}^3}{EI} = \frac{6250 \text{ k-ft}^3}{EI}\downarrow$$

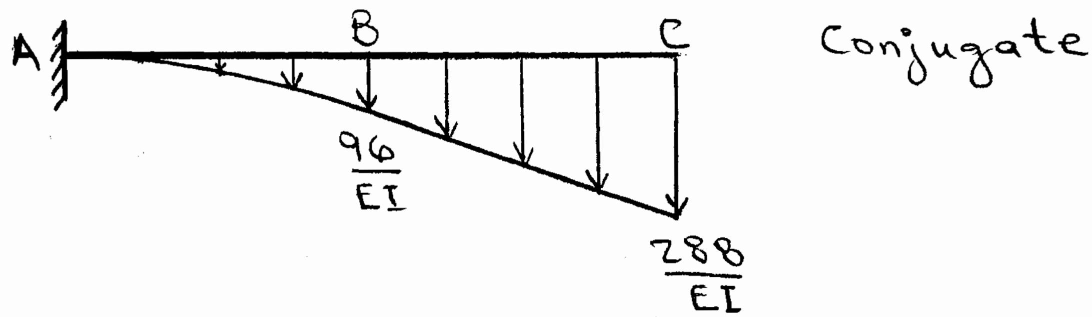
$$\Delta_{max} = \frac{L}{360}$$

$$\frac{6250(12)^3}{29000(I)} = \frac{20(12)}{360}$$

from which, $\underline{I = 559 \text{ in}^4}$

6.46





$$\Delta_{max} = \Delta_A = \frac{1}{EI}\left[-\frac{1}{3}(96)(4)(3) - 96(4)(6) - \frac{1}{2}(192)(4)\left(\frac{8}{3}+4\right)\right] = -\frac{5248 \text{ kN.m}^3}{EI}$$

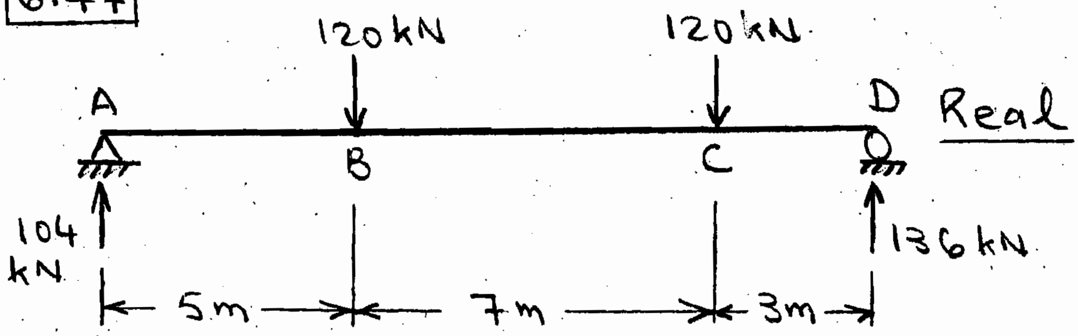
$$= \frac{5248 \text{ kN.m}^3}{EI} \downarrow$$

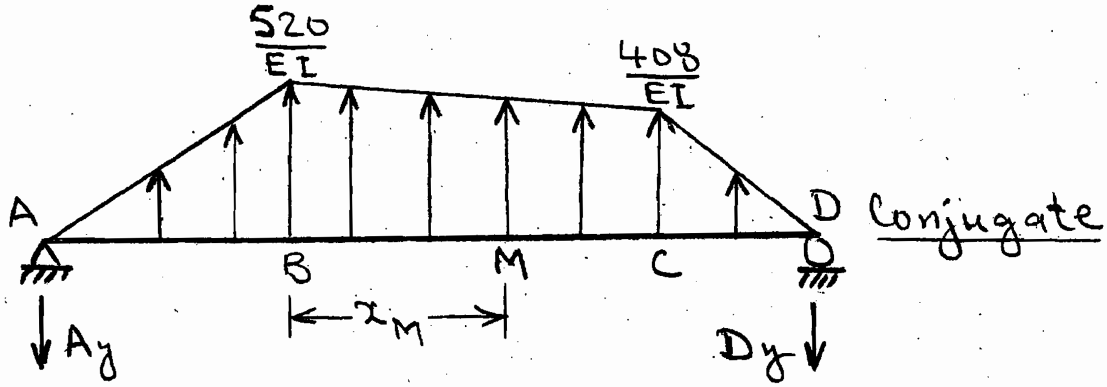
$$\Delta_{max} = \frac{L}{360}$$

$$\frac{5248}{70(10^6)I} = \frac{8}{360}$$

from which, $I = 3374(10^{-6})\,m^4 = \underline{3374\,(10^6)\,mm^4}$

6.47





$$+\circlearrowleft \sum M_D = 0$$

$$A_y(15) - \frac{1}{2}\left(\frac{520}{EI}\right)5\left(\frac{5}{3}+10\right) - \left(\frac{408}{EI}\right)(7)\left(\frac{7}{2}+3\right)$$
$$-\frac{1}{2}\left(\frac{112}{EI}\right)7\left(\frac{14}{3}+3\right) - \frac{1}{2}\left(\frac{408}{EI}\right)3\,(2) = 0$$

$$A_y = \frac{2530.67 \text{ kN.m}^2}{EI}$$

Let the maximum bending moment in the conjugate beam occur at point M, at a distance x_M from point B. Then the shear at M must be zero. Thus,

$$S_M = \frac{1}{EI}\left[-2530.67 + \frac{1}{2}(520)(5) + (520)x_M - \frac{1}{2}\left(\frac{112}{7}x_M\right)x_M\right] = 0$$

$$8x_M^2 - 520x_M + 1230.67 = 0$$

$$x_M = 2.46 \text{ m}$$

6.47 (contd.)

$$\Delta_{max} = \Delta_M = \frac{1}{EI}\Big[-2530.67(7.46) + \frac{1}{2}(520)(5)$$

$$\left(\frac{5}{3} + 2.46\right) + (520)(2.46)\left(\frac{2.46}{2}\right) - \frac{1}{2}(39.36)$$

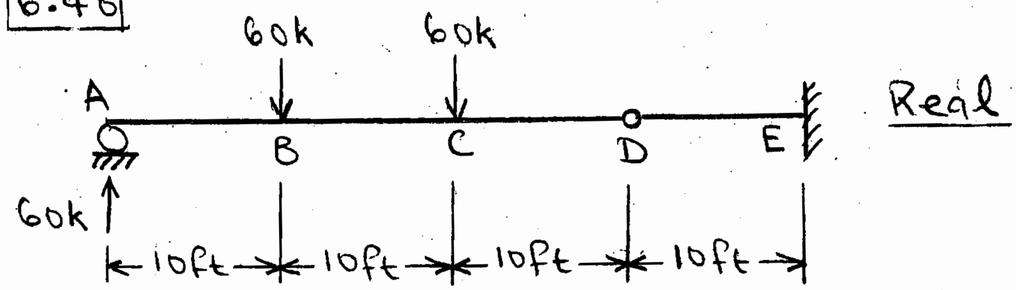
$$(2.46)\left(\frac{2.46}{3}\right)\Big] = -\frac{11980.41}{EI} = \frac{11980.41 \text{ kN.m}^3}{EI} \downarrow$$

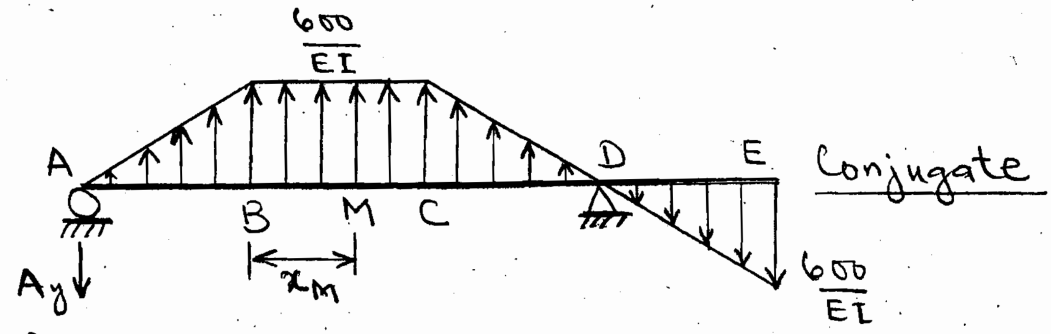
$$\Delta_{max} = \frac{L}{360}$$

$$\frac{11980.41}{30(10^6)I} = \frac{15}{360}$$

from which, $I = 9584(10^{-6})\text{m}^4 = \underline{9584(10^6)\text{ mm}^4}$

6.48





$+\circlearrowright \Sigma M_D = 0$

$$A_y(30) - \frac{1}{2}\left(\frac{600}{EI}\right)10\left(\frac{10}{3}+20\right) - \left(\frac{600}{EI}\right)(10)(15)$$
$$- \frac{1}{2}\left(\frac{600}{EI}\right)(10)\left(\frac{20}{3}\right) - \frac{1}{2}\left(\frac{600}{EI}\right)10\left(\frac{20}{3}\right) = 0$$

$$A_y = \frac{6666.67 \text{ k-ft}^2}{EI}$$

Let the maximum bending moment in the conjugate beam occur at point M, at a distance x_M from point B. Then the shear at M must be zero. Thus,

$$S_M = \frac{1}{EI}\left[-6666.67 + \frac{1}{2}(600)10 + (600)x_M\right] = 0$$
$$x_M = 6.11 \text{ ft}$$

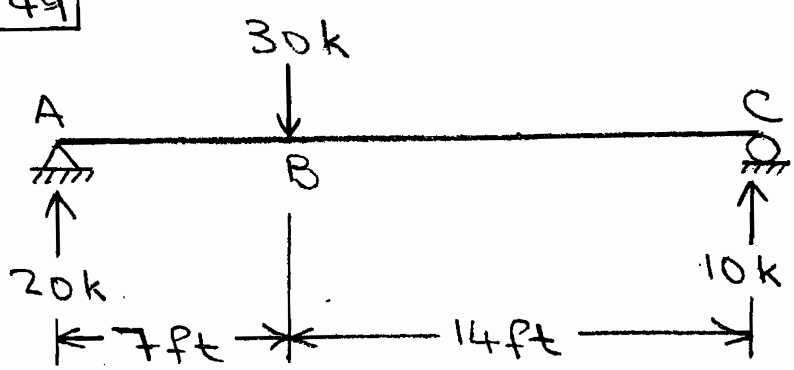
$$\Delta_{max} = \Delta_M = \frac{1}{EI}\left[-6666.67(16.11) + \frac{1}{2}(600)(10)\left(\frac{10}{3}+6.11\right) + (600)(6.11)\left(\frac{6.11}{2}\right)\right]$$
$$= -\frac{67870}{EI} = \frac{67870 \text{ k-ft}^3}{EI} \downarrow$$

$$\Delta_{max} = \frac{L}{360}$$

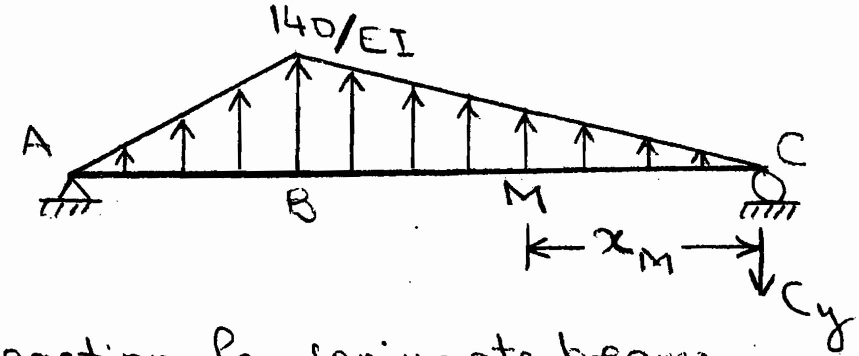
$$\frac{67870(12)^3}{29000(I)} = \frac{40(12)}{360}$$

from which $\underline{I = 3033 \text{ in}^4}$

$\boxed{6.49}$



Real



Conjugate

Reaction for conjugate beam:

$$+\circlearrowleft \sum M_A = 0$$

$$-C_y(21) + \frac{1}{2}\left(\frac{140}{EI}\right)14\left(\frac{14}{3}+7\right) + \frac{1}{2}\left(\frac{140}{EI}\right)7\left(\frac{14}{3}\right) = 0$$

$$C_y = \frac{653.33 \text{ k-ft}^2}{EI}$$

Let the maximum bending moment in the conjugate beam occur at point M, at a distance x_M from point C. Then, the shear at M must be zero. Thus,

$$S_M = \frac{1}{EI}\left[653.33 - \frac{1}{2}(10x_M)x_M\right] = 0$$

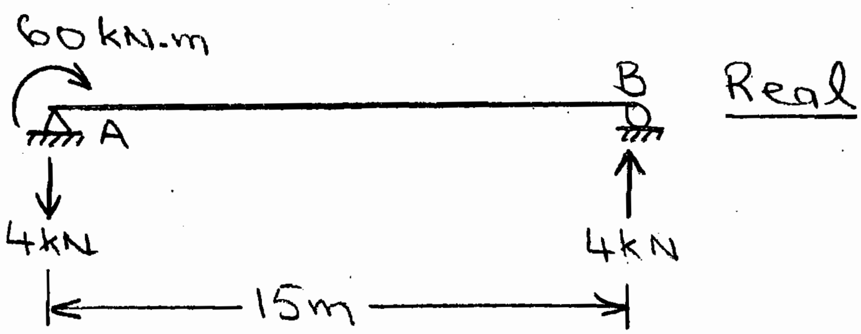
$$x_M = 11.43 \text{ ft}$$

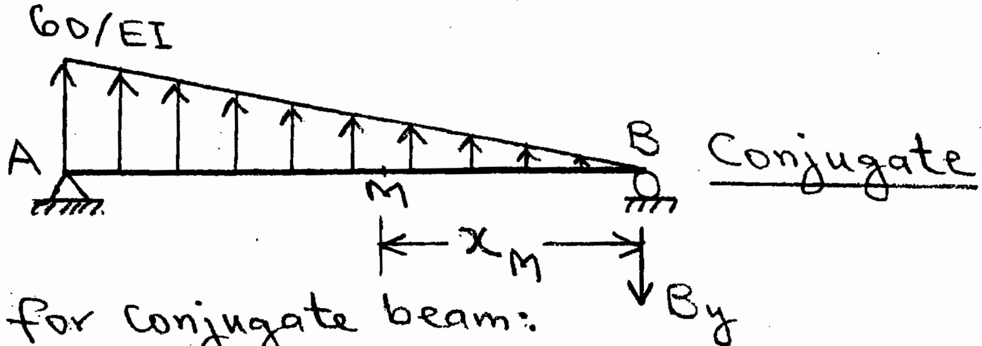
$$\Delta_{max} = \Delta_M = \frac{1}{EI}\left[-653.33(11.43) + \frac{1}{2}(114.3)11.43\left(\frac{11.43}{3}\right)\right]$$

$$= -\frac{4978.81 \text{ k-ft}^3}{EI} = -\frac{4978.81(12)^3}{10000(500)}$$

$$= -1.72 \text{ in.} = \underline{1.72 \text{ in.}\downarrow}$$

$\boxed{6.50}$





Reaction for conjugate beam:

$$+\circlearrowright \Sigma M_A = 0$$

$$-B_y(15) + \frac{1}{2}\left(\frac{60}{EI}\right)15(5) = 0$$

$$B_y = \frac{150 \text{ kN}\cdot\text{m}^2}{EI}$$

Let the maximum bending moment in the conjugate beam occur at point M, at a distance x_M from point B. Then the shear at M must be zero. Thus,

$$S_M = \frac{1}{EI}\left[150 - \frac{1}{2}(4x_M)x_M\right] = 0$$

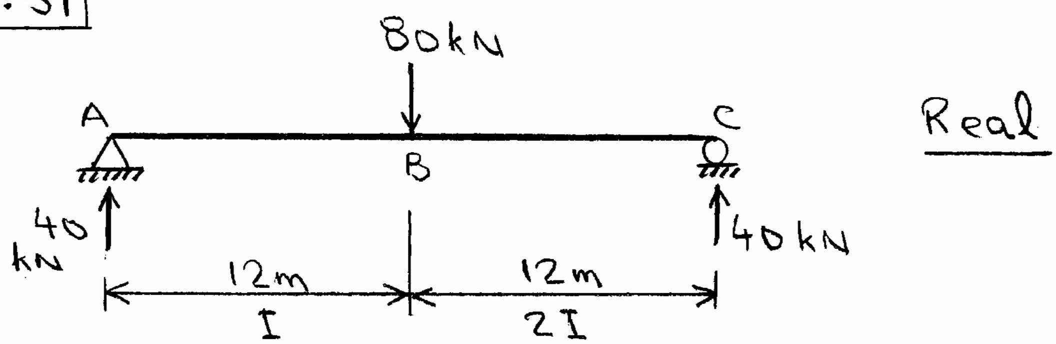
$$x_M = 8.66 \text{ m}$$

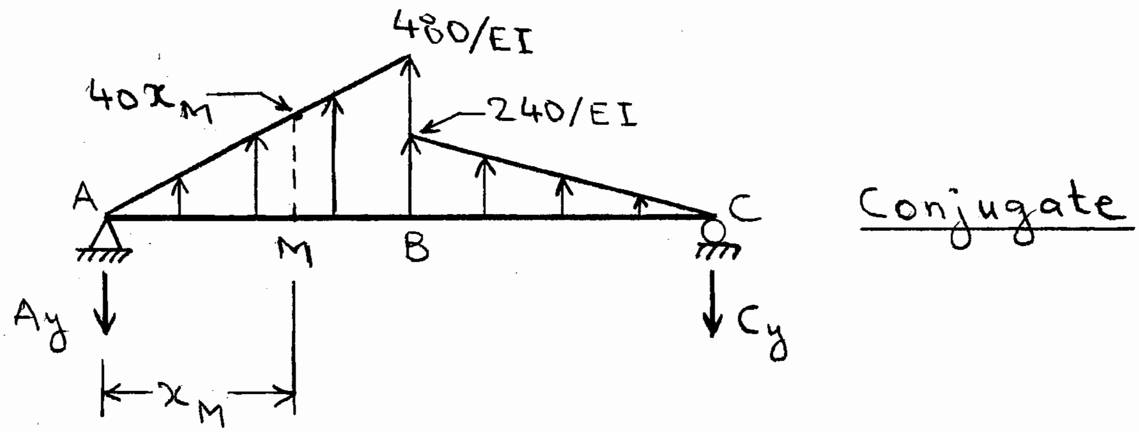
$$\Delta_{max} = \Delta_M = \frac{1}{EI}\left[-150(8.66) + \frac{1}{2}(34.64)(8.66)\left(\frac{8.66}{3}\right)\right]$$

$$= -\frac{866 \text{ kN}\cdot\text{m}^3}{EI} = -\frac{866}{70(712)} = -0.0174 \text{ m}$$

$$= \underline{17.4 \text{ mm}\downarrow}$$

$\boxed{6.51}$





$+\circlearrowleft \Sigma M_C = 0$

$$A_y(24) - \frac{1}{2}\left(\frac{480}{EI}\right)(12)(16) - \frac{1}{2}\left(\frac{240}{EI}\right)(12)(8) = 0$$

$$A_y = \frac{2400\ \text{kN.m}^2}{EI}$$

$S_M = 0$

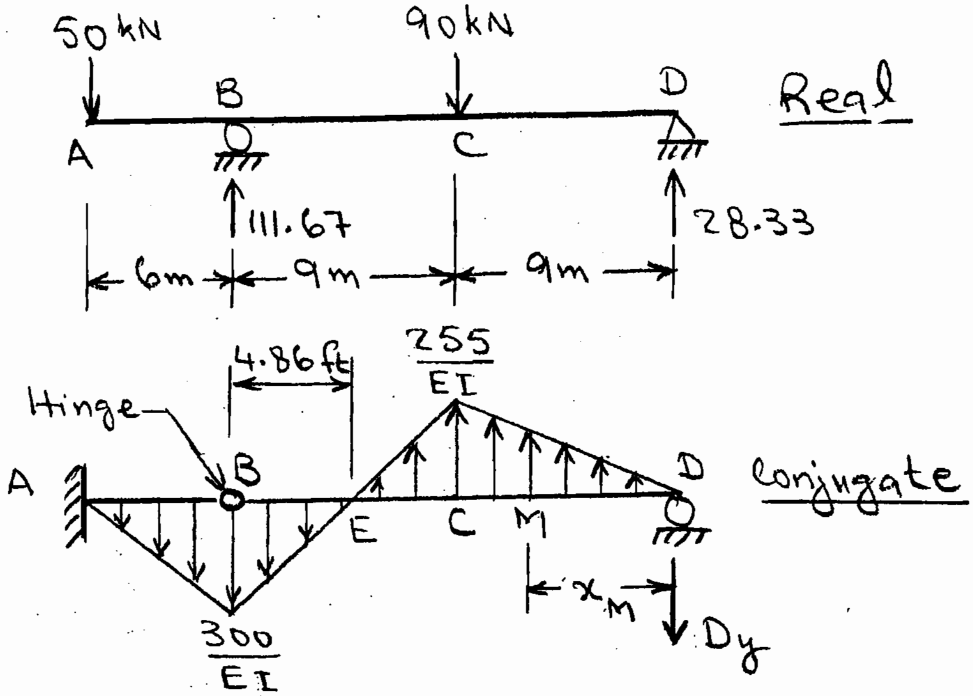
$$\frac{1}{EI}\left[-2400 + \frac{1}{2}(40x_M)x_M\right] = 0 \qquad \underline{x_M = 10.95\ m}$$

$$\Delta_{max} = \Delta_M = \frac{1}{EI}\left[-2400(10.95) + \frac{1}{2}(438)(10.95)\left(\frac{10.95}{3}\right)\right]$$

$$= -\frac{17527\ \text{kN.m}^3}{EI} = -\frac{17527}{200(600)} = -0.146\ m$$

$$\underline{\Delta_{max} = 146\ mm \downarrow}$$

$\boxed{6.52}$



$$+\circlearrowleft \Sigma M_B^{BD} = 0$$

$$-Dy(18) + \frac{1}{2}\left(\frac{255}{EI}\right)(9)(12) + \frac{1}{2}\left(\frac{255}{EI}\right)(4.14)(7.62)$$

$$-\frac{1}{2}\left(\frac{300}{EI}\right)(4.86)\left(\frac{4.86}{3}\right) = 0$$

$$Dy = \frac{922.85 \text{ kN.m}^2}{EI}$$

Let the maximum bending moment in the conjugate beam occur at point M, at a distance x_M from point D. Then the shear at M must be zero. Thus,

$$S_M = \frac{1}{EI}\left[922.85 - \frac{1}{2}\left(\frac{255}{9}x_M\right)x_M\right] = 0$$

$$x_M = 8.07 \text{ m}$$

$$\Delta_{max} = \Delta_M = \frac{1}{EI}\left[-922.85(8.07) + \frac{1}{2}\left(\frac{255}{9}\right)(8.07)\cdot\right.$$

$$\left.(8.07)\left(\frac{8.07}{3}\right)\right] = -\frac{4965 \text{ kN.m}^3}{EI}$$

$$= -\frac{4965}{70(95)} = -0.746 \text{ m} = \underline{746 \text{ mm} \downarrow}$$

$\boxed{6.52 \text{ (contd.)}}$

$$1.67\,x_M^2 + 233.33\,x_M - 1314.8 = 0$$

$$x_M = 5.42 \text{ ft}$$

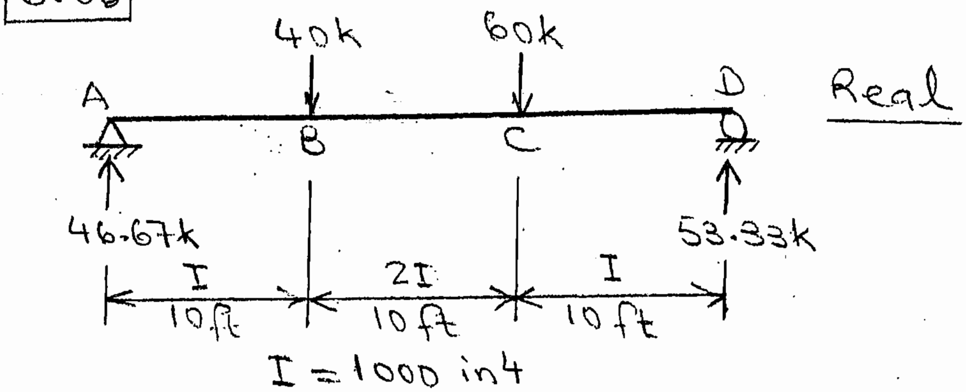
$$\Delta_{max} = \Delta_M = \frac{1}{EI}\Big[-3648.15(15.42)$$

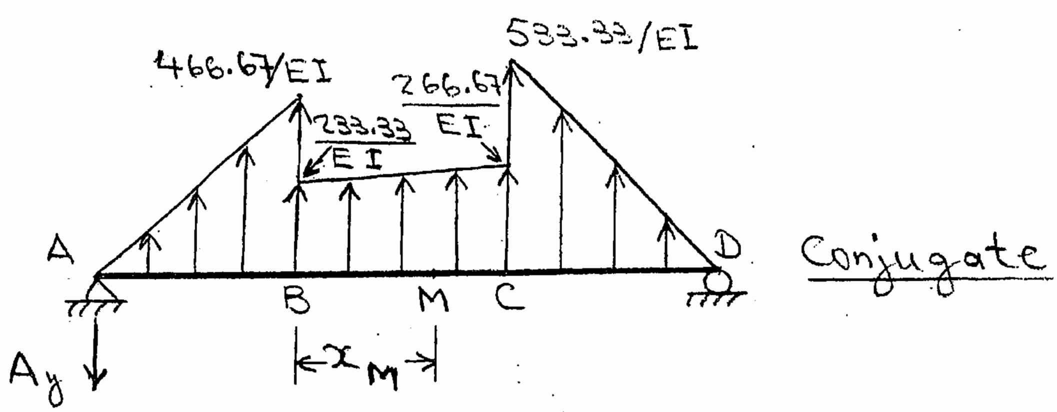
$$+ \frac{1}{2}(466.67)10\left(\frac{10}{3}+5.42\right) + 233.33(5.42)\left(\frac{5.42}{2}\right)$$

$$+ \frac{1}{2}(18.06)5.42\left(\frac{5.42}{3}\right)\Big] = -\frac{32314 \text{ k-ft}^3}{EI}$$

$$= -\frac{32314(12)^3}{29000(1000)} = -1.92 \text{ in.} = \underline{1.92 \text{ in.}\downarrow}$$

$\boxed{6.53}$





6.53 (contd.)

Reaction for conjugate beam:

$$+\circlearrowleft \Sigma M_D = 0$$

$$A_y(30) - \frac{1}{EI}\left[\frac{1}{2}(466.67)10\left(\frac{10}{3}+20\right) + 233.33(10)15 + \frac{1}{2}(33.33)10\left(\frac{10}{3}+10\right) + \frac{1}{2}(533.33)10\left(\frac{20}{3}\right)\right] = 0$$

$$A_y = \frac{3648.15 \text{ k-ft}^2}{EI}$$

Let the maximum bending moment in the conjugate beam occur at point M, at a distance x_M from point B. Then the shear at M must be zero. Thus,

$$S_M = \frac{1}{EI}\left[-3648.15 + \frac{1}{2}(466.67)10 + 233.33x_M + \frac{1}{2}(3.33x_M)x_M\right] = 0$$

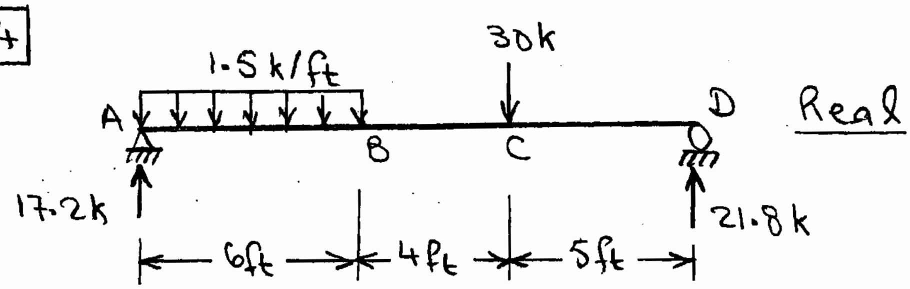
$$1.67x_M^2 + 233.33x_M - 1314.8 = 0$$

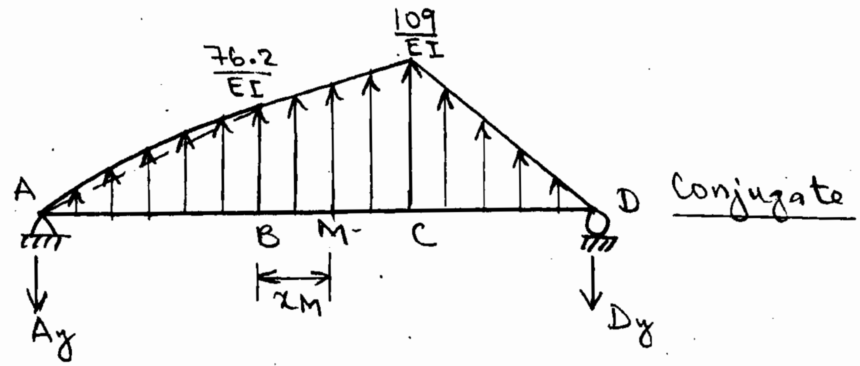
$$x_M = 5.42 \text{ ft}$$

$$\Delta_{max} = \Delta_M = \frac{1}{EI}\left[-3648.15(15.42) + \frac{1}{2}(466.67)10\left(\frac{10}{3}+5.42\right) + 233.33(5.42)\left(\frac{5.42}{2}\right) + \frac{1}{2}(18.06)5.42\left(\frac{5.42}{3}\right)\right] = -\frac{32314 \text{ k-ft}^3}{EI}$$

$$= -\frac{32314(12)^3}{29000(1000)} = -1.92 \text{ in.} = \underline{1.92 \text{ in.}\downarrow}$$

6.54





$$+\circlearrowleft \sum M_D = 0$$

$$A_y(15) - \frac{1}{EI}\left[\frac{1}{2}(76.2)6(11) + \frac{2}{3}\frac{(1.5)(6)^2}{8}(6)(12) + 76.2(4)7 + \frac{1}{2}(32.8)(4)\left(\frac{4}{3}+5\right) + \frac{1}{2}(109)5\left(\frac{10}{3}\right)\right] = 0$$

$$A_y = \frac{419.73\ \text{k-ft}^2}{EI}$$

Let the maximum bending moment in the conjugate beam occur at point M. Thus,

$$S_M = \frac{1}{EI}\left[-419.73 + \frac{1}{2}(76.2)6 + \frac{2}{3}\frac{(1.5)(6)^2}{8}(6) + 76.2x_M + \frac{1}{2}(8.2x_M)x_M\right] = 0$$

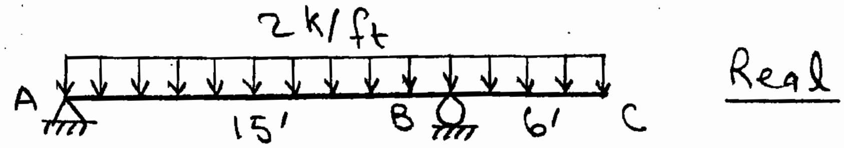
$$4.1x_M^2 + 76.2x_M - 164.13 = 0$$

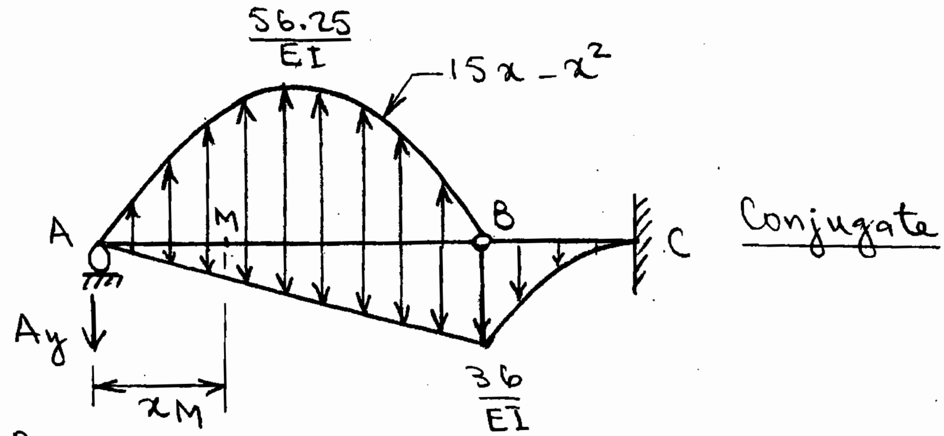
$$x_M = 1.95 \text{ ft}$$

$$\Delta_{max} = \Delta_M = \frac{1}{EI}\left[-419.73(7.95) + \frac{1}{2}(76.2)6(3.95) + \frac{2}{3}\frac{(1.5)(6)^2}{8}(6)(4.95) + 76.2\frac{(1.95)^2}{2} + \frac{1}{2}(15.99)(1.95)\left(\frac{1.95}{3}\right)\right] = -\frac{2145.2\ \text{k-ft}^3}{EI}$$

$$= -\frac{2145.2(12)^3}{1500(20000)} = -0.124 \text{ in} = \underline{0.124 \text{ in.}\downarrow}$$

6.55





$$+\circlearrowleft \sum M_B^{AB} = 0$$

$$A_y(15) - \frac{1}{EI}\left[\frac{2}{3}(56.25)15(7.5) - \frac{1}{2}(36)15(5)\right] = 0$$

$$A_y = \frac{191.25 \text{ k-ft}^2}{EI}$$

$$S_M = 0$$

$$\frac{1}{EI}\left[-191.25 + \frac{1}{2}(15x_M - x_M^2)x_M + \frac{2}{3}\left(\frac{2x_M^2}{8}\right)x_M - \frac{1}{2}(2.4x_M)x_M\right] = 0$$

$$-\frac{x_M^3}{3} + 6.3x_M^2 - 191.25 = 0$$

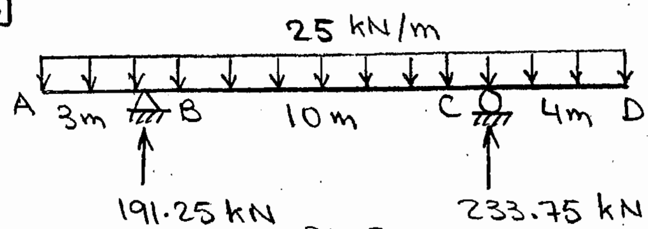
from which $\quad x_M = 6.92$ ft

$$\Delta_{max} = \Delta_M = \frac{1}{EI}\left[-191.25(6.92) + \frac{1}{2}(55.91)\frac{(6.92)^2}{3} + \frac{2}{3}(11.97)\frac{(6.92)^2}{2} - \frac{1}{2}(16.61)6.92\left(\frac{6.92}{3}\right)\right]$$

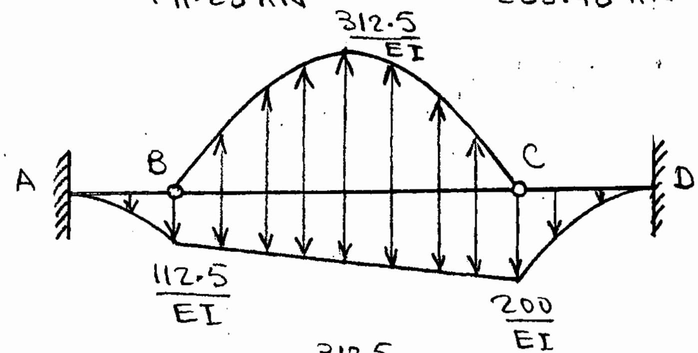
$$= -\frac{818.73 \text{ k-ft}^3}{EI} = -0.0139 \text{ in}$$

$$= \underline{0.0139 \text{ in.} \downarrow}$$

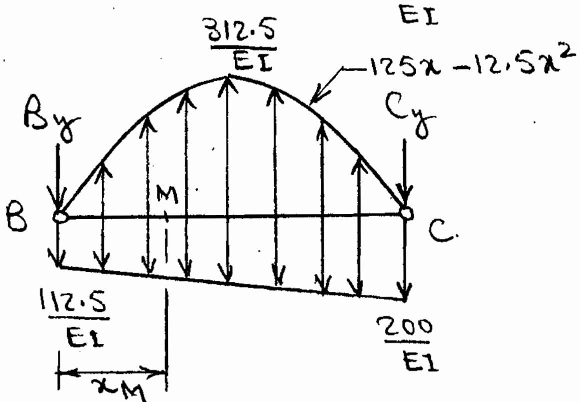
6.56



Real



Conjugate



$$+\circlearrowleft \Sigma M_C^{BC} = 0$$

$$B_y(10) - \frac{1}{EI}\left[\frac{2}{3}(312.5)10(5) - 112.5(10)5 - \frac{1}{2}(87.5)10\left(\frac{10}{3}\right)\right] = 0$$

$$B_y = \frac{333.33 \text{ kN-m}^2}{EI}$$

$$\delta_M = 0$$

$$\frac{1}{EI}\left[-333.33 + \frac{1}{2}(125x_M - 12.5x_M^2)x_M + \frac{2}{3}\left(\frac{25x_M^2}{8}\right)x_M - 112.5x_M - \frac{1}{2}(8.75x_M)x_M\right] = 0$$

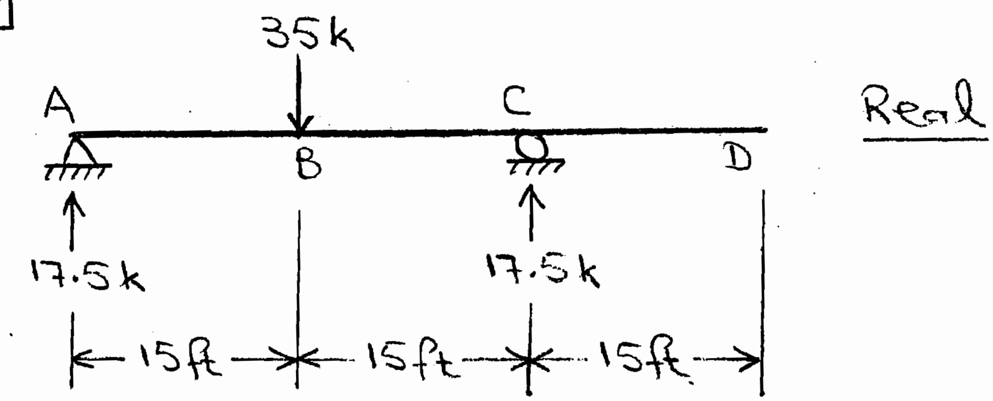
$$-4.167x_M^3 + 58.125x_M^2 - 112.5x_M - 333.33 = 0$$

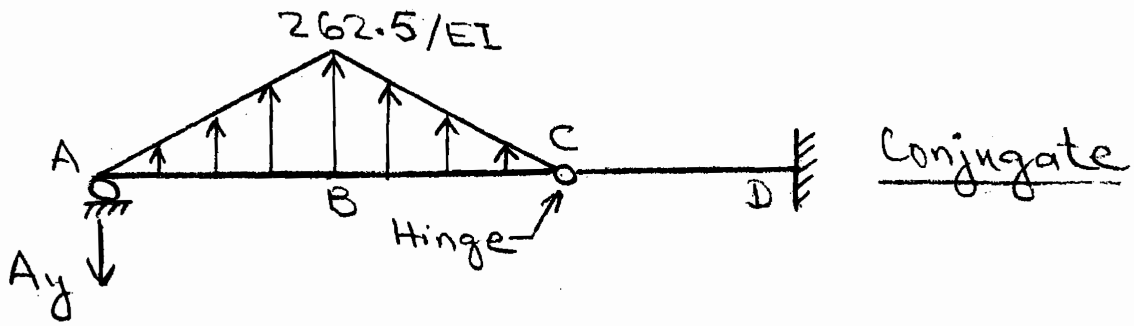
from which $x_M = 4.77$ m

$$\Delta_{max} = \Delta_M = \frac{1}{EI}\left[-333.33(4.77) + \frac{1}{2}(311.84)\frac{(4.77)^2}{3} + \frac{2}{3}(71.1)\frac{(4.77)^2}{2} - 112.5\frac{(4.77)^2}{2} - \frac{1}{2}(41.74)\frac{(4.77)^2}{3}\right]$$

$$= -\frac{1306.3 \text{ kN-m}^3}{EI} = -0.0131 \text{ m} = \underline{13.1 \text{ mm} \downarrow}$$

6.57





$$+\circlearrowleft \Sigma M_C^{AC} = 0$$

$$A_y(30) - \frac{1}{2}\left(\frac{262.5}{EI}\right)30\,(15) = 0$$

$$A_y = \frac{1968.75\ \text{k-ft}^2}{EI}$$

$$\theta_D = \frac{1}{EI}\left[-1968.75 + \frac{1}{2}(262.5)30\right] = \frac{1968.75\ \text{k-ft}^2}{EI}$$

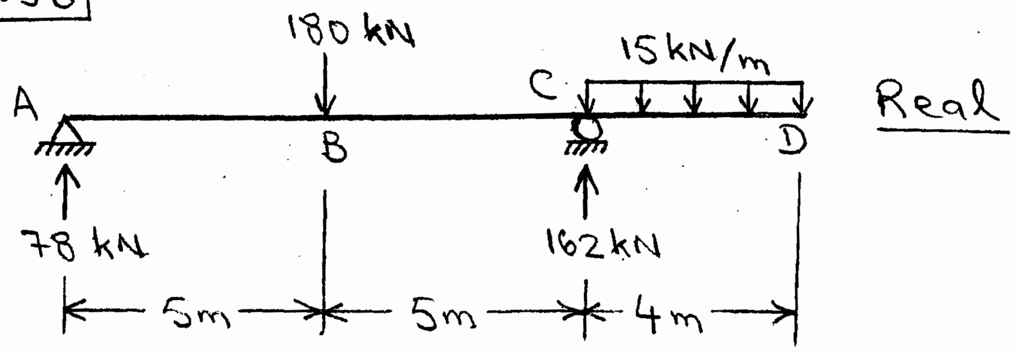
$$= \frac{1968.75\,(12)^2}{10000\,(2500)} = \underline{0.01134\ \text{rad}}$$

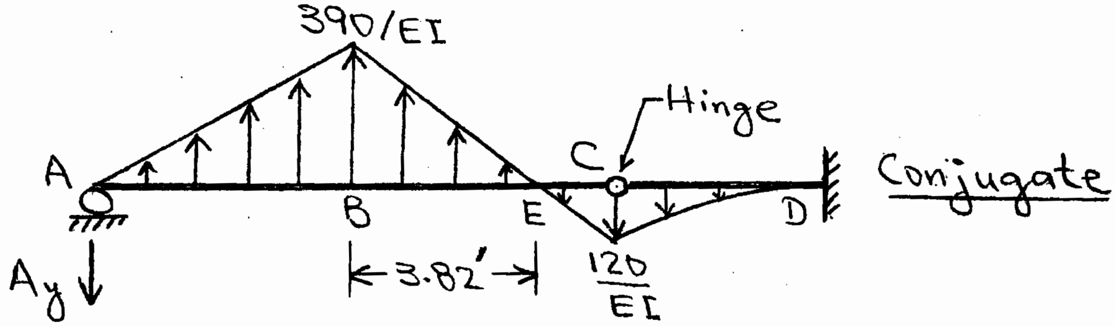
$$\Delta_D = \frac{1}{EI}\left[-1968.75(45) + \frac{1}{2}(262.5)30(30)\right]$$

$$= \frac{29531.25\ \text{k-ft}^3}{EI} = \frac{29531.25\,(12)^3}{10000\,(2500)}$$

$$= \underline{2.04\ \text{in.}\uparrow}$$

$\boxed{6.58}$





$$+\circlearrowleft \Sigma M_C^{AC} = 0$$

$$A_y(10) - \frac{1}{EI}\left[\frac{1}{2}(390)5\left(\frac{5}{3}+5\right) + \frac{1}{2}(390)(3.82)(3.73) - \frac{1}{2}(120)1.18\left(\frac{1.18}{3}\right)\right] = 0$$

$$A_y = \frac{925.1 \text{ kN}\cdot\text{m}^2}{EI}$$

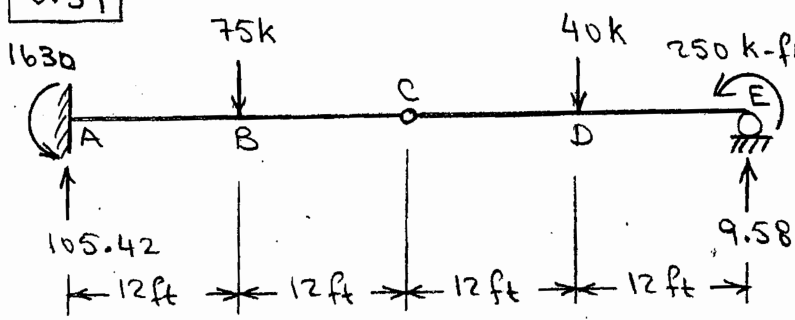
$$\theta_D = \frac{1}{EI}\left[-925.1 + \frac{1}{2}(390)8.82 - \frac{1}{2}(120)1.18 - \frac{1}{3}(120)4\right]$$

$$= \frac{564 \text{ kN}\cdot\text{m}^2}{EI} = \frac{564}{70(2340)} = \underline{0.00344 \text{ rad}}$$

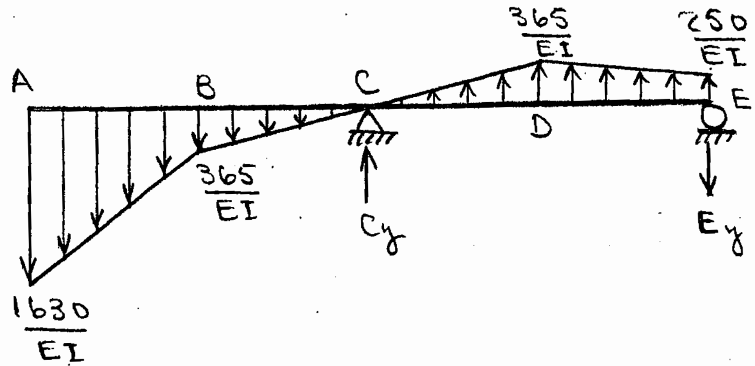
$$\Delta_D = \frac{1}{EI}\left[-925.1(14) + \frac{1}{2}(390)5(10.67) + \frac{1}{2}(390)(3.82)(7.727) - \frac{1}{2}(120)1.18(4.393) - \frac{1}{3}(120)4(3)\right] = \frac{2417 \text{ kN}\cdot\text{m}^3}{EI}$$

$$= \frac{2417}{70(2340)} = 0.01476 \text{ m} = \underline{14.76 \text{ mm} \uparrow}$$

6.59



Real



Conjugate

$+\circlearrowright \Sigma M_E = 0$

$$\frac{1}{EI}\left[\frac{1}{2}(1265)12(44) + 365(12)42 + \frac{1}{2}(365)12(32) - \frac{1}{2}(365)12(16) - \frac{1}{2}(115)12(8) - 250(12)6\right]$$

$$-C_y(24) = 0$$

$$C_y = \frac{22060 \text{ k-ft}^2}{EI} \uparrow$$

$+\uparrow \Sigma F_y = 0$

$$\frac{1}{EI}\left[-\frac{1}{2}(1630+365)12 + \frac{1}{2}(365+250)12 + 22060\right]$$

$$-E_y = 0$$

$$E_y = \frac{13780 \text{ k-ft}^2}{EI} \downarrow$$

$$\theta_B = -\frac{1}{EI}\left[\frac{1}{2}(1630+365)12\right] = -\frac{11970 \text{ k-ft}^2}{EI} = \underline{0.0099 \text{ rad}}$$

$$\Delta_B = -\frac{1}{EI}\left[365(12)6 + \frac{1}{2}(1265)12(8)\right] = -\frac{87000 \text{ k-ft}^3}{EI}$$

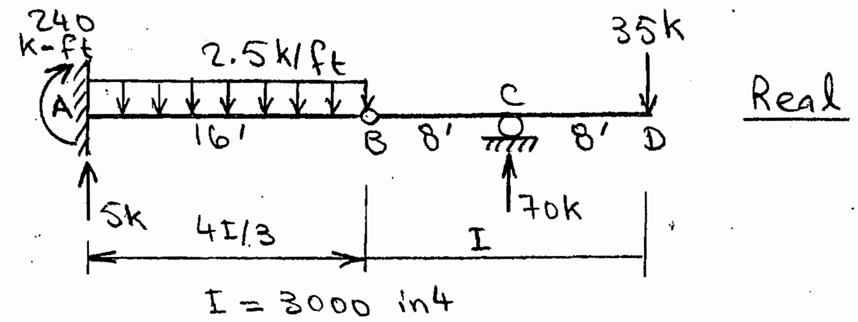
$$\underline{\Delta_B = 0.86 \text{ in} \downarrow}$$

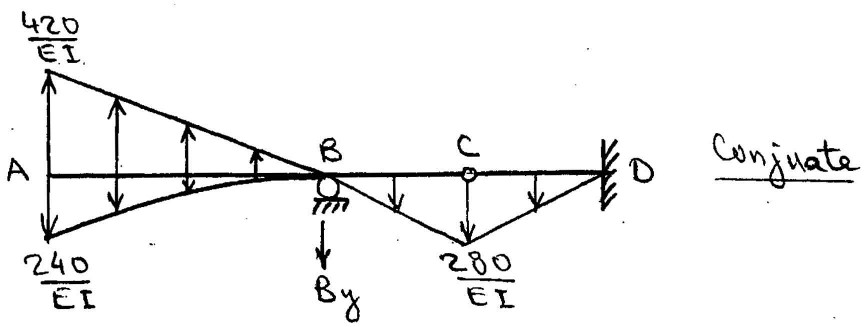
$$\theta_D = \frac{1}{EI}\left[13780 - \frac{1}{2}(365+250)12\right] = \frac{10090}{EI} = \underline{0.0084 \text{ rad}}$$

$$\Delta_D = \frac{1}{EI}\left[-13780(12) + 250(12)6 + \frac{1}{2}(115)12(4)\right]$$

$$= -\frac{144600 \text{ k-ft}^3}{EI} = 1.44 \text{ in} \downarrow$$

6.60





$+\circlearrowright \Sigma M_C^{AC} = 0$

$$\frac{1}{EI}\left[\frac{1}{3}(240)16(20) - \frac{1}{2}(420)16(18.67) + \frac{1}{2}(280)8\left(\frac{8}{3}\right)\right] + B_y(8) = 0$$

$$B_y = \frac{4266.67 \text{ k-ft}^2}{EI} \downarrow$$

$$\theta_{B,Left} = \frac{1}{EI}\left[\frac{1}{2}(420)16 - \frac{1}{3}(240)16\right] = \frac{2080 \text{ k-ft}^2}{EI}$$

$$= \frac{2080(12)^2}{30000(3000)} = \underline{0.0033 \text{ rad.}}$$

$$\theta_{B,Right} = \frac{1}{EI}(2080 - 4266.67) = -\frac{2186.67 \text{ k-ft}^2}{EI}$$

$$= \underline{-0.0035 \text{ rad.}}$$

$$\Delta_B = \frac{1}{EI}\left[\frac{1}{2}(420)16(10.67) - \frac{1}{3}(240)16(12)\right]$$

$$= \frac{20480 \text{ k-ft}^3}{EI} = \frac{20480(12)^3}{30000(3000)} = \underline{0.39 \text{ in} \uparrow}$$

$$\theta_D = \frac{1}{EI}\left[\frac{1}{2}(420)16 - \frac{1}{3}(240)16 - 4266.67 - \frac{1}{2}(280)16\right]$$

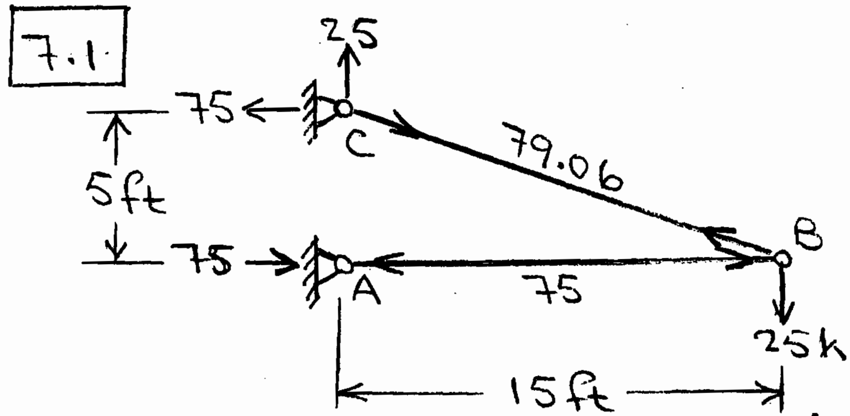
$$= -\frac{4426.67 \text{ k-ft}^2}{EI} = -0.0071 \text{ rad.} = \underline{0.0071 \text{ rad.}}$$

$$\Delta_D = \frac{1}{EI}\left[\frac{1}{2}(420)16(26.67) - \frac{1}{3}(240)16(28) - 4266.67(16) - \frac{1}{2}(280)16(8)\right] = -\frac{32426.67}{EI} = \underline{0.62 \text{ in} \downarrow}$$

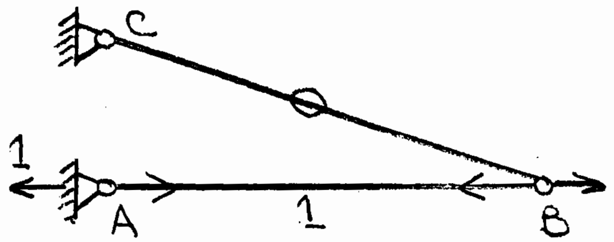
Chapter Seven
Deflections of Trusses, Beams, and Frames: Work-Energy Methods

CHAPTER 7

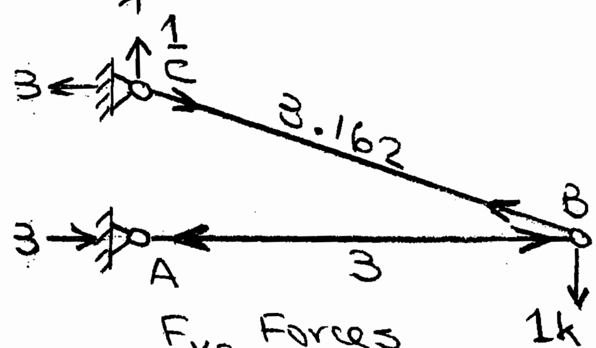
7.1



F Forces



F_{v1} Forces



F_{v2} Forces

Member	L (in)	F (k)	F_{v1} (k)	$F_{v1}(FL)$ (k^2-in)	F_{v2} (k)	$F_{v2}(FL)$ (k^2-in)
AB	180	−75	1	−13500	−3	40500
BC	189.74	79.06	0	0	3.162	47432.67
			Σ	−13500		87932.67

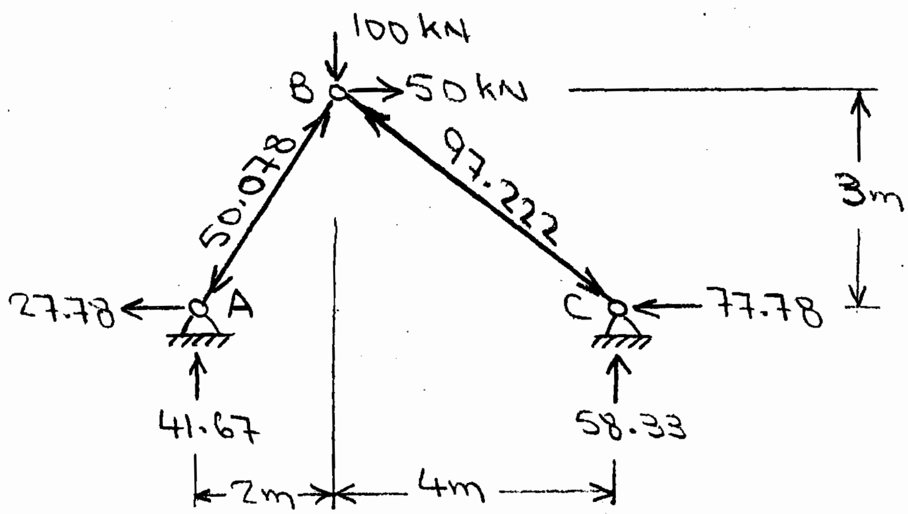
$$(1k)\,\Delta_{BH} = -\frac{13500}{10000(6)} = -0.225 \text{ k-in.}$$

$$\Delta_{BH} = -0.225 \text{ in.} = \underline{0.225 \text{ in.} \leftarrow}$$

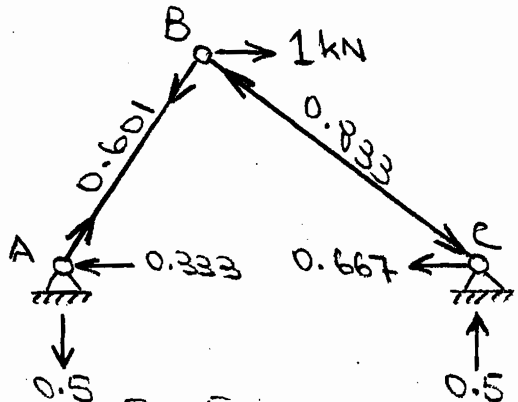
$$(1k)\,\Delta_{BV} = \frac{87932.67}{10000(6)} = 1.466 \text{ k-in.}$$

$$\underline{\Delta_{BV} = 1.466 \text{ in.} \downarrow}$$

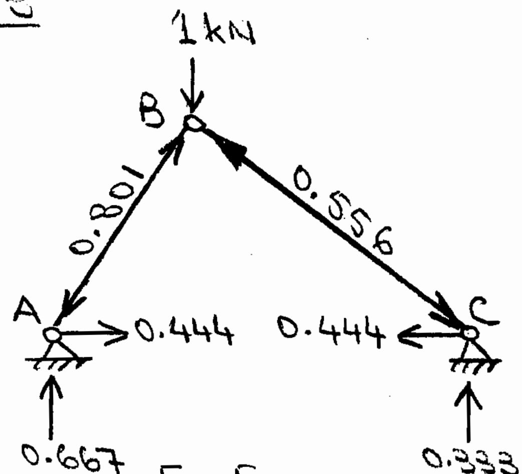
7.2



F Forces



F_{V1} Forces



F_{V2} Forces

Member	L (m)	F (kN)	F_{V1} (kN)	$F_{V1}(FL)$ (kN^2-m)	F_{V2} (kN)	$F_{V2}(FL)$ (kN^2-m)
AB	3.606	−50.078	0.601	−108.53	−0.801	144.65
BC	5	−97.222	−0.833	405.09	−0.556	270.06
			Σ	296.56		414.71

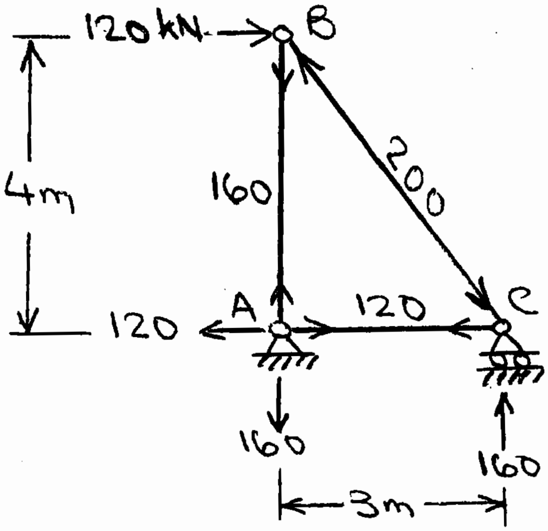
$$(1\text{kN})\,\Delta_{BH} = \frac{296.56}{70(10^6)(0.001)} = 0.00424 \text{ kN-m}$$

$$\Delta_{BH} = 0.00424\text{ m} = \underline{4.24\text{ mm} \rightarrow}$$

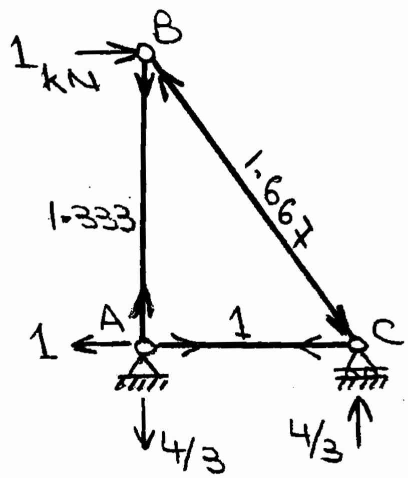
$$(1\text{kN})\,\Delta_{BV} = \frac{414.71}{70(10^6)(0.001)} = 0.00592 \text{ kN-m}$$

$$\Delta_{BV} = 0.00592\text{ m} = \underline{5.92\text{ mm}\downarrow}$$

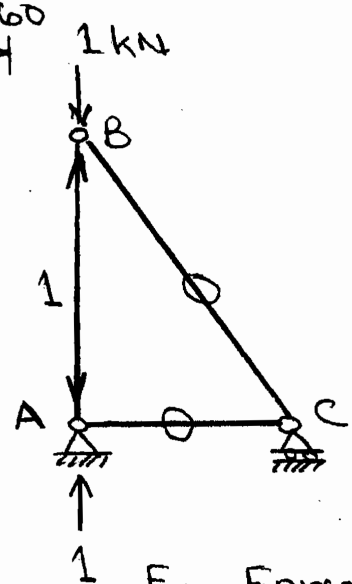
7.3



F Forces



F_{V_1} Forces



F_{V_2} Forces

Member	L (m)	F (kN)	F_{V1} (kN)	$F_{V1}(FL)$ (kN^2-m)	F_{V2} (kN)	$F_{V2}(FL)$ (kN^2-m)
AC	3	120	1	360	0	0
AB	4	160	1.333	853.33	−1	−640
BC	5	−200	−1.667	1666.67	0	0
			Σ	2880		−640

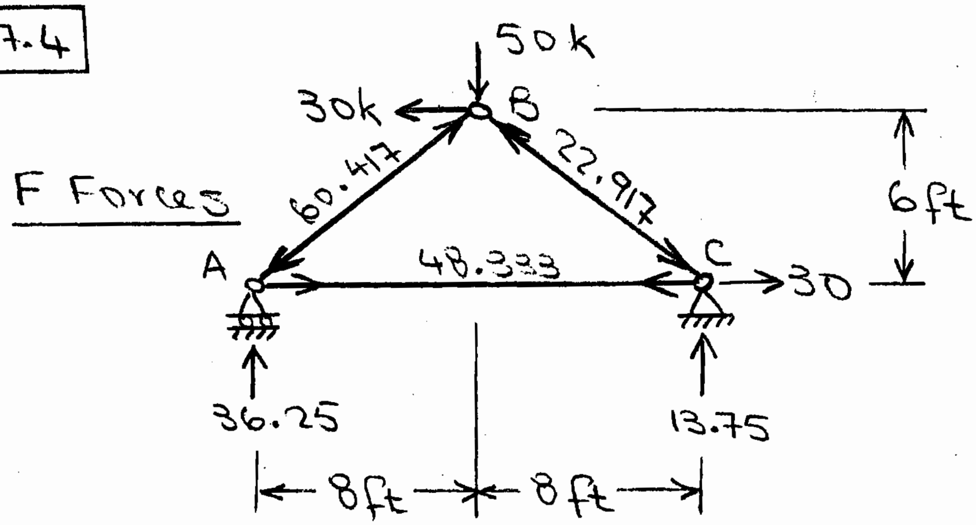
$$(1\text{kN})\,\Delta_{BH} = \frac{2880}{200(10^6)(0.0015)} = 0.0096 \text{ kN.m}$$

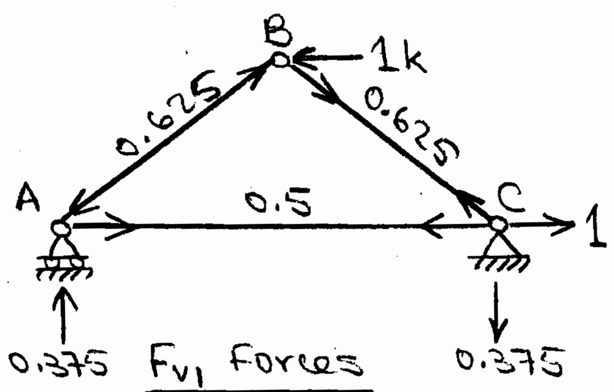
$$\Delta_{BH} = 0.0096 \text{ m} = \underline{9.6 \text{ mm} \rightarrow}$$

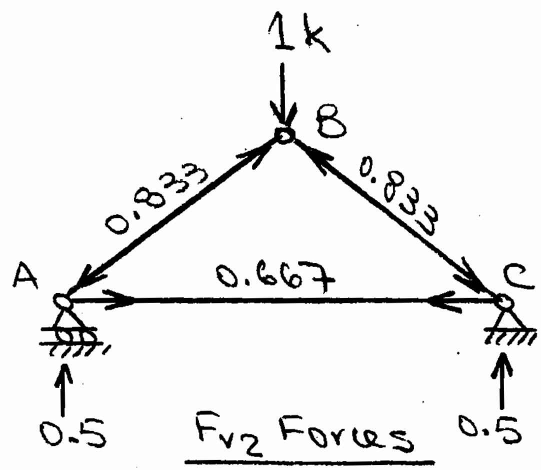
$$(1\text{kN})\,\Delta_{BV} = \frac{-640}{200(10^6)(0.0015)} = -0.00213 \text{ kN.m}$$

$$\Delta_{BV} = -0.00213 \text{ m} = \underline{2.13 \text{ mm} \uparrow}$$

7.4







Member	L (in.)	F (k)	Fv1 (k)	Fv1 (FL) (k²-in.)	Fv2 (k)	Fv2 (FL) (k²-in.)
AC	192	48.333	0.5	4640	0.667	6186.67
AB	120	-60.417	-0.625	4531.25	-0.833	6041.67
BC	120	-22.917	0.625	-1718.75	-0.833	2291.67
			Σ	7452.5		14520

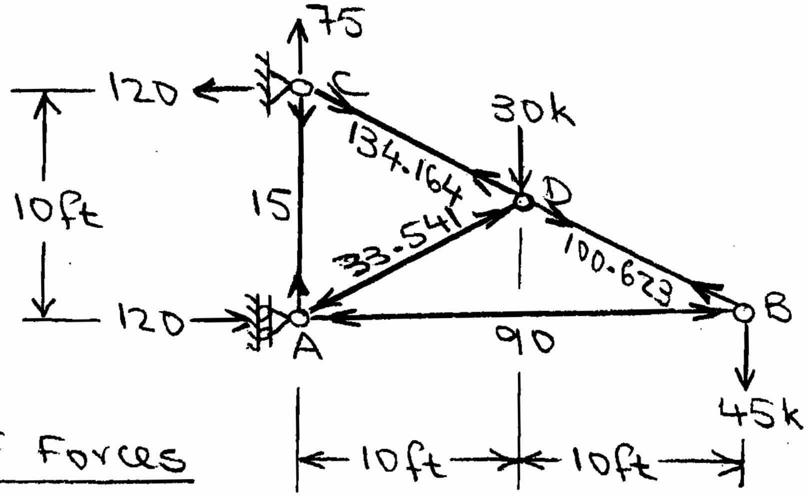
$$(1k)\,\Delta_{BH} = \frac{7452.5}{29000(3)} = 0.0857 \text{ k-in.}$$

$$\underline{\Delta_{BH} = 0.0857 \text{ in.} \leftarrow}$$

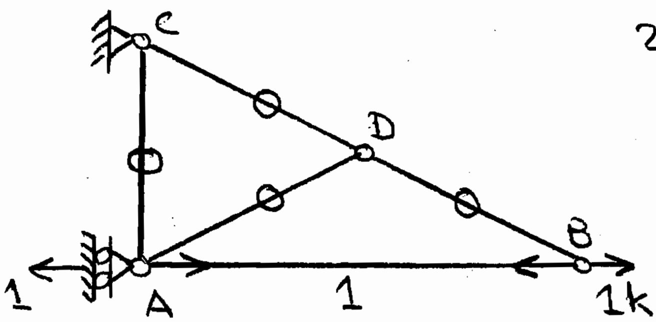
$$(1k)\,\Delta_{BV} = \frac{14520}{29000(3)} = 0.167 \text{ k-in.}$$

$$\underline{\Delta_{BV} = 0.167 \text{ in.} \downarrow}$$

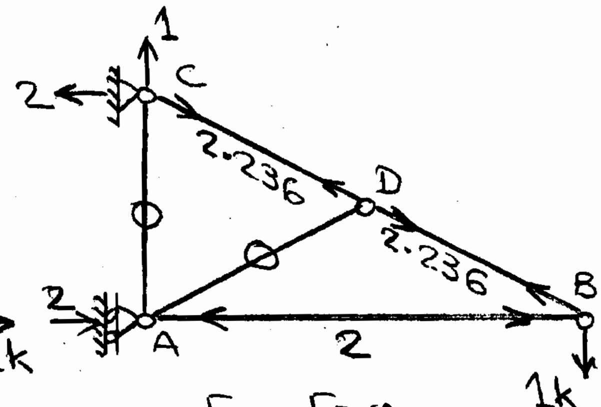
7.5



F Forces



F_{V1} Forces



F_{V2} Forces

Member	L (in.)	A (in.²)	F (k)	F_{V1} (k)	$F_{V1}(FL/A)$ (k²/in.)	F_{V2} (k)	$F_{V2}(FL/A)$ (k²/in.)
AB	240	6	−90	1	−3600	−2	7200
AC	120	4	15	0	0	0	0
AD	134.16	4	−33.541	0	0	0	0
CD	134.16	6	134.164	0	0	2.236	6707.99
BD	134.16	6	100.623	0	0	2.236	5030.99
				Σ	−3600		18938.98

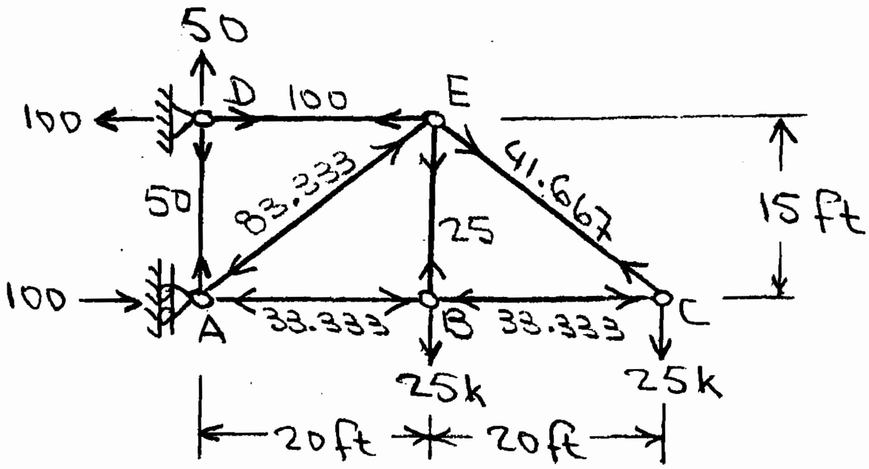
$$(1k)\Delta_{BH} = \frac{-3600}{10000} = -0.36\text{ k-in}$$

$$\Delta_{BH} = 0.36\text{ in.}\leftarrow$$

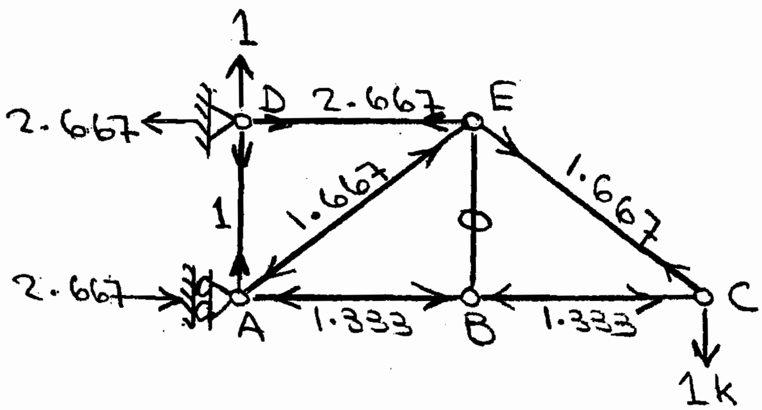
$$(1k)\Delta_{BV} = \frac{18938.99}{10000} = 1.894\text{ k-in.}$$

$$\Delta_{BV} = 1.894\text{ in.}\downarrow$$

7.6



F Forces

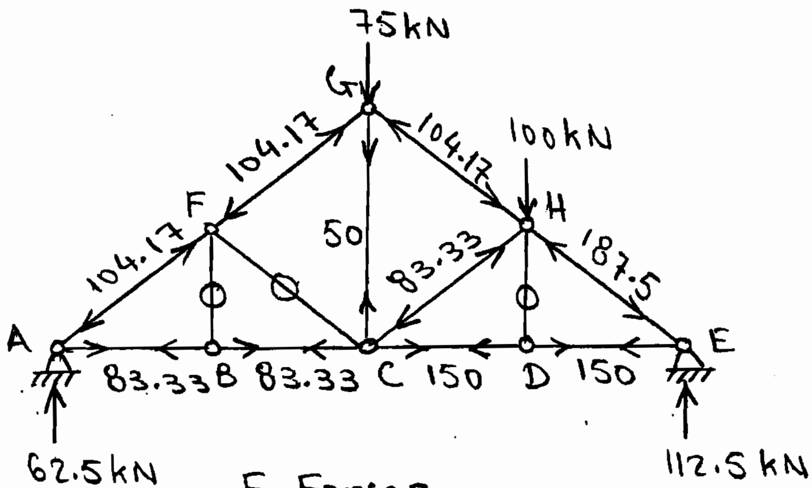


F_v Forces

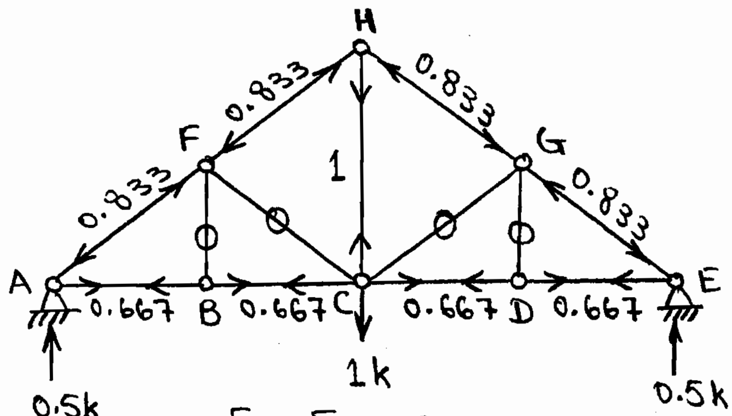
Member	L (in.)	A (in².)	F (k)	F_v (k)	F_v (FL/A) (k²/in.)
AB	240	4	−33.333	−1.333	2666.67
BC	240	4	−33.333	−1.333	2666.67
DE	240	4	100	2.667	16000
AD	180	3	50	1	3000
BE	180	3	25	0	0
AE	300	3	−83.333	−1.667	13888.89
CE	300	4	41.667	1.667	5208.33
				Σ	43430.56

$$\Delta_C = \frac{43430.56}{29000} = 1.498 \text{ in.} \downarrow$$

7.7



F Forces



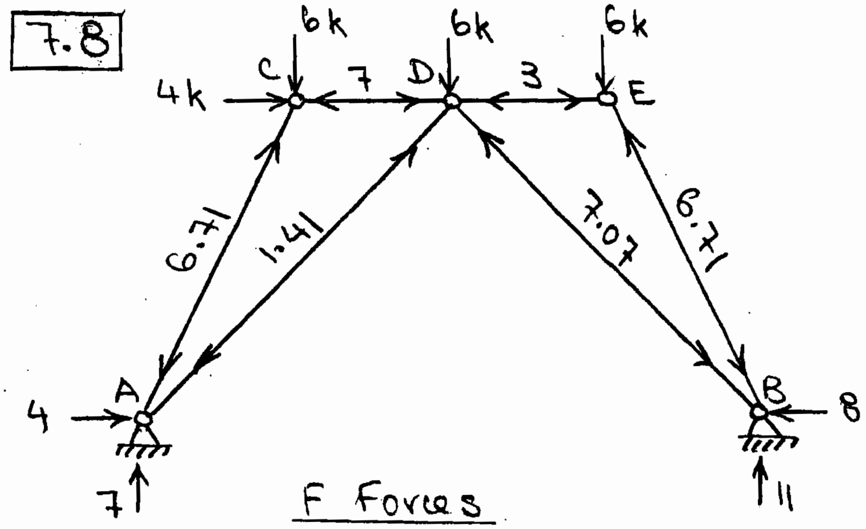
F_v Forces

Member	L (m)	F (kN)	F_v (kN)	$F_v(FL)$ (kN^2-m)
AB	6	83.33	0.667	333.5
BC	6	83.33	0.667	333.5
CD	6	150	0.667	600.3
DE	6	150	0.667	600.3
AF	7.5	−104.17	−0.833	650.8
FG	7.5	−104.17	−0.833	650.8
GH	7.5	−104.17	−0.833	650.8
EH	7.5	−187.5	−0.833	1171.4
CG	9	50	1	450
			Σ	5441.4

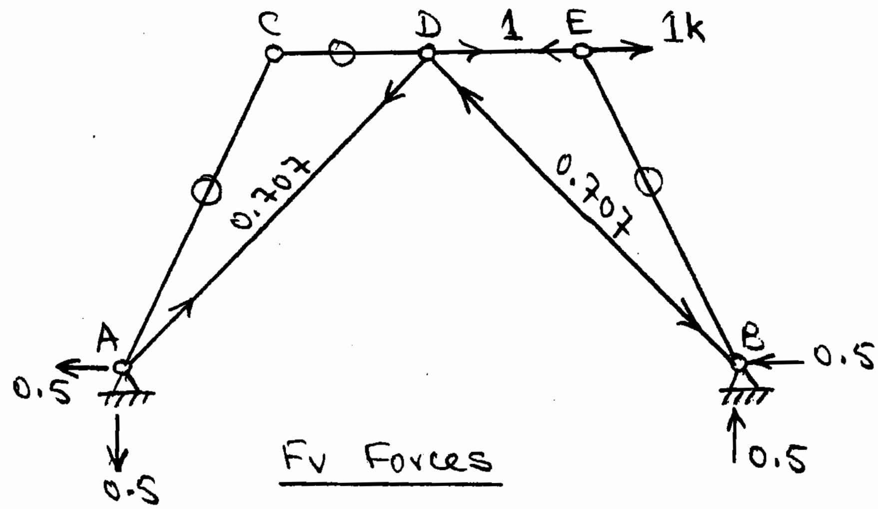
$$(1\text{kN})\,\Delta_C = \frac{5441.4}{200(10^6)(0.003)} = 0.0091 \text{ kN-m}$$

$$\Delta_C = 0.0091\text{ m} = \underline{9.1\text{ mm}\downarrow}$$

7.8



F Forces



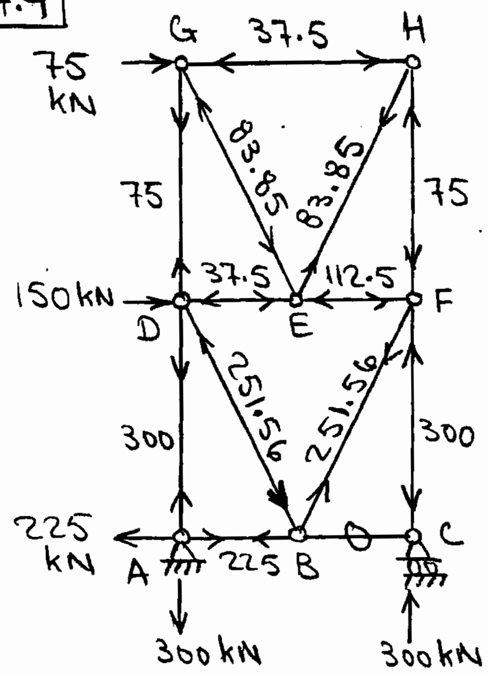
Fv Forces

Member	L (ft)	F (k)	Fv (k)	Fv (FL) (k^2-ft)
AD	14.14	-1.41	0.707	-14.1
BD	14.14	-7.07	-0.707	70.68
DE	5	-3	1	-15
			Σ	41.58

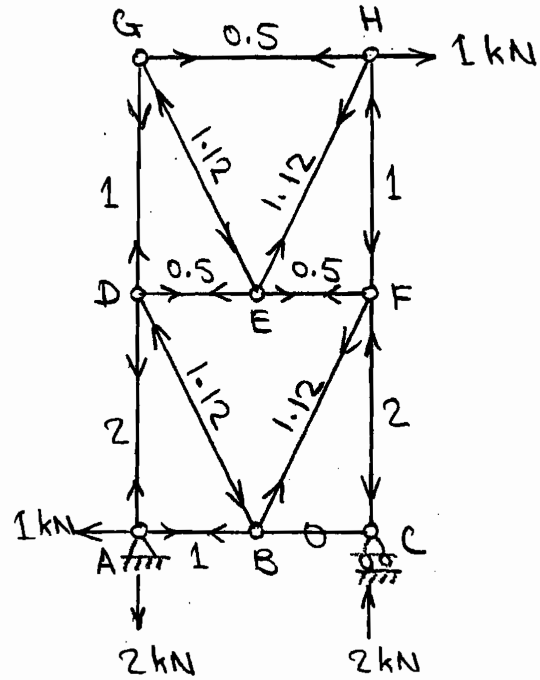
$$\Delta_{EH} = \frac{41.58}{(29000)6} = 0.000239\ \text{ft.}$$

$$= \underline{0.0029\ \text{in.} \rightarrow}$$

7.9



F Forces



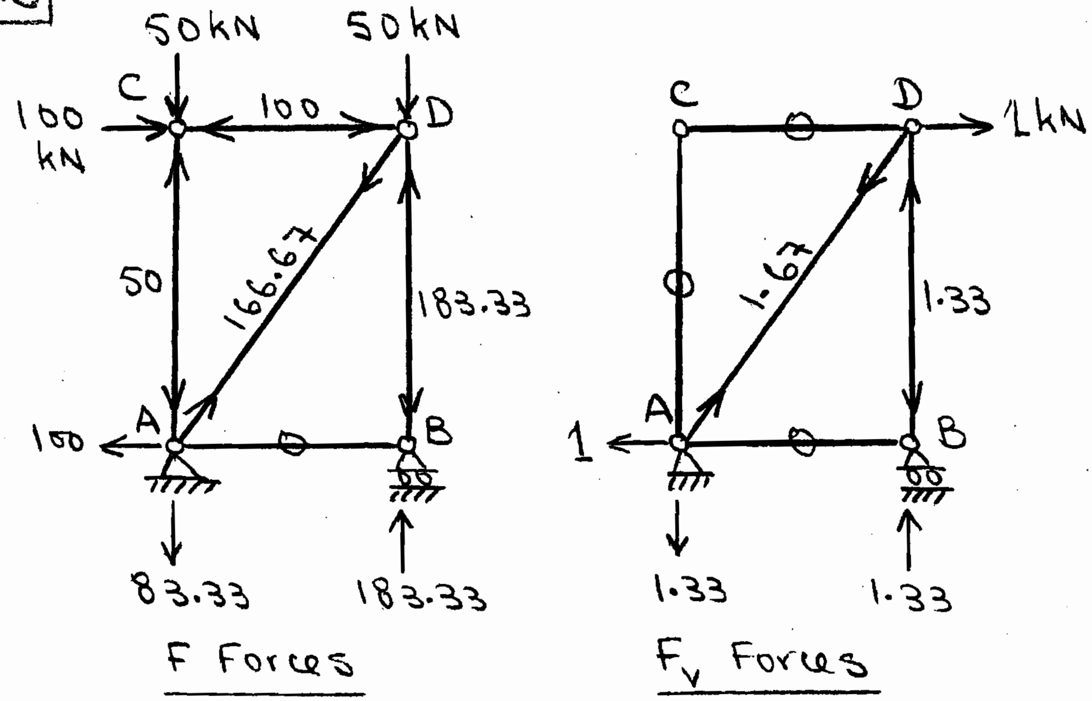
F_v Forces

Member	L (m)	A (m^2)	F (kN)	F_v (kN)	$F_v(FL/A)$ (kN^2/m)
AD	4	0.0025	300	2	960 000
CF	4	0.0025	−300	−2	960 000
DG	4	0.0025	75	1	120 000
FH	4	0.0025	−75	−1	120 000
GH	4	0.0015	−37.5	0.5	−50 000
AB	2	0.0015	225	1	300 000
DE	2	0.0015	−37.5	0.5	−25 000
EF	2	0.0015	−112.5	0.5	−75 000
BD	4.47	0.0015	−251.56	−1.12	839607
BF	4.47	0.0015	251.56	1.12	839607
EG	4.47	0.0015	−83.85	−1.12	279858
EH	4.47	0.0015	83.85	1.12	279858
				Σ	4548930

$$(1\,\text{kN})\,\Delta_H = \frac{4548930}{200(10^6)} = 0.023 \text{ kN-m}$$

$$\Delta_H = 0.023 \text{ m} = \underline{23. \text{ mm} \rightarrow}$$

7.10



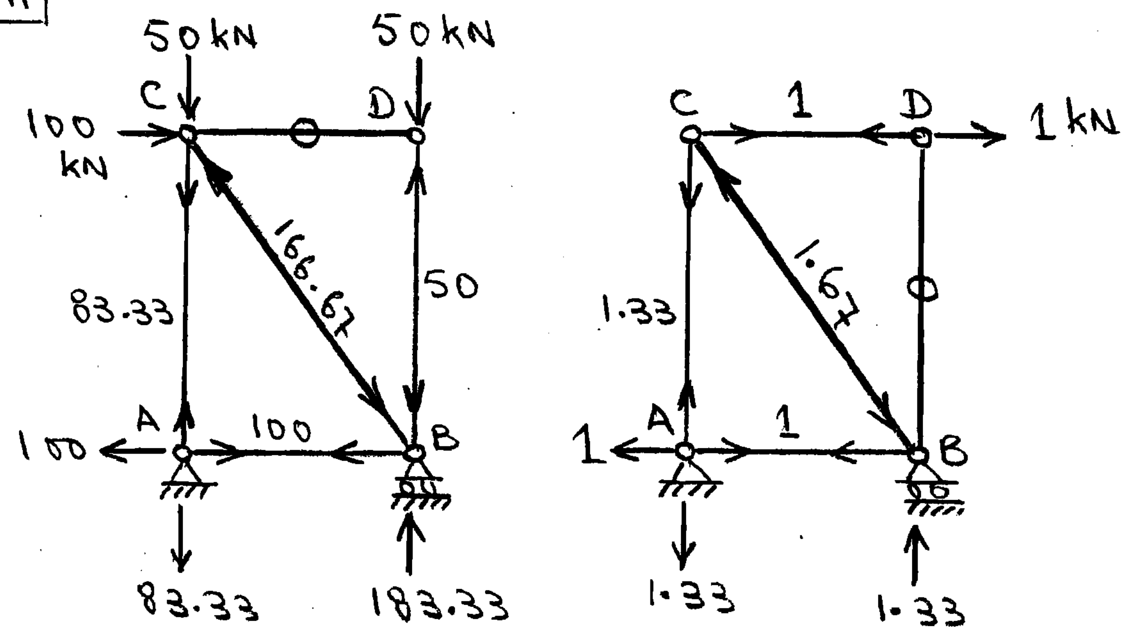
Member	L (m)	F (kN)	F_v (kN)	$F_v(FL)$ ($kN^2 \cdot m$)
AB	3	0	0	0
CD	3	−100	0	0
AC	4	−50	0	0
BD	4	−183.33	−1.33	975.32
AD	5	166.67	1.67	1391.69
			Σ	2367

$$(1\,kN)\Delta_D = \frac{2367\,kN^2 \cdot m}{EA}$$

$$\Delta_D = \frac{2367\,kN \cdot m}{EA} = \frac{2367}{70(10^6)A} = 0.01\,m$$

from which, $A = 0.003381\,m^2 = \underline{3381\,mm^2}$

7.11



F Forces $\qquad$ F_v Forces

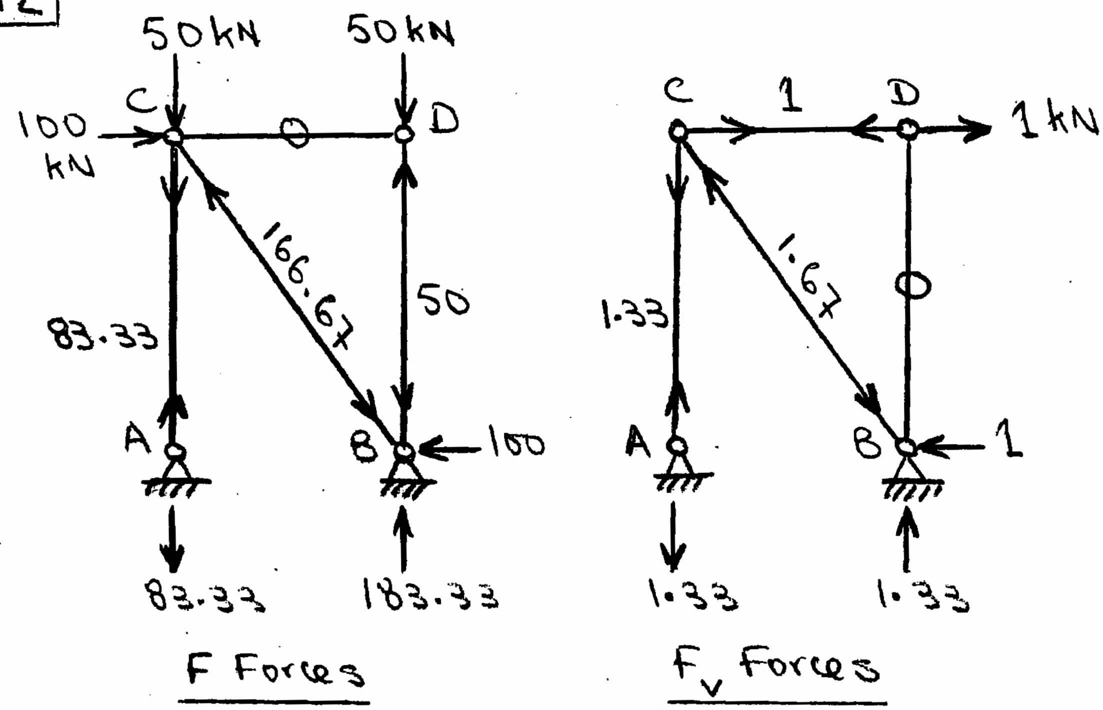
Member	L (m)	F (kN)	F_v (kN)	$F_v(FL)$ ($kN^2 \cdot m$)
AB	3	100	1	300
CD	3	0	1	0
AC	4	83.33	1.33	443.32
BD	4	-50	0	0
BC	5	-166.67	-1.67	1391.69
			Σ	2135

$$(1\,kN)\Delta_D = \frac{2135\ kN^2 \cdot m}{EA}$$

$$\Delta_D = \frac{2135\ kN \cdot m}{EA} = \frac{2135}{70(10^6)A} = 0.01\ m$$

from which, $A = 0.00305\ m^2 = \underline{3050\ mm^2}$

7.12



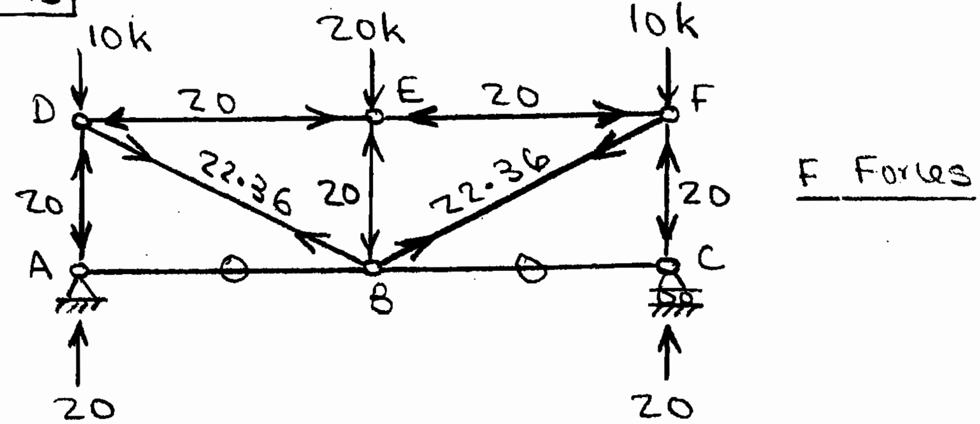
Member	L (m)	F (kN)	F_v (kN)	$F_v(FL)$ ($kN^2 \cdot m$)
CD	3	0	1	0
AC	4	83.33	1.33	443.32
BD	4	-50	0	0
BC	5	-166.67	-1.67	1391.69
			Σ	1835

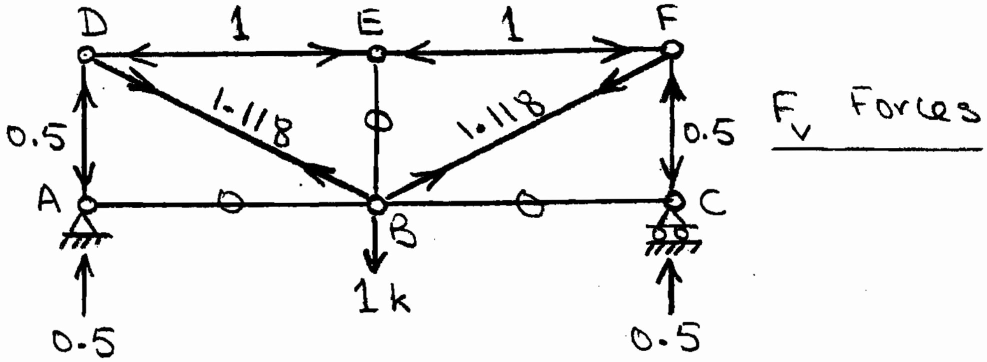
$$(1\,kN)\,\Delta_D = \frac{1835\ kN^2 \cdot m}{EA}$$

$$\Delta_D = \frac{1835\,kN \cdot m}{EA} = \frac{1835}{70(10^6)A} = 0.01\ m$$

from which, $A = 0.002621\ m^2 = \underline{2621\ mm^2}$

7.13





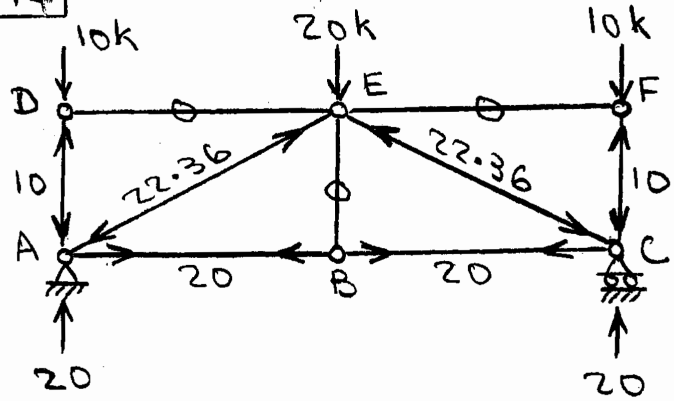
Member	L (in)	F (k)	F_v (k)	$F_v(FL)$ (k^2-in)
AB	72	0	0	0
BC	72	0	0	0
DE	72	-20	-1	1440
EF	72	-20	-1	1440
AD	36	-20	-0.5	360
BE	36	-20	0	0
CF	36	-20	-0.5	360
BD	80.5	22.36	1.118	2012.4
BF	80.5	22.36	1.118	2012.4
			Σ	7624.8

$$(1k)\Delta_B = \frac{7624.8 \ k^2\text{-in}}{EA}$$

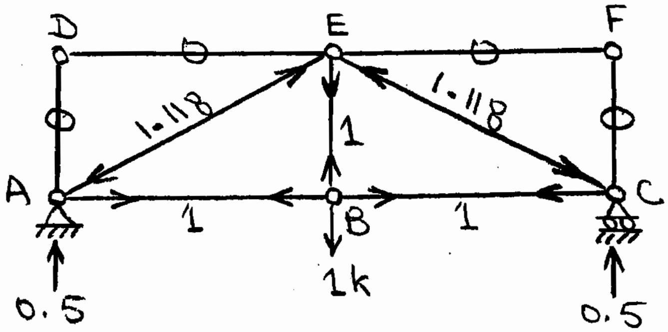
$$\Delta_B = \frac{7624.8 \ k\text{-in}}{EA} = \frac{7624.8}{1600\,(A)} = 0.4 \text{ in}$$

from which, $\underline{A = 11.91 \text{ in}^2}$

$\boxed{7\text{-}14}$



F Forces



F_v Forces

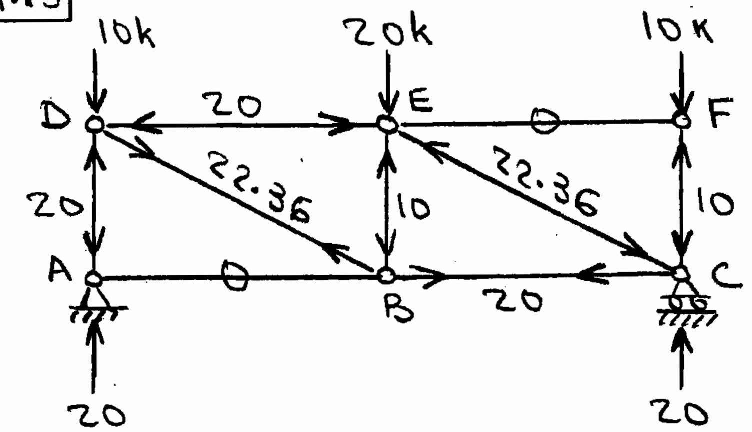
Member	L (in.)	F (k)	F_v (k)	F_v (FL) (k^2-in)
AB	72	20	1	1440
BC	72	20	1	1440
DE	72	0	0	0
EF	72	0	0	0
AD	36	-10	0	0
BE	36	0	1	0
CF	36	-10	0	0
AE	80.5	-22.36	-1.118	2012.4
CE	80.5	-22.36	-1.118	2012.4
			Σ	6904.8

$$(1k)\Delta_B = \frac{6904.8 \text{ k}^2\text{-in}}{EA}$$

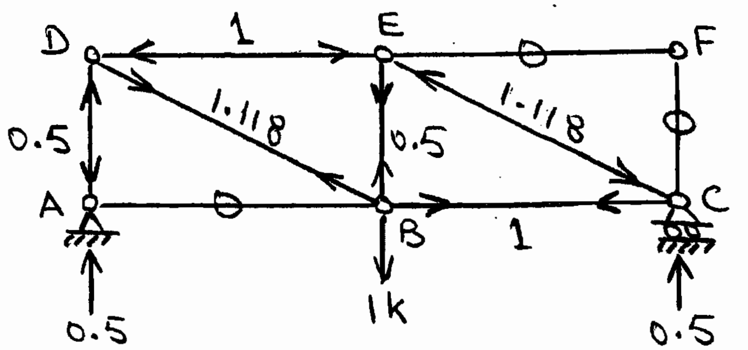
$$\Delta_B = \frac{6904.8 \text{ k-in}}{EA} = \frac{6904.8}{1600(A)} = 0.4 \text{ in}$$

from which, $\quad A = \underline{10.79 \text{ in}^2}$

7.15



F Forces



F_v Forces

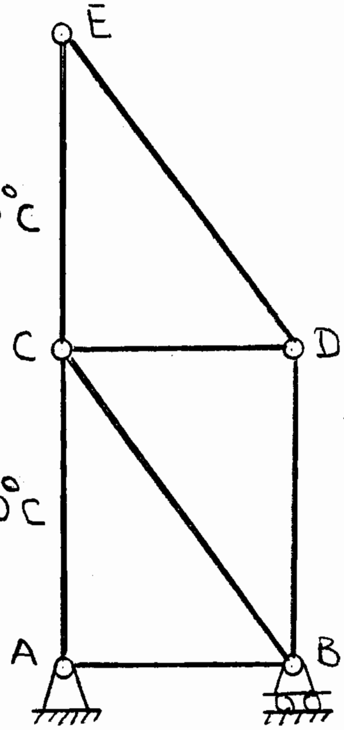
Member	L (in.)	F (k)	F_v (k)	$F_v(FL)$ (k^2-in.)
AB	72	0	0	0
BC	72	20	1	1440
DE	72	−20	−1	1440
EF	72	0	0	0
AD	36	−20	−0.5	360
BE	36	−10	0.5	−180
CF	36	−10	0	0
BD	80.5	22.36	1.118	2012.4
CE	80.5	−22.36	−1.118	2012.4
			Σ	7084.8

$$(1k)\Delta_B = \frac{7084.8 \text{ k}^2\text{-in.}}{EA}$$

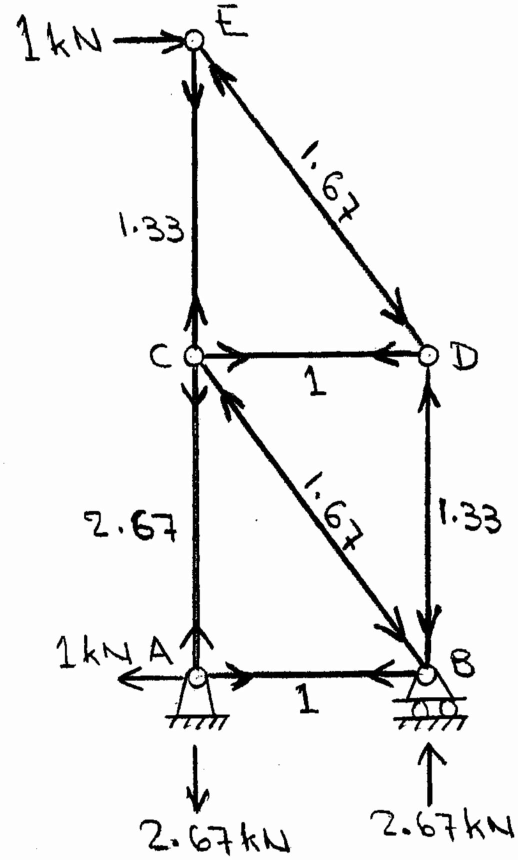
$$\Delta_B = \frac{7084.8 \text{ k-in.}}{EA} = \frac{7084.8}{1600(A)} = 0.4 \text{ in}$$

from which, $\underline{A = 11.07 \text{ in}^2}$

7.16



Real System – ΔT



Virtual System – F_v Forces

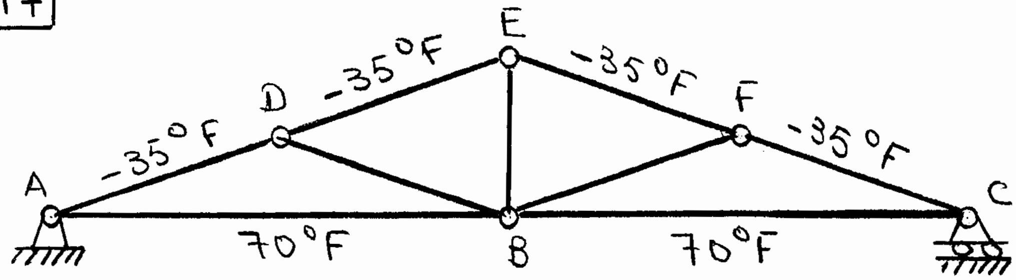
Member	F_v (kN)
AC	2.67
CE	1.33
Σ	4

$$(1\text{ kN})\,\Delta_E = \alpha(\Delta T)L\sum F_v = 1.2(10^{-5})(50)(4)(4)$$

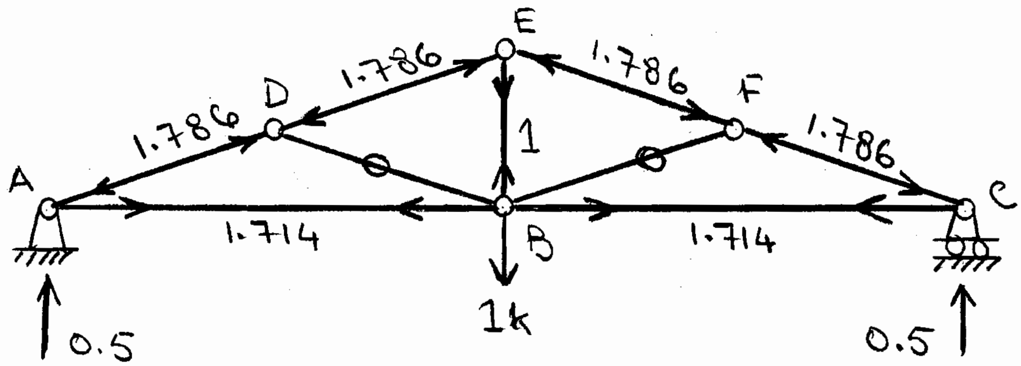
$$= 0.0096 \text{ kN.m}$$

$$\Delta_E = 0.0096\text{ m} = \underline{9.6\text{ mm} \rightarrow}$$

7.17



Real System – ΔT



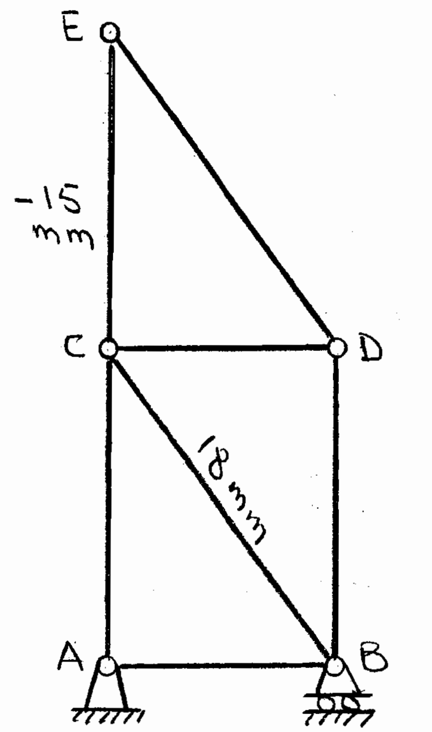
Virtual System – F_v Forces

Member	L (in.)	ΔT (°F)	F_v (k)	$F_v(\Delta T)L$ (k-in-°F)
AB	288	70	1.714	34560
BC	288	70	1.714	34560
AD	150	-35	-1.786	9375
DE	150	-35	-1.786	9375
EF	150	-35	-1.786	9375
CF	150	-35	-1.786	9375
			Σ	106620

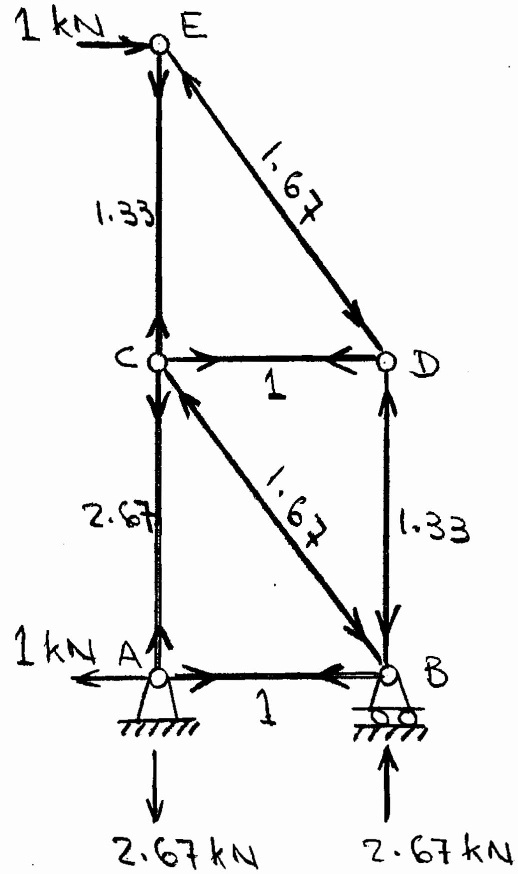
$$(1k)\,\Delta_B = \alpha \Sigma F_v(\Delta T)L = 6.5(10^{-6})(106620)$$

$$\Delta_B = 0.693 \text{ in.} \downarrow$$

7.18



Real System – δ



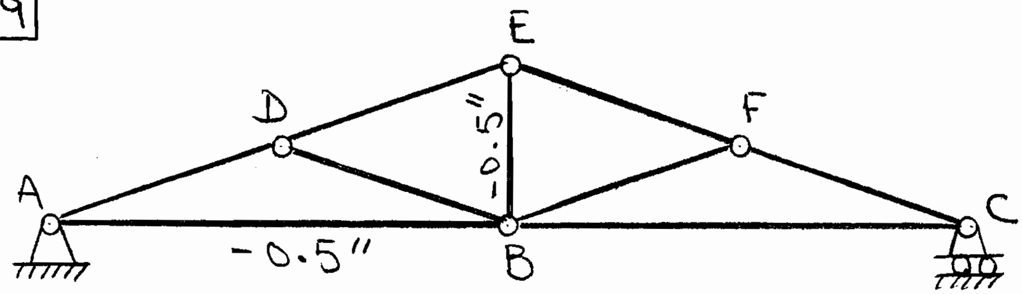
Virtual System – F_v Forces

Member	δ (mm)	F_v (kN)	$F_v(\delta)$ (kN.mm)
BC	18	-1.67	-30
CE	-15	1.33	-20
		Σ	-50

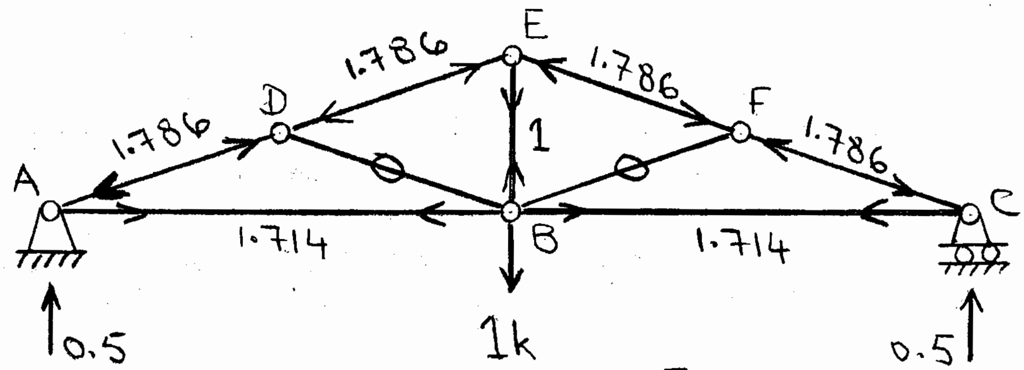
$$(1\,kN)\Delta_E = \Sigma F_v(\delta) = -50\ kN.mm$$

$$\Delta_E = -50\,mm = \underline{50\ mm \leftarrow}$$

7.19



Real System - δ



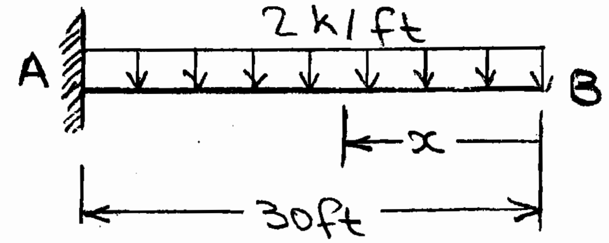
Virtual System - F_v Forces

Member	F_v (k)
AB	1.714
BE	1
Σ	2.714

$$(1k)\ \Delta_B = \delta \Sigma F_v = (-0.5)2.714 = -1.357 \text{ k-in.}$$

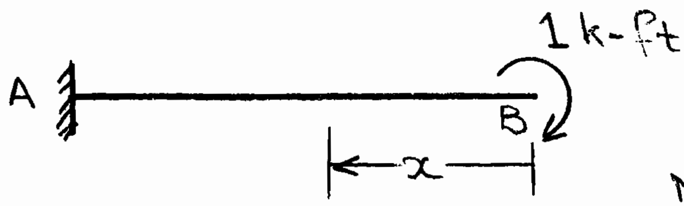
$$\Delta_B = -1.357 \text{ in.} = \underline{1.357 \text{ in.}\uparrow}$$

$\boxed{7.20}$



$$M = -\frac{2x^2}{2} = -x^2$$

Real System – M

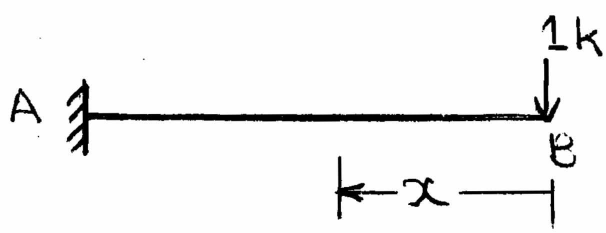


$$M_{v1} = -1$$

Virtual System – M_{v1}

$$\theta_B = \frac{1}{EI}\int_0^{30} -1(-x^2)\,dx = \frac{9000 \text{ k-ft}^2}{EI}$$

$$= \frac{9000(12)^2}{29000(3000)} = \underline{0.0149 \text{ rad}}$$



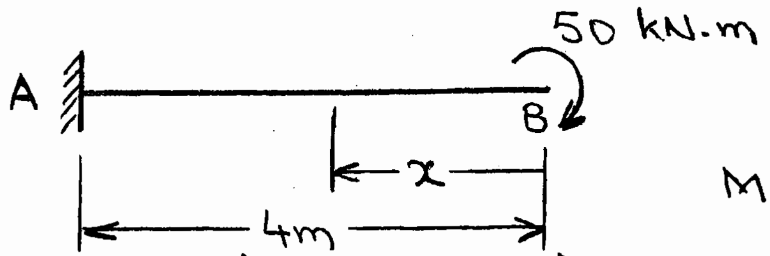
$$M_{v2} = -1(x) = -x$$

Virtal System M_{v2}

$$\Delta_B = \frac{1}{EI}\int_0^{30} (-x)(-x^2)\,dx = \frac{202500 \text{ k-ft}^3}{EI}$$

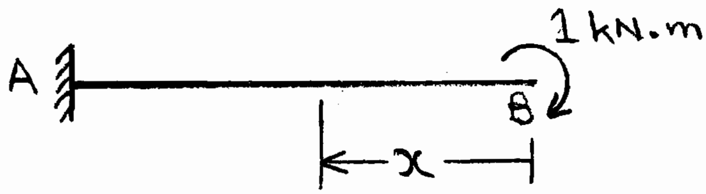
$$= \frac{202500(12)^3}{29000(3000)} = \underline{4.022 \text{ in.} \downarrow}$$

$\boxed{7.21}$



$M = -50$

Real System – M

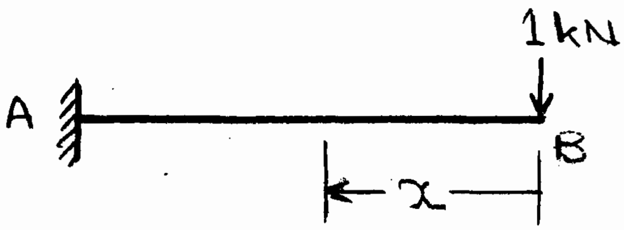


$M_{v1} = -1$

Virtual System – M_{v1}

$$\theta_B = \frac{1}{EI}\int_0^4 (-1)(-50)\,dx = \frac{200 \text{ kN.m}^2}{EI}$$

$$= \frac{200}{70(164)} = \underline{0.0174 \text{ rad}}$$



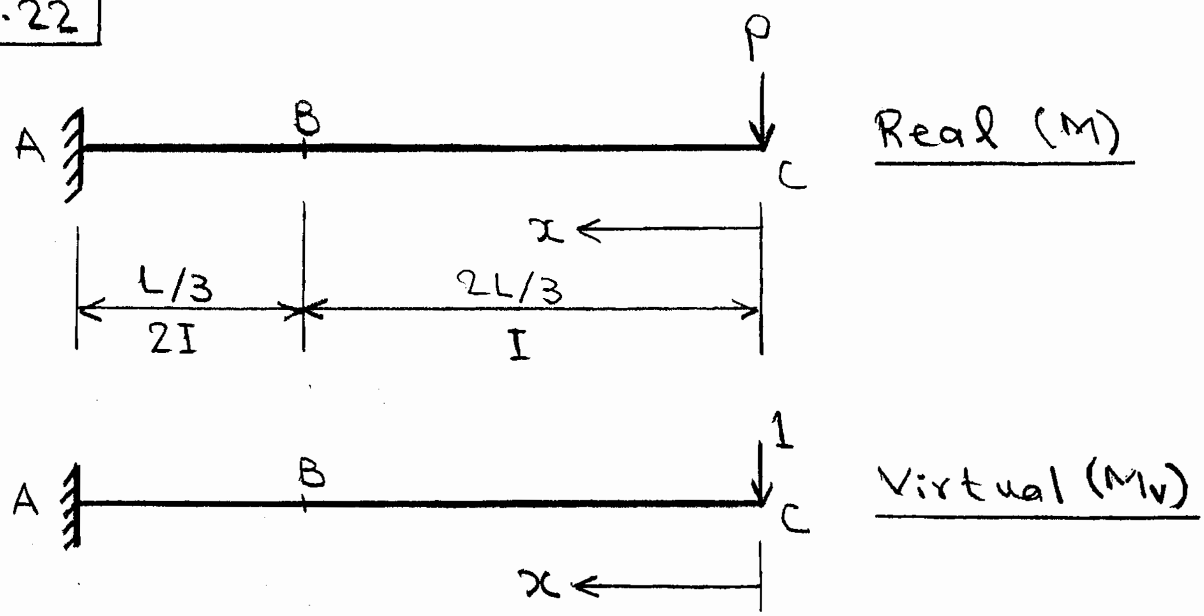
$M_{v2} = -1(x) = -x$

Virtual System – M_{v2}

$$\Delta_B = \frac{1}{EI}\int_0^4 (-x)(-50)\,dx = \frac{400 \text{ kN.m}^3}{EI}$$

$$= \frac{400}{70(164)} = 0.0348 \text{ m} = \underline{34.8 \text{ mm}\downarrow}$$

7.22

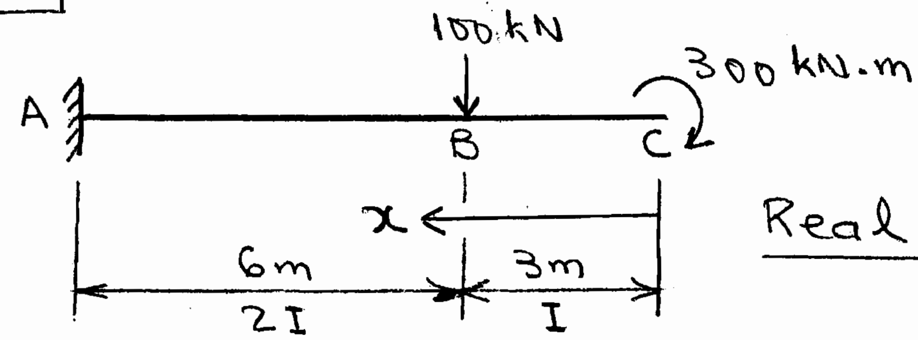


Segment	x Coordinate Origin	x Coordinate Limits	M	M_v
CB	C	$0 - \frac{2L}{3}$	$-Px$	$-1x$
BA	C	$\frac{2L}{3} - L$	$-Px$	$-1x$

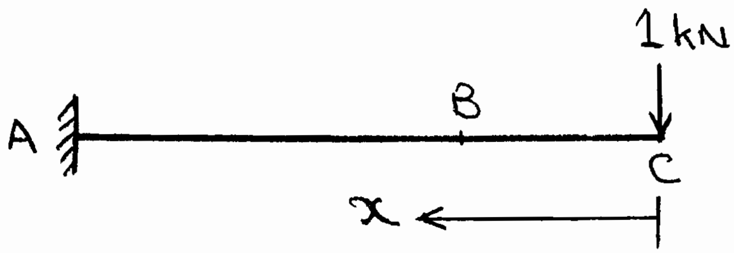
$$\Delta_C = \frac{1}{EI}\left[\int_0^{\frac{2L}{3}} (-x)(-Px)\,dx + \frac{1}{2}\int_{\frac{2L}{3}}^{L} (-x)(-Px)\,dx\right]$$

$$= \underline{\frac{35PL^3}{162EI}\downarrow}$$

$\boxed{7.23}$



Real (M)



Virtual (M_v)

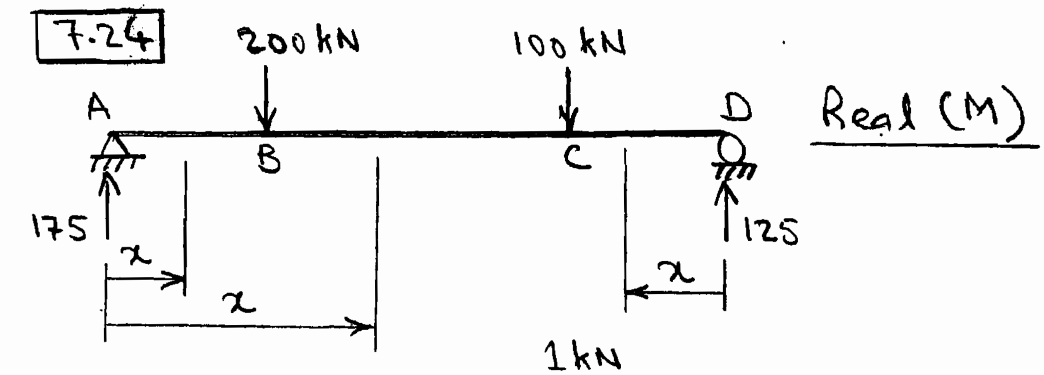
Segment	x Coordinate		M (kN.m)	M_v (kN.m)
	Origin	Limits (m)		
CB	C	0 – 3	−300	−x
BA	C	3 – 9	−300 −100(x−3)	−x

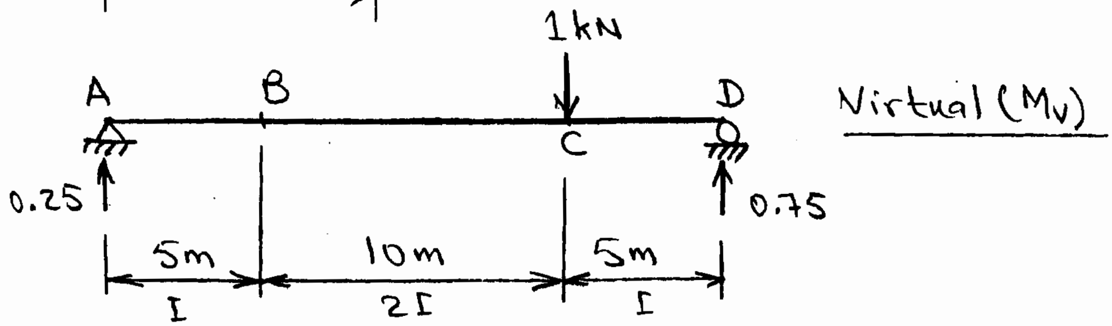
$$\Delta_C = \frac{1}{EI}\left[\int_0^3 (-x)(-300)\,dx + \frac{1}{2}\int_3^9 (-x)(-100x)\,dx\right]$$

$$= \frac{13050\ \text{kN.m}^3}{EI} = \frac{13050}{70(500)} = 0.373\ \text{m}$$

$$\underline{\Delta_C = 373\ \text{mm} \downarrow}$$

7.24





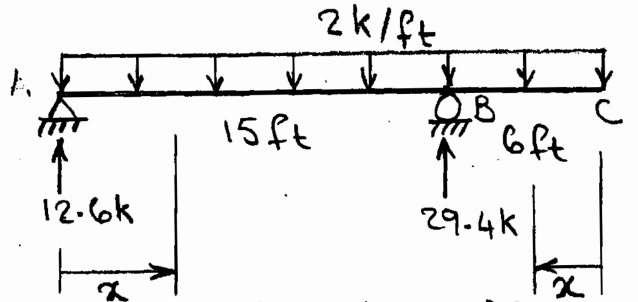
Segment	x Coordinate Origin	Limits (m)	M (kN-m)	M_v (kN-m)
AB	A	0 – 5	$175x$	$0.25x$
BC	A	5 – 15	$175x - 200(x-5)$	$0.25x$
DC	D	0 – 5	$125x$	$0.75x$

$$\Delta_C = \frac{1}{EI}\left[\int_0^5 (0.25x)(175x)\,dx + \frac{1}{2}\int_5^{15} 0.25x(-25x+1000)\,dx + \int_0^5 0.75x(125x)\,dx\right]$$

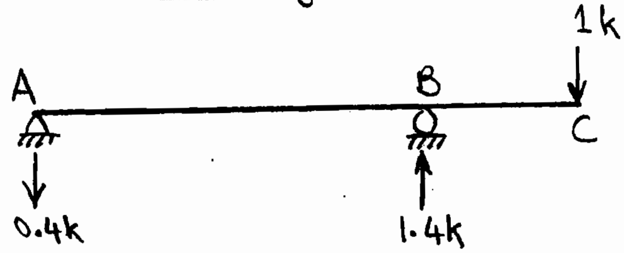
$$= \frac{14843.75\ \text{kN-m}^3}{EI} = \frac{14843.75}{250(600)}$$

$$= 0.099\ \text{m} = \underline{99\ \text{mm}\downarrow}$$

7.25



Real System – M



Virtual System – M_v

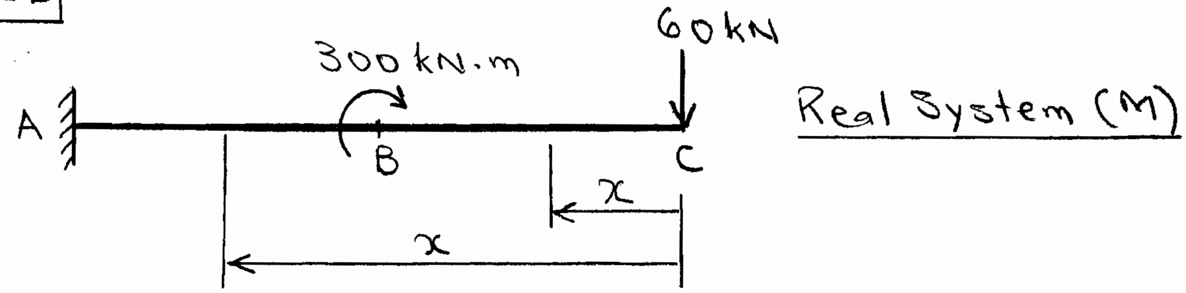
Segment	x Coordinate		M (k-ft)	M_v (k-ft)
	Origin	Limits (ft)		
AB	A	0–15	$12.6x - x^2$	$-0.4x$
CB	C	0–6	$-x^2$	$-x$

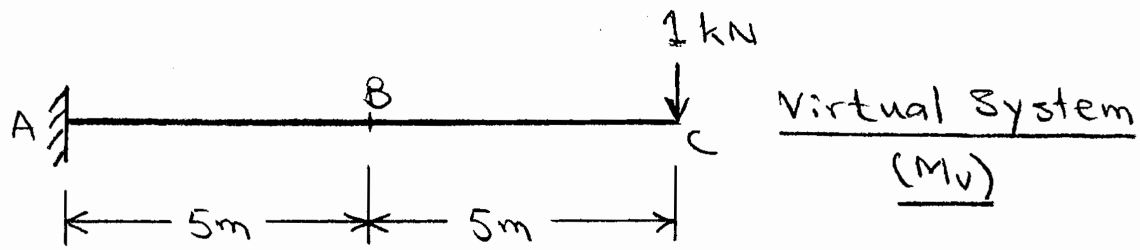
$$(1k)\Delta_C = \frac{1}{EI}\left[\int_0^{15} -0.4x(12.6x - x^2)\,dx + \int_0^{6} -x(-x^2)\,dx\right]$$

$$= -\frac{283.5\ k^2\text{-}ft^3}{EI}$$

$$\Delta_C = -\frac{283.5(12)^3}{(29000)(3500)} = -0.0048\text{ in.} = \underline{0.0048\text{ in.}\uparrow}$$

7.26





Segment	x coordinate Origin	Limits (m)	M (kN.m)	M_v (kN.m)
CB	C	0 – 5	$-60x$	$-1x$
BA	C	5 – 10	$-60x-300$	$-1x$

$$\Delta_{max} = \Delta_C = \frac{1}{EI}\left[\int_0^5 (-1x)(-60x)\,dx + \int_5^{10}(-1x)(-60x-300)\,dx\right]$$

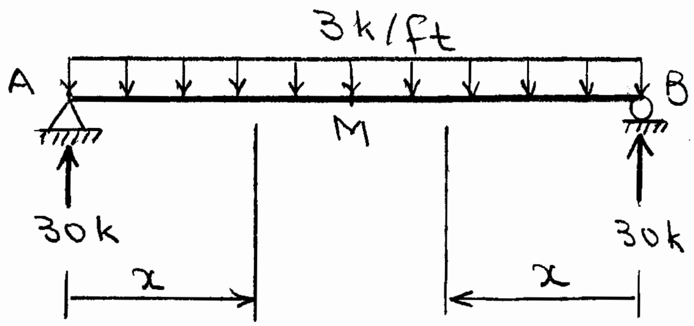
$$= \frac{31250 \text{ kN.m}^3}{EI} \downarrow$$

$$\Delta_{max} = \frac{L}{360}$$

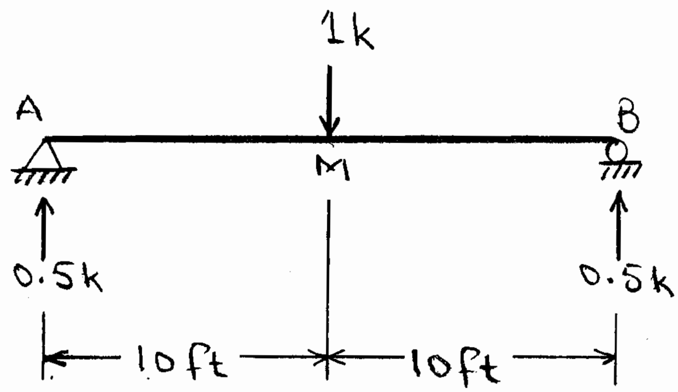
$$\frac{31250}{200(10^6)I} = \frac{10}{360}$$

from which, $I = 5625(10^{-6})\,m^4 = \underline{5625(10^6)\,mm^4}$

$\boxed{7.27}$



Real System (M)



Virtual System (M_v)

Segment	x Coordinate		M (k-ft)	M_v (k-ft)
	Origin	Limits (ft)		
AM	A	0 – 10	$30x - \frac{3x^2}{2}$	$0.5x$
BM	B	0 – 10	$30x - 3\frac{x^2}{2}$	$0.5x$

$$\Delta_{max} = \Delta_M = \frac{1}{EI}\left[2\int_0^{10}(0.5x)\left(30x - \frac{3x^2}{2}\right)dx\right]$$

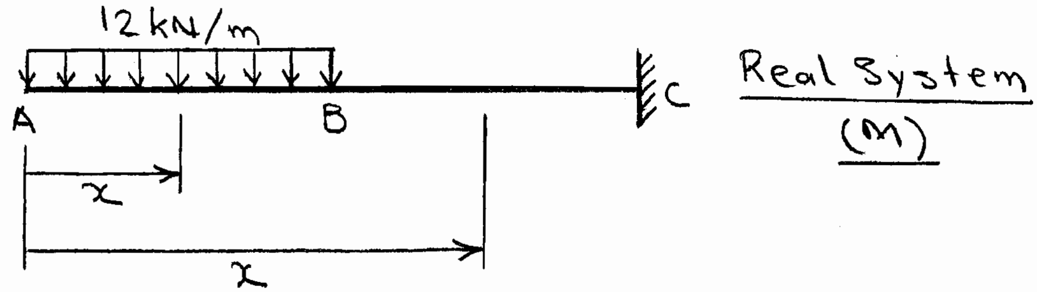
$$= \frac{6250 \text{ k-ft}^3}{EI}$$

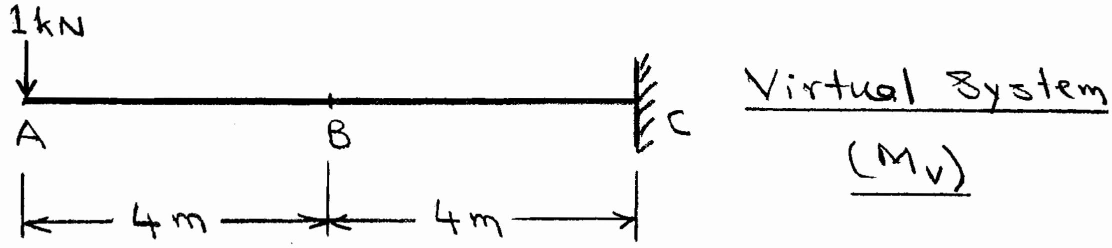
$$\Delta_{max} = \frac{L}{360}$$

$$\frac{6250\,(12)^3}{29000\,(I)} = \frac{20(12)}{360}$$

from which, $\quad \underline{I = 559 \text{ in}^4}$

$\boxed{7.28}$





Segment	x Coordinate		M (kN.m)	M_v (kN.m)
	Origin	Limits (m)		
AB	A	0 – 4	$-12x^2/2$	$-1x$
BC	A	4 – 8	$-12(4)(x-2)$	$-1x$

$$\Delta_{max} = \Delta_A = \frac{1}{EI}\left[\int_0^4 (-1x)(-6x^2)\,dx + \int_4^8 (-1x)(-48)(x-2)\,dx\right]$$

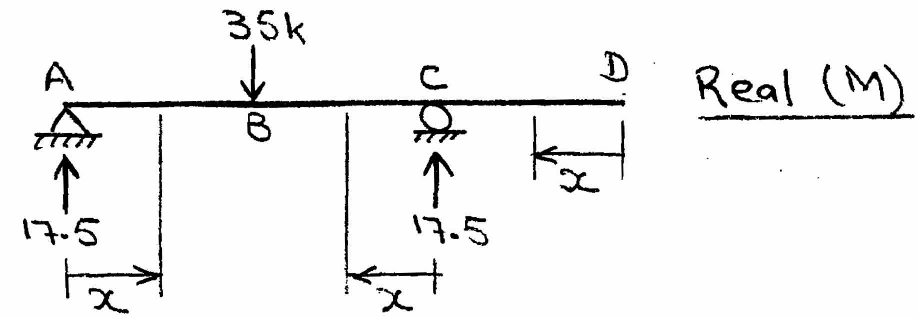
$$= \frac{5248 \text{ kN.m}^3}{EI}$$

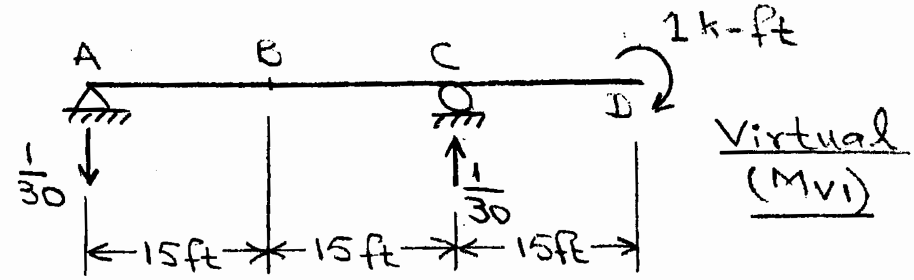
$$\Delta_{max} = \frac{L}{360}$$

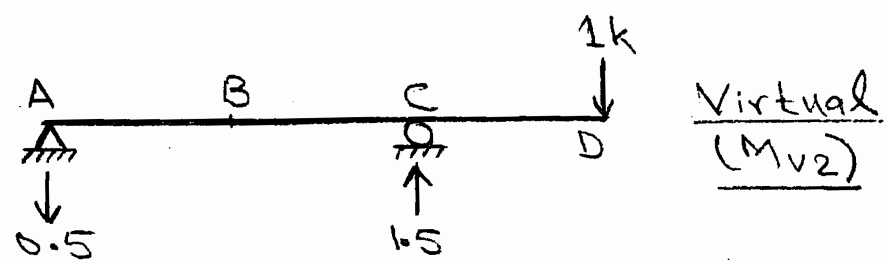
$$\frac{5248}{70(10^6)I} = \frac{8}{360}$$

From which, $I = 3374(10^{-6})\,m^4 = \underline{3374(10^6)\,mm^4}$

7.29







Segment	x coordinate Origin	Limits (ft)	M (k-ft)	M_{v1} (k-ft)	M_{v2} (k-ft)
AB	A	0–15	$17.5x$	$-x/30$	$-0.5x$
CB	C	0–15	$17.5x$	$-1+(x/30)$	$-1(x+15)+1.5x$
DC	D	0–15	0	-1	$-1x$

$$\theta_D = \frac{1}{EI}\left[\int_0^{15} -\frac{x}{30}(17.5x)\,dx + \int_0^{15}\left(-1+\frac{x}{30}\right)(17.5x)\,dx\right]$$

$$= -\frac{1968.75\text{ k-ft}^2}{EI} = -\frac{1968.75(12)^2}{10000(2500)} = -0.01134\text{ rad}$$

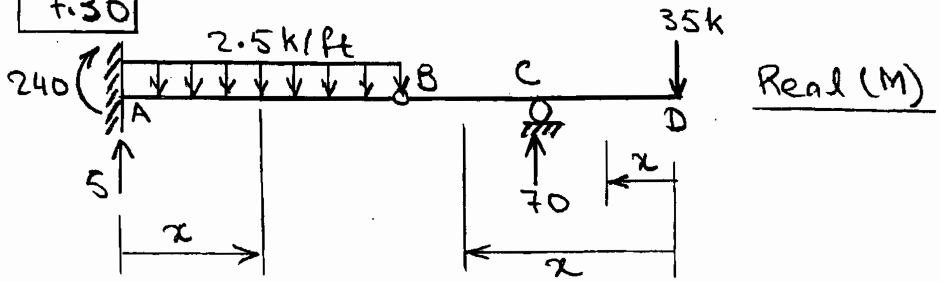
$$\theta_D = 0.01134\text{ rad}$$

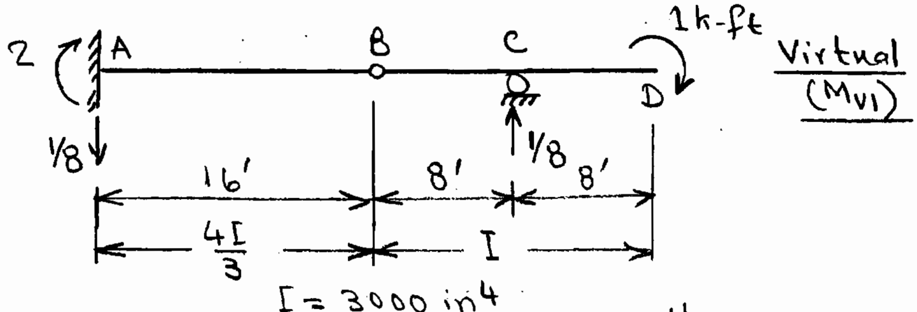
$$\Delta_D = \frac{1}{EI}\left[\int_0^{15}(-0.5x)(17.5x)\,dx + \int_0^{15}(0.5x-15)(17.5x)\,dx\right]$$

$$= -\frac{29531.25\text{ k-ft}^3}{EI} = -\frac{29531.25(12)^3}{10000(2500)}$$

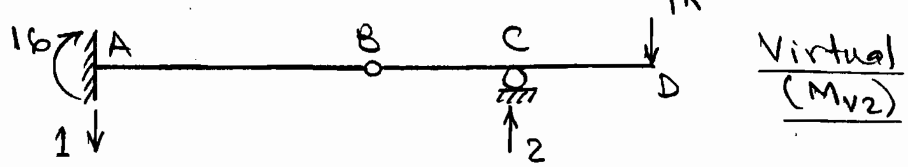
$$= -2.04\text{ in.} = 2.04\text{ in.}\uparrow$$

7.30





$I = 3000 \text{ in}^4$



Segment	x Coordinate: Origin	x Coordinate: Limits (ft)	M (k-ft)	M_{v1} (k-ft)	M_{v2} (k-ft)
DC	D	0 – 8	$-35x$	-1	$-1x$
CB	D	8 – 16	$-35x+70(x-8)$	$-1+\frac{1}{8}(x-8)$	$x-16$
AB	A	0 – 16	$240+5x-1.25x^2$	$2-x/8$	$16-x$

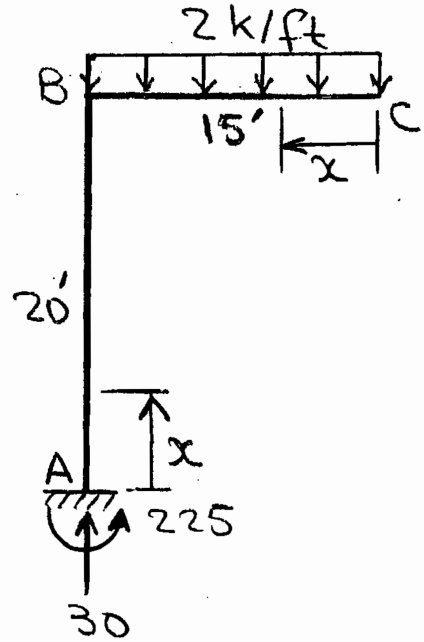
$$\theta_D = \frac{1}{EI}\left[\int_0^8 -1(-35x)\,dx + \int_8^{16}\left(\frac{x}{8}-2\right)(35x-560)\,dx + \frac{3}{4}\int_0^{16}\left(2-\frac{x}{8}\right)(240+5x-1.25x^2)\,dx\right]$$

$$= \frac{4426.67 \text{ k-ft}^2}{EI} = \frac{4426.67(12)^2}{30\,000(3000)} = \underline{0.0071 \text{ rad.}}$$

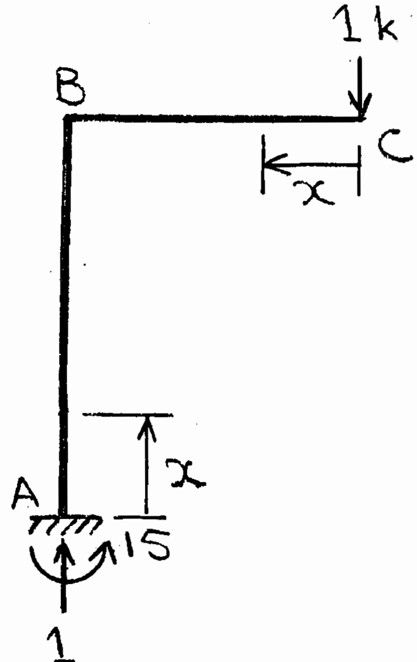
$$\Delta_D = \frac{1}{EI}\left[\int_0^8 -x(-35x)\,dx + \int_8^{16}(x-16)(35x-560)\,dx + \frac{3}{4}\int_0^{16}(16-x)(240+5x-1.25x^2)\,dx\right]$$

$$= \frac{32426.67 \text{ k-ft}^3}{EI} = \frac{32426.67(12)^3}{30000(3000)} = \underline{0.62 \text{ in.} \downarrow}$$

7.31



Real System – M



Virtual System – M_v

Segment	x Coordinate		M (k-ft)	M_v (k-ft)
	Origin	Limits (ft.)		
AB	A	0–20	−225	−15
CB	C	0–15	$-x^2$	$-x$

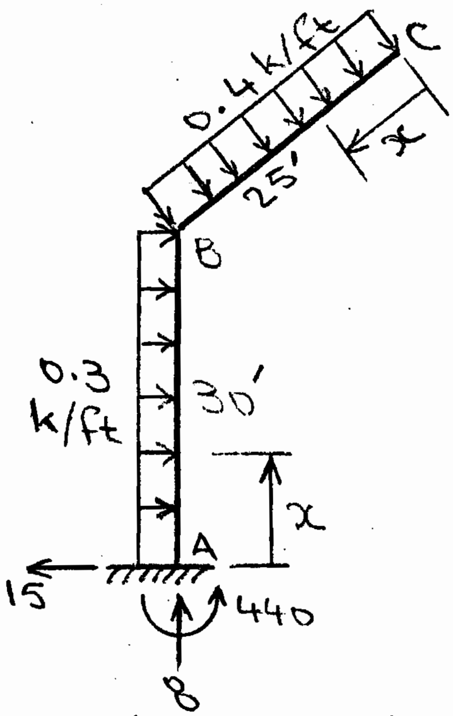
$$(1k)\,\Delta_C = \frac{1}{EI}\left[\int_0^{20}(-15)(-225)\,dx + \int_0^{15}(-x)(-x^2)\,dx\right]$$

$$= \frac{80156.25 \text{ k}^2\text{-ft}^3}{EI}$$

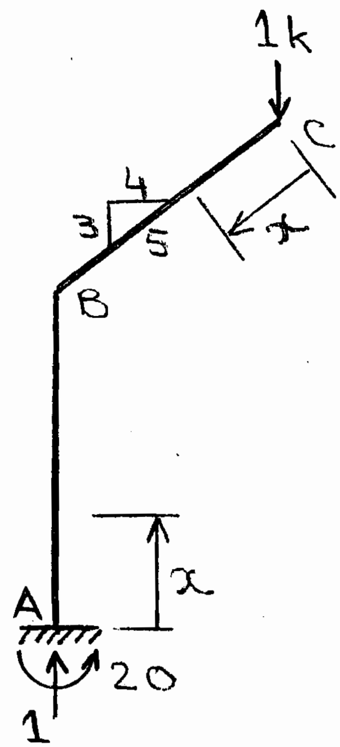
$$\Delta_C = \frac{80156.25 \text{ k-ft}^3}{EI} = \frac{80156.25\,(12)^3}{29000\,(2000)}$$

$$= \underline{2.388 \text{ in.} \downarrow}$$

$\boxed{7.32}$



Real System – M



Virtual System – M_v

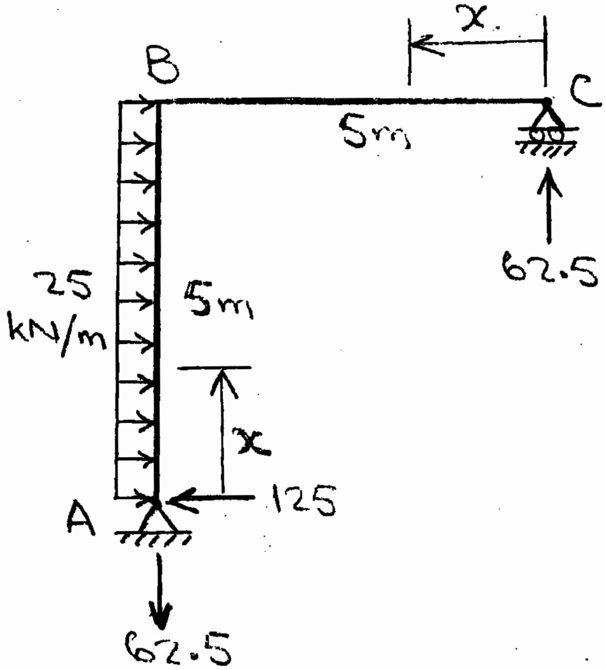
Segment	x Coordinate		M (k-ft)	M_v (k-ft)
	Origin	Limits (ft)		
AB	A	0–30	$-440+15x-0.15x^2$	-20
CB	C	0–25	$-0.2x^2$	$-\frac{4}{5}x$

$$(1k)\,\Delta_C = \frac{1}{EI}\left[\int_0^{30}(-20)(-440+15x-0.15x^2)\,dx + \int_0^{25}\left(-\frac{4}{5}x\right)(-0.2x^2)\,dx\right]$$

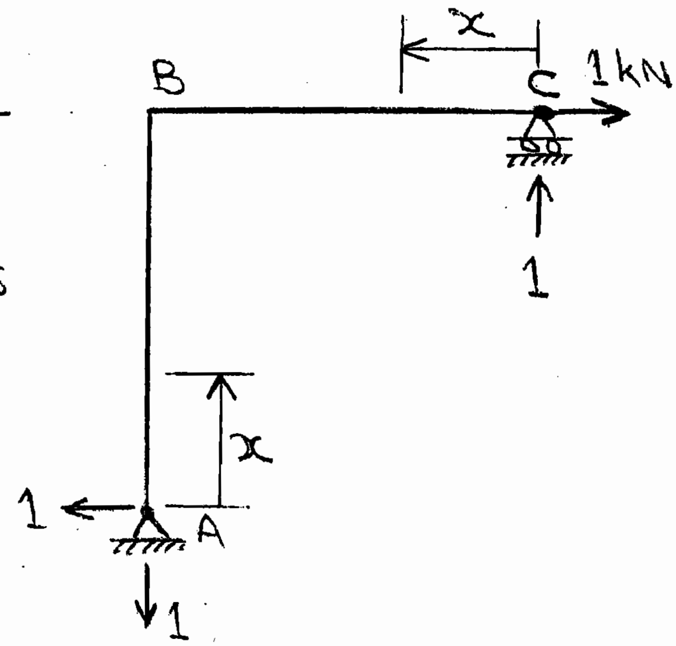
$$= \frac{171625\ k^2\text{-}ft^3}{EI}$$

$$\Delta_C = \frac{171625\ k\text{-}ft^3}{EI} = \frac{171625(12)^3}{10000(8160)} = \underline{3.63\text{ in.}\downarrow}$$

7.33



Real System - M



Virtual System - M_v

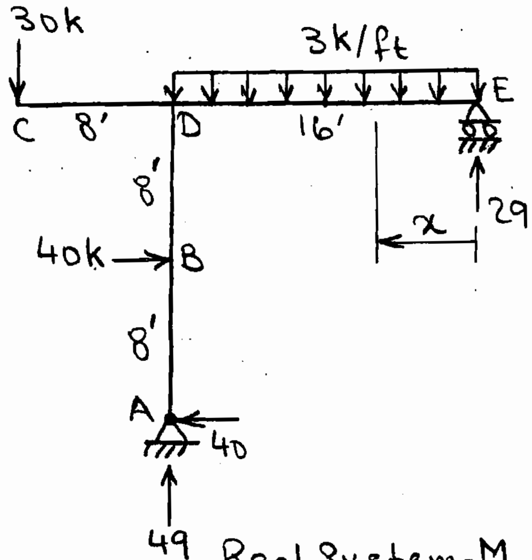
Segment	x Coordinate		M (kN.m)	M_v (kN.m)
	Origin	Limits (m)		
AB	A	0 - 5	$125x - 12.5x^2$	$1x$
CB	C	0 - 5	$62.5x$	$1x$

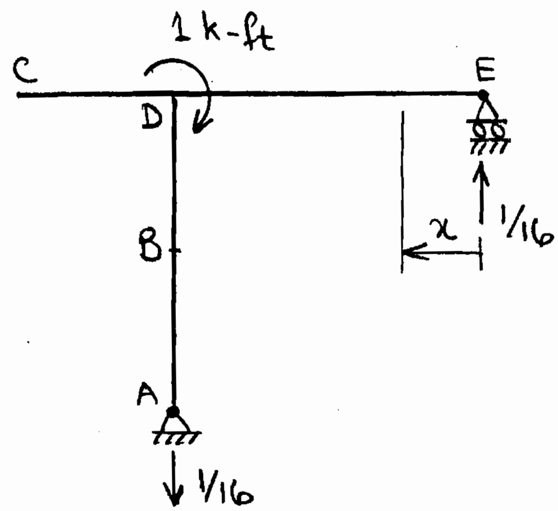
$$\Delta_C = \frac{1}{EI}\left[\int_0^5 x(125x - 12.5x^2)\,dx + \int_0^5 x(62.5x)\,dx\right]$$

$$= \frac{5859.375 \text{ kN.m}^3}{EI} = \frac{5859.375}{70(1030)}$$

$$= 0.0813 \text{ m} = \underline{81.3 \text{ mm} \rightarrow}$$

$\boxed{7\text{-}34}$





Real System-M

$M = 29x - 1.5x^2$

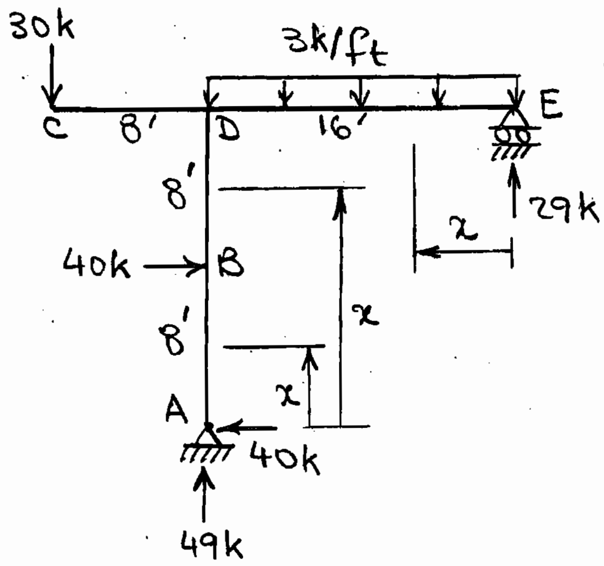
Virtual System - M_v

$M_v = x/16$

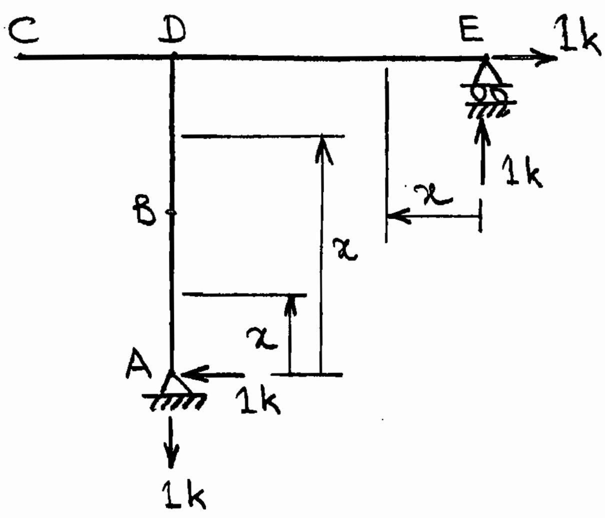
$$\theta_D = \frac{1}{2EI}\int_0^{16}\left(\frac{x}{16}\right)(29x - 1.5x^2)\,dx = \frac{469.33 \text{ k-ft}^2}{EI}$$

$$= \frac{469.33\,(12)^2}{2000(10000)} = \underline{0.0034 \text{ rad}}$$

7.35



Real System – M



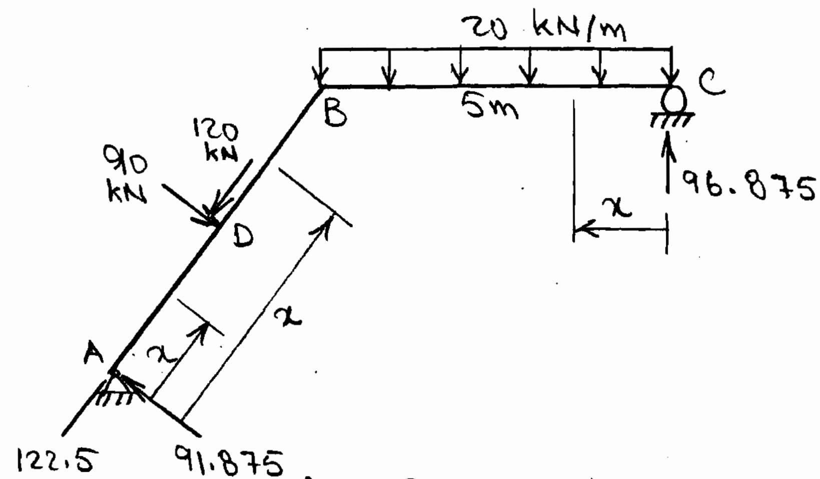
Virtual System – M_v

Segment	x Coordinate		M (k-ft)	M_v (k-ft)
	Origin	Limits (ft)		
AB	A	0–8	$40x$	$1x$
BD	A	8–16	320	$1x$
ED	E	0–16	$29x-\frac{3}{2}x^2$	$1x$

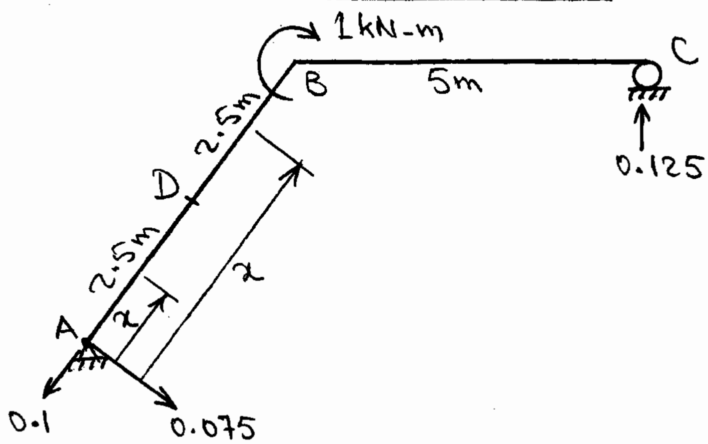
$$(1k)\,\Delta_E = \frac{1}{2EI}\left[\int_0^8 x(40x)\,dx + \int_8^{16} x(320)\,dx + \int_0^{16} x\left(29x-\frac{3}{2}x^2\right)dx\right] = \frac{26282.67\ k^2\text{-}ft^3}{EI}$$

$$\Delta_E = \frac{26282.67(12)^3}{(2000)(10000)} = \underline{2.27\ \text{in.} \rightarrow}$$

7.36



Real System – M



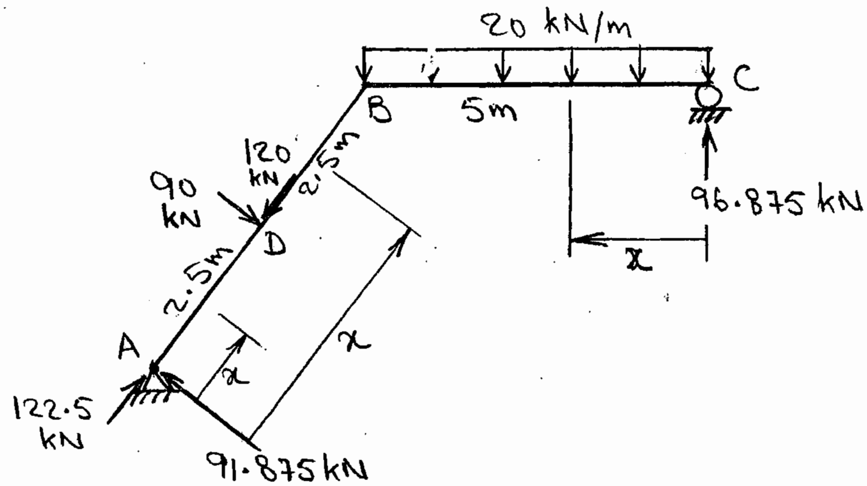
Virtual System – M_v

Segment	x Coordinate Origin	Limits (m)	M (kN.m)	M_v (kN-m)
AD	A	0–2.5	$91.875x$	$-0.075x$
DB	A	2.5–5	$1.875x+225$	$-0.075x$
CB	C	0–5	$96.875x-10x^2$	$0.125x$

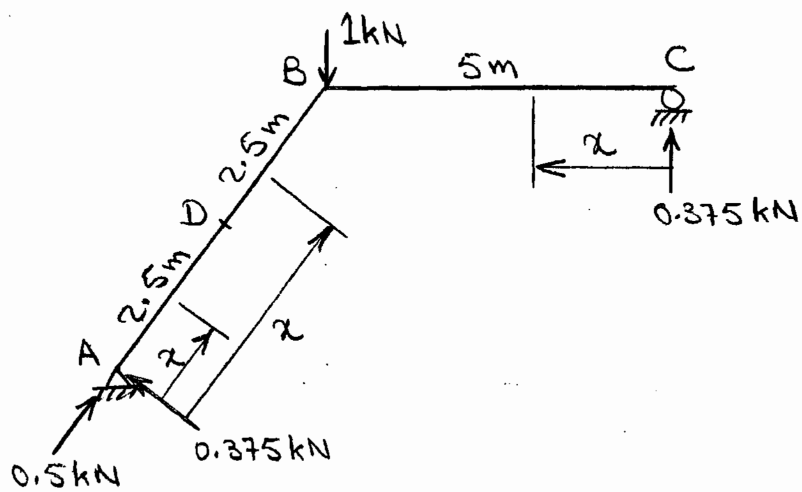
$$\theta_B = \frac{1}{EI}\left[\int_0^{2.5} -0.075x(91.875x)\,dx + \int_{2.5}^{5} -0.075x\,(1.875x+225)\,dx + \int_0^{5} 0.125x\,(96.875x-10x^2)\,dx\right.$$

$$= \frac{110.01\text{ kN-m}^2}{EI} = \frac{110.01}{200(500)} = \underline{0.0011\text{ rad.}}$$

7.37



Real System – M



Virtual System – M_v

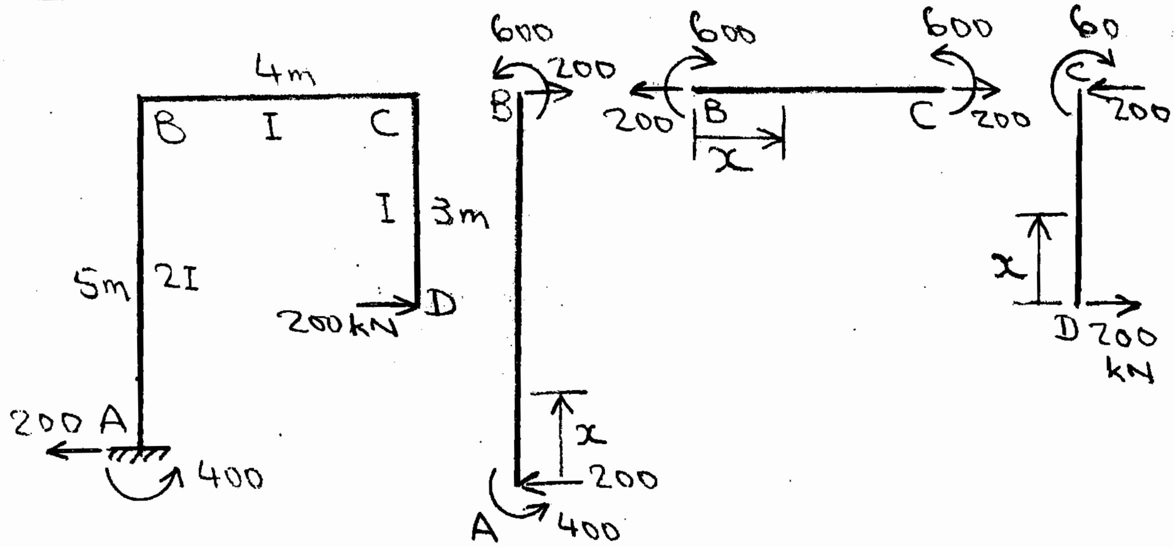
Segment	x Coordinate		M (kN.m)	M_v (kN.m)
	Origin	Limits (m)		
AD	A	0 – 2.5	$91.875x$	$0.375x$
DB	A	2.5 – 5	$1.875x + 225$	$0.375x$
CB	C	0 – 5	$96.875x - 10x^2$	$0.375x$

$$(1\,kN)\,\Delta_B = \frac{1}{EI}\left[\int_0^{2.5} 0.375x(91.875x)\,dx + \int_{2.5}^{5} 0.375x(1.875x + 225)\,dx + \int_0^{5} 0.375x(96.875x - 10x^2)\,dx\right]$$

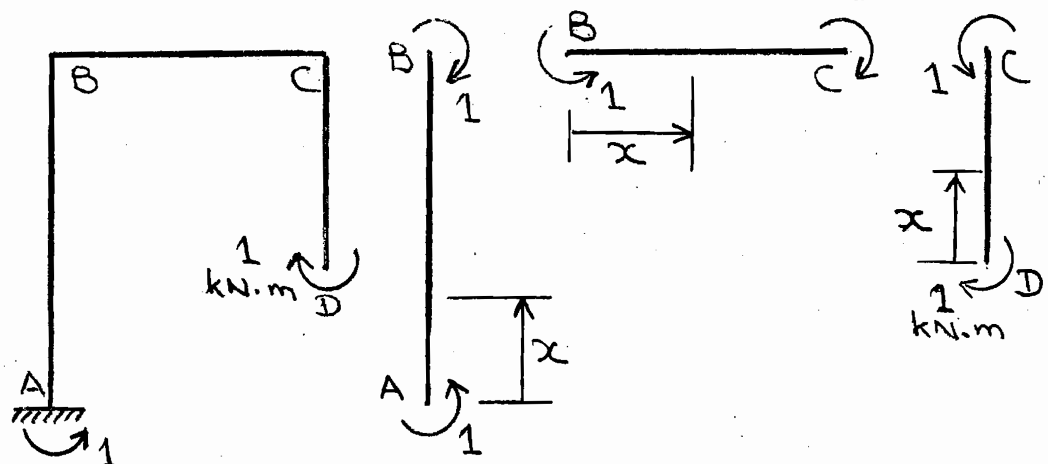
$$= \frac{1923.83\ kN^2 \cdot m^3}{EI}$$

$$\Delta_B = \frac{1923.83}{200(500)} = 0.0192\,m = \underline{19.2\ mm \downarrow}$$

7.38



Real System – M



Virtual System – M_v

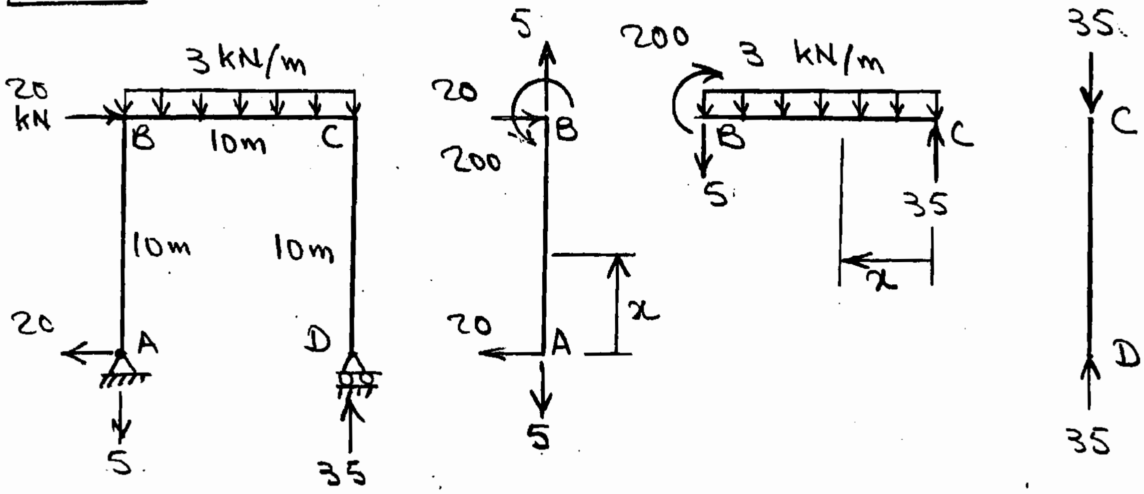
Segment	x Coordinate		M (kN.m)	M_v (kN.m)
	Origin	Limits (m)		
AB	A	0–5	$-400+200x$	-1
BC	B	0–4	600	-1
DC	D	0–3	$-200x$	1

$$\theta_D = \frac{1}{EI}\left[\frac{1}{2}\int_0^5 -1(-400+200x)\,dx + \int_0^4 -1(600)\,dx + \int_0^3 1(-200x)\,dx\right]$$

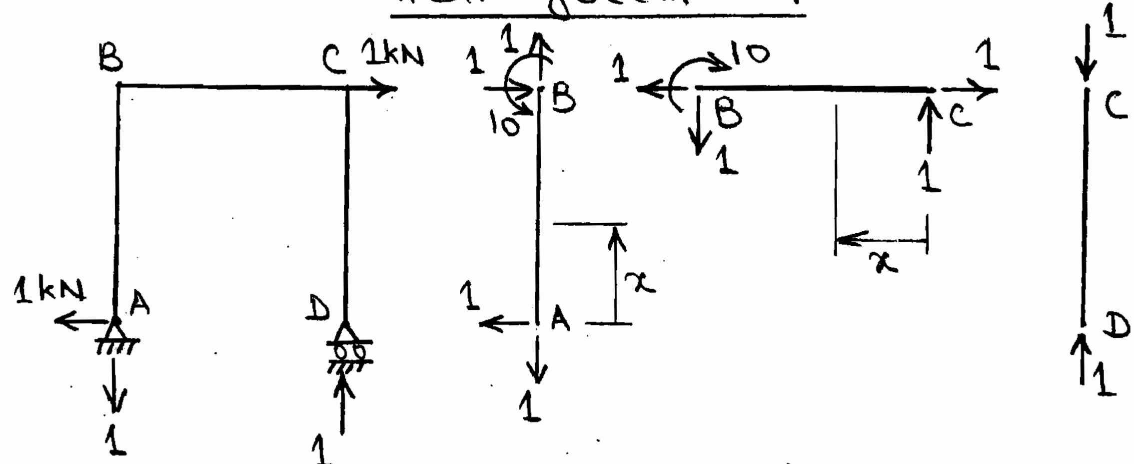
$$= -\frac{3550 \text{ kN.m}^2}{EI} = -\frac{3550}{70(1290)} = -0.0393 \text{ rad}$$

$$= \underline{0.0393 \text{ rad}}$$

7-39



Real System – M



Virtual System – M_v

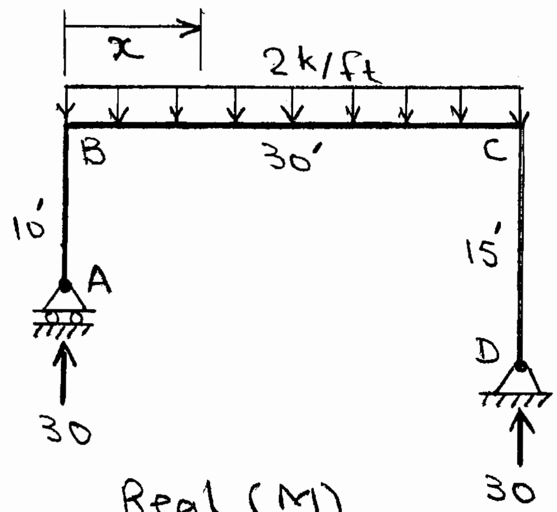
Segment	x Coordinate		M (kN.m)	M_v (kN.m)
	Origin	Limits (m)		
AB	A	0–10	$20x$	$1x$
CB	C	0–10	$35x-1.5x^2$	$1x$

$$(1\text{kN})\,\Delta_C = \frac{1}{EI}\left[\int_0^{10} x(20x)\,dx + \int_0^{10} x(35x - 1.5x^2)\,dx\right]$$

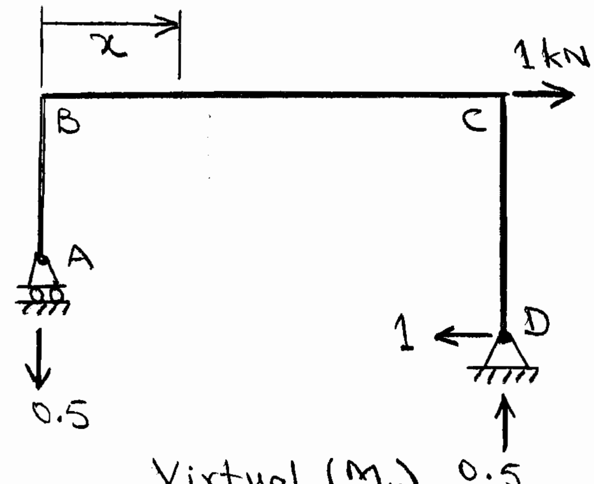
$$= \frac{14583.33\ \text{kN}^2\text{.m}^3}{EI}$$

$$\Delta_C = \frac{14583.33}{200(400)} = \underline{0.182\ \text{m} \rightarrow}$$

7.40



Real (M)



Virtual (M_v)

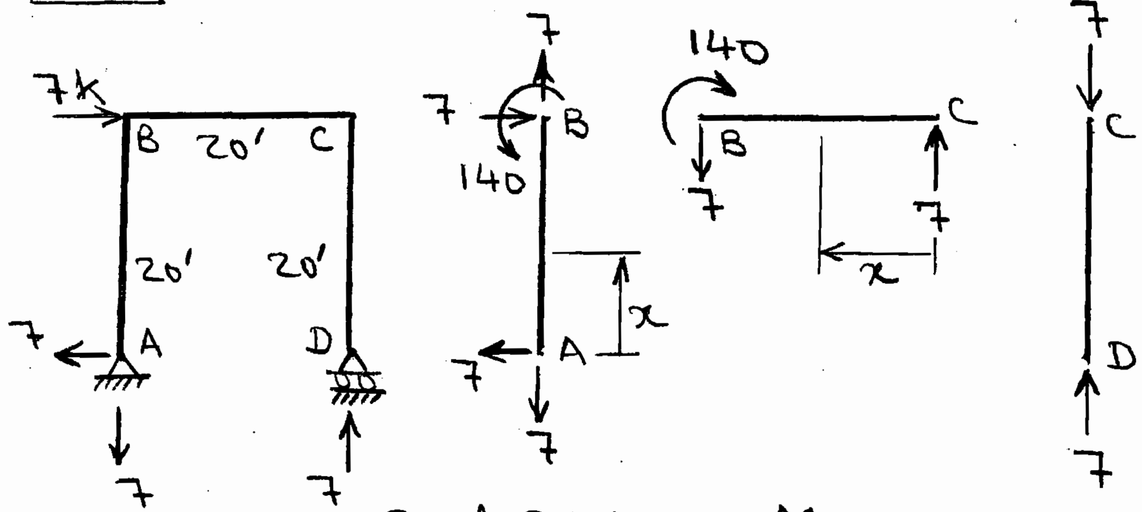
$$M = 30x - \frac{2x^2}{2}$$

$$M_v = -0.5x$$

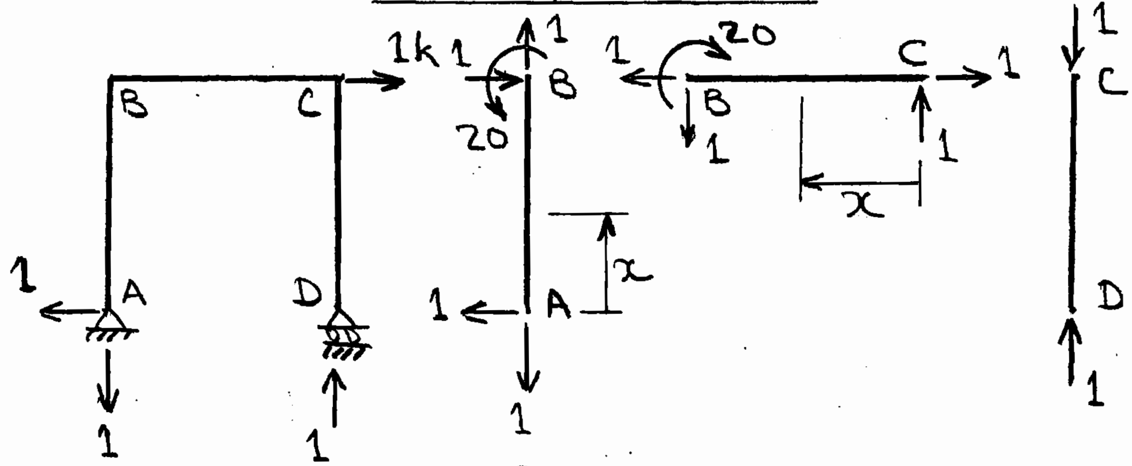
$$\Delta_C = \frac{1}{EI}\int_0^{30}(-0.5x)(30x - x^2)\,dx = -\frac{33750\ \text{k-ft}^3}{EI}$$

$$= -\frac{33750\,(12)^3}{29000\,(1500)} = -1.34\ \text{in.} = \underline{1.34\ \text{in.} \leftarrow}$$

7.41



Real System – M



Virtual System – M_v

Segment	x coordinate		M (k-ft)	M_v (k-ft)
	Origin	Limits (ft)		
AB	A	0–20	$7x$	$1x$
CB	C	0–20	$7x$	$1x$

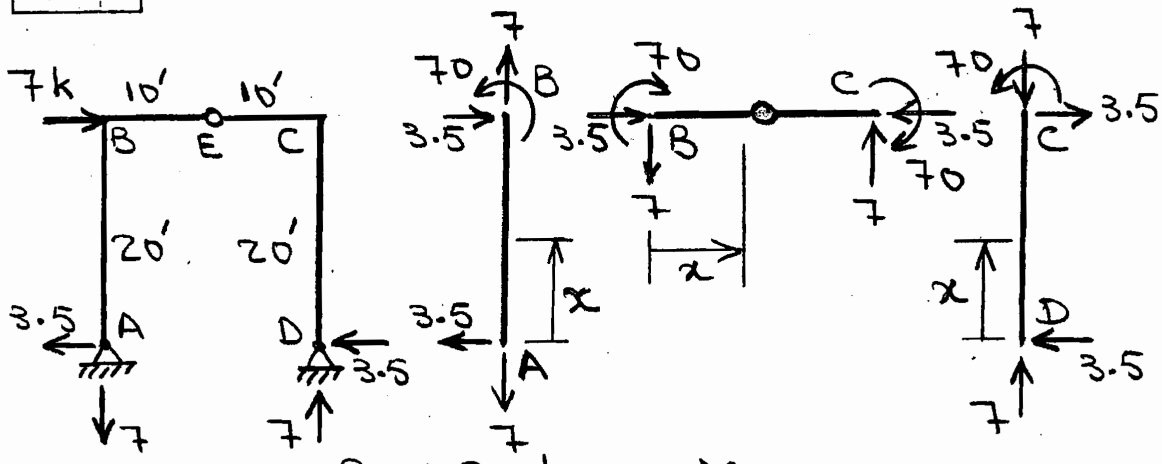
$$(1k)\Delta_C = \frac{1}{EI}\left[\int_0^{20}(1x)(7x)dx + \int_0^{20}(1x)(7x)dx\right]$$

$$= \frac{37333\ k^2\text{-}ft^3}{EI}$$

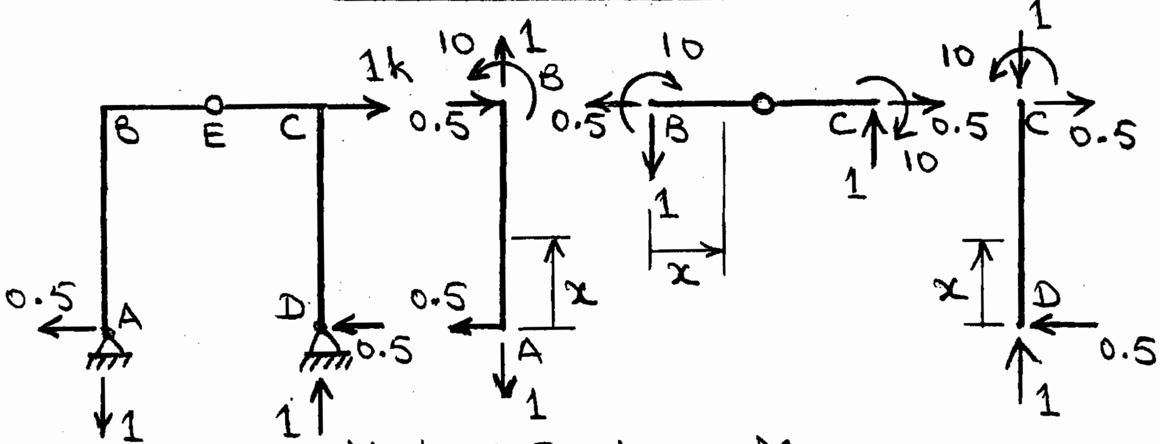
$$\Delta_C = \frac{37333\ k\text{-}ft^3}{EI} = \frac{37333\,(12)^3}{29000(I)} = 1\ \text{in.}$$

from which, $\underline{I = 2225\ in^4}$

7.42



Real System – M



Virtual System – M_v

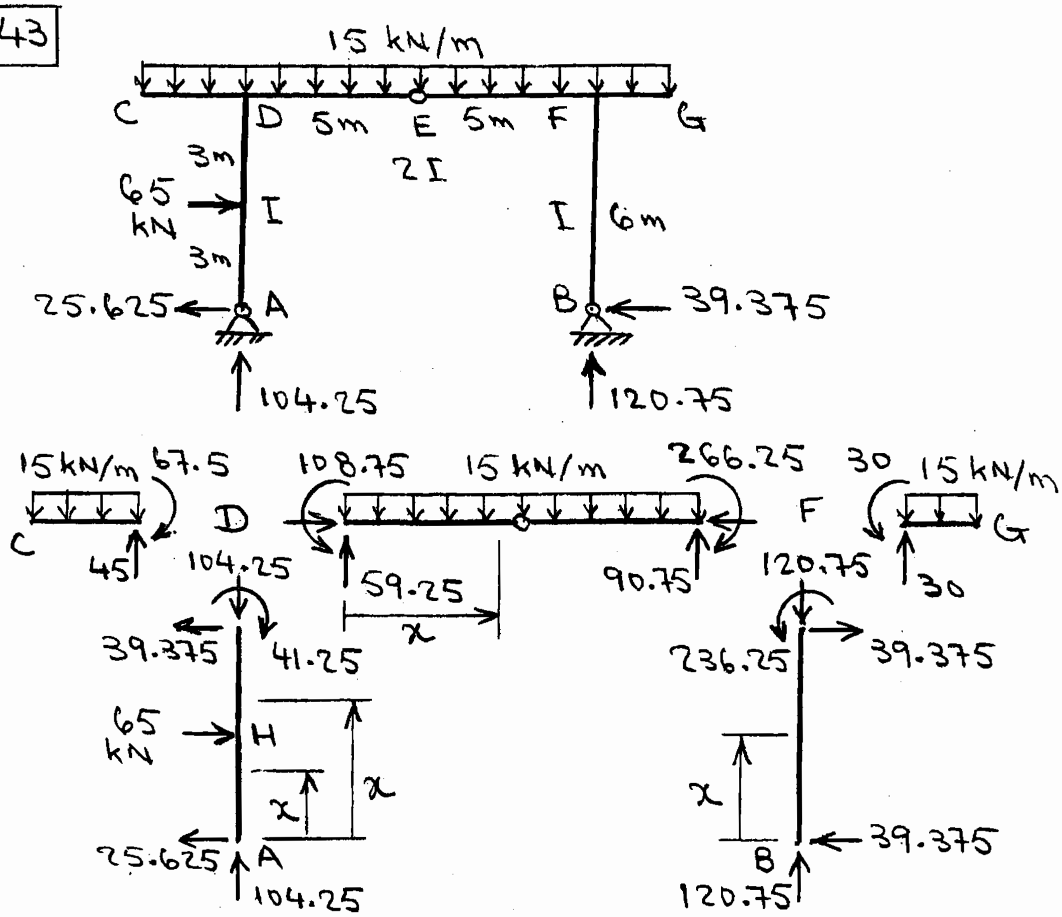
Segment	x Coordinate		M (k-ft)	M_v (k-ft)
	Origin	Limits (ft)		
AB	A	0–20	$3.5x$	$0.5x$
BC	B	0–20	$70-7x$	$10-1x$
DC	D	0–20	$3.5x$	$0.5x$

$$(1k)\Delta_C = \frac{1}{EI}\left[2\int_0^{20}(0.5x)(3.5x)\,dx + \int_0^{20}(10-x)(70-7x)\,dx\right] = \frac{14000\ k^2\text{-}ft^3}{EI}$$

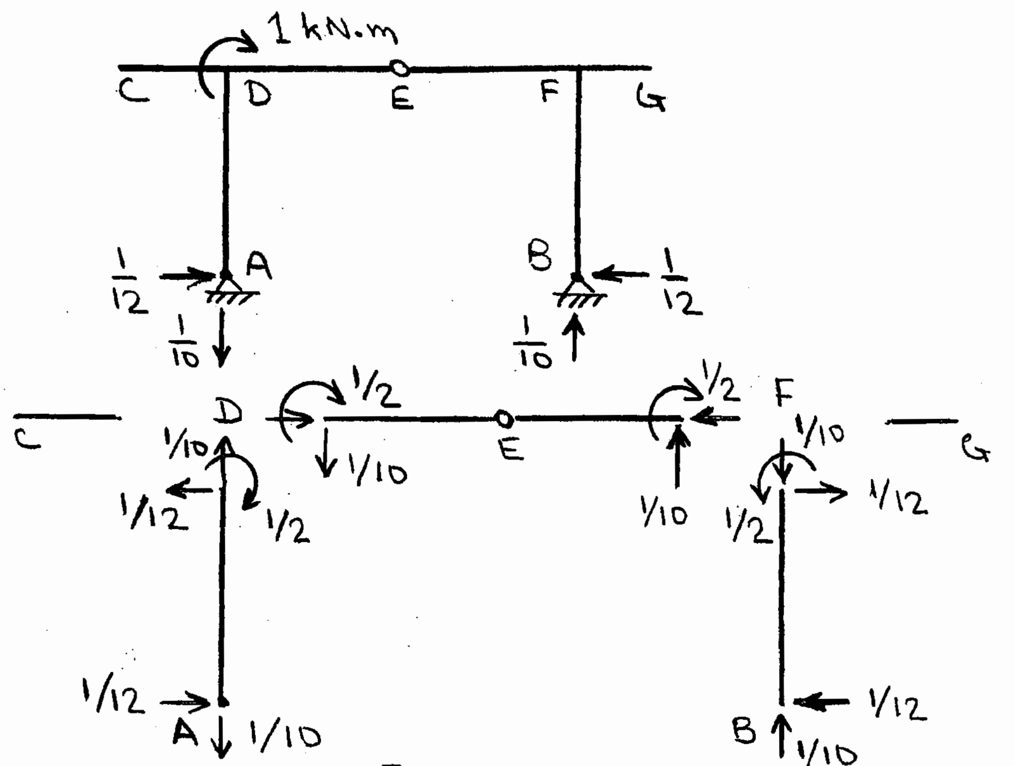
$$\Delta_C = \frac{14000\ k\text{-}ft^3}{EI} = \frac{14000(12)^3}{29000(I)} = 1\ \text{in.}$$

from which, $\underline{I = 834\ in^4}$

7.43



Real System – M



Virtual System – Mv

7.43 (contd.)

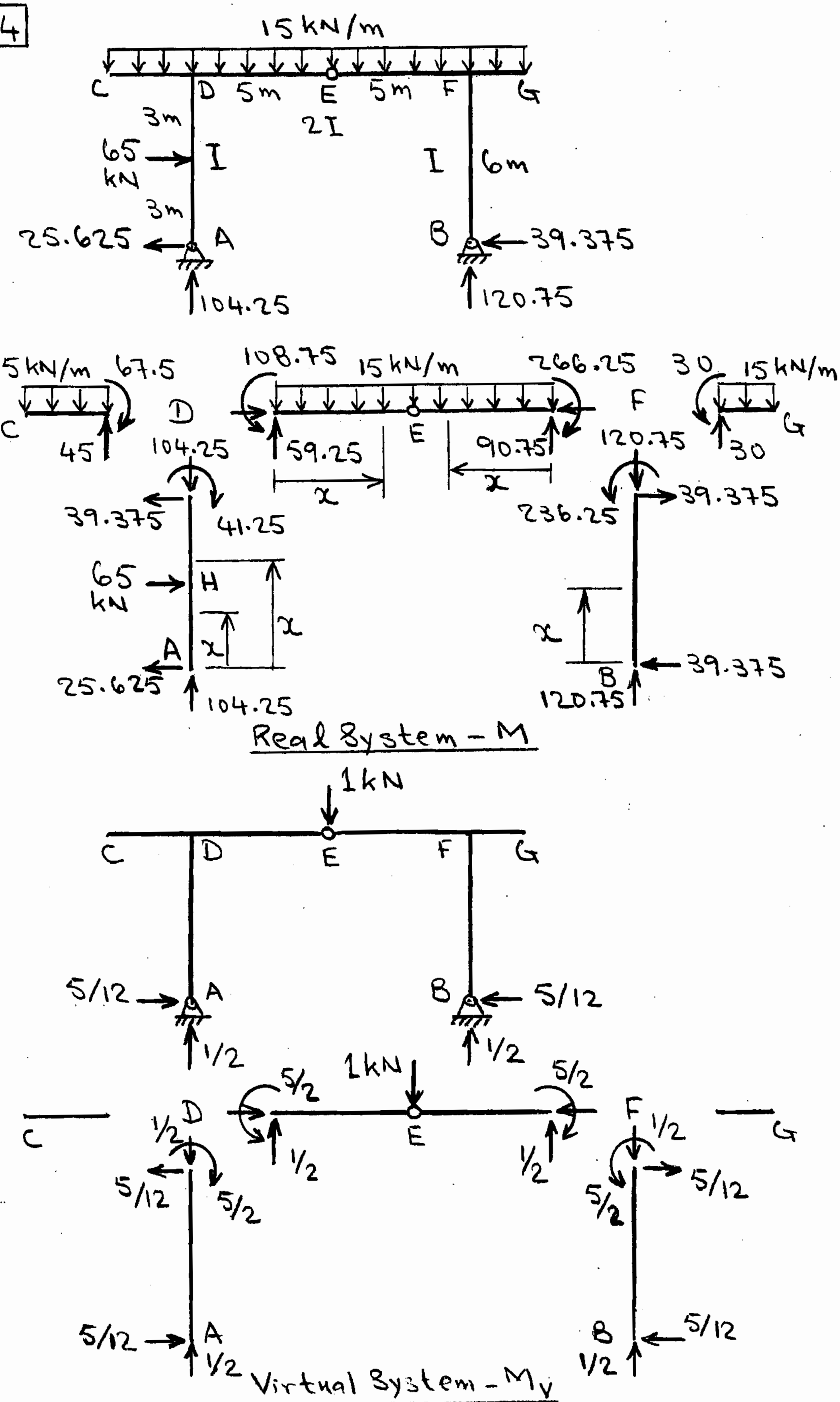
Segment	x Coordinate		M (kN.m)	M_v (kN.m)
	Origin	Limits (m)		
AH	A	0–3	$25.625x$	$-x/12$
HD	A	3–6	$25.625x - 65(x-3)$	$-x/12$
BF	B	0–6	$39.375x$	$x/12$
DF	D	0–10	$59.25x - 108.75 - 7.5x^2$	$\frac{1}{2} - \frac{x}{10}$

$$(1\,\text{kN.m})\,\theta_D = \frac{1}{EI}\left[\int_0^3 \left(-\frac{x}{12}\right)(25.625x)\,dx + \int_3^6 \left(-\frac{x}{12}\right)(25.625x - 65x + 195)\,dx + \int_0^6 \frac{x}{12}(39.375x)\,dx + \frac{1}{2}\int_0^{10} \left(\frac{1}{2} - \frac{x}{10}\right)(59.25x - 108.75 - 7.5x^2)\,dx\right]$$

$$= \frac{270\,\text{kN}^2\cdot\text{m}^3}{EI}$$

$$\theta_D = \frac{270}{200(350)} = \underline{0.00386 \text{ rad.}}$$

7.44


15 kN/m
C
D
5m
E
5m
F
G
3m
2I
65 kN
I
I
6m
3m
25.625
A
B
39.375
104.25
120.75
15 kN/m
67.5
108.75
15 kN/m
266.25
30
15 kN/m
D
F
C
E
G
45
104.25
59.25
90.75
120.75
30
39.375
41.25
x
236.25
39.375
65 kN
H
x
x
A
B
39.375
25.625
104.25
120.75
Real System – M
1kN
C
D
E
F
G
5/12
A
B
5/12
1/2
1/2
5/2
1kN
5/2
D
F
1/2
C
1/2
E
G
1/2
1/2
5/12
5/2
5/12
5/2
5/12
A
B
5/12
1/2
1/2
Virtual System – M_v

7.44 (contd.)

Segment	x coordinate		M (kN.m)	M_v (kN.m)
	Origin	Limits (m)		
AH	A	0 – 3	$25.625x$	$-5x/12$
HD	A	3 – 6	$25.625x - 65(x-3)$	$-5x/12$
BF	B	0 – 6	$39.375x$	$5x/12$
DE	D	0 – 5	$59.25x - 108.75 - 7.5x^2$	$\frac{x}{2} - \frac{5}{2}$
FE	F	0 – 5	$90.75x - 266.25 - 7.5x^2$	$\frac{x}{2} - \frac{5}{2}$

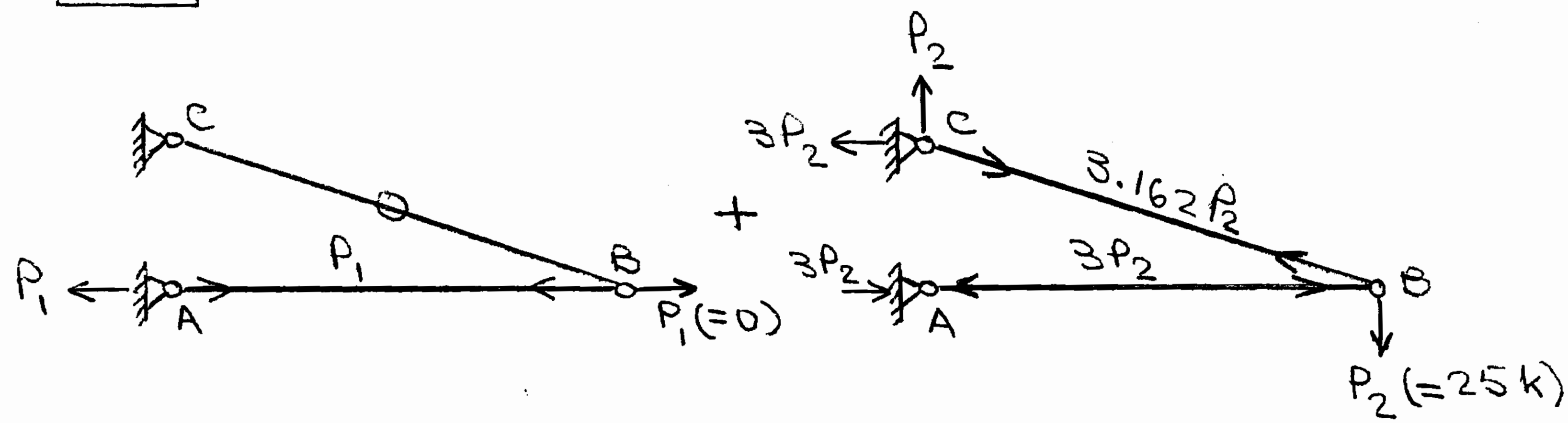
$$(1\,kN)\,\Delta_E = \frac{1}{EI}\left[\int_0^3 \left(-\frac{5x}{12}\right)(25.625x)\,dx + \int_3^6 \left(-\frac{5x}{12}\right)(25.625x - 65x + 195)\,dx + \int_0^6 \frac{5x}{12}(39.375x)\,dx + \frac{1}{2}\int_0^5 \left(\frac{x}{2} - \frac{5}{2}\right)(59.25x - 108.75 - 7.5x^2)\,dx + \frac{1}{2}\int_0^5 \left(\frac{x}{2} - \frac{5}{2}\right)(90.75x - 266.25 - 7.5x^2)\,dx\right]$$

$$= \frac{1607.8125\ kN^2\text{-}m^3}{EI}$$

$$\Delta_E = \frac{1607.8125}{200(350)} = 0.023\ m \downarrow$$

$$\underline{\Delta_E = 23\ mm \downarrow}$$

7.45

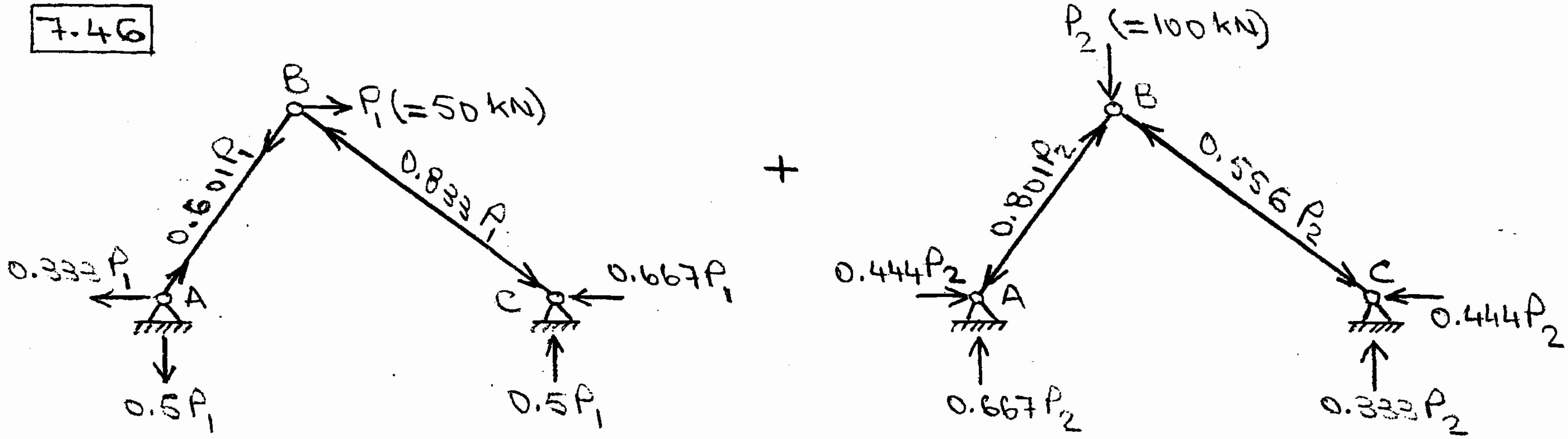


Member	L (in.)	F	$\frac{\partial F}{\partial P_1}$	$\frac{\partial F}{\partial P_2}$	$P_1=0,\ P_2=25\text{k}$: $\frac{\partial F}{\partial P_1}FL$	$P_1=0,\ P_2=25\text{k}$: $\frac{\partial F}{\partial P_2}FL$
AB	180	P_1-3P_2	1	−3	−13500	40500
BC	189.74	$3.162P_2$	0	3.162	0	47432.67
				Σ	−13500	87932.67

$$\Delta_{BH} = -\frac{13500}{10000(6)} = -0.225 \text{ in.} = \underline{0.225 \text{ in.} \rightarrow}$$

$$\Delta_{BV} = \frac{87932.67}{10000(6)} = \underline{1.466 \text{ in.} \downarrow}$$

7.46

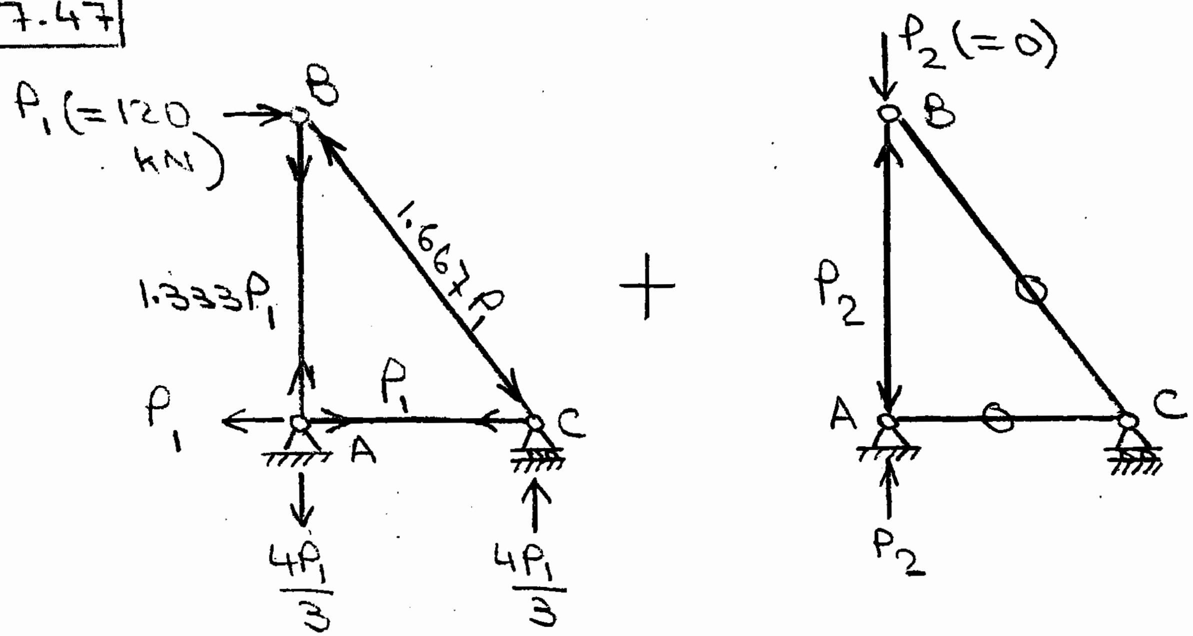


Member	L (m)	F	$\frac{\partial F}{\partial P_1}$	$\frac{\partial F}{\partial P_2}$	$P_1 = 50$ kN, $P_2 = 100$ kN: $\frac{\partial F}{\partial P_1}FL$	$\frac{\partial F}{\partial P_2}FL$
AB	3.606	$0.601P_1 - 0.801P_2$	0.601	−0.801	−108.53	144.65
BC	5	$-0.833P_1 - 0.556P_2$	−0.833	−0.556	405.09	270.06
				Σ	296.56	414.71

$$\Delta_{BH} = \frac{296.56}{70(10^6)(0.001)} = 0.00424\text{ m} = \underline{4.24\text{ mm} \rightarrow}$$

$$\Delta_{BV} = \frac{414.71}{70(10^6)(0.001)} = 0.00592\text{ m} = \underline{5.92\text{ mm} \downarrow}$$

7.47

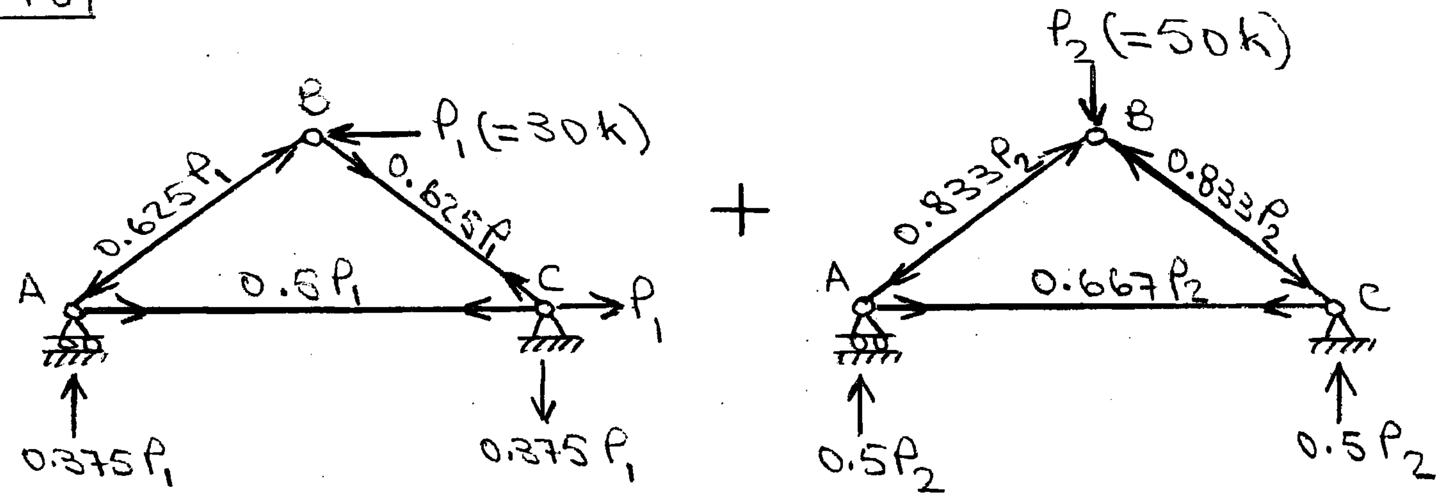


Member	L (m)	F	$\frac{\partial F}{\partial P_1}$	$\frac{\partial F}{\partial P_2}$	$P_1=120$ kN, $P_2=0$: $\frac{\partial F}{\partial P_1}FL$	$P_1=120$ kN, $P_2=0$: $\frac{\partial F}{\partial P_2}FL$
AC	3	P_1	1	0	360	0
AB	4	$1.333P_1-P_2$	1.333	−1	853.33	−640
BC	5	$-1.667P_1$	−1.667	0	1666.67	0
				Σ	2880	−640

$$\Delta_{BH}=\frac{2880}{200(10^6)(0.0015)}=0.0096\text{ m}=\underline{9.6\text{ mm}\rightarrow}$$

$$\Delta_{BV}=-\frac{640}{200(10^6)(0.0015)}=-0.00213\text{ m}=\underline{2.13\text{ mm}\uparrow}$$

7.48

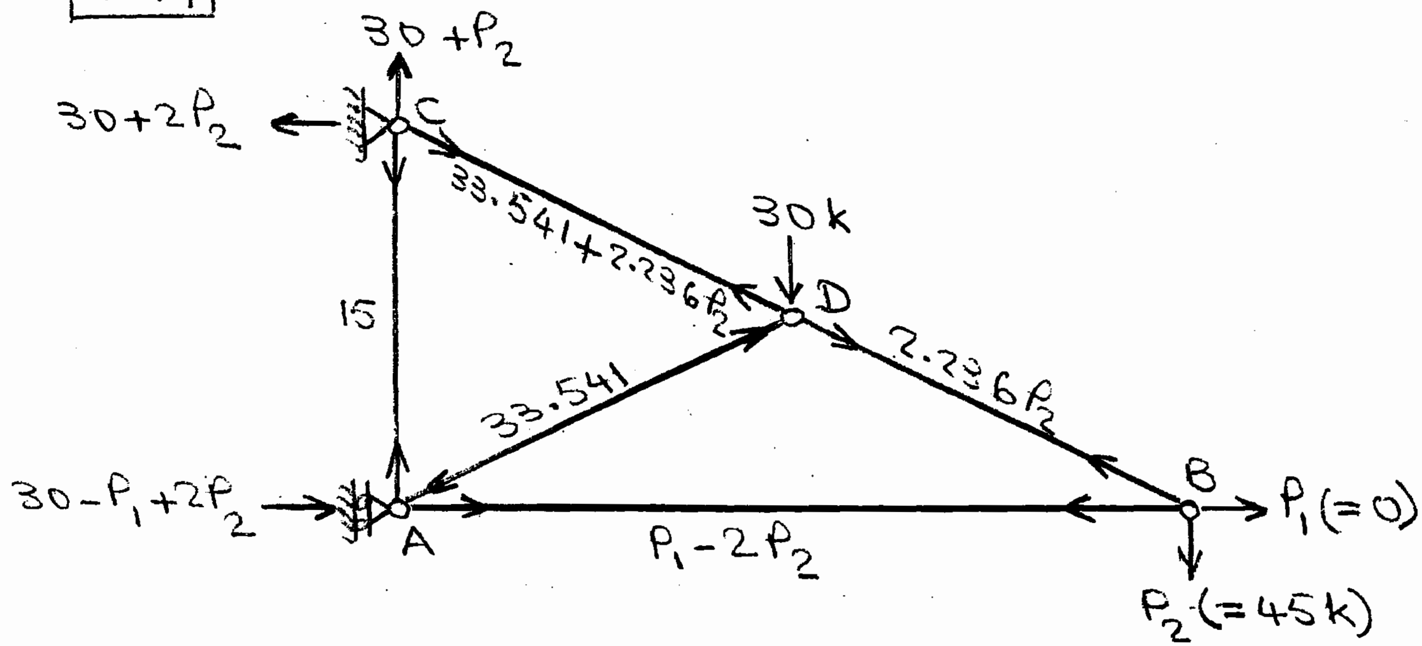


Member	L (in.)	F	$\frac{\partial F}{\partial P_1}$	$\frac{\partial F}{\partial P_2}$	$P_1=30k$, $P_2=50k$	
AC	192	$0.5P_1+0.667P_2$	0.5	0.667	4640	6186.67
AB	120	$-0.625P_1-0.833P_2$	-0.625	-0.833	4531.25	6041.67
BC	120	$0.625P_1-0.833P_2$	0.625	-0.833	-1718.75	2291.67
				Σ	7452.5	14520

$$\Delta_{BH} = \frac{7452.5}{29000\,(3)} = \underline{0.0857 \text{ in.} \leftarrow}$$

$$\Delta_{BV} = \frac{14520}{29000\,(3)} = \underline{0.167 \text{ in.} \downarrow}$$

$\boxed{7.49}$

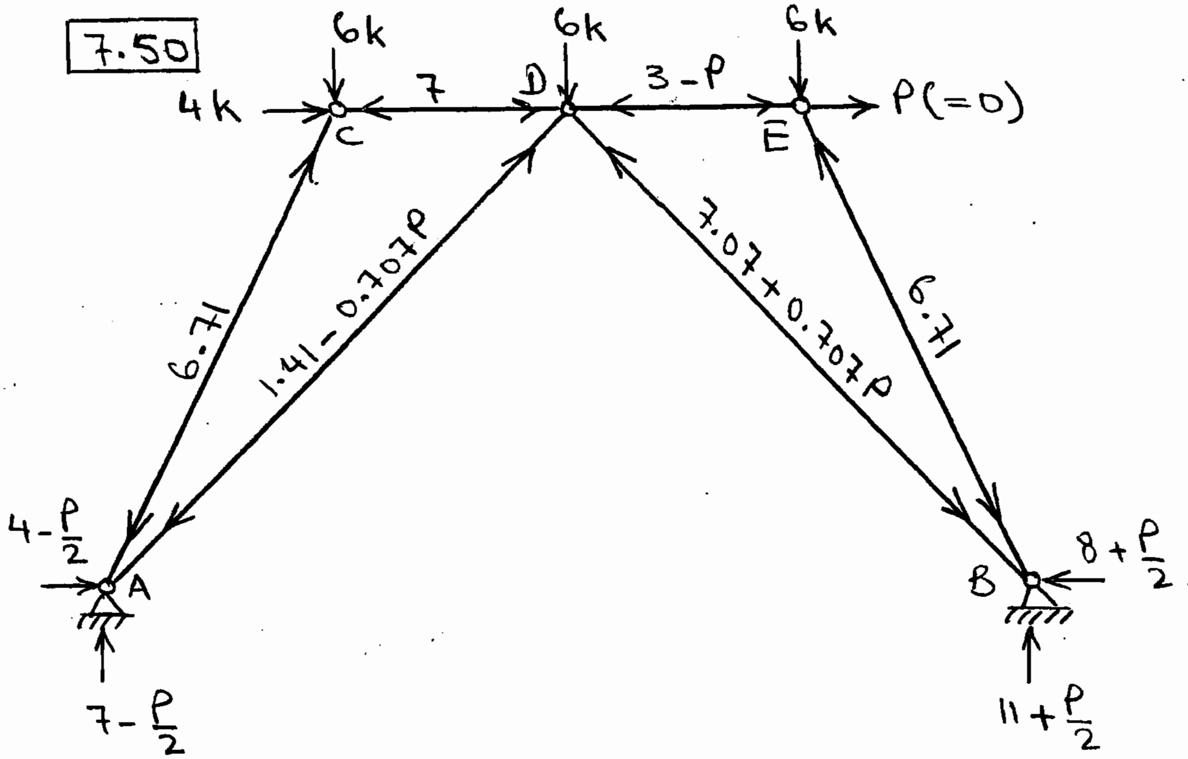


Member	L (in.)	A (in.^{2})	F	$\frac{\partial F}{\partial P_1}$	$\frac{\partial F}{\partial P_2}$	$P_1 = 0$, $P_2 = 45$ k $\frac{\partial F}{\partial P_1}\left(\frac{FL}{A}\right)$	$\frac{\partial F}{\partial P_2}\left(\frac{FL}{A}\right)$
AB	240	6	P_1-2P_2	1	−2	−3600	7200
AC	120	4	15	0	0	0	0
AD	134.16	4	−33.541	0	0	0	0
CD	134.16	6	$33.541+2.236P_2$	0	2.236	0	6707.99
BD	134.16	6	$2.236P_2$	0	2.236	0	5030.99
					Σ	−3600	18938.98

$$\Delta_{BH} = \frac{-3600}{10000} = -0.36 \text{ in.} = \underline{0.36 \text{ in} \leftarrow}$$

$$\Delta_{BV} = \frac{18938.98}{10000} = \underline{1.894 \text{ in.} \downarrow}$$

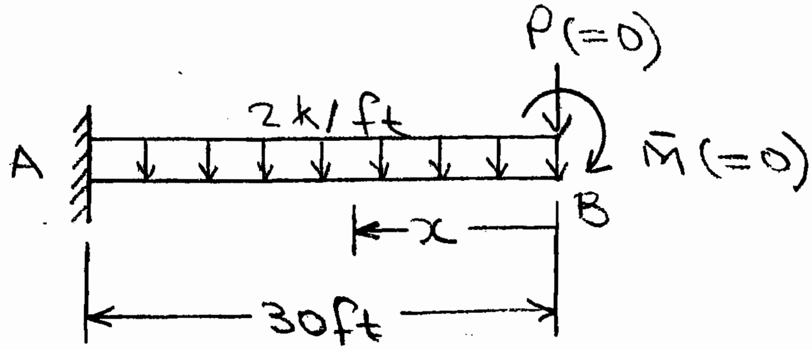
7.50



Member	L (in.)	F	$\frac{\partial F}{\partial P}$	P = 0: $\frac{\partial F}{\partial P}(FL)$
AC	134.16	-6.71	0	0
BE	134.16	-6.71	0	0
AD	169.71	-1.41 + 0.707P	0.707	-169.18
BD	169.71	-7.07 - 0.707P	-0.707	848.29
CD	60	-7	0	0
DE	60	-3 + P	1	-180
			Σ	499.11

$$\Delta_{EH} = \frac{499.11}{29000(6)} = \underline{0.0029 \text{ in.} \rightarrow}$$

$\boxed{7.51}$



$$M = -x^2 - \bar{M} - Px$$

$$\frac{\partial M}{\partial \bar{M}} = -1$$

$$\theta_B = \frac{1}{EI}\left[\int_0^{30}(-1)(-x^2)\,dx\right] = \frac{9000\text{ k-ft}^2}{EI}$$

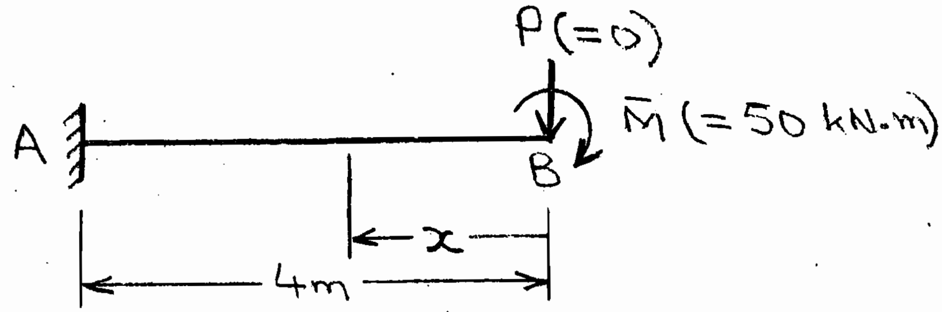
$$= \frac{9000(12)^2}{29000(3000)} = \underline{0.0149\text{ rad}}$$

$$\frac{\partial M}{\partial P} = -x$$

$$\Delta_B = \frac{1}{EI}\left[\int_0^{30}(-x)(-x^2)\,dx\right] = \frac{202500\text{ k-ft}^3}{EI}$$

$$= \frac{202500(12)^3}{29000(3000)} = \underline{4.022\text{ in.}\downarrow}$$

$\boxed{7.52}$



$$M = -\bar{M} - Px$$

$$\frac{\partial M}{\partial \bar{M}} = -1$$

$$\theta_B = \frac{1}{EI}\left[\int_0^4 (-1)(-50)\,dx\right] = \frac{200\text{ kN.m}^2}{EI}$$

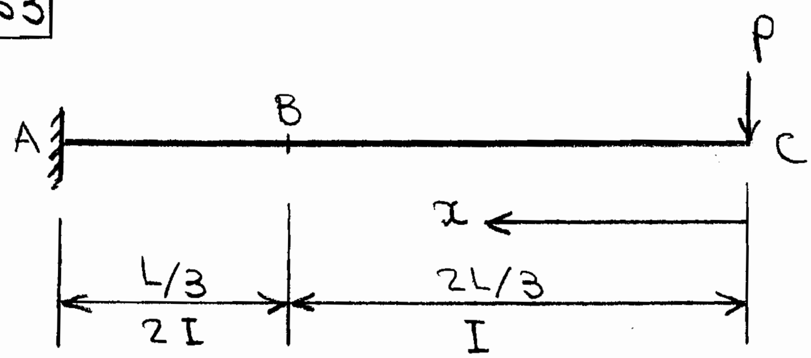
$$= \frac{200}{70(164)} = \underline{0.0174\text{ rad}}$$

$$\frac{\partial M}{\partial P} = -x$$

$$\Delta_B = \frac{1}{EI}\left[\int_0^4 (-x)(-50)\,dx\right] = \frac{400\text{ kN.m}^3}{EI}$$

$$= \frac{400}{70(164)} = 0.0348\text{ m} = \underline{34.8\text{ mm}\downarrow}$$

7.53

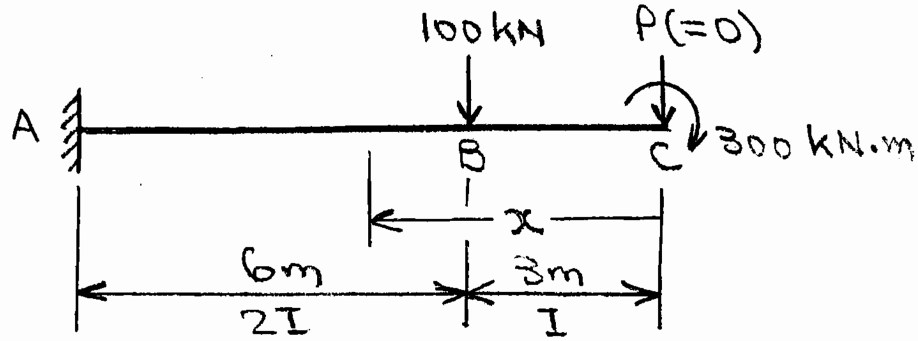


Segment	x Coordinate Origin	x Coordinate Limits	M	$\frac{\partial M}{\partial P}$
CB	C	$0 - \frac{2L}{3}$	$-Px$	$-x$
BA	C	$\frac{2L}{3} - L$	$-Px$	$-x$

$$\Delta_C = \frac{1}{EI}\left[\int_0^{\frac{2L}{3}} (-x)(-Px)\,dx + \frac{1}{2}\int_{\frac{2L}{3}}^{L} (-x)(-Px)\,dx\right]$$

$$= \underline{\frac{35PL^3}{162EI}\downarrow}$$

7.54



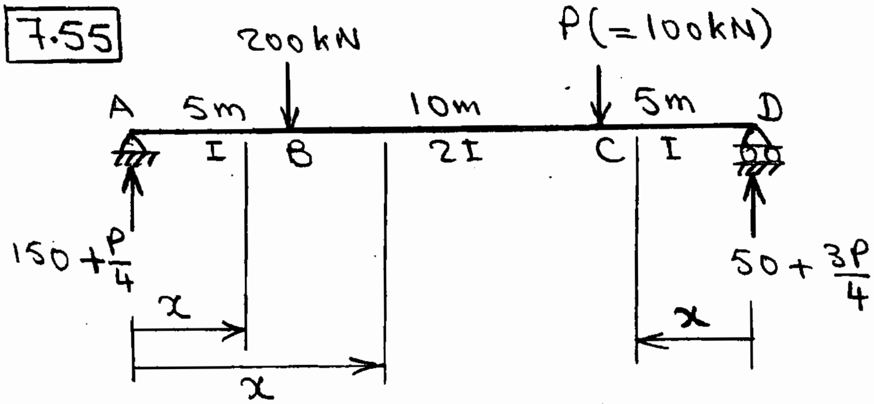
Segment	x Coordinate Origin	x Coordinate Limits (m)	M	$\frac{\partial M}{\partial P}$
CB	C	0–3	$-300-Px$	$-x$
BA	C	3–9	$-300-Px$ $-100(x-3)$	$-x$

$$\Delta_C = \frac{1}{EI}\left[\int_0^3(-x)(-300)\,dx + \frac{1}{2}\int_3^9(-x)(-100x)\,dx\right]$$

$$= \frac{13050\ \text{kN}\cdot\text{m}^3}{EI} = \frac{13050}{70(500)}$$

$$= 0.373\ \text{m} = \underline{373\ \text{mm}\downarrow}$$

7.55



Segment	x coordinate		M	$\frac{\partial M}{\partial P}$
	Origin	Limits		
AB	A	0 – 5	$(150+P/4)x$	$0.25x$
BC	A	5 – 15	$(150+P/4)x-200(x-5)$	$0.25x$
DC	D	0 – 5	$(50+3/4P)x$	$0.75x$

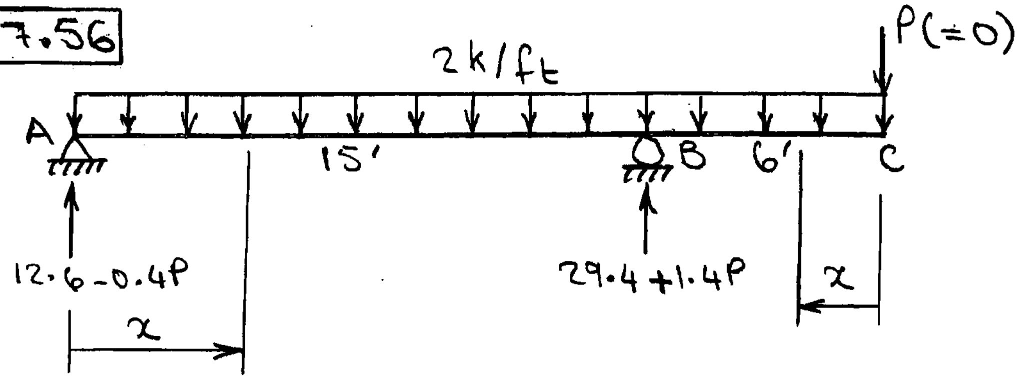
Substituting P = 100 kN and integrating, we obtain

$$\Delta_C = \frac{1}{EI}\left[\int_0^5 (0.25x)(175x)\,dx + \frac{1}{2}\int_5^{15} 0.25x(-25x+1000)\,dx + \int_0^5 0.75x(125x)\,dx\right]$$

$$= \frac{14843.75 \text{ kN-m}^3}{EI} = \frac{14843.75}{250(600)} = 0.099\text{ m}$$

$$\underline{\Delta_C = 99\text{ mm}\downarrow}$$

7.56



Segment	x Coordinate Origin	Limits	M	$\frac{\partial M}{\partial P}$
AB	A	0−15	$(12.6-0.4P)x - x^2$	$-0.4x$
CD	C	0−6	$-Px - x^2$	$-x$

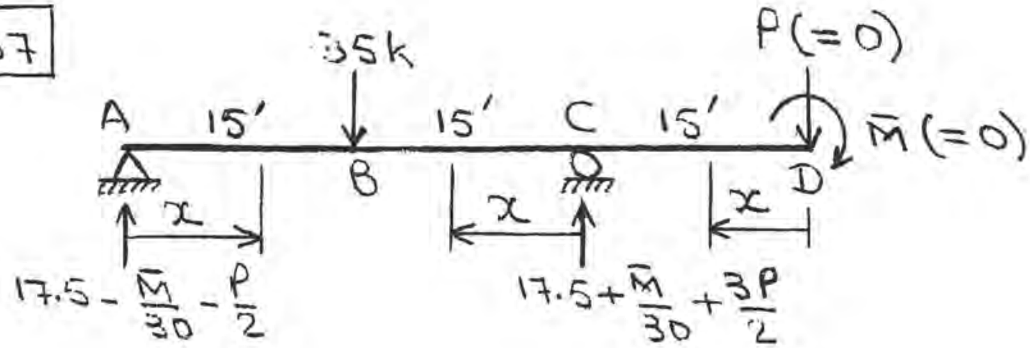
Substituting $P=0$ and integrating, we obtain

$$\Delta_C = \frac{1}{EI}\left[\int_0^{15}(-0.4x)(12.6x - x^2)\,dx + \int_0^{6} -x(-x^2)\,dx\right]$$

$$= -\frac{283.5\text{ k-ft}^3}{EI} = -\frac{283.5(12)^3}{29000(3500)}$$

$$= -0.0048\text{ in.} = \underline{0.0048\text{ in.}\uparrow}$$

7.57



Segment	x Coordinate		M	$\frac{\partial M}{\partial \bar{M}}$	$\frac{\partial M}{\partial P}$
	Origin	Limits (ft)			
AB	A	0–15	$\left(17.5-\frac{\bar{M}}{30}-\frac{P}{2}\right)x$	$-x/30$	$-x/2$
CB	C	0–15	$\left(17.5+\frac{\bar{M}}{30}+\frac{3P}{2}\right)x-\bar{M}-P(x+15)$	$\frac{x}{30}-1$	$\frac{x}{2}-15$
DC	D	0–15	$-\bar{M}-Px$	-1	$-x$

Substituting $\bar{M}=P=0$, and integrating, we obtain

$$\theta_D=\frac{1}{EI}\left[\int_0^{15}-\frac{x}{30}(17.5x)\,dx+\int_0^{15}\left(\frac{x}{30}-1\right)(17.5x)\,dx\right]$$

$$=-\frac{1968.75\text{ k-ft}^2}{EI}=-\frac{1968.75(12)^2}{10000(2500)}$$

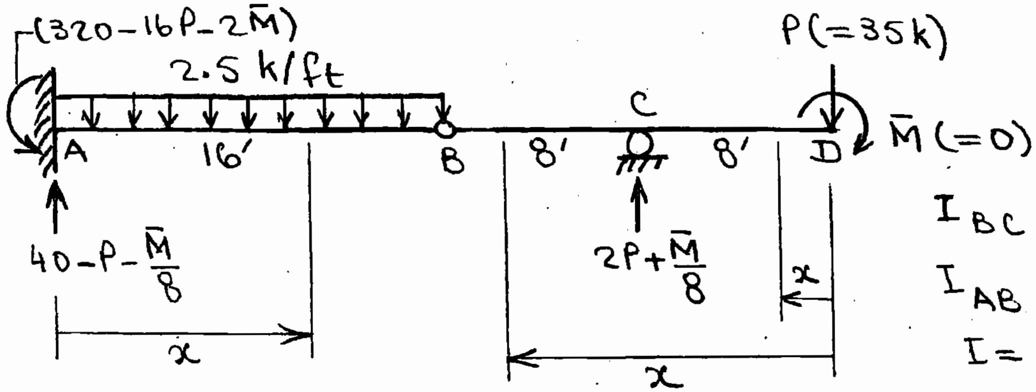
$$=-0.01134\text{ rad}=\underline{0.01134\text{ rad}}$$

$$\Delta_D=\frac{1}{EI}\left[\int_0^{15}-\frac{x}{2}(17.5x)\,dx+\int_0^{15}\left(\frac{x}{2}-15\right)(17.5x)\,dx\right]$$

$$=-\frac{29531.25\text{ k-ft}^3}{EI}=-\frac{29531.25(12)^3}{10000(2500)}$$

$$=-2.04\text{ in.}=\underline{2.04\text{ in.}\uparrow}$$

7.58



$I_{BC} = I_{CD} = I$

$I_{AB} = \frac{4}{3} I$

$I = 3000 \text{ in.}^4$

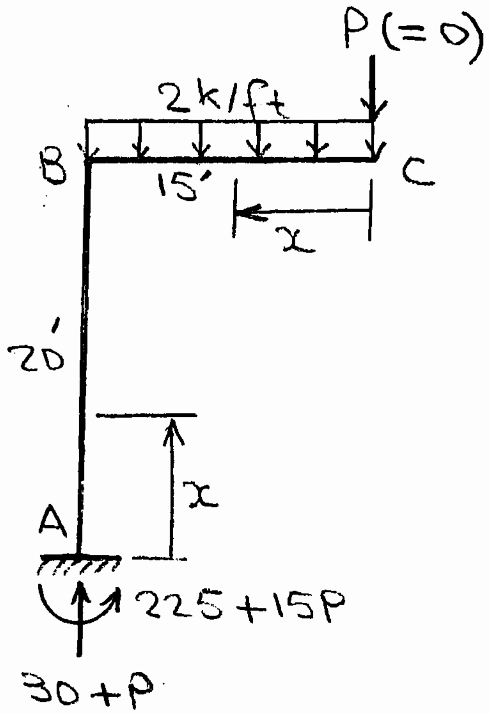
Segment	x Coordinate		M	$\frac{\partial M}{\partial \bar{M}}$	$\frac{\partial M}{\partial P}$
	Origin	Limits			
DC	D	0−8	$-Px - \bar{M}$	-1	$-x$
CB	D	8−16	$-Px - \bar{M} + (2P + \frac{\bar{M}}{8})(x-8)$	$\frac{x}{8} - 2$	$x - 16$
AB	A	0−16	$-(320 - 16P - 2\bar{M}) + (40 - P - \frac{\bar{M}}{8})x - 1.25x^2$	$2 - \frac{x}{8}$	$16 - x$

Substituting $\bar{M} = 0$ and $P = 35$ k, and integrating, we obtain

$$\theta_D = \frac{1}{EI}\left[\int_0^8 -1(-35x)\,dx + \int_8^{16}\left(\frac{x}{8} - 2\right)(35x - 560)\,dx + \frac{3}{4}\int_0^{16}\left(2 - \frac{x}{8}\right)(240 + 5x - 1.25x^2)\,dx\right] = \frac{4426.67 \text{ k-ft}^2}{EI} = \underline{0.0071 \text{ rad.}}$$

$$\Delta_D = \frac{1}{EI}\left[\int_0^8 -x(-35x)\,dx + \int_8^{16}(x - 16)(35x - 560)\,dx + \frac{3}{4}\int_0^{16}(16 - x)(240 + 5x - 1.25x^2)\,dx\right] = \frac{32426.67 \text{ k-ft}^3}{EI} = \underline{0.62 \text{ in.}} \downarrow$$

7.59



Segment	x Coordinate		M	$\frac{\partial M}{\partial P}$
	Origin	Limits (ft)		
AB	A	0-20	$-(225+15P)$	-15
CB	C	0-15	$-x^2-Px$	$-x$

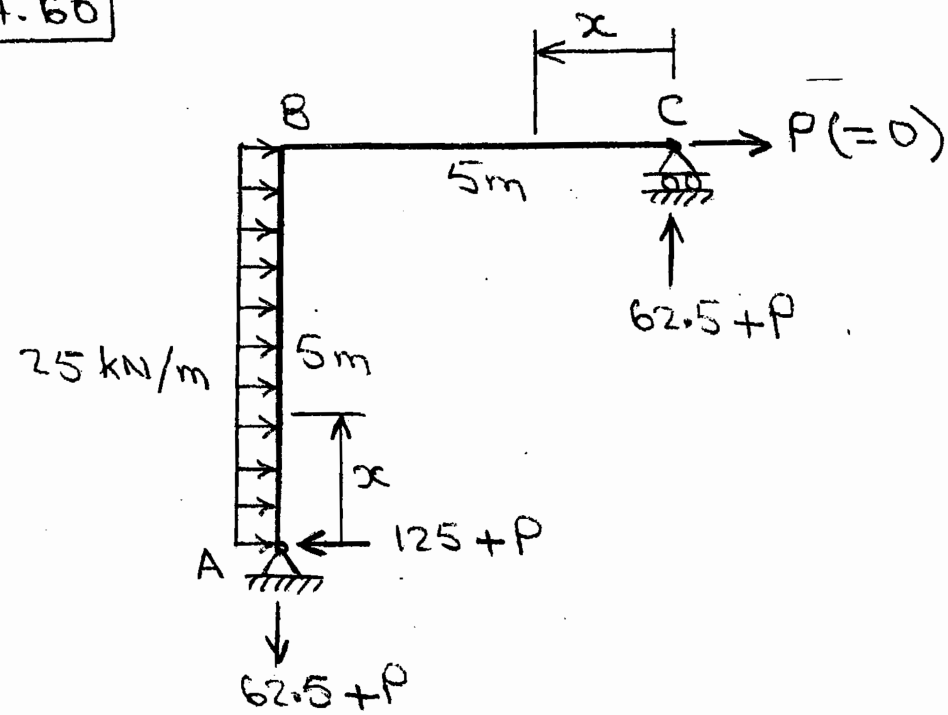
Substituting P = 0 and integrating, we obtain

$$\Delta_C = \frac{1}{EI}\left[\int_0^{20}(-15)(-225)\,dx + \int_0^{15}(-x)(-x^2)\,dx\right]$$

$$= \frac{80156.25 \text{ k-ft}^3}{EI} = \frac{80156.25(12)^3}{29000(2000)}$$

$$= \underline{2.388 \text{ in.}\downarrow}$$

7.60



Segment	x Coordinate		M	$\frac{\partial M}{\partial P}$
	Origin	Limits (m)		
AB	A	0-5	$(125+P)x-12.5x^2$	x
CB	C	0-5	$(62.5+P)x$	x

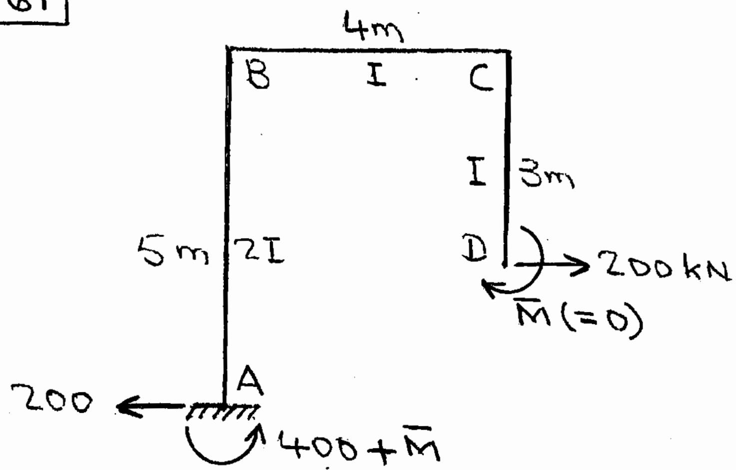
Substituting $P=0$ and integrating, we obtain

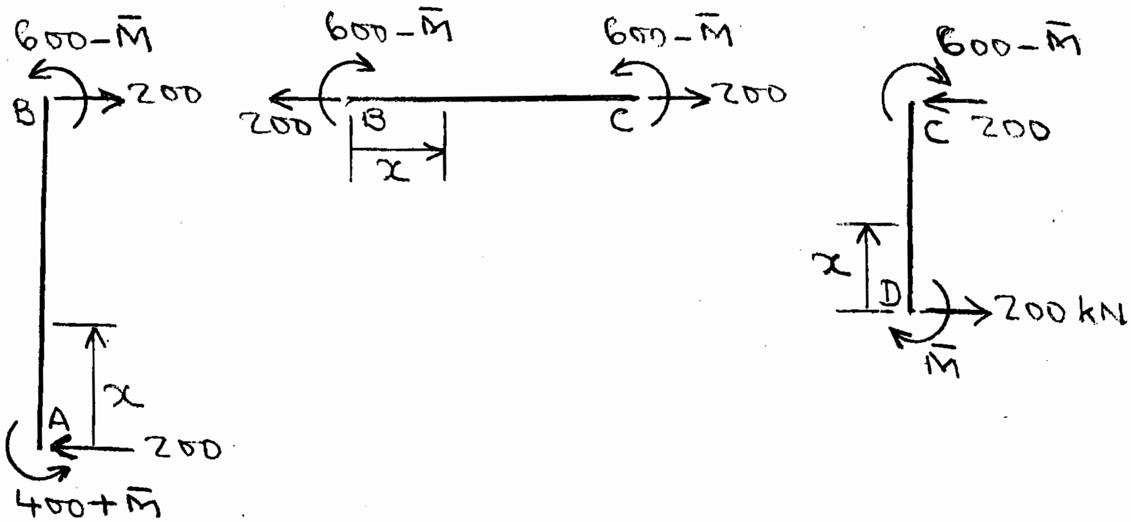
$$\Delta_C = \frac{1}{EI}\left[\int_0^5 x(125x-12.5x^2)\,dx + \int_0^5 x(62.5x)\,dx\right]$$

$$= \frac{5859.375 \text{ kN}\cdot\text{m}^3}{EI} = \frac{5859.375}{70(1030)}$$

$$= 0.0813 \text{ m} = \underline{81.3 \text{ mm} \rightarrow}$$

7.61





Segment	x Coordinate		M	$\frac{\partial M}{\partial \bar{M}}$
	Origin	Limits (m)		
AB	A	0 − 5	$-(400+\bar{M})+200x$	−1
BC	B	0 − 4	$600-\bar{M}$	−1
DC	D	0 − 3	$-200x+\bar{M}$	1

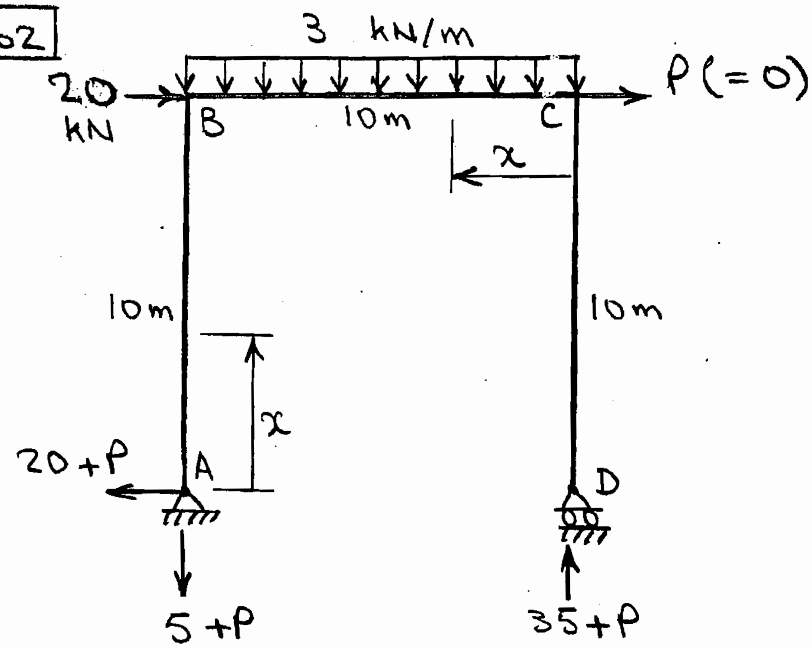
Substituting $\bar{M}=0$ and integrating, we obtain

$$\theta_D = \frac{1}{EI}\left[\frac{1}{2}\int_0^5 -1(-400+200x)\,dx + \int_0^4 -1(600)\,dx + \int_0^3 1(-200x)\,dx\right]$$

$$= -\frac{3550\ \text{kN.m}^2}{EI} = -\frac{3550}{70(1290)} = -0.0393\ \text{rad}$$

$$= \underline{0.0393\ \text{rad}}$$

7.62



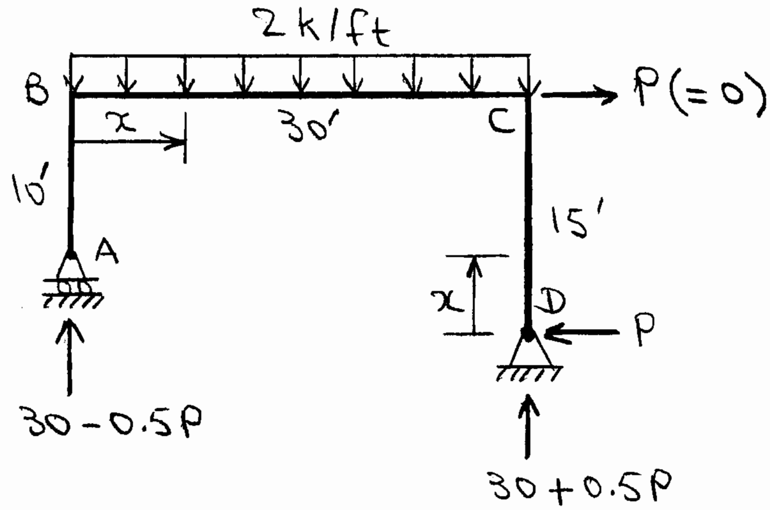
Segment	x Coordinate		M	$\frac{\partial M}{\partial P}$
	Origin	Limits		
AB	A	0–10	$(20+P)x$	x
CB	C	0–10	$(35+P)x-1.5x^2$	x

Substituting $P=0$ and integrating, we obtain

$$\Delta_C = \frac{1}{EI}\left[\int_0^{10} x(20x)\,dx + \int_0^{10} x(35x-1.5x^2)\,dx\right]$$

$$= \frac{14583.33 \text{ kN-m}^3}{EI} = \underline{0.182 \text{ m} \rightarrow}$$

7.63



Segment	x Coordinate		M	$\frac{\partial M}{\partial P}$
	Origin	Limits		
BC	B	0 – 30	$(30-0.5P)x - x^2$	$-0.5x$
DC	D	0 – 15	Px	x

Substituting $P = 0$ and integrating, we obtain

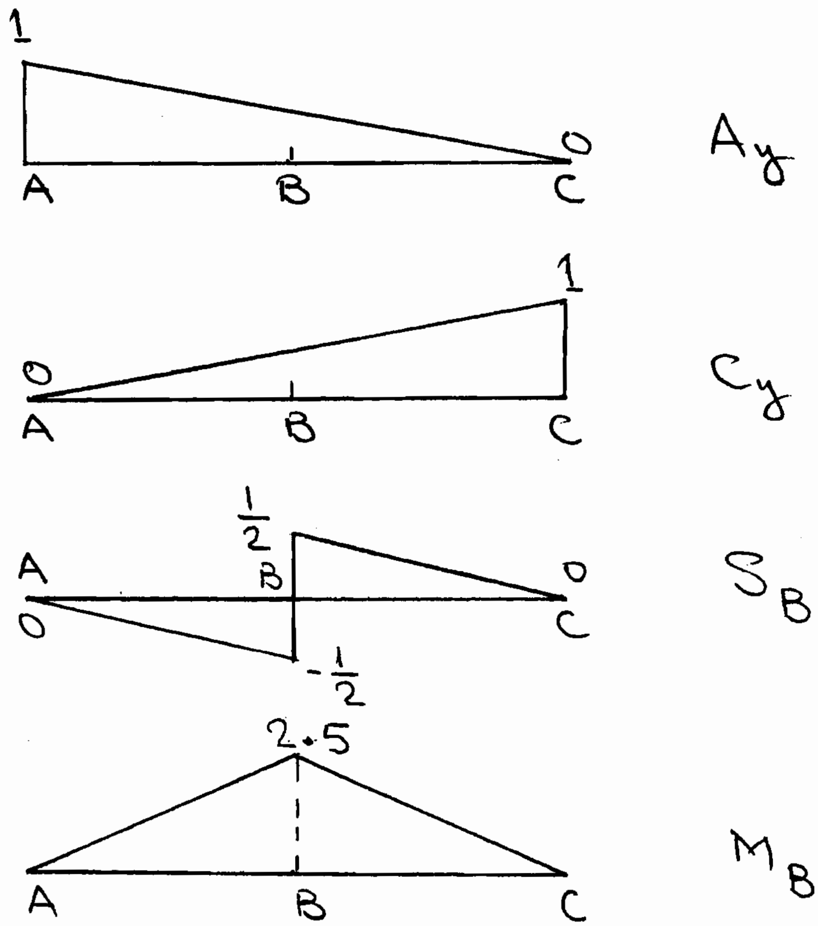
$$\Delta_C = \frac{1}{EI}\int_0^{30}(-0.5x)(30x - x^2)\,dx = -\frac{33750 \text{ k-ft}^3}{EI}$$

$$= -\frac{33750\,(12)^3}{29000\,(1500)} = -1.34 \text{ in.} = \underline{1.34 \text{ in.} \leftarrow}$$

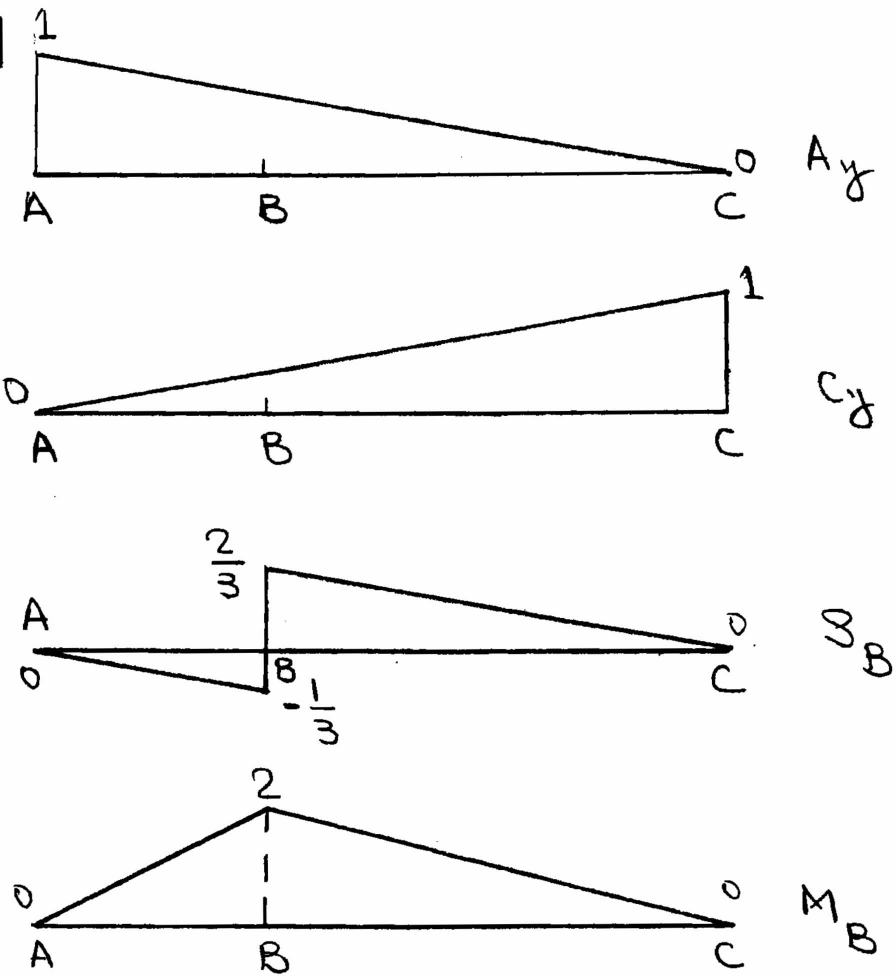
Chapter Eight
Influence Lines

CHAPTER 8

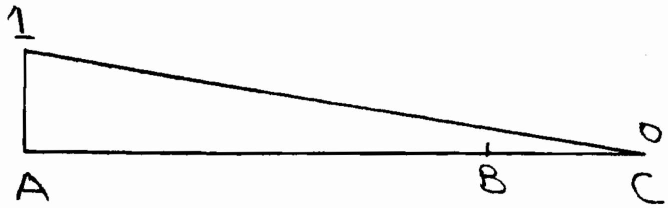
8-1



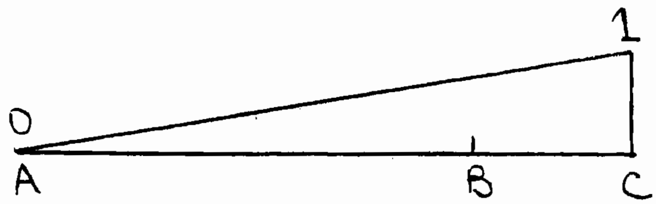
8.2



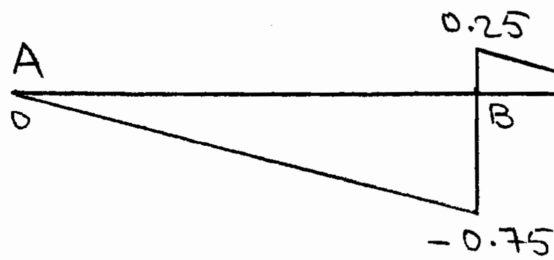
$\boxed{8.3}$



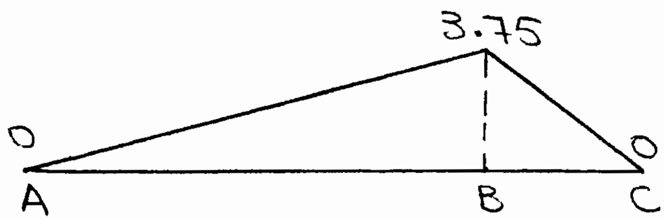
A_y



C_y

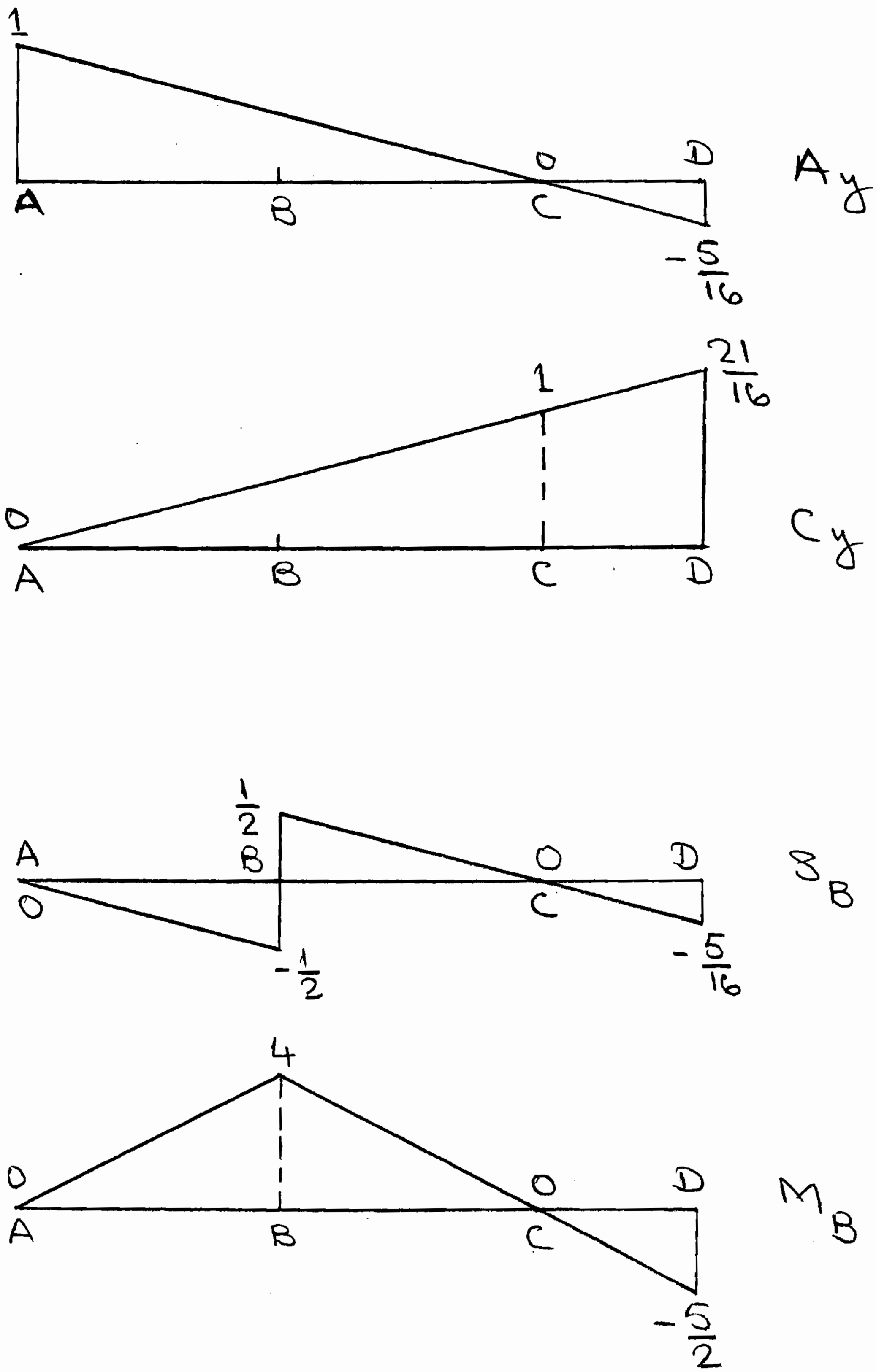


S_B

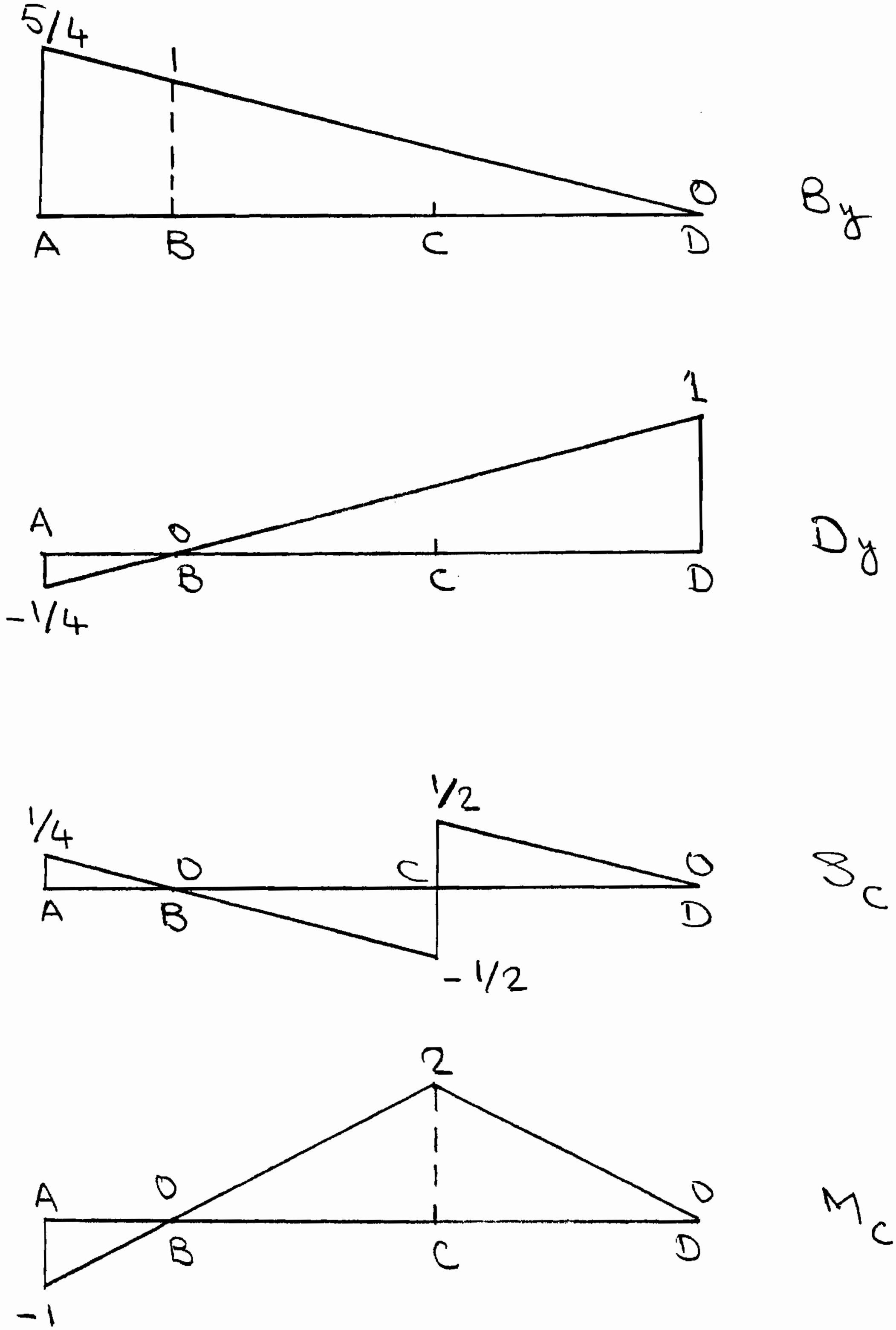


M_B

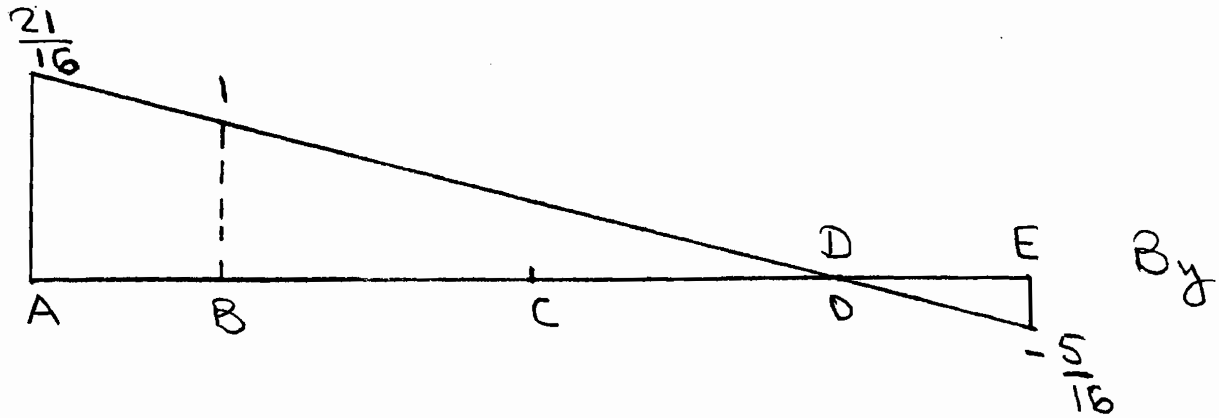
8.4

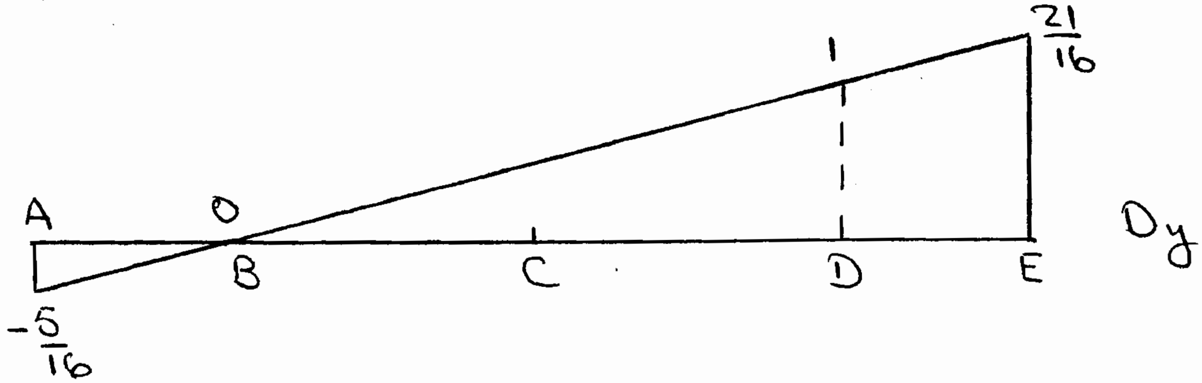

1
0
D
A
B
C
$-\frac{5}{16}$
A_y
$\frac{21}{16}$
1
0
A
B
C
D
C_y
$\frac{1}{2}$
A
B
0
D
0
C
$-\frac{1}{2}$
$-\frac{5}{16}$
S_B
4
0
0
D
A
B
C
$-\frac{5}{2}$
M_B

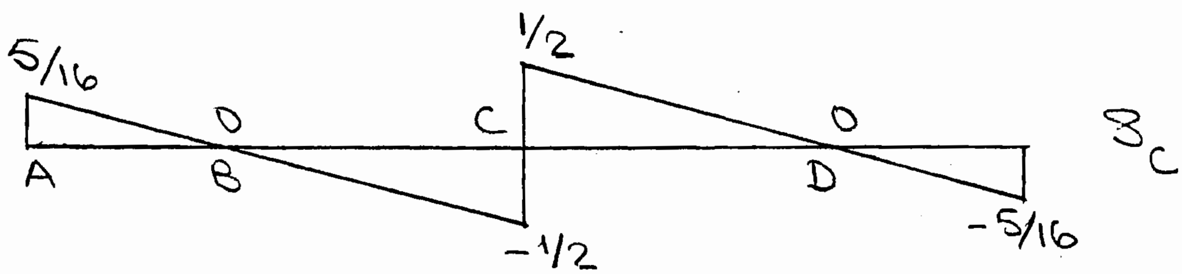
8.5

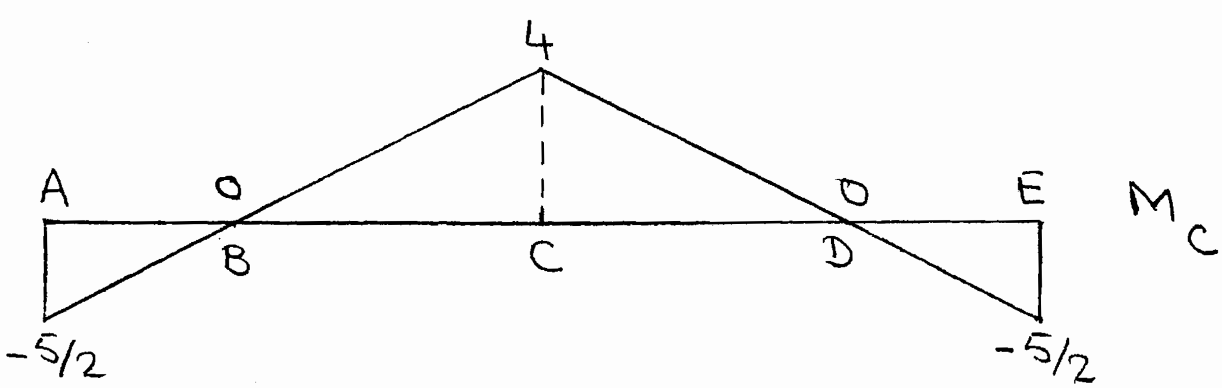


8.6

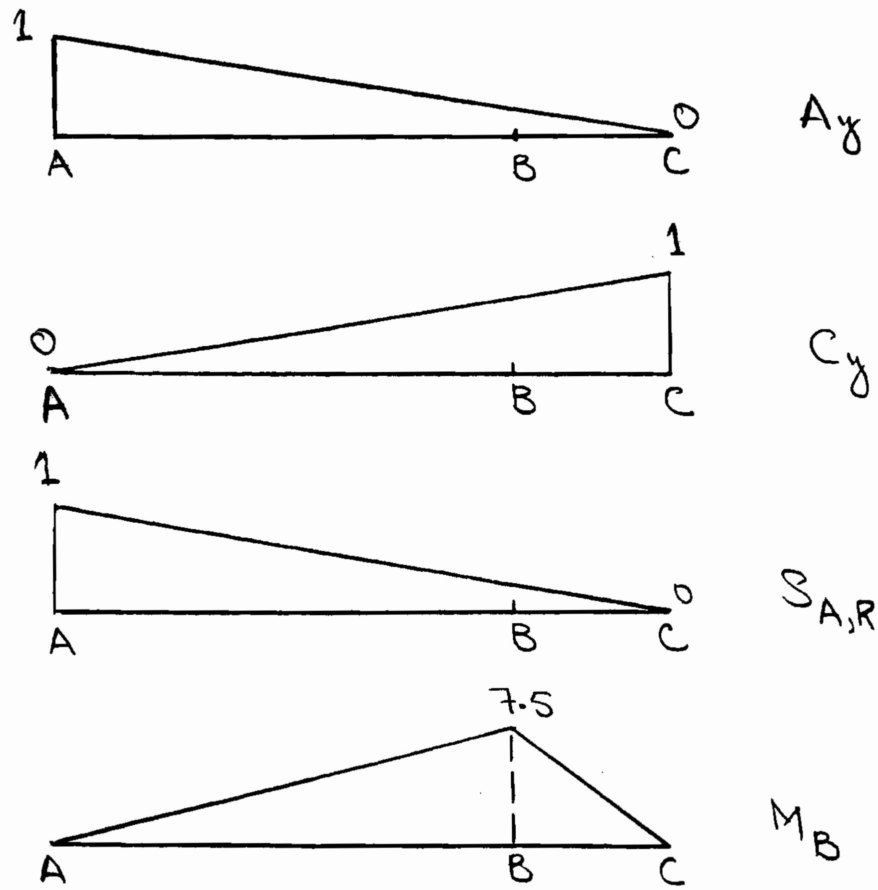

21/16
1
D
E
By
A
B
C
0
-5/16


21/16
1
A
0
Dy
B
C
D
E
-5/16

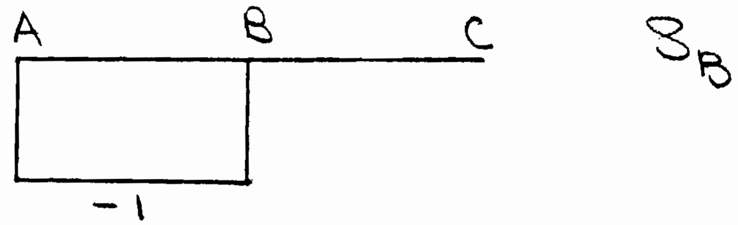

5/16
1/2
0
C
0
Sc
A
B
D
-1/2
-5/16

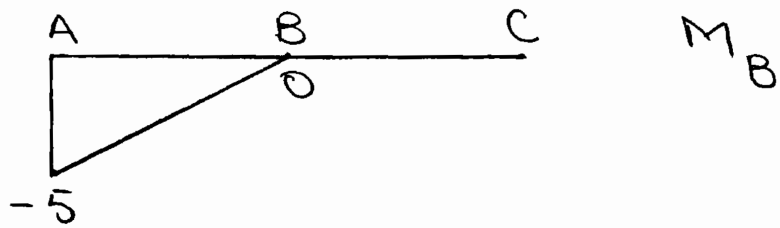

4
A
0
0
E
Mc
B
C
D
-5/2
-5/2

8.7

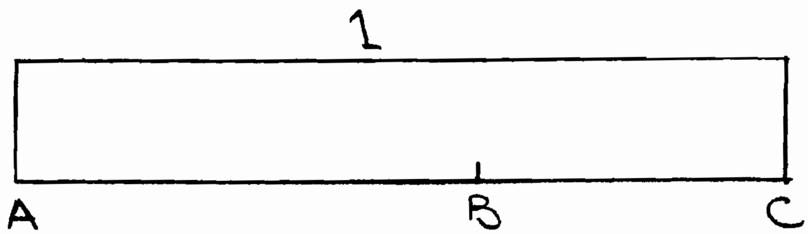


$\boxed{8.8}$

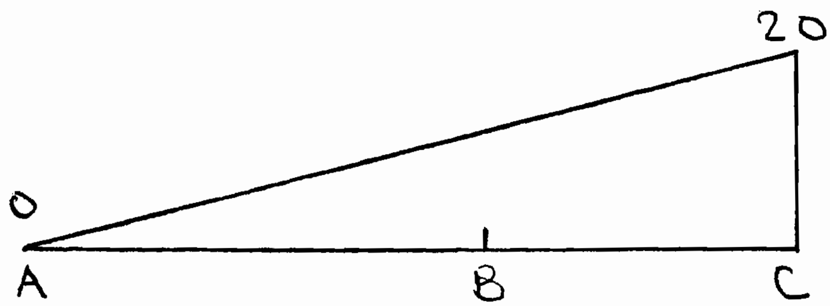




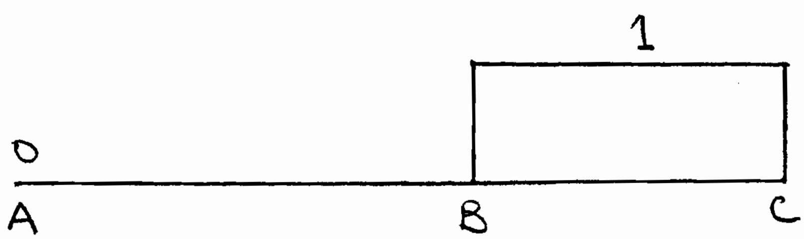
8.9



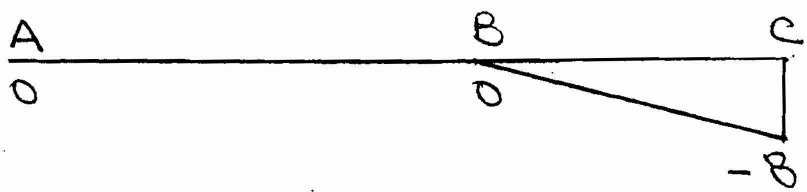
A_y



M_A (+↻)

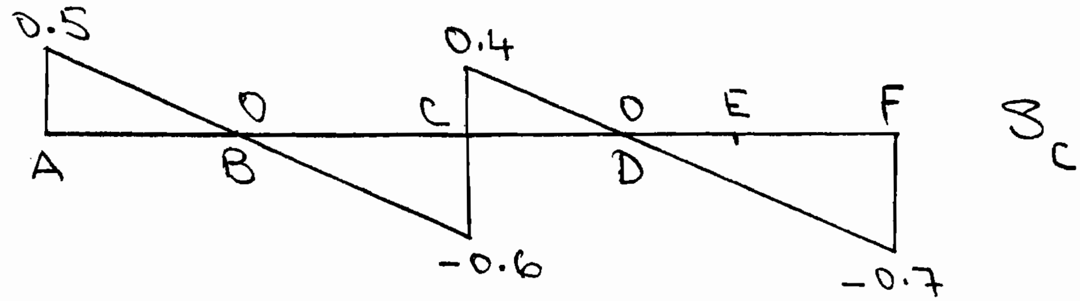


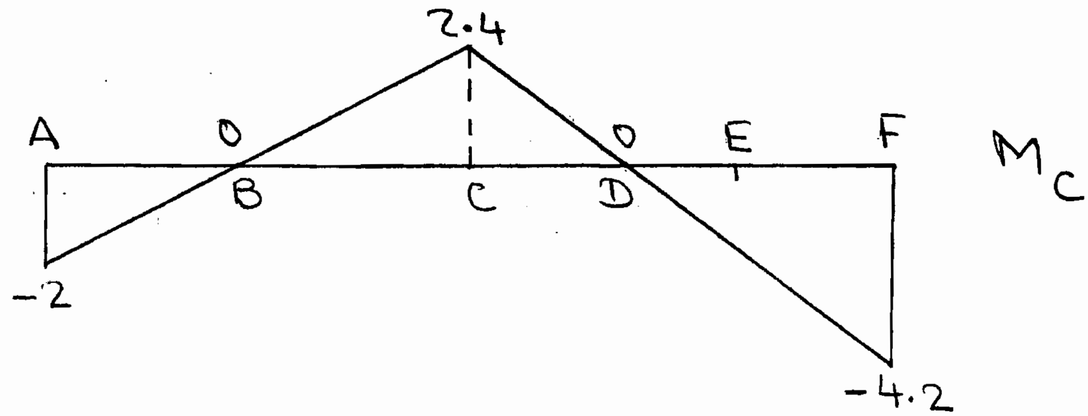
S_B

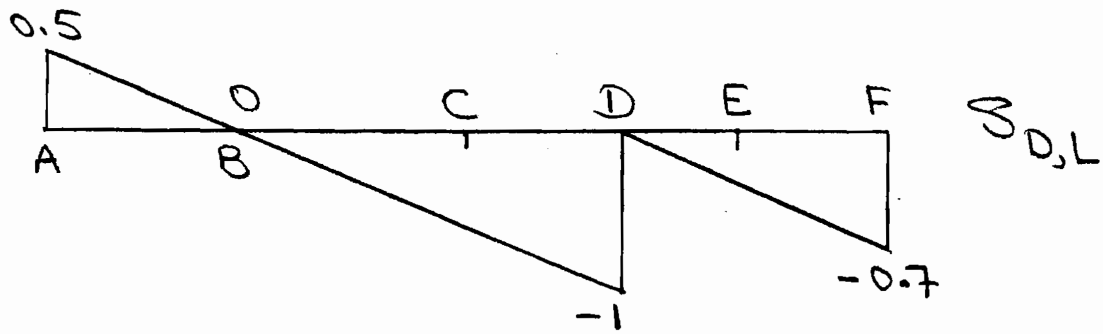


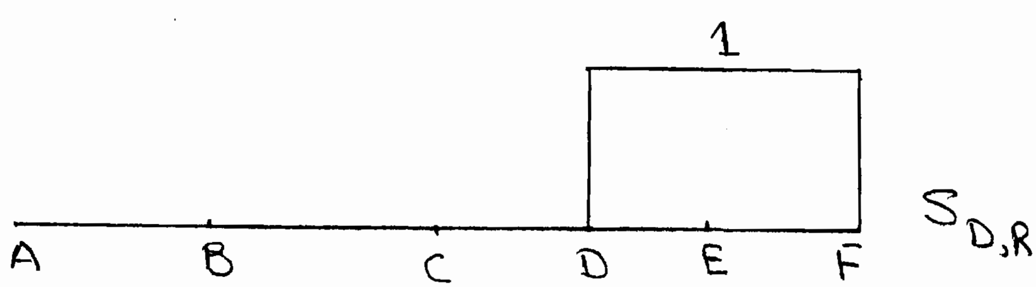
M_B

8.10

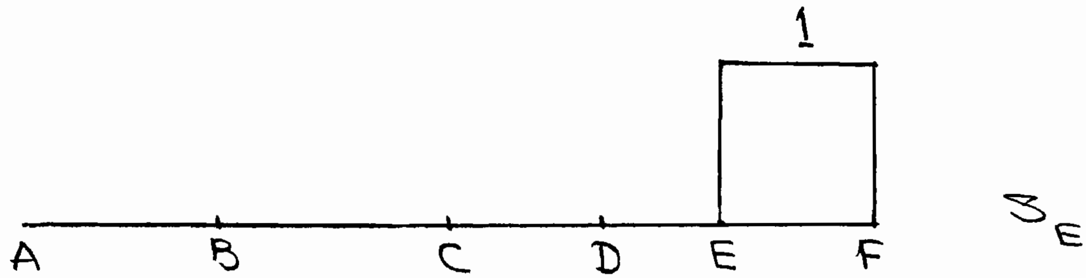

0.5
0.4
0
C
0
E
F
S_C
A
B
D
-0.6
-0.7

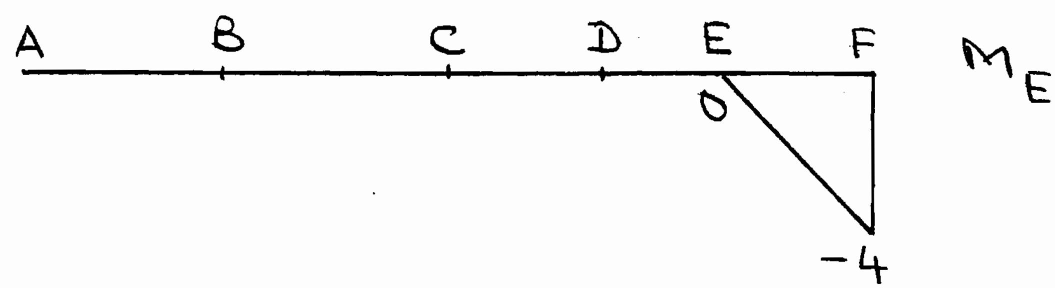

2.4
A
0
0
E
F
M_C
B
C
D
-2
-4.2


0.5
0
C
D
E
F
$S_{D,L}$
A
B
-1
-0.7

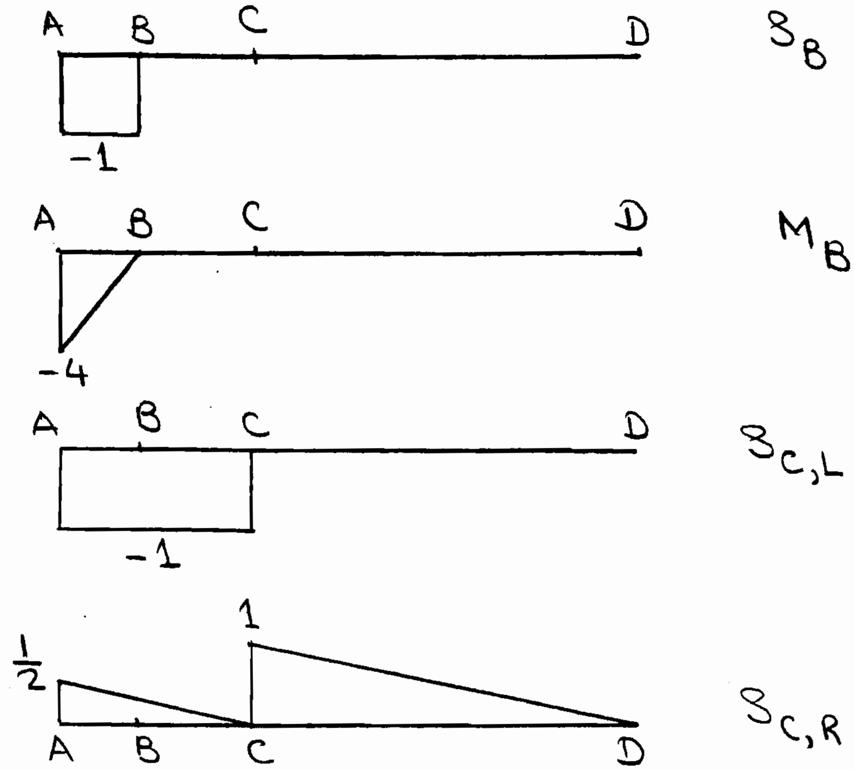

1
$S_{D,R}$
A
B
C
D
E
F

8.11

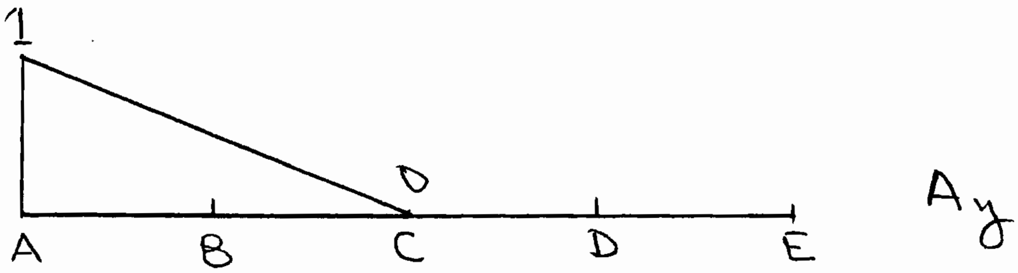


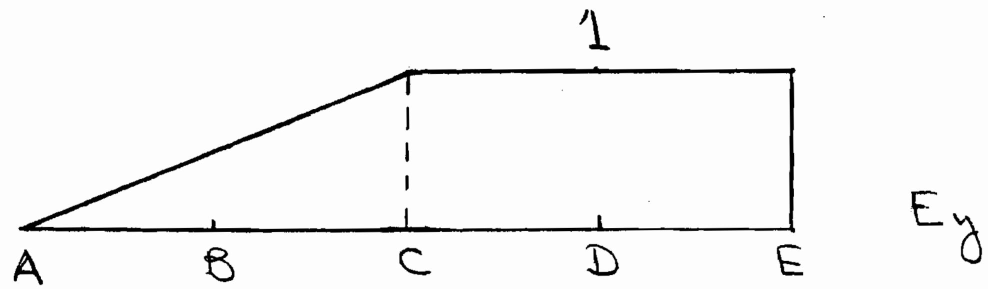


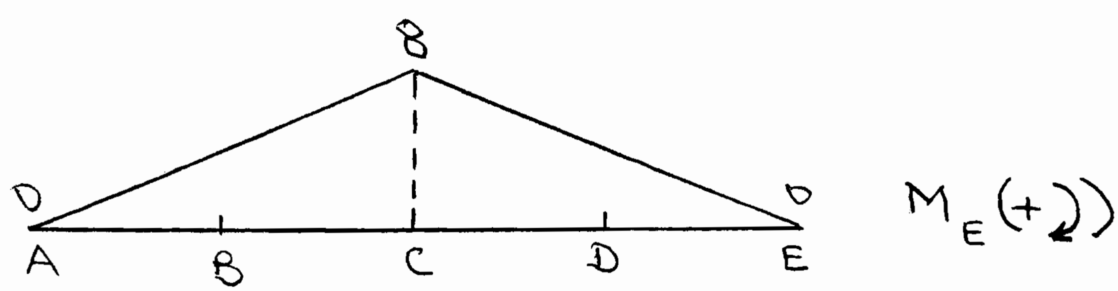
8.12



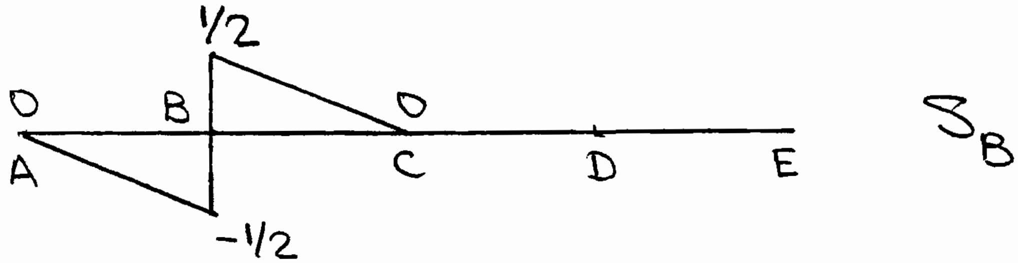
8.13

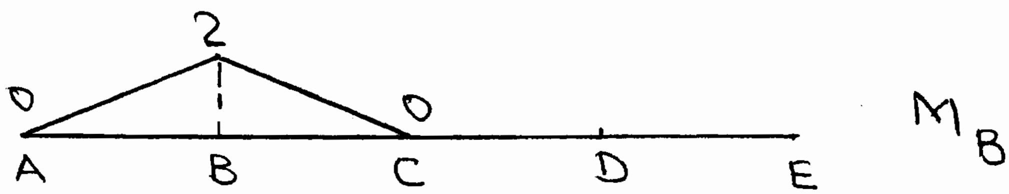




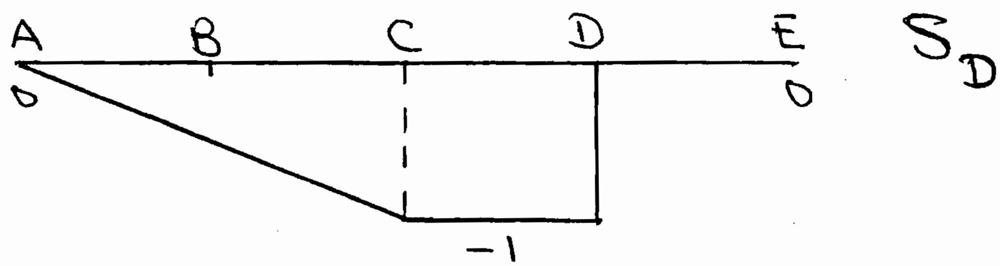


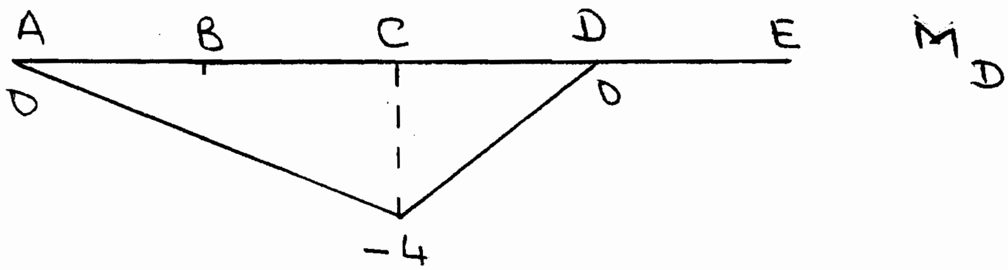
8.14



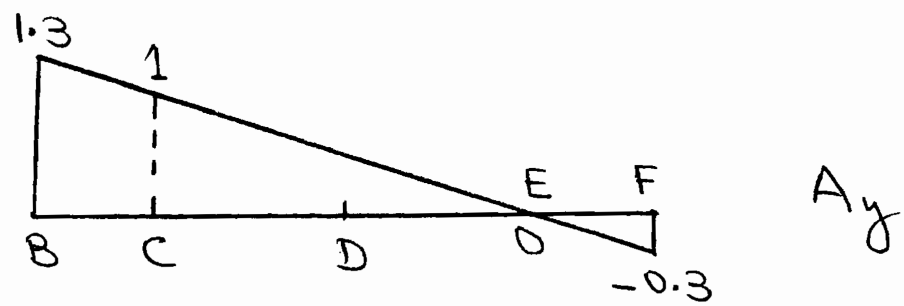


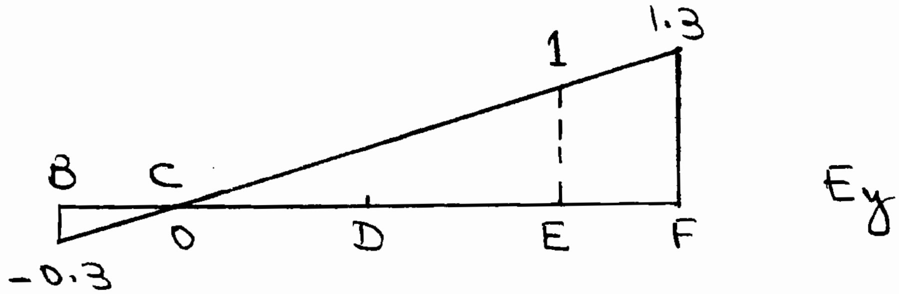
$\boxed{8.15}$

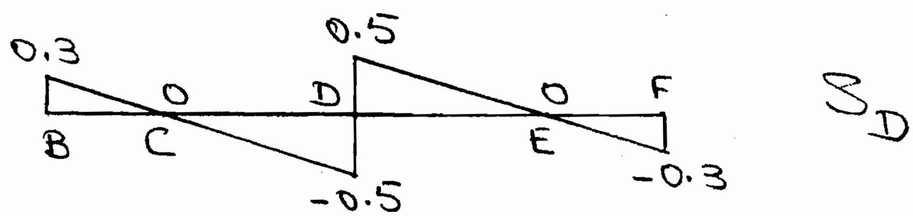


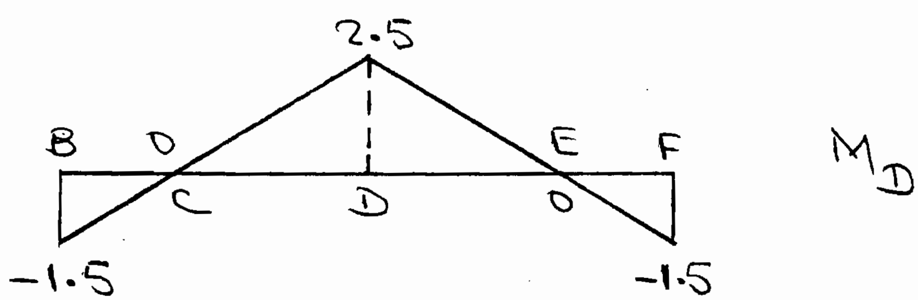


8.16

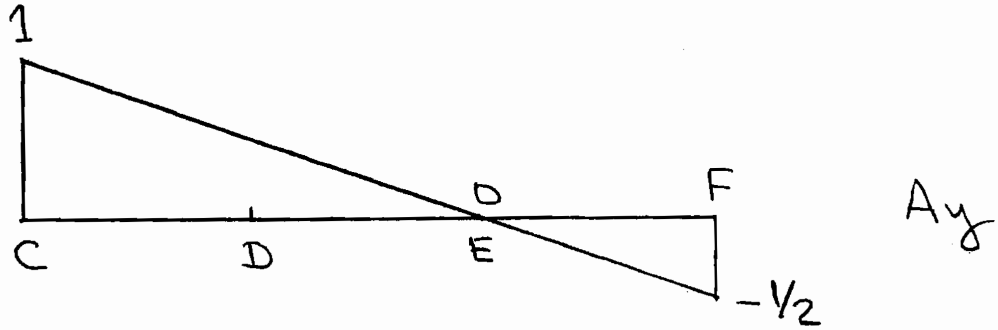

1.3
1
E
F
Ay
B
C
D
0
-0.3

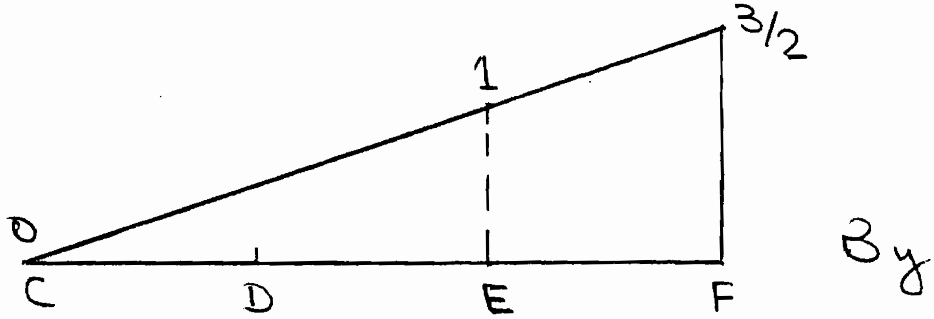

1.3
1
B
C
Ey
0
D
E
F
-0.3

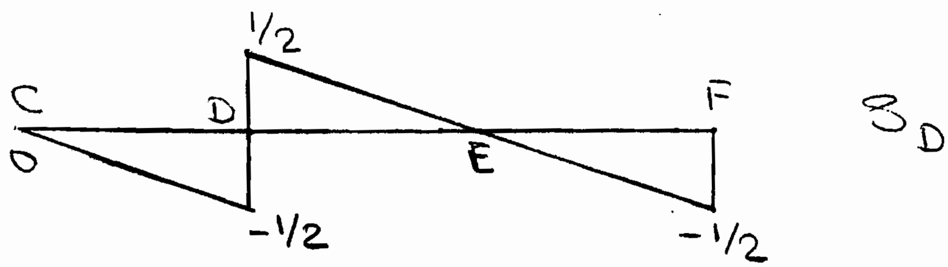

0.5
0.3
0
D
0
F
SD
B
C
E
-0.3
-0.5

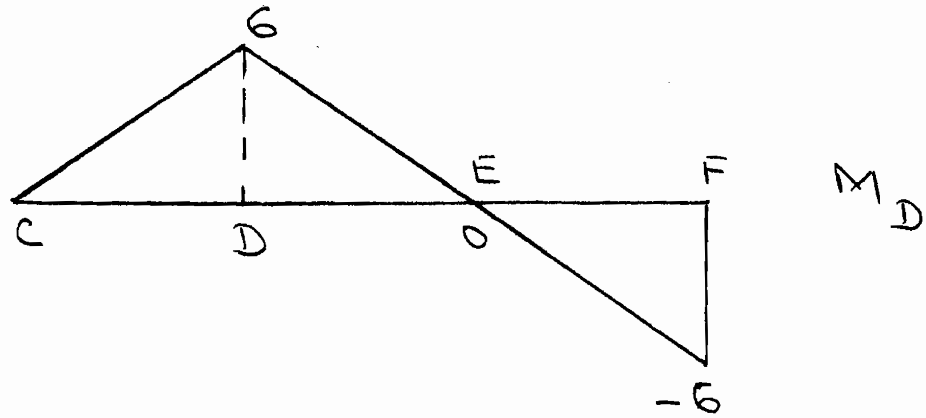

2.5
B
0
E
F
MD
C
D
0
-1.5
-1.5

$\boxed{8.17}$









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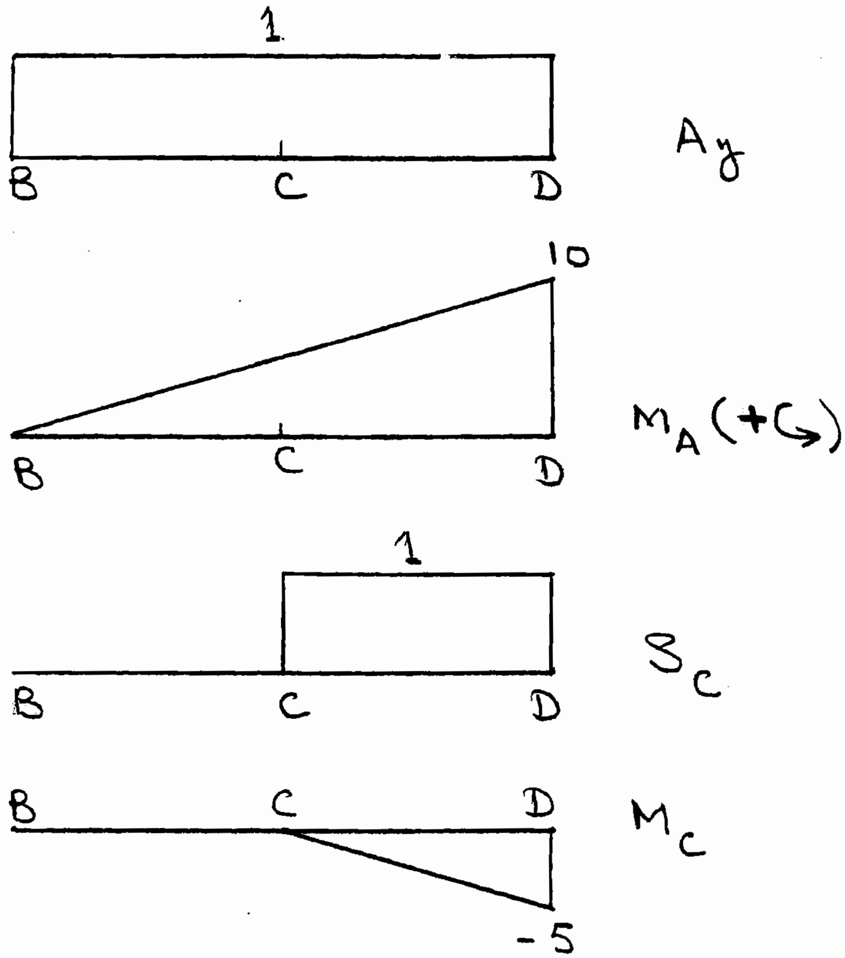

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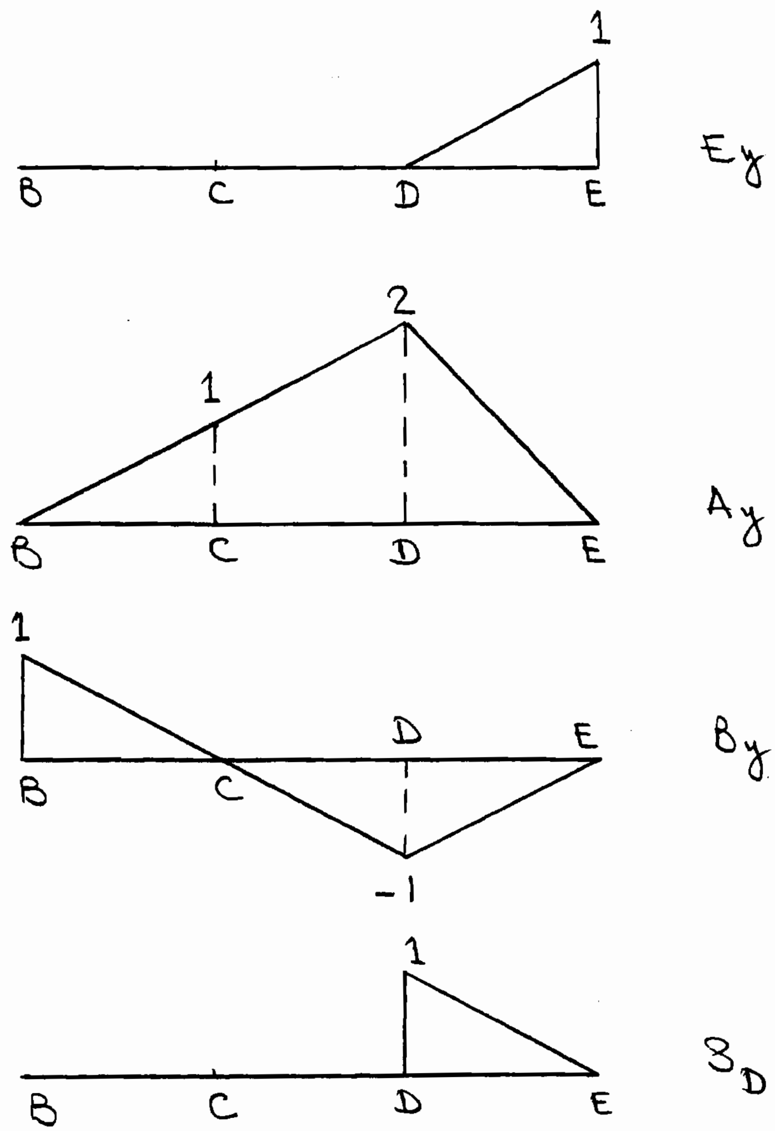

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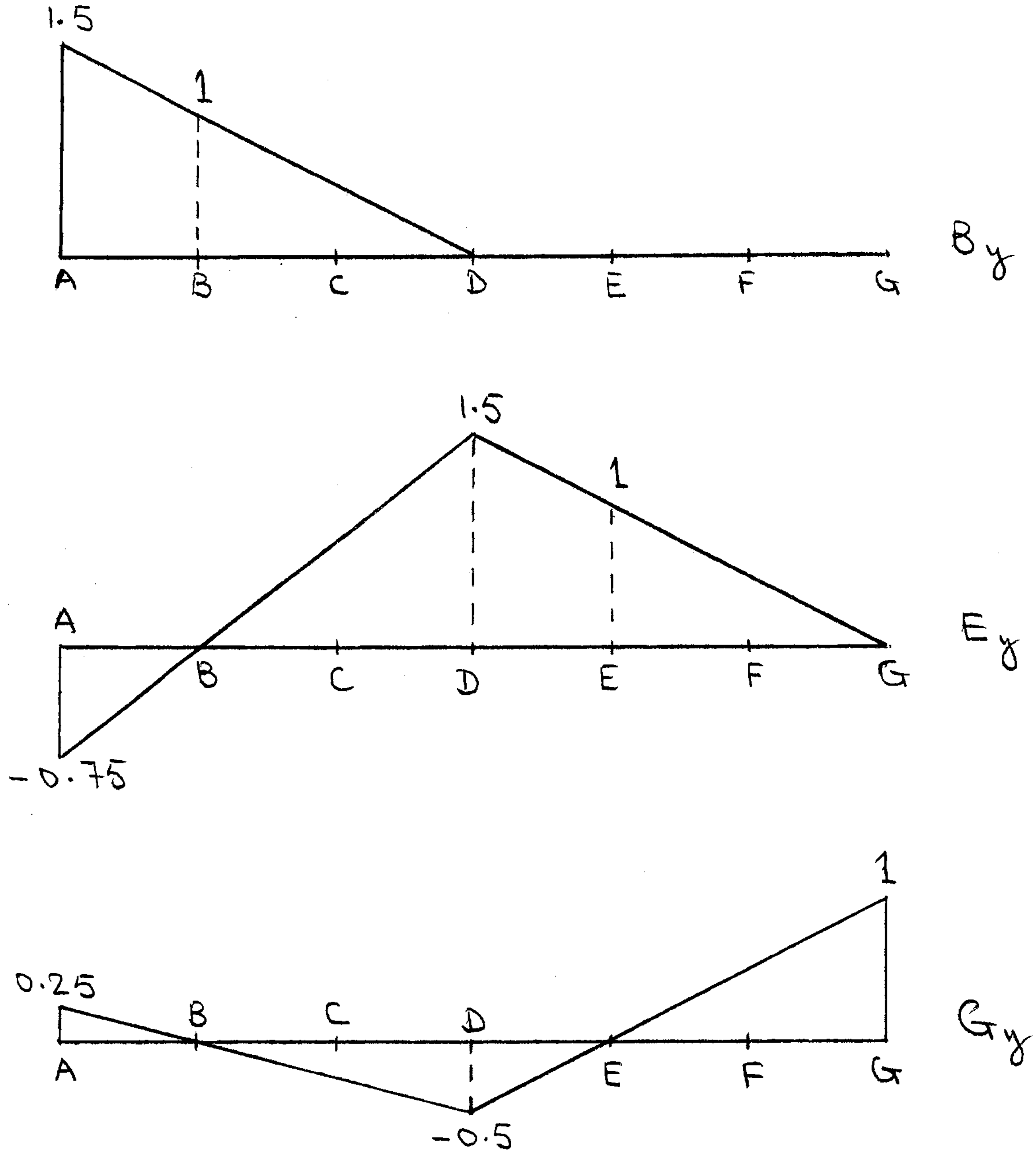
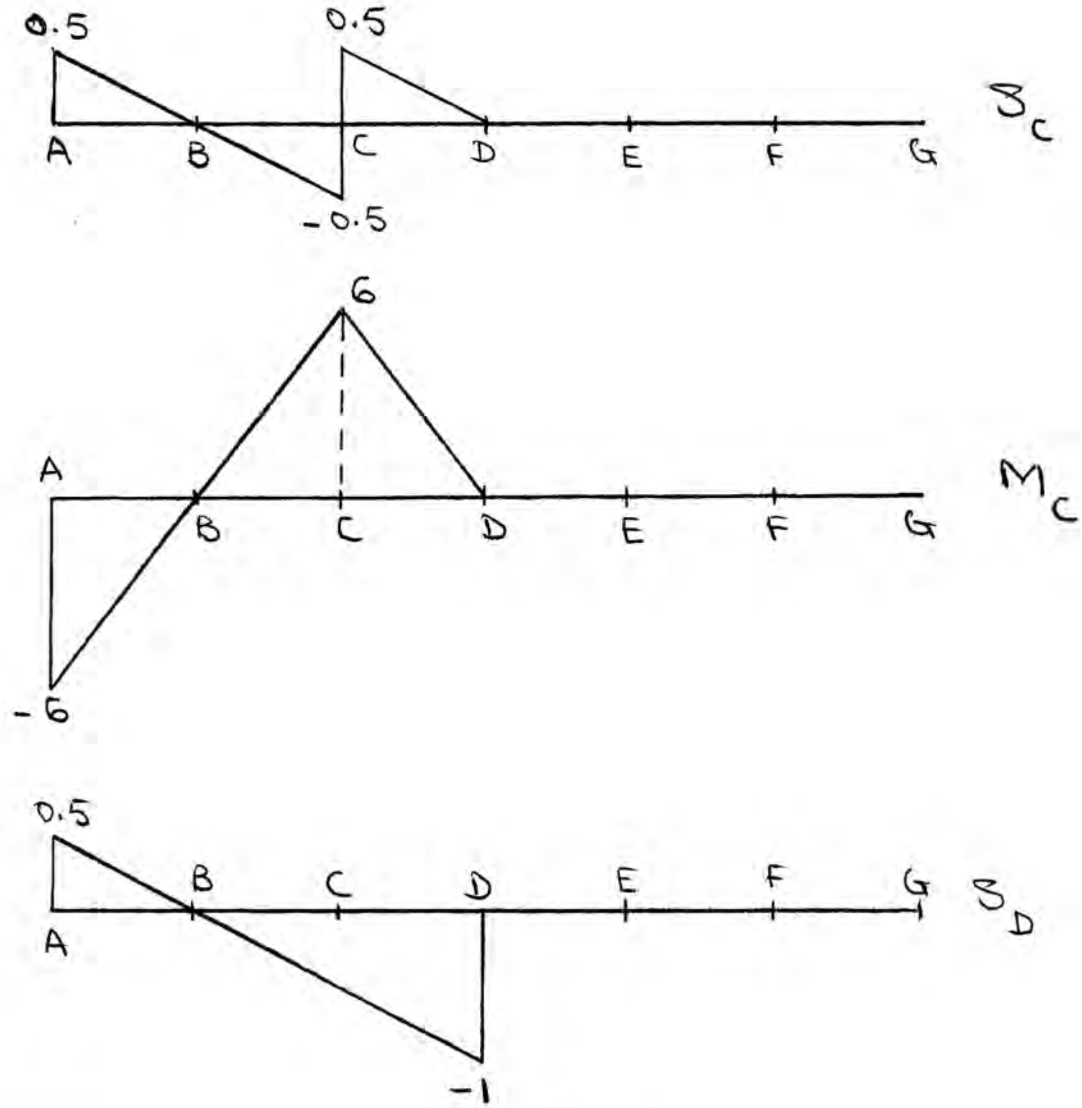
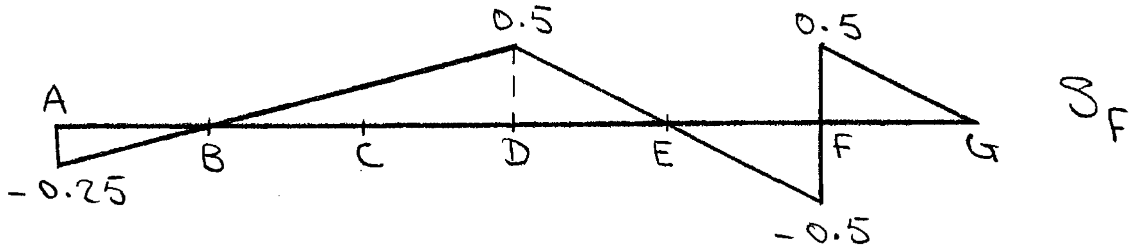
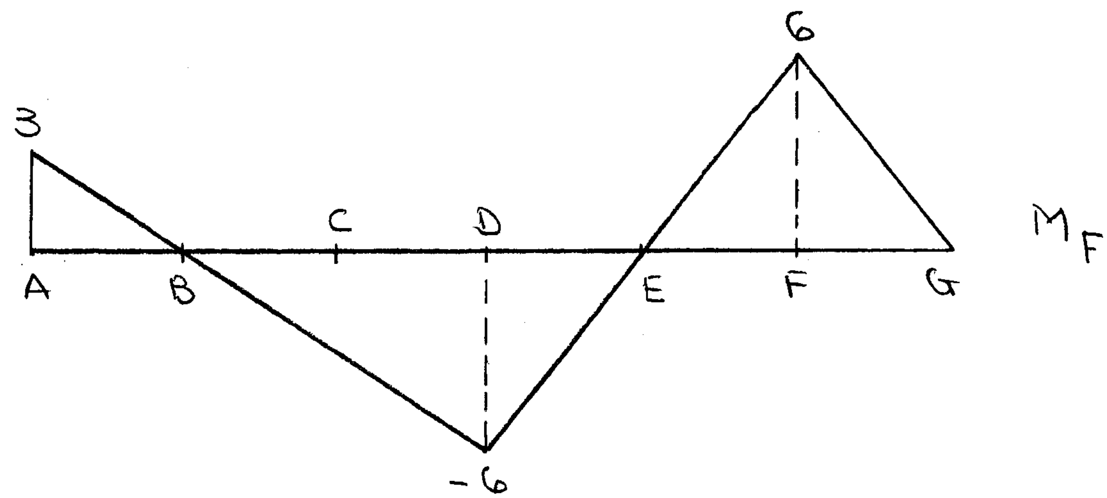


8.21

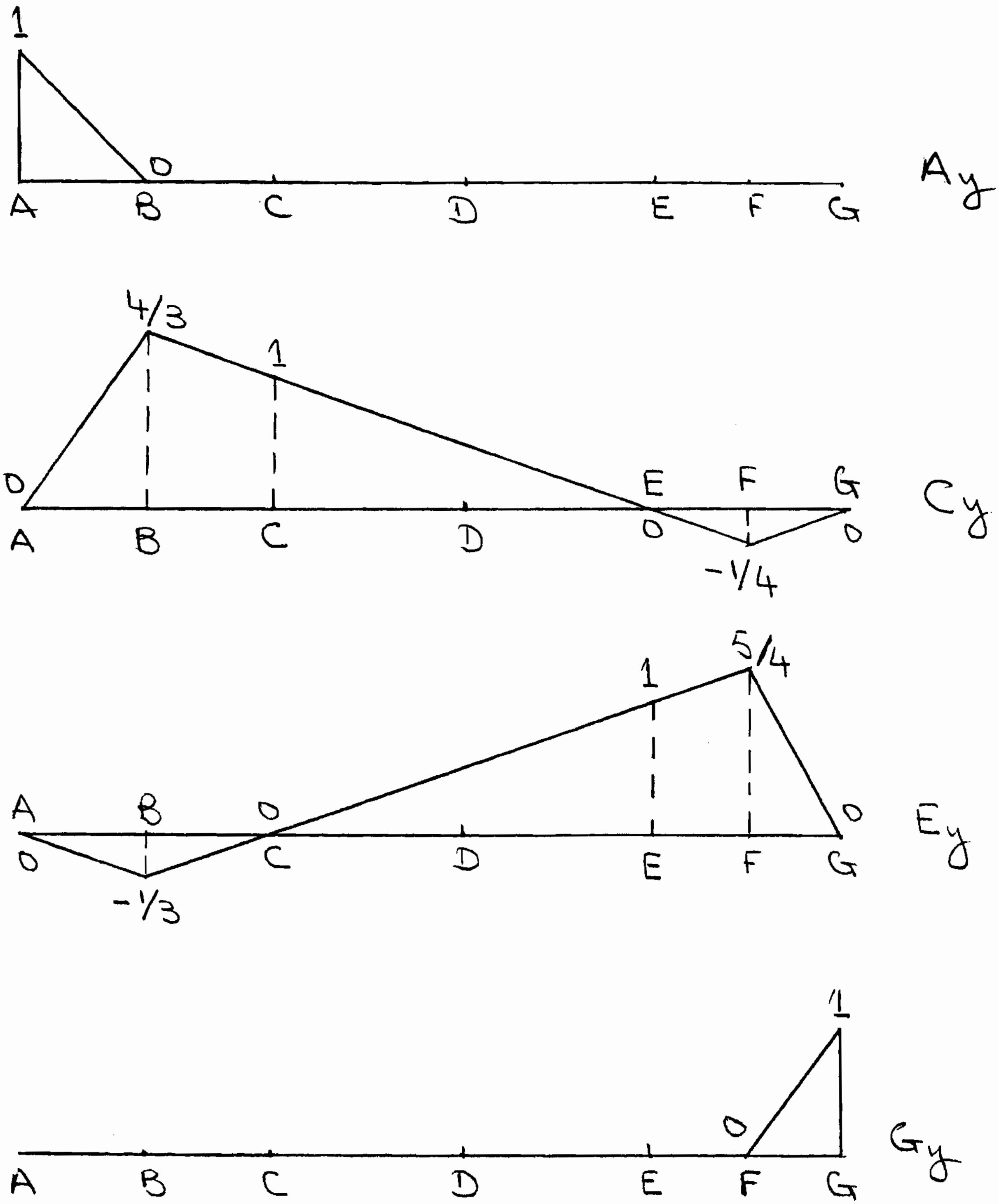


8.22

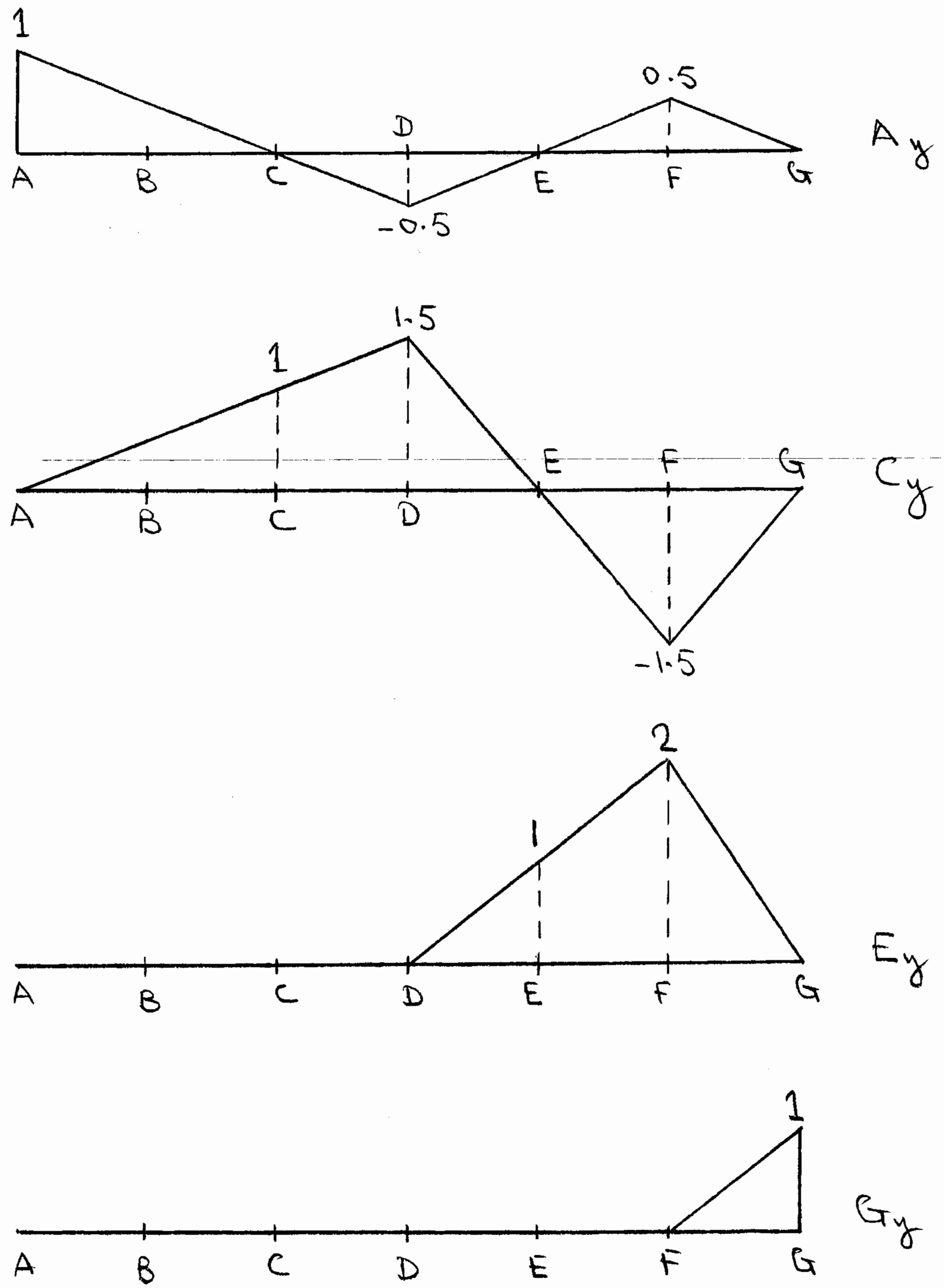




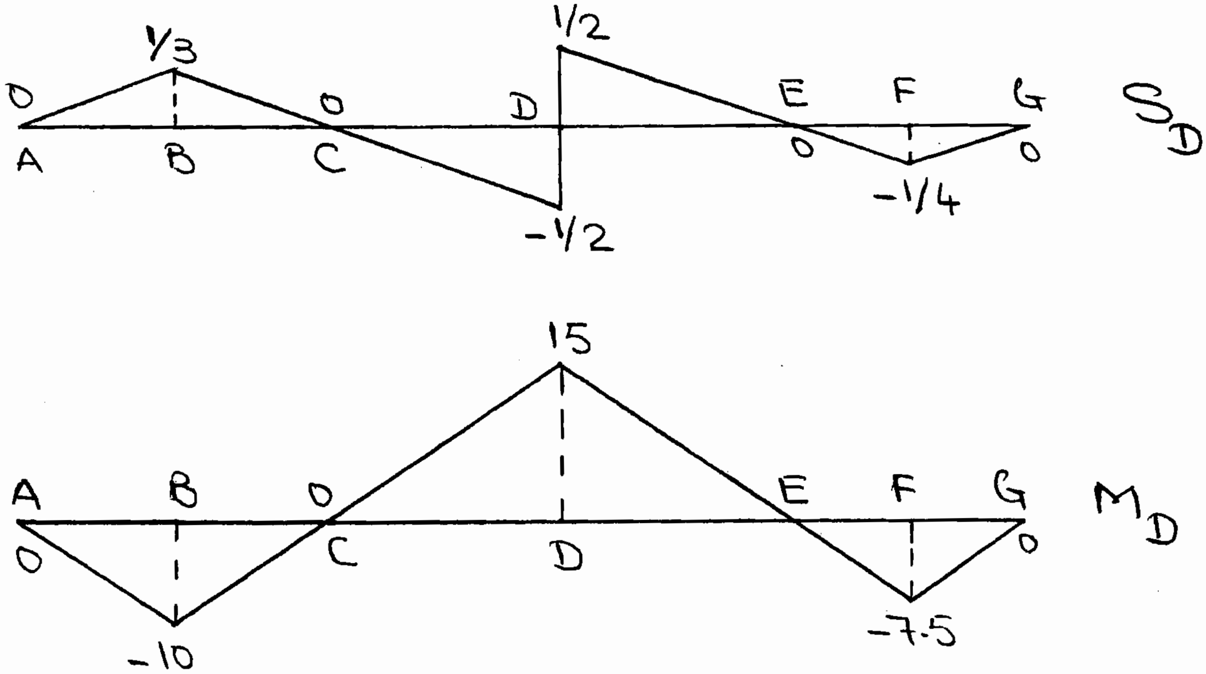
8.23


1
0
A B C D E F G
A_y
4/3
1
0
E F G
0
0
A B C D
-1/4
C_y
1
5/4
A B
0
0
0
C D E F G
-1/3
E_y
1
0
A B C D E F G
G_y

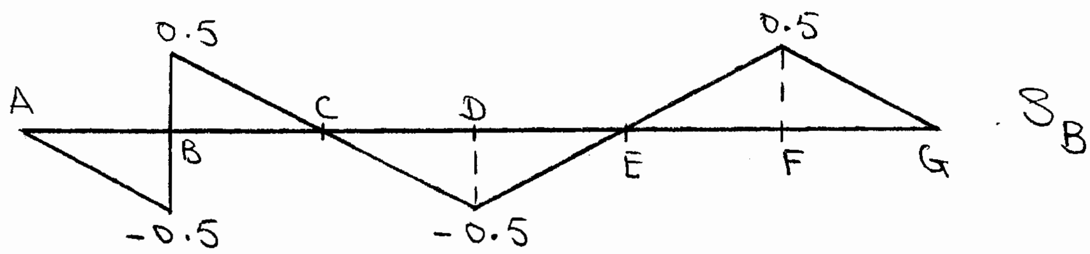
8.24

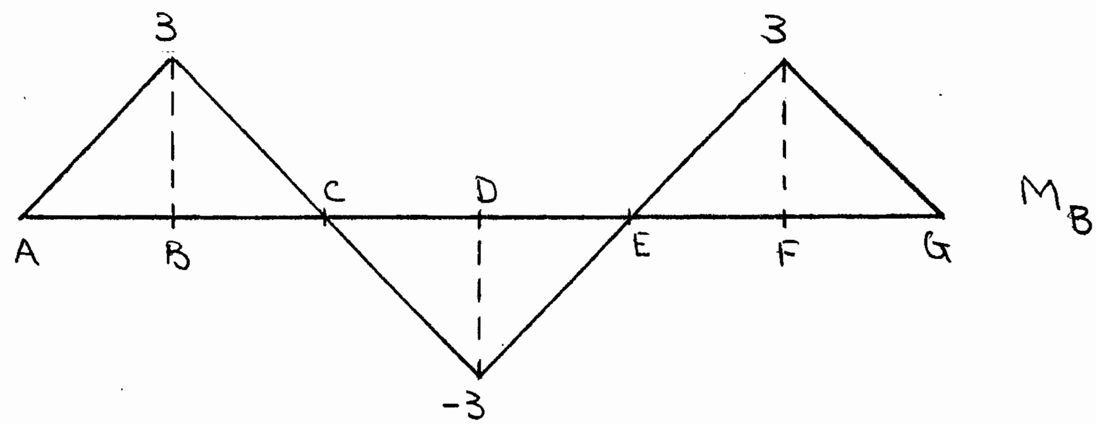


8.25

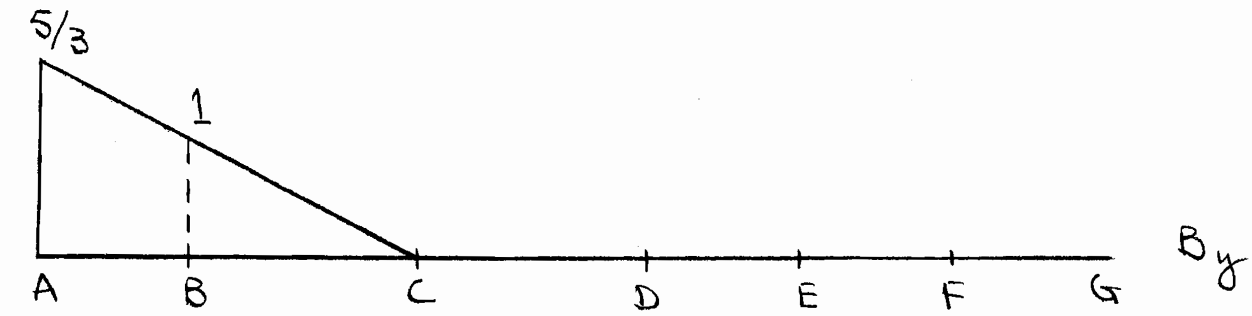


8.26

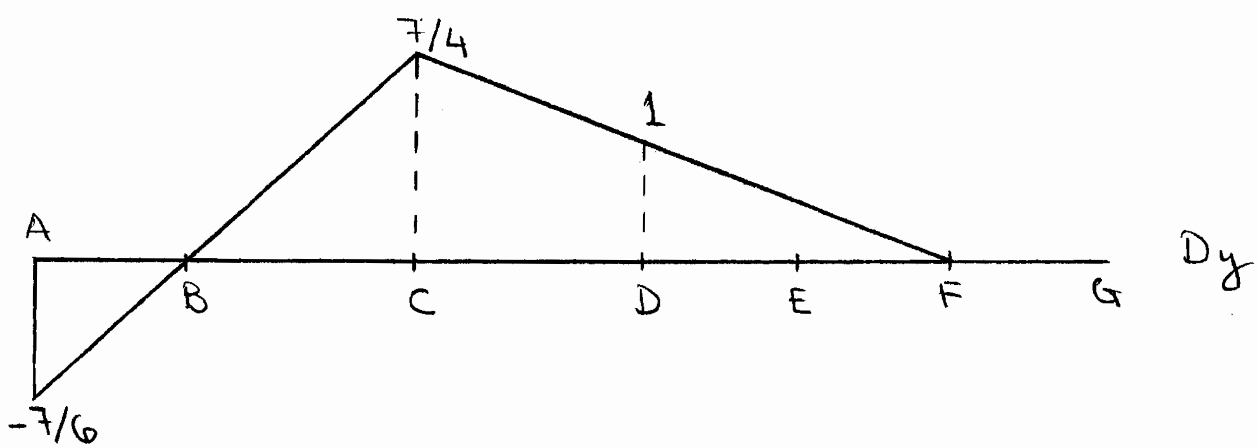




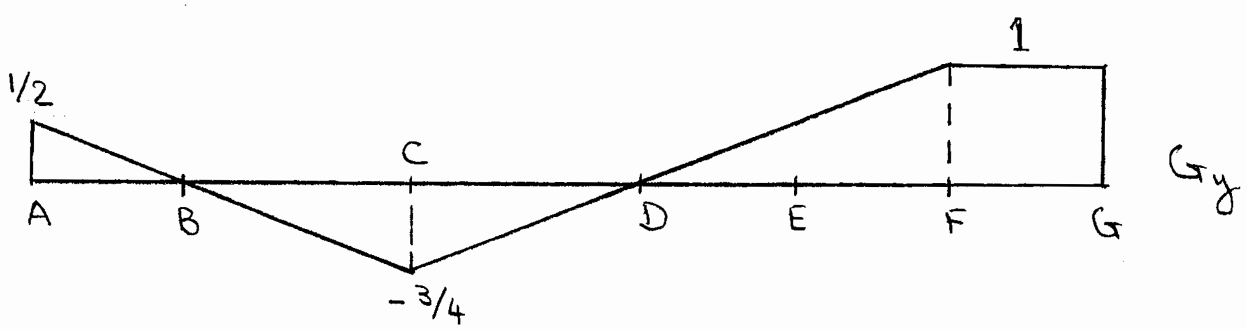
8.27



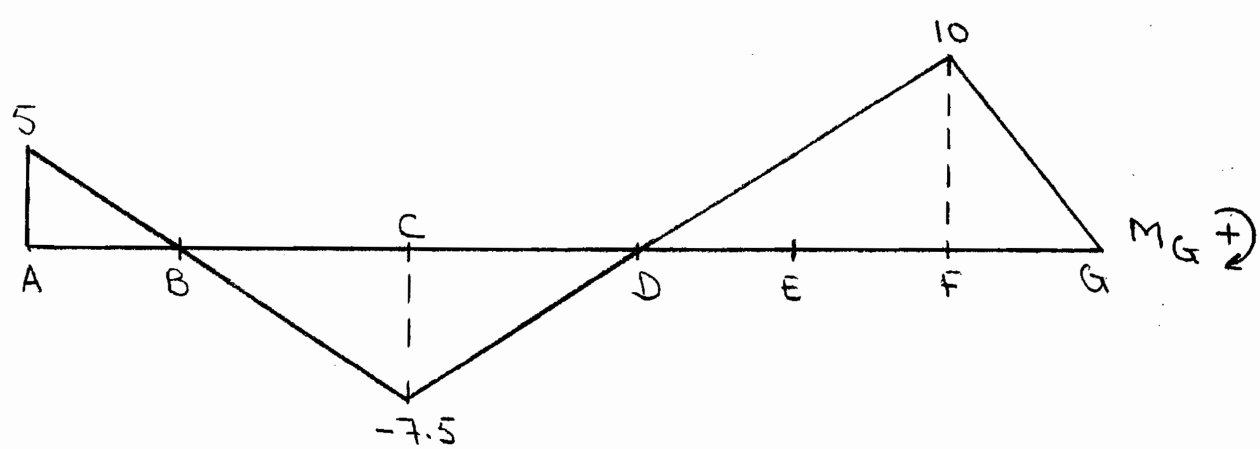
5/3
1
A
B
C
D
E
F
G
By



7/4
1
A
B
C
D
E
F
G
Dy
-7/6

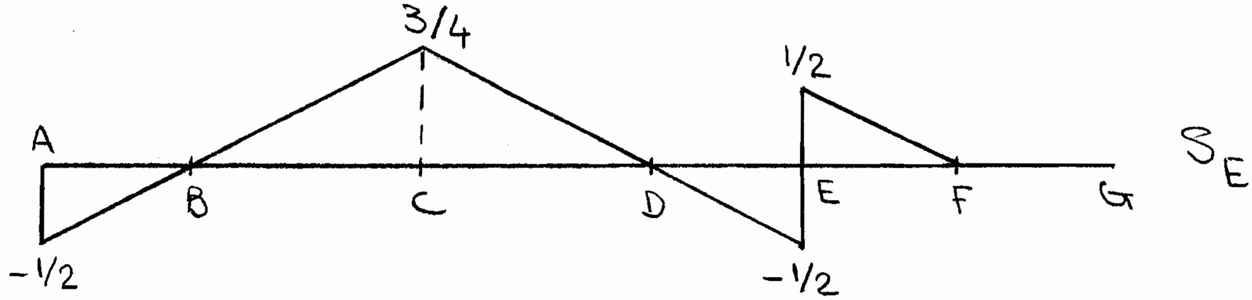


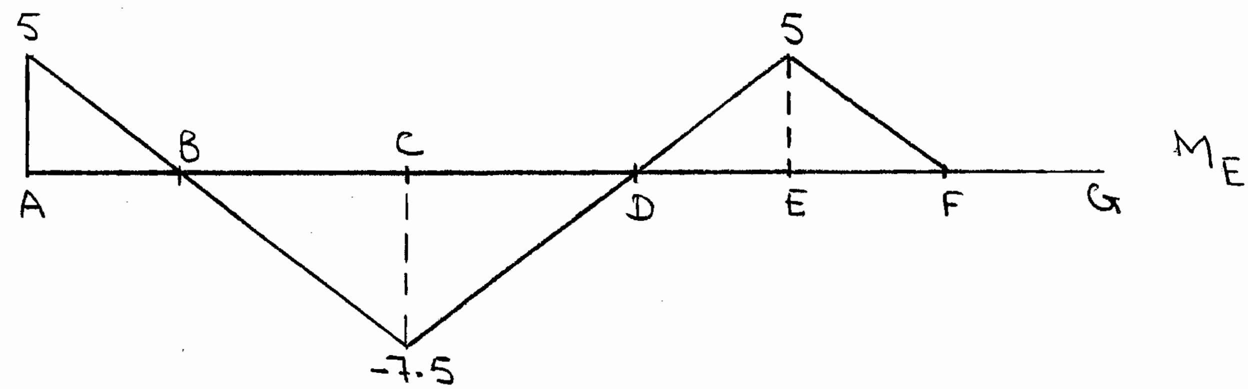
1
1/2
C
A
B
D
E
F
G
Gy
-3/4



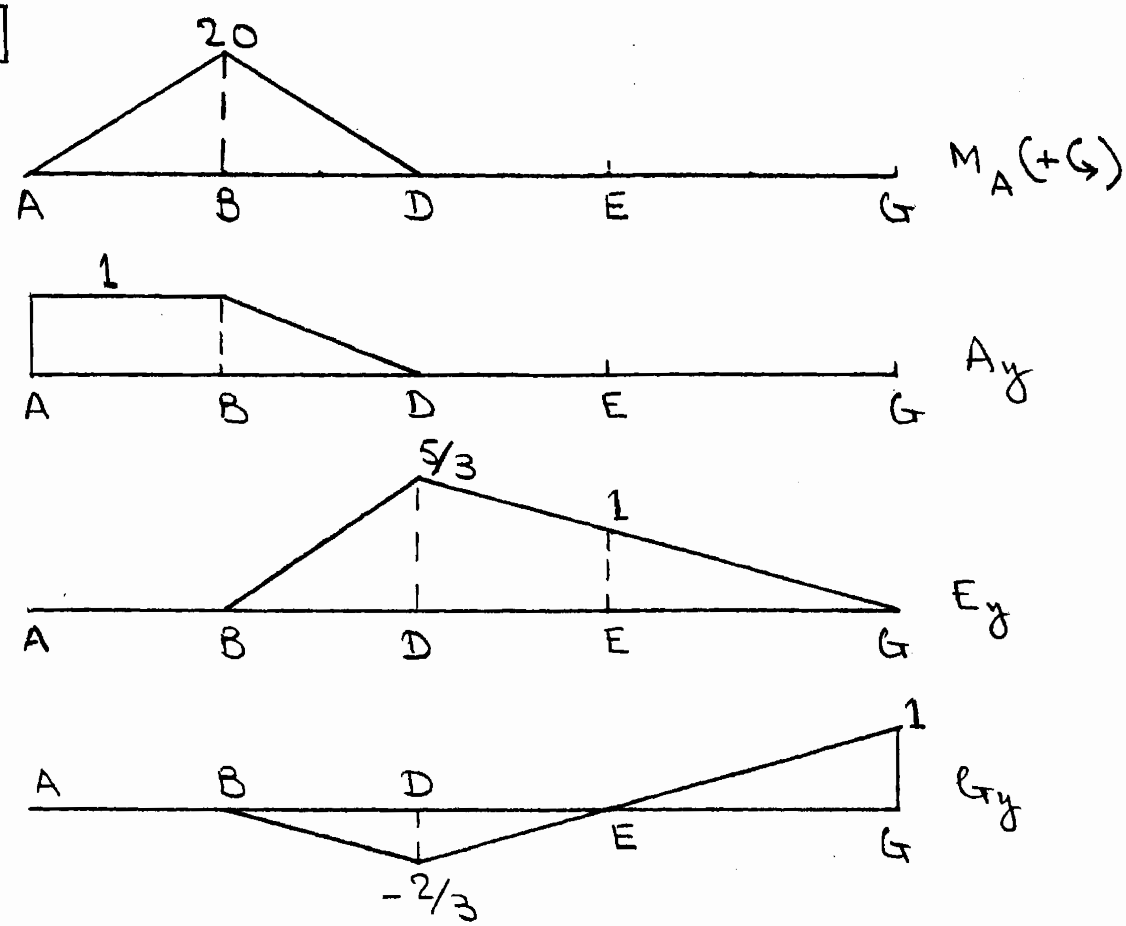
10
5
C
A
B
D
E
F
G
MG +
-7.5

8.28

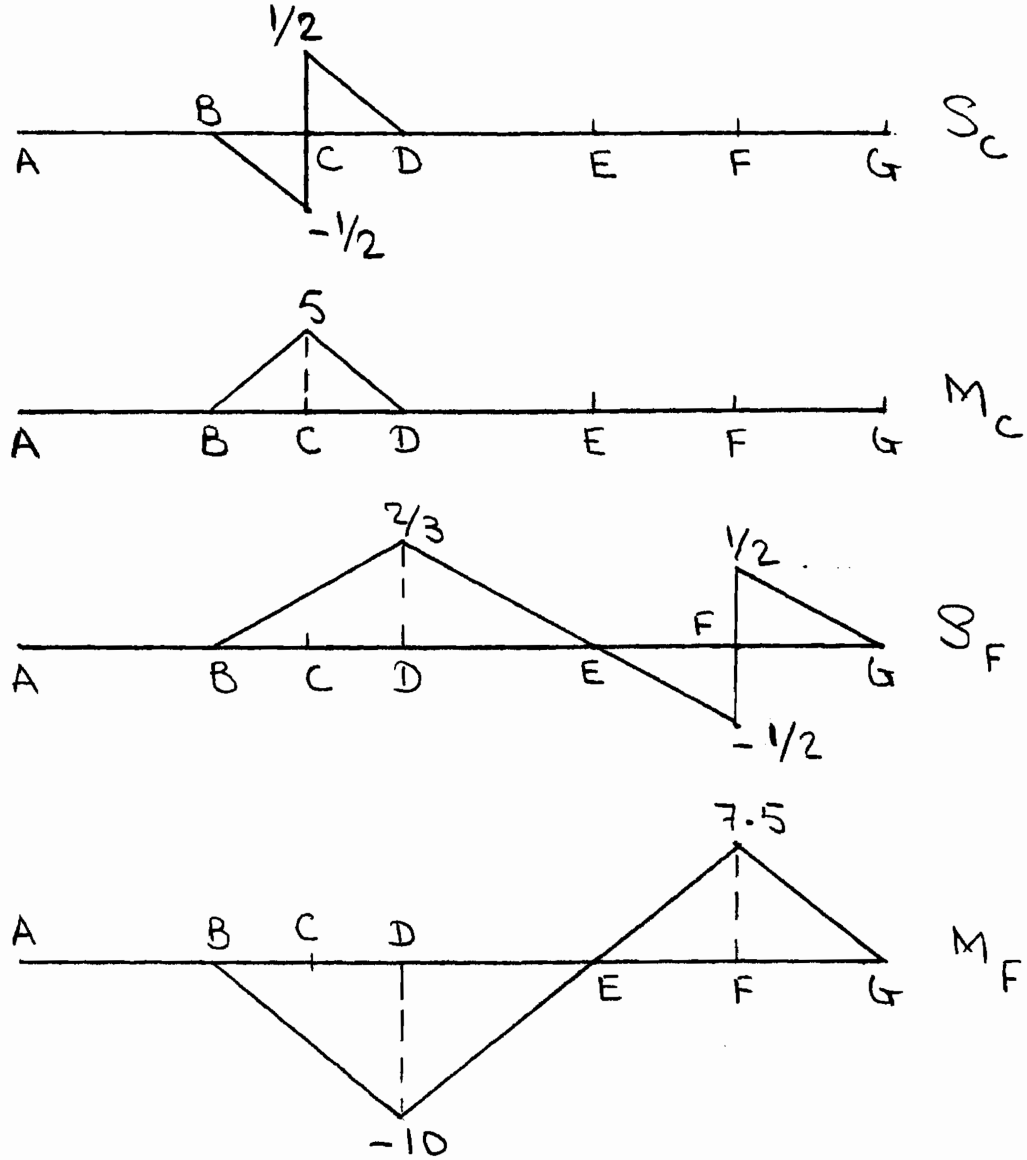




$\boxed{8.29}$

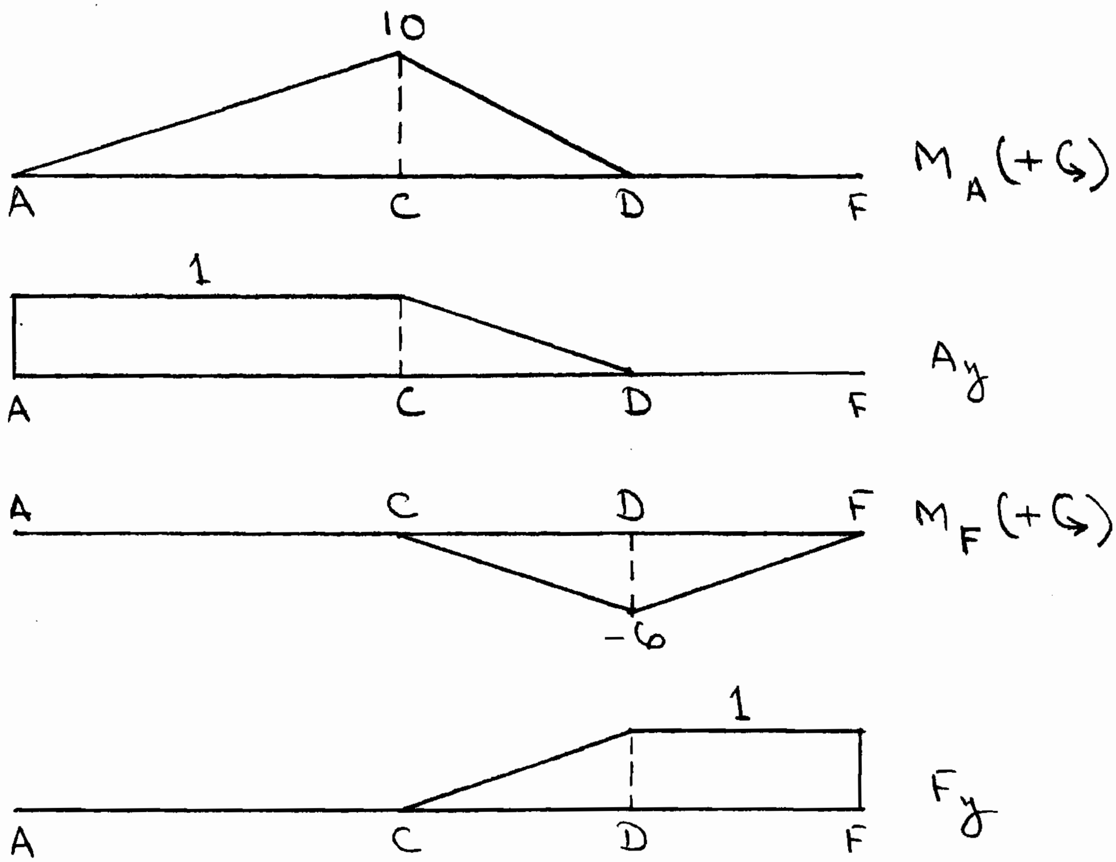


8.30

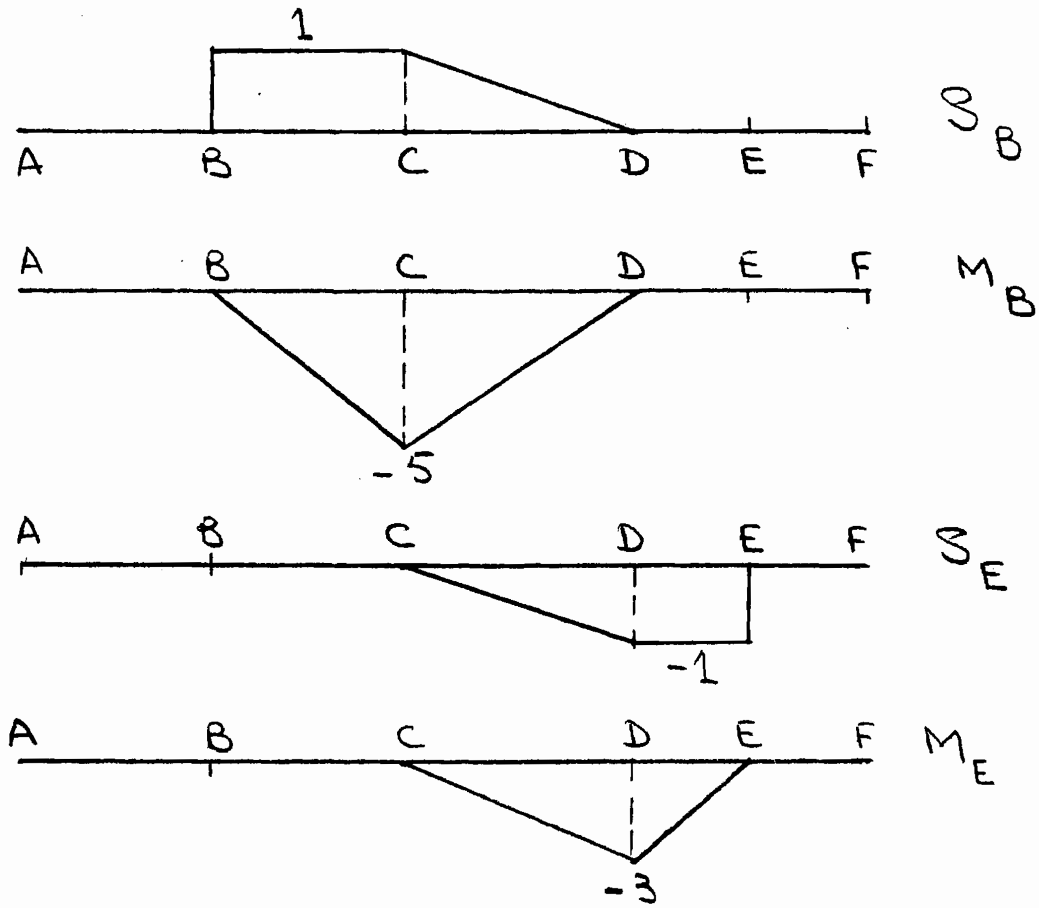


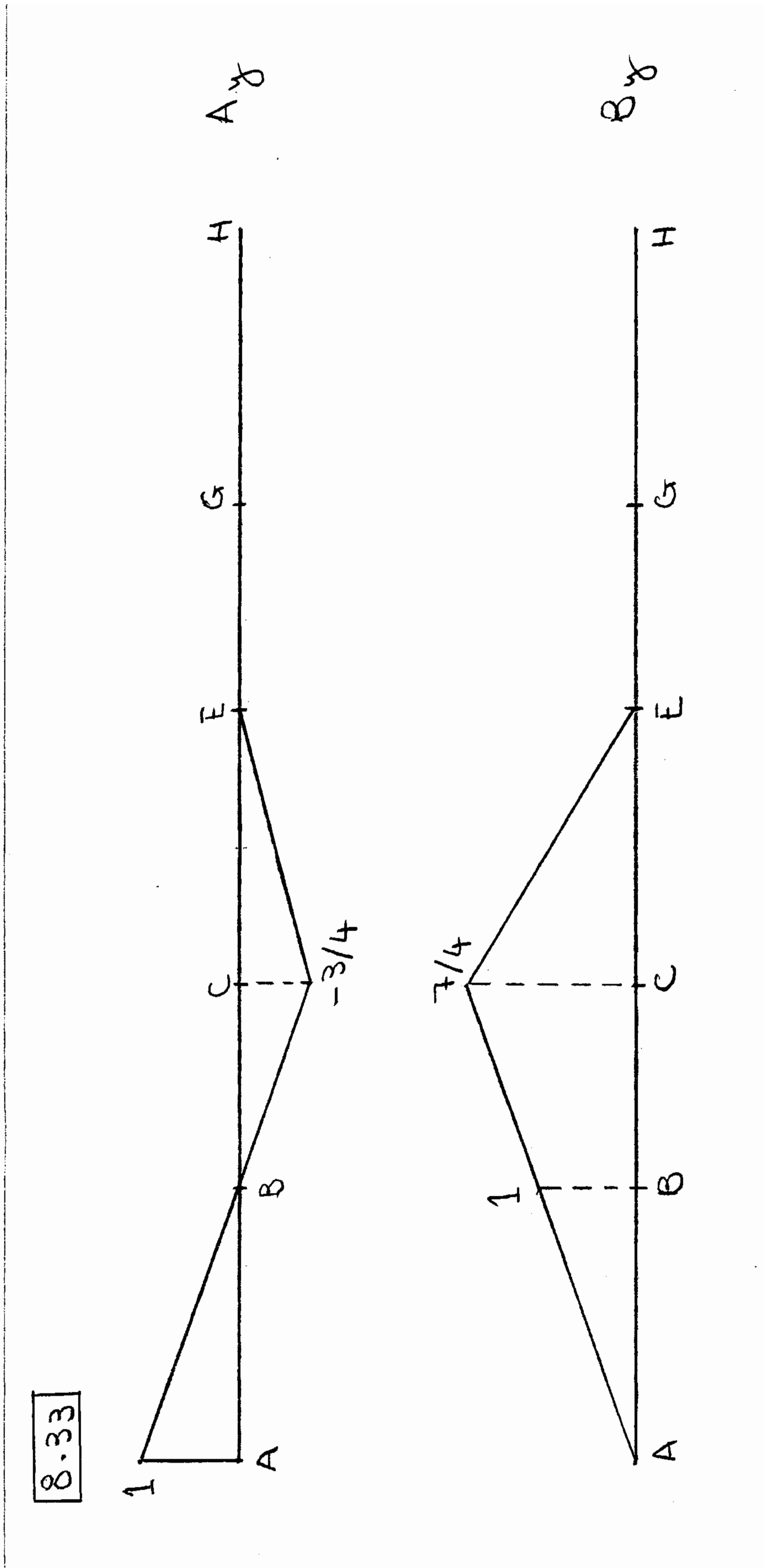
1/2
B
A
C
D
E
F
G
S_C
-1/2
5
M_C
A
B
C
D
E
F
G
2/3
1/2
F
S_F
A
B
C
D
E
G
-1/2
7.5
A
B
C
D
M_F
E
F
G
-10

8.31

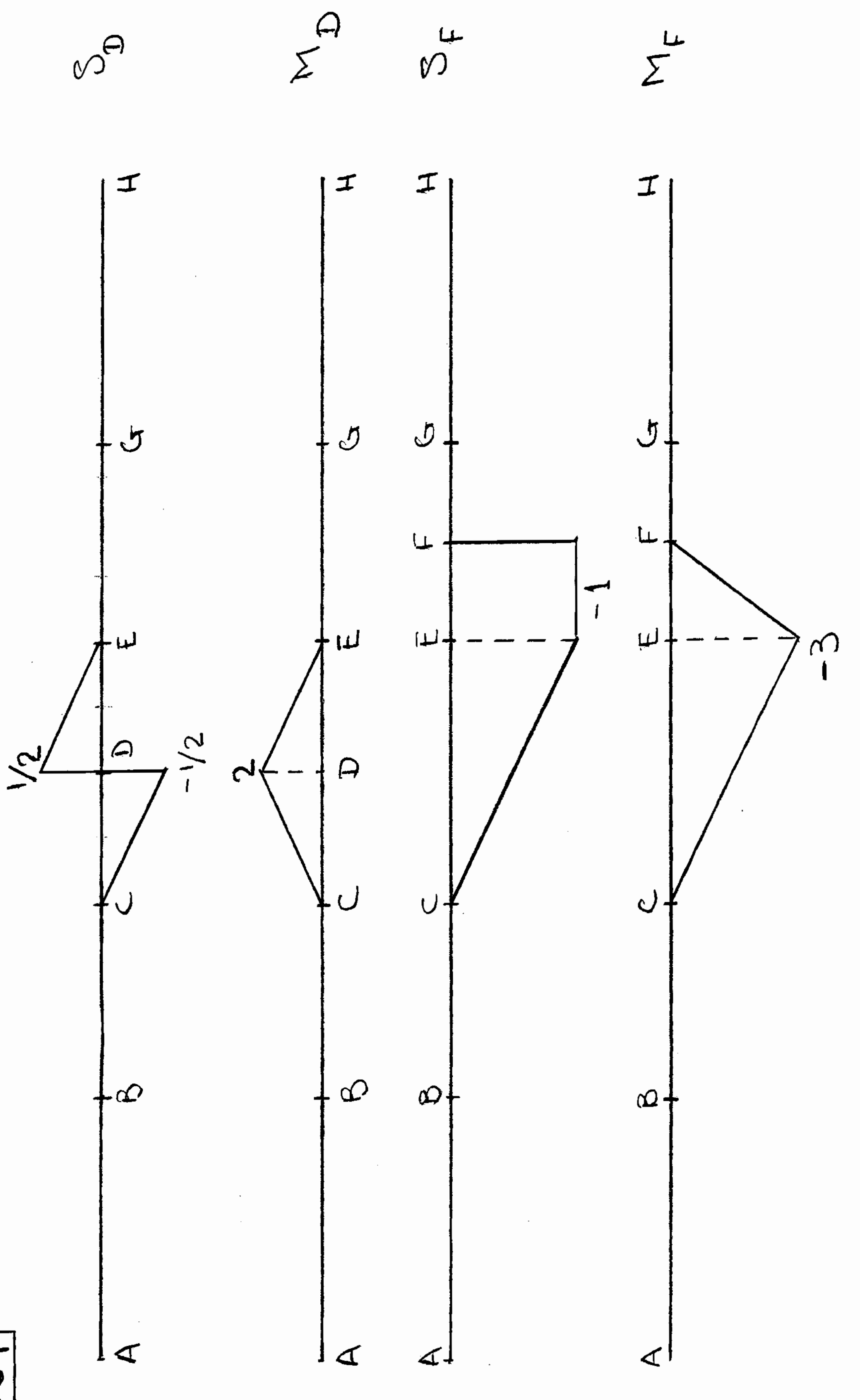


8.32




8.33
A_y
B_y
A
B
C
E
G
H
1
-3/4
7/4
1

8.34



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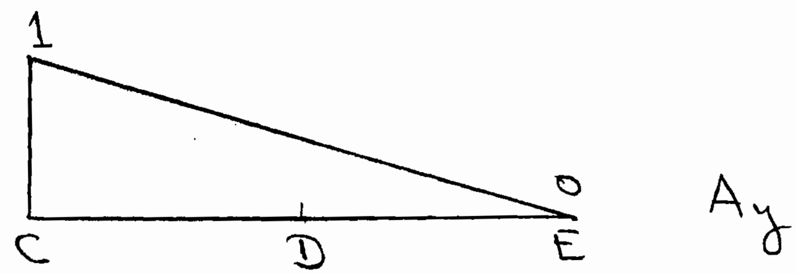
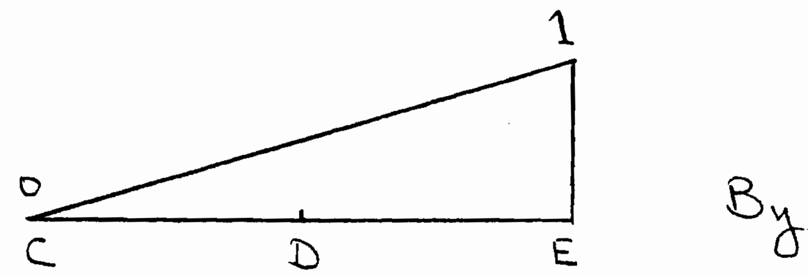
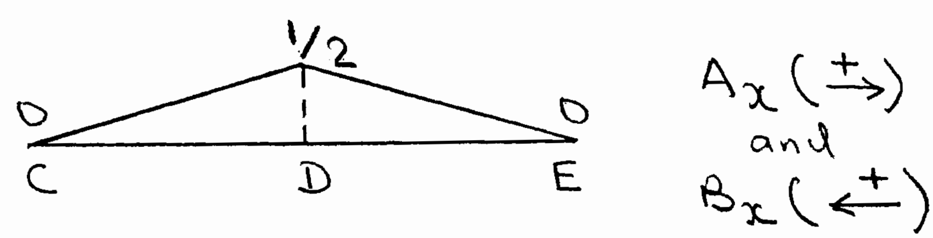
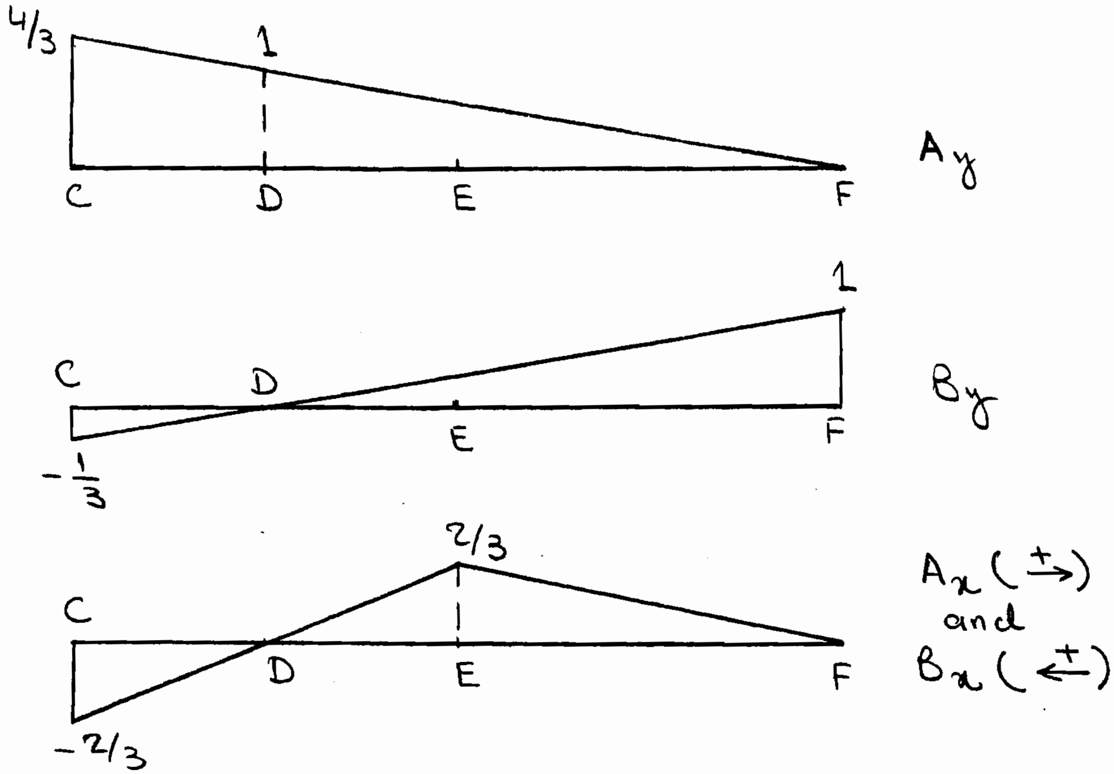


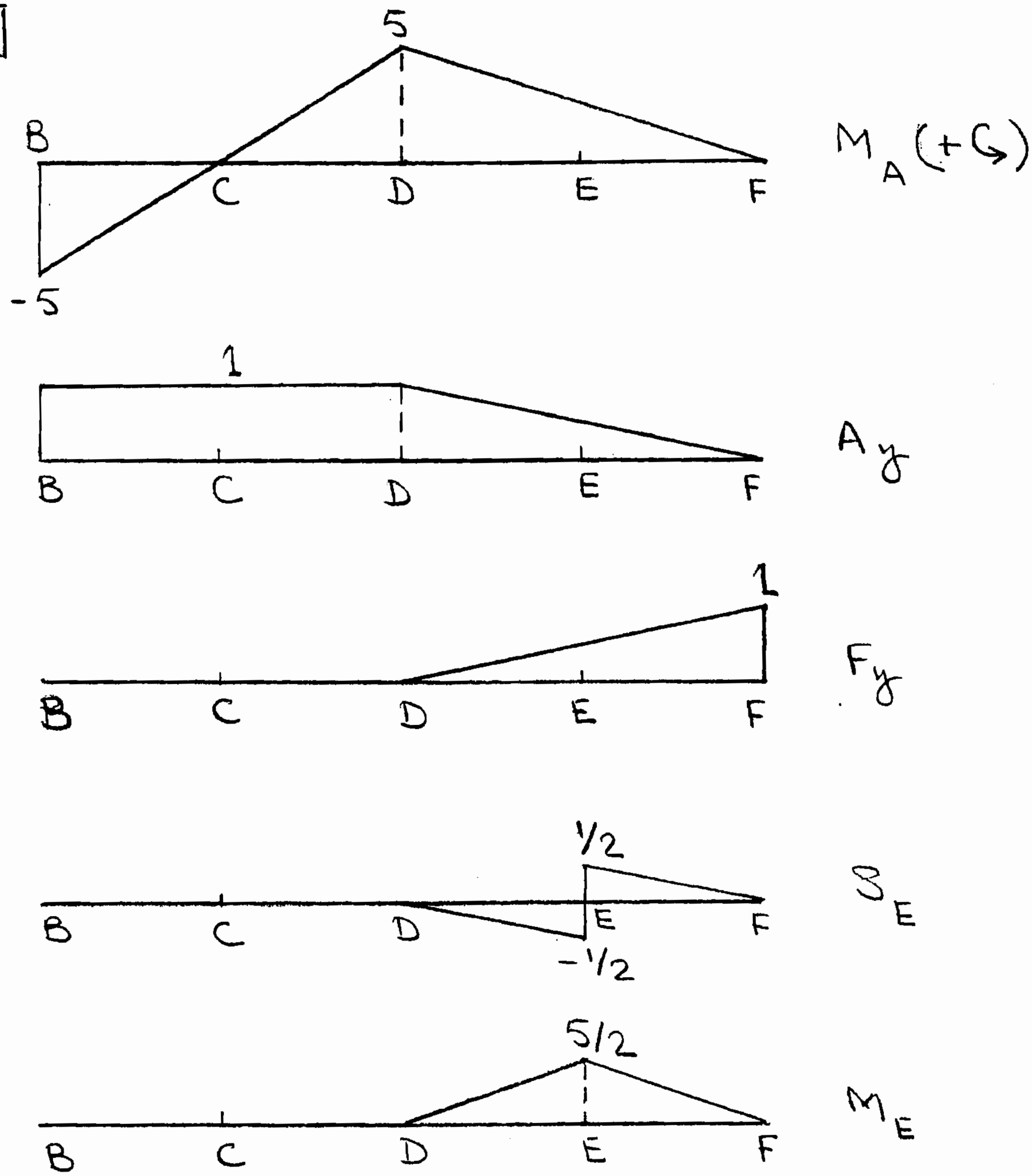




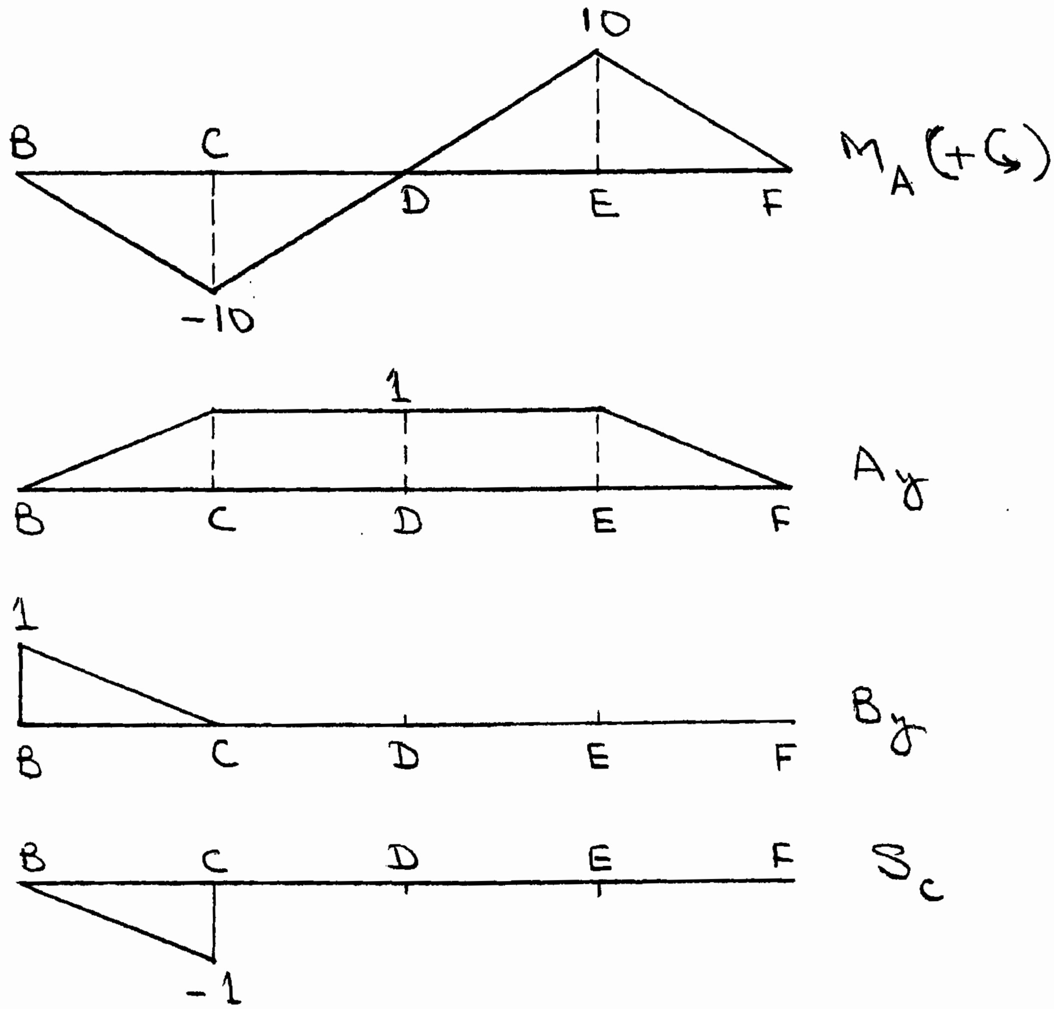
8.36



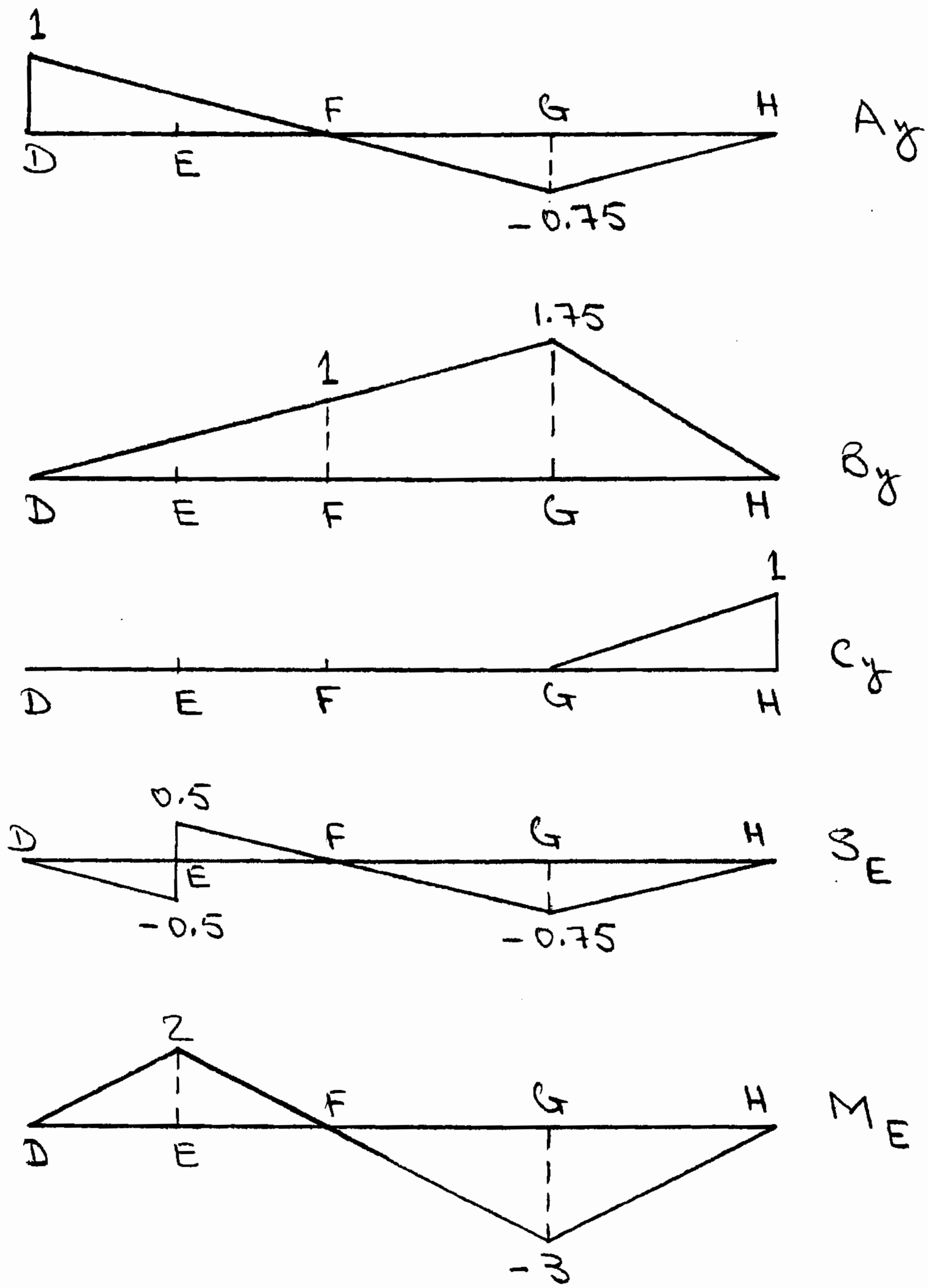
8.37



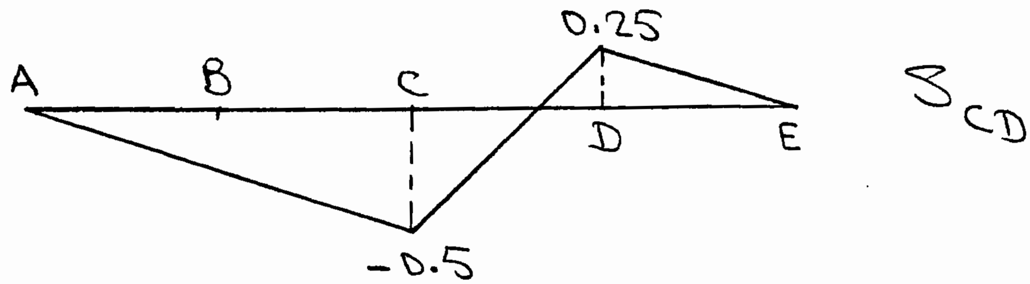
8.38

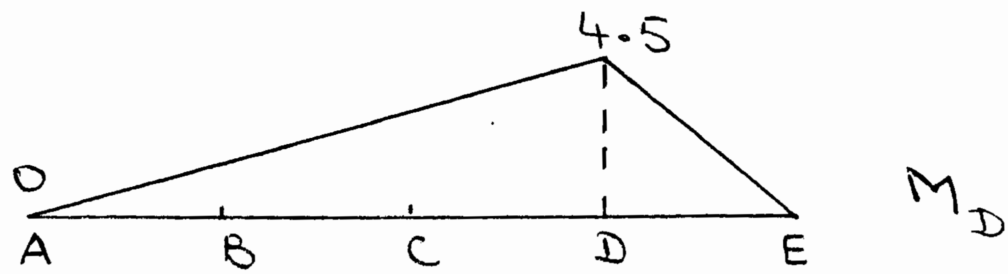


8.39

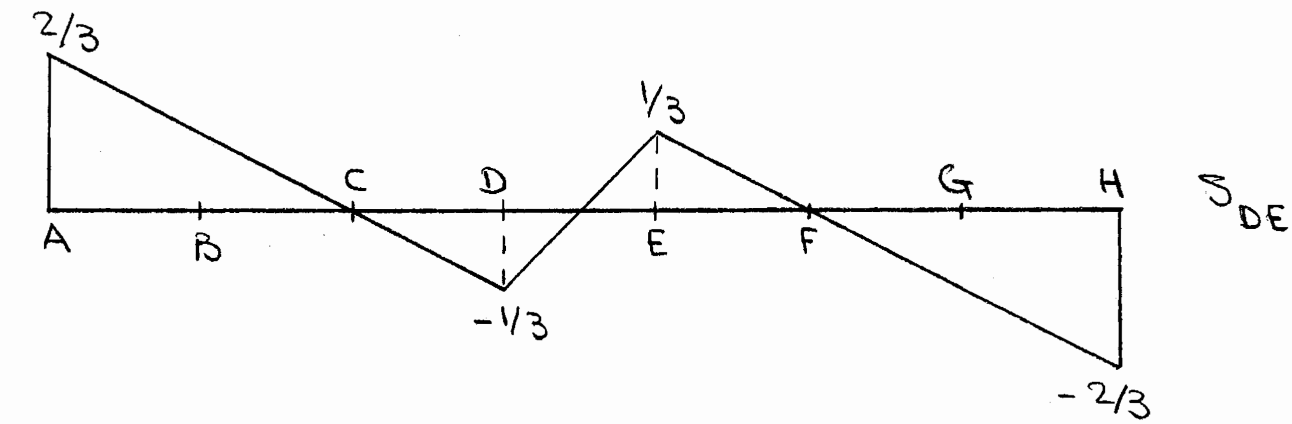

1
D
E
F
G
H
-0.75
A_y
1.75
1
D
E
F
G
H
B_y
1
D
E
F
G
H
C_y
0.5
D
E
F
G
H
-0.5
-0.75
S_E
2
D
E
F
G
H
-3
M_E

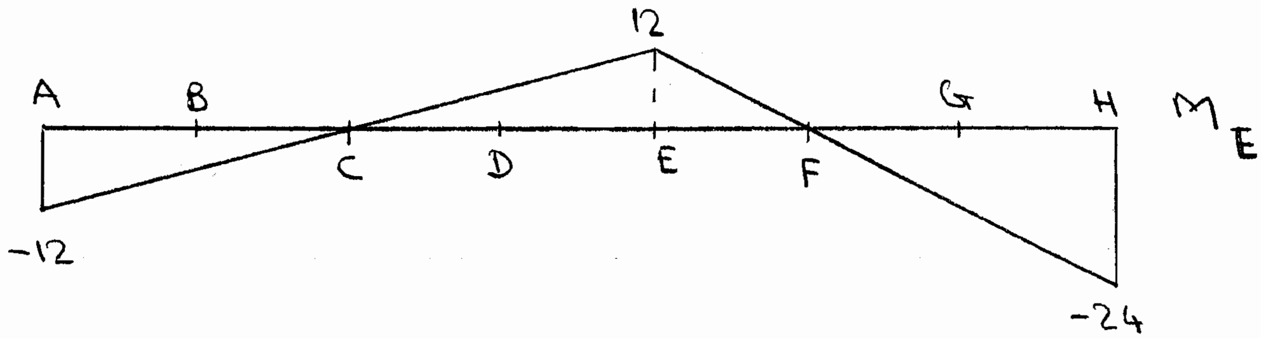
8.40



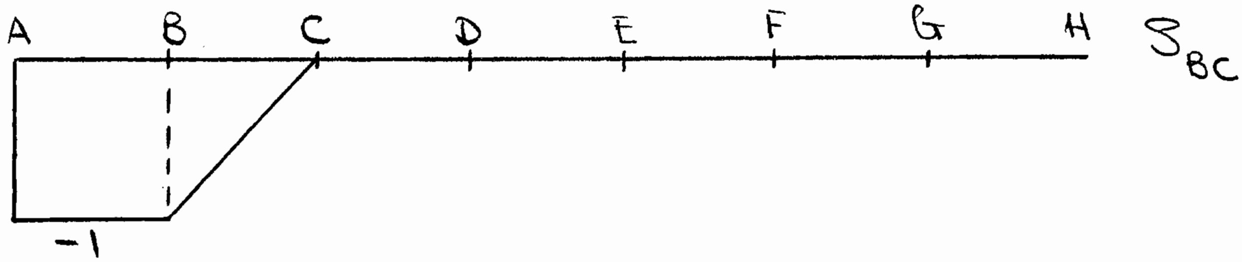


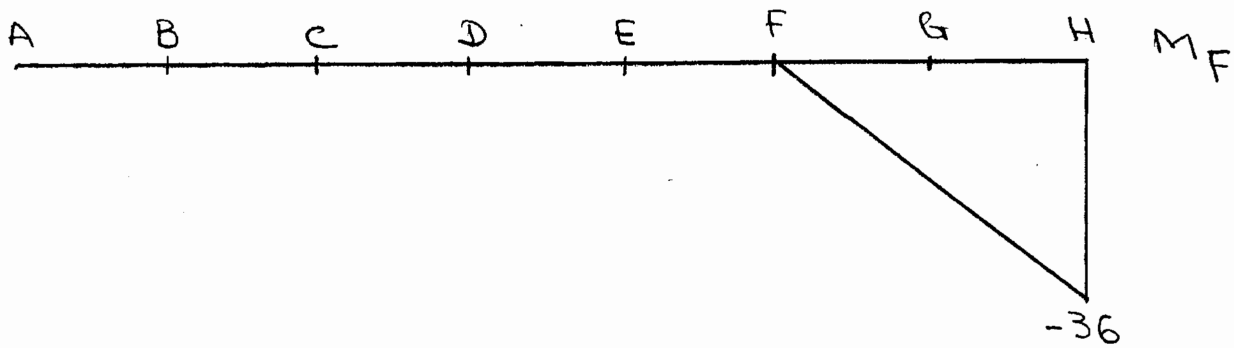
8.41



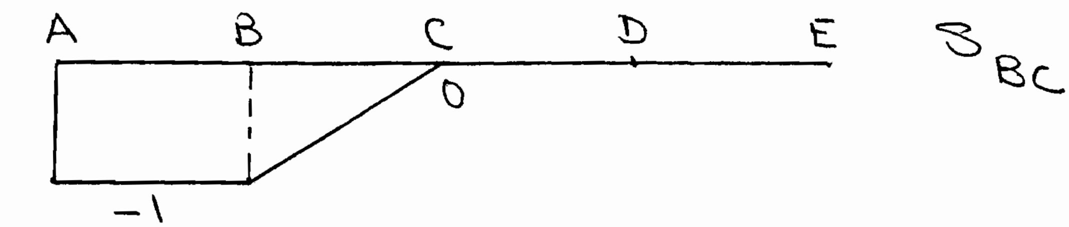


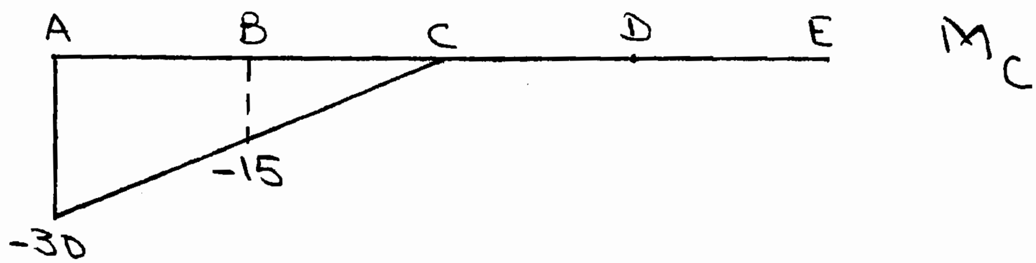
$\boxed{8.42}$



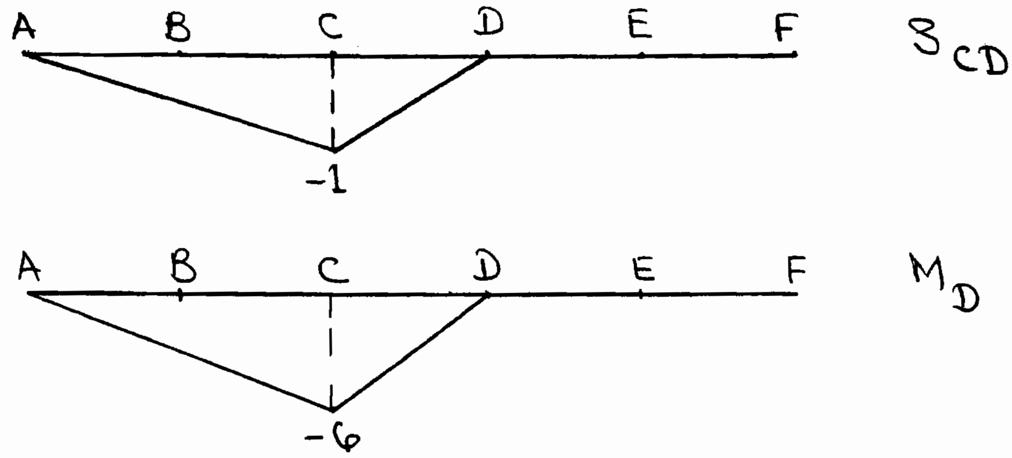


$\boxed{8.43}$

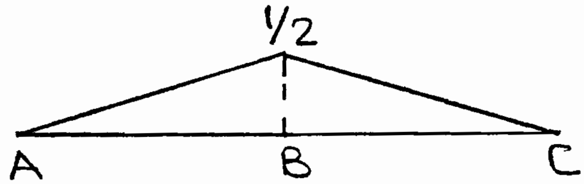




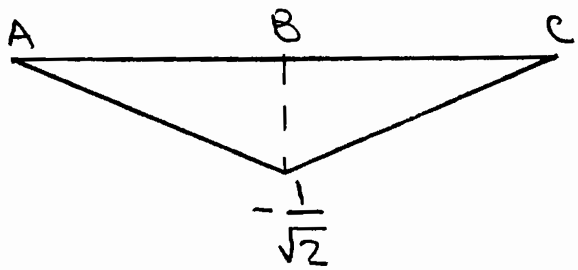
8.44



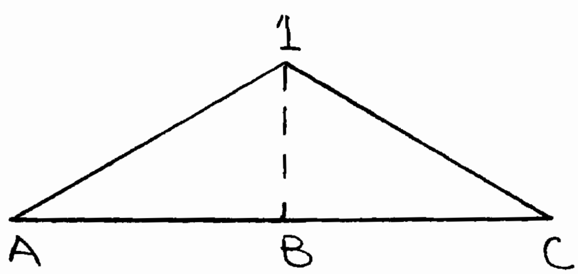
$\boxed{8.45}$



F_{AB}

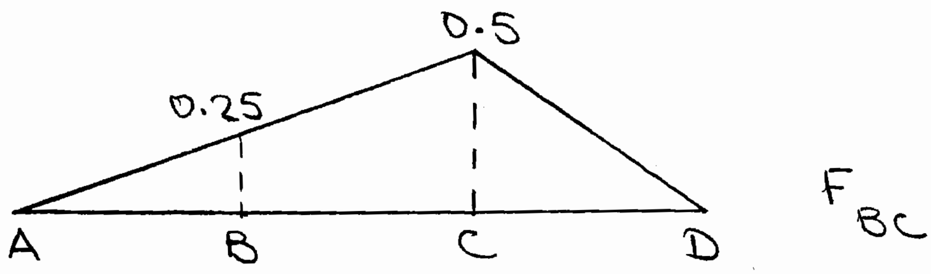


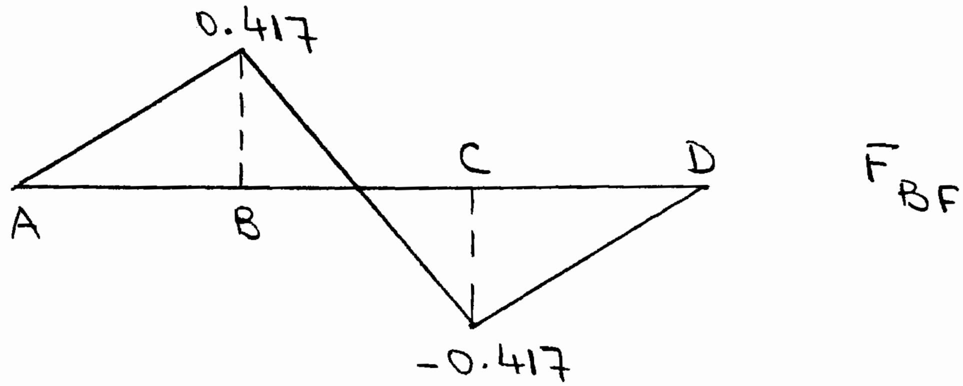
F_{AD}

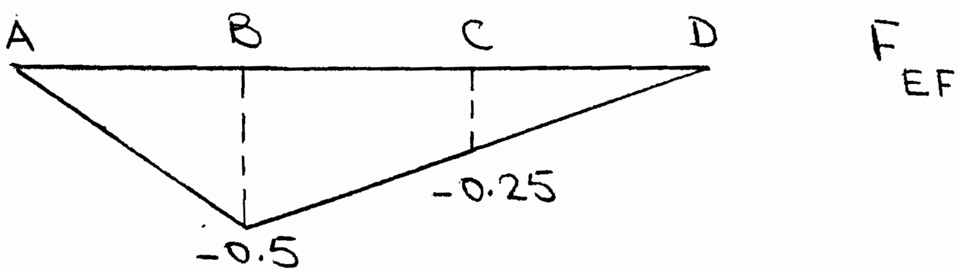


F_{BD}

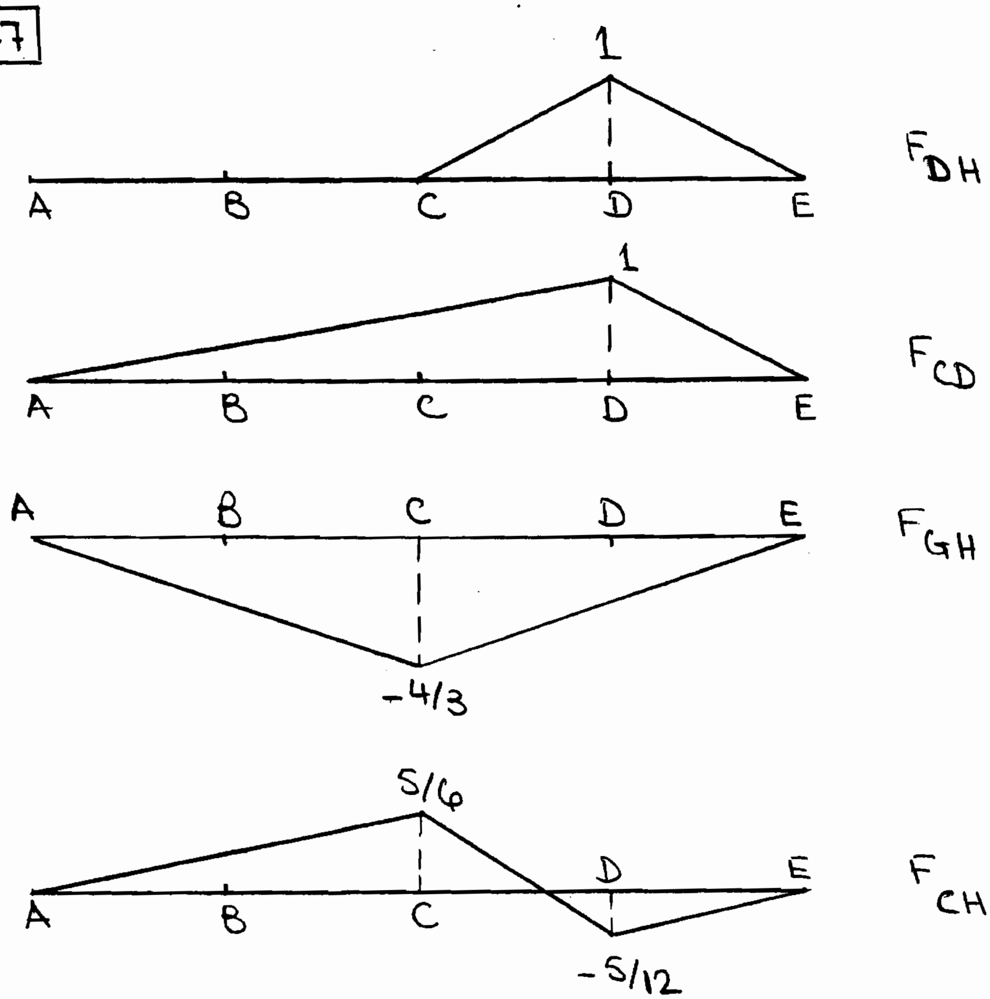
8.46



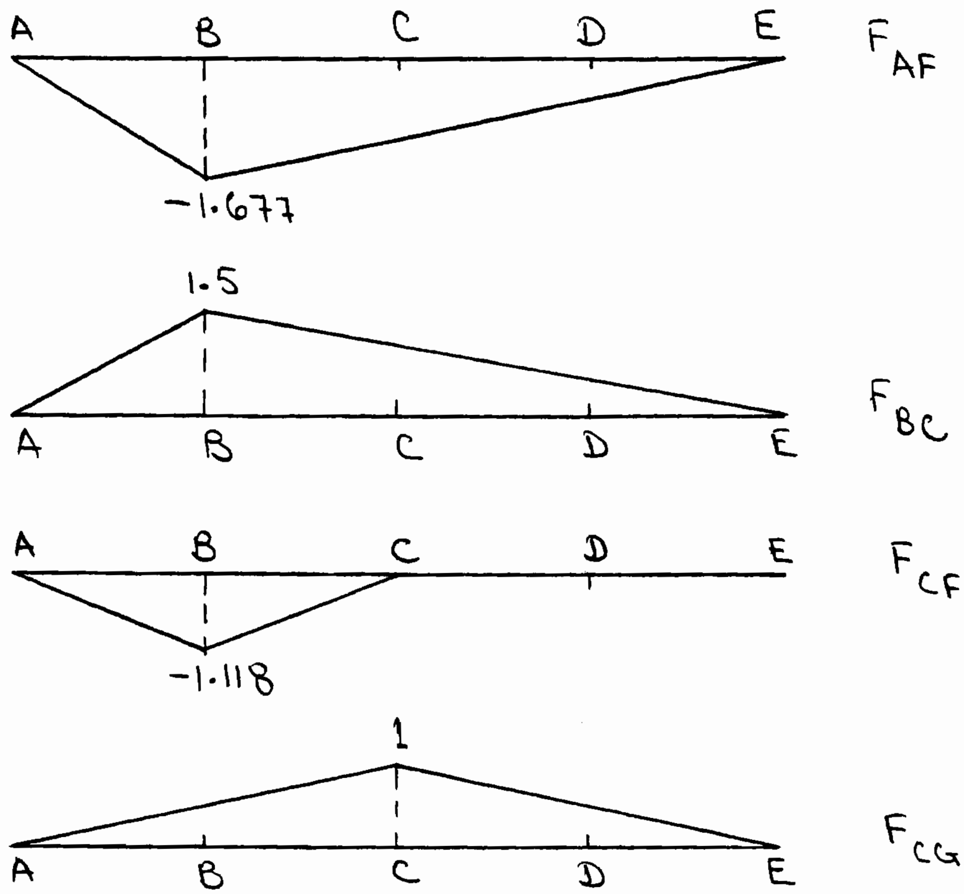




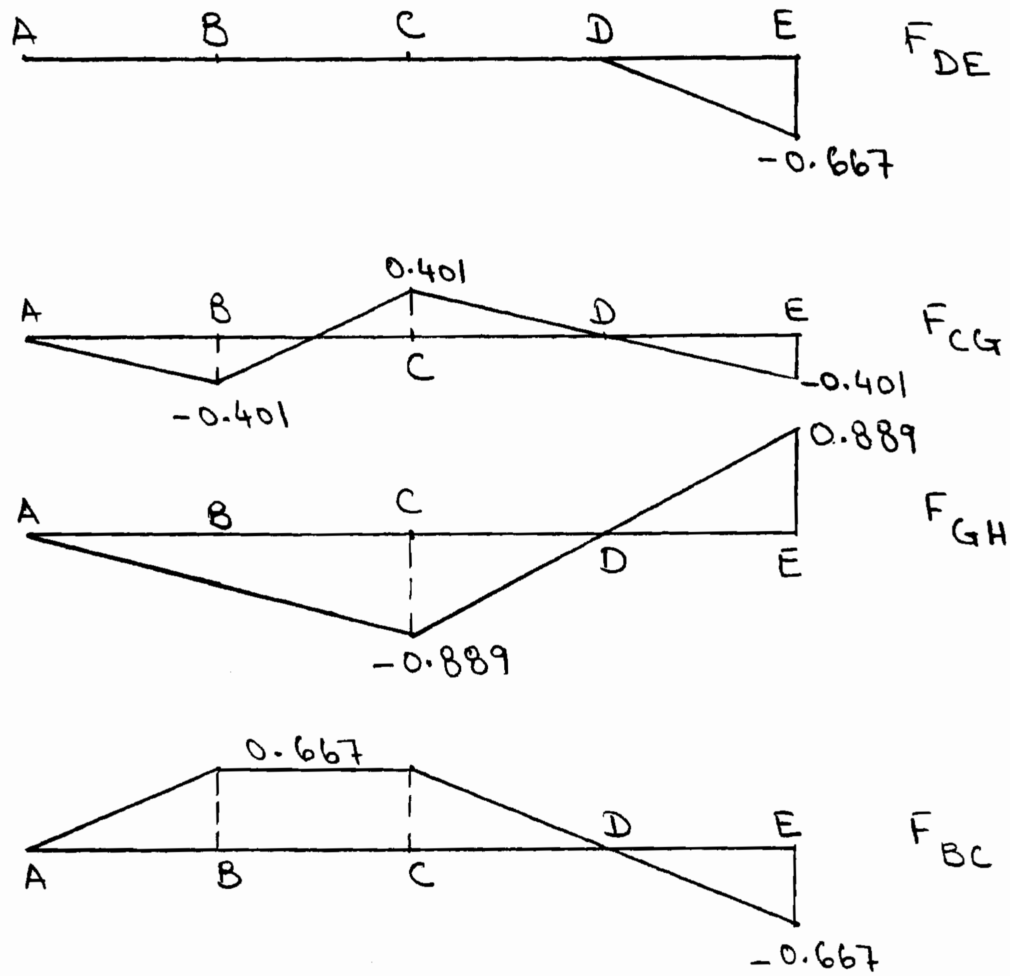
8.47



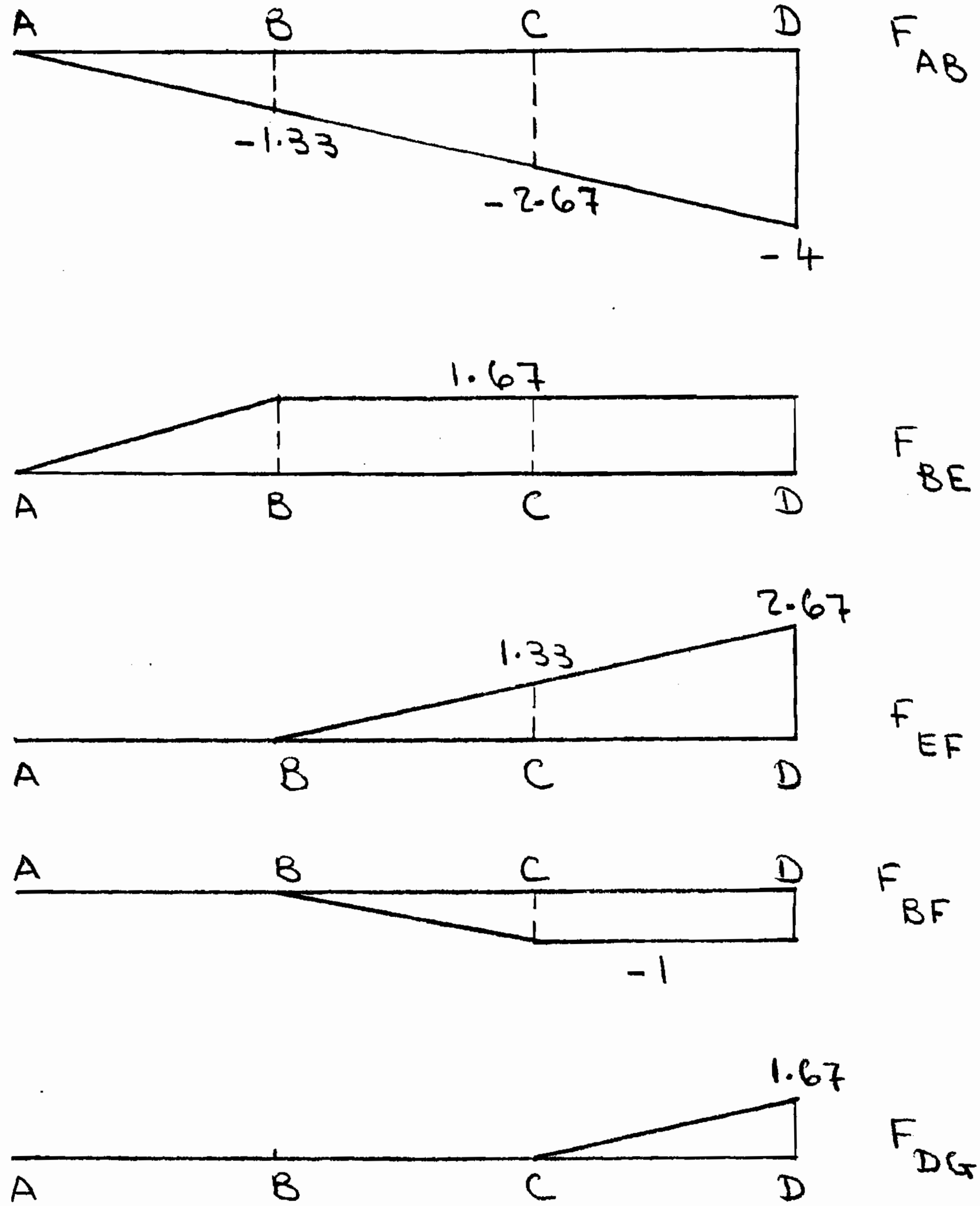
$\boxed{8.48}$



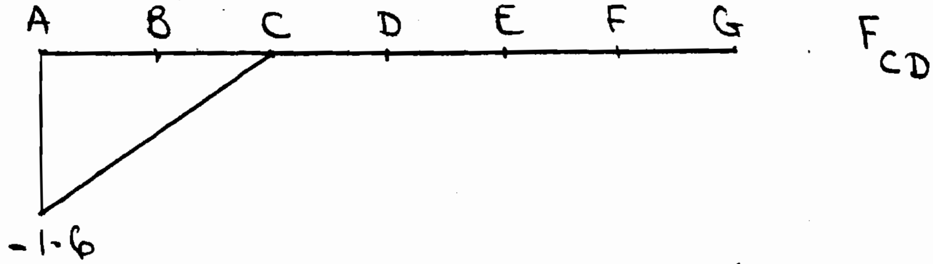
8.49


A B C D E
F_{DE}
-0.667
0.401
A B D E
F_{CG}
C
-0.401
-0.401
0.889
A B C
F_{GH}
D E
-0.889
0.667
D E
F_{BC}
A B C
-0.667

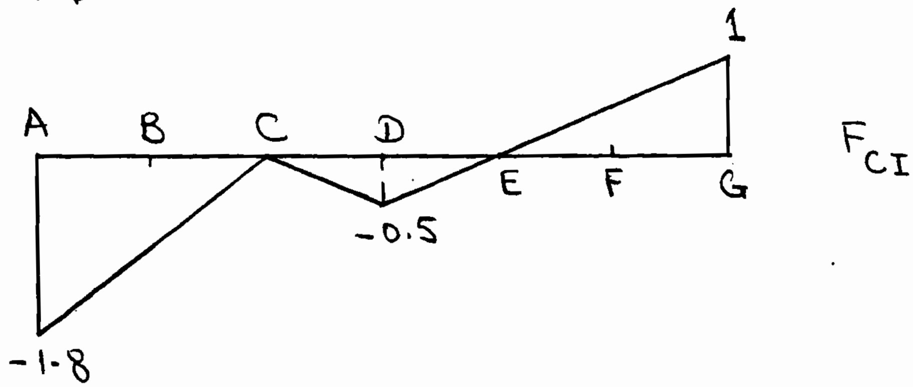
8.50


A
B
C
D
F_{AB}
-1.33
-2.67
-4
1.67
F_{BE}
A
B
C
D
2.67
1.33
F_{EF}
A
B
C
D
A
B
C
D
F_{BF}
-1
1.67
F_{DG}
A
B
C
D

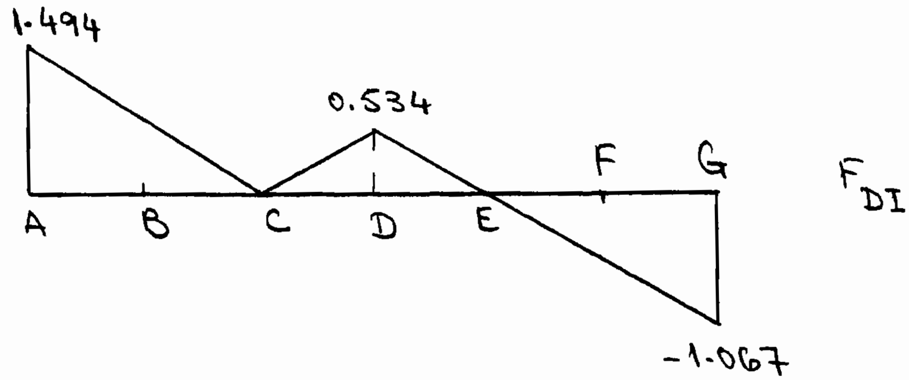
$\boxed{8.51}$



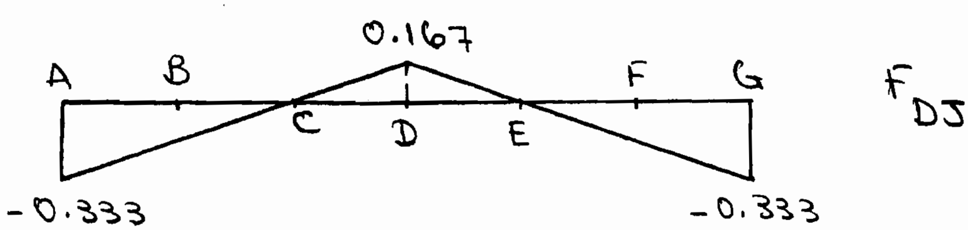
A B C D E F G
F_{CD}
-1.6



1
A B C D
F_{CI}
E F G
-0.5
-1.8

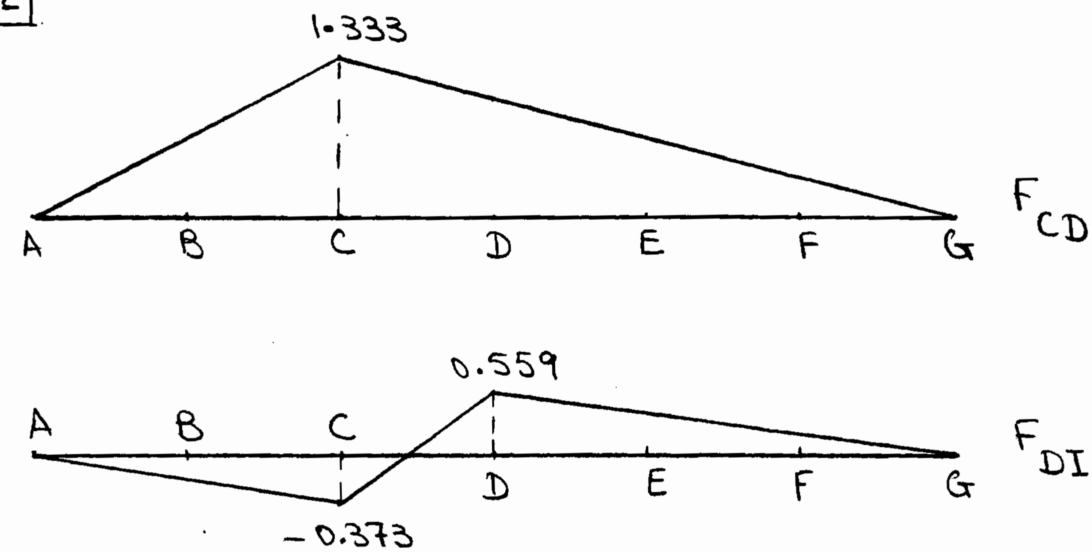


1.494
0.534
F G
F_{DI}
A B C D E
-1.067

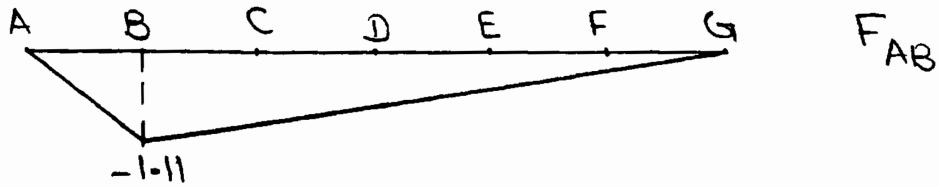


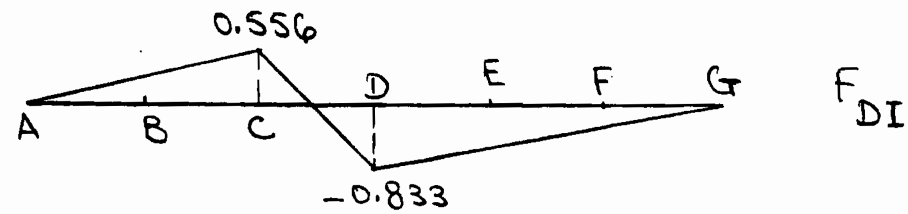
0.167
A B F G
F_{DJ}
C D E
-0.333
-0.333

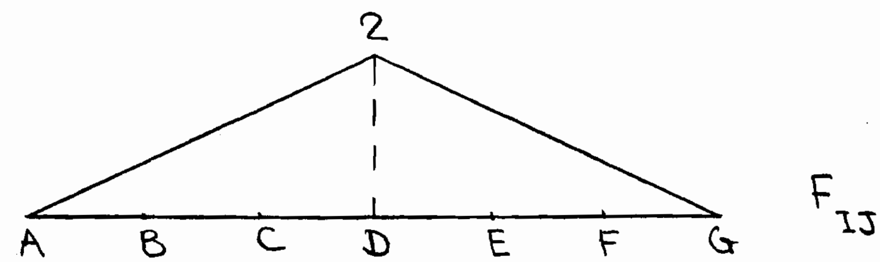
8.52

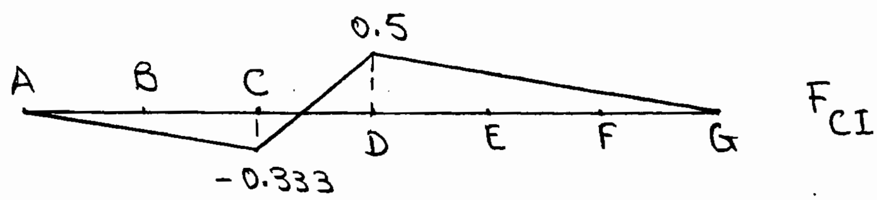


8.53

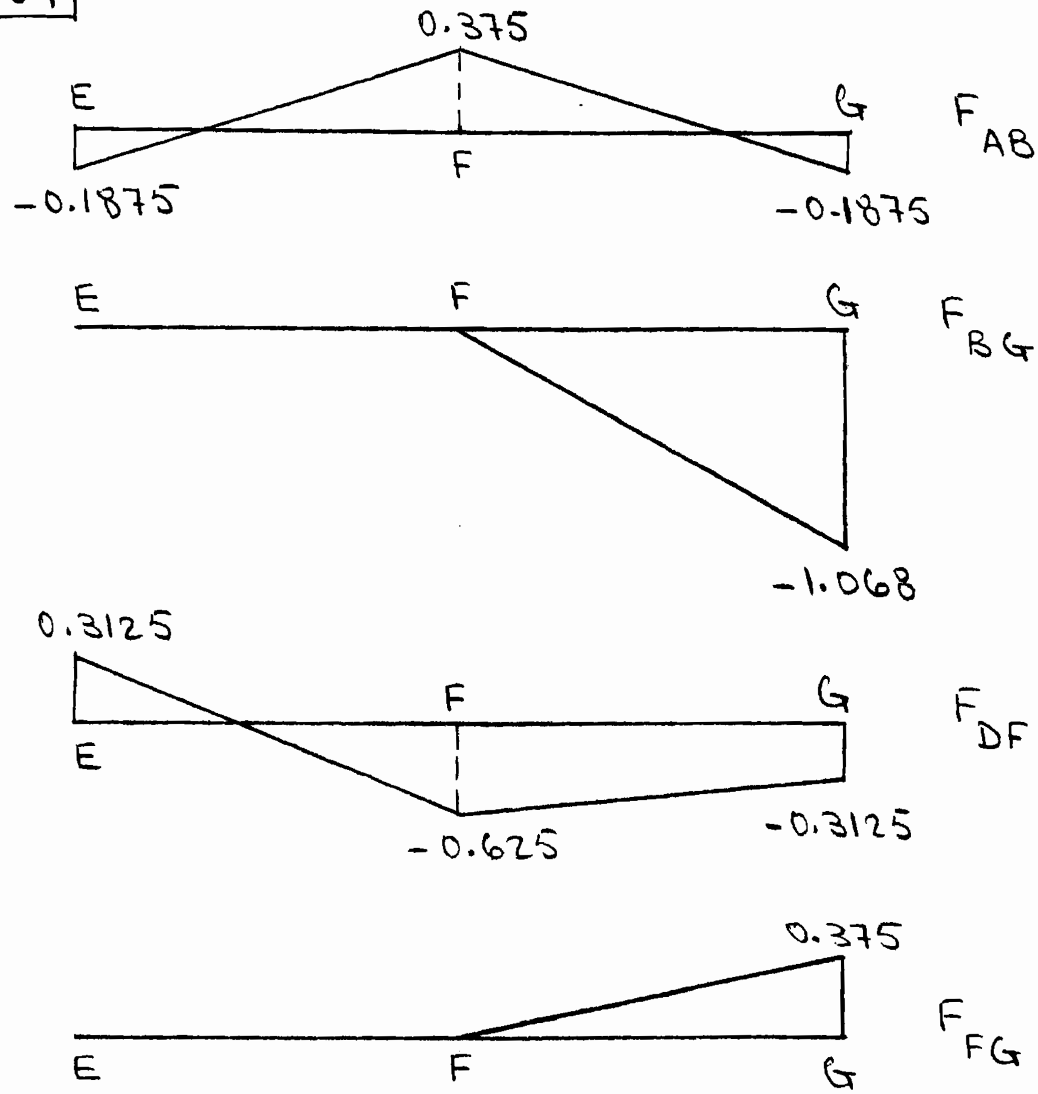

A B C D E F G
F_{AB}
-1.11


0.556
A B C D E F G
F_{DI}
-0.833

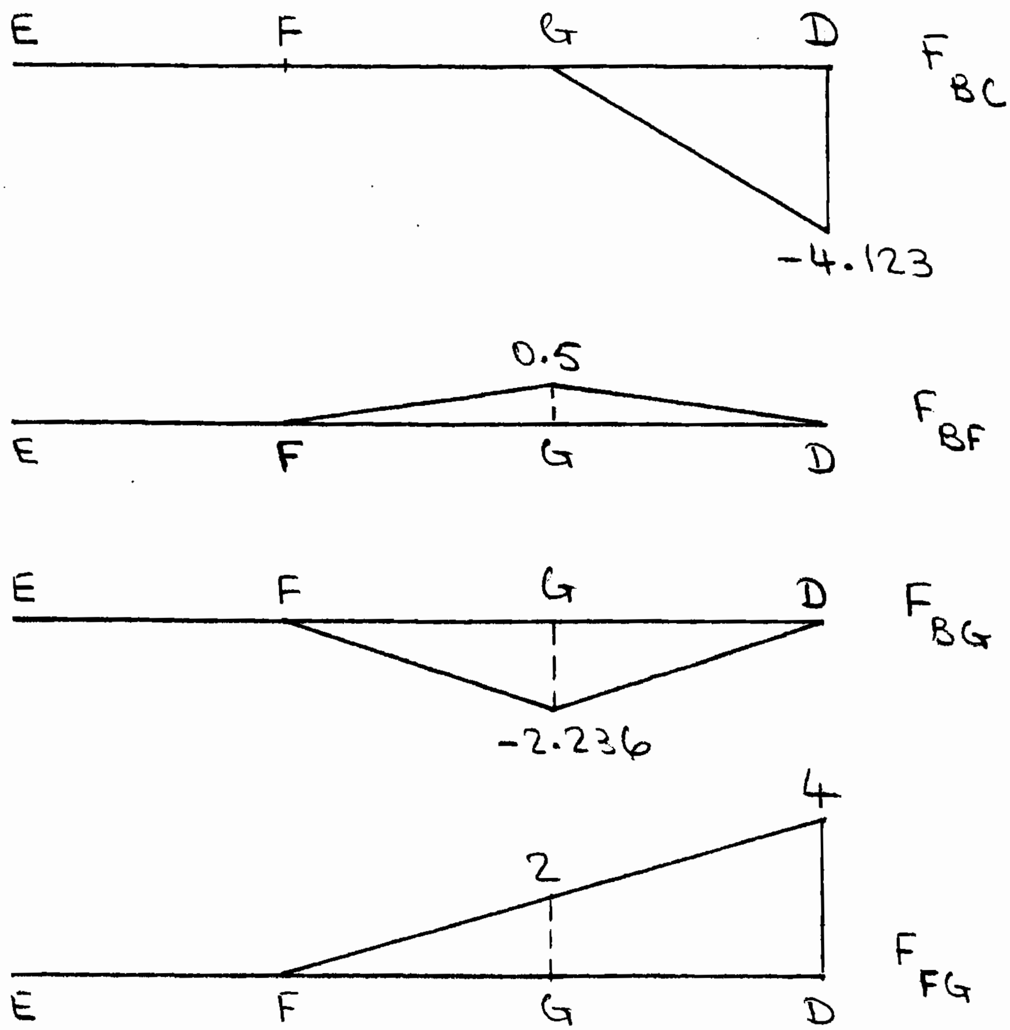

2
A B C D E F G
F_{IJ}


0.5
A B C D E F G
F_{CI}
-0.333

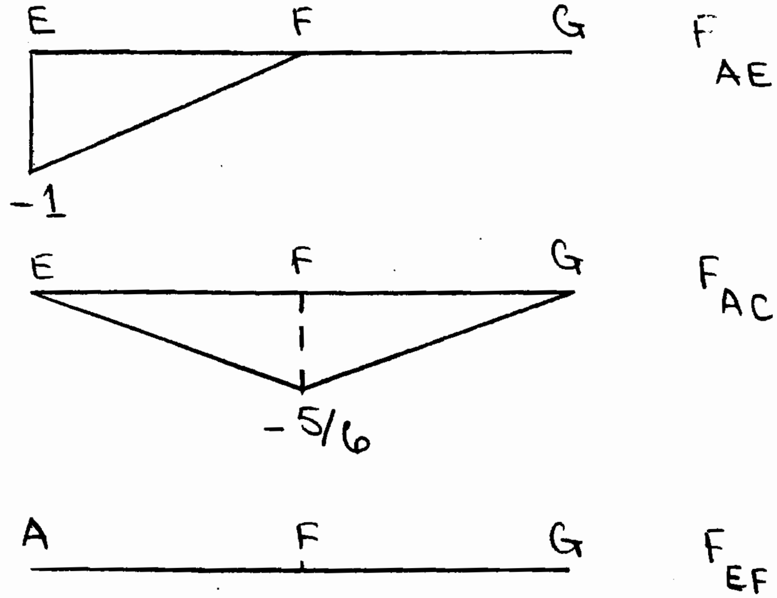
8.54


0.375
E
G
F_{AB}
F
-0.1875
-0.1875
E
F
G
F_{BG}
-1.068
0.3125
F
G
F_{DF}
E
-0.625
-0.3125
0.375
F_{FG}
E
F
G

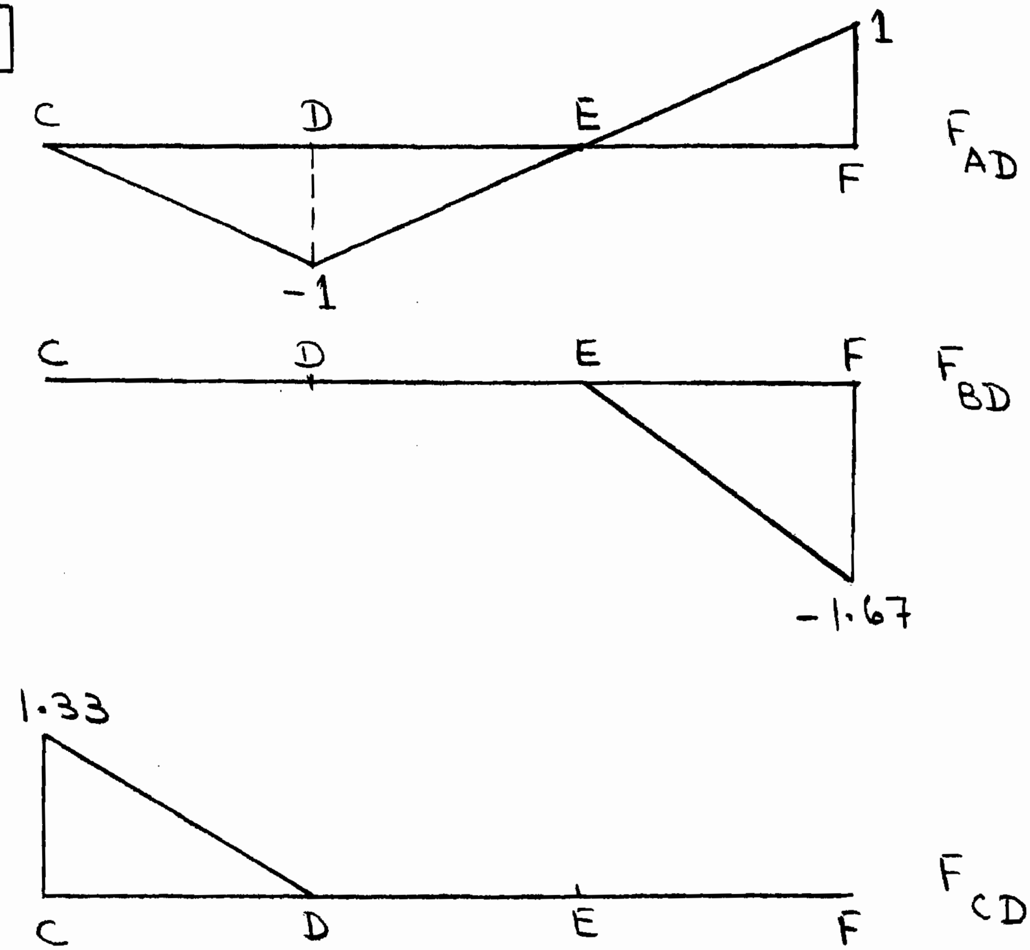
$\boxed{8.55}$



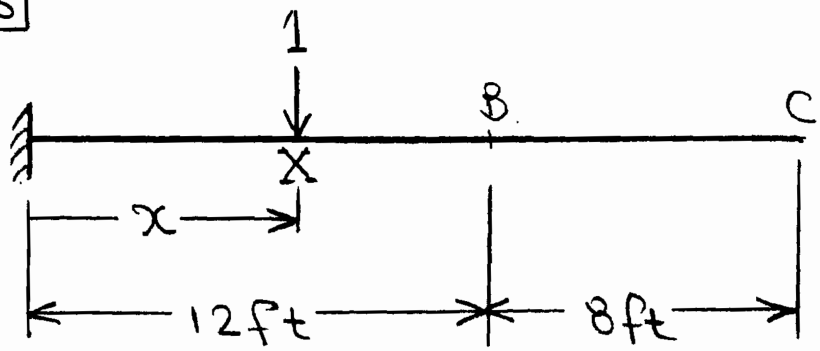
8.56



8.57

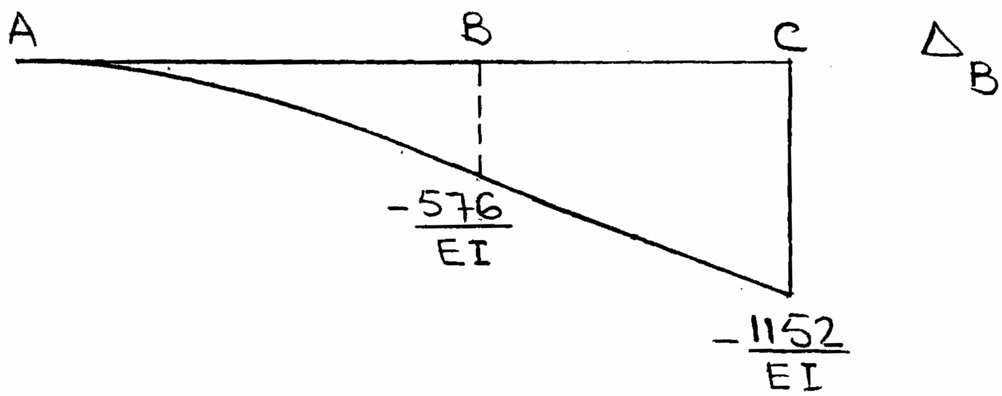


8.58

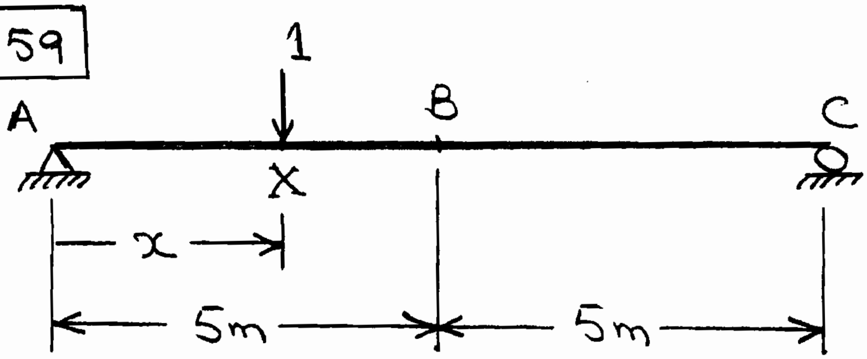


$0 \leqslant x \leqslant 12'$: $\quad f_{BX} = \frac{1}{6EI}(x^3 - 36x^2)$

$12' \leqslant x \leqslant 20'$: $\quad f_{BX} = \frac{72}{EI}(4 - x)$

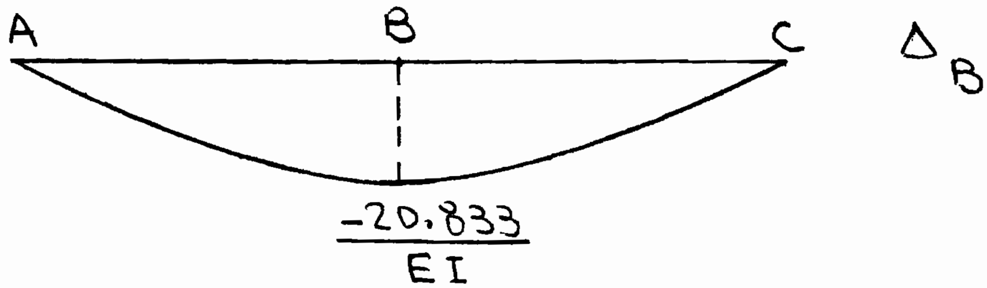


$\boxed{8.59}$

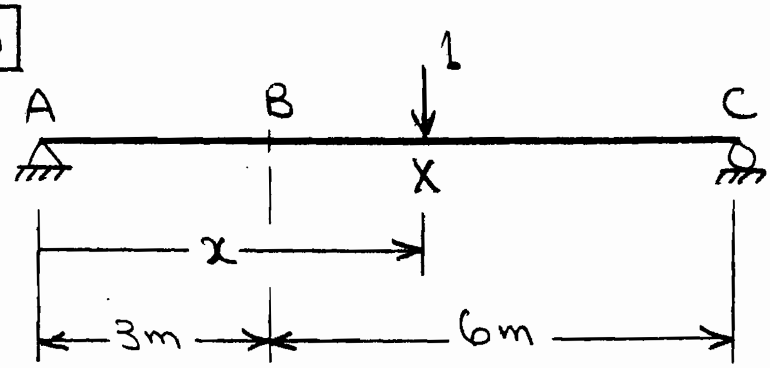


$$0 \leqslant x \leqslant 5\text{m}: \quad f_{BX} = \frac{1}{12EI}\left(x^3 - 75x\right)$$

$$5\text{m} \leqslant x \leqslant 10\text{m}: \quad f_{BX} = \frac{1}{12EI}\left[(10-x)^3 - 75(10-x)\right]$$

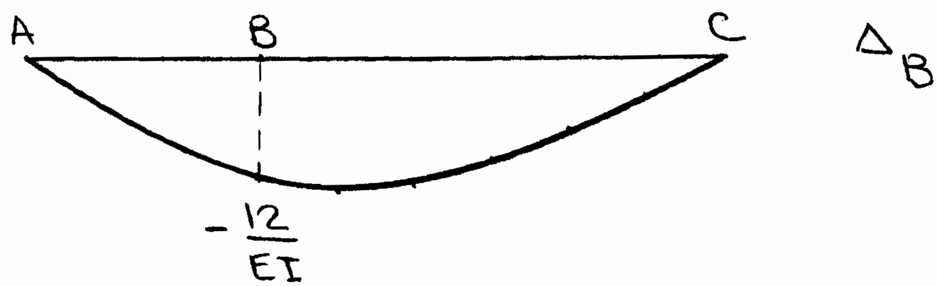


8.60

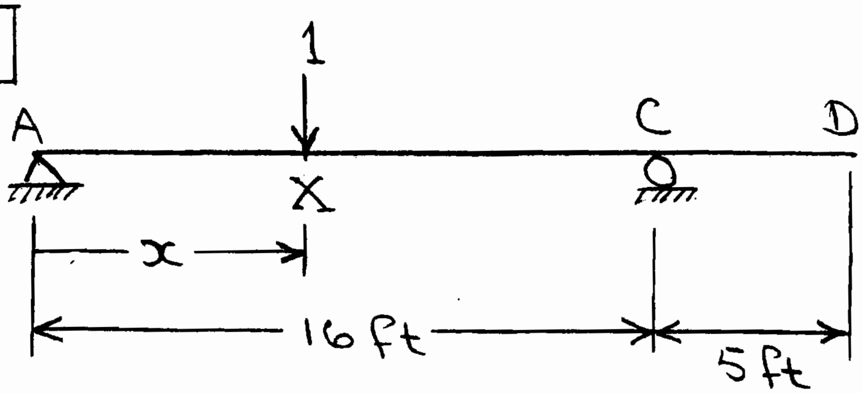


$$0 \leqslant x \leqslant 3\text{m} \qquad f_{BX} = \frac{1}{9EI}\left(x^3 - 45x\right)$$

$$3\text{m} \leqslant x \leqslant 9\text{m} \qquad f_{BX} = \frac{1}{18EI}\left(-x^3 + 27x^2 - 171x + 81\right)$$

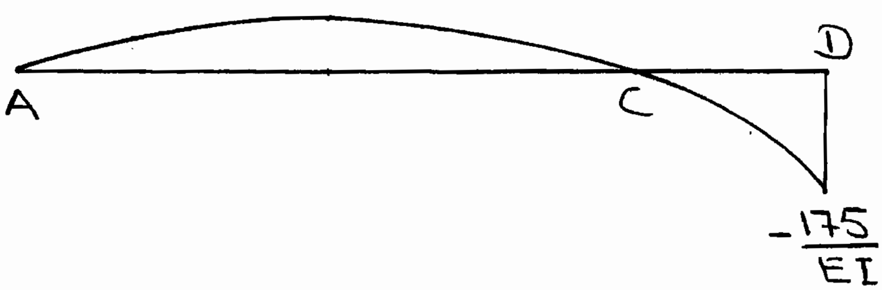


8.61



$$0 \leqslant x \leqslant 16': \quad f_{DX} = \frac{5x}{96EI}(256 - x^2)$$

$$16' \leqslant x \leqslant 21': \quad f_{DX} = \frac{1}{6EI}(-5376 + 1088x - 63x^2 + x^3)$$

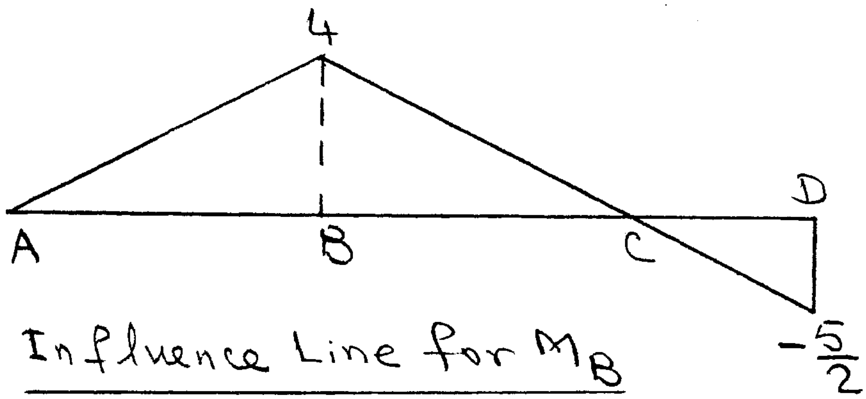


Δ_D

Chapter Nine
Application of Influence Lines

CHAPTER 9

9.1

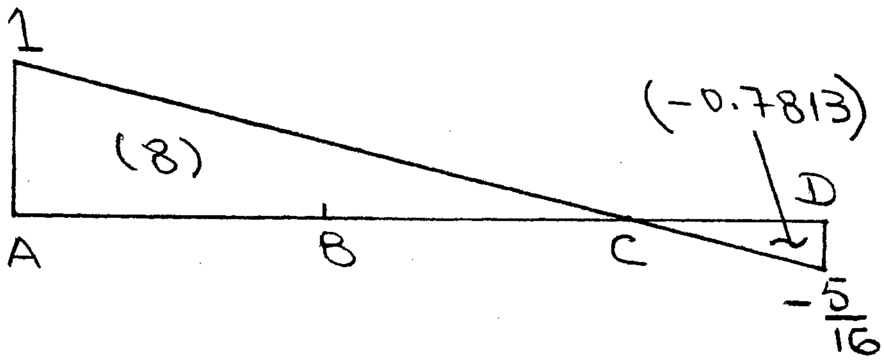


Influence Line for M_B

$$\text{Maximum Negative } M_B = 15\left(-\frac{5}{2}\right)$$

$$= -37.5 \text{ k-ft}$$

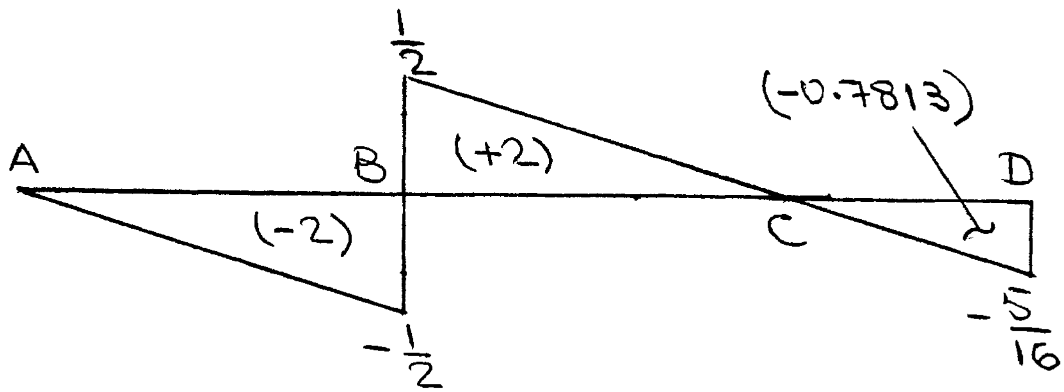
9.2



Influence Line for A_y

Maximum Upward $A_y = 3(8) = \underline{24\text{ k}\uparrow}$

$\boxed{9.3}$

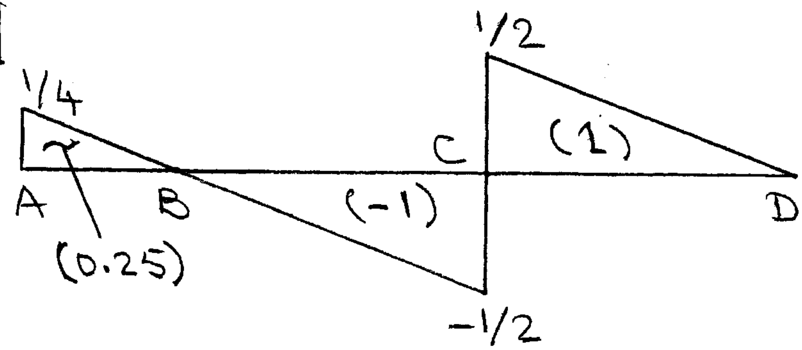


Influence Line for S_B

Maximum Negative $S_B = 3(-2 - 0.7813)$

$= \underline{-8.344 \text{ k}}$

9.4



Influence Line for S_C

Maximum Positive $S_C = 100\left(\frac{1}{2}\right) + 50(0.25+1)$

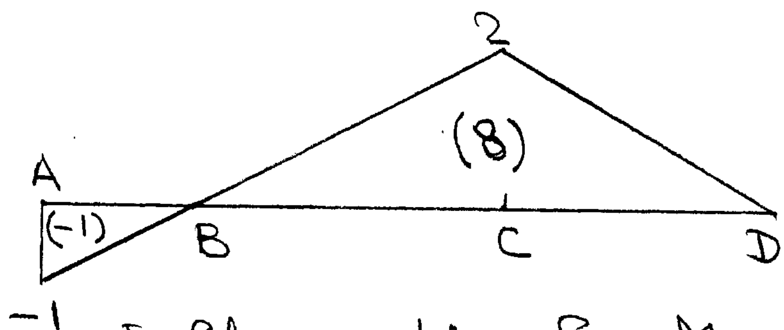
$+ 20(0.25 - 1 + 1)$

$= \underline{117.5 \text{ kN}}$

Maximum Negative $S_C = 100\left(-\frac{1}{2}\right) + 50(-1)$

$+ 20(0.25 - 1 + 1)$

$= \underline{-95 \text{ kN}}$



Influence Line for M_C

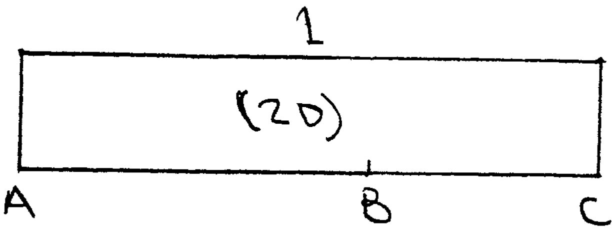
Maximum Positive $M_C = 100(2) + 50(8) + 20(7)$

$= \underline{740 \text{ kN.m}}$

Maximum Negative $M_C = 100(-1) + 50(-1) + 20(7)$

$= \underline{-10 \text{ kN.m}}$

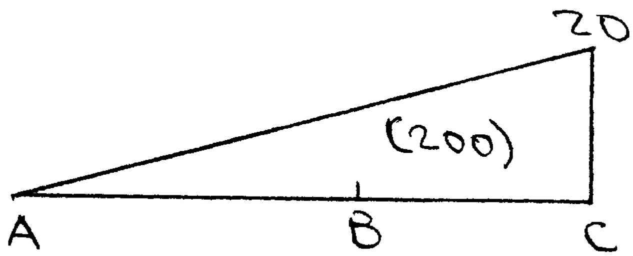
9.5



Influence Line for A_y

Maximum Upward $A_y = 25(1) + 2(20) + 0.5(20)$

$= \underline{75 \text{ k} \uparrow}$



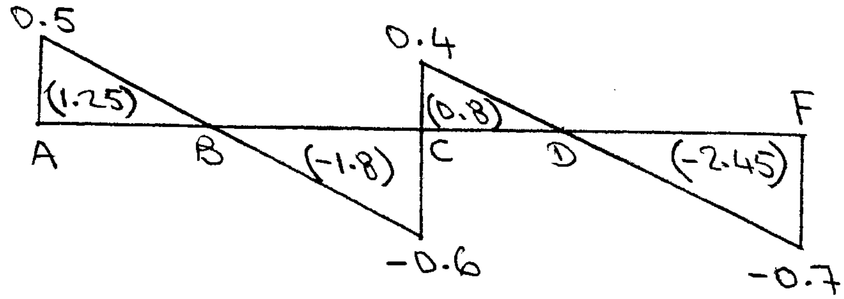
Influence Line for M_A (+↺)

Maximum Counterclockwise M_A

$= 25(20) + 2(200) + 0.5(200)$

$= \underline{1000 \text{ k-ft} \circlearrowleft}$

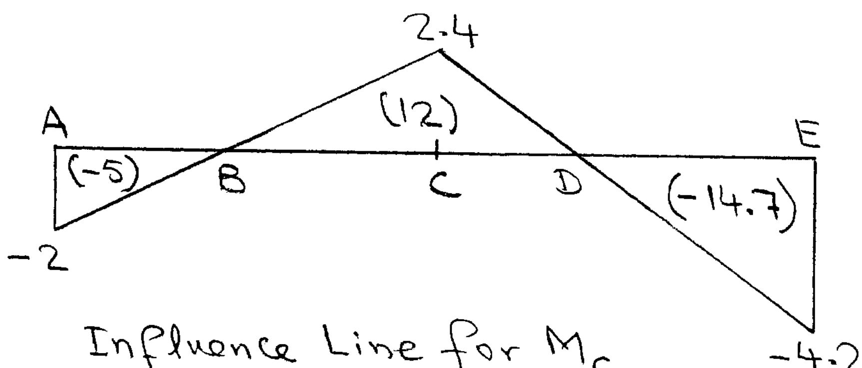
9.6



Influence Line for S_C

Maximum Positive $S_C = 150(0.5) + 50(1.25 + 0.8)$
$+25(1.25 - 1.8 + 0.8 - 2.45)$
$= \underline{122.5 \text{ kN}}$

Maximum Negative $S_C = 150(-0.7) + 50(-1.8 - 2.45)$
$+25(1.25 - 1.8 + 0.8 - 2.45)$
$= \underline{-372.5 \text{ kN}}$

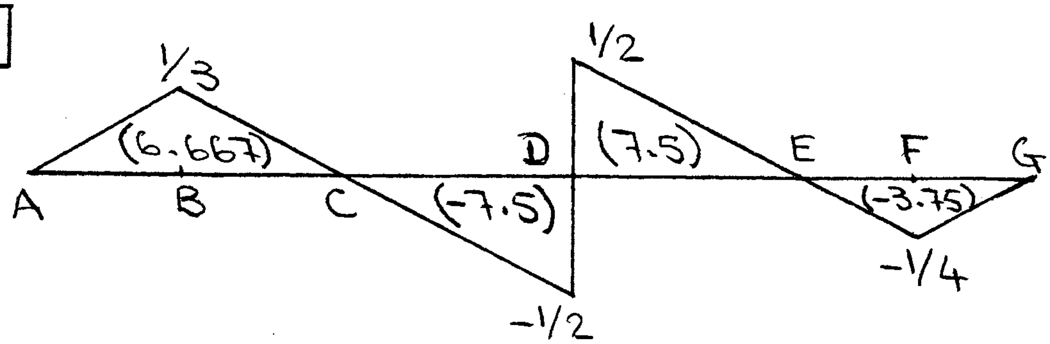


Influence Line for M_C

Maximum Positive $M_C = 150(2.4) + 50(12)$
$+25(-5 + 12 - 14.7)$
$= \underline{767.5 \text{ kN.m}}$

Maximum Negative $M_C = 150(-4.2) + 50(-5 - 14.7)$
$+25(-5 + 12 - 14.7)$
$= \underline{-1807.5 \text{ kN.m}}$

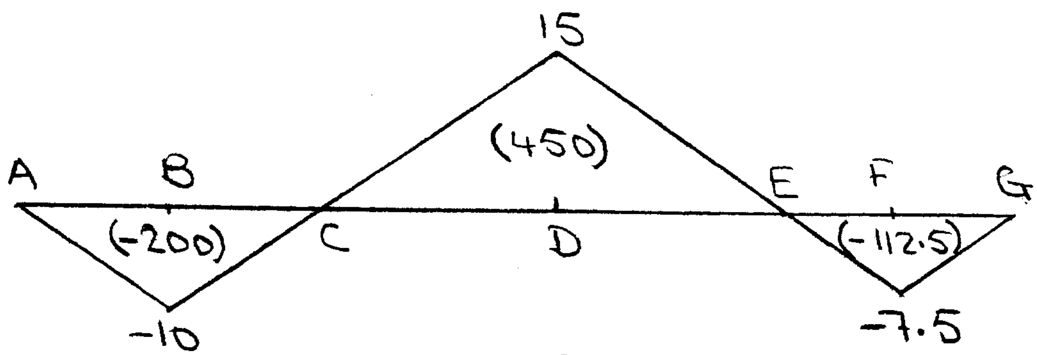
9.7



Influence Line for S_D

Maximum Positive $S_D = 30(\frac{1}{2}) + 3(6.667 + 7.5)$
$+ 1(6.667 - 7.5 + 7.5 - 3.75)$
$= \underline{60.417\text{ k}}$

Maximum Negative $S_D = 30(-\frac{1}{2}) + 3(-7.5 - 3.75)$
$+ 1(6.667 - 7.5 + 7.5 - 3.75)$
$= \underline{-45.833\text{ k}}$

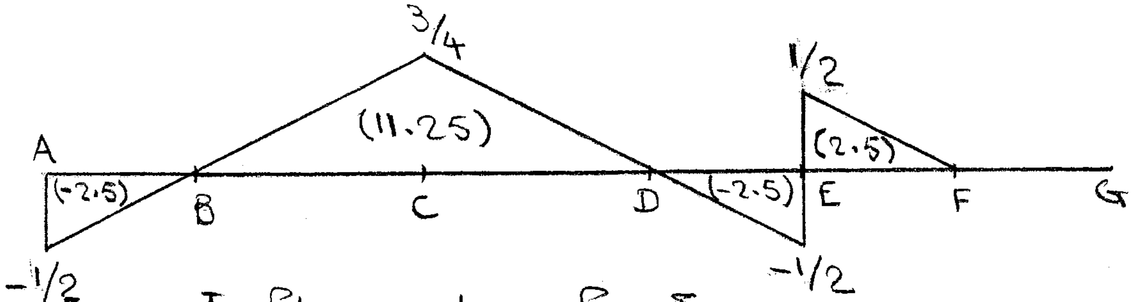


Influence Line for M_D

Maximum Positive $M_D = 30(15) + 3(450)$
$+ 1(-200 + 450 - 112.5)$
$= \underline{1937.5\text{ k-ft}}$

Maximum Negative $M_D = 30(-10) + 3(-200 - 112.5)$
$+ 1(-200 + 450 - 112.5)$
$= \underline{-1100\text{ k-ft}}$

9.8



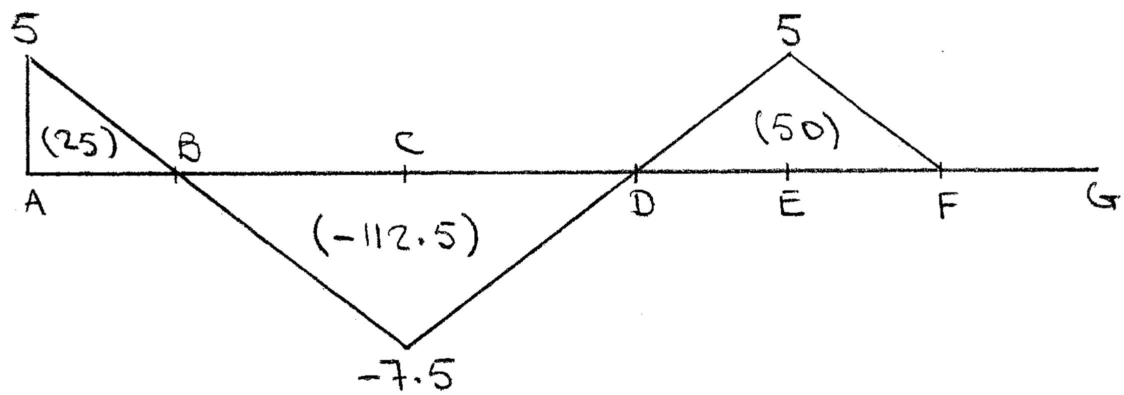
Influence Line for S_E

Maximum Positive $S_E = 40\left(\frac{3}{4}\right) + 2(11.25 + 2.5) + 1(-2.5 + 11.25)$

$= \underline{66.25\ k}$

Maximum Negative $S_E = 40\left(-\frac{1}{2}\right) + 2(-2.5 - 2.5) + 1(-2.5 + 11.25)$

$= \underline{-21.25\ k}$



Influence Line for M_E

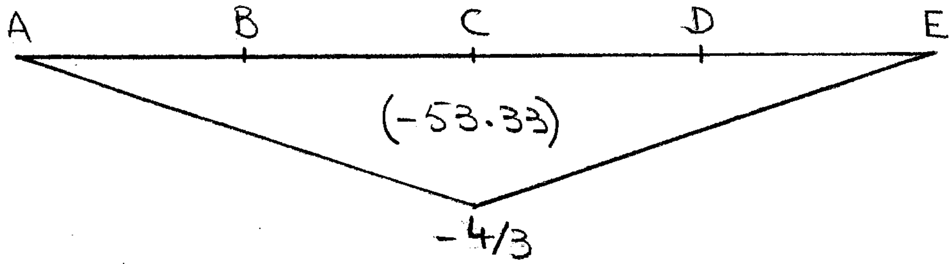
Maximum Positive $M_E = 40(5) + 2(25 + 50) + 1(25 - 112.5 + 50)$

$= \underline{312.5\ k\text{-}ft}$

Maximum Negative $M_E = 40(-7.5) + 2(-112.5) + 1(25 - 112.5 + 50)$

$= \underline{-562.5\ k\text{-}ft}$

9.9

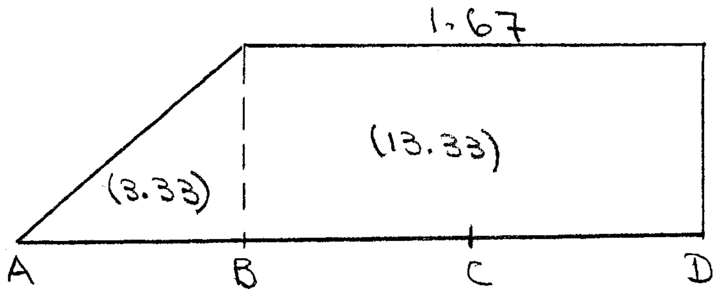


Influence Line for F_{GH}

Maximum Compressive $F_{GH} = 30\left(-\frac{4}{3}\right) + 3(-53.33)$

$= -200\text{ k} = \underline{200\text{ k (C)}}$

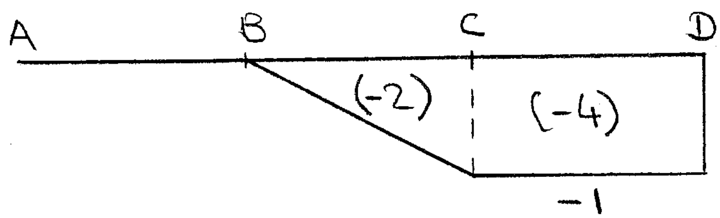
9.10



Influence Line for F_{BE}

Maximum Tensile $F_{BE} = 120(1.67) + 60(3.33 + 13.33)$

$= \underline{1200 \text{ kN (T)}}$



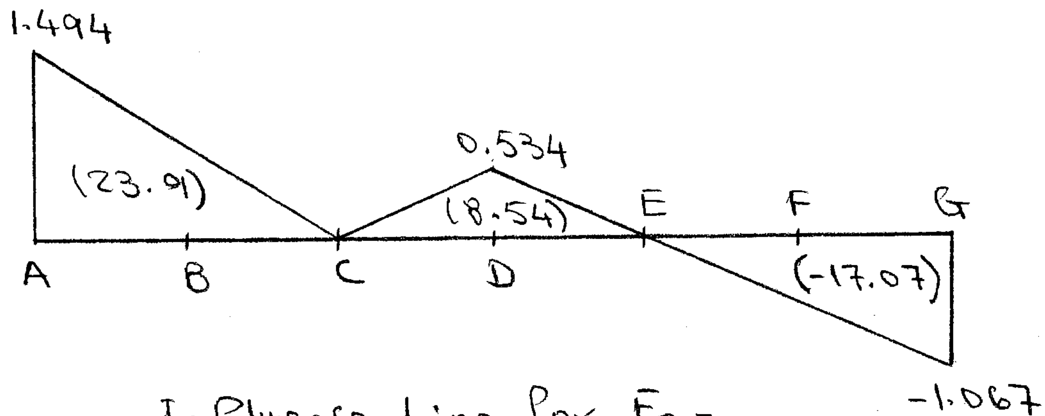
Influence Line for F_{BF}

Maximum Compressive $F_{BF} = 120(-1) + 60(-2-4)$

$= -480 \text{ kN}$

$= \underline{480 \text{ kN (C)}}$

9.11

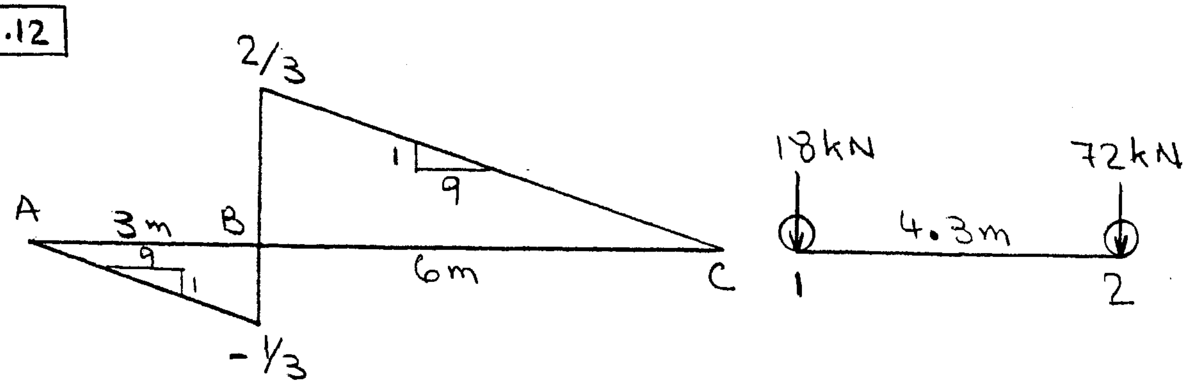


Influence Line for F_{DI}

Maximum Tensile $F_{DI} = 40(1.494) + 4(23.9 + 8.54)$
$+ 2(23.9 + 8.54 - 17.07) = \underline{220.3\ k\ (T)}$

Maximum Compressive $F_{DI} = 40(-1.067) + 4(-17.07)$
$+ 2(23.9 + 8.54 - 17.07) = -80.2\ k$
$= \underline{80.2\ k\ (C)}$

9.12



Influence Line for S_B

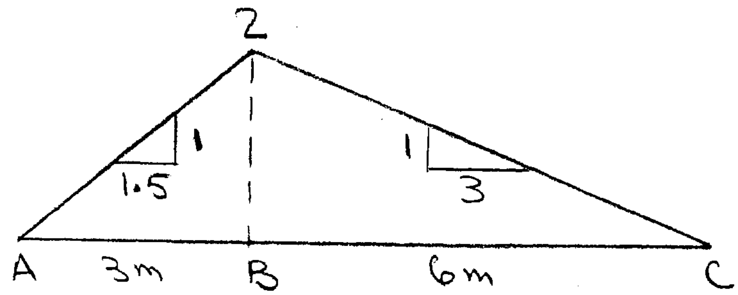
Loading position 1:

$$S_B = 18\left(\frac{2}{3}\right) + 72\left(\frac{1.7}{9}\right) = 25.6 \text{ kN}$$

Loading position 2:

$$S_B = 72\left(\frac{2}{3}\right) = 48 \text{ kN}$$

Max. Positive S_B = 48 kN



Influence Line for M_B

Loading position 1:

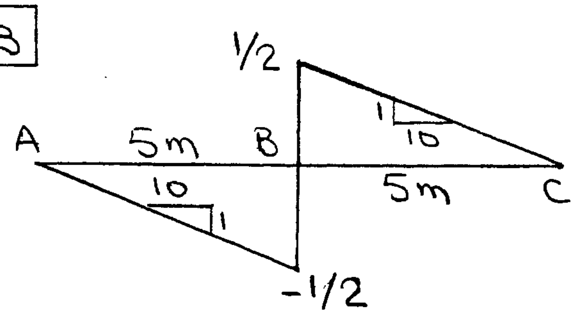
$$M_B = 18(2) + 72\left(\frac{1.7}{3}\right) = 76.8 \text{ kN.m}$$

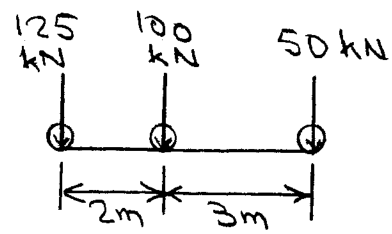
Loading position 2:

$$M_B = 72(2) = 144 \text{ kN.m}$$

Max. Positive M_B = 144 kN.m

$\boxed{9.13}$





Influence Line for S_B

Loading position 1:

$$S_B = 125\left(\frac{1}{2}\right) + 100\left(\frac{3}{10}\right) = 92.5 \text{ kN}$$

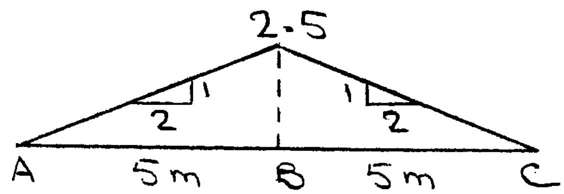
Loading position 2:

$$S_B = 125\left(-\frac{3}{10}\right) + 100\left(\frac{1}{2}\right) + 50\left(\frac{2}{10}\right) = 22.5 \text{ kN}$$

Loading position 3:

$$S_B = 100\left(-\frac{2}{10}\right) + 50\left(\frac{1}{2}\right) = 5 \text{ kN}$$

Max. Positive $S_B = \underline{92.5 \text{ kN}}$



Influence Line for M_B

Loading position 1:

$$M_B = 125(2.5) + 100(1.5) = 462.5 \text{ kN.m}$$

Loading position 2:

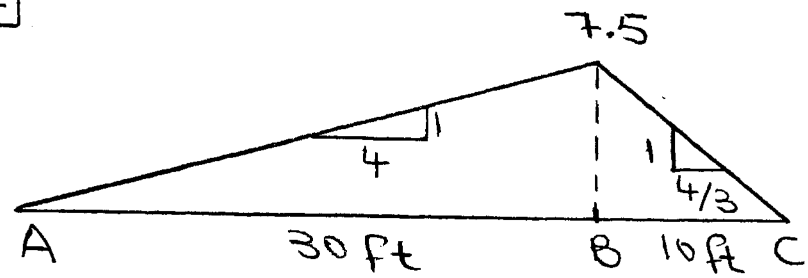
$$M_B = 125(1.5) + 100(2.5) + 50(1) = 487.5 \text{ kN.m}$$

Loading position 3:

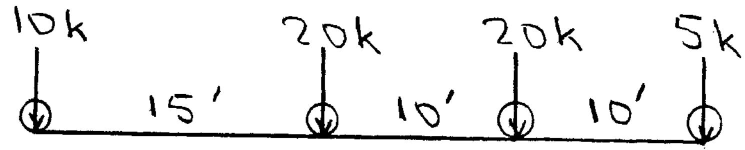
$$M_B = 100(1) + 50(2.5) = 225 \text{ kN.m}$$

Max. Positive $\underline{M_B = 487.5 \text{ kN.m}}$

9.14



Influence Line for M_B



Loading position 1:

$$M_B = 10(7.5) = 75 \text{ k-ft}$$

Loading position 2:

$$M_B = 10\left(\frac{15}{4}\right) + 20(7.5) = 187.5 \text{ k-ft}$$

Loading position 3:

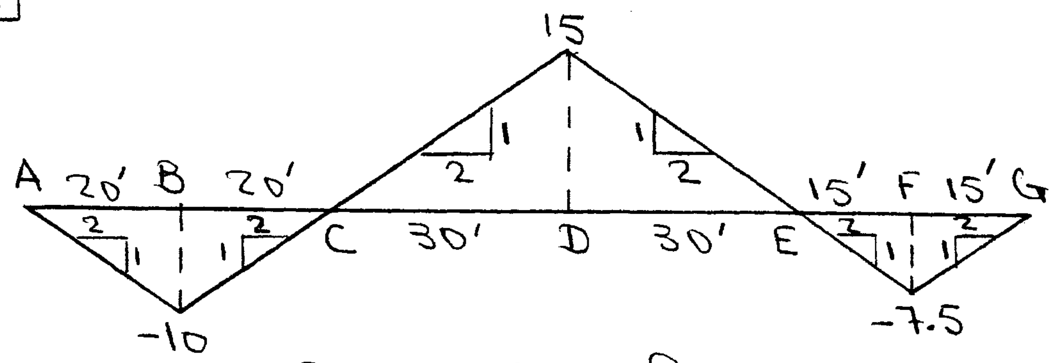
$$M_B = 10\left(\frac{5}{4}\right) + 20(5) + 20(7.5) = 262.5 \text{ k-ft}$$

Loading position 4:

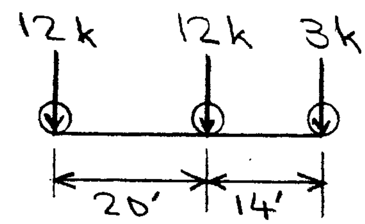
$$M_B = 20\left(\frac{10}{4}\right) + 20(5) + 5(7.5) = 187.5 \text{ k-ft}$$

Max. Positive $M_B = 262.5$ k-ft

$\boxed{9.15}$



<u>Influence Line for M_D</u>



<u>Loading position 1:</u>

$$M_D = 12(15) + 12(5) + 3\left(-\frac{4}{2}\right) = 234 \text{ k-ft}$$

<u>Loading position 2:</u>

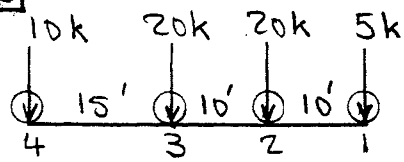
$$M_D = 12(5) + 12(15) + 3\left(\frac{16}{2}\right) = 264 \text{ k-ft}$$

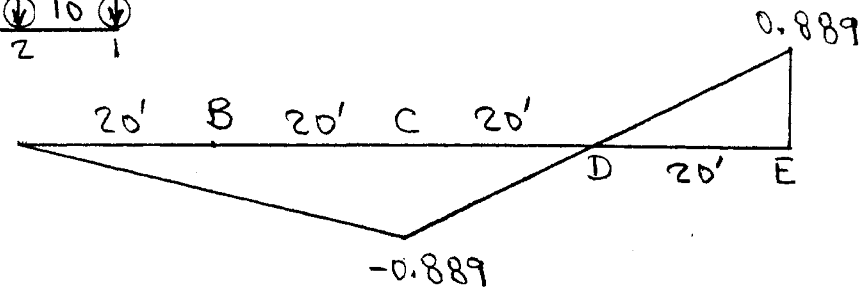
<u>Loading position 3:</u>

$$M_D = 12\left(-\frac{4}{2}\right) + 12\left(\frac{16}{2}\right) + 3(15) = 117 \text{ k-ft}$$

Max. Positive $\underline{M_D = 264 \text{ k-ft}}$

9.16





Influence Line for F_{GH}

Loading position 1:

$$F_{GH} = [5(40) + 20(30+20) + 10(5)]\left(\frac{-0.889}{40}\right) = -27.78\text{ k}$$

Loading position 2:

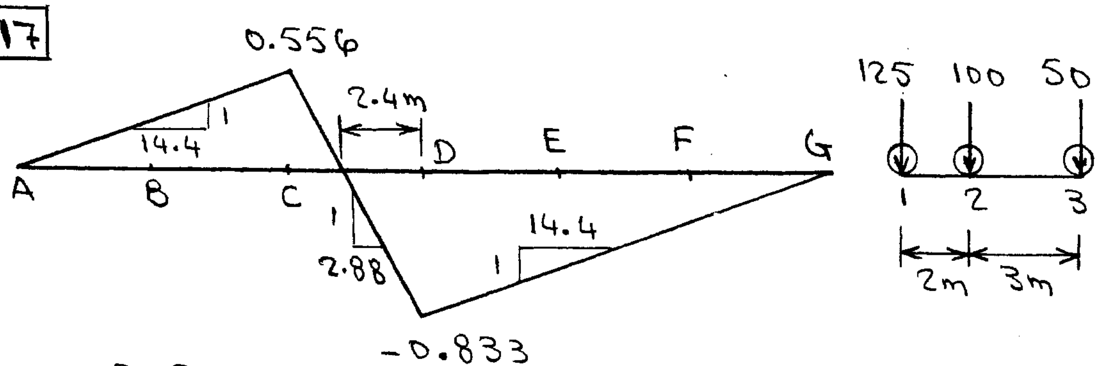
$$F_{GH} = 5(10)\left(\frac{-0.889}{20}\right) + [20(40+30) + 10(15)]\left(\frac{-0.889}{40}\right)$$
$$= -36.67\text{ k}$$

Loading position 3:

$$F_{GH} = [5(0) + 20(10)]\left(\frac{-0.889}{20}\right) + [20(40) + 10(25)]\left(\frac{-0.889}{40}\right)$$
$$= \underline{-32.23\text{ k}}$$

Maximum compressive $F_{GH} = -36.67\text{k} = \underline{36.67\text{k (C)}}$

9.17



Influence Line for F_{DI}

Loading position 1:

$$F_{DI} = 125(0.556) - 100\left(\frac{0.4}{2.88}\right) - 50\left(\frac{11}{14.4}\right) = 17.42 \text{ kN}$$

Loading position 2:

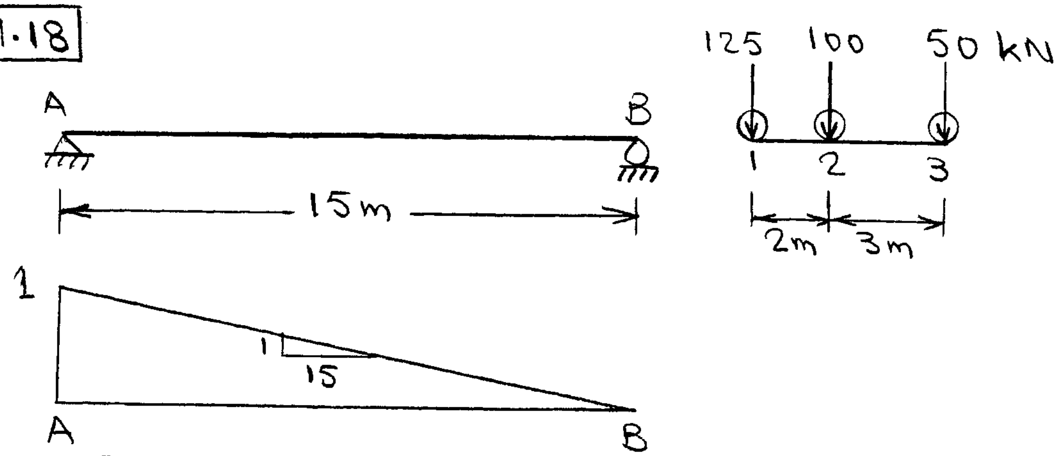
$$F_{DI} = 125\left(\frac{6}{14.4}\right) + 100(0.556) - 50\left(\frac{1.4}{2.88}\right) = 83.38 \text{ kN}$$

Loading position 3:

$$F_{DI} = 125\left(\frac{3}{14.4}\right) + 100\left(\frac{5}{14.4}\right) + 50(0.556) = 88.56 \text{ kN}$$

Maximum Tensile $F_{DI} = \underline{88.56 \text{ kN (T)}}$

9.18



Influence Line $S_{A,R}$

Loading position 1:

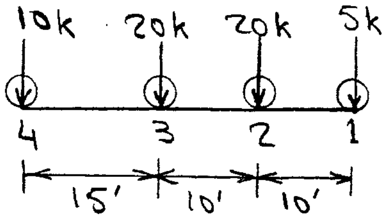
$$S_{A,R} = [125(15) + 100(13) + 50(10)]\frac{1}{15} = 245 \text{ kN}$$

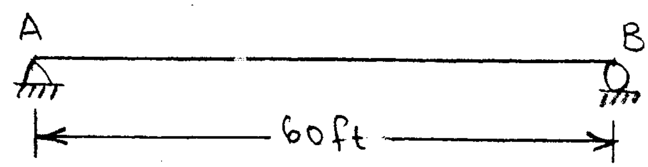
Loading position 2:

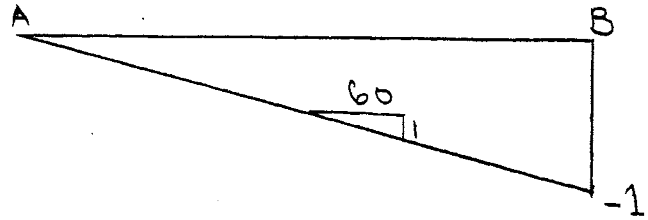
$$S_{A,R} = [100(15) + 50(12)]\frac{1}{15} = 140 \text{ kN}$$

Absolute maximum shear = 245 kN

9.19







Influence Line for $S_{B,L}$

Loading position 1:

$$S_{B,L} = [5(60) + 20(50+40) + 10(25)]\left(-\frac{1}{60}\right) = -39.17 \text{ k}$$

Loading position 2:

$$S_{B,L} = [20(60+50) + 10(35)]\left(\frac{-1}{60}\right) = -42.5 \text{ k}$$

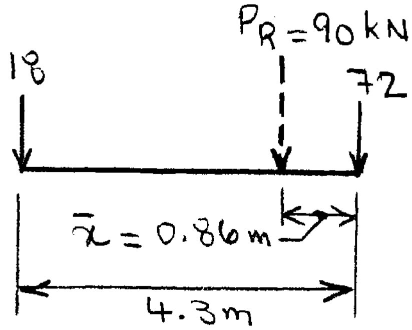
Loading position 3:

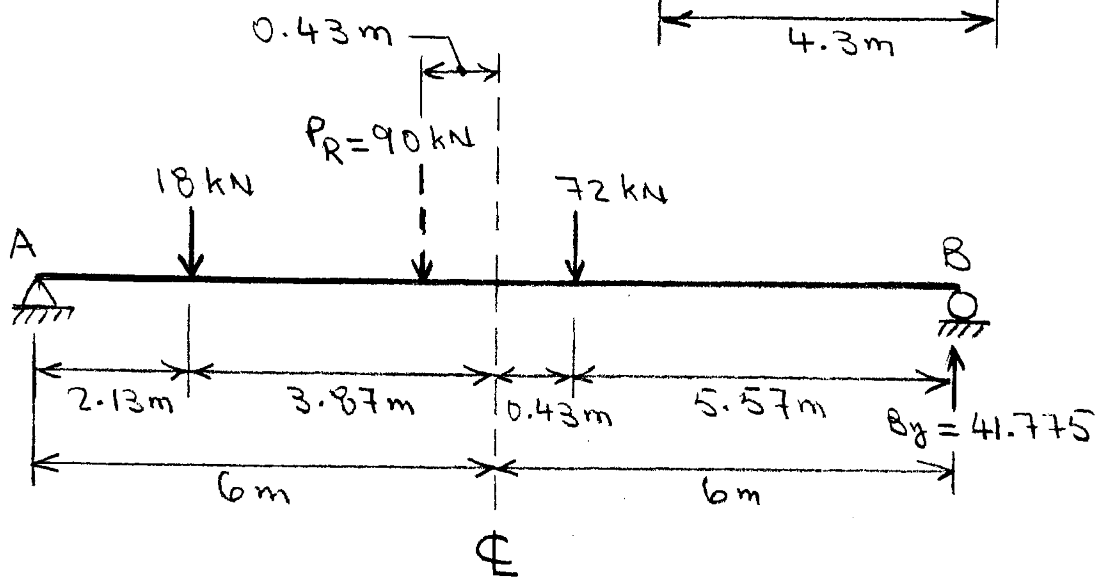
$$S_{B,L} = [20(60) + 10(45)]\left(\frac{-1}{60}\right) = -27.5 \text{ k}$$

Absolute maximum shear = 42.5 k

9.20

$$\bar{x} = \frac{18(4.3)}{90} = 0.86\text{ m}$$



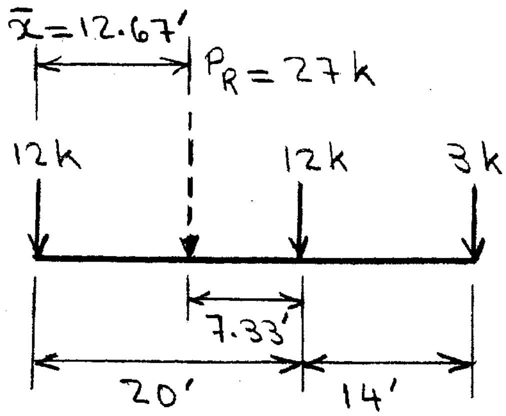


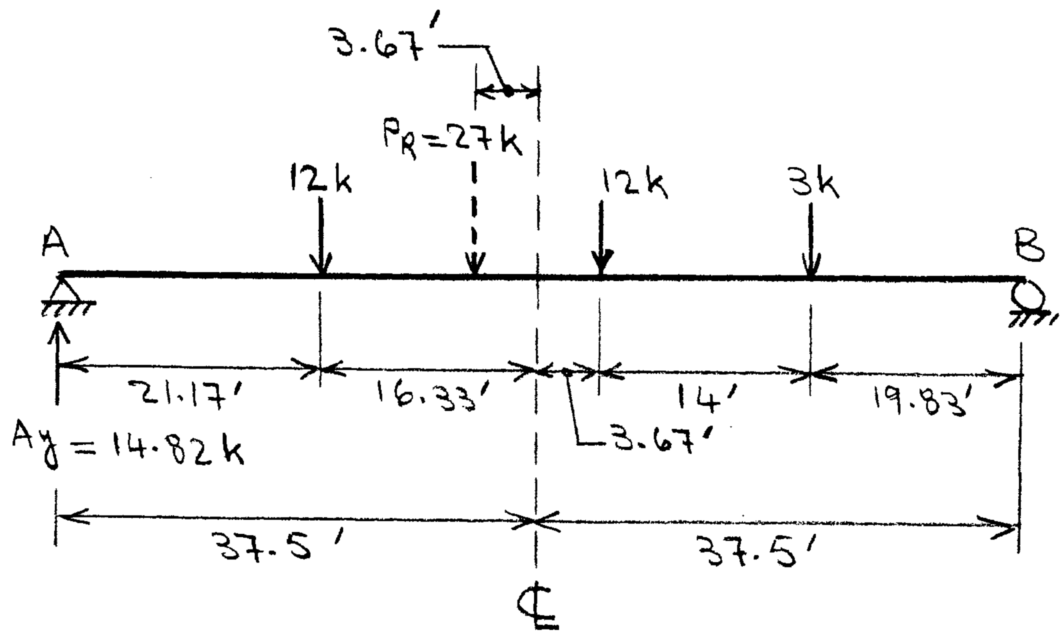
$$B_y = \frac{90(6 - 0.43)}{12} = 41.775\text{ kN}$$

$$M_{max} = 41.775\,(5.57) = \underline{232.7\text{ kN.m}}$$

9.21

$$\bar{x} = \frac{12(20)+3(34)}{27} = 12.67 \text{ ft}$$

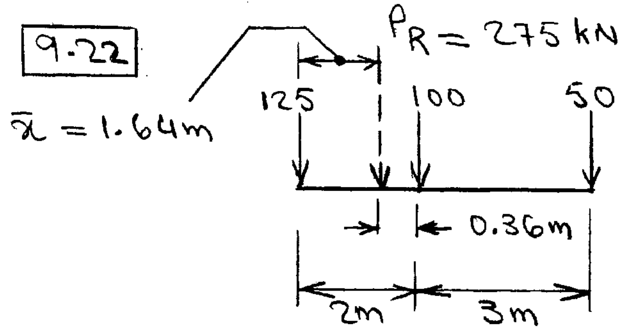




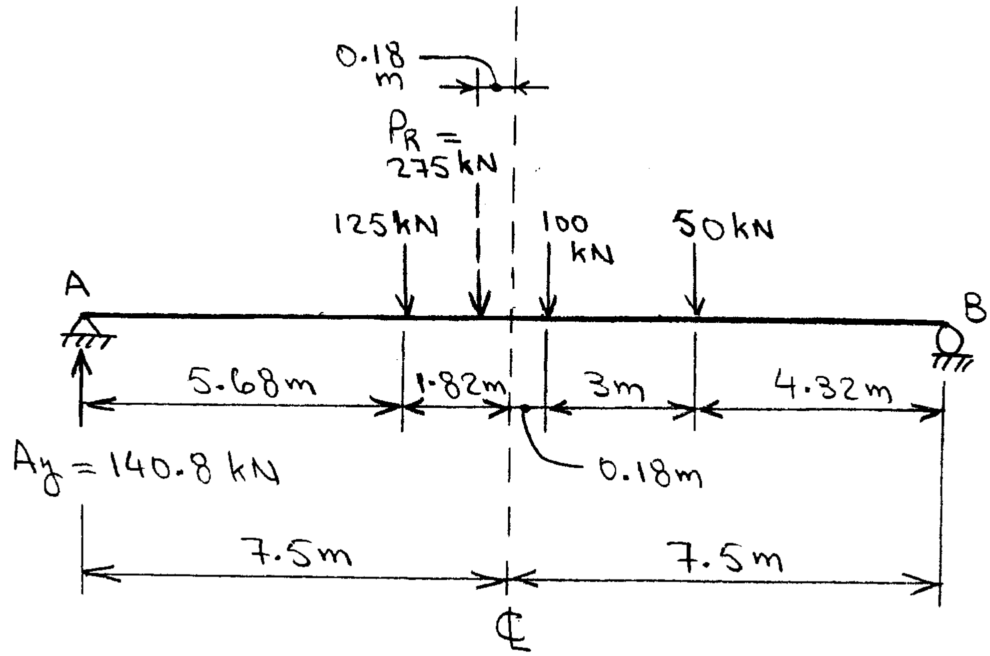
$$A_y = \frac{27(37.5+3.67)}{75} = 14.82 \text{ k}$$

$$M_{max} = 14.82(37.5+3.67) - 12(20) = \underline{370.1 \text{ k-ft}}$$

9.22



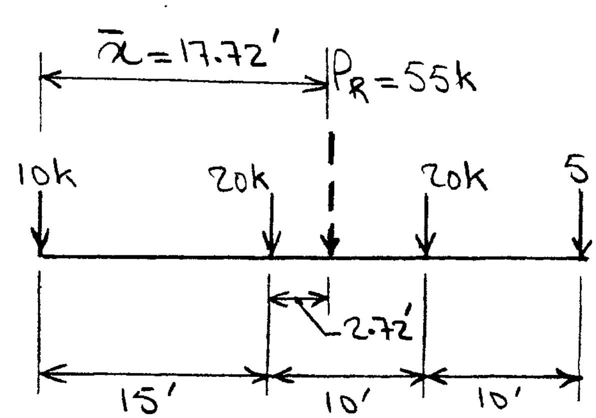
$$\bar{x} = \frac{100(2) + 50(5)}{275} = 1.64 \text{ m}$$



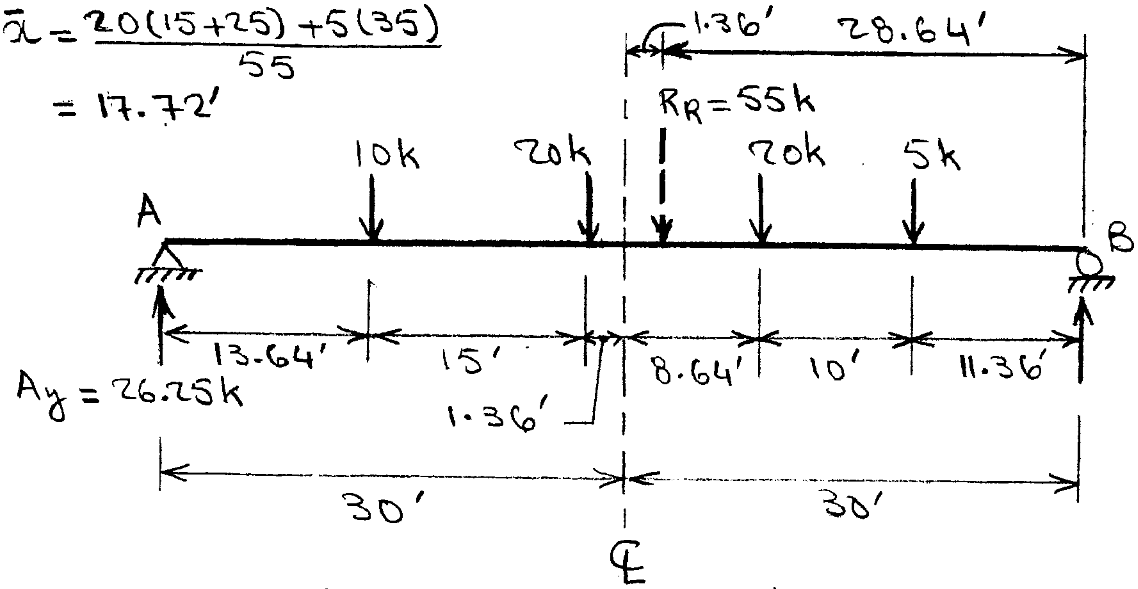
$$A_y = 275\left(\frac{0.18 + 7.5}{15}\right) = 140.8 \text{ kN}$$

$$M_{max} = 140.8(7.5 + 0.18) - 125(2) = \underline{831.3 \text{ kN}\cdot\text{m}}$$

$\boxed{9.23}$



$$\bar{x} = \frac{20(15+25)+5(35)}{55} = 17.72'$$



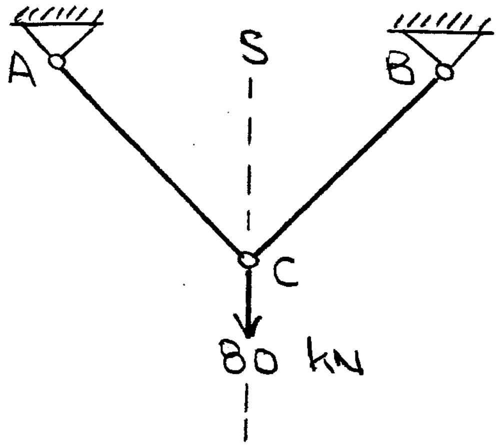
$$A_y = 55(28.64/60) = 26.25k$$

$$M_{max} = 26.25(28.64) - 10(15) = \underline{601.8 \text{ k-ft}}$$

Chapter Ten
Analysis of Symmetric Structures

CHAPTER 10

10.1



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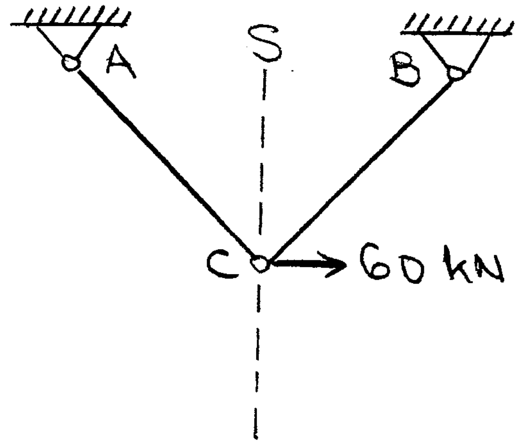
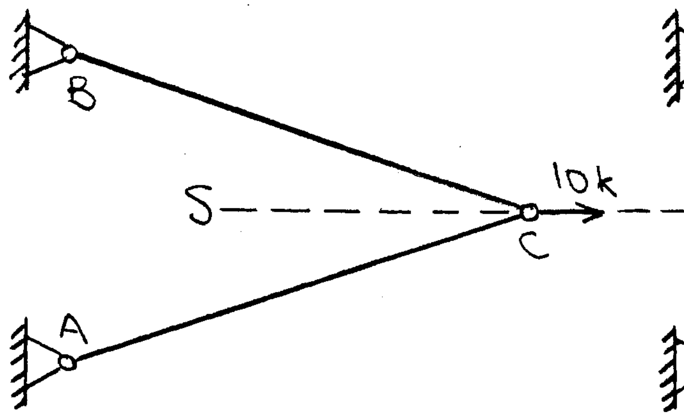
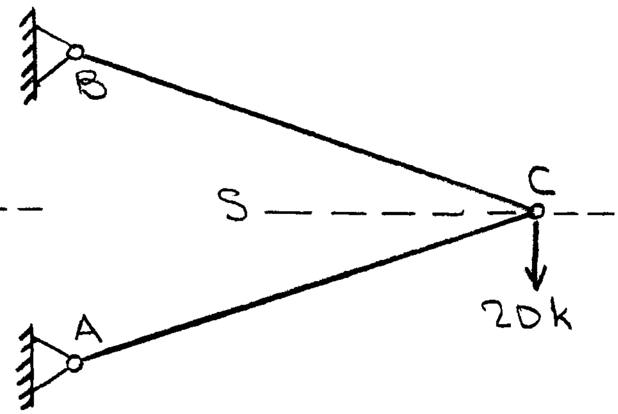


Antisymmetric Loading Component

10.2

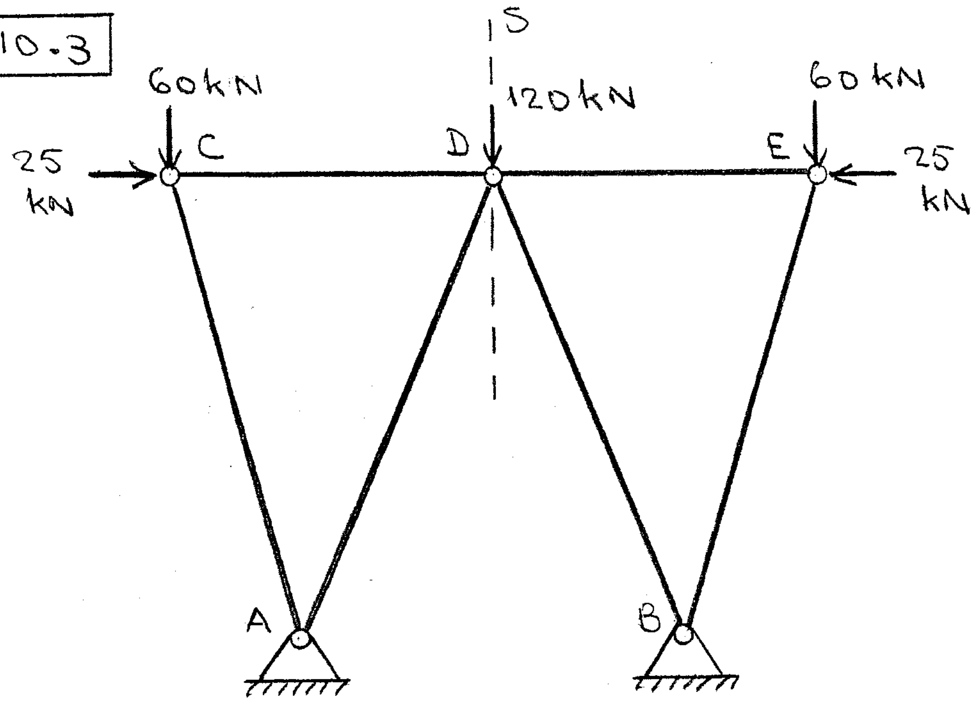


Symmetric Loading Component

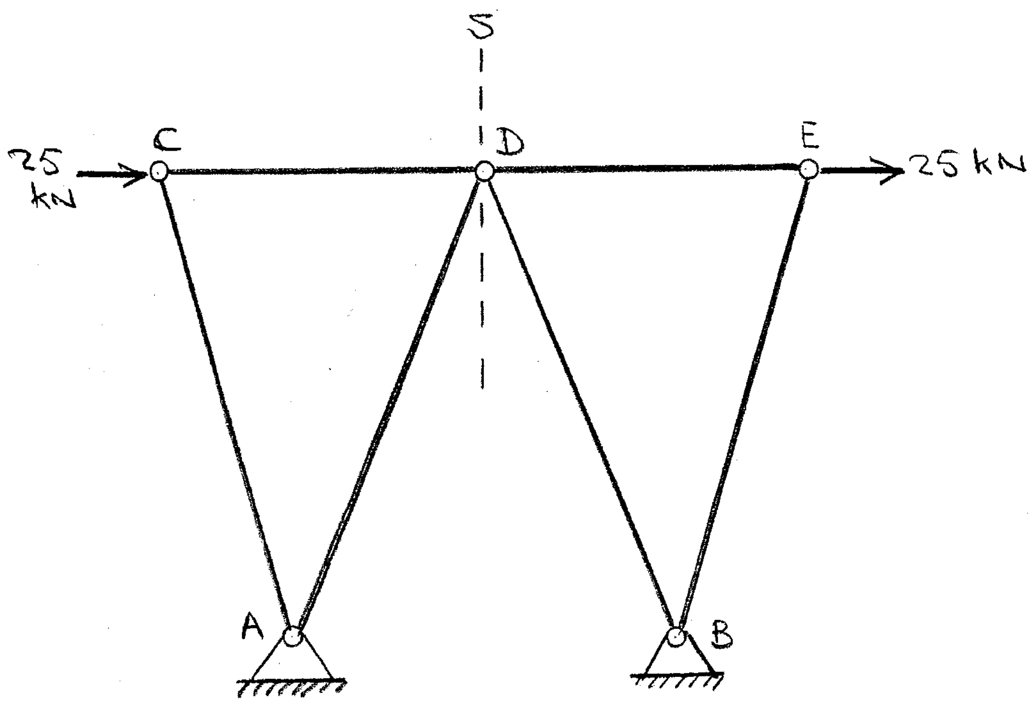


Antisymmetric Loading Component

10.3



Symmetric Loading Component



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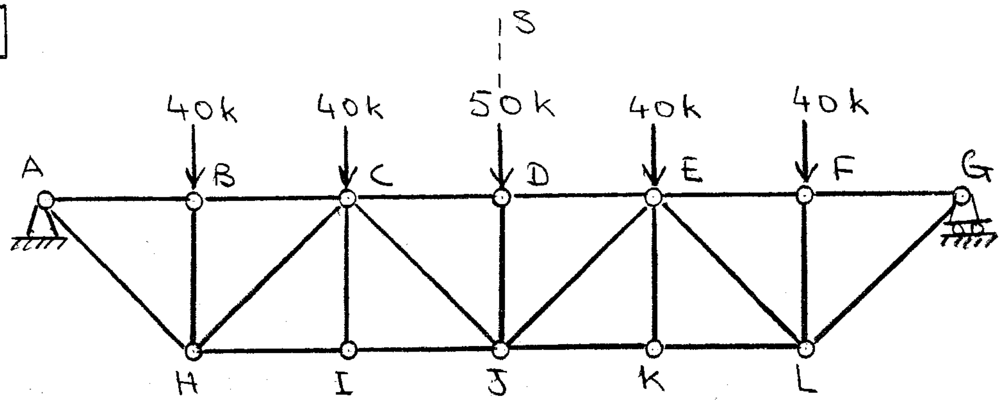
10.4



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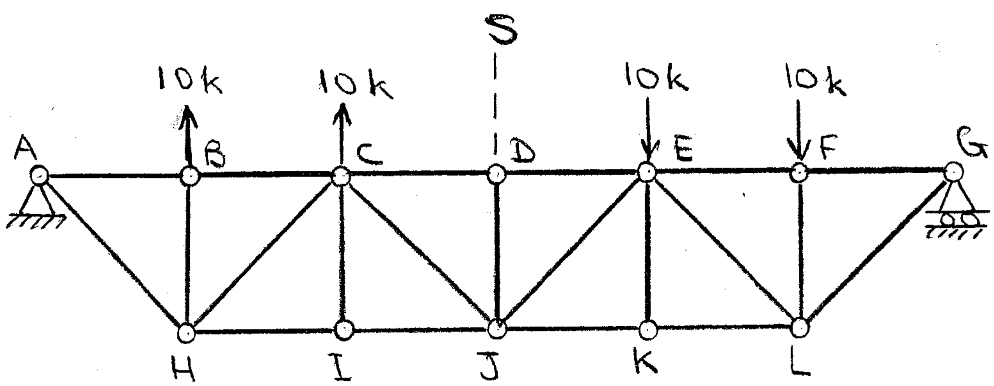
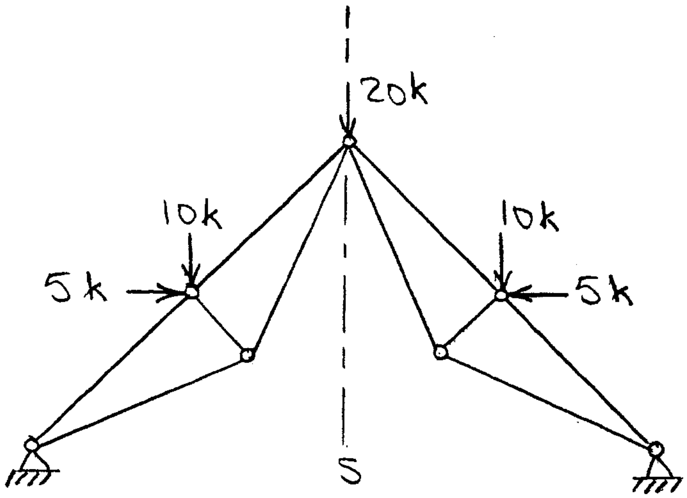


Antisymmetric Loading Component

10.5



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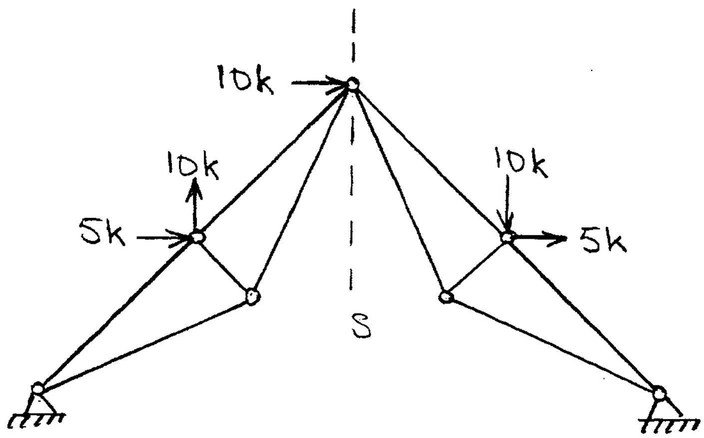
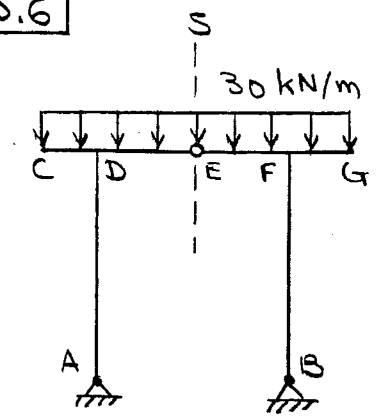


Antisymmetric Loading Component

10.6



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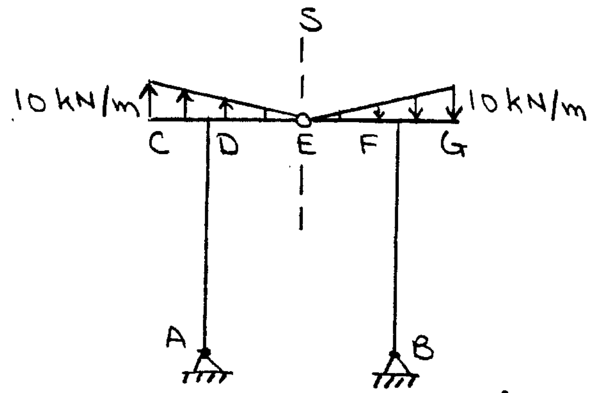
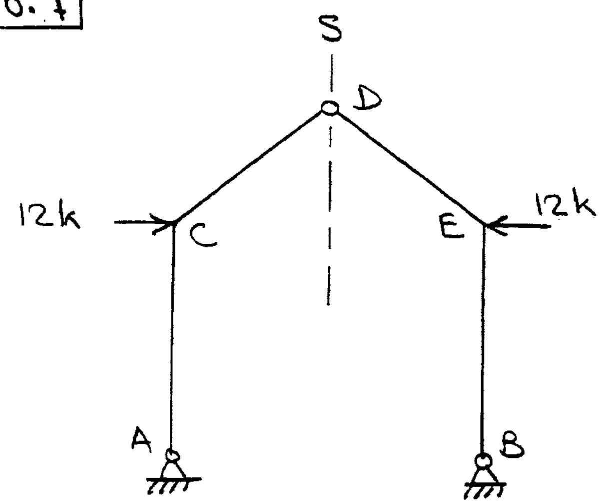


Antisymmetric Loading Component

10.7



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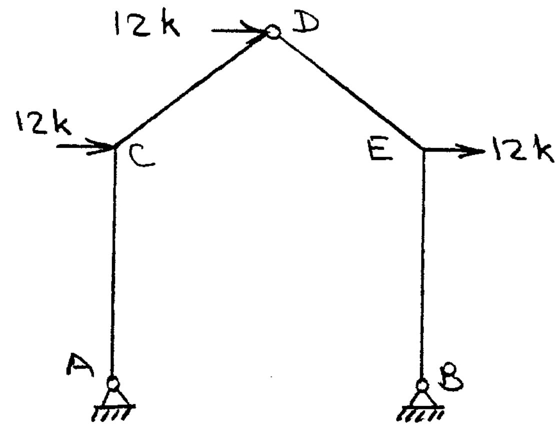
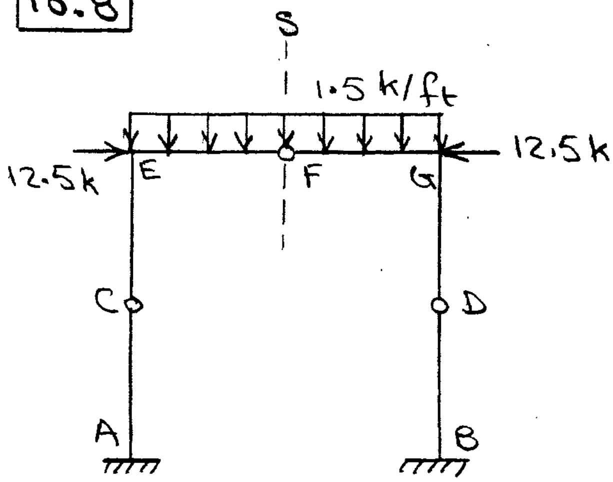


Antisymmetric Loading Component

10.8



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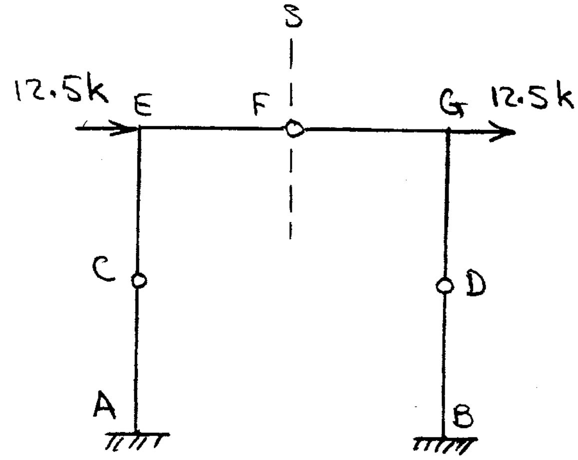
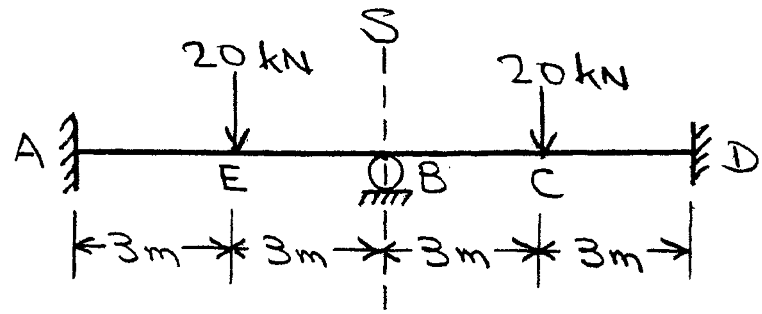


Antisymmetric Loading Component

10.9



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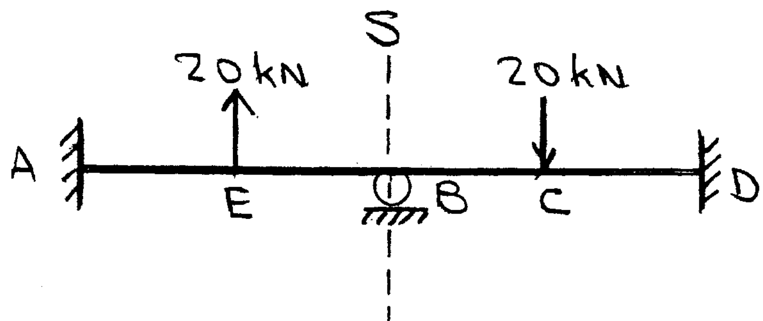

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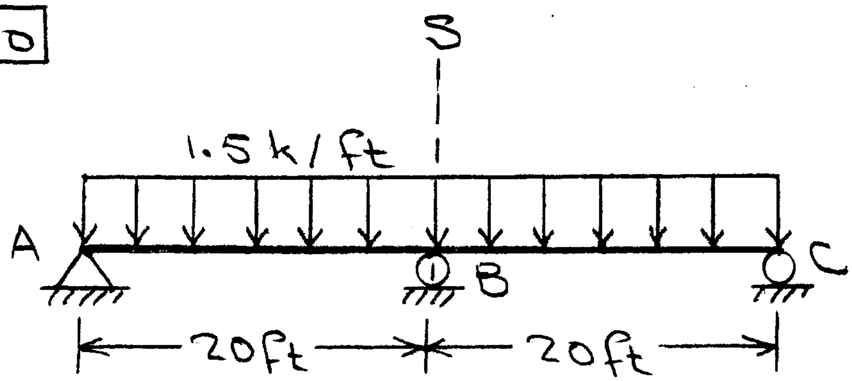

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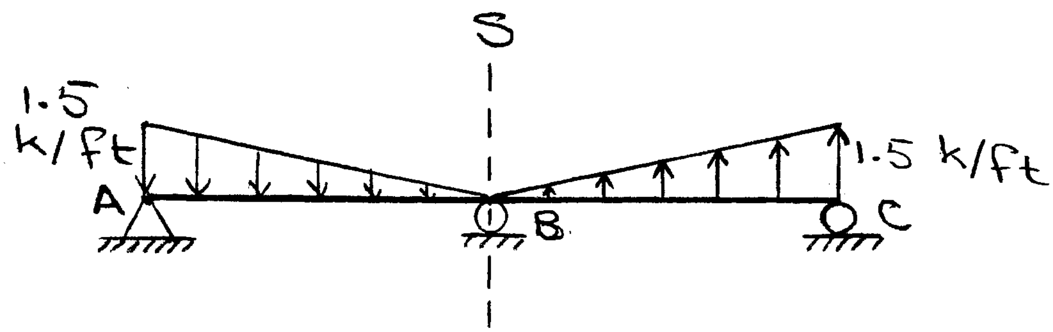

Antisymmetric Loading Component

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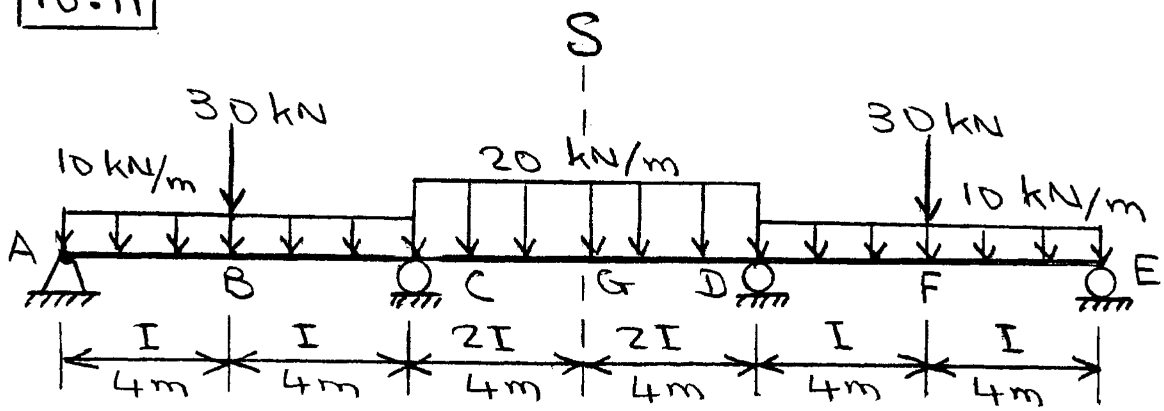

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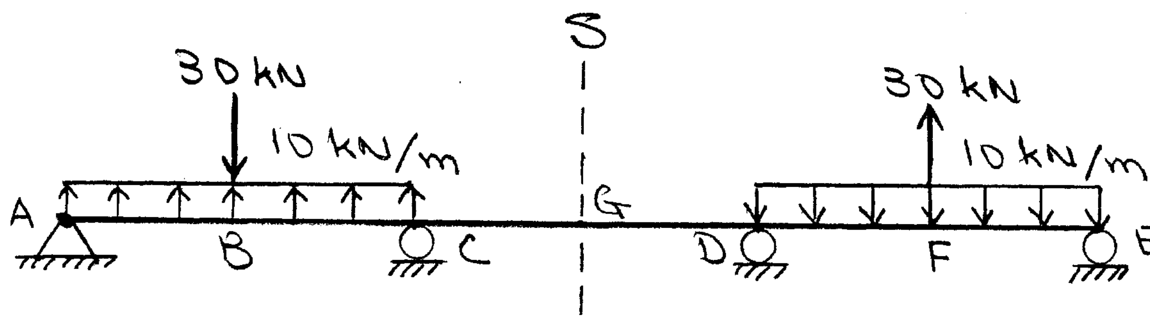

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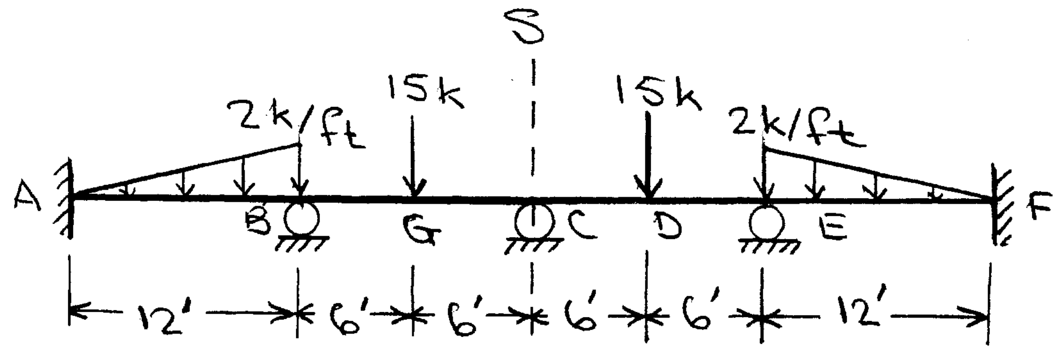

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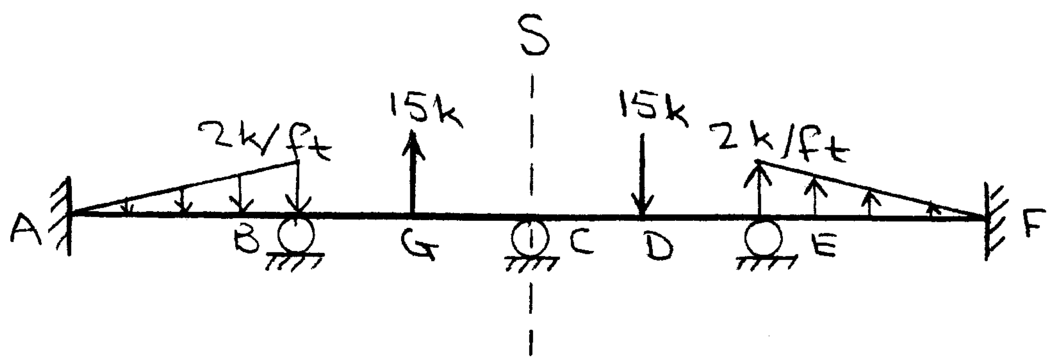

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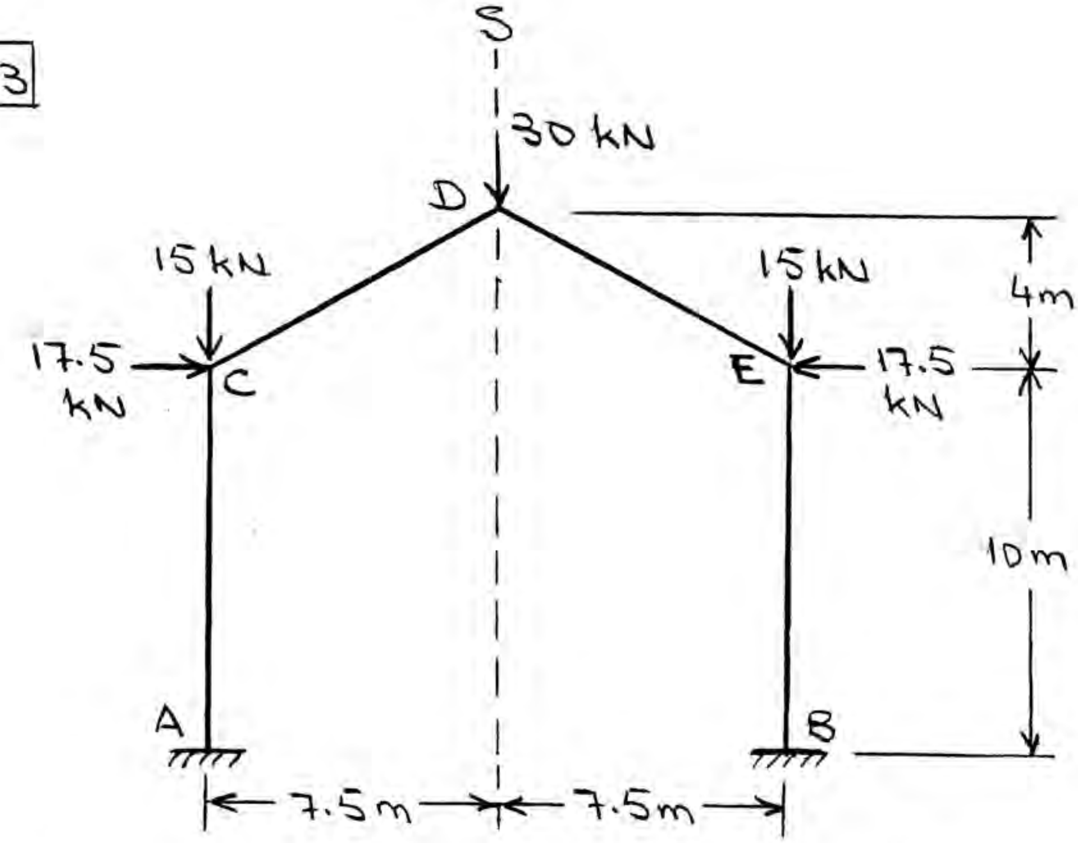

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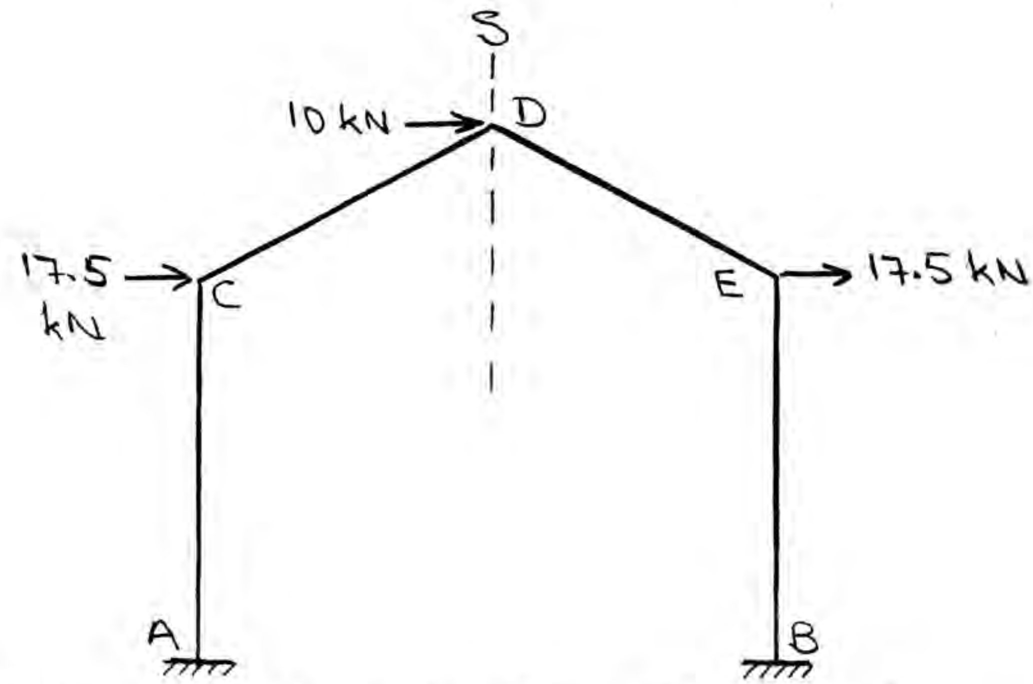

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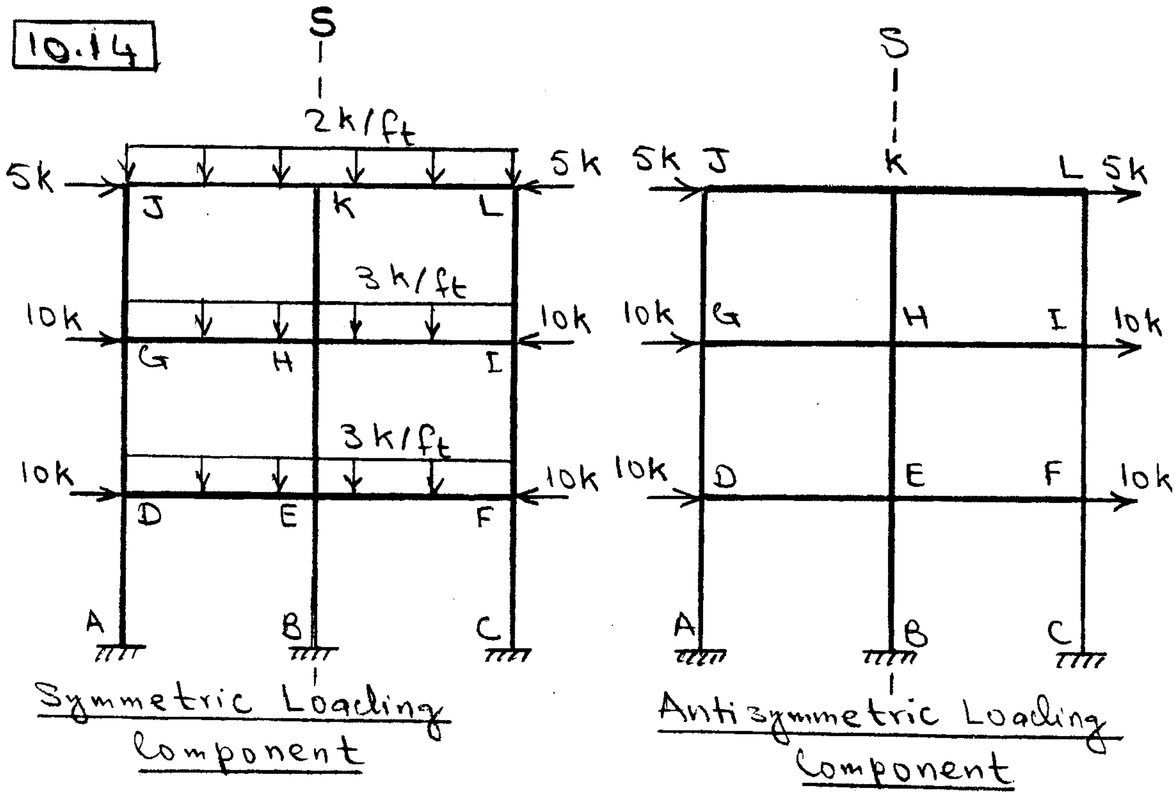
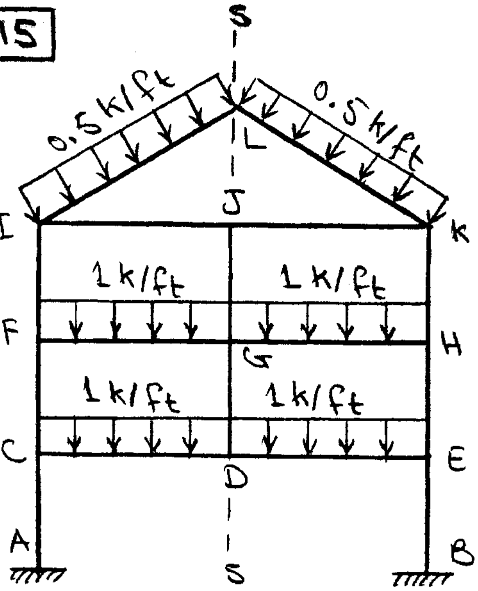

S
2 k/ft
5k
5k
J
K
L
3 k/ft
10k
10k
G
H
I
3 k/ft
10k
10k
D
E
F
A
B
C
Symmetric Loading Component
S
5k
J
K
L
5k
10k
G
H
I
10k
10k
D
E
F
10k
A
B
C
Antisymmetric Loading Component

10.15



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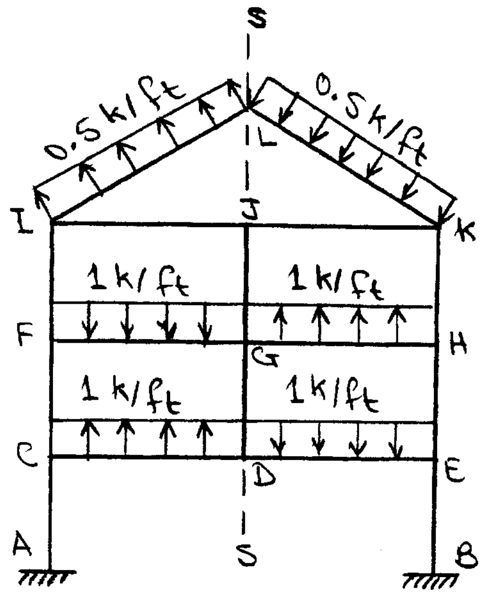
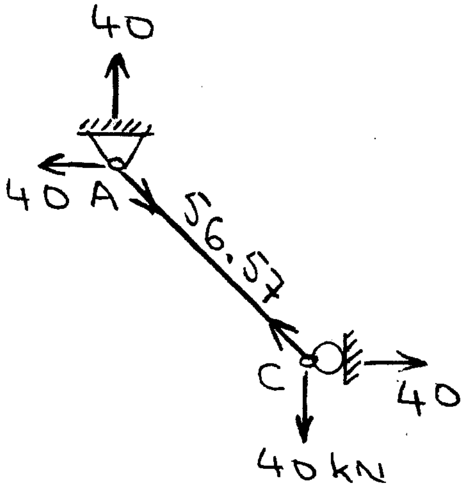
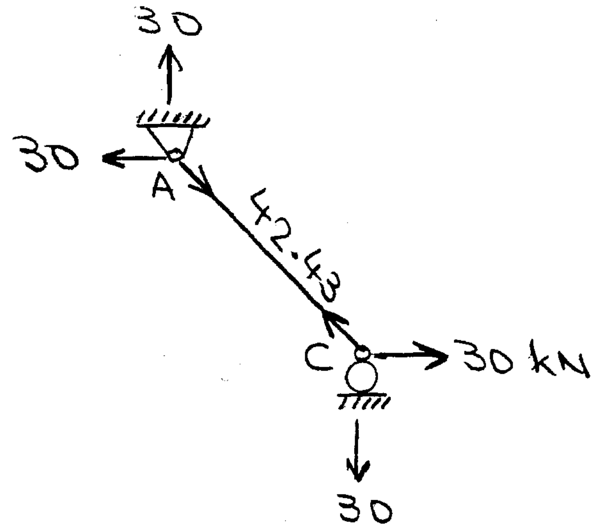


Antisymmetric Loading Component

10.16



Symmetric



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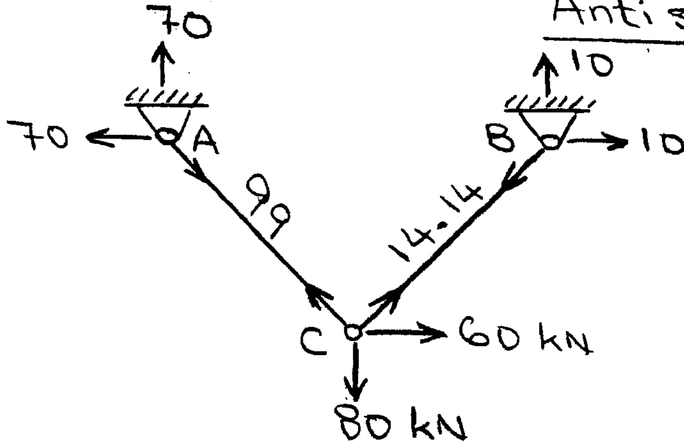
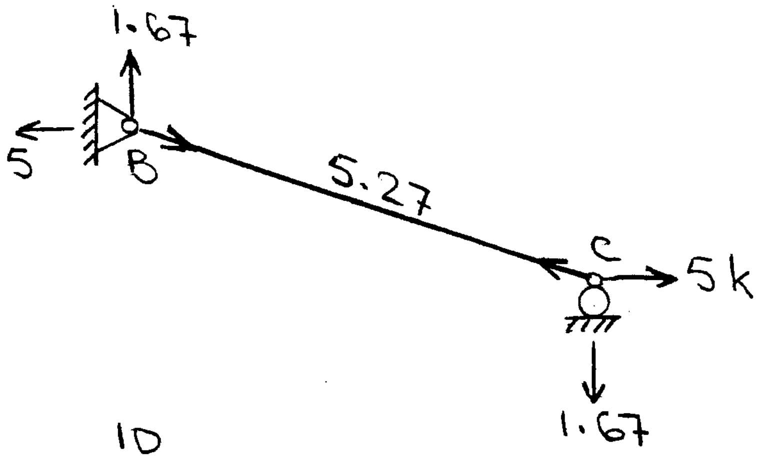


Member Forces

10.17



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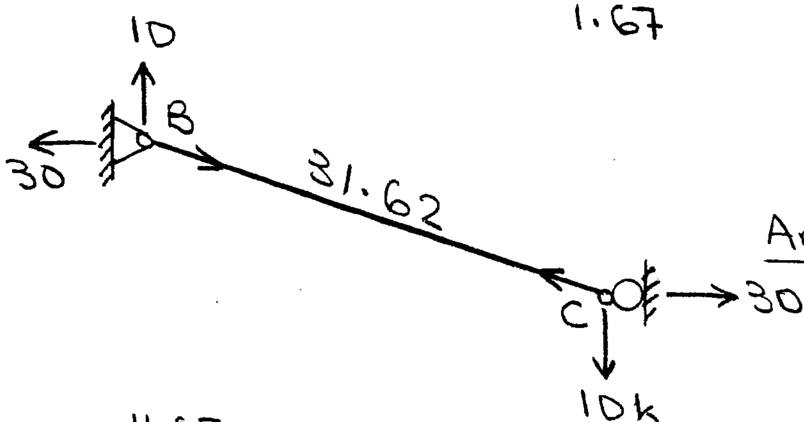

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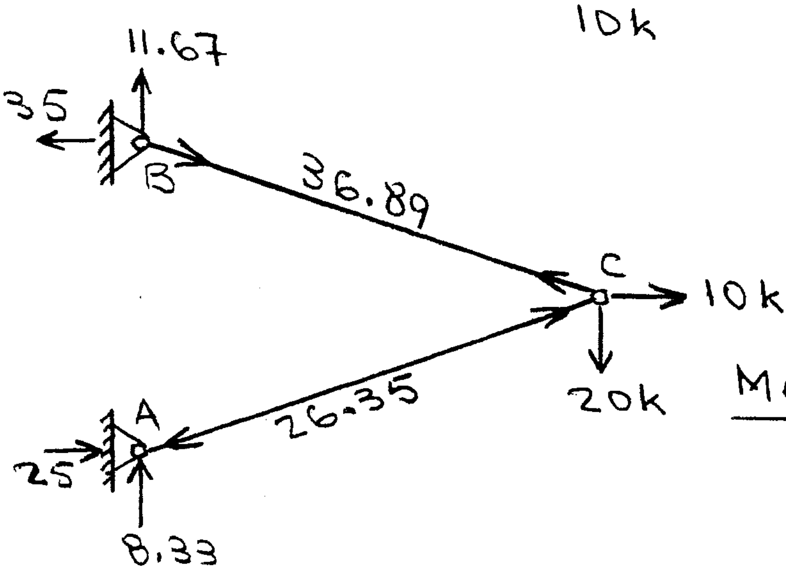
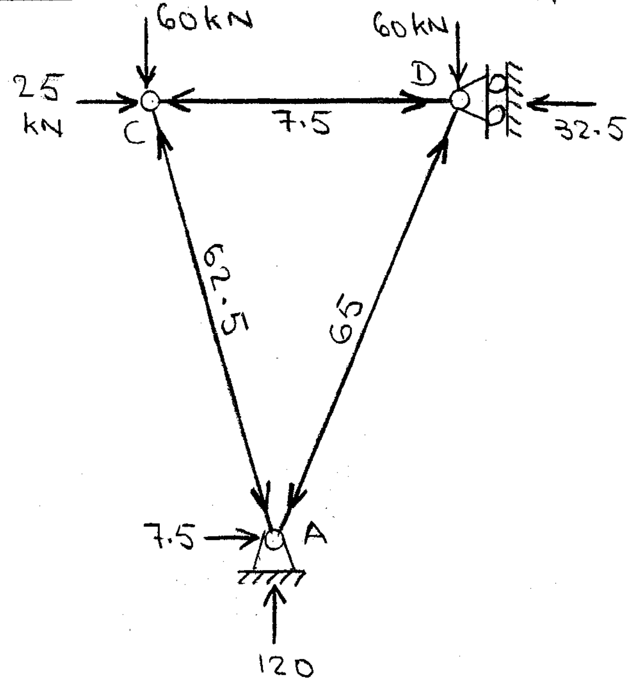
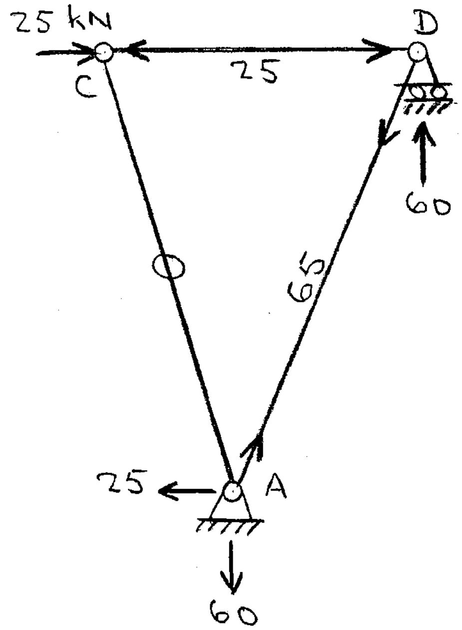


Member Forces

10.18 See Solution of Problem 10.3



Symmetric



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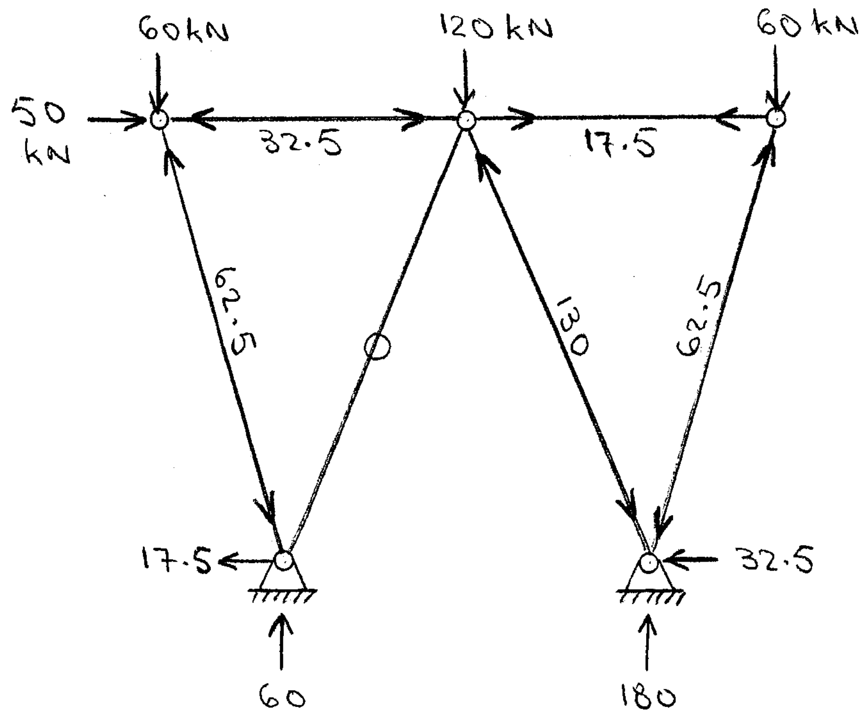
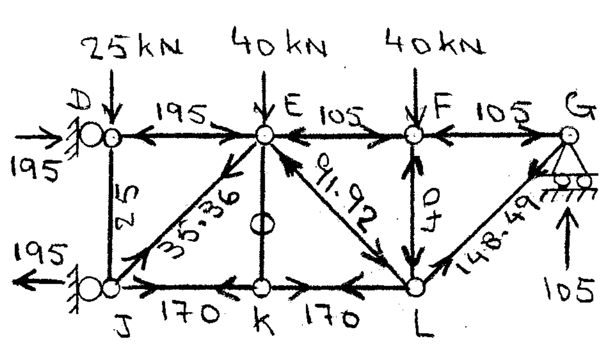


Member Forces

10.19 See solution of Problem 10.4.



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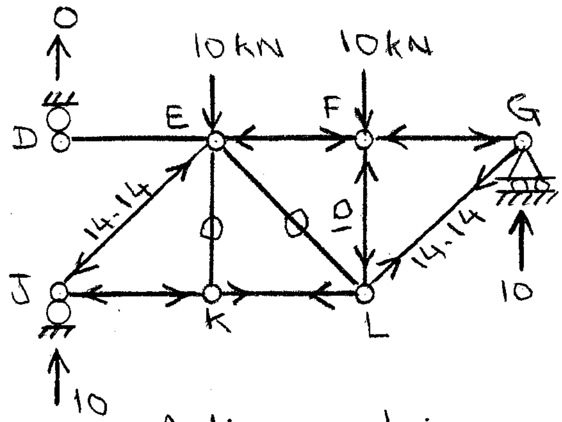

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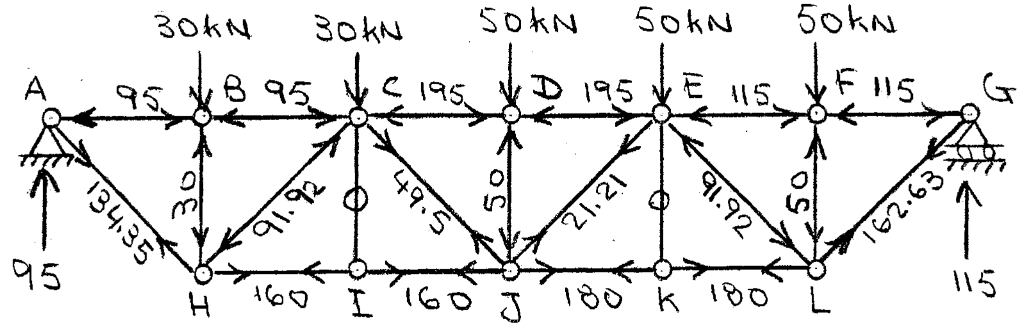
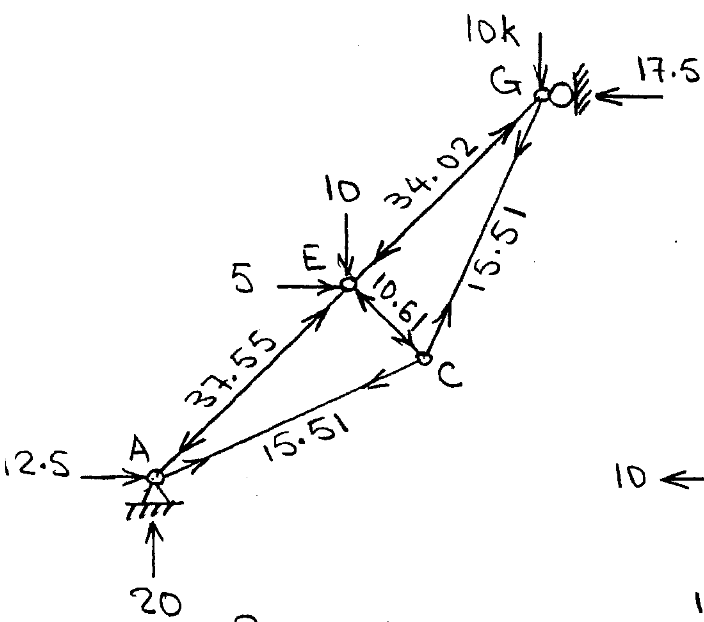


Member Forces

10.20 See solution of Problem 10.5.



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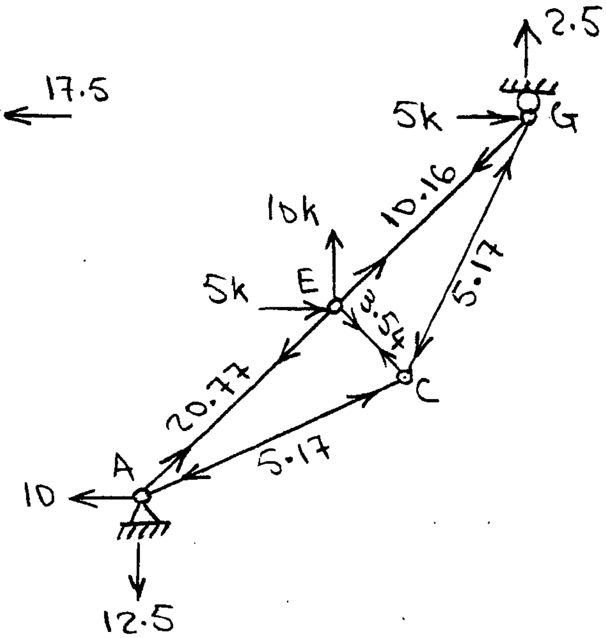

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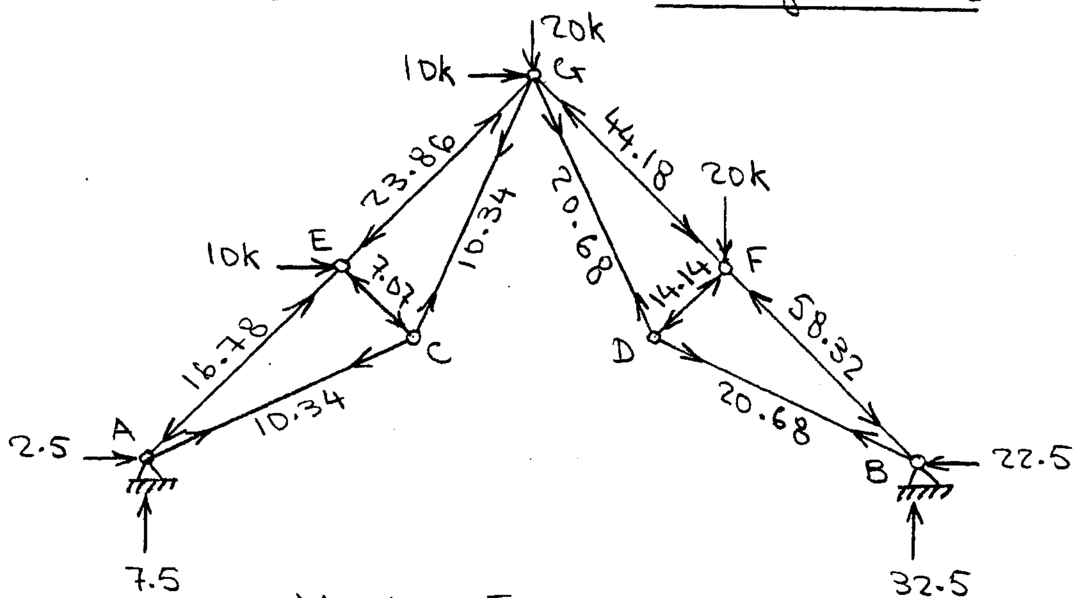
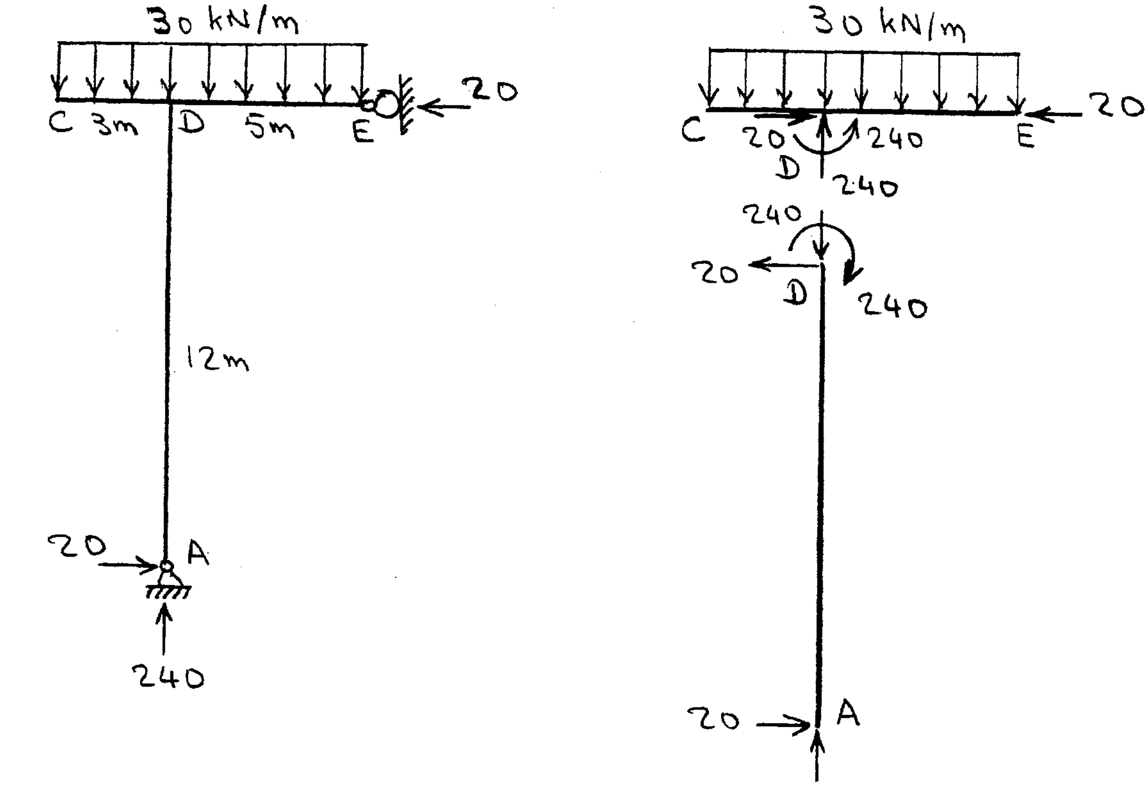


Member Forces

10.21 See solution of Problem 10.6.



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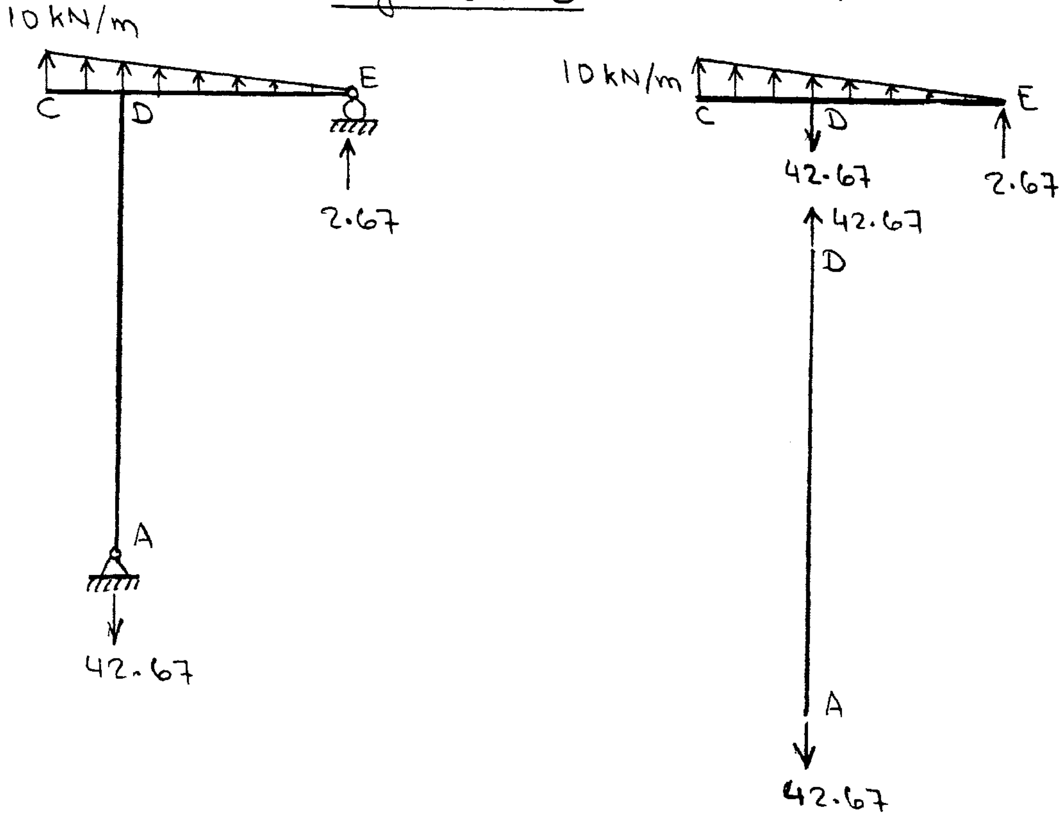

Antisymmetric

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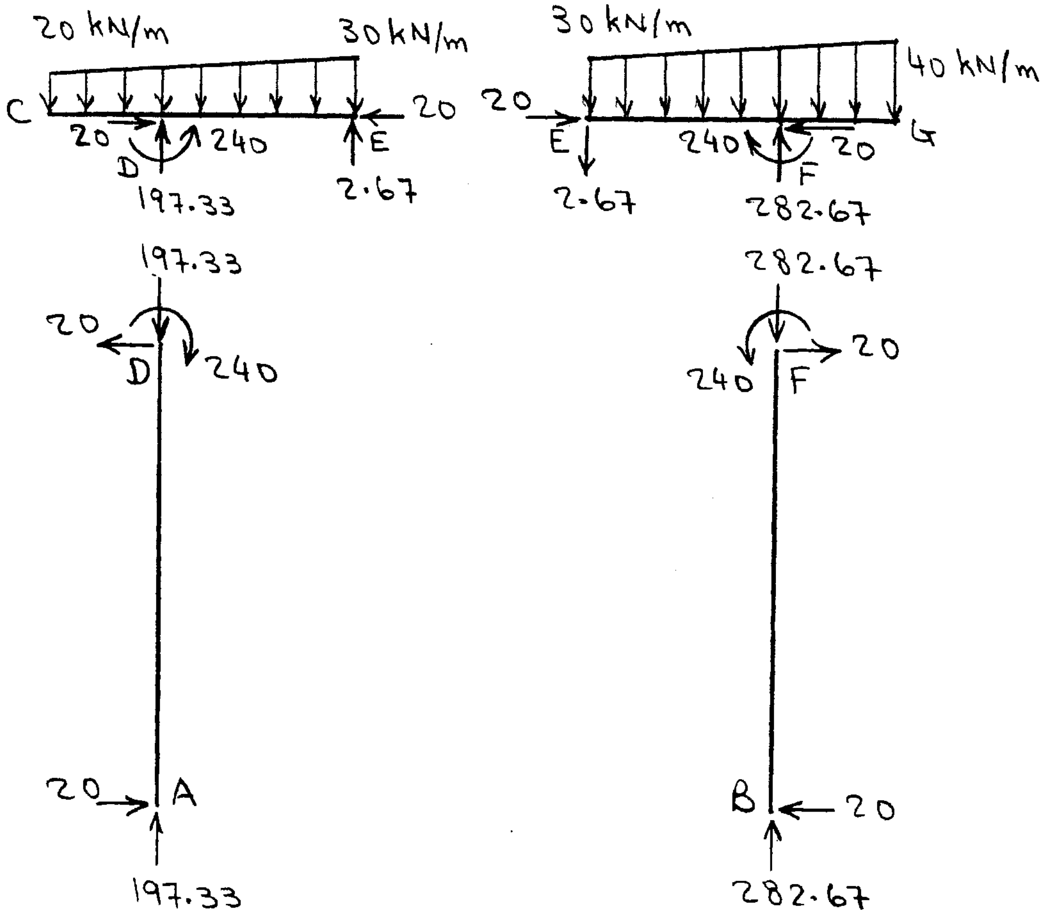
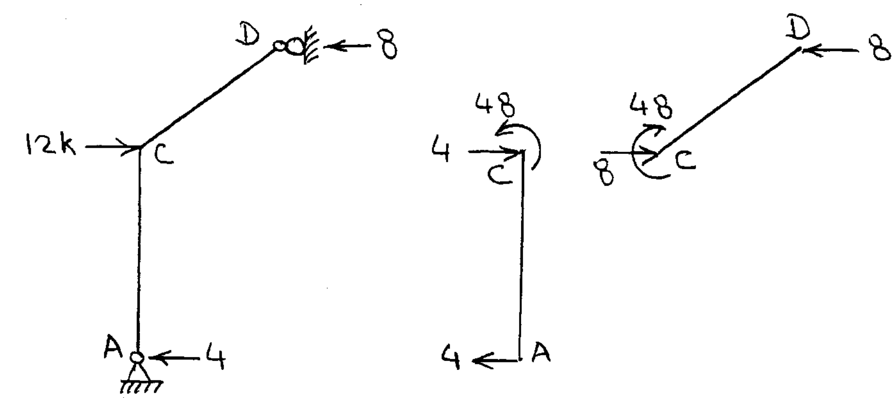
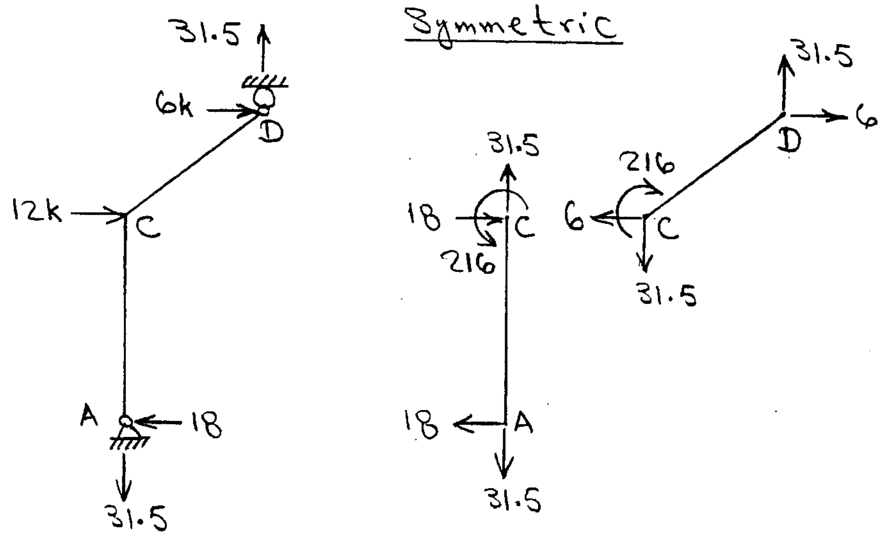


Member End Forces

10.22 See solution of Problem 10.7.



Symmetric



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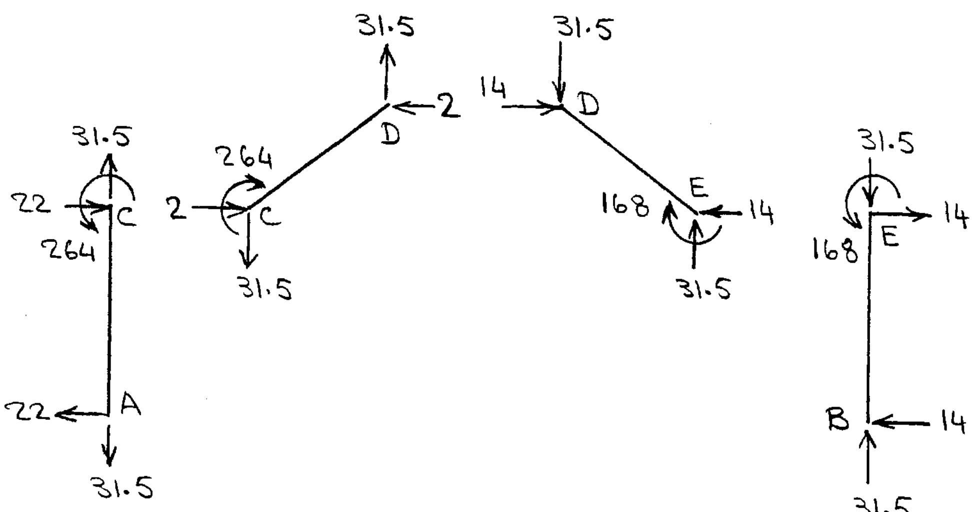
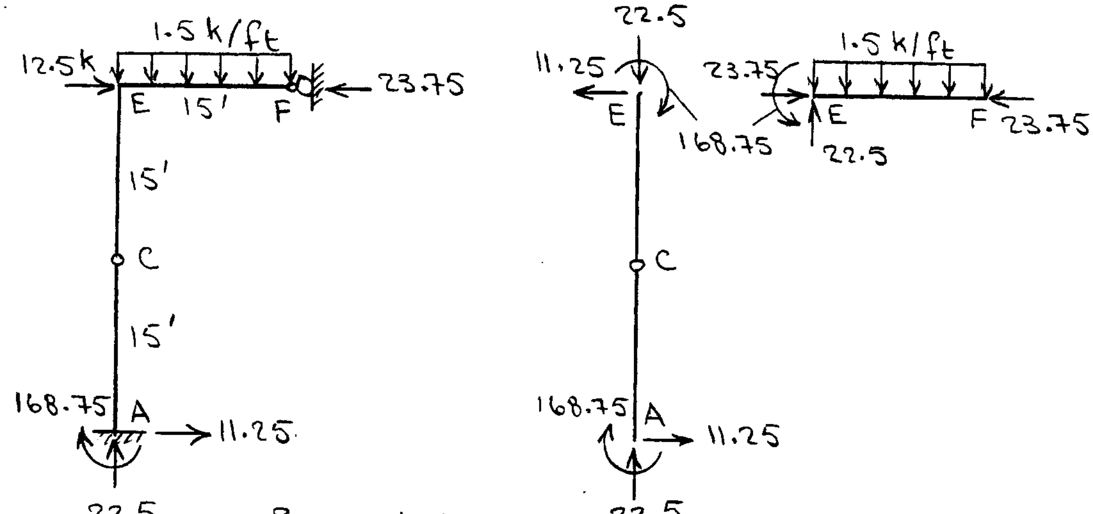
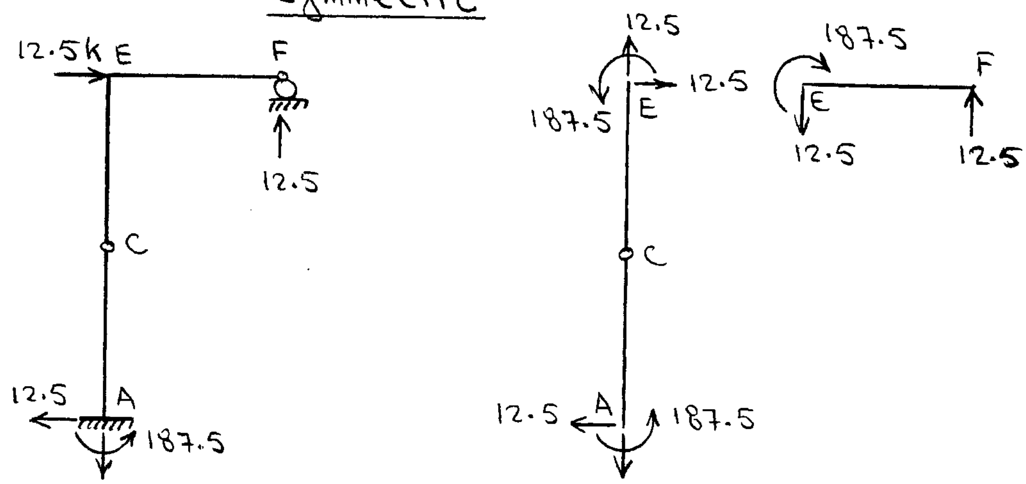


Member End Forces

$\boxed{10.23}$ See solution of Problem 10.8.



Symmetric



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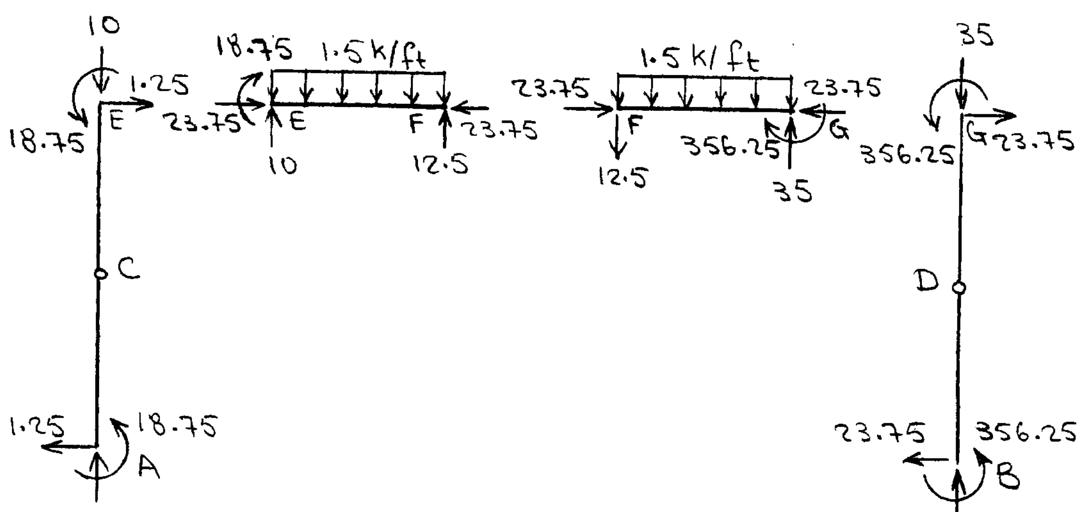
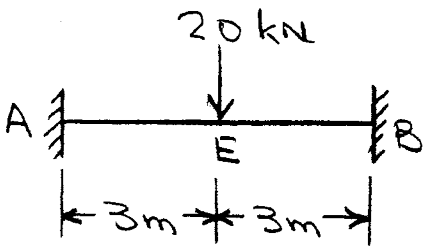
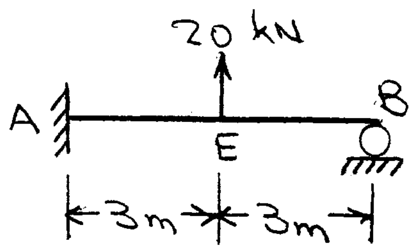


Member End Forces

10.24 See solution of Problem 10.9.

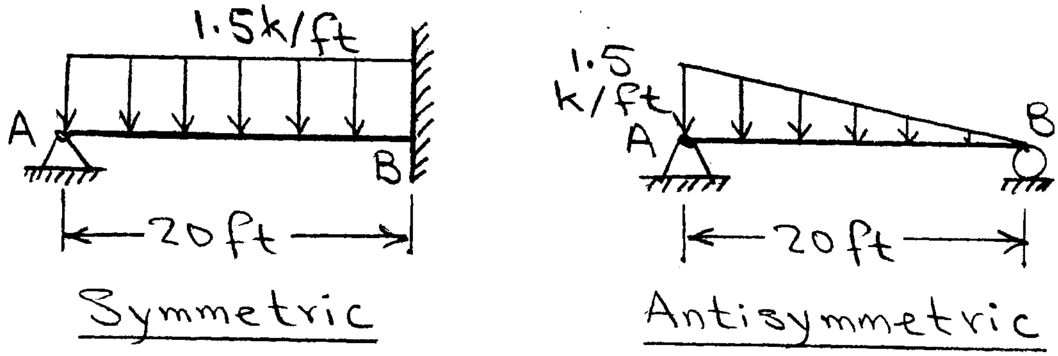


Symmetric

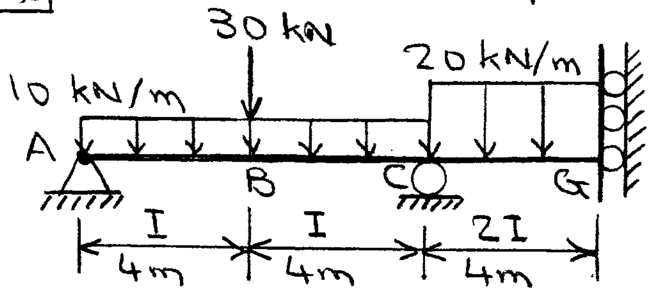


Antisymmetric

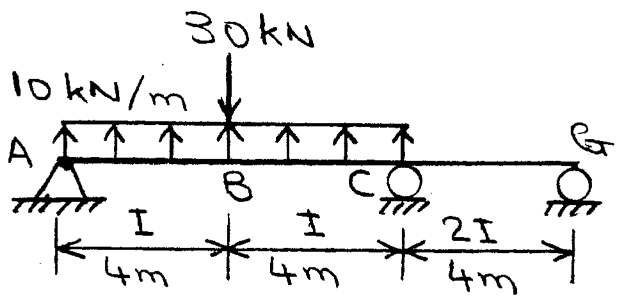
10.25 See solution of Problem 10.10.



10.26 See solution of Problem 10.11

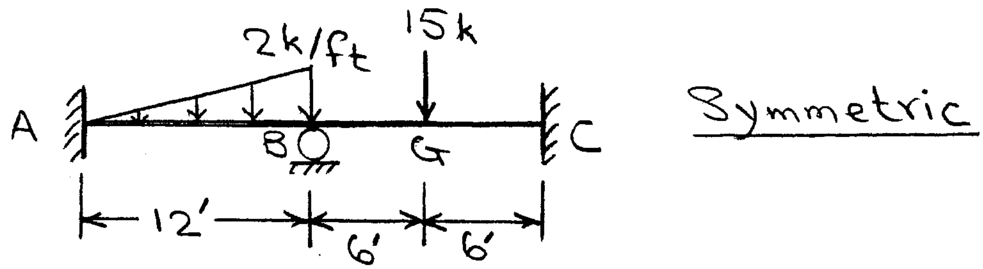


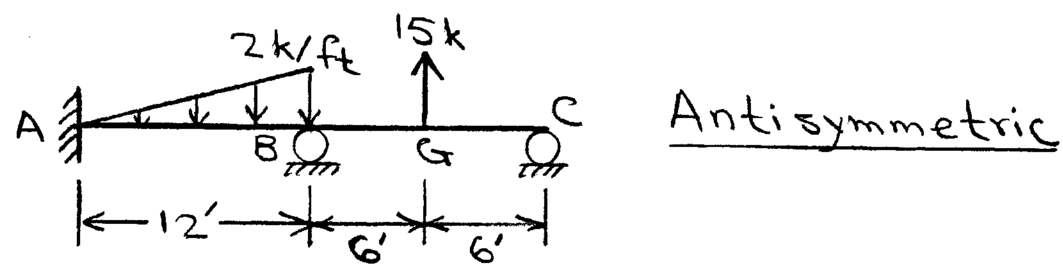
Symmetric



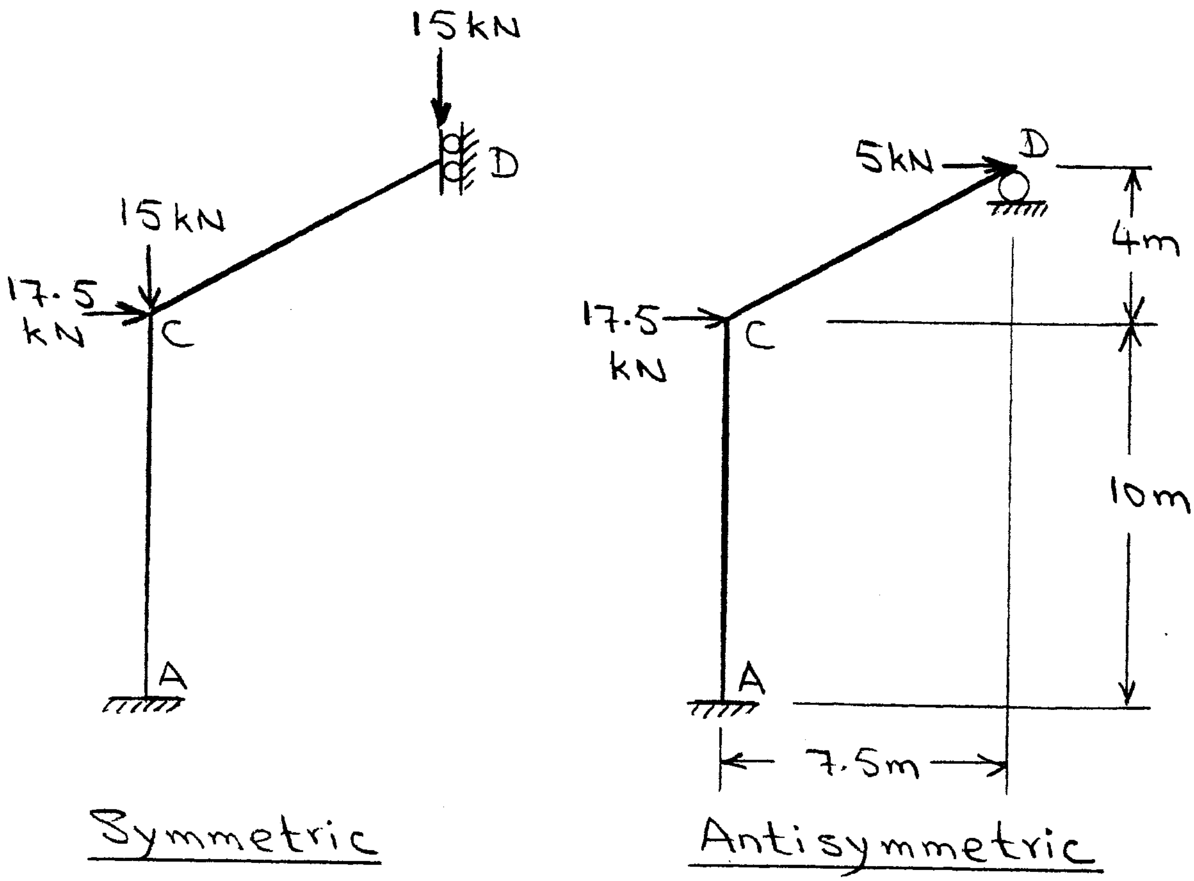
Antisymmetric

$\boxed{10.27}$ See solution of Problem 10.12.

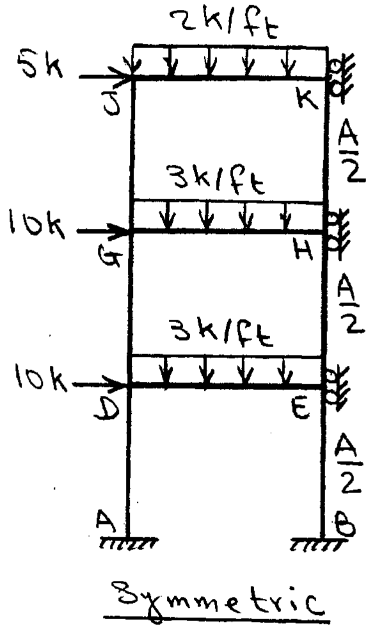


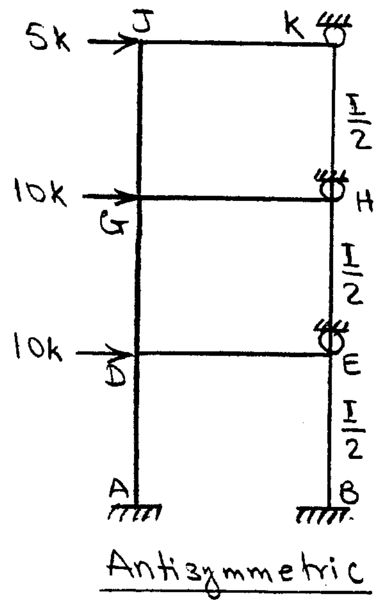


10.28 See solution of Problem 10.13.

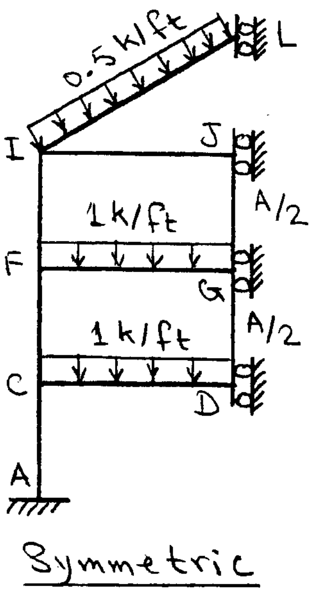


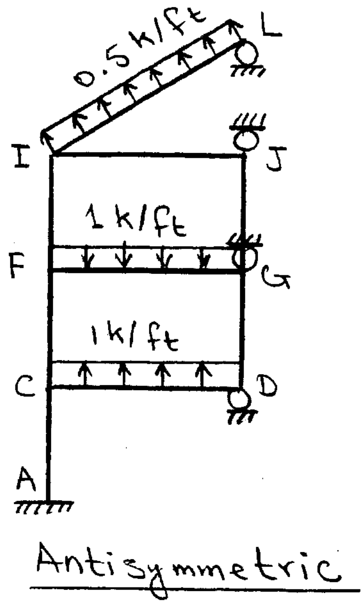
10.29 See solution of Problem 10.14.





10.30 See solution of Problem 10.15.

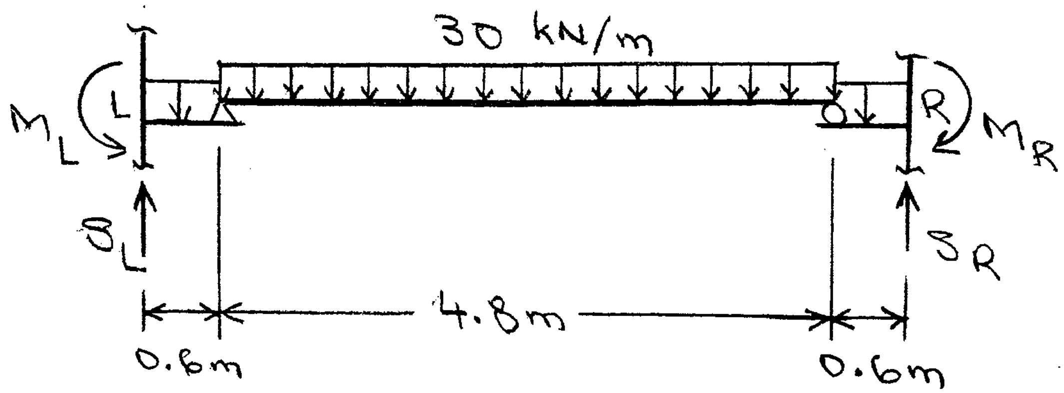


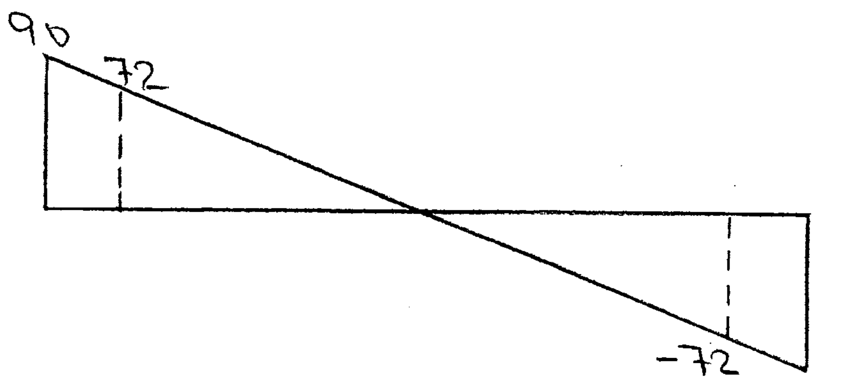


Chapter Twelve
Approximate Analysis of Rectangular Building Frames

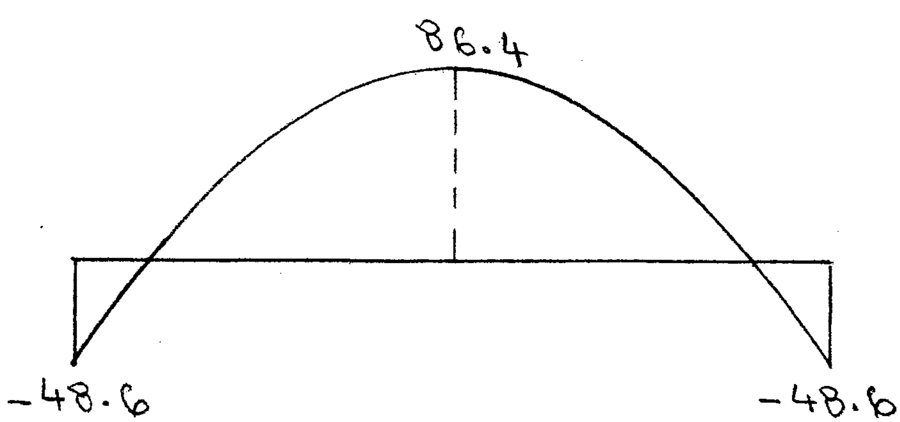
CHAPTER 12

12.1



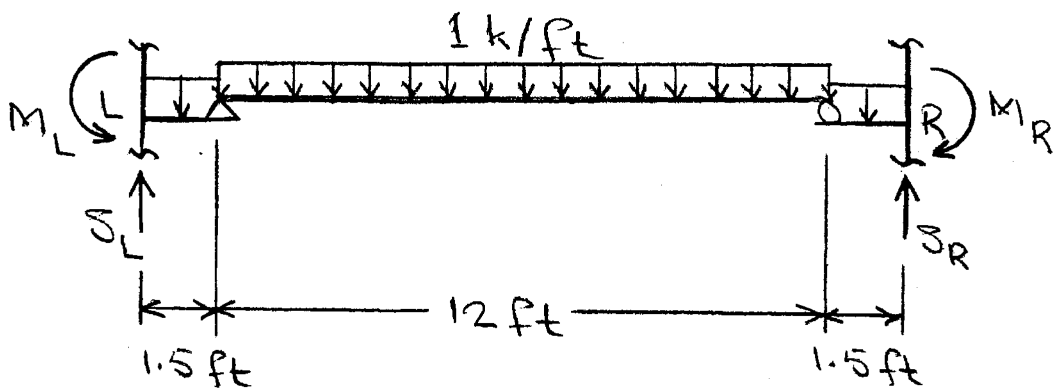


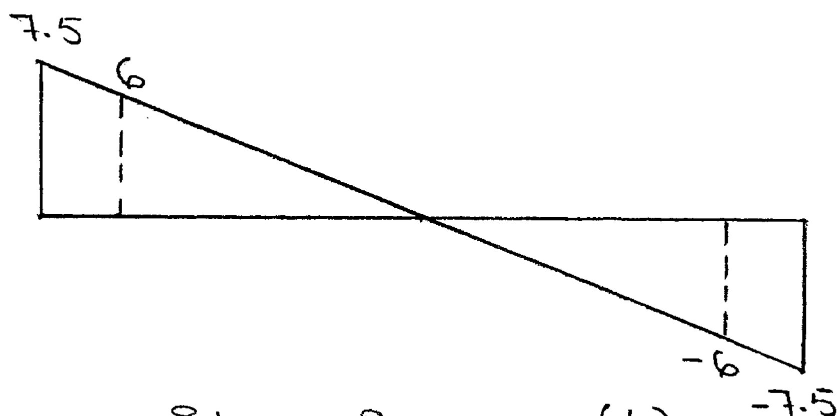
Shear Diagram (kN)



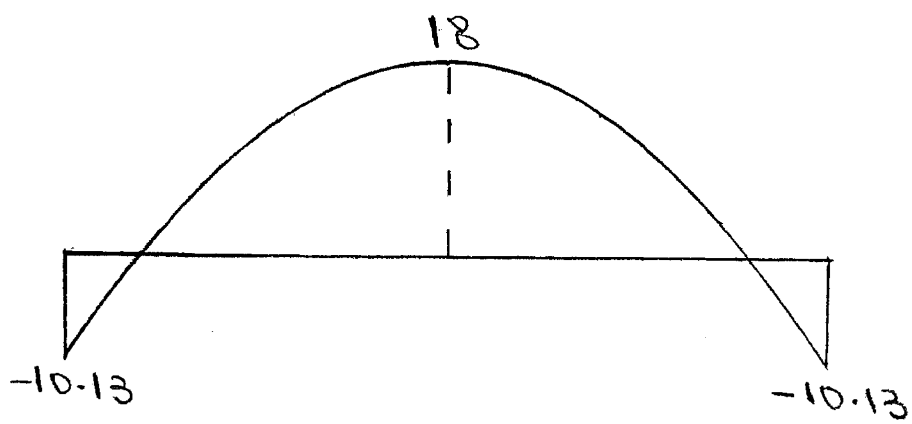
Bending Moment Diagram (kN.m)

12.2



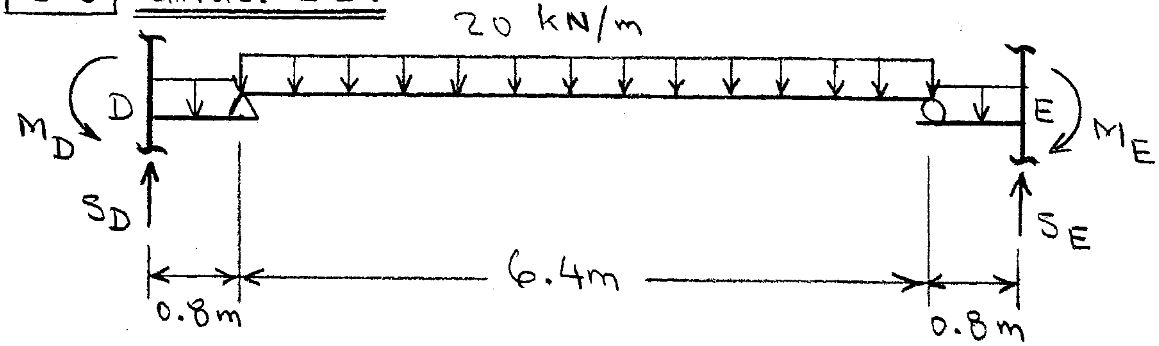


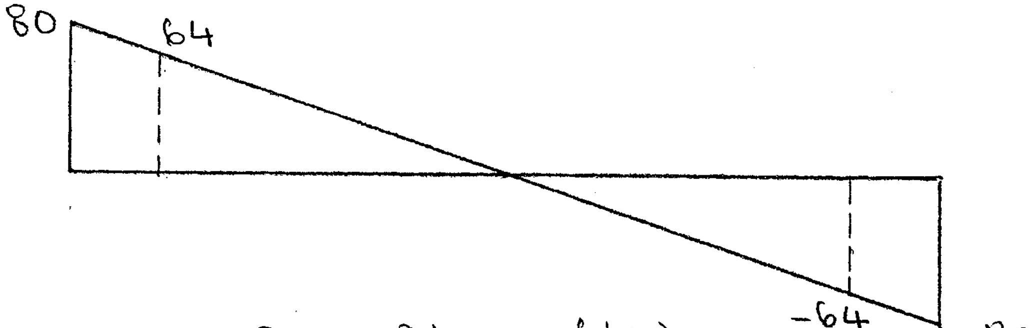
Shear Diagram (k)



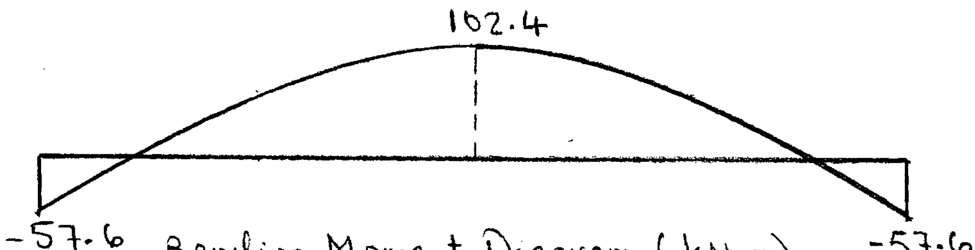
Bending Moment Diagram (k-ft)

12.3 Girder DE:



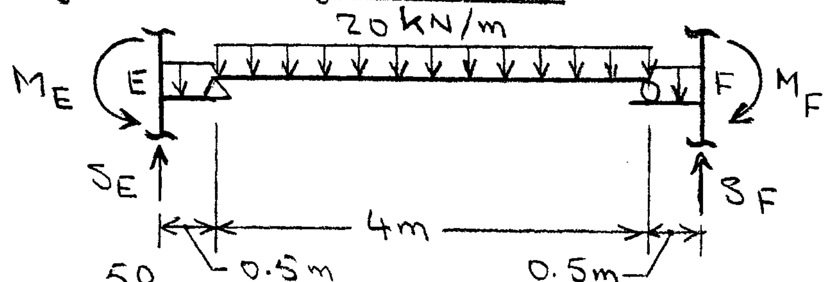


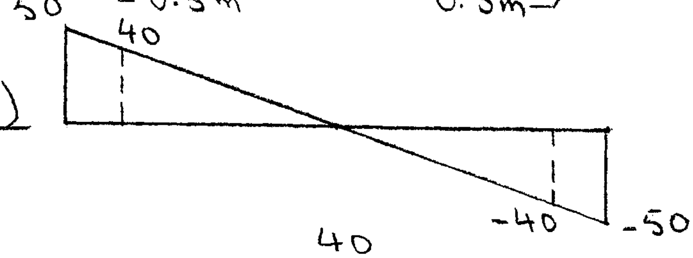
Shear Diagram (kN)



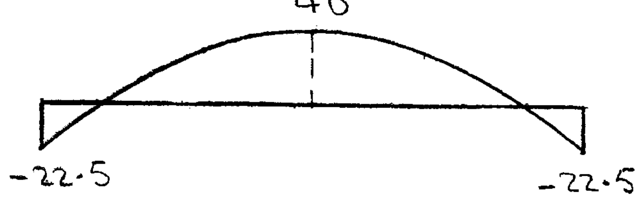
Bending Moment Diagram (kN.m)

Girder EF:



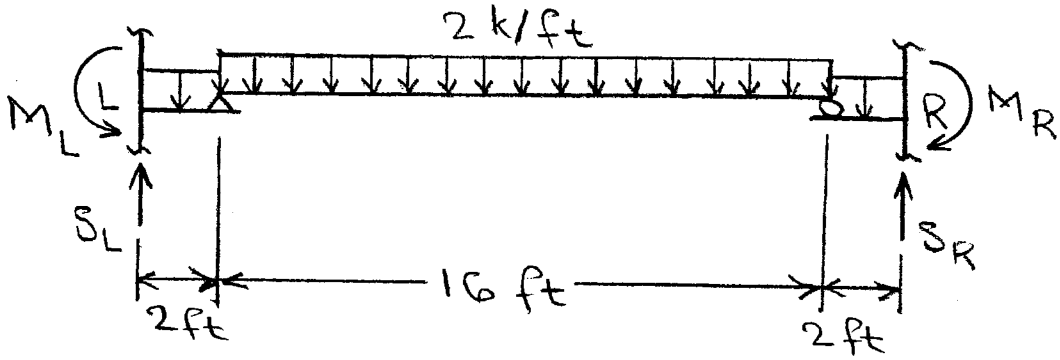


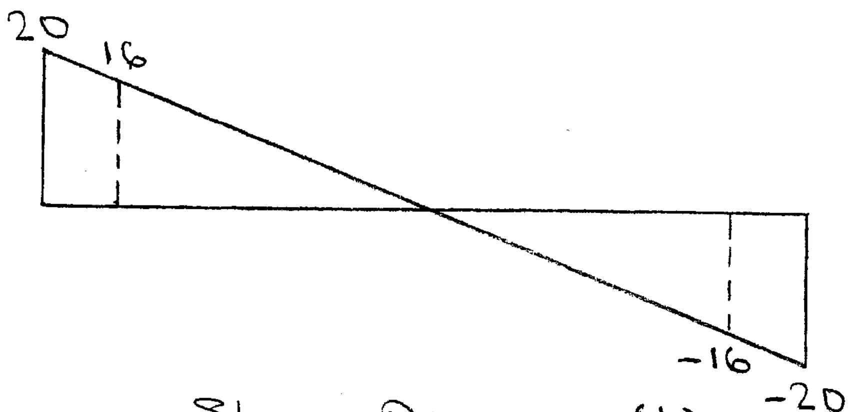
Shear Diagram (kN)



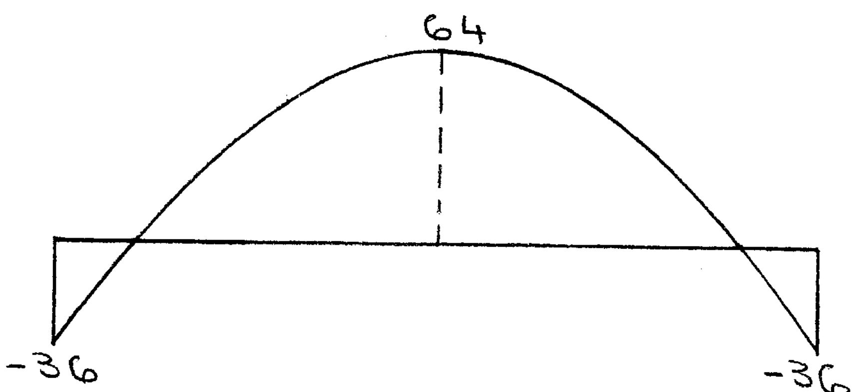
Bending Moment Diagram (kN.m)

12.4



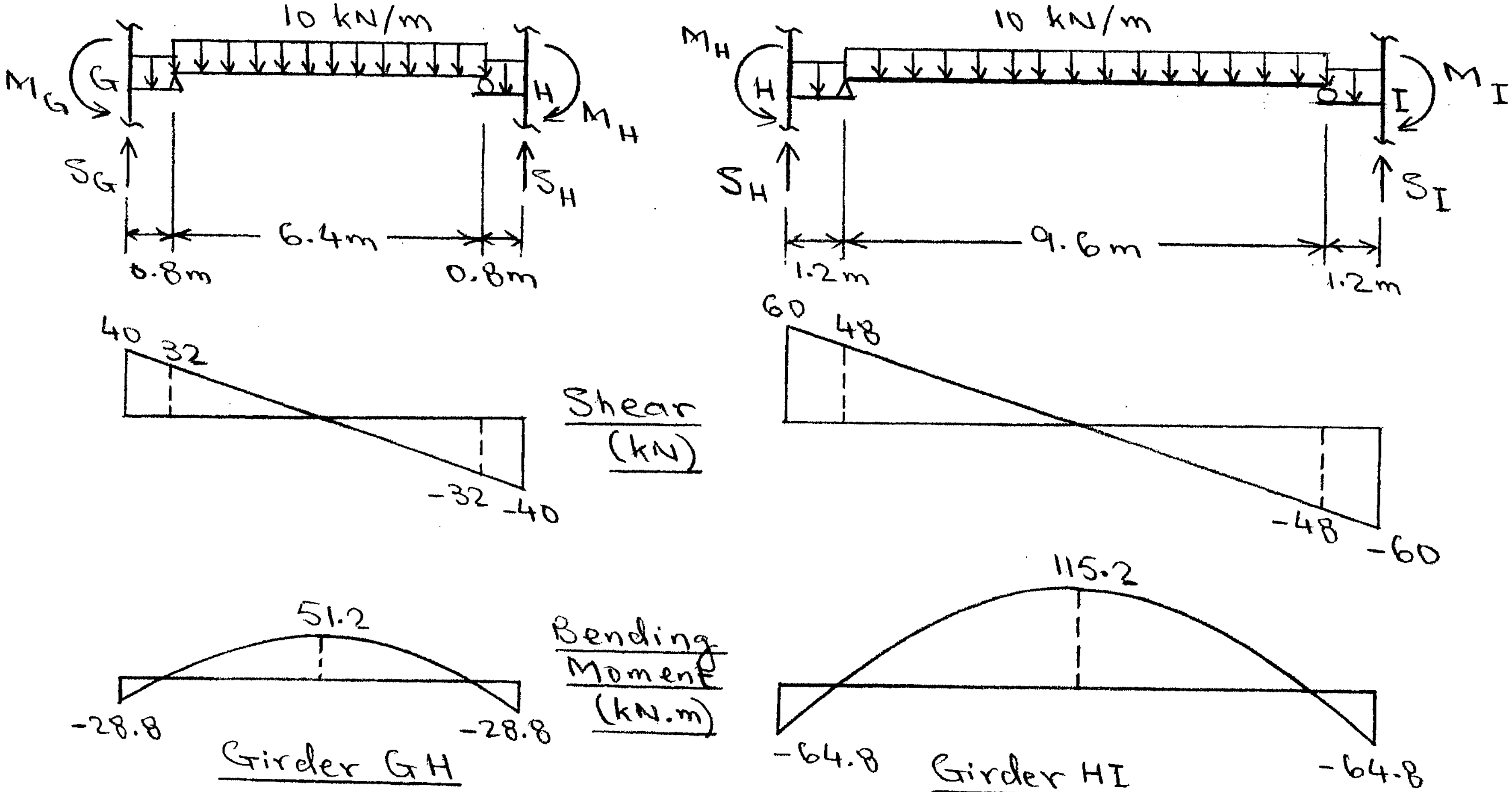


Shear Diagram (k)



Bending Moment Diagram (k-ft)

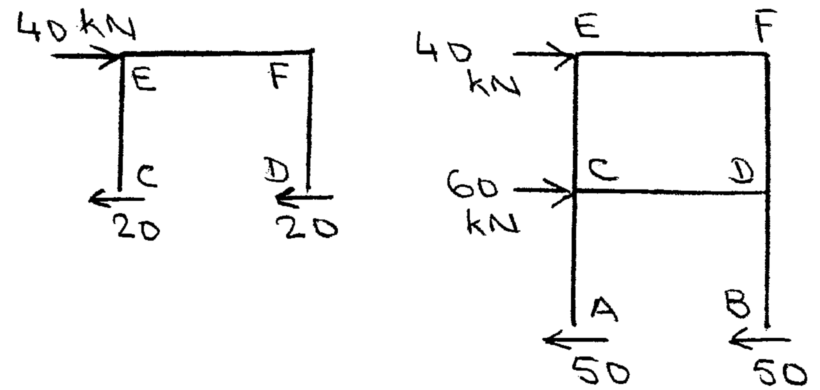
12-5



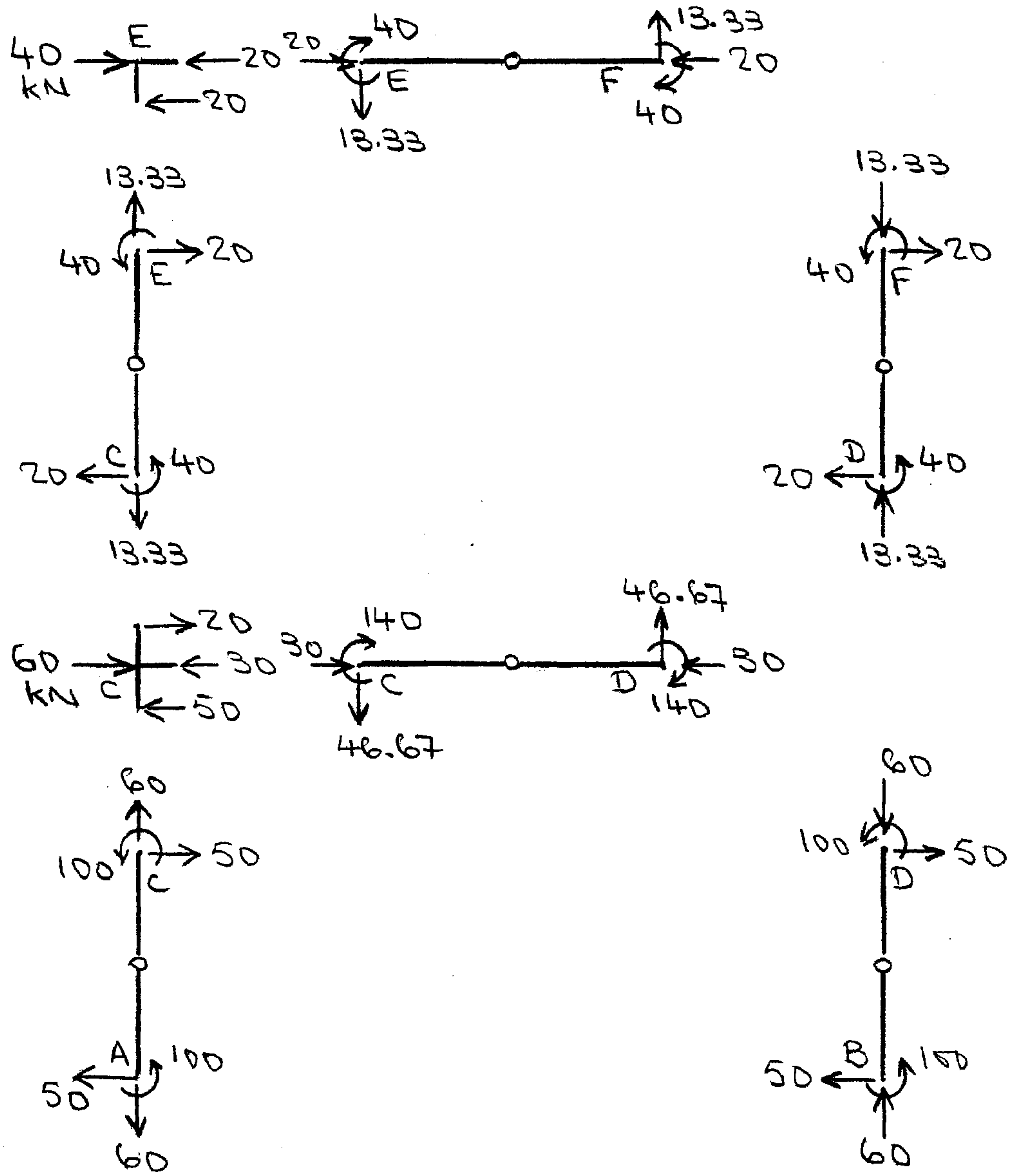
Shear in Girder DE = 2 x Shear in Girder GH.
Moment in Girder DE = 2 x Moment in Girder GH.

Shear in Girder EF = 2 x Shear in Girder HI.
Moment in Girder EF = 2 x Moment in Girder HI.

12.6

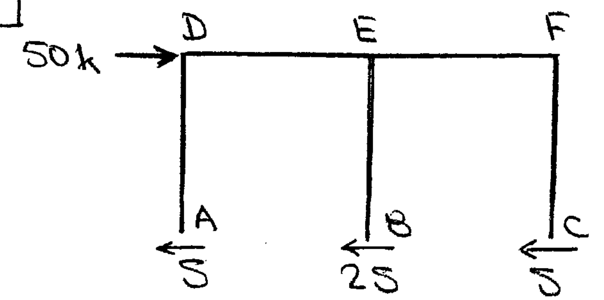


Column Shears



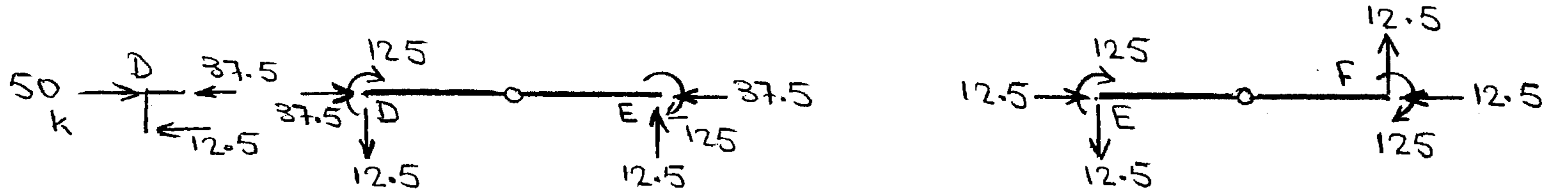
Member Axial Forces, Shears, and Moments

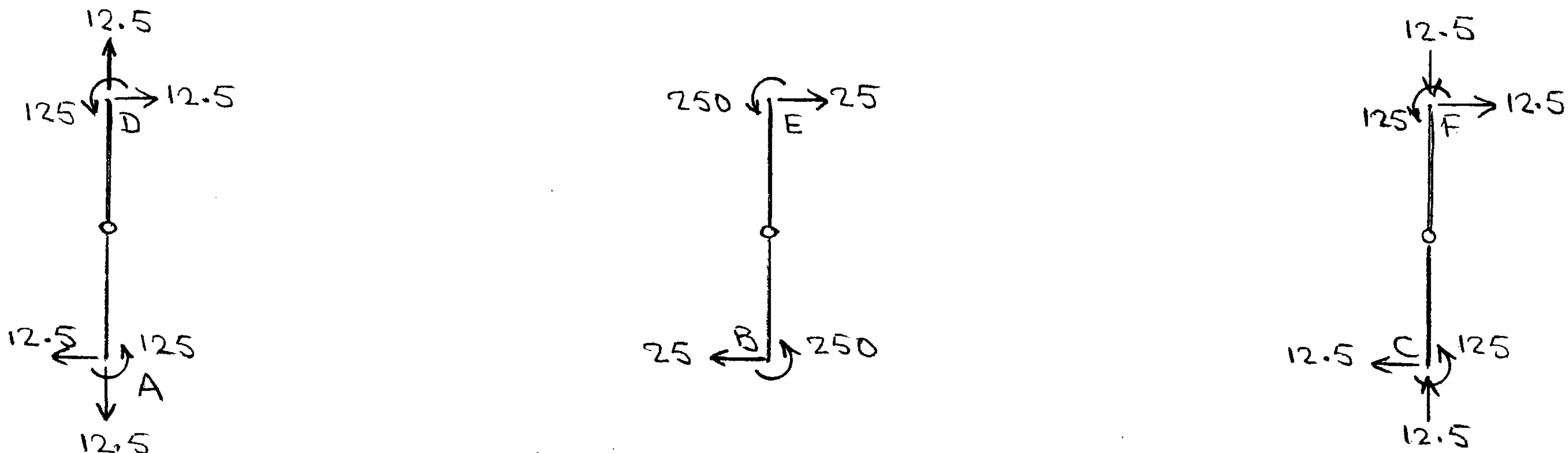
12.7



Column Shears

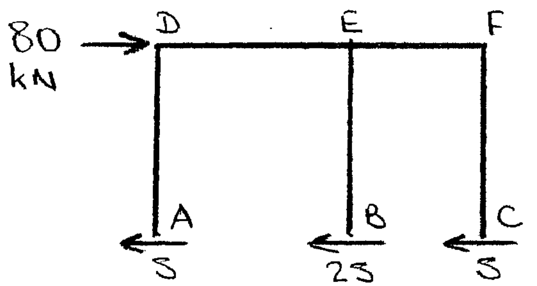
$4S = 50 \qquad S = 12.5\ k$





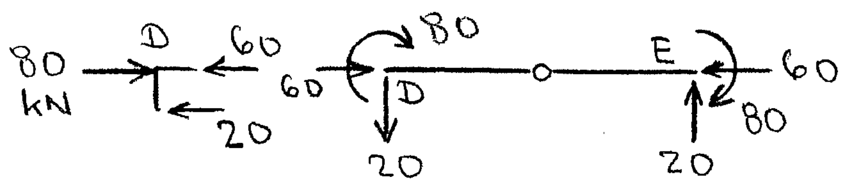
Member Axial Forces, Shears, and Moments

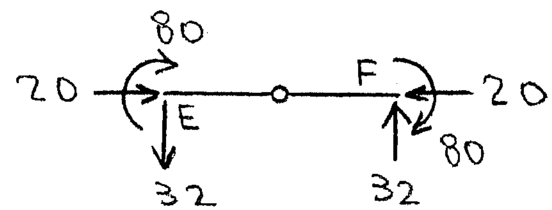
12.8

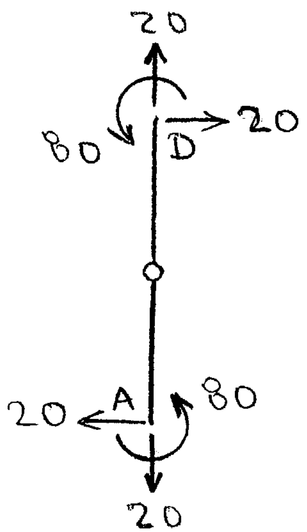


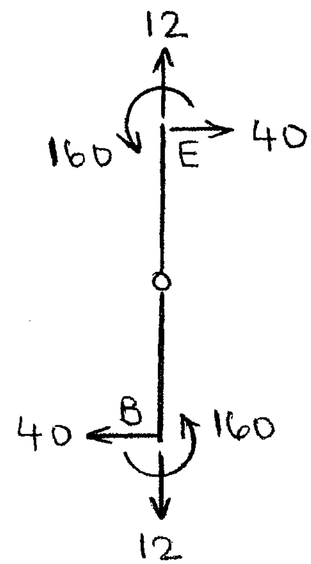
Column Shears

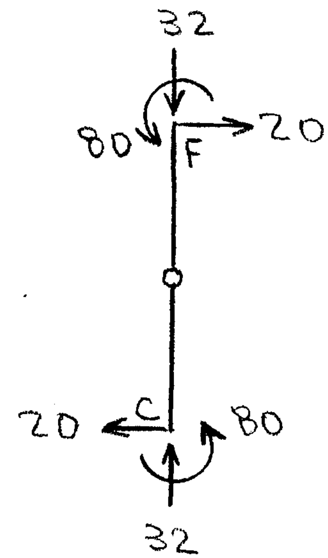
4S = 80   S = 20 kN







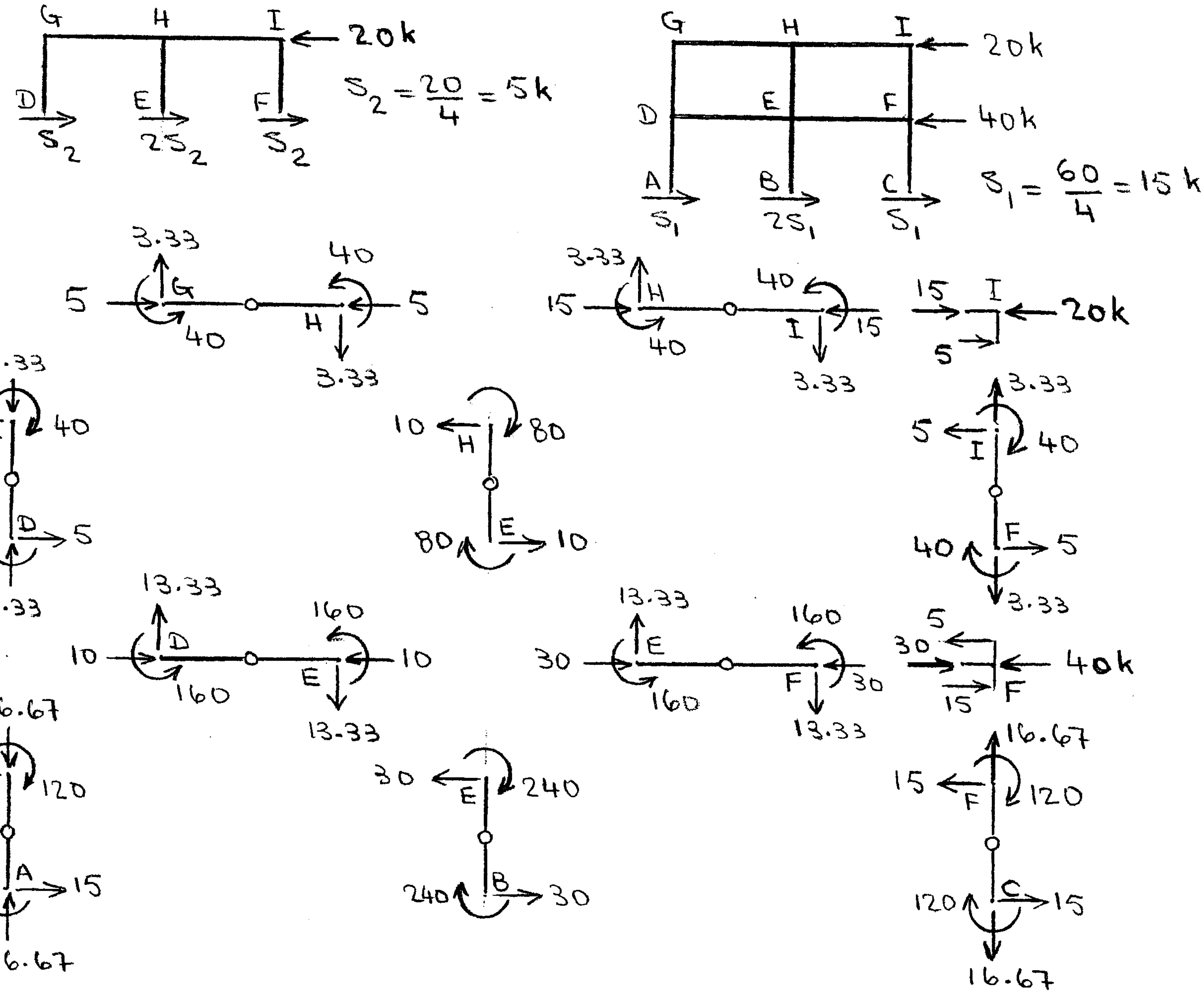




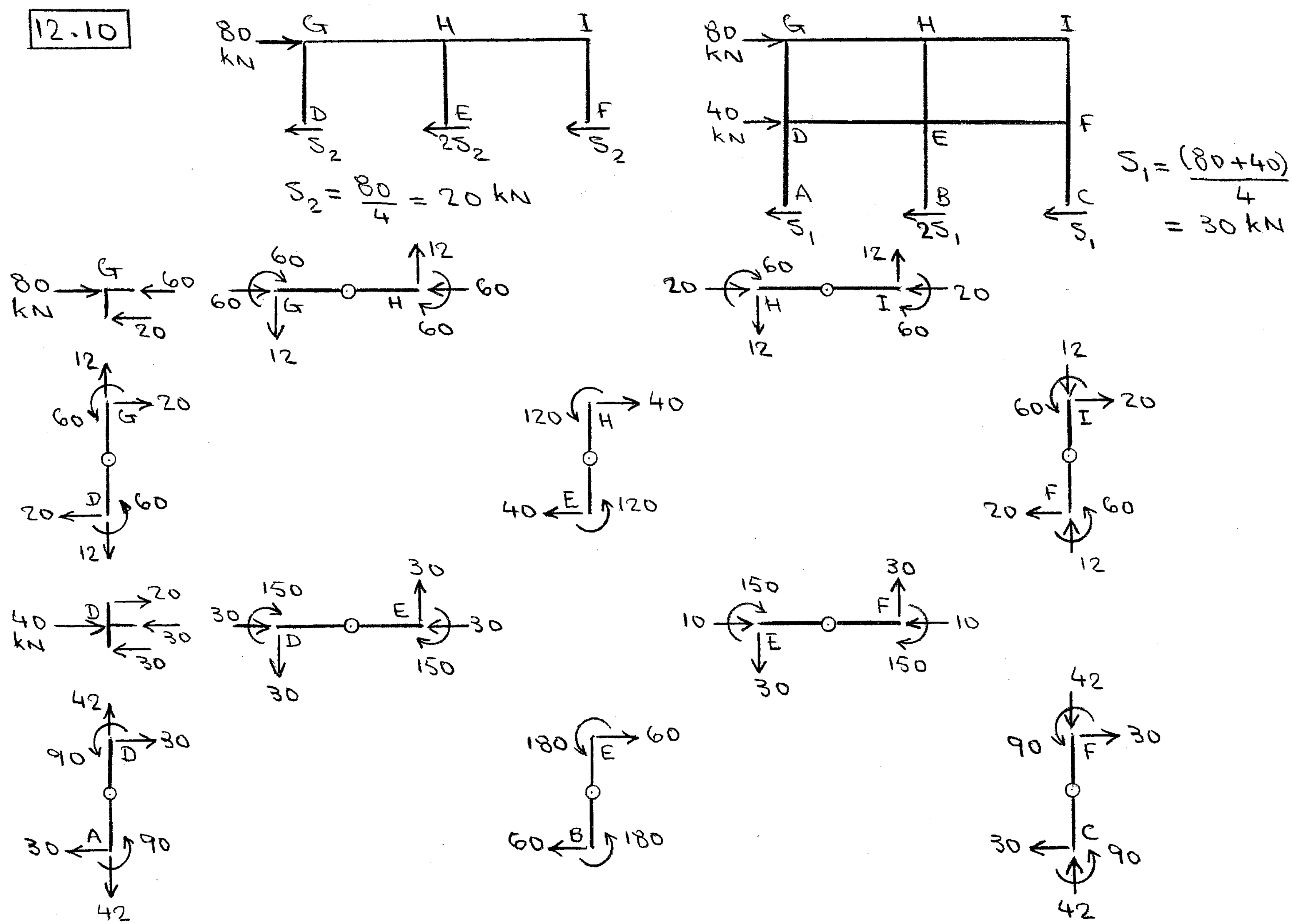
12.9

$S_2 = \frac{20}{4} = 5\text{k}$

$S_1 = \frac{60}{4} = 15\text{ k}$



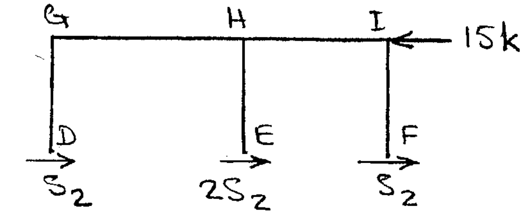
12.10



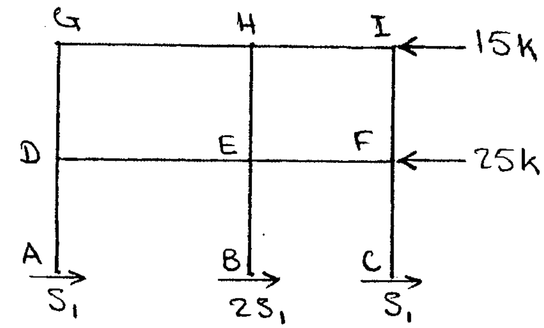
$$S_2 = \frac{80}{4} = 20 \text{ kN}$$

$$S_1 = \frac{(80+40)}{4} = 30 \text{ kN}$$

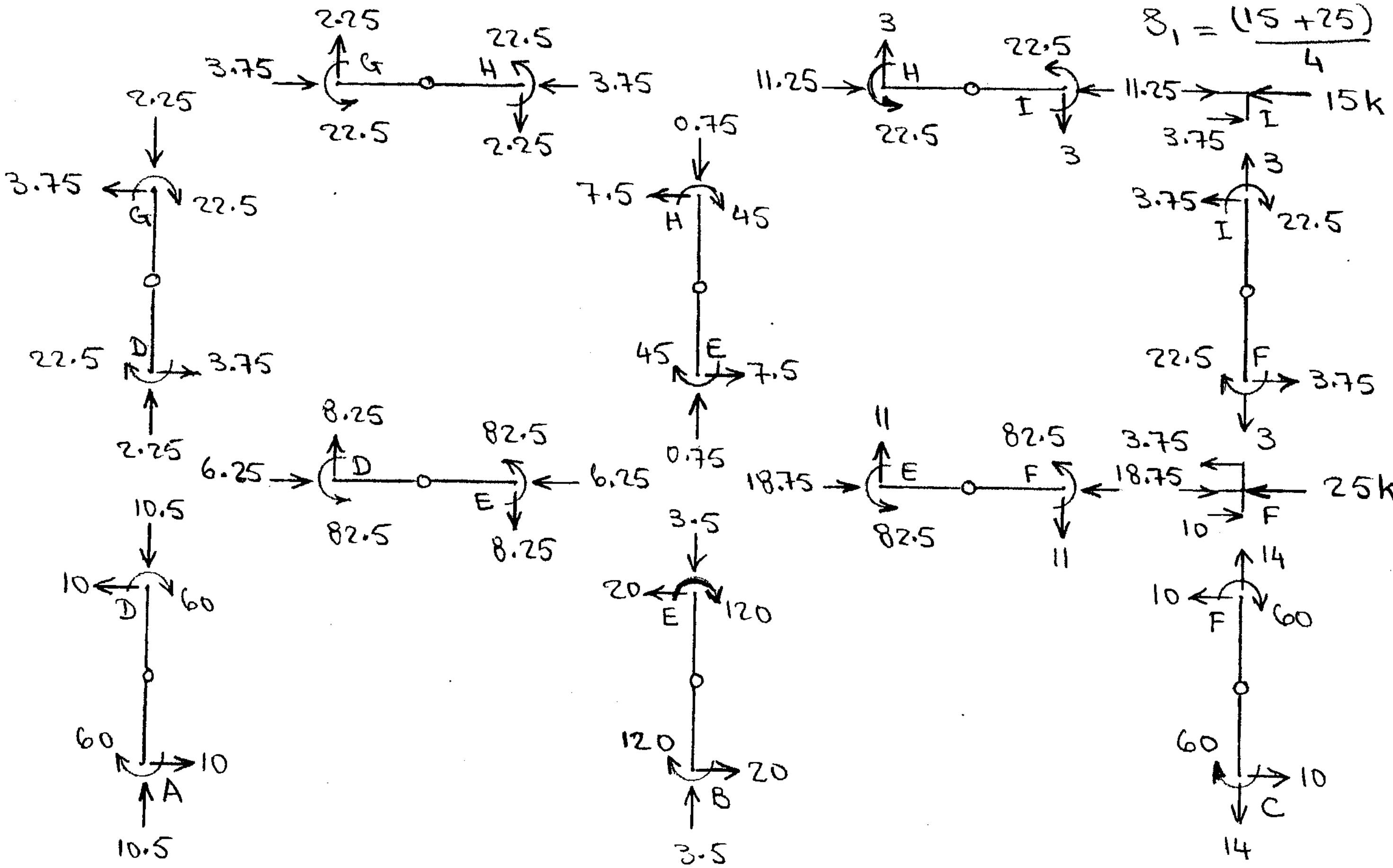
12.11



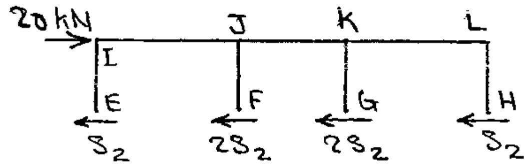
$$S_2 = \frac{15}{4} = 3.75\ k$$



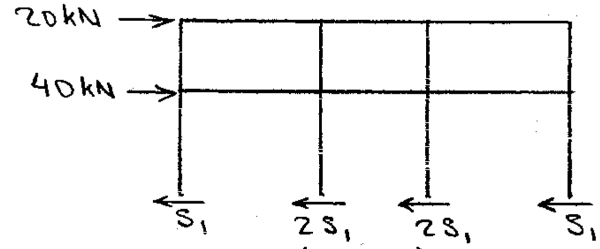
$$S_1 = \frac{(15+25)}{4} = 10\ k$$



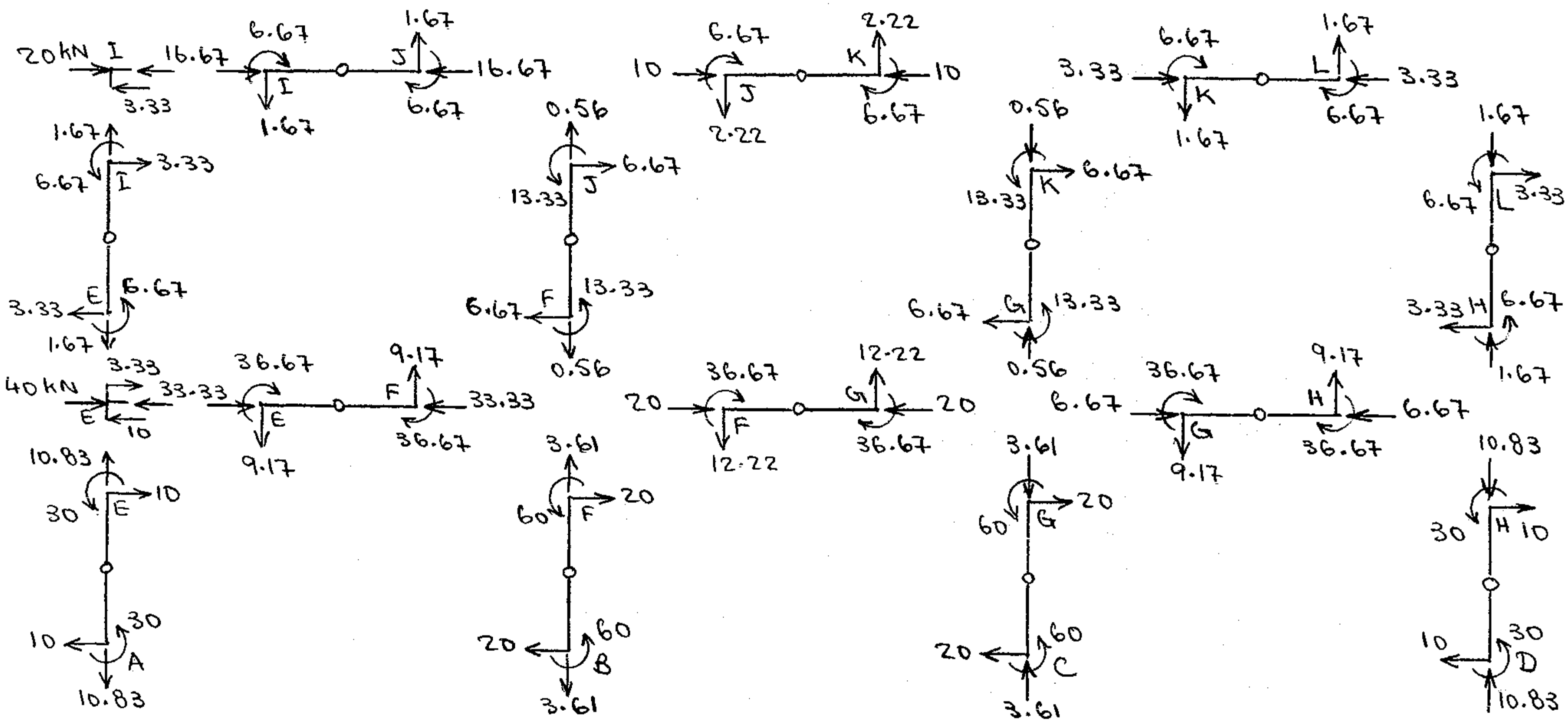
12-12



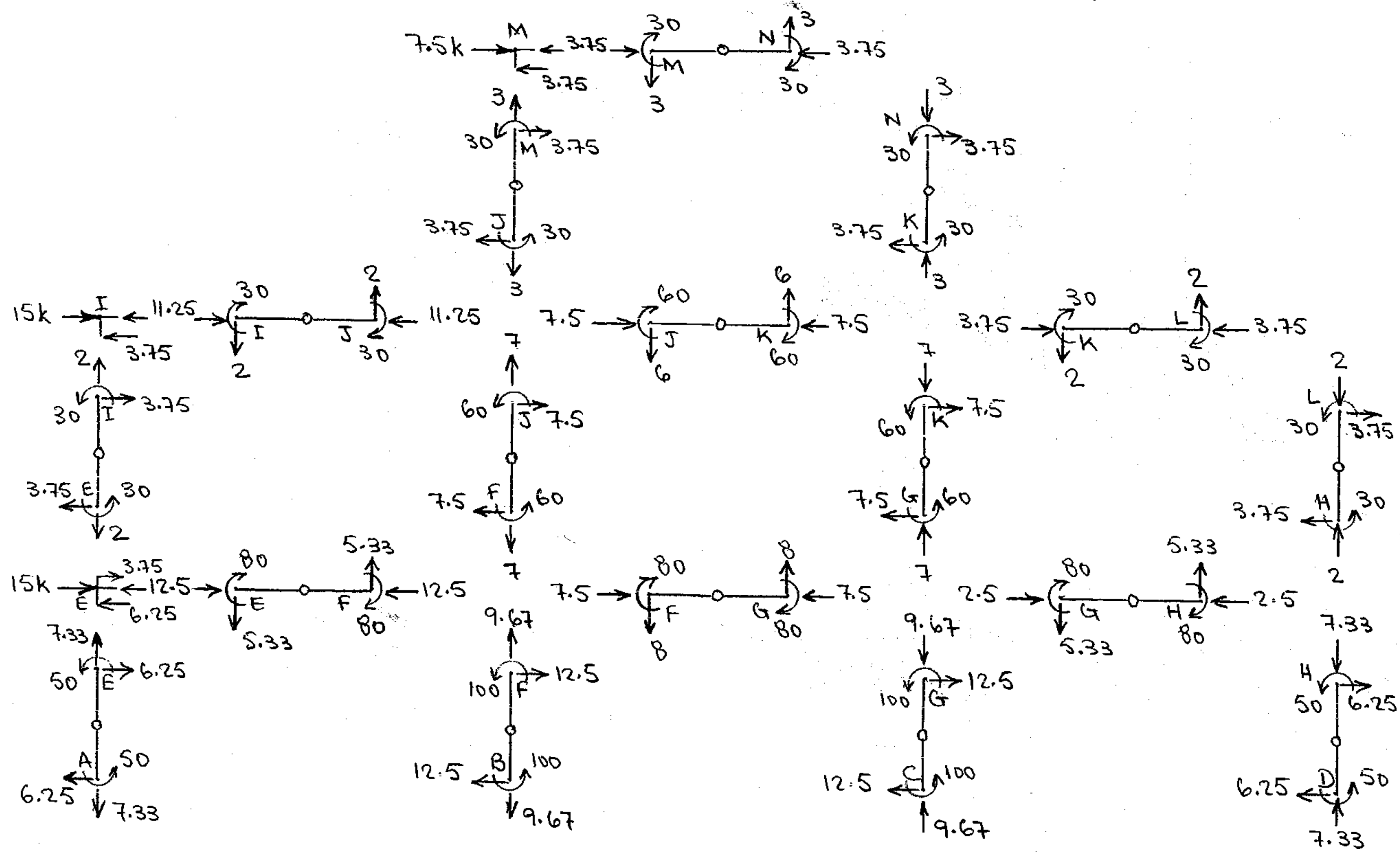
$$S_2 = \frac{20}{6} = 3.33 \text{ kN}$$



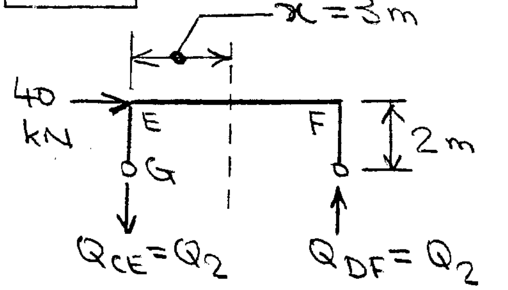
$$S_1 = \frac{(20+40)}{6} = 10 \text{ kN}$$



12.13 $S_3 = \dfrac{7.5}{2} = 3.75\text{ k};\quad S_2 = \dfrac{(7.5+15)}{6} = 3.75\text{ k};\quad S_1 = \dfrac{(7.5+15+15)}{6} = 6.25\text{ k}$

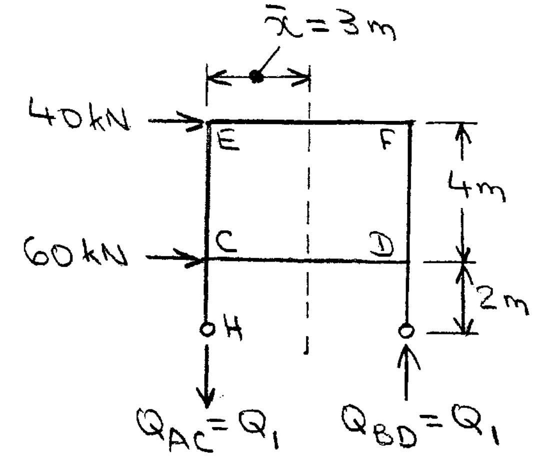


12.14



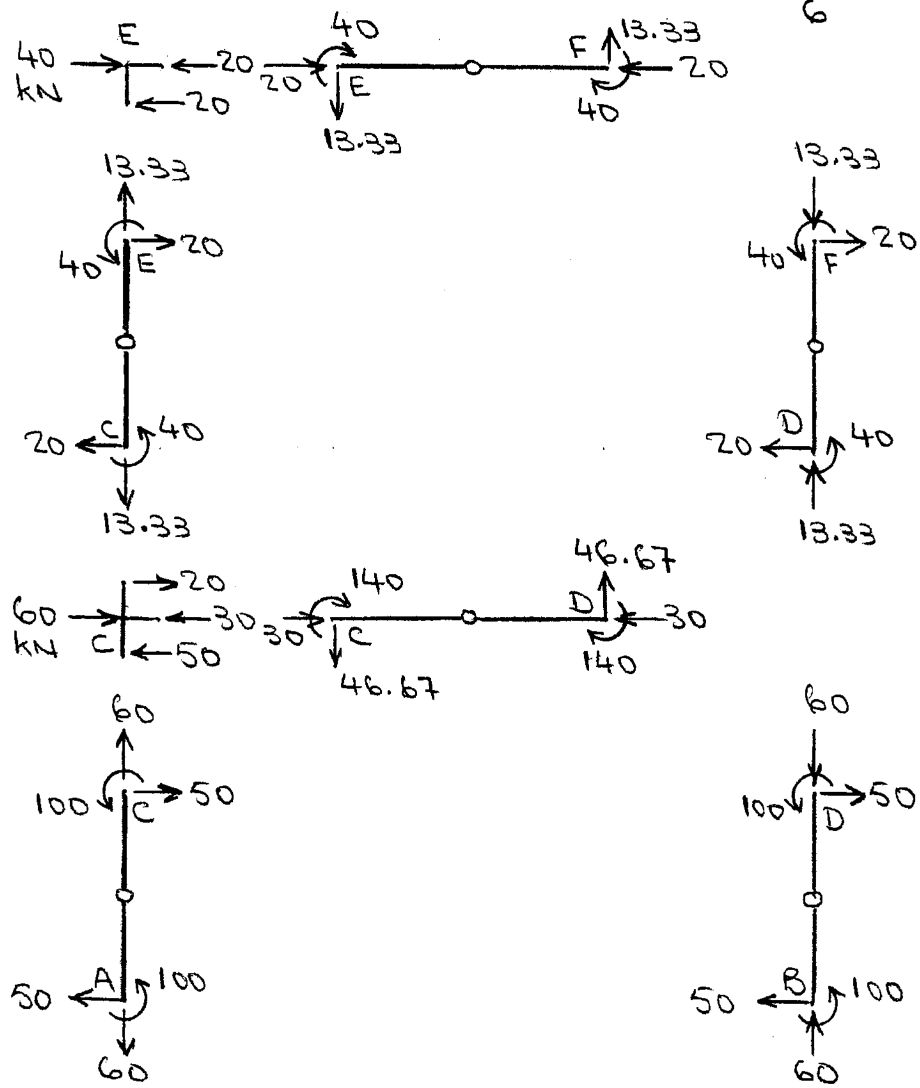
$\Sigma M_G = 0$ $\quad Q_2 = \dfrac{40(2)}{6}$

$= 13.33$ kN

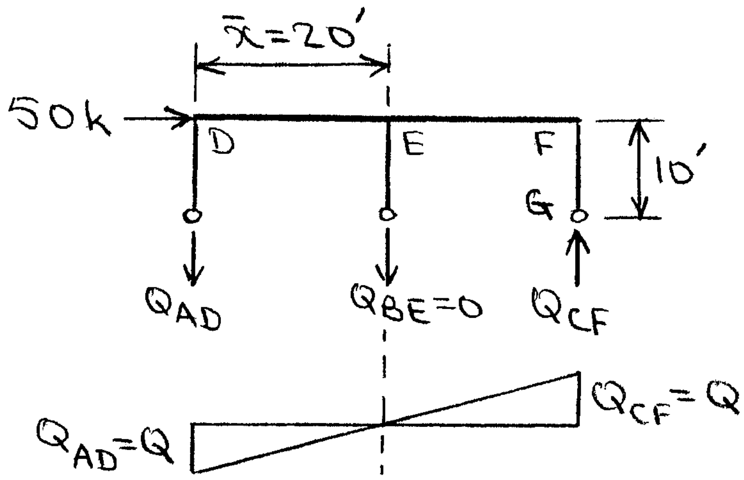


$\Sigma M_H = 0$

$$Q_1 = \frac{40(6) + 60(2)}{6} = 60 \text{ kN}$$



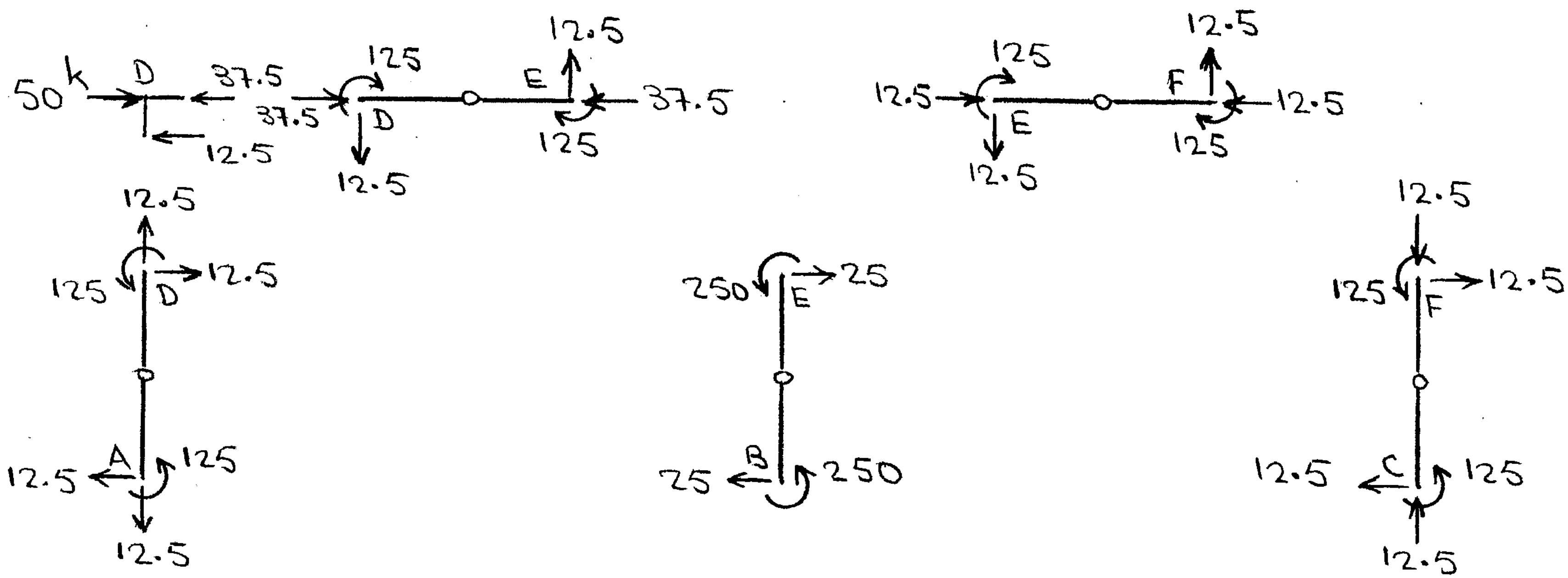
12.15



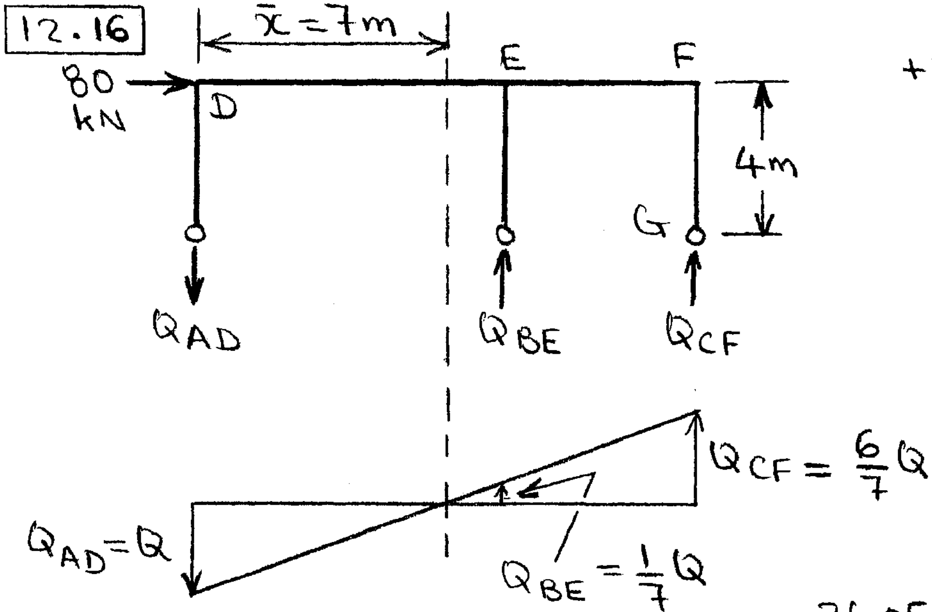
$$\circlearrowleft \Sigma M_G = 0$$

$$-50(10) + Q(40) = 0$$

$$Q = 12.5 \text{ k}$$



12.16



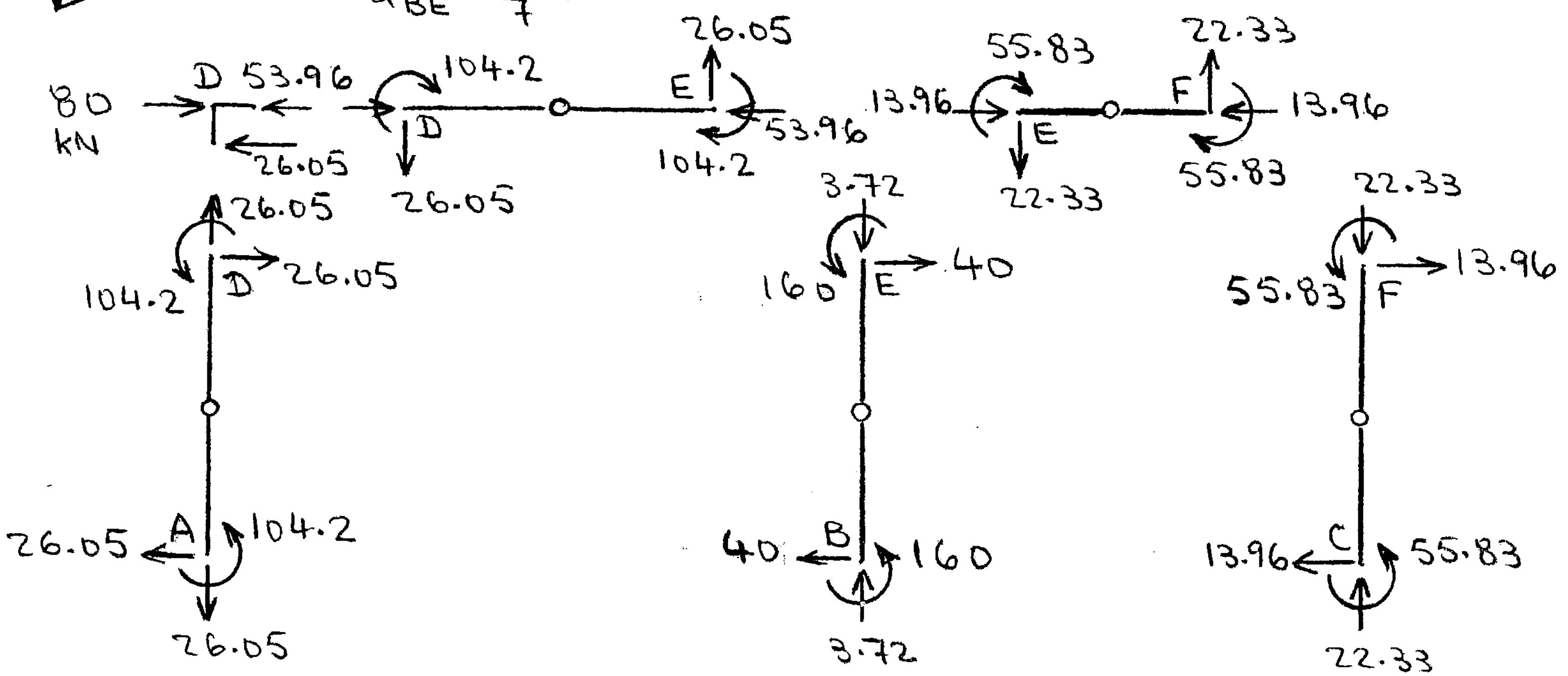
$$+\circlearrowleft \Sigma M_G = 0$$

$$-80(4) + Q(13) - \frac{1}{7}Q(5) = 0$$

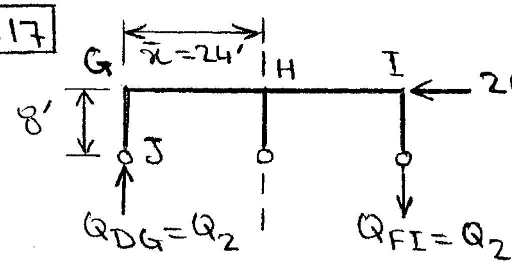
$$Q = 26.05 \text{ kN}$$

$$Q_{AD} = 26.05 \text{ kN}, \quad Q_{BE} = 3.72 \text{ kN}$$

$$Q_{CF} = 22.33 \text{ kN}$$



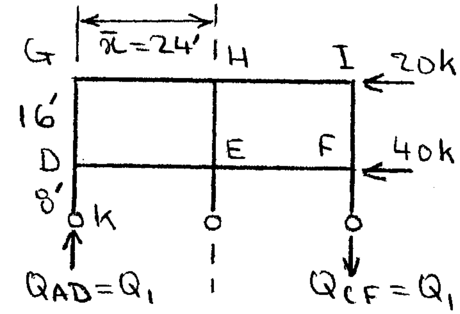
12.17



$\Sigma M_J = 0$

$Q_2 = \frac{20(8)}{48}$

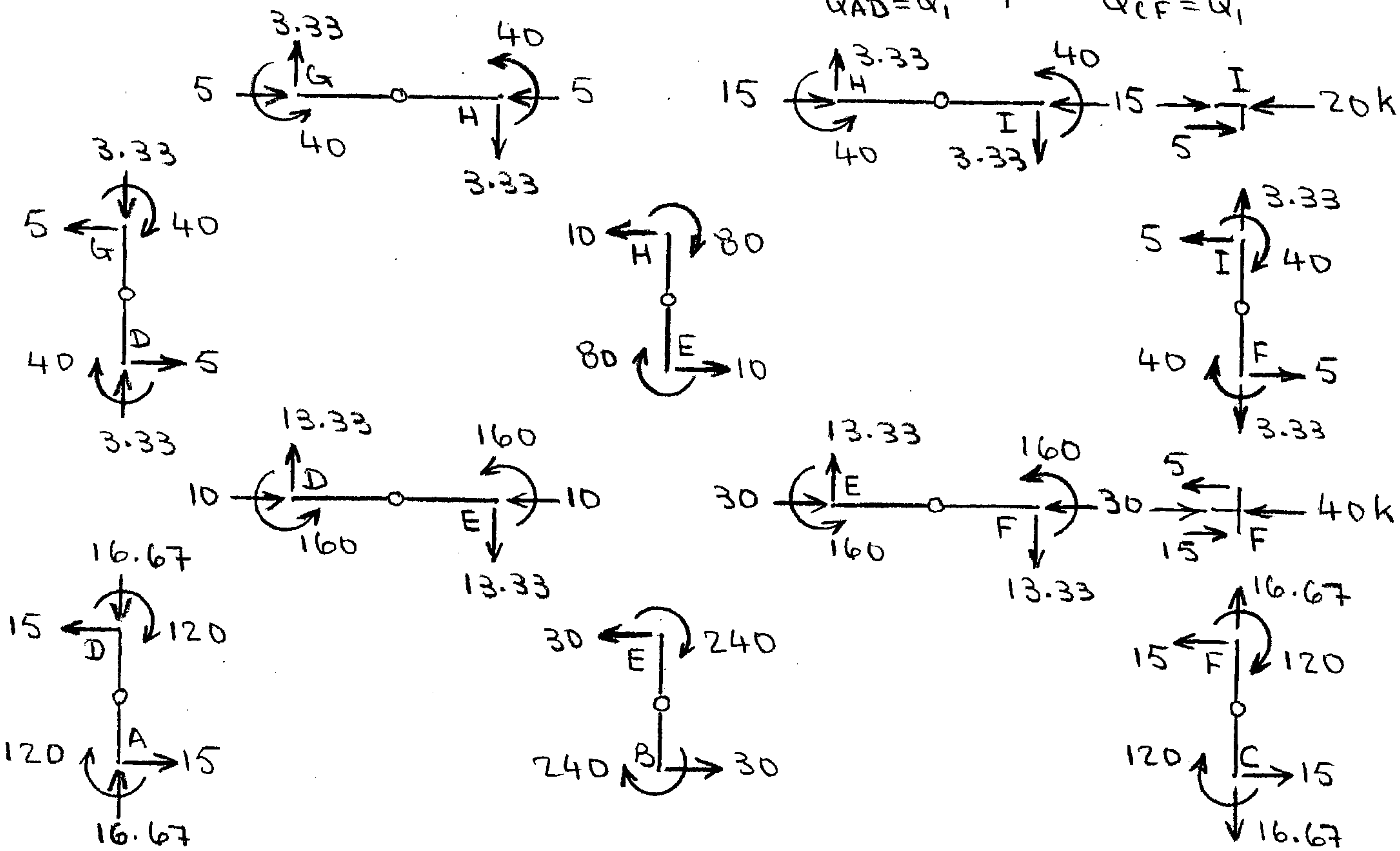
$= 3.33k$



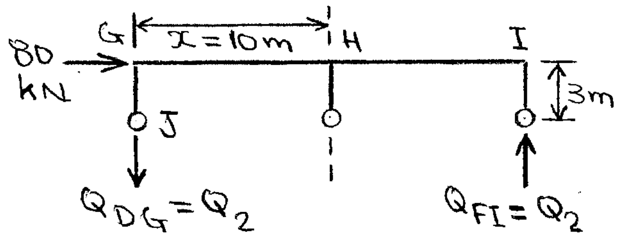
$\Sigma M_K = 0$

$Q_1 = \frac{20(24)+40(8)}{48}$

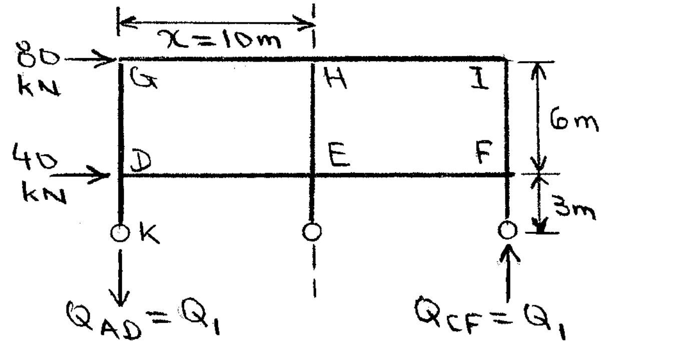
$= 16.67k$



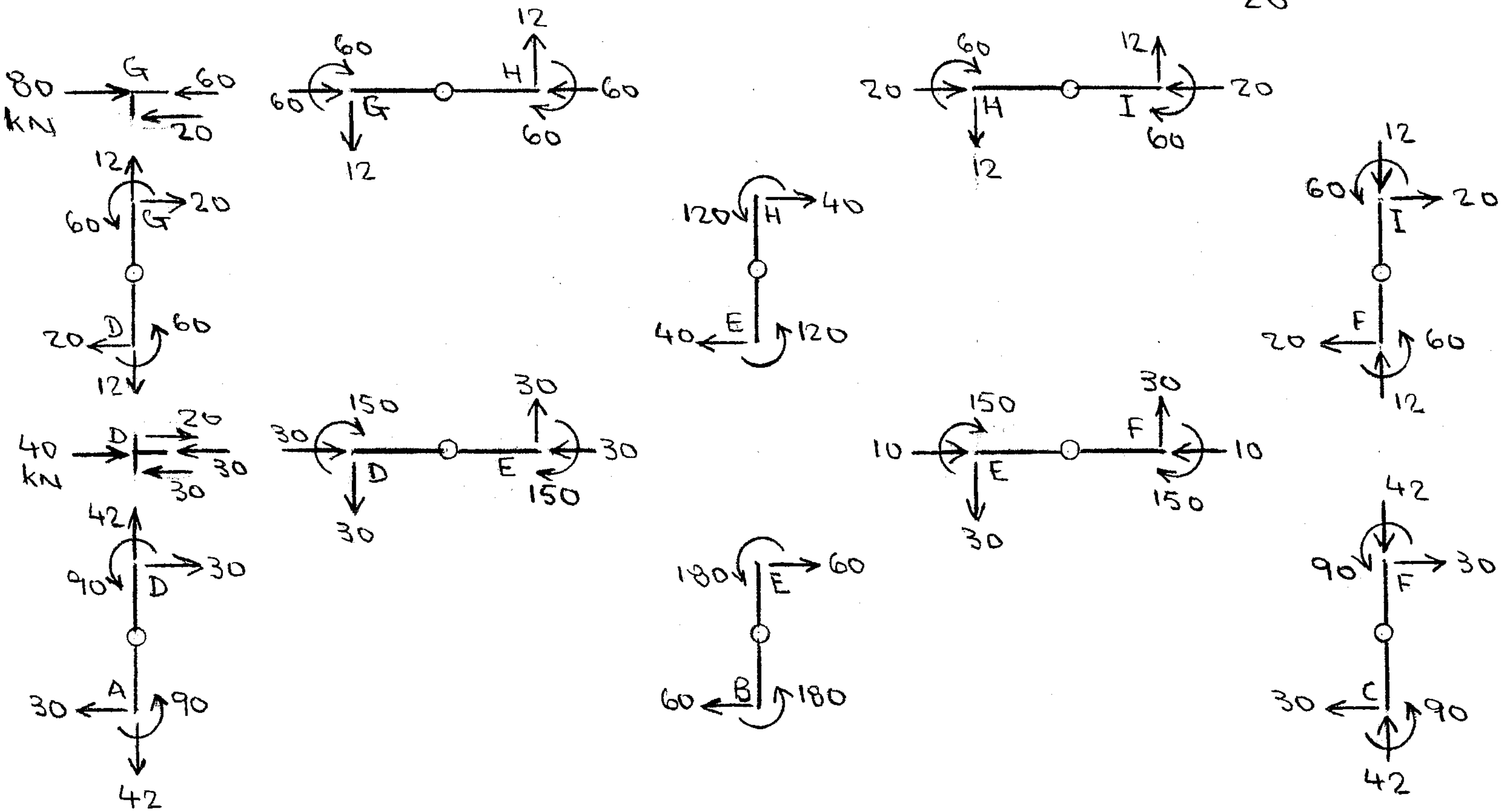
12.18



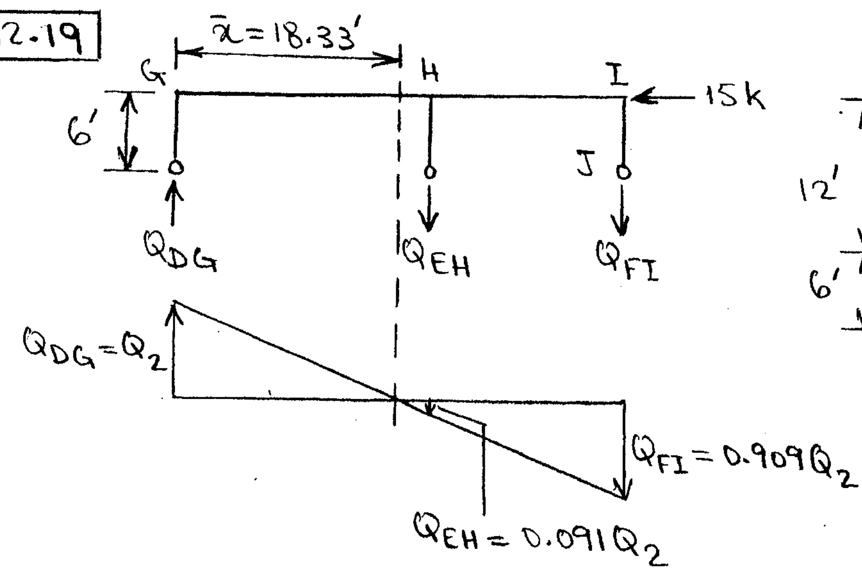
$$\Sigma M_J = 0 \qquad Q_2 = \frac{80(3)}{20} = 12\ \text{kN}$$

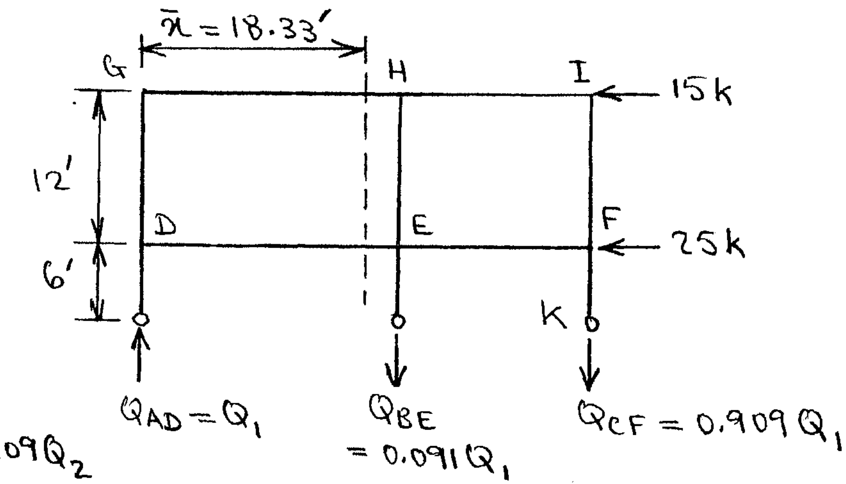


$$\Sigma M_K = 0 \qquad Q_1 = \frac{80(9) + 40(3)}{20} = 42\ \text{kN}$$



12.19





$+\circlearrowright \sum M_J = 0$

$$15(6) - Q_2(35) + 0.091 Q_2 (20) = 0$$

$$Q_2 = 2.71\,k$$

$Q_{DG} = 2.71\,k$, $Q_{EH} = 0.247\,k$,

$Q_{FI} = 2.46\,k$

$+\circlearrowright \sum M_K = 0$

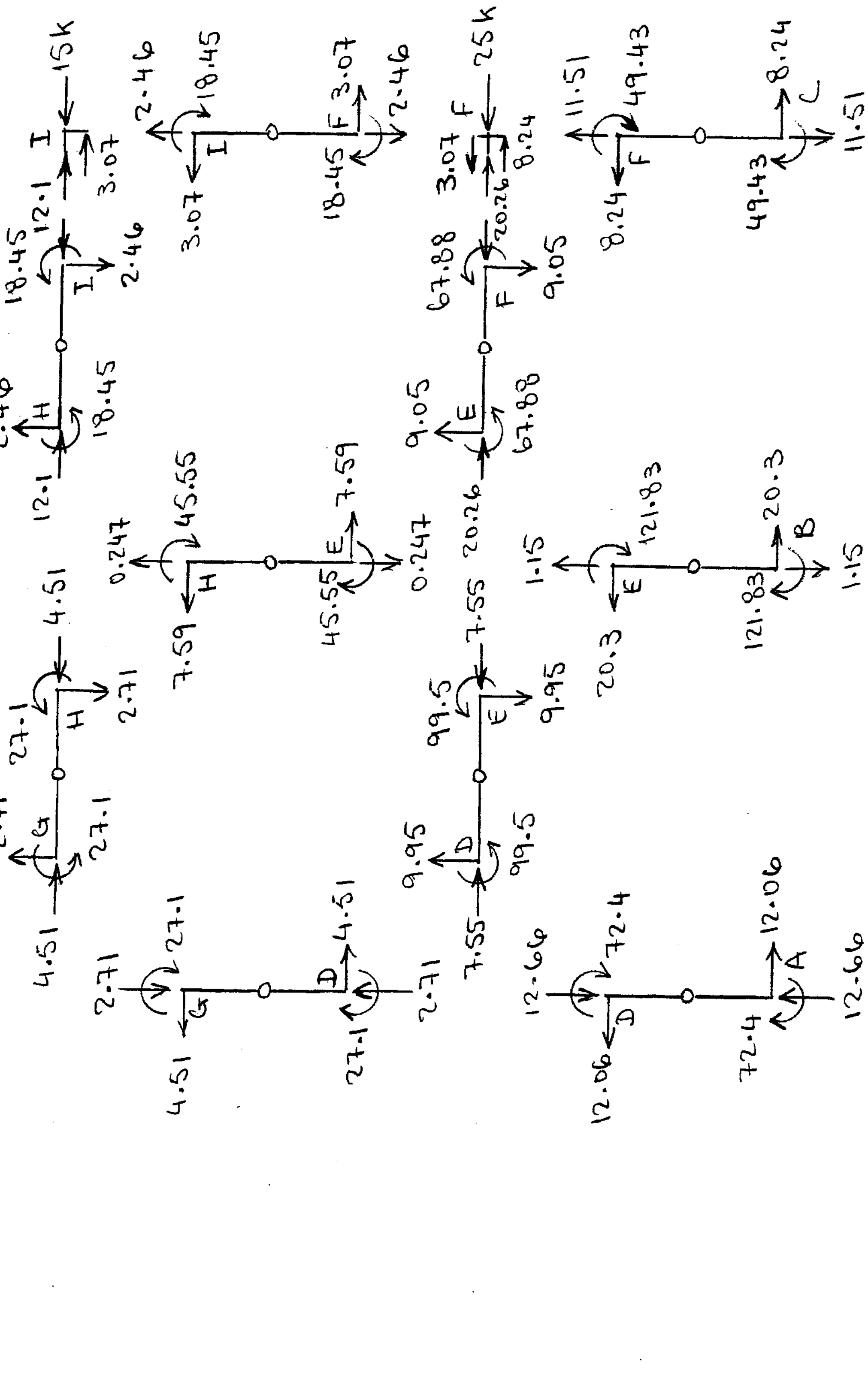
$$15(18) + 25(6) - Q_1(35) + 0.091 Q_1 (20) = 0$$

$$Q_1 = 12.66\,k$$

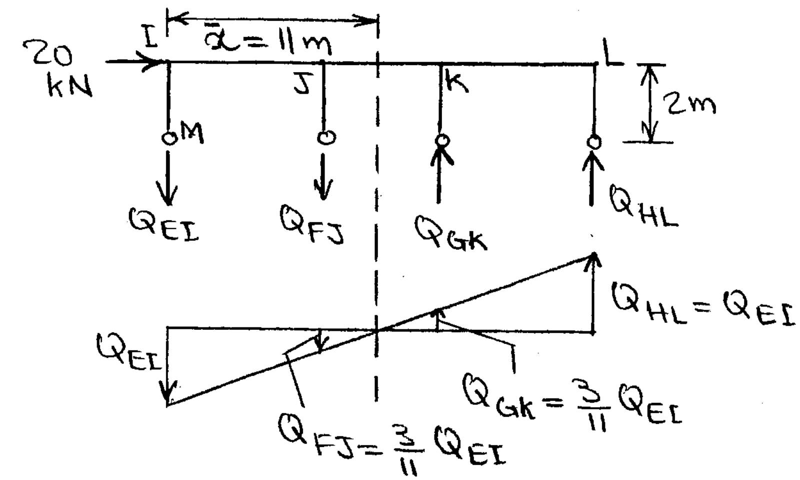
$Q_{AD} = 12.66\,k$, $Q_{BE} = 1.15\,k$

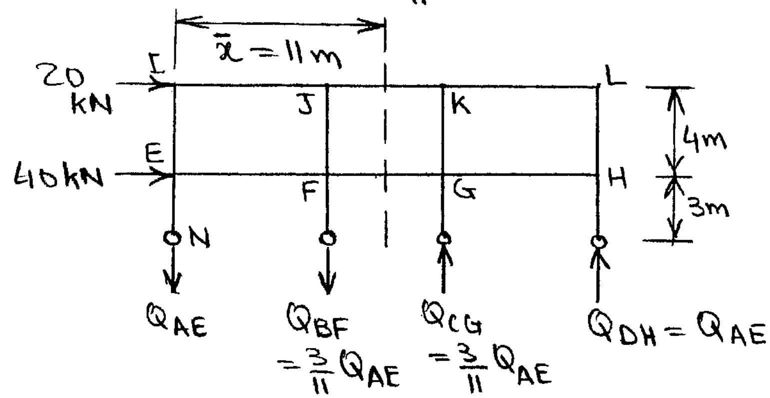
$Q_{CF} = 11.51\,k$

12.19 (Contd.)



12.20





$+\circlearrowright \Sigma M_M = 0$

$$-20(2) - \frac{3}{11} Q_{EI}(8) + \frac{3}{11} Q_{EI}(14) + Q_{EI}(22) = 0$$

$Q_{EI} = Q_{HL} = 1.69$ kN

$Q_{FJ} = Q_{GK} = \frac{3}{11}(1.69) = 0.46$ kN

$+\circlearrowright \Sigma M_N = 0$

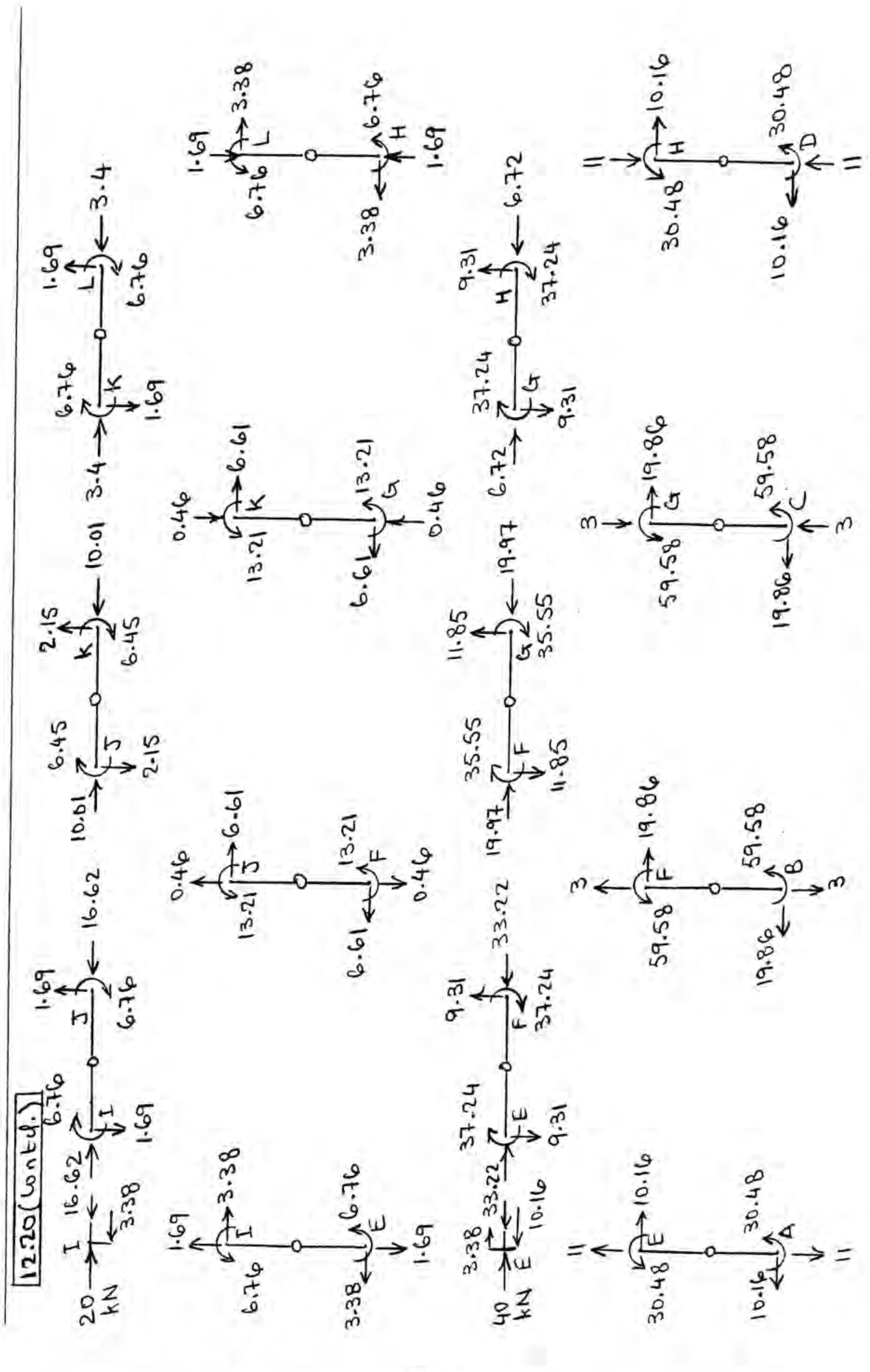
$$-20(7) - 40(3) - \frac{3}{11} Q_{AE}(8) + \frac{3}{11} Q_{AE}(14) + Q_{AE}(22) = 0$$

$Q_{AE} = Q_{DH} = 11$ kN

$Q_{BF} = Q_{CG} = 3$ kN

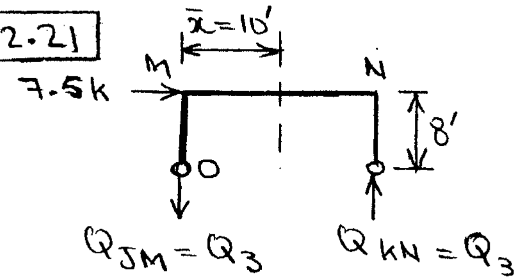
<u>Column Axial Forces</u>

12.20 (Contd.)



Member Axial Forces, Shears, and Moments

12.21

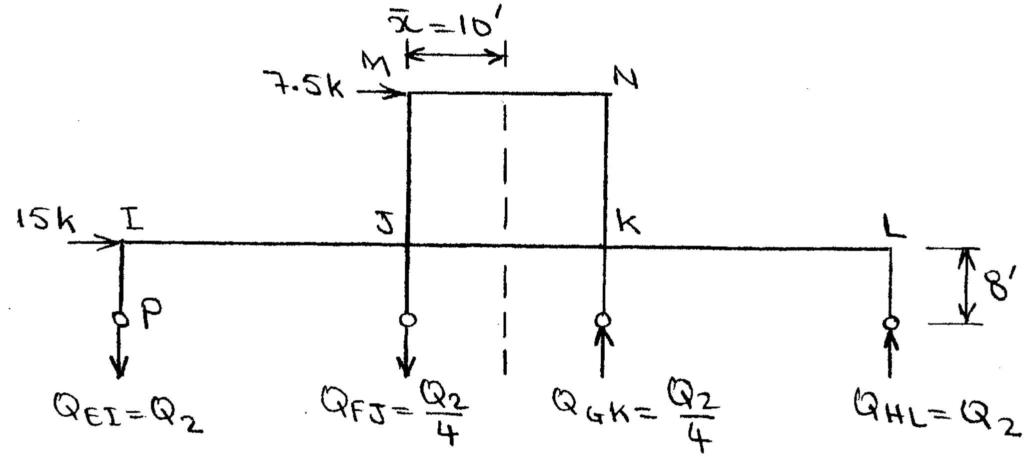


$+\circlearrowright \Sigma M_O = 0$

$-7.5(8) + Q_3(20) = 0$

$Q_3 = 3\text{ k}$

$Q_{JM} = Q_{KN} = 3\text{ k}$

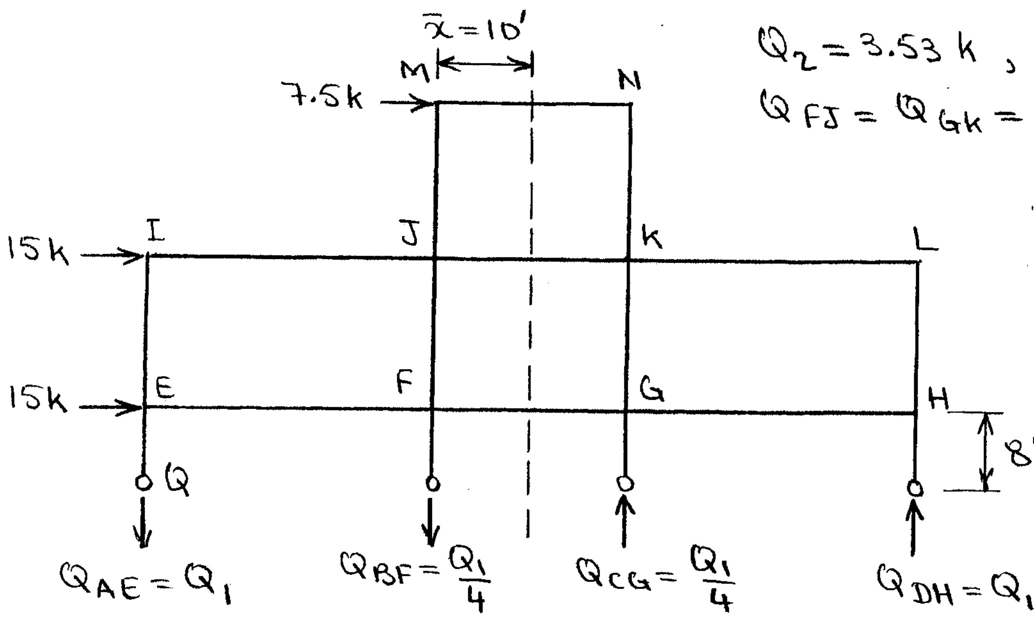


$+\circlearrowright \Sigma M_P = 0$

$-7.5(24) - 15(8) - \frac{Q_2}{4}(30) + \frac{Q_2}{4}(50) + Q_2(80) = 0$

$Q_2 = 3.53\text{ k}, \quad Q_{EI} = Q_{HL} = 3.53\text{ k}$

$Q_{FJ} = Q_{GK} = 0.882\text{ k}$



$+\circlearrowright \Sigma M_Q = 0$

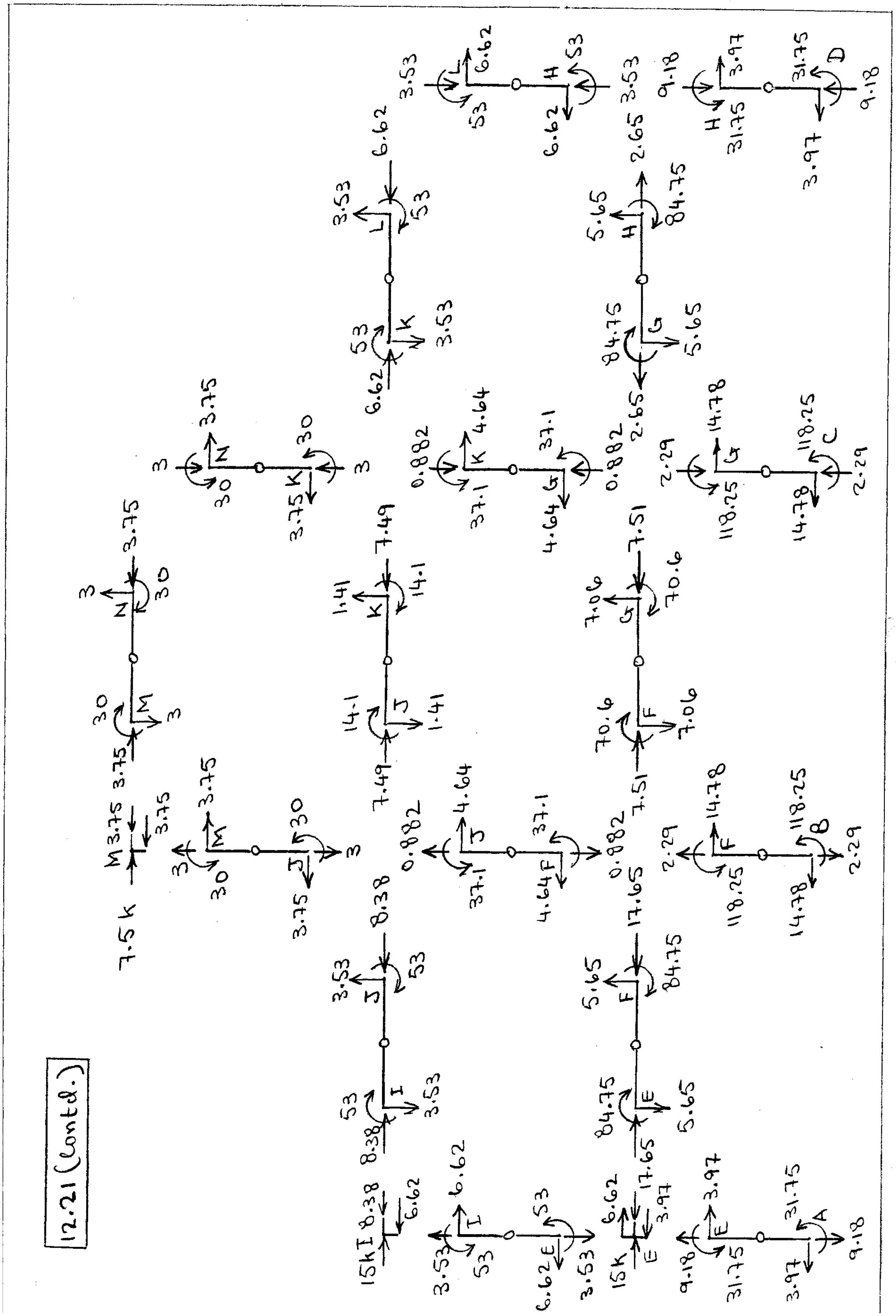
$-7.5(40) - 15(24) - 15(8) - \frac{Q_1}{4}(30)$
$+ \frac{Q_1}{4}(50) + Q_1(80) = 0$

$Q_1 = 9.18\text{ k}$

$Q_{AE} = Q_{DE} = 9.18\text{ k}$

$Q_{BF} = Q_{CG} = 2.29\text{ k}$

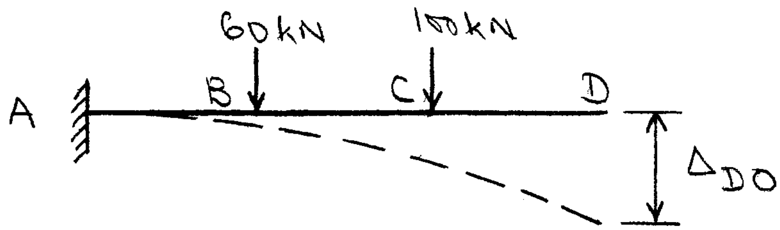
12.21 (Contd.)



Chapter Thirteen
Method of Consistent Deformations – Force Method

CHAPTER 13

13.1

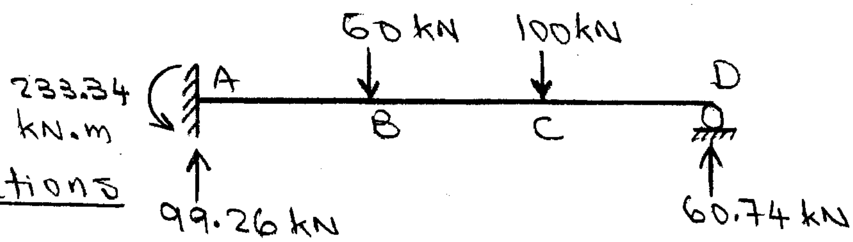


Using beam deflection formulas:

$$\Delta_{DO} = -\frac{14760 \text{ kN}\cdot\text{m}^3}{EI} \qquad f_{DD} = \frac{243 \text{ kN}\cdot\text{m}^3/\text{kN}}{EI}$$

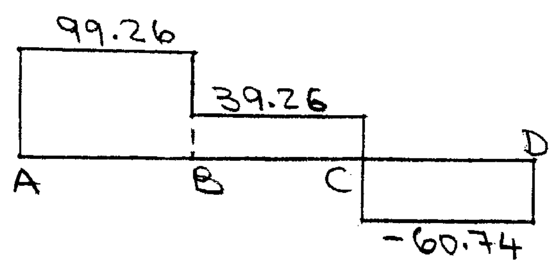
Compatibility Equation:

$$\Delta_{DO} + f_{DD} D_y = 0 \qquad D_y = \frac{14760}{243} = \underline{60.74 \text{ kN}\uparrow}$$

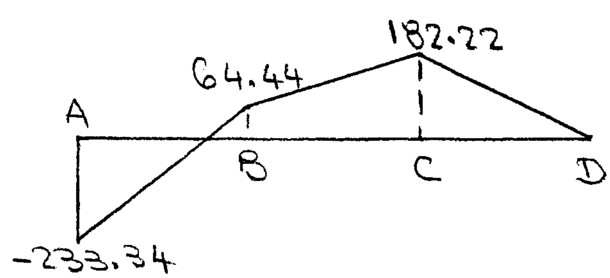


Reactions

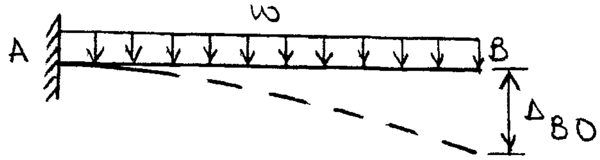
Shear Diagram (kN)



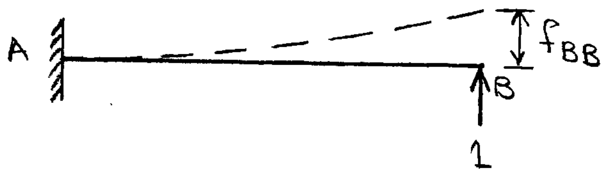
Bending Moment Diagram (kN.m)



$\boxed{13.2}$

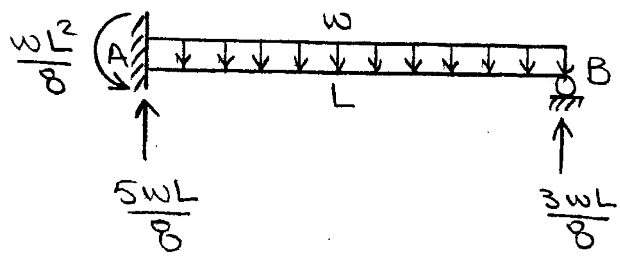


$$\Delta_{BO} = -\frac{wL^4}{8EI}$$

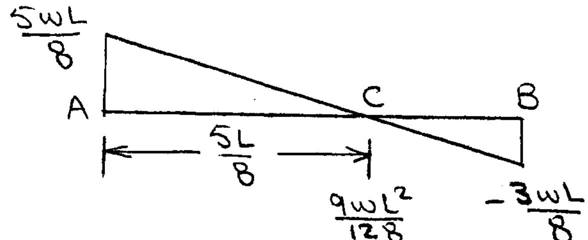


$f_{BB} \times B_y$ $\qquad$ $f_{BB} = \dfrac{L^3}{3EI}$

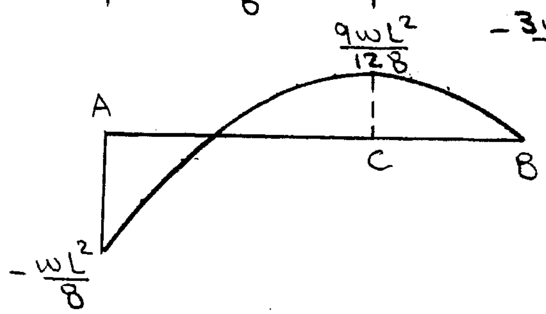
$$\Delta_{BO} + f_{BB} B_y = 0 \qquad B_y = \left(\frac{wL^4}{8EI}\right)\frac{3EI}{L^3} = \underline{\frac{3wL}{8}\uparrow}$$



Reactions

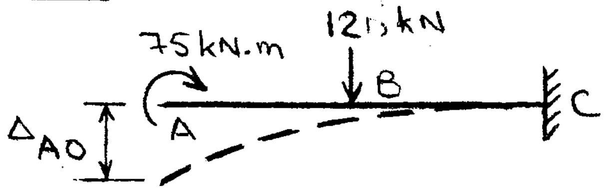


Shear Diagram



Bending Moment Diagram

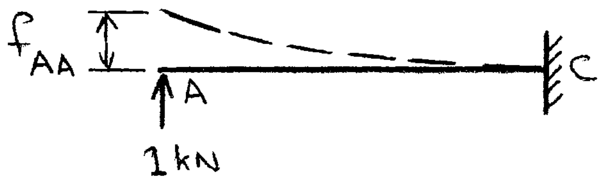
13.3



Using beam deflection formulas:

$$\Delta_{AO} = \frac{5400}{EI} - \frac{21600}{EI}$$

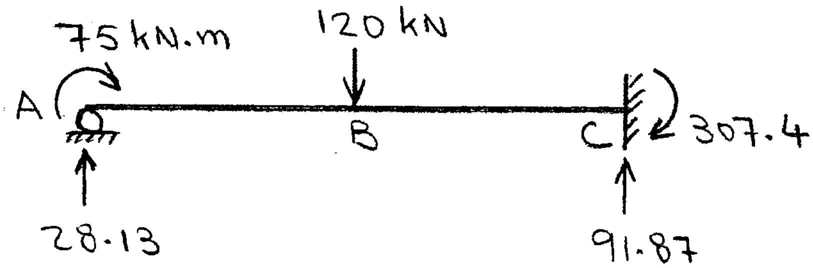
$$= -\frac{16200 \text{ kN.m}^3}{EI}$$



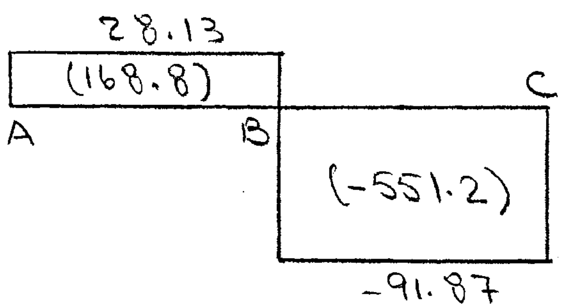
$\times A_y$ $\quad f_{AA} = \dfrac{576 \text{ kN.m}^3/\text{kN}}{EI}$

$$\Delta_{AO} + f_{AA} A_y = 0 \qquad A_y = \frac{16200}{576} = \underline{28.13 \text{ kN} \uparrow}$$

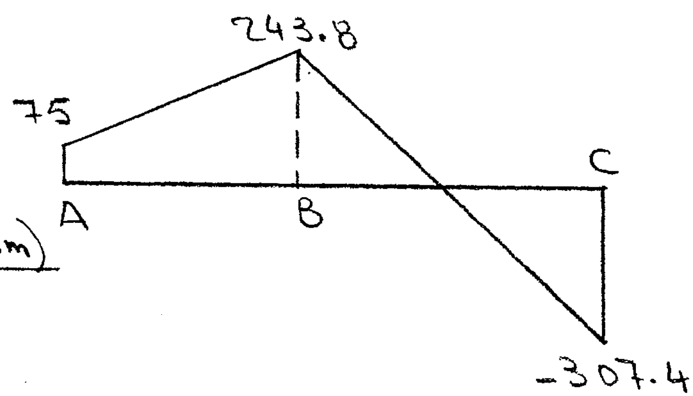
Reactions



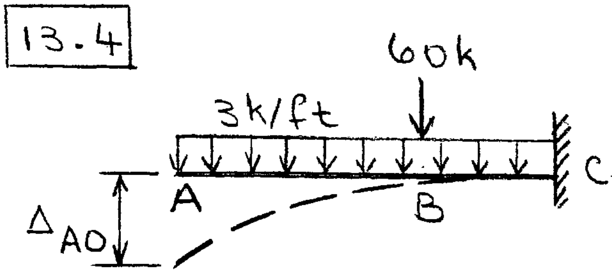
Shear Diagram (kN)



Bending Moment Diagram (kN.m)



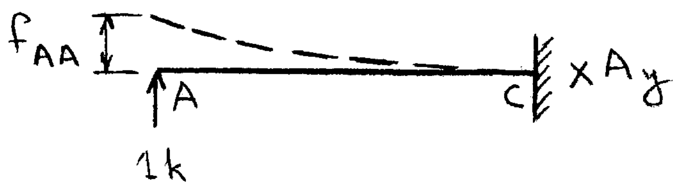
13.4



Using beam deflection formulas:

$$\Delta_{AO} = -\frac{303750}{EI} - \frac{80000}{EI}$$

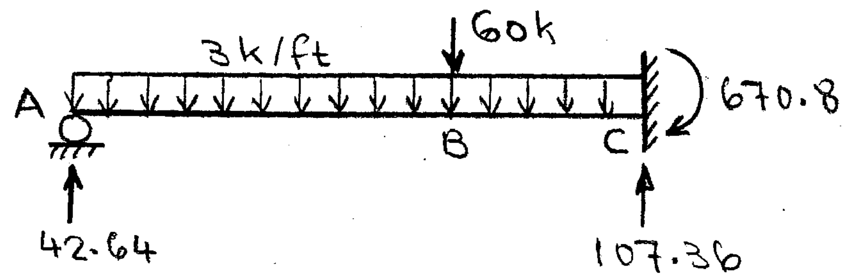
$$= -\frac{383750 \text{ k-ft}^3}{EI}$$



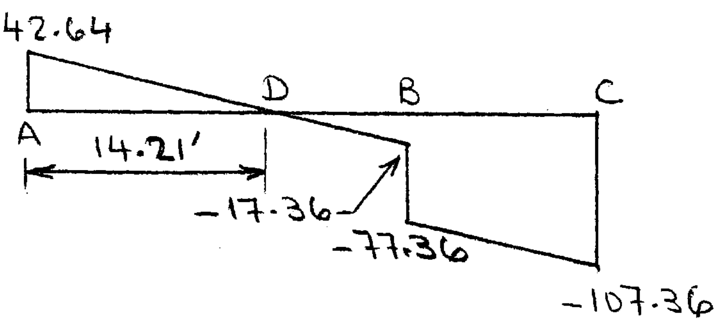
$$f_{AA} = \frac{9000 \text{ k-ft}^3/\text{k}}{EI}$$

$$\Delta_{AO} + f_{AA} A_y = 0 \qquad A_y = \frac{383750}{9000} = \underline{42.64\text{k} \uparrow}$$

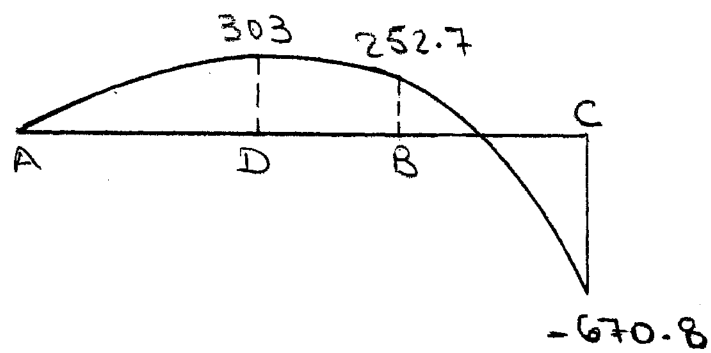
Reactions



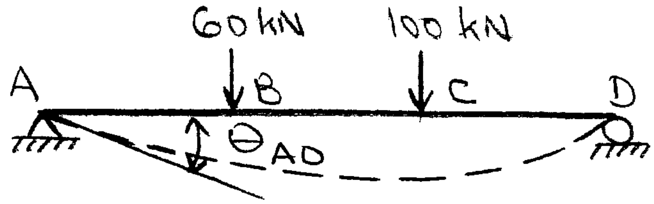
Shear Diagram (k)

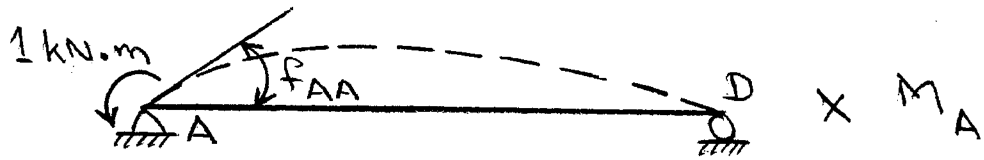


Bending Moment Diagram (k-ft)



13.5





Using beam deflection formulas:

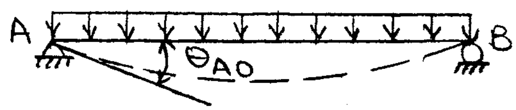
$$\theta_{AD} = -\frac{700\ \text{kN.m}^2}{EI} \qquad f_{AA} = \frac{3\ \text{kN.m}^2/\text{kN.m}}{EI}$$

Compatibility Equations:

$$\theta_{AD} + f_{AA} M_A = 0 \qquad M_A = \frac{700}{3} = \underline{233.33\ \text{kN.m}}$$

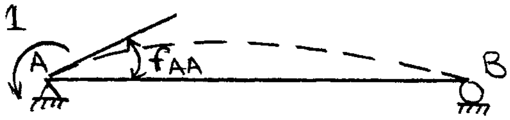
For reactions, and shear and moment diagrams, see solution of Problem 13.1.

13.6



$$\theta_{A0} = -\frac{wL^3}{24EI}$$

1



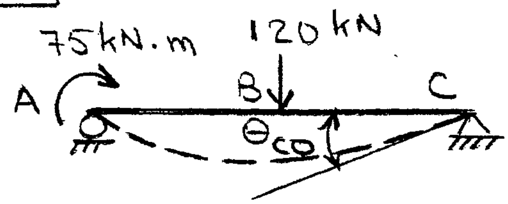
$\times M_A$

$$f_{AA} = \frac{L}{3EI}$$

$$\theta_{A0} + f_{AA} M_A = 0 \qquad M_A = \frac{wL^3}{24EI}\left(\frac{3EI}{L}\right) = \frac{wL^2}{8}$$

For reactions, and shear and moment diagrams, see solution of Problem 13.2.

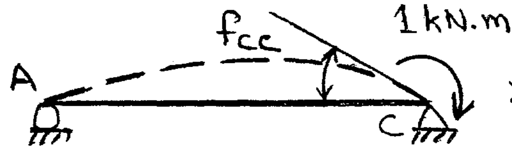
13.7



Using beam deflection formulas:

$$\theta_{CO} = -\frac{150}{EI} - \frac{1080}{EI}$$

$$= -\frac{1230 \text{ kN.m}^2}{EI}$$

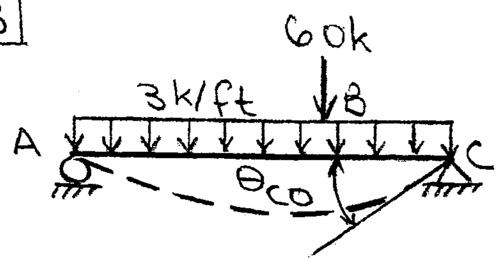


$\times M_C$ $\quad f_{CC} = \dfrac{4 \text{ kN.m}^2/\text{kN.m}}{EI}$

$$\theta_{CO} + f_{CC} M_C = 0 \qquad M_C = \frac{1230}{4} = \underline{307.5 \text{ kN.m}}$$

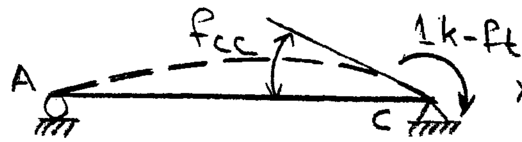
For reactions, and shear and moment diagrams, see solution of Problem 13.3.

$\boxed{13.8}$



Using beam deflection formula:

$$\theta_{CO} = -\frac{3375}{EI} - \frac{3333}{EI}$$
$$= -\frac{6708 \text{ k-ft}^2}{EI}$$

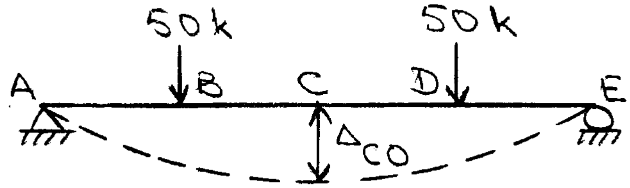


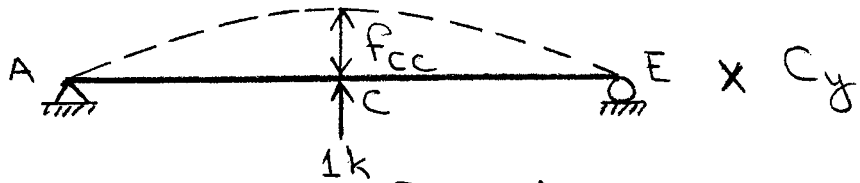
$\times M_C$ $\quad f_{CC} = \frac{10 \text{ k-ft}^2/\text{k-ft}}{EI}$

$$\theta_{CO} + f_{CC} M_C = 0 \qquad M_C = \frac{6708}{10} = \underline{670.8 \text{ k-ft}}$$

For reactions, and shear and bending moment diagrams, see solution of Problem 13.4.

13.9



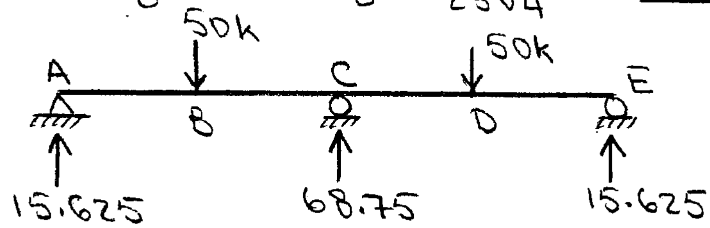


Using beam deflection formulas:

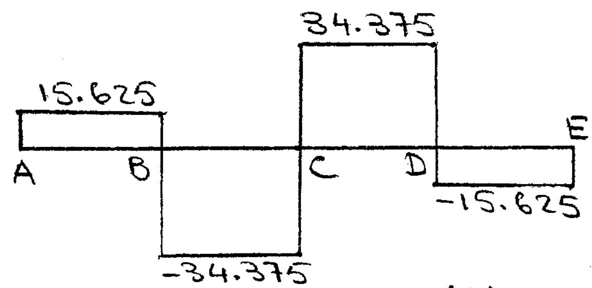
$$\Delta_{CO} = -\frac{158400 \text{ k-ft}^3}{EI} \qquad f_{CC} = \frac{2304 \text{ k-ft}^3/\text{k}}{EI}$$

Compatibility Equation:

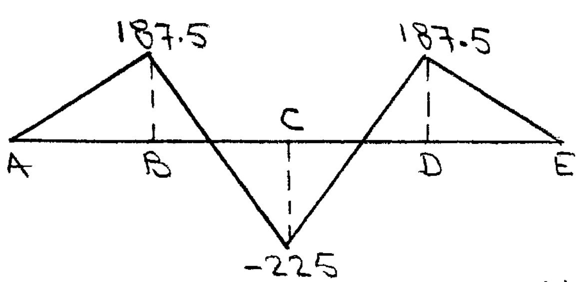
$$\Delta_{CO} + f_{CC} C_y = 0 \qquad C_y = \frac{158400}{2304} = \underline{68.75 \text{ k} \uparrow}$$



Reactions

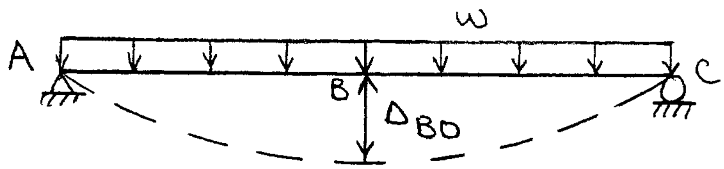


Shear Diagram (k)

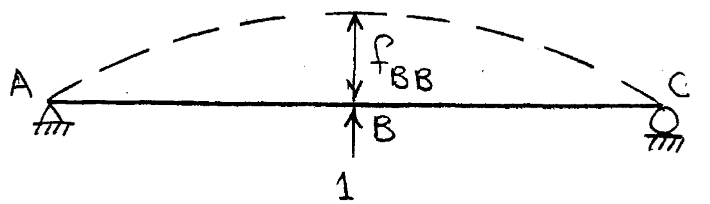


Bending Moment Diagram (k-ft)

13.10



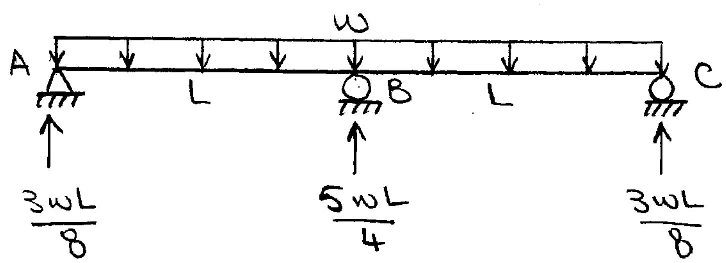
$$\Delta_{BO} = -\frac{5wL^4}{24EI}$$



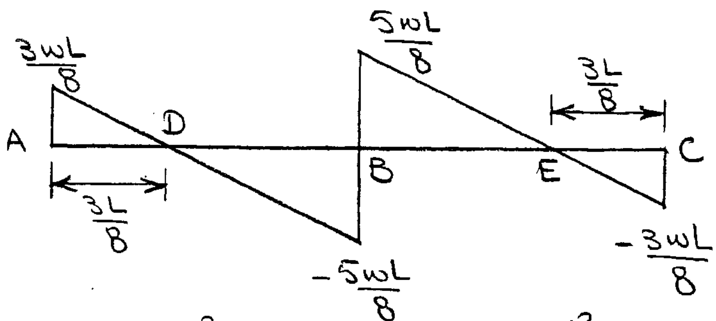
$\times B_y$

$$f_{BB} = \frac{L^3}{6EI}$$

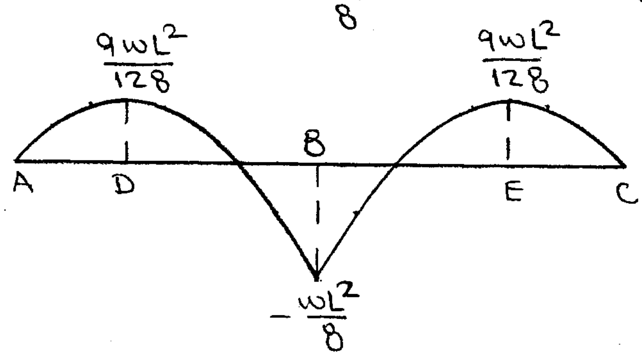
$$\Delta_{BO} + f_{BB} B_y = 0 \qquad B_y = \frac{5wL^4}{24EI}\left(\frac{6EI}{L^3}\right) = \underline{\frac{5wL}{4}} \uparrow$$



Reactions

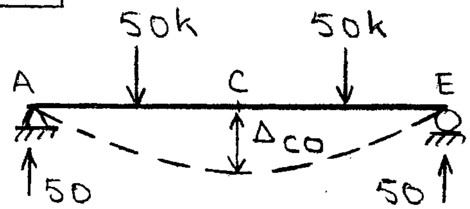


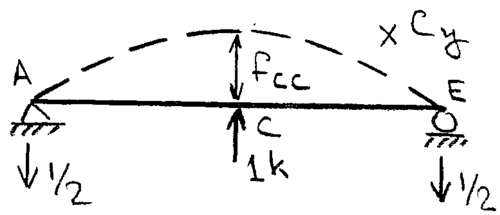
Shear Diagram

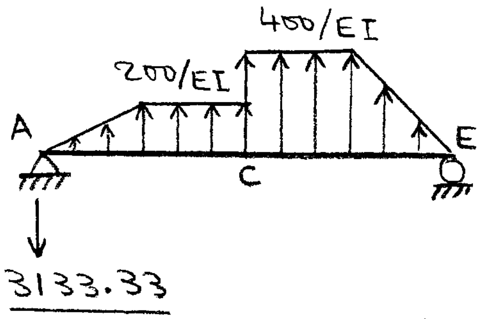


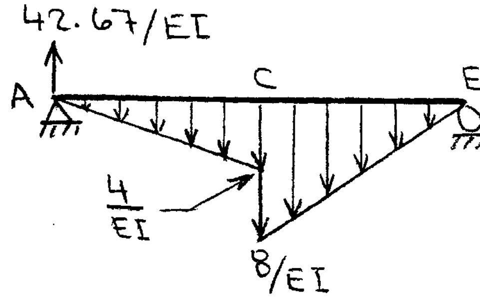
Bending Moment Diagram

13.11









Conjugate Beam for External Loading

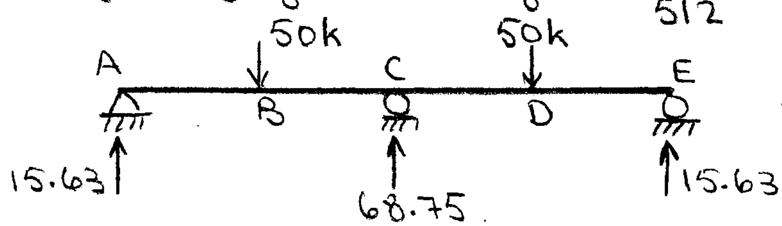
Conjugate Beam for Unit Value of Redundant C_y

$$\Delta_{CO} = -\frac{3133.33}{EI}(16) + \frac{1}{2}(8)\left(\frac{200}{EI}\right)\left(\frac{8}{3}+8\right) + \left(\frac{200}{EI}\right)8(4)$$

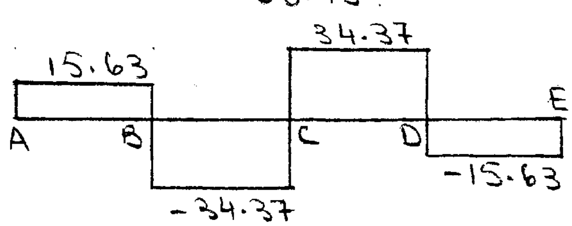
$$= -\frac{35200}{EI}$$

$$f_{CC} = \frac{42.67}{EI}(16) - \frac{1}{2}(16)\left(\frac{4}{EI}\right)\left(\frac{16}{3}\right) = \frac{512}{EI}$$

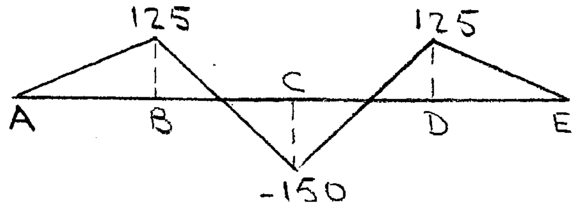
$$\Delta_{CO} + f_{CC}C_y = 0 \qquad C_y = \frac{35200}{512} = \underline{68.75\text{ k}\uparrow}$$



Reactions

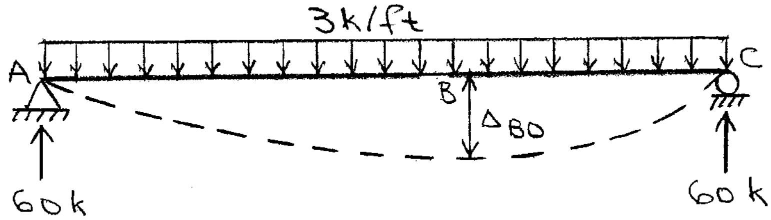


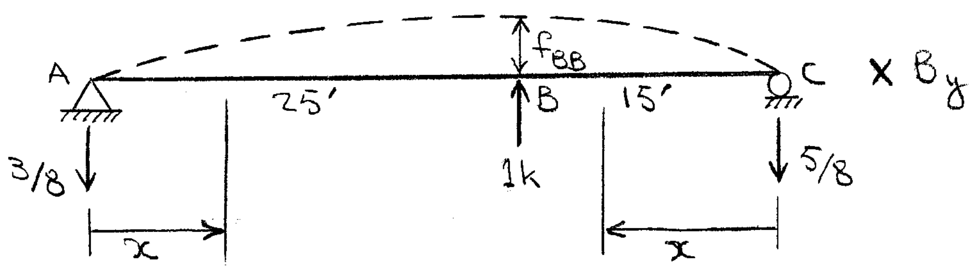
Shear Diagram (k)



Bending Moment Diagram (k-ft)

13.12





Using the virtual work method:

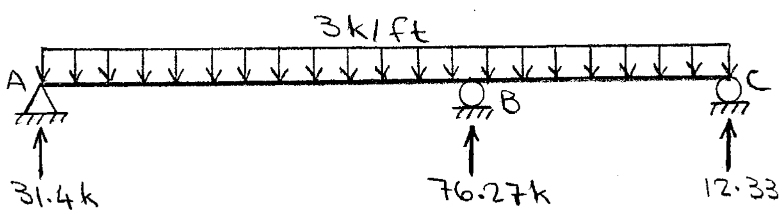
$$\Delta_{BO} = \frac{1}{EI}\left[\frac{1}{2}\int_0^{25}\left(60x - \frac{3x^2}{2}\right)\left(-\frac{3x}{8}\right)dx + \int_0^{15}\left(60x - \frac{3x^2}{2}\right)\left(-\frac{5x}{8}\right)dx\right]$$

$$= -\frac{61450.2 \text{ k-ft}^3}{EI}$$

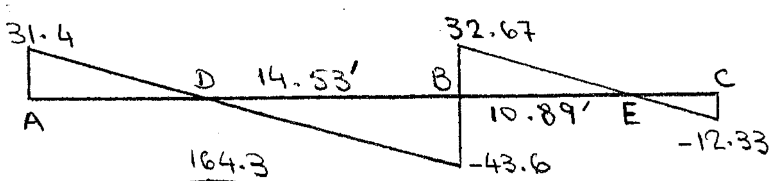
$$f_{BB} = \frac{1}{EI}\left[\frac{1}{2}\int_0^{25}\left(-\frac{3x}{8}\right)^2 dx + \int_0^{15}\left(-\frac{5x}{8}\right)^2 dx\right] = \frac{805.66 \text{ k-ft}^3/\text{k}}{EI}$$

Compatibility Equation:

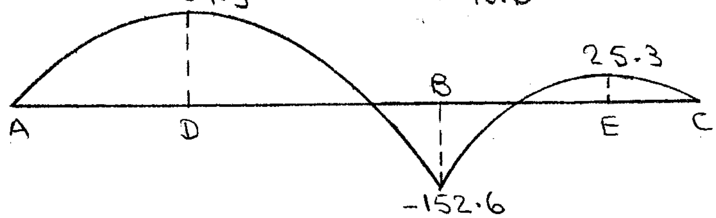
$$\Delta_{BO} + f_{BB}B_y = 0 \qquad B_y = \frac{61450.2}{805.66} = \underline{76.27 \text{ k}\uparrow}$$



Reactions

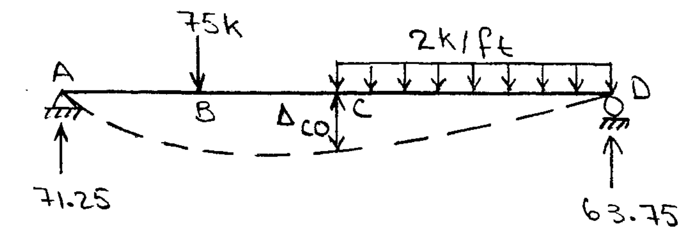


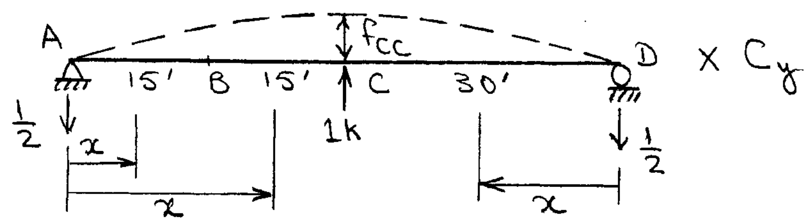
Shear Diagram (k)



Bending Moment Diagram (k-ft)

$\boxed{13.13}$





Using the method of virtual work:

$$\Delta_{CO} = \frac{1}{EI}\left[\frac{1}{3}\int_0^{15}(71.25x)\left(-\frac{x}{2}\right)dx\right.$$

$$+\frac{1}{3}\int_{15}^{30}\{71.25x - 75(x-15)\}\left(-\frac{x}{2}\right)dx$$

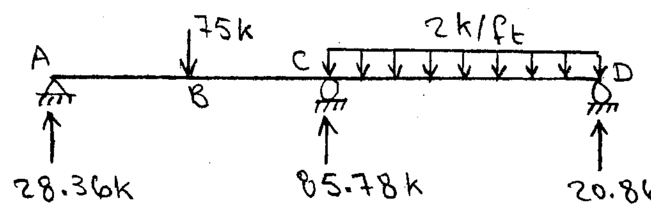
$$\left.+\int_0^{30}(63.75x - x^2)\left(-\frac{x}{2}\right)dx\right]$$

$$= -\frac{257343.75 \text{ k-ft}^3}{EI}$$

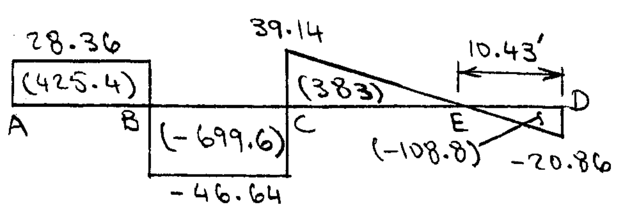
$$f_{CC} = \frac{1}{EI}\left[\frac{1}{3}\int_0^{30}\left(-\frac{x}{2}\right)^2 dx + \int_0^{30}\left(-\frac{x}{2}\right)^2 dx\right] = \frac{3000 \text{ k-ft}^3/\text{k}}{EI}$$

Compatibility Equation: $\Delta_{CO} + f_{CC}\,C_y = 0$

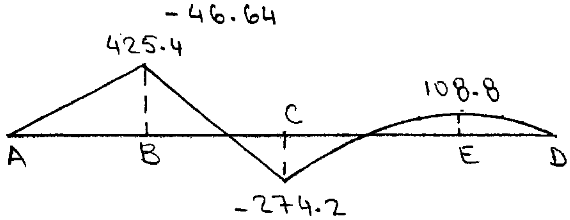
$$C_y = \frac{257343.75}{3000} = \underline{85.78 \text{ k} \uparrow}$$



Reactions

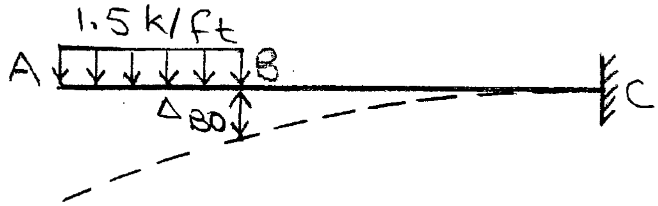


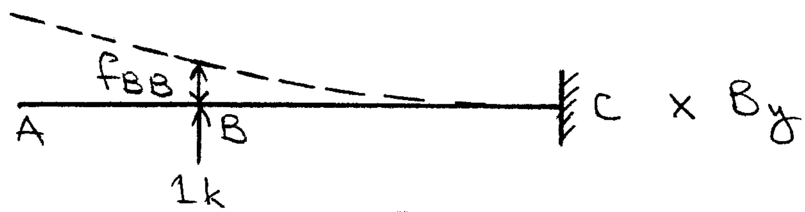
Shear Diagram (k)



Bending Moment Diagram (k-ft)

$\boxed{13.14}$



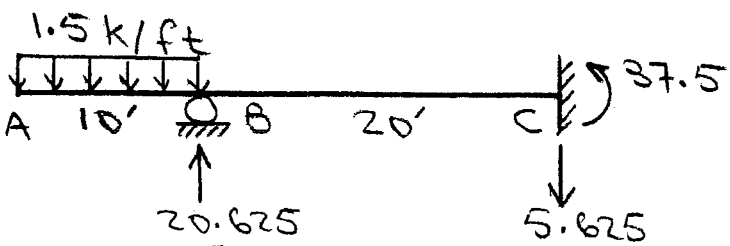


Using beam deflection formulas:

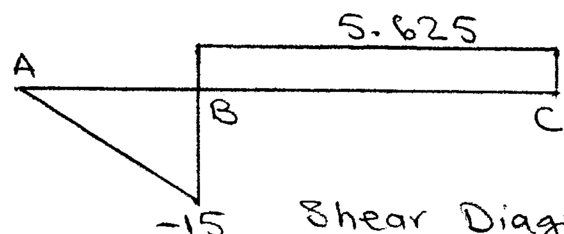
$$\Delta_{BO} = -\frac{55000\ \text{k-ft}^3}{EI} \qquad f_{BB} = \frac{2666.67\ \text{k-ft}^3/\text{k}}{EI}$$

Compatibility Equation:

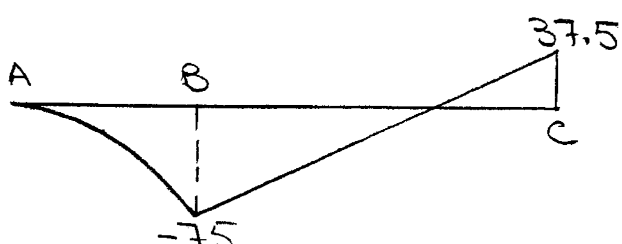
$$\Delta_{BO} + f_{BB} B_y = 0 \qquad B_y = \frac{55000}{2666.67} = \underline{20.625\ \text{k}\uparrow}$$



Reactions

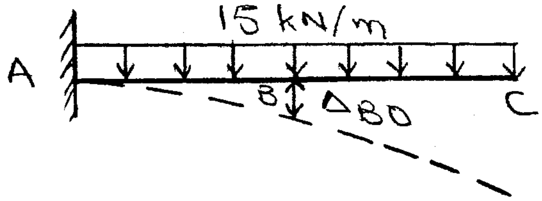


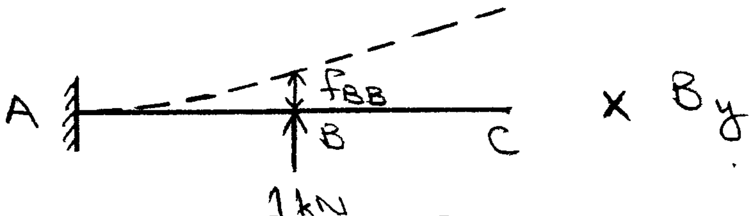
Shear Diagram (k)



Bending Moment Diagram (k-ft)

$\boxed{13.15}$



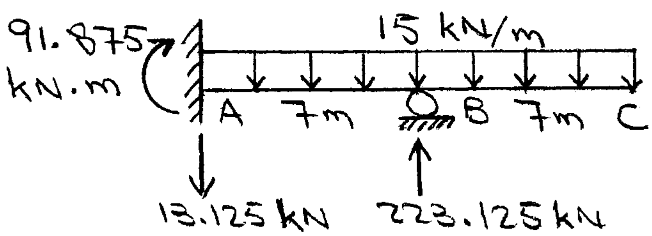


Using beam deflection formulas:

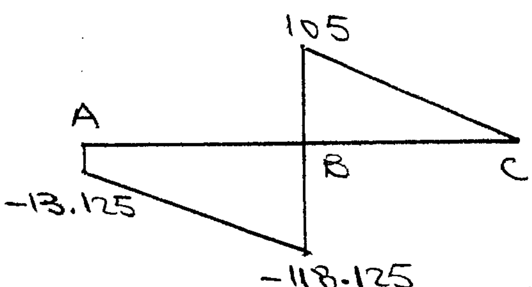
$$\Delta_{BO} = -\frac{25510.625 \text{ kN}\cdot\text{m}^3}{EI} \qquad f_{BB} = \frac{114.333 \text{ kN}\cdot\text{m}^3/\text{kN}}{EI}$$

Compatibility Equation: $\Delta_{BO} + f_{BB} B_y = 0$

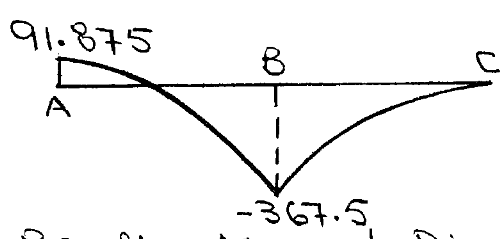
$$B_y = \frac{25510.625}{114.333} = \underline{223.125 \text{ kN} \uparrow}$$



Reactions

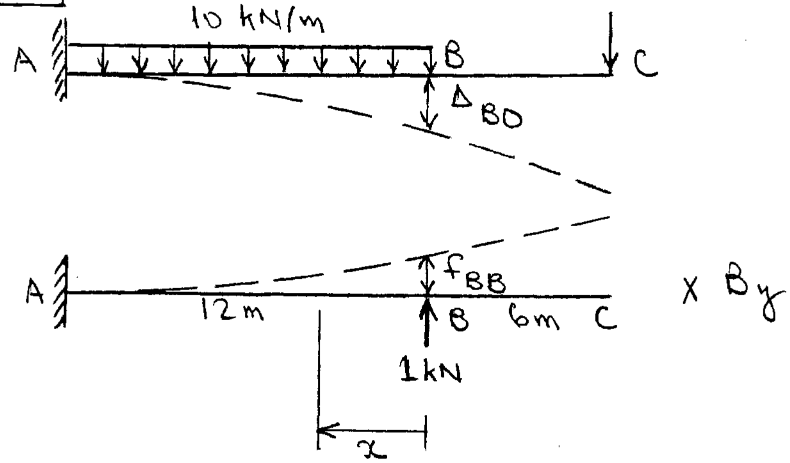


Shear Diagram (kN)



Bending Moment Diagram (kN.m)

B.16



Using the virtual work method:

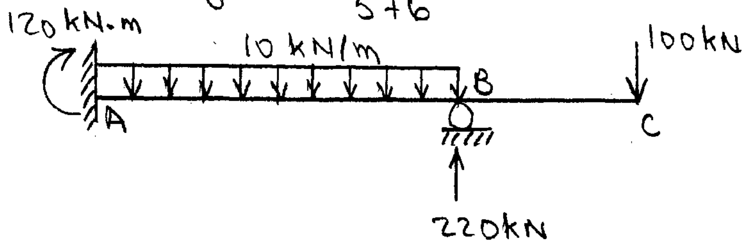
$$\Delta_{BO} = \frac{1}{EI}\left[\int_0^{12}\left\{-100(6+x) - 5x^2\right\} x\, dx\right]$$

$$= -\frac{126720 \text{ kN-m}^3}{EI}$$

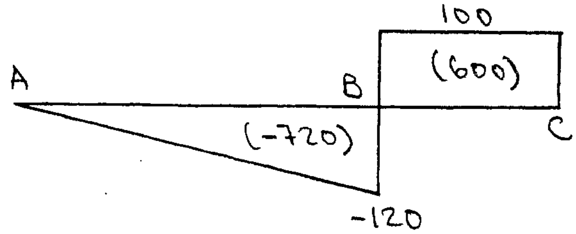
$$f_{BB} = \frac{1}{EI}\left[\int_0^{12} x^2\, dx\right] = \frac{576 \text{ kN-m}^3/\text{kN}}{EI}$$

Compatibility Equation: $\Delta_{BO} + f_{BB} B_y = 0$

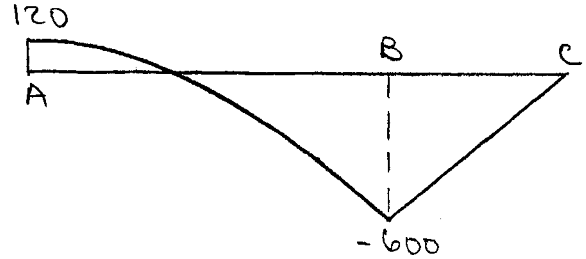
$$B_y = \frac{126720}{576} = \underline{220 \text{ kN} \uparrow}$$



Reactions

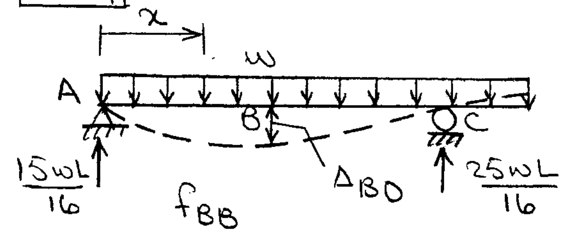


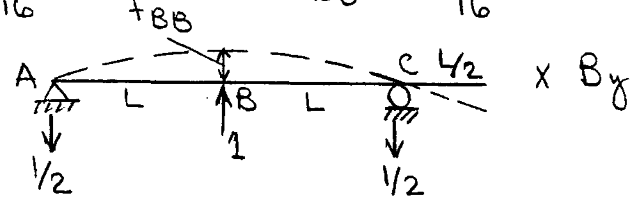
Shear Diagram (kN)



Bending Moment Diagram (kN.m)

13.17





Using the virtual work method:

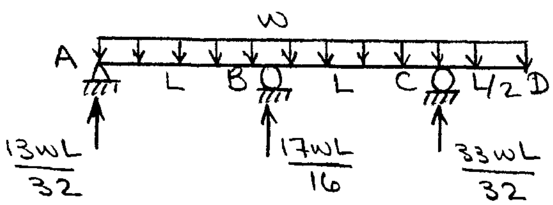
$$\Delta_{BO} = \frac{1}{EI}\left[\int_0^L \left(\frac{15}{16}wLx - \frac{wx^2}{2}\right)\left(-\frac{x}{2}\right)dx + \int_L^{2L}\left(\frac{15}{16}wLx - \frac{wx^2}{2}\right)\left\{-\frac{x}{2} + 1(x-L)\right\}dx\right]$$

$$= -\frac{17wL^4}{96EI}$$

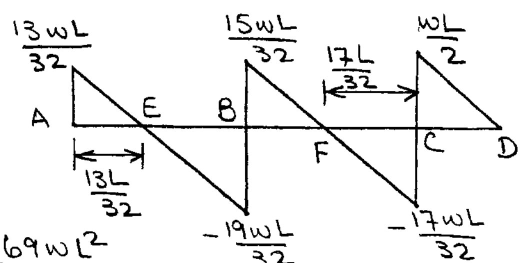
$$f_{BB} = \frac{2}{EI}\int_0^L \left(-\frac{x}{2}\right)^2 dx = \frac{L^3}{6EI}$$

Compatibility Equation: $\Delta_{BO} + f_{BB}B_y = 0$

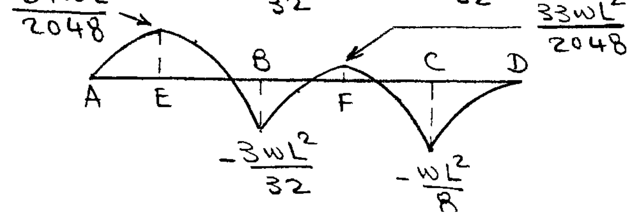
$$B_y = \frac{17wL^4}{96EI}\left(\frac{6EI}{L^3}\right) = \underline{\frac{17wL}{16}} \uparrow$$



Reactions

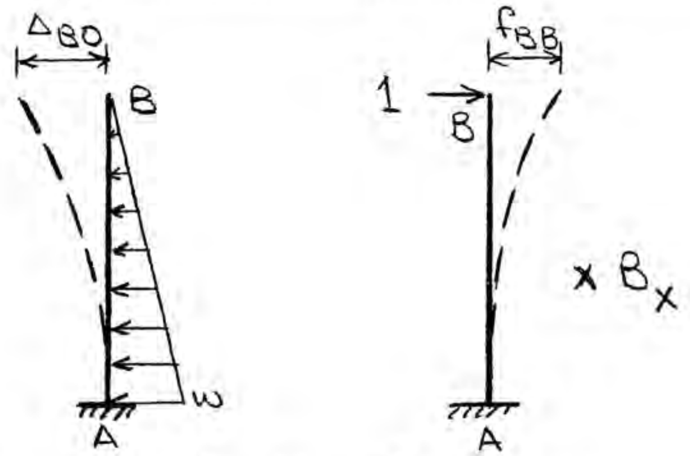


Shear Diagram



Bending Moment Diagram

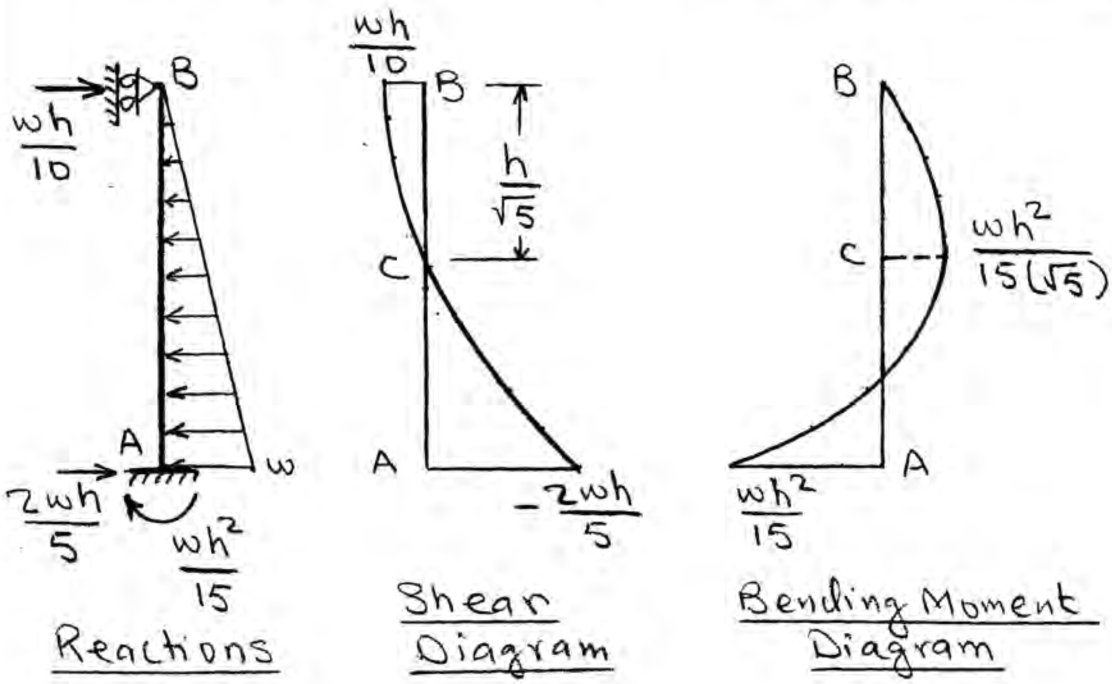
13.18



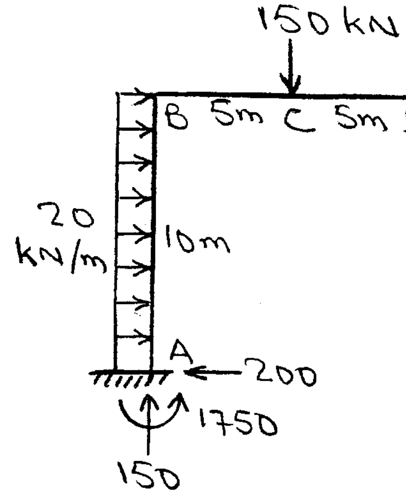
Using beam deflection formulas:

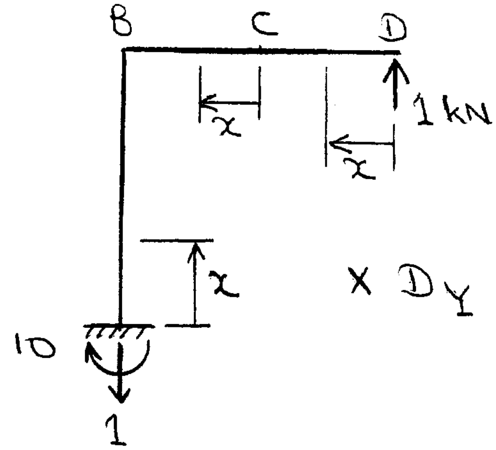
$$\Delta_{BO} = -\frac{wh^4}{30EI} \;; \qquad f_{BB} = \frac{h^3}{3EI}$$

$$\Delta_{BO} + f_{BB} B_x = 0 \qquad B_x = \left(\frac{wh^4}{30EI}\right)\left(\frac{3EI}{h^3}\right) = \frac{wh}{10} \rightarrow$$



13.19





Using the virtual work method:

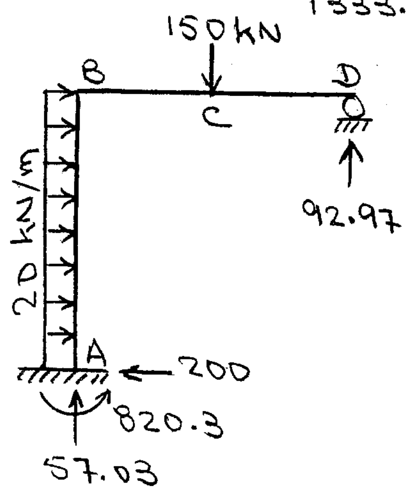
$$\Delta_{DO} = \frac{1}{EI}\left[\int_0^{10}(-1750 + 200x - 10x^2)(10)\,dx + \int_0^{5}(-150x)\,1\,(x+5)\,dx\right]$$

$$= -\frac{123958.33 \text{ kN.m}^3}{EI}$$

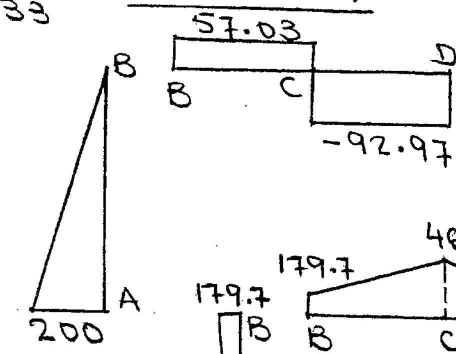
$$f_{DD} = \frac{1}{EI}\left[\int_0^{10}(10)^2 dx + \int_0^{10}(x)^2 dx\right] = \frac{1333.33 \text{ kN.m}^3/\text{kN}}{EI}$$

Compatibility Equation: $\Delta_{DO} + f_{DD} D_Y = 0$

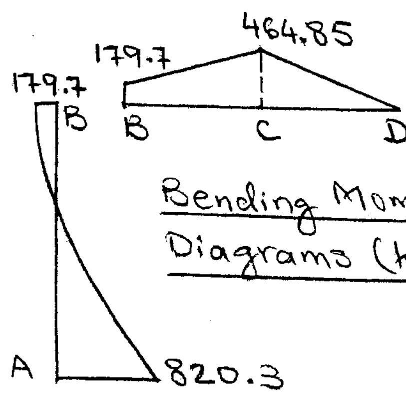
$$D_Y = \frac{123958.33}{1333.33} = 92.97 \text{ kN} \uparrow$$



Reactions

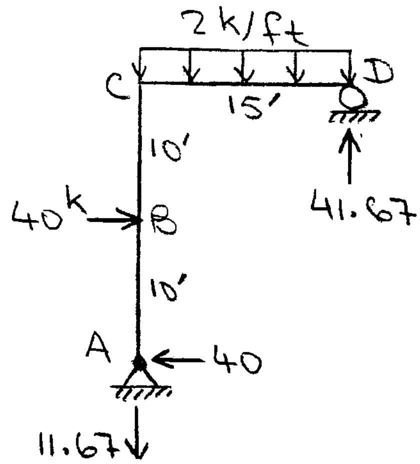


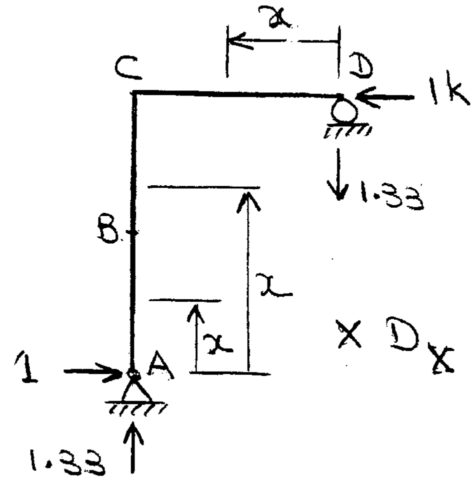
Shear Diagrams (kN)



Bending Moment Diagrams (kN.m)

13.20





× D_X

Using the virtual work method:

$$\Delta_{DO} = \frac{1}{EI}\left[\int_0^{10}(40x)(-x)\,dx + \int_{10}^{20}(400)(-x)\,dx + \int_0^{15}(41.67x - x^2)(-1.33x)\,dx\right]$$

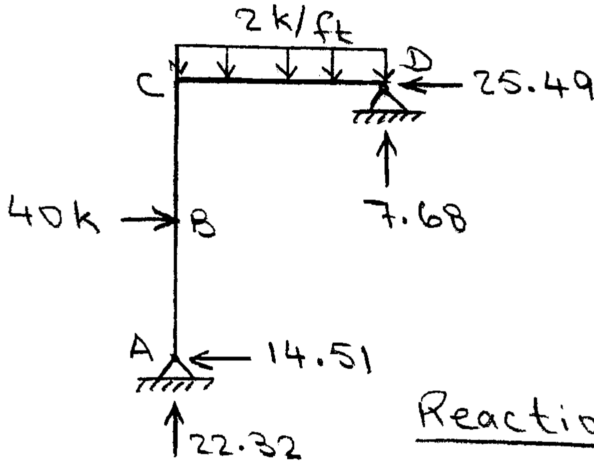
$$= -\frac{118958.33 \text{ k-ft}^3}{EI}$$

$$f_{DD} = \frac{1}{EI}\left[\int_0^{20}(-x)^2\,dx + \int_0^{15}(-1.33x)^2\,dx\right]$$

$$= \frac{4666.67 \text{ k-ft}^3/\text{k}}{EI}$$

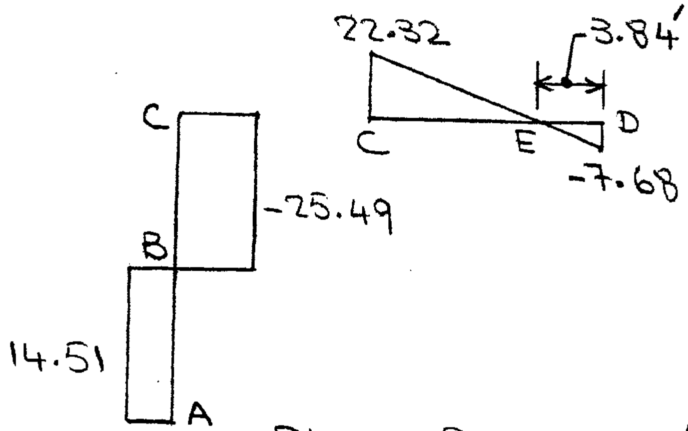
Compatibility Equation: $\Delta_{DO} + f_{DD} D_X = 0$

$$D_X = \frac{118958.33}{4666.67} = \underline{25.49 \text{ k} \leftarrow}$$

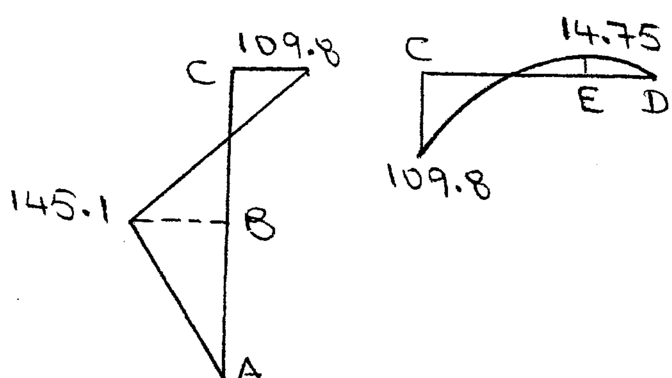


Reactions

13.20 (contd.)

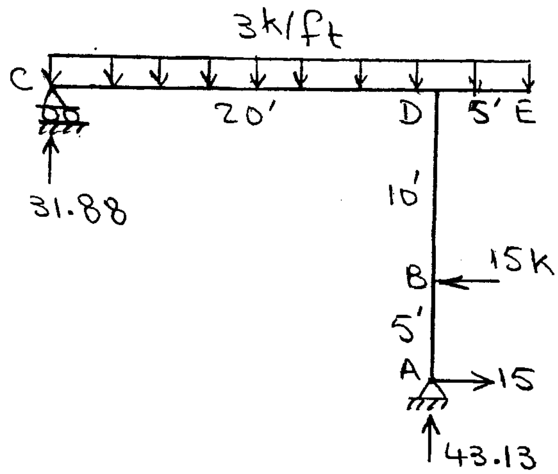


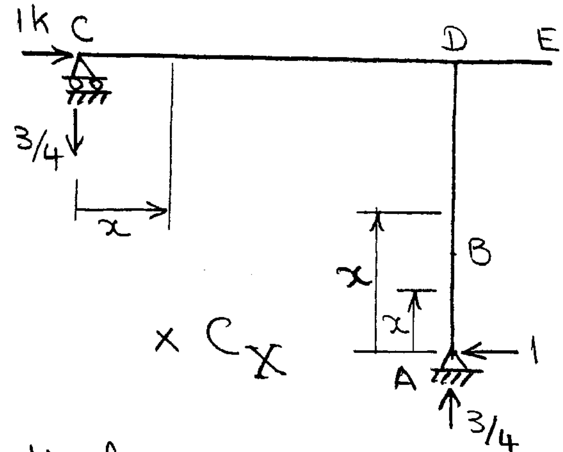
Shear Diagrams (k)



Bending Moment Diagrams (k-ft)

13.21





$\times C_X$

Using the virtual work method:

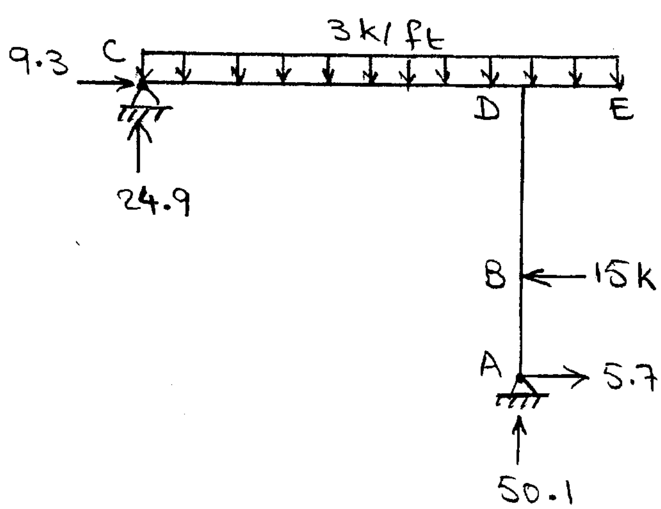
$$\Delta_{CO} = \frac{1}{EI}\left[\frac{1}{2}\int_0^{20}(31.88x - 1.5x^2)\left(-\frac{3}{4}x\right)dx + \int_0^5(-15x)\,x\,dx + \int_5^{15}\{-15x + 15(x-5)\}\,x\,dx\right]$$

$$= -\frac{17505\ \text{k-ft}^3}{EI}$$

$$f_{CC} = \frac{1}{EI}\left[\frac{1}{2}\int_0^{20}\left(-\frac{3}{4}x\right)^2 dx + \int_0^{15}(x)^2\,dx\right] = \frac{1875\ \text{k-ft}^3/\text{k}}{EI}$$

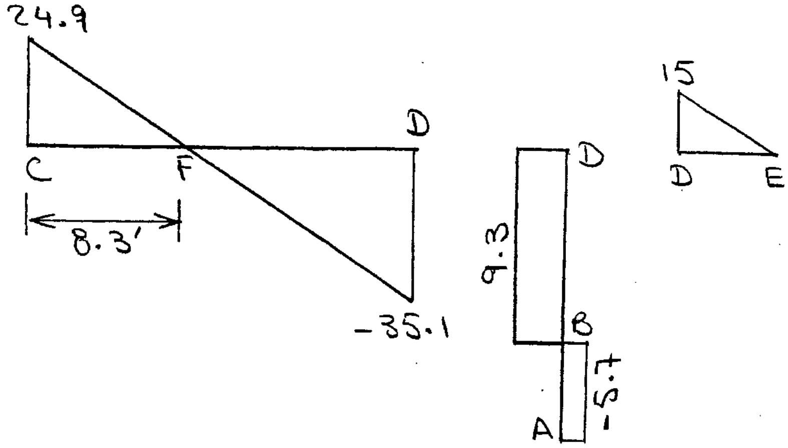
Compatibility Equation: $\Delta_{CO} + f_{CC}C_X = 0$

$$C_X = \frac{17505}{1875} = \underline{9.3\text{ k} \rightarrow}$$

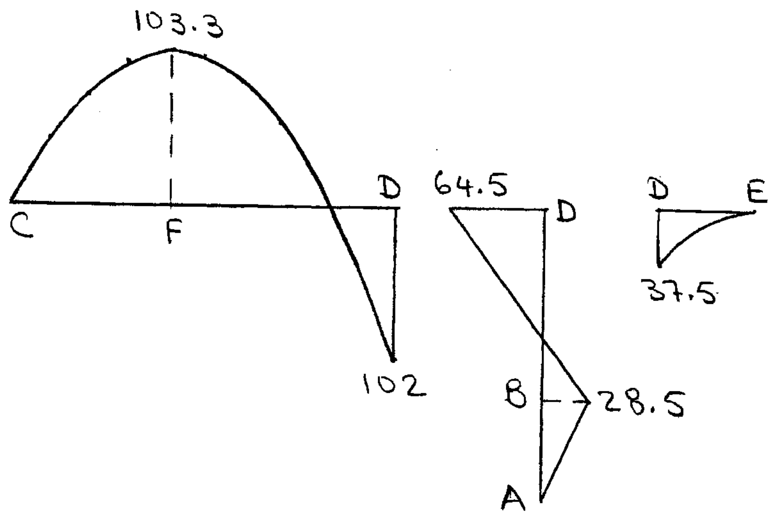


Reactions

13.21 (Contd.)

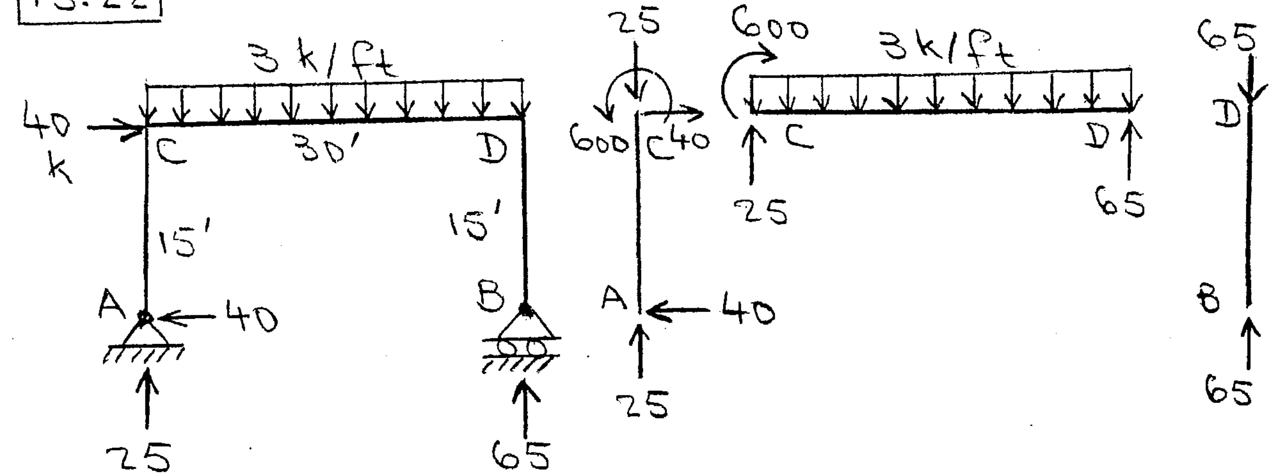


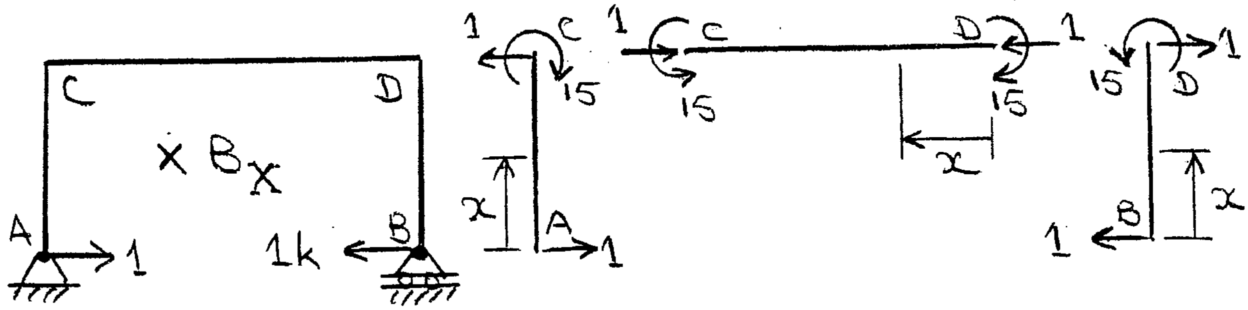
Shear Diagrams (k)



Bending Moment Diagrams (k-ft)

13.22





Using the virtual work method:

$$\Delta_{BO} = \frac{1}{EI}\left[\int_0^{15}(40x)(-x)\,dx + \int_0^{30}(65x - 1.5x^2)(-15)\,dx\right]$$

$$= -\frac{281250\ \text{k-ft}^3}{EI}$$

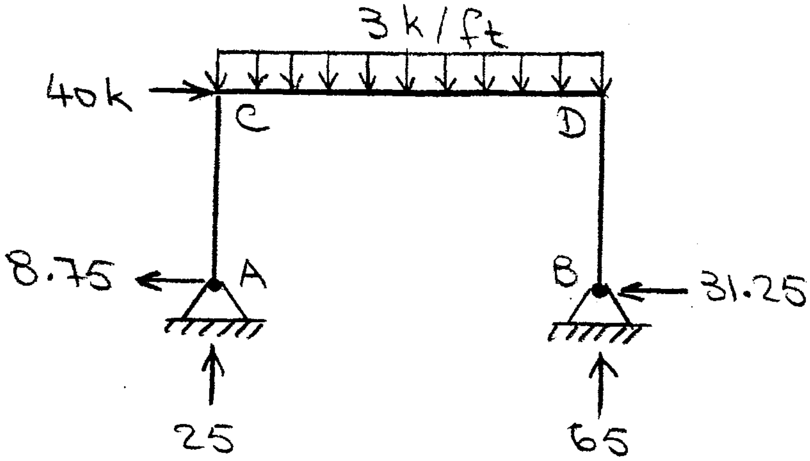
$$f_{BB} = \frac{1}{EI}\left[\int_0^{15}(-x)^2\,dx + \int_0^{30}(-15)^2\,dx + \int_0^{15}x^2\,dx\right]$$

$$= \frac{9000\ \text{k-ft}^3/\text{k}}{EI}$$

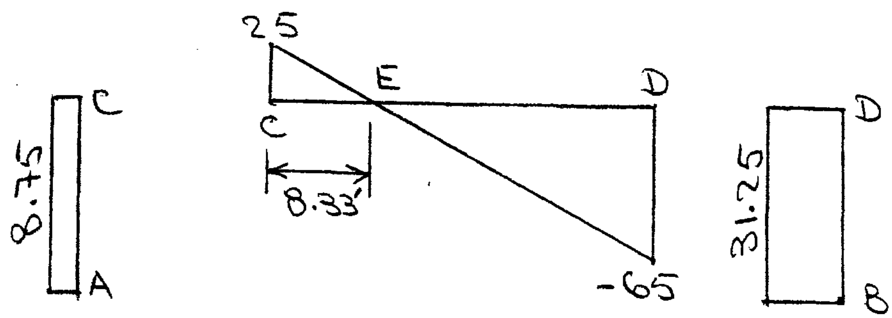
Compatibility Equation: $\Delta_{BO} + f_{BB}B_X = 0$

$$B_X = \frac{281250}{9000} = \underline{31.25\ \text{k} \leftarrow}$$

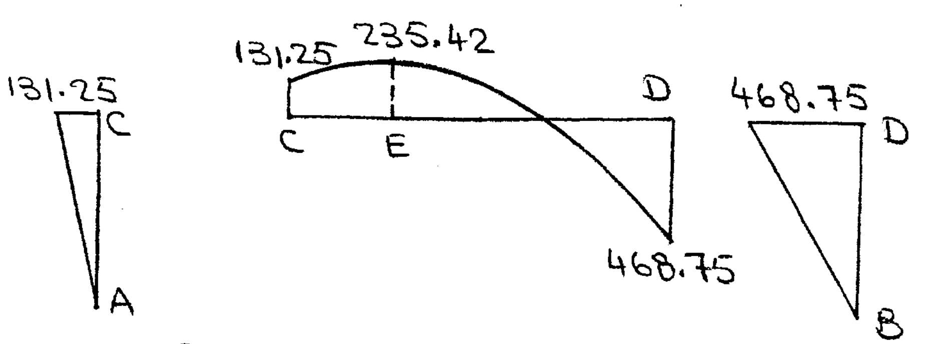
13.22 (Cont'd.)



Reactions

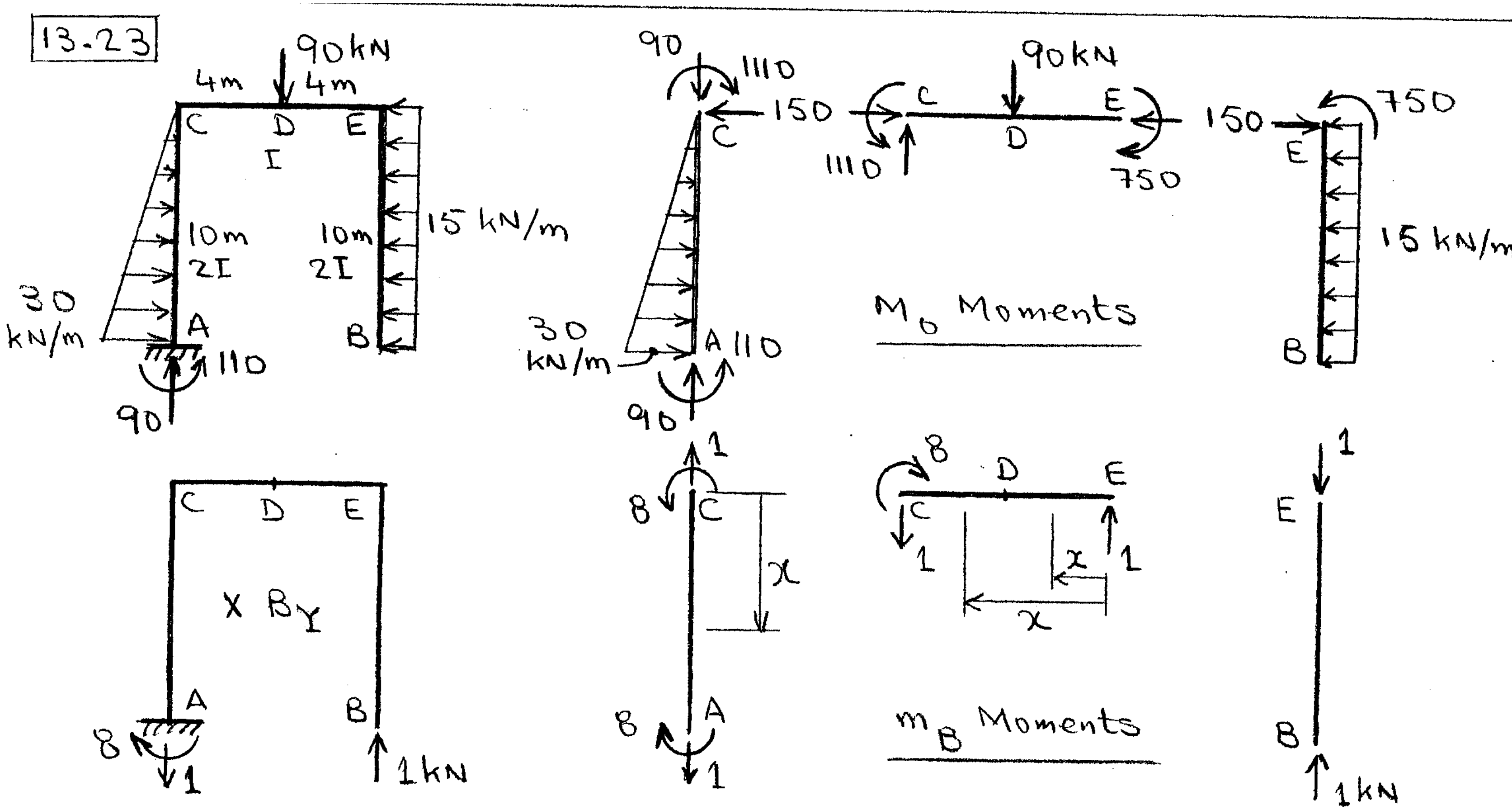


Shear Diagrams (k)



Bending Moment Diagrams (k-ft)

13-23



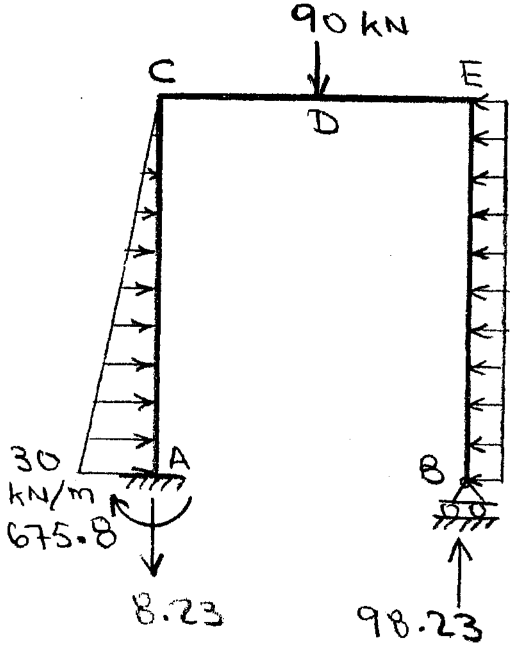
Segment	x coordinate		M_0 (kN.m)	m_B (kN.m/kN)
	Origin	Limits (m)		
CA	C	0–10	$150x - 1110 - \frac{1}{2}(x)3x\left(\frac{x}{3}\right)$	8
ED	E	0–4	-750	$1x$
DC	E	4–8	$-750 - 90(x-4)$	$1x$

$\boxed{\text{13.23 (contd.)}}$ Using the virtual work method:

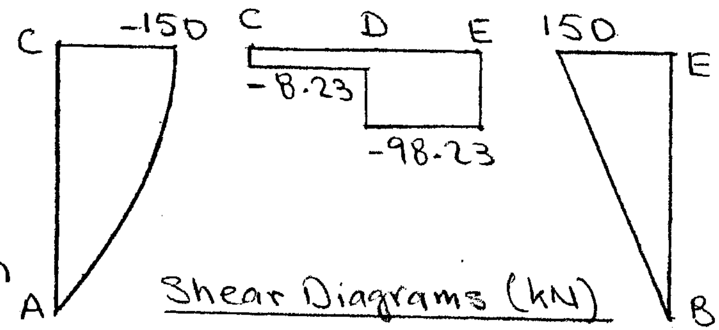
$$\Delta_{BO} = \Sigma \int \frac{M_O m_B}{EI} dx = \frac{1}{EI}\left[\frac{1}{2}\int_0^{10}\left(150x - 1110 - \frac{x^3}{2}\right)(8)\,dx + \int_0^4 (-750)x\,dx + \int_4^8 (-390 - 90x)x\,dx\right] = -\frac{48200 \text{ kN.m}^3}{EI}$$

$$f_{BB} = \Sigma \int \frac{m_B^2}{EI} dx = \frac{1}{EI}\left[\frac{1}{2}\int_0^{10} 64\,dx + \int_0^8 x^2\,dx\right] = \frac{490.67 \text{ kN.m}^3/\text{kN}}{EI}$$

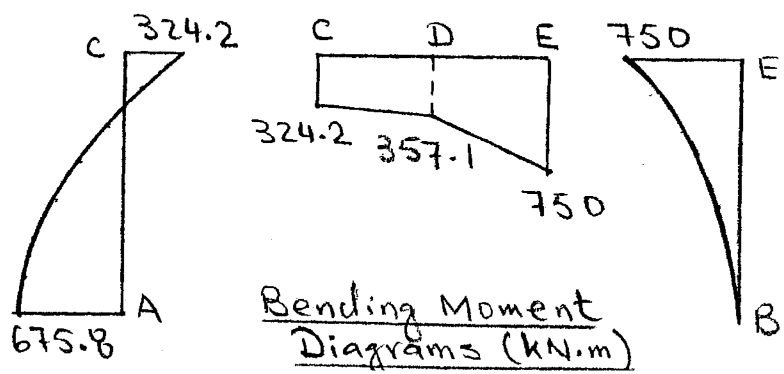
Compatibility Equation: $\Delta_{BO} + f_{BB} B_Y = 0$ $\quad B_Y = \frac{48200}{490.67} = \underline{98.23 \text{ kN} \uparrow}$



Reactions

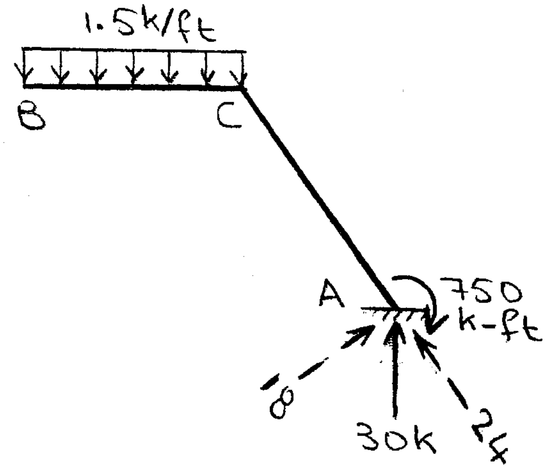


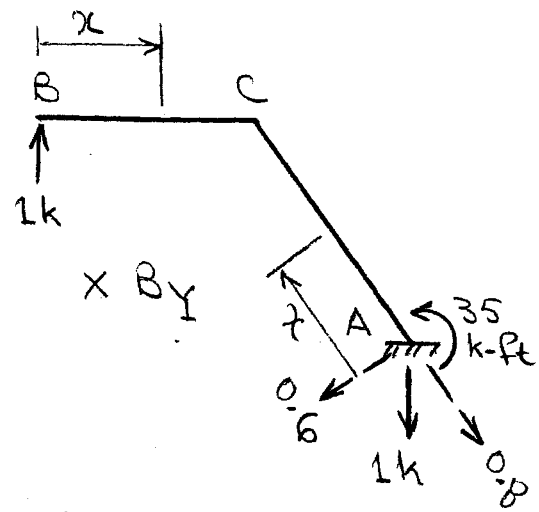
Shear Diagrams (kN)



Bending Moment Diagrams (kN.m)

13.24





Using the virtual work method:

$$\Delta_{BO} = \frac{1}{EI}\left[\int_0^{20}\left(-\frac{1.5x^2}{2}\right)(1x)\,dx + \int_0^{25}(18x-750)(-0.6x+35)\,dx\right]$$

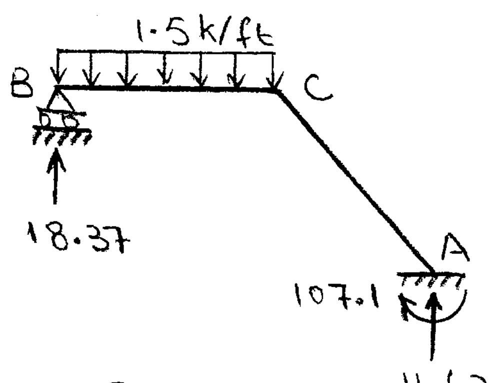
$$= -\frac{405000 \text{ k-ft}^3}{EI}$$

$$f_{BB} = \frac{1}{EI}\left[\int_0^{20}(1x)^2\,dx + \int_0^{25}(-0.6x+35)^2\,dx\right]$$

$$= \frac{22041.667 \text{ k-ft}^3/\text{k}}{EI}$$

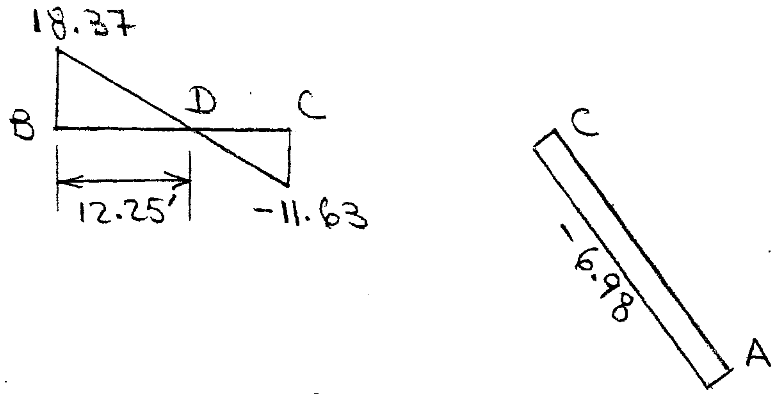
Compatibility Equation: $\Delta_{BO} + f_{BB}B_Y = 0$

$$B_Y = \frac{405000}{22041.667} = \underline{18.37 \text{ k} \uparrow}$$

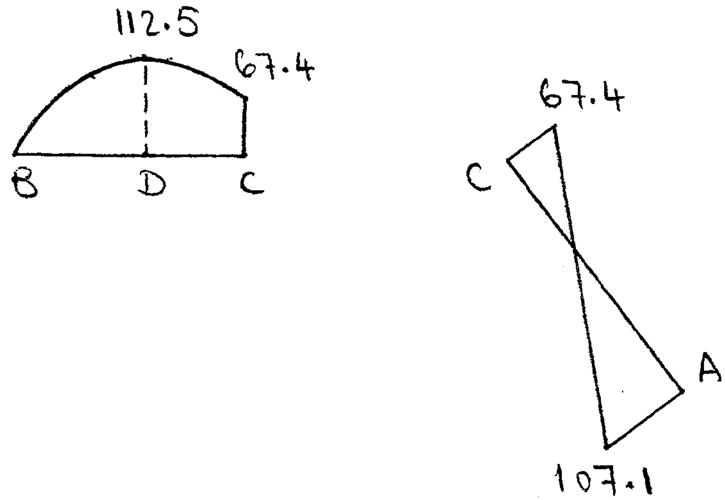


Reactions

13.24 (Contd.)

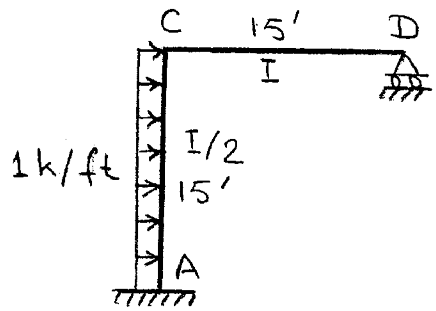


Shear Diagrams (k)

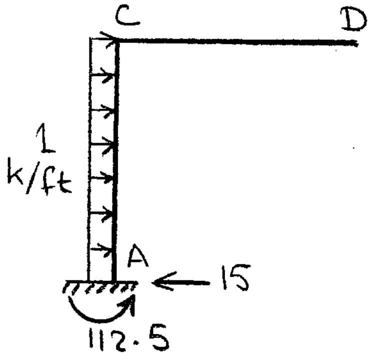


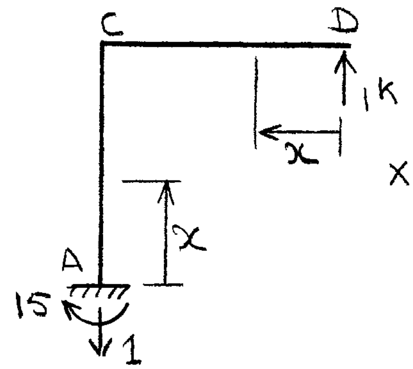
Bending Moment Diagrams (k-ft)

13-25 As the frame is symmetric subjected to antisymmetric loading, only a half of the structure needs to be analyzed.



Substructure for Analysis





$\times D_Y$

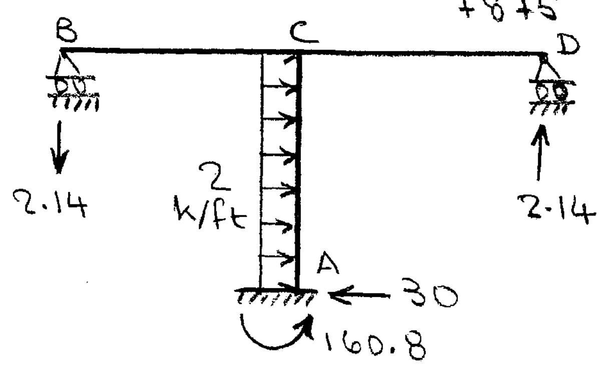
Using the virtual work method:

$$\Delta_{DO} = \frac{1}{EI}\left[2\int_0^{15}\left(-112.5 + 15x - \frac{x^2}{2}\right)(15)\,dx\right] = -\frac{16875}{EI}$$

$$f_{DD} = \frac{1}{EI}\left[2\int_0^{15}(15)^2\,dx + \int_0^{15}(1x)^2\,dx\right] = \frac{7875}{EI}$$

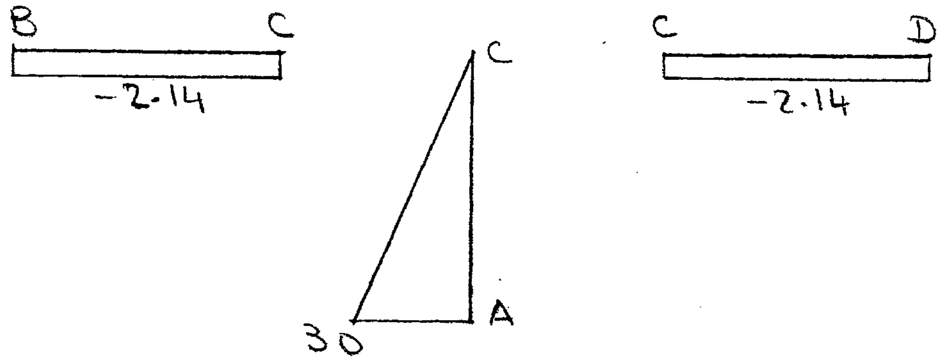
Compatibility Equation:

$$\Delta_{DO} + f_{DD} D_Y = 0 \qquad D_Y = \frac{16875}{7875} = \underline{2.14\text{ k}\uparrow}$$

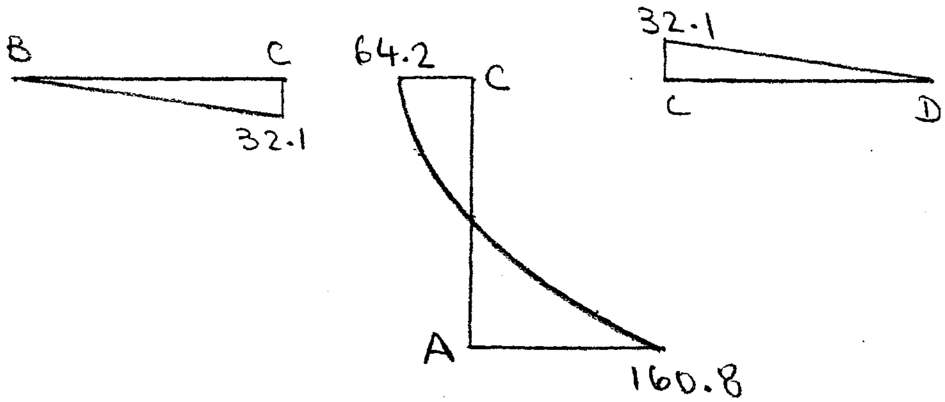


Reactions

13.25 (contd.)

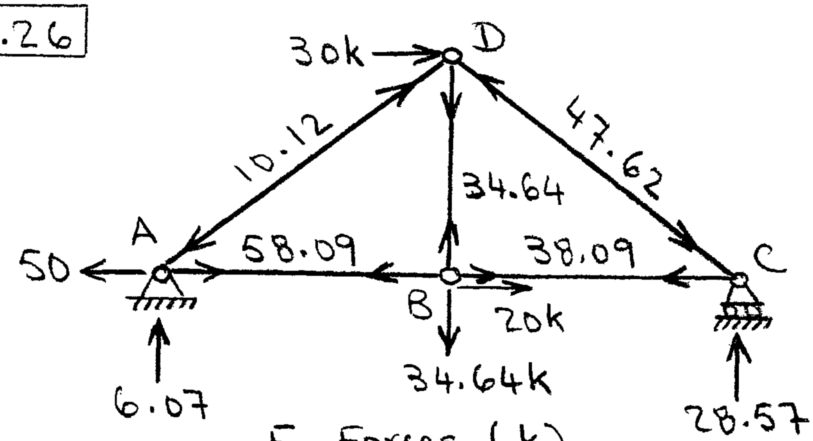


Shear Diagrams (k)

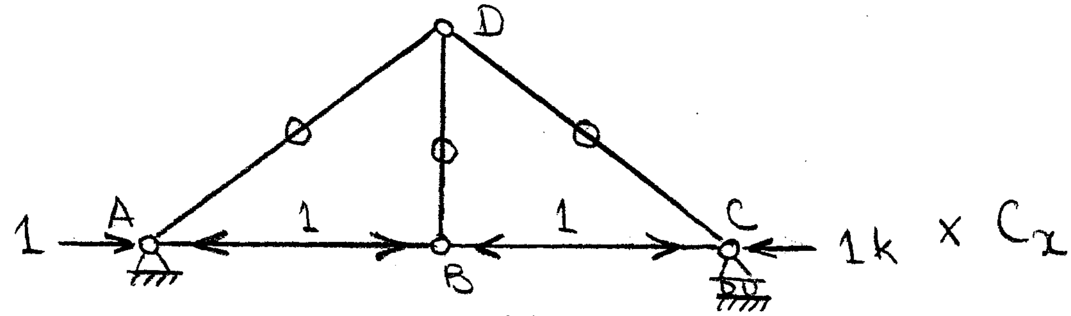


Bending Moment Diagrams (k-ft)

13.26



F_0 Forces (k)



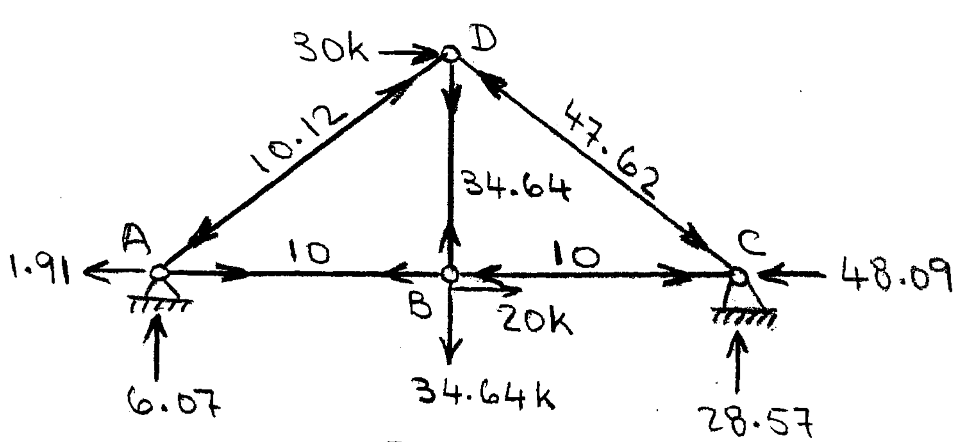
u_C Forces (k/k)

$$\Delta_{CO} = \frac{1}{EA}\sum F_0 u_C L = \frac{1}{EA}\left[58.09(-1)8 + 38.09(-1)8\right]$$

$$= -\frac{769.44 \text{ k-ft}}{EA}$$

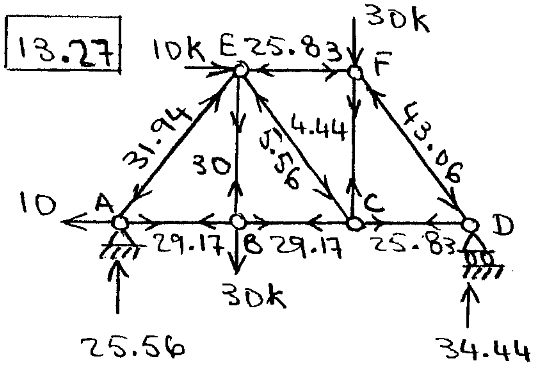
$$f_{CC} = \frac{1}{EA}\sum u_C^2 L = \frac{1}{EA}\left[2(-1)^2 8\right] = \frac{16 \text{ ft}}{EA}$$

$$C_x = -\frac{\Delta_{CO}}{f_{CC}} = \frac{769.44}{16} = \underline{48.09 \text{ k} \leftarrow}$$

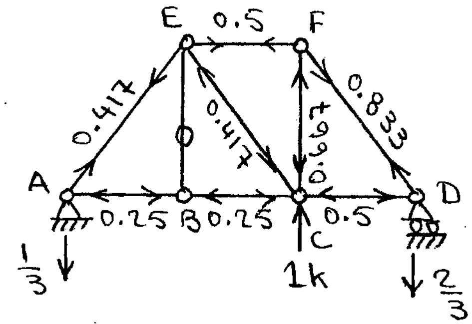


Member Forces and Reactions

13.27



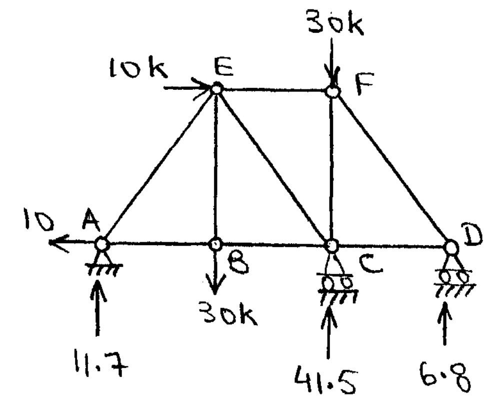
F_o Forces



u_C Forces

$\times\ C_y$

Member	L (in.)	F_o (k)	u_C (k/k)	$F_o u_C L$ (k-in.)	$u_C^2 L$ (in.)	$F = F_o + u_C C_y$ (k)
AB	180	29.17	−0.25	−1312.65	11.25	18.8
BC	180	29.17	−0.25	−1312.65	11.25	18.8
CD	180	25.83	−0.5	−2324.7	45	5.1
EF	180	−25.83	0.5	−2324.7	45	−5.1
AE	300	−31.94	0.417	−3995.69	52.17	−14.6
CE	300	−5.56	−0.417	695.56	52.17	−22.9
DF	300	−43.06	0.833	−10760.69	208.17	−8.5
BE	240	30	0	0	0	30
CF	240	4.44	−0.667	−710.76	106.77	−23.2
			Σ	−22046.28	531.78	

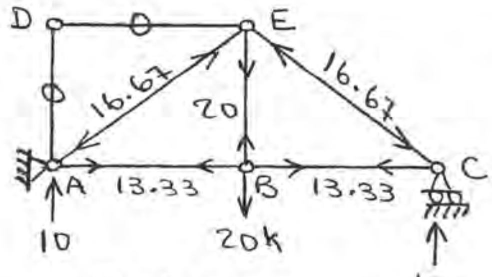


Reactions

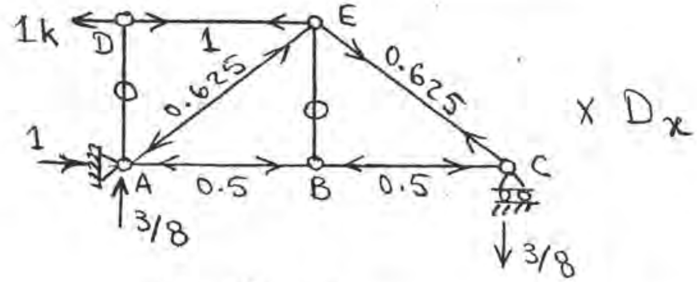
$$\Delta_{CO} = \frac{1}{EA}\Sigma F_o u_C L = -\frac{22046.28\ (\text{k-in.})}{EA}\ ; \quad f_{CC} = \frac{1}{EA}\Sigma u_C^2 L = \frac{531.78\ (\text{in.})}{EA}$$

$$C_y = -\frac{\Delta_{CO}}{f_{CC}} = \frac{22046.28}{531.78} = \underline{41.5\ \text{k}\uparrow}$$

13.28



F_0 Forces



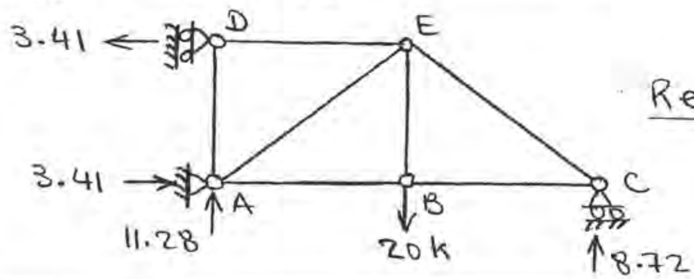
$\times D_x$

u_D Forces

Member	L (in.)	A (in.^{2})	F_0 (k)	u_D (k/k)	$F_0 u_D L/A$ (k/in.)	$u_D^2 L/A$ (1/in.)	$F = F_0 + u_D D_x$ (k)
AB	192	8	13.33	−0.5	−159.96	6	11.63
BC	192	8	13.33	−0.5	−159.96	6	11.63
DE	192	8	0	1	0	24	3.41
AE	240	6	−16.67	−0.625	416.75	15.63	−18.8
CE	240	8	−16.67	0.625	−312.56	11.72	−14.54
AD	144	6	0	0	0	0	0
BE	144	6	20	0	0	0	20
				Σ	−215.73	63.35	

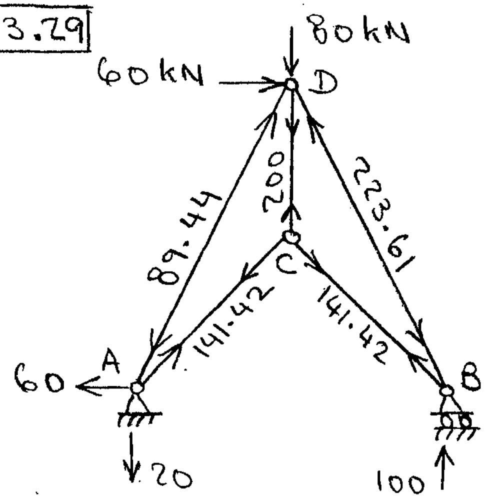
$$\Delta_{DO} = \frac{1}{E}\Sigma \frac{F_0 u_D L}{A} = -\frac{215.73\ (\text{k/in.})}{E} \quad ; \quad f_{DD} = \frac{1}{E}\Sigma \frac{u_D^2 L}{A} = \frac{63.35\ (1/\text{in.})}{E}$$

$$D_x = -\frac{\Delta_{DO}}{f_{DD}} = \frac{215.73}{63.35} = 3.41\text{ k} \leftarrow$$

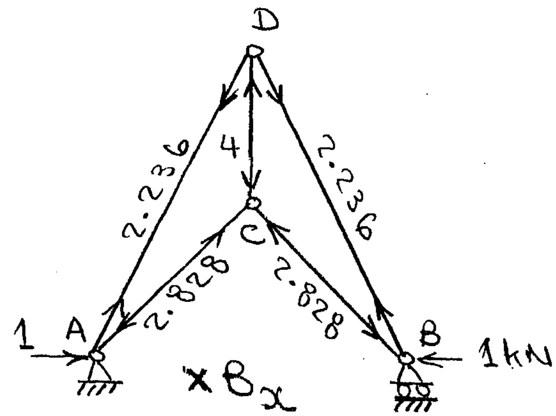


Reactions

13.29

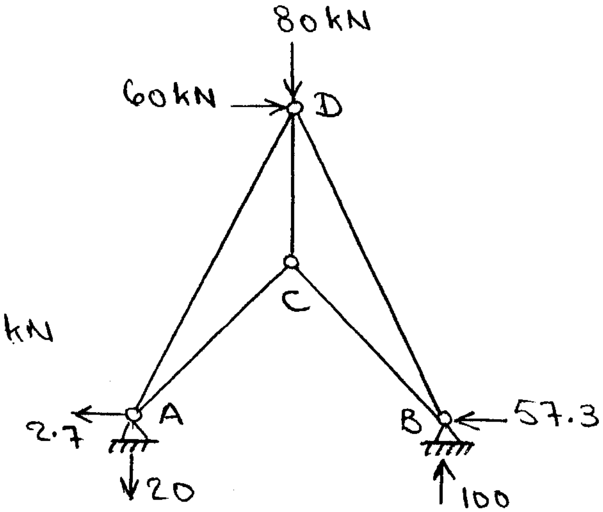


F_0 Forces



$\times B_x$

u_B Forces



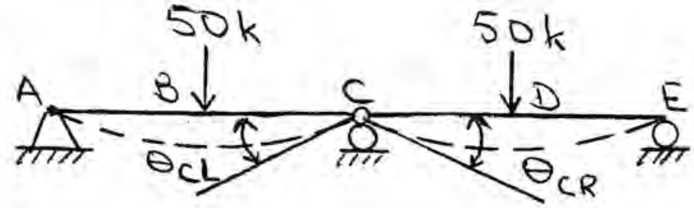
Reactions

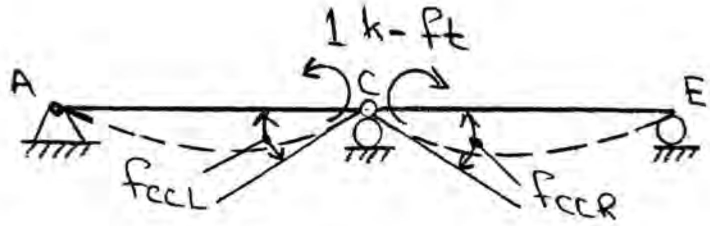
Member	L (m)	F_0 (kN)	u_B (kN/kN)	$F_0 u_B L$ (kN-m)	$u_B^2 L$ (m)	$F = F_0 + u_B B_x$ (kN)
AD	11.18	-89.44	2.236	-2235.86	55.9	38.7
BD	11.18	-223.61	2.236	-5589.91	55.9	-95.5
AC	7.07	141.42	-2.828	-2827.55	56.54	-20.6
BC	7.07	141.42	-2.828	-2827.55	56.54	-20.6
CD	5	200	-4	-4000	80	-29.2
			Σ	-17480.87	304.88	

$$\Delta_{BO} = \frac{1}{EA}\Sigma F_0 u_B L = -\frac{17480.87\ (\text{kN-m})}{EA} \quad ; \quad f_{BB} = \frac{1}{EA}\Sigma u_B^2 L = \frac{304.88\ (\text{m})}{EA}$$

$$B_x = -\frac{\Delta_{BO}}{f_{BB}} = \frac{17480.87}{304.88} = \underline{57.3\ \text{kN} \leftarrow}$$

13.30





$\times M_C$

Using beam deflection formulas:

$$\theta_{CL} = \theta_{CR} = \frac{1800 \text{ k-ft}^2}{EI}$$

$$\theta_{CO\,rel.} = \theta_{CL} + \theta_{CR} = \frac{3600 \text{ k-ft}^2}{EI}$$

$$f_{CCL} = f_{CCR} = \frac{8 \text{ ft}}{EI}$$

$$f_{CC\,rel.} = f_{CCL} + f_{CCR} = \frac{16 \text{ ft}}{EI}$$

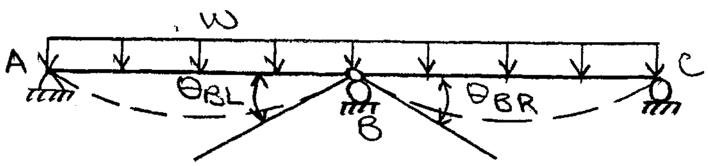
Compatibility Equation:

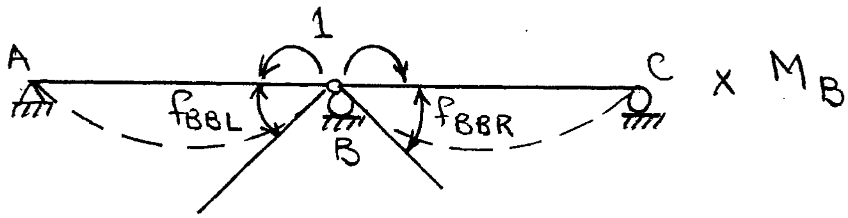
$$\theta_{CO\,rel.} + f_{CC\,rel.} M_C = 0$$

$$M_C = -\frac{3600}{16} = \underline{-225 \text{ k-ft}}$$

For reactions, and shear and bending moment diagrams, see solution of Problem 13.9.

13.31





Using beam deflection formulas:

$$\theta_{BL} = \theta_{BR} = \frac{wL^3}{24EI}$$

$$\theta_{BO\,rel.} = \theta_{BL} + \theta_{BR} = \frac{wL^3}{12EI}$$

$$f_{BBL} = f_{BBR} = \frac{L}{3EI}$$

$$f_{BB\,rel.} = f_{BBL} + f_{BBR} = \frac{2L}{3EI}$$

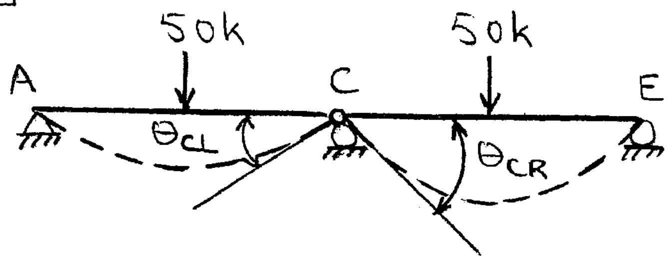
Compatibility Equation:

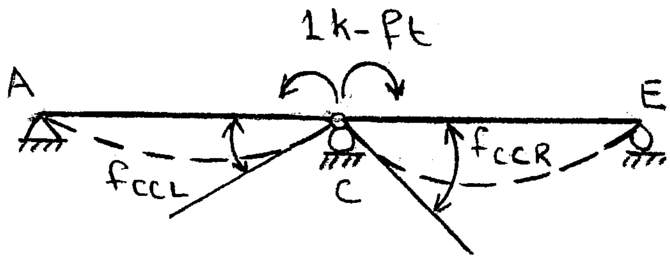
$$\theta_{BO\,rel.} + f_{BB\,rel.}\, M_B = 0$$

$$M_B = -\frac{wL^3}{12EI}\left(\frac{3EI}{2L}\right) = -\frac{wL^2}{8}$$

For reactions, and shear and bending moment diagrams, see solution of Problem 13.10.

$\boxed{13.32}$





$\times M_C$

Using beam deflection formulas:

$$\theta_{CL} = \frac{400}{EI} \; ; \; \theta_{CR} = \frac{800}{EI}$$

$$\theta_{CO\,rel.} = \theta_{CL} + \theta_{CR} = \frac{1200 \text{ k-ft}^2}{EI}$$

$$f_{CCL} = \frac{2.67}{EI} \; ; \; f_{CCR} = \frac{5.33}{EI}$$

$$f_{CC\,rel.} = f_{CCL} + f_{CCR} = \frac{8 \text{ ft}}{EI}$$

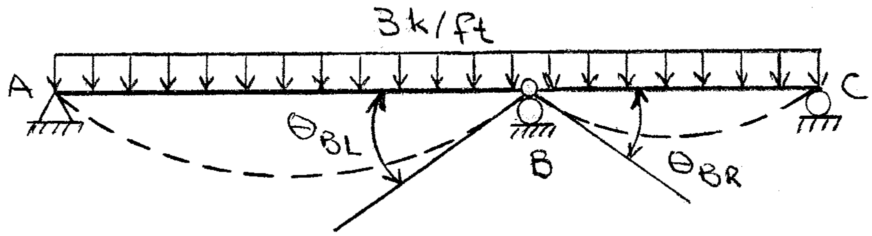
Compatibility Equation:

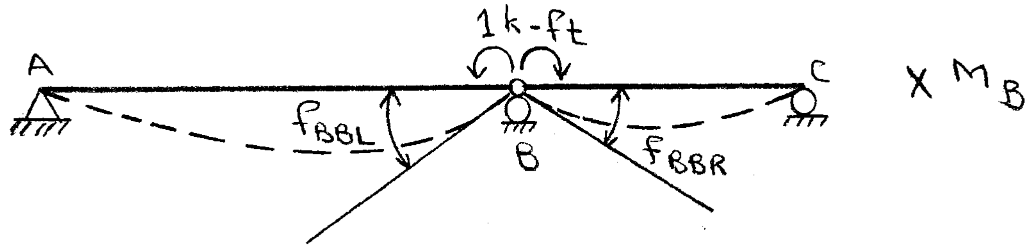
$$\theta_{CO\,rel.} + f_{CC\,rel.} M_C = 0$$

$$M_C = -\frac{1200}{8} = \underline{-150 \text{ k-ft}}$$

For reactions, and shear and bending moment diagrams, see solution of Problem 13.11.

13.33





Using beam deflection formulas:

$$\theta_{BL} = \frac{976.56 \text{ k-ft}^2}{EI} \quad ; \quad \theta_{BR} = \frac{421.88 \text{ k-ft}^2}{EI}$$

$$\theta_{BO\,rel.} = \theta_{BL} + \theta_{BR} = \frac{1398.44 \text{ k-ft}^2}{EI}$$

$$f_{BBL} = \frac{4.17 \text{ ft}}{EI} \quad ; \quad f_{BBR} = \frac{5 \text{ ft}}{EI}$$

$$f_{BB\,rel.} = f_{BBL} + f_{BBR} = \frac{9.17 \text{ ft}}{EI}$$

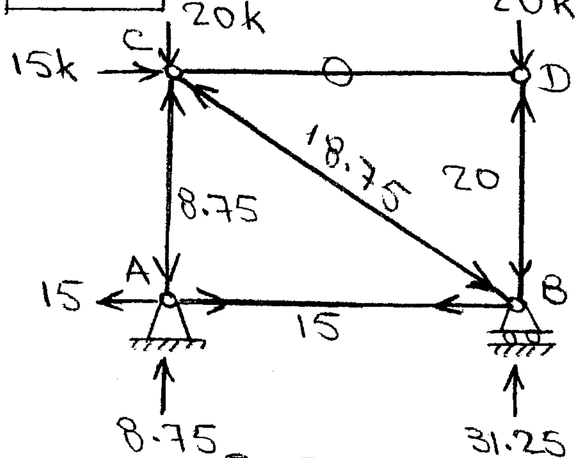
Compatibility Equation:

$$\theta_{BO\,rel.} + f_{BB\,rel.} M_B = 0$$

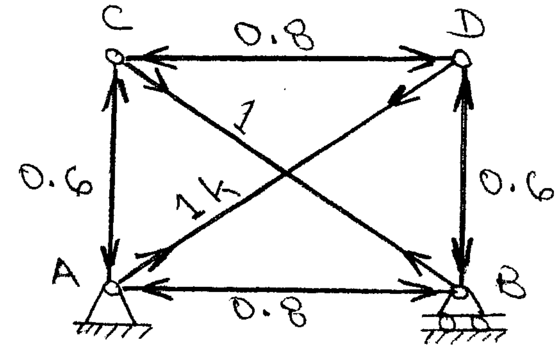
$$M_B = -\frac{1398.44}{9.17} = \underline{-152.6 \text{ k-ft}}$$

For reactions, and shear and bending moment diagrams, see solution of Problem 13.12.

13.34



F_o Forces



$\times F_{AD}$

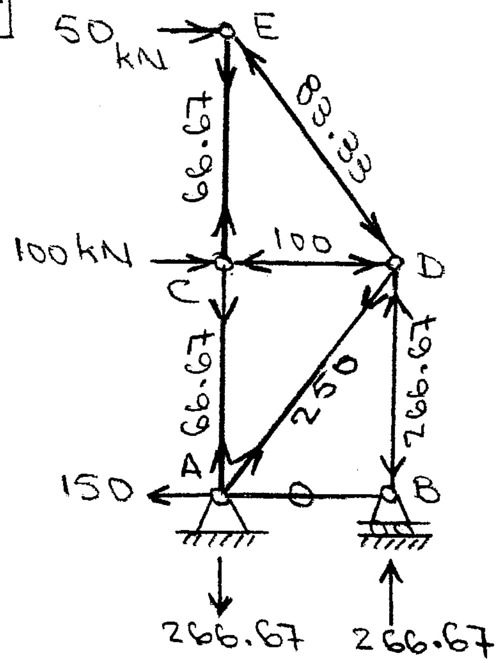
u_{AD} Forces

Member	L (in.)	A (in.²)	F_o (k)	u_{AD} (k/k)	$F_o u_{AD} L/A$ (k/in.)	$u_{AD}^2 L/A$ (1/in.)	$F = F_o + u_{AD} F_{AD}$ (k)
AB	192	6	15	−0.8	−384	20.48	11.4
CD	192	6	0	−0.8	0	20.48	−3.6
AC	144	6	−8.75	−0.6	126	8.64	−11.5
BD	144	6	−20	−0.6	288	8.64	−22.7
BC	240	8	−18.75	1	−562.5	30	−14.2
AD	240	8	0	1	0	30	4.5
				Σ	−532.5	118.24	

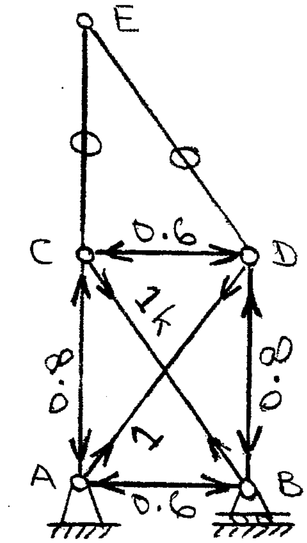
$$\Delta_{AD0} = \frac{1}{E}\sum \frac{F_o u_{AD} L}{A} = -\frac{532.5\ (\text{k-in.})}{E}\ ; \quad f_{AD,AD} = \frac{1}{E}\sum \frac{u_{AD}^2 L}{A} = \frac{118.24\ (1/\text{in.})}{E}$$

$$F_{AD} = -\frac{\Delta_{AD0}}{f_{AD,AD}} = \frac{532.5}{118.24} = \underline{4.5\ \text{k}\ (T)}$$

13.35



F_0 Forces



u_{BC} Forces

Member	L (m)	F_0 (kN)	u_{BC} (kN/kN)	$F_0 u_{BC} L$ (kN.m)	$u_{BC}^2 L$ (m)	$F = F_0 + u_{BC} F_{BC}$ (kN)
AB	3	0	−0.6	0	1.08	71.9
CD	3	−100	−0.6	180	1.08	−28.1
AC	4	66.67	−0.8	−213.33	2.56	162.5
BD	4	−266.67	−0.8	853.33	2.56	−170.8
CE	4	66.67	0	0	0	66.7
AD	5	250	1	1250	5	130.2
DE	5	−83.33	0	0	0	−83.3
BC	5	0	1	0	5	−119.8
			Σ	2070	17.28	

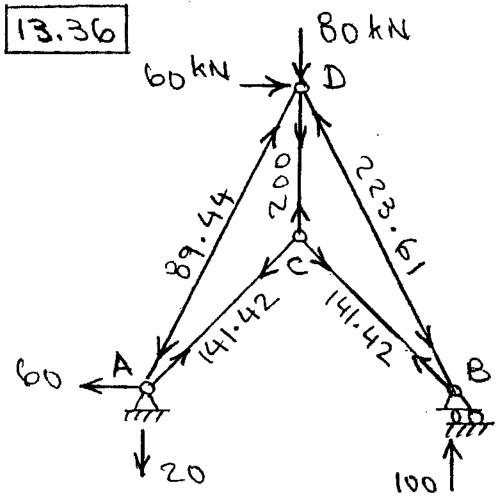
$$\Delta_{BC0} = \frac{1}{EA}\sum F_0 u_{BC} L = \frac{2070\ (\text{kN.m})}{EA}$$

$$f_{BC,BC} = \frac{1}{EA}\sum u_{BC}^2 L = \frac{17.28\ (\text{m})}{EA}$$

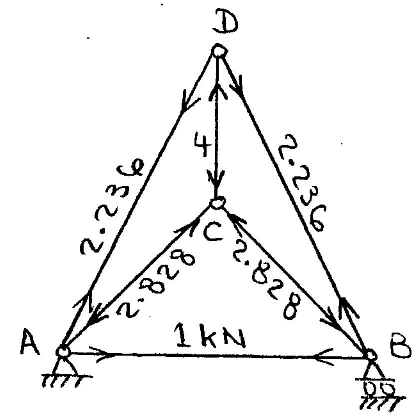
$$F_{BC} = -\frac{\Delta_{BC0}}{f_{BC,BC}} = -\frac{2070}{17.28} = -119.8\ \text{kN}$$

$$= \underline{119.8\ \text{kN (C)}}$$

13.36



F_0 Forces



u_{AB} Forces

Member	L (m)	F_0 (kN)	u_{AB} (kN/kN)	$F_0 u_{AB} L$ (kN-m)	$u_{AB}^2 L$ (m)	$F = F_0 + u_{AB} F_{AB}$
AD	11.18	-89.44	2.236	-2235.86	55.9	34.7
BD	11.18	-223.61	2.236	-5589.91	55.9	-99.5
AC	7.07	141.42	-2.828	-2827.55	56.54	-15.6
BC	7.07	141.42	-2.828	-2827.55	56.54	-15.6
CD	5	200	-4	-4000	80	-22
AB	10	0	1	0	10	55.5
			Σ	-17480.87	314.88	

$$\Delta_{AB0} = \frac{1}{EA} \Sigma F_0 u_{AB} L = -\frac{17480.87}{EA} \; ; \quad f_{AB,AB} = \frac{1}{EA} \Sigma u_{AB}^2 L = \frac{314.88}{EA}$$

$$F_{AB} = -\frac{\Delta_{AB0}}{f_{AB,AB}} = \frac{17480.87}{314.88} = \underline{55.5 \text{ kN (T)}}$$

13.37 Using beam deflection formulas:

$$\Delta_{BO} = -\frac{17wL^4}{384EI} = -\frac{17(25)(16)^4}{384EI} = -\frac{72533.33 \text{ kN.m}^3}{EI}$$

$$\Delta_{CO} = -\frac{wL^4}{8EI} = -\frac{25(16)^4}{8EI} = -\frac{204800 \text{ kN.m}^3}{EI}$$

$$f_{BB} = \frac{L^3}{24EI} = \frac{(16)^3}{24EI} = \frac{170.67 \text{ kN.m}^3/\text{kN}}{EI}$$

$$f_{BC} = f_{CB} = \frac{5L^3}{48EI} = \frac{5(16)^3}{48EI} = \frac{426.67 \text{ kN.m}^3/\text{kN}}{EI}$$

$$f_{CC} = \frac{L^3}{3EI} = \frac{(16)^3}{3EI} = \frac{1365.33 \text{ kN.m}^3/\text{kN}}{EI}$$

Compatibility Equations:

$$\Delta_{BO} + f_{BB}B_y + f_{BC}C_y = 0$$

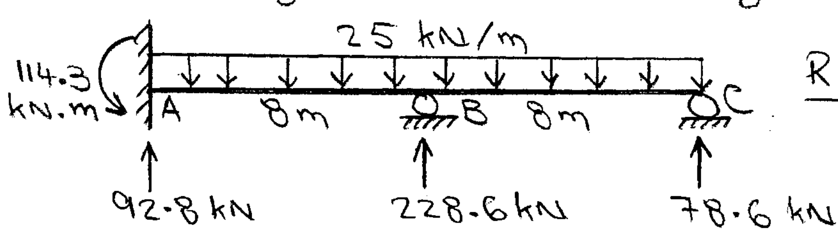
$$\Delta_{CO} + f_{CB}B_y + f_{CC}C_y = 0$$

$$-72533.33 + (170.67)B_y + (426.67)C_y = 0$$

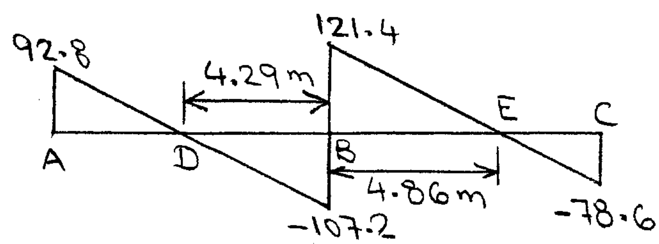
$$-204800 + (426.67)B_y + (1365.33)C_y = 0$$

Solving these equations, we obtain

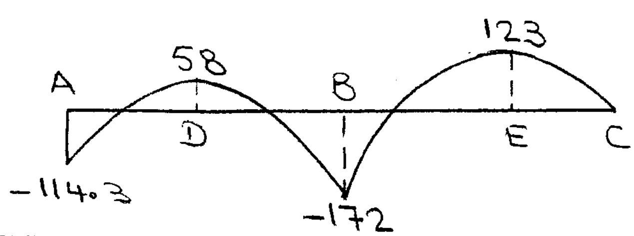
$\underline{B_y = 228.6 \text{ kN} \uparrow}$ $\quad$ $\underline{C_y = 78.6 \text{ kN} \uparrow}$



Reactions



Shear Diagram (kN)



Bending Moment Diagram (kN.m)

13.38 Using beam deflection formulas:

$$\Delta_{BO} = -\frac{819166.67\text{ k-ft}^3}{EI}; \quad \Delta_{DO} = -\frac{280000\text{ k-ft}^3}{EI}$$

$$f_{BB} = \frac{21333.33\text{ k-ft}^3/\text{k}}{EI}; \quad f_{BD} = f_{DB} = \frac{6666.67\text{ k-ft}^3/\text{k}}{EI}$$

$$f_{DD} = \frac{2666.67\text{ k-ft}^3/\text{k}}{EI}$$

Compatibility Equations:

$$\Delta_{BO} + f_{BB}B_y + f_{BD}D_y = 0$$

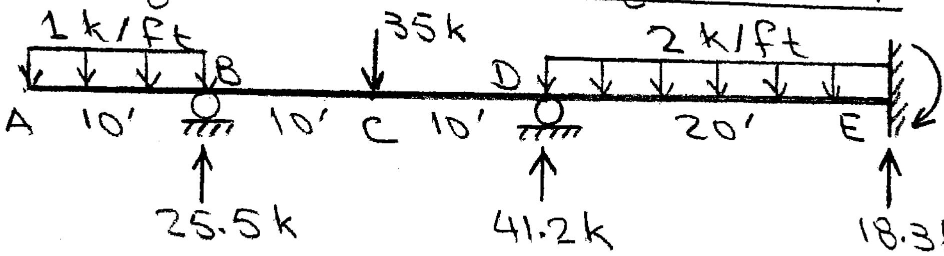
$$\Delta_{DO} + f_{DB}B_y + f_{DD}D_y = 0$$

$$-819166.67 + (21333.33)B_y + (6666.67)D_y = 0$$

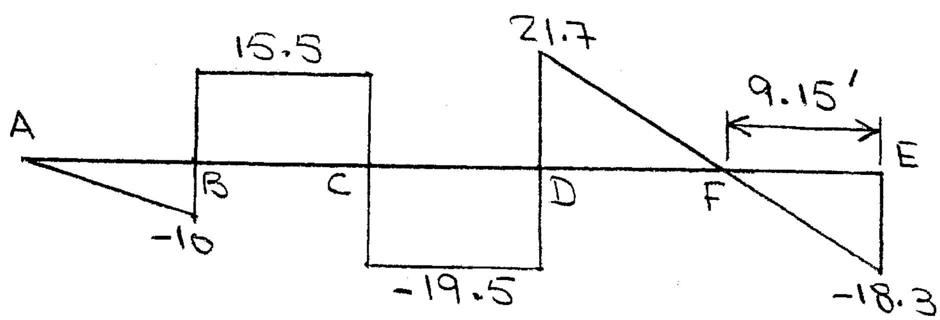
$$-280000 + (6666.67)B_y + (2666.67)D_y = 0$$

Solving these equations, we obtain

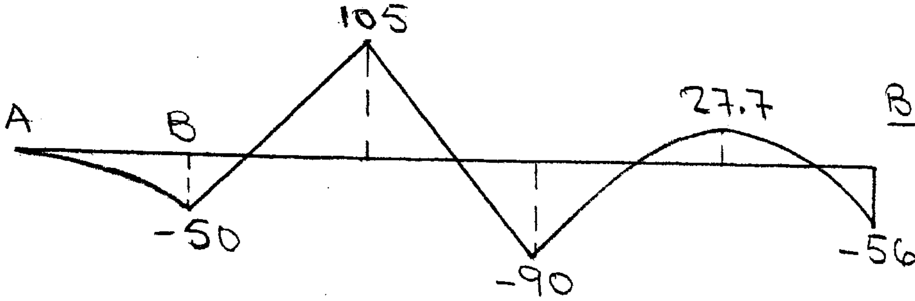
$B_y = 25.5\text{ k}\uparrow$ $\quad$ $D_y = 41.2\text{ k}\uparrow$



Reactions



Shear Diagram (k)



Bending Moment Diagram (k-ft)

$\boxed{13.39}$ The bending moments at C and E are selected as the redundants. Using beam deflection formulas, we obtain:

$$\theta_{CO\,rel.} = \theta_{CL} + \theta_{CR} = \frac{1}{EI}(768 + 336) = \frac{1104\ \text{kN.m}^2}{EI}$$

$$\theta_{EO\,rel.} = \theta_{EL} + \theta_{ER} = \frac{1}{EI}(384 + 600) = \frac{984\ \text{kN.m}^2}{EI}$$

$$f_{CC\,rel.} = f_{CCL} + f_{CCR} = \frac{1}{EI}\left(\frac{10}{3} + \frac{10}{6}\right) = \frac{5\ \text{kN.m}^2/\text{kN.m}}{EI}$$

$$f_{CE} = f_{EC} = \frac{10}{12EI} = \frac{0.833\ \text{kN.m}^2/\text{kN.m}}{EI}$$

$$f_{EE\,rel.} = f_{EEL} + f_{EER} = \frac{1}{EI}\left(\frac{10}{6} + \frac{8}{3}\right) = \frac{4.33\ \text{kN.m}^2/\text{kN.m}}{EI}$$

Compatibility Equations: $\theta_{CO\,rel.} + f_{CC\,rel.} M_C + f_{CE} M_E = 0$

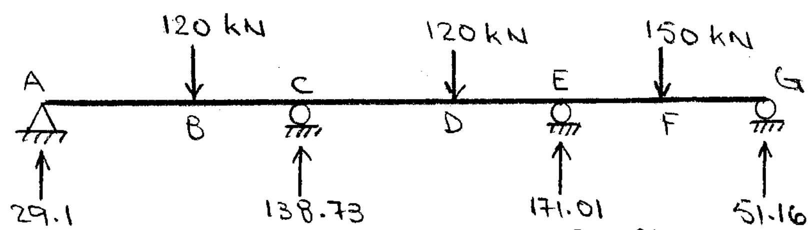
$$\theta_{EO\,rel.} + f_{EC} M_C + f_{EE\,rel.} M_E = 0$$

$$1104 + 5M_C + 0.833 M_E = 0$$

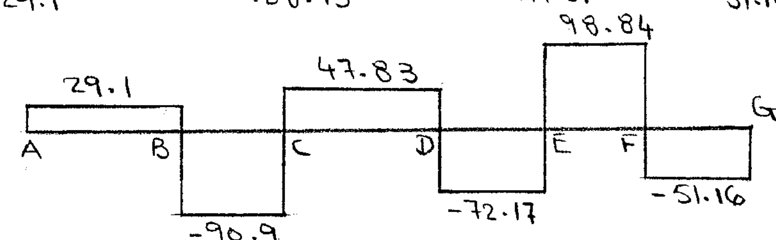
$$984 + 0.833 M_C + 4.33 M_E = 0$$

Solving these equations, we obtain

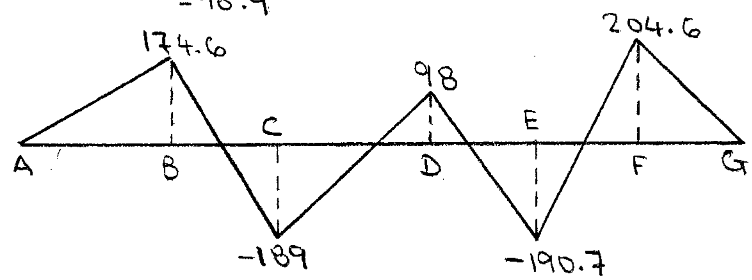
$$\underline{M_C = -189\ \text{kN.m}} \qquad \underline{M_E = -190.7\ \text{kN.m}}$$



Reactions



Shear Diagram (kN)



Bending Moment Diagram (kN.m)

$\boxed{13.40}$ The bending moments at B and C are selected as the redundants. Using beam deflection formulas, we obtain:

$$\theta_{BOrel.} = \theta_{BL} + \theta_{BR} = \frac{1}{EI}(1440 + 720) = \frac{2160 \text{ k-ft}^2}{EI}$$

$$\theta_{COrel.} = \theta_{CL} + \theta_{CR} = \frac{1}{EI}(720 + 1440) = \frac{2160 \text{ k-ft}^2}{EI}$$

$$f_{BBrel.} = f_{BBL} + f_{BBR} = \frac{1}{EI}(8 + 4) = \frac{12 \text{ k-ft}^2/\text{k-ft}}{EI}$$

$$f_{CCrel.} = f_{CCL} + f_{CCR} = \frac{1}{EI}(4 + 8) = \frac{12 \text{ k-ft}^2/\text{k-ft}}{EI}$$

$$f_{BC} = f_{CB} = \frac{2 \text{ k-ft}^2/\text{k-ft}}{EI}$$

Compatibility Equations:

$$\theta_{BOrel.} + f_{BBrel.} M_B + f_{BC} M_C = 0$$

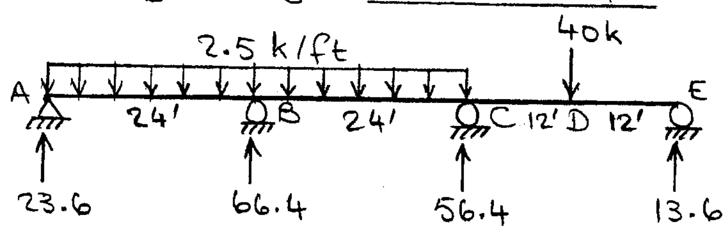
$$\theta_{COrel.} + f_{CB} M_B + f_{CCrel.} M_C = 0$$

$$2160 + 12 M_B + 2 M_C = 0$$

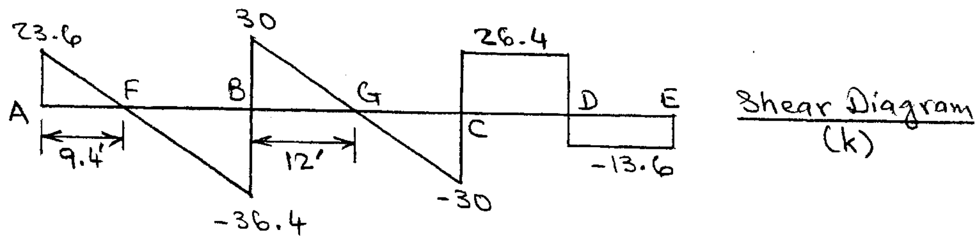
$$2160 + 2 M_B + 12 M_C = 0$$

Solving these equations, we obtain

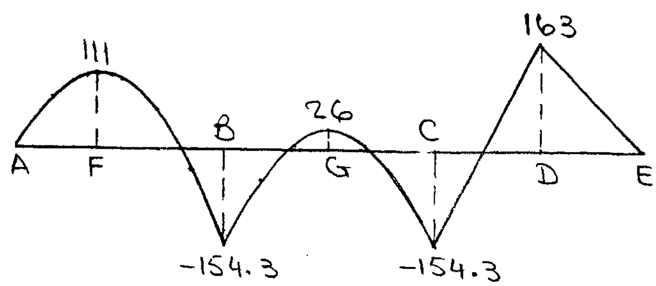
$$M_B = M_C = \underline{-154.3 \text{ k-ft}}$$



Reactions

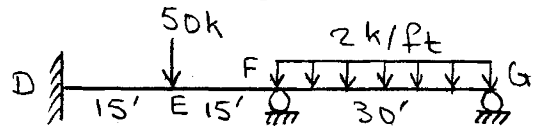


Shear Diagram (k)



Bending Moment Diagram (k-ft)

$\boxed{13.41}$ As the beam and the loading are symmetric, we will analyze only the right half DG of the beam.



Using beam deflection formulas:

$$\Delta_{FO} = -\frac{1085625 \text{ k-ft}^3}{EI} \; ; \; \Delta_{GO} = -\frac{3076875 \text{ k-ft}^3}{EI}$$

$$f_{FF} = \frac{9000 \text{ k-ft}^3/\text{k}}{EI} \; ; \; f_{FG} = f_{GF} = \frac{22500 \text{ k-ft}^3/\text{k}}{EI}$$

$$f_{GG} = \frac{72000 \text{ k-ft}^3/\text{k}}{EI}$$

Compatibility Equations:

$$\Delta_{FO} + f_{FF} F_y + f_{FG} G_y = 0$$

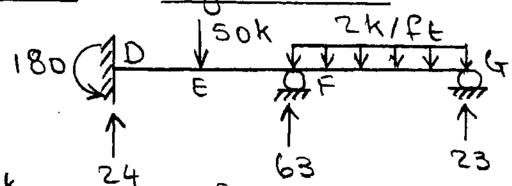
$$\Delta_{GO} + f_{GF} F_y + f_{GG} G_y = 0$$

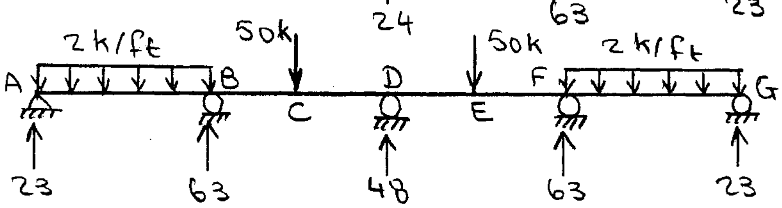
$$-1085625 + 9000 F_y + 22500 G_y = 0$$

$$-3076875 + 22500 F_y + 72000 G_y = 0$$

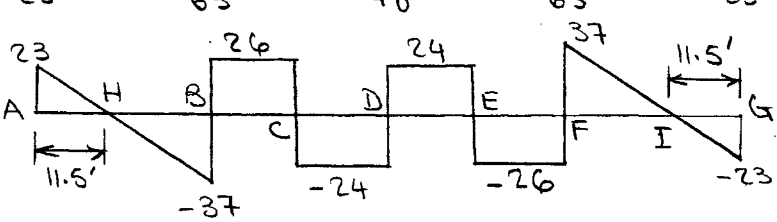
Solving these equations, we obtain

$\underline{F_y = 63 \text{ k} \uparrow}$ $\quad\quad$ $\underline{G_y = 23 \text{ k} \uparrow}$

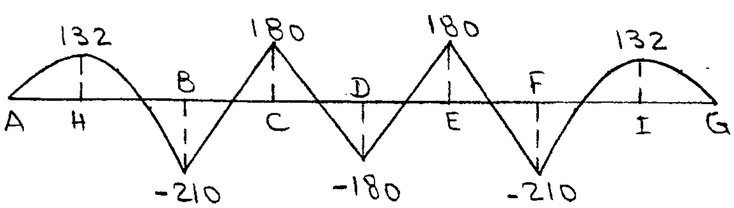




Reactions

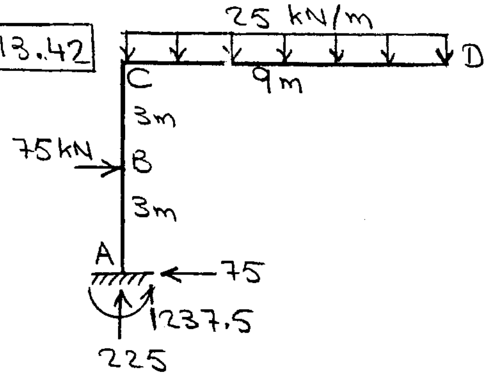


Shear Diagram (k)

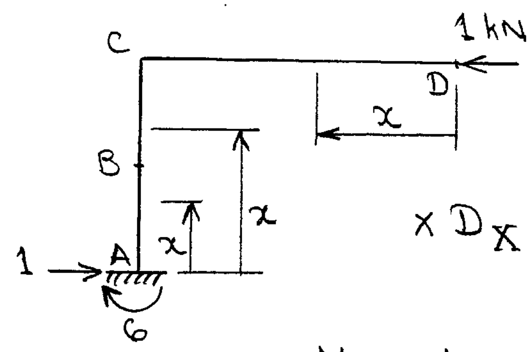


Bending Moment Diagram (k-ft)

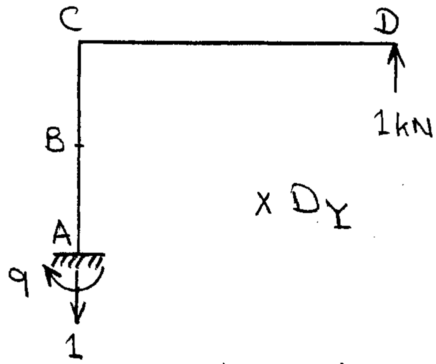
13.42



M_0 Moments



m_{DX} Moments



m_{DY} Moments

Segment	x Coordinate		M_0 (kN.m)	m_{DX} (kN.m/kN)	m_{DY} (kN.m/kN)
	Origin	Limits (m)			
AB	A	0–3	$-1237.5+75x$	$6-x$	9
BC	A	3–6	-1012.5	$6-x$	9
DC	D	0–9	$-12.5x^2$	0	x

$$\Delta_{DXO} = \Sigma \int \frac{M_0 m_{DX}}{EI}\, dx = -\frac{19912.5 \text{ kN-m}^3}{EI}$$

$$\Delta_{DYO} = \Sigma \int \frac{M_0 m_{DY}}{EI}\, dx = -\frac{78215.625 \text{ kN-m}^3}{EI}$$

$$f_{DX,DX} = \Sigma \int \frac{m_{DX}^2}{EI}\, dx = \frac{72 \text{ kN-m}^3/\text{kN}}{EI}$$

$$f_{DY,DY} = \Sigma \int \frac{m_{DY}^2}{EI}\, dx = \frac{729 \text{ kN-m}^3/\text{kN}}{EI}$$

$$f_{DX,DY} = f_{DY,DX} = \Sigma \int \frac{m_{DX} m_{DY}}{EI}\, dx = \frac{162 \text{ kN-m}^3/\text{kN}}{EI}$$

13.42 (contd.) Compatibility Equations:

$$\Delta_{DX0} + f_{DX,DX} D_X + f_{DX,DY} D_Y = 0$$

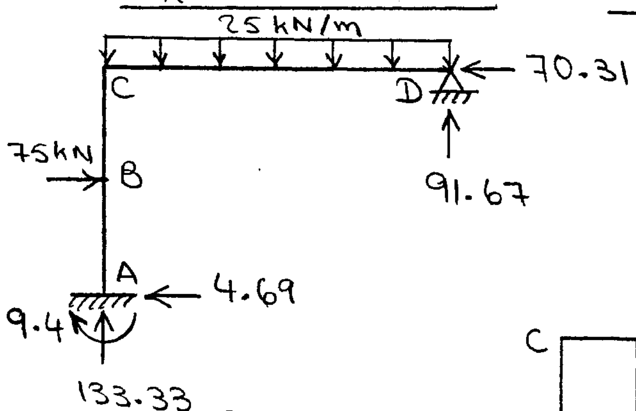
$$\Delta_{DY0} + f_{DY,DX} D_X + f_{DY,DY} D_Y = 0$$

$$-19912.5 + 72 D_X + 162 D_Y = 0$$

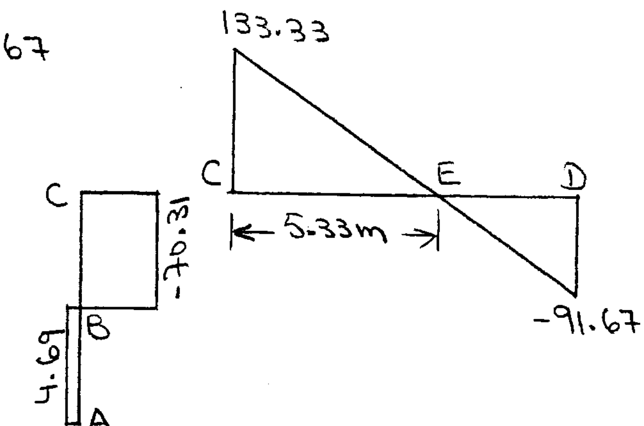
$$-78215.625 + 162 D_X + 729 D_Y = 0$$

Solving these equations, we obtain

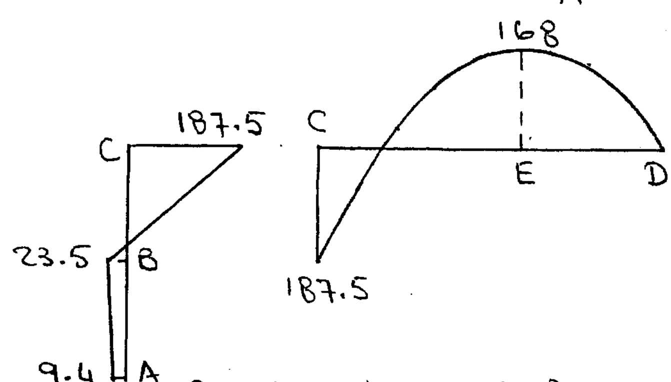
$D_X = 70.3$ kN ← $\quad$ $D_Y = 91.7$ kN ↑



Reactions

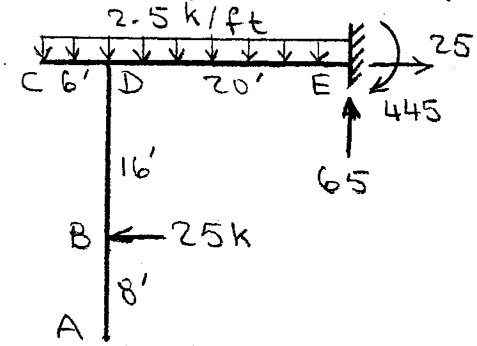


Shear Diagrams (kN)

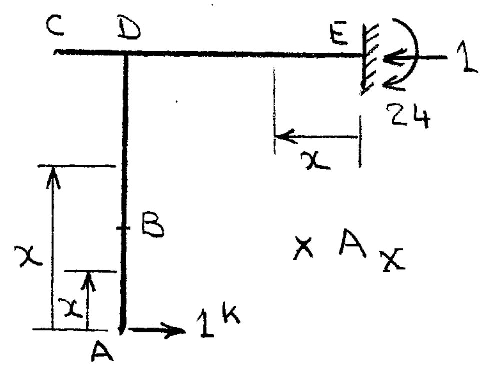


Bending Moment Diagrams (kN.m)

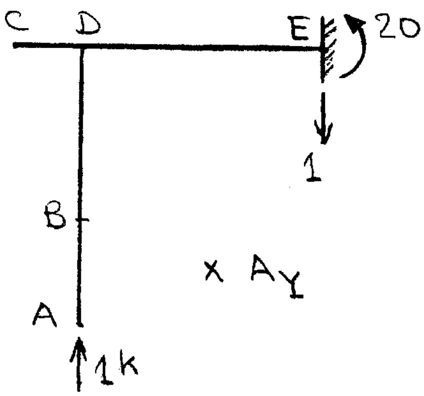
13.43 The reactions A_X and A_Y are selected as the redundants.



M_0 Moments



m_{AX} Moments



m_{AY} Moments

Segment	x Coordinate		M_0 (k-ft)	m_{AX} (k-ft/k)	m_{AY} (k-ft/k)
	Origin	Limits (ft)			
AB	A	0-8	0	$-1x$	0
BD	A	8-24	$25(x-8)$	$-1x$	0
ED	E	0-20	$-445+65x-1.25x^2$	-24	$20-x$

$$\Delta_{AXO} = \sum\int \frac{M_0 m_{AX}}{EI}dx = -\frac{78133.33\ \text{k-ft}^3}{EI}$$

$$\Delta_{AYO} = \sum\int \frac{M_0 m_{AY}}{EI}dx = -\frac{19000\ \text{k-ft}^3}{EI}$$

$$f_{AX,AX} = \sum\int \frac{m_{AX}^2}{EI}dx = \frac{16128\ \text{k-ft}^3/\text{k}}{EI}$$

$$f_{AY,AY} = \sum\int \frac{m_{AY}^2}{EI}dx = \frac{2666.67\ \text{k-ft}^3/\text{k}}{EI}$$

$$f_{AX,AY} = f_{AY,AX} = \sum\int \frac{m_{AX}m_{AY}}{EI}dx = -\frac{4800\ \text{k-ft}^3/\text{k}}{EI}$$

13.43 (contd.) Compatibility Equations:

$$\Delta_{AX0} + f_{AX,AX} A_X + f_{AX,AY} A_Y = 0$$

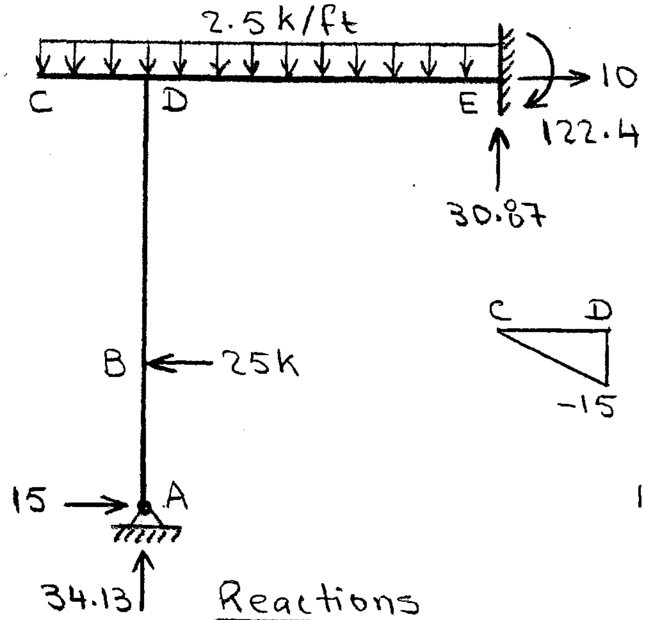
$$\Delta_{AY0} + f_{AY,AX} A_X + f_{AY,AY} A_Y = 0$$

$$-78133.33 + 16128 A_X - 4800 A_Y = 0$$

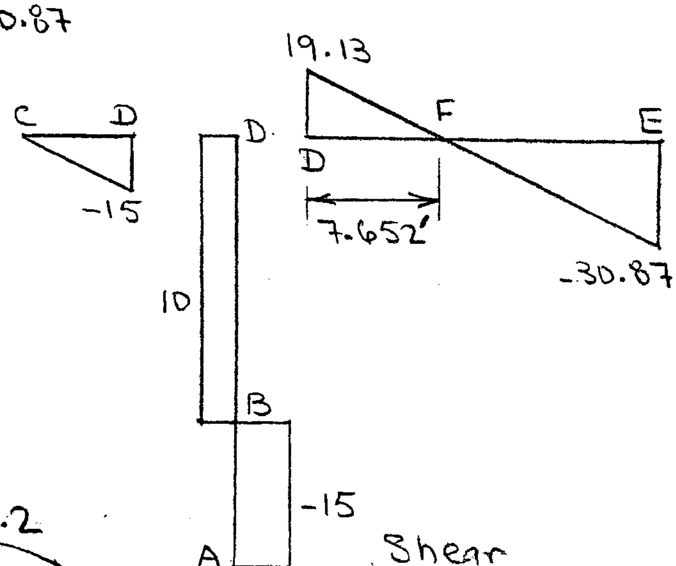
$$-19000 - 4800 A_X + 2666.67 A_Y = 0$$

Solving these equations, we obtain:

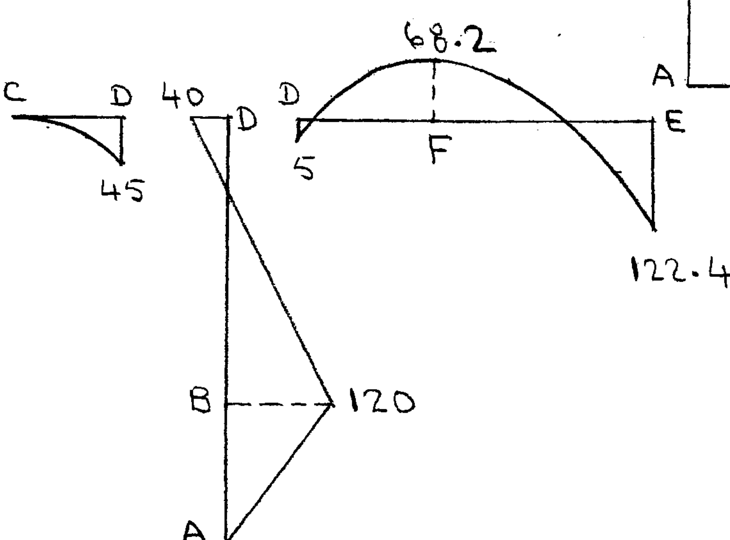
$A_X = 15\text{ k} \rightarrow$ $\qquad$ $A_Y = 34.13\text{ k} \uparrow$



Reactions

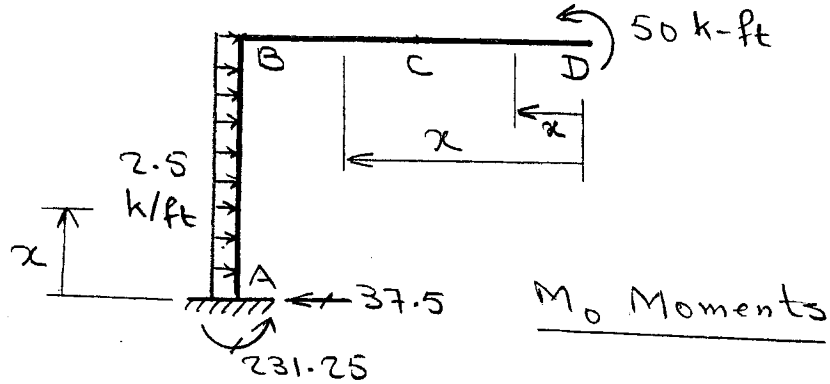


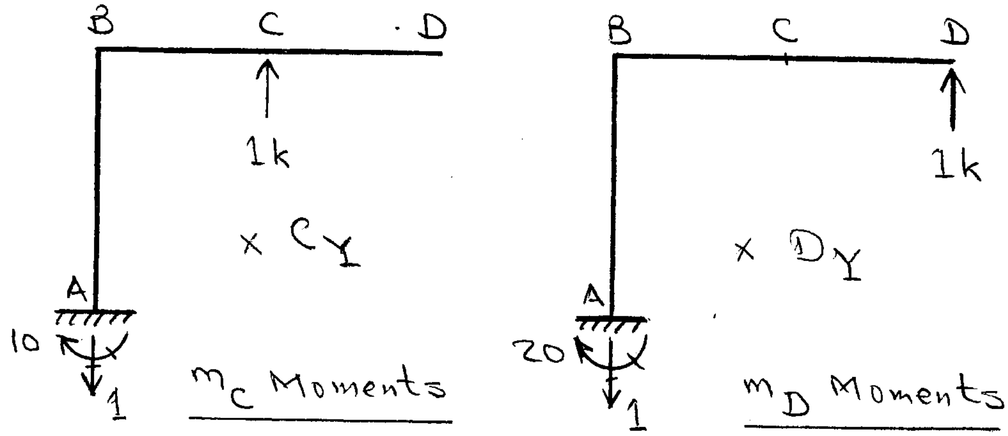
Shear Diagrams (k)



Bending Moment Diagrams (k-ft)

13.44





Member	x coordinate		M_0 (k-ft)	m_C	m_D
	Origin	Limits (ft)			
AB	A	0 - 15	$-231.25+37.5x-1.25x^2$	10	20
CB	D	10 - 20	50	$1(x-10)$	$1x$
DC	D	0 - 10	50	0	$1x$

$$\Delta_{CO} = \Sigma\int \frac{M_0 m_C}{EI}dx = -\frac{4062.5 \text{ k-ft}^3}{EI}$$

$$\Delta_{DO} = \Sigma\int \frac{M_0 m_D}{EI}dx = -\frac{3125 \text{ k-ft}^3}{EI}$$

$$f_{CC} = \Sigma\int \frac{m_C^2}{EI}dx = \frac{1833.33 \text{ k-ft}^3/\text{k}}{EI}$$

$$f_{DD} = \Sigma\int \frac{m_D^2}{EI}dx = \frac{8666.67 \text{ k-ft}^3/\text{k}}{EI}$$

$$f_{CD} = f_{DC} = \Sigma\int \frac{m_C m_D}{EI}dx = \frac{3833.33 \text{ k-ft}^3/\text{k}}{EI}$$

13.44 (Contd.) Compatibility Equations:

$$\Delta_{CO} + f_{CC} C_Y + f_{CD} D_Y = 0$$

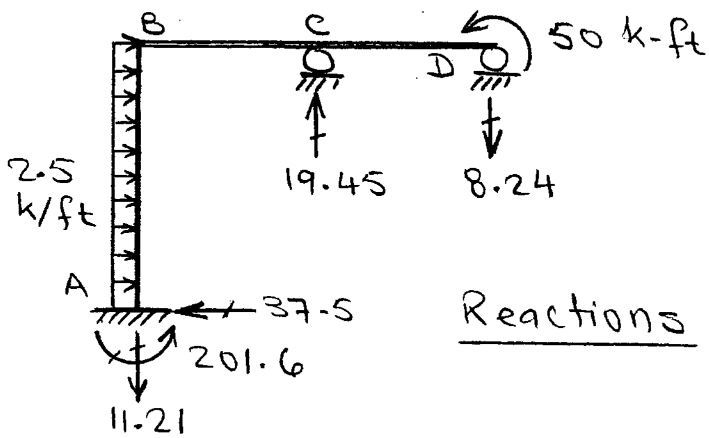
$$\Delta_{DO} + f_{DC} C_Y + f_{DD} D_Y = 0$$

$$-4062.5 + 1833.33\, C_Y + 3833.33\, D_Y = 0$$

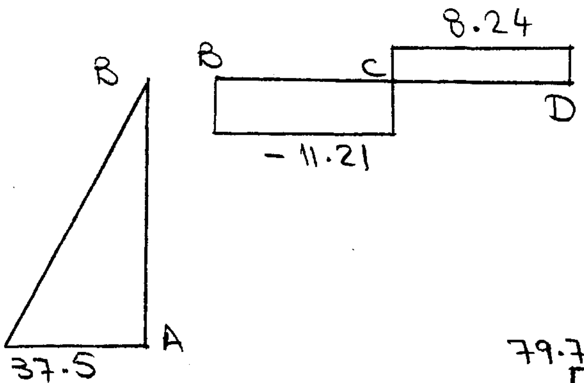
$$-3125 + 3833.33\, C_Y + 8666.67\, D_Y = 0$$

Solving these equations, we obtain:

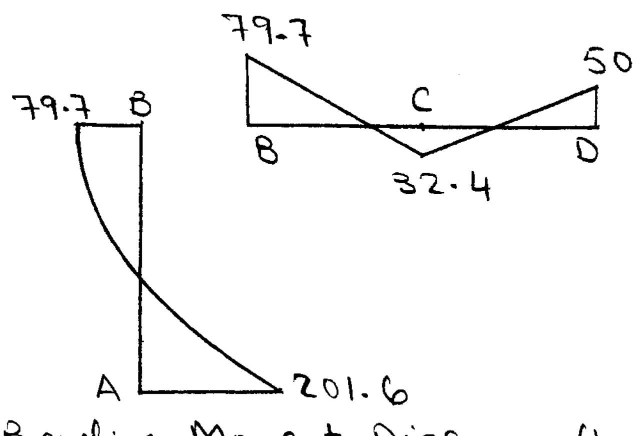
$C_Y = 19.45 \text{ k} \uparrow$ $\quad D_Y = -8.24 \text{ k} = 8.24 \text{ k} \downarrow$



Reactions

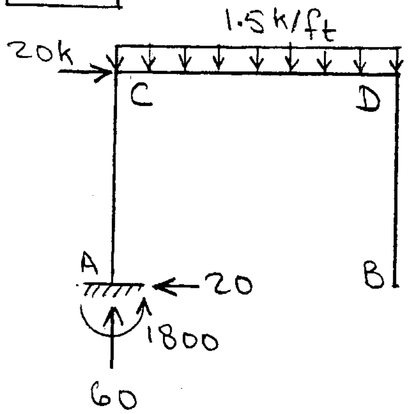


Shear Diagrams (k)

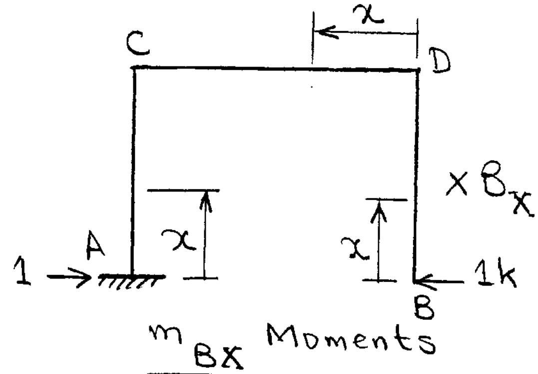


Bending Moment Diagrams (k-ft)

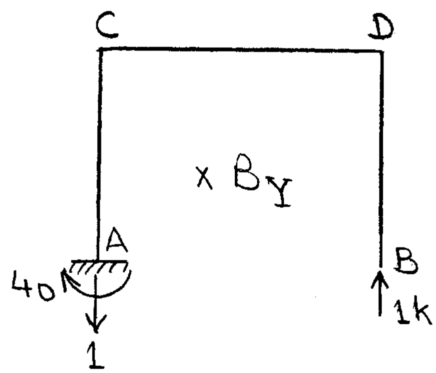
13.45



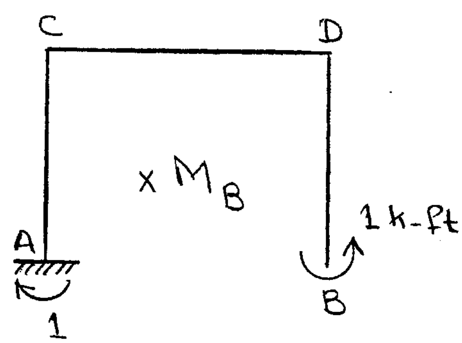
M_0 Moments



m_{BX} Moments



m_{BY} Moments



m_{MB} Moments

Member	x Coordinate		Moment of Inertia	M_0 (k-ft)	m_{BX}	m_{BY}	m_{MB}
	Origin	Limits (ft)					
AC	A	0-30	I	$-1800+20x$	$-x$	40	1
DC	D	0-40	2I	$-0.75x^2$	-30	x	1
BD	B	0-30	I	0	x	0	-1

$$\Delta_{BXO} = \sum \int \frac{M_0 m_{DX}}{EI} dx = \frac{870000 \text{ k-ft}^3}{EI}$$

$$\Delta_{BYO} = \sum \int \frac{M_0 m_{DY}}{EI} dx = -\frac{2040000 \text{ k-ft}^3}{EI}$$

$$\Theta_{BO} = \sum \int \frac{M_0 m_{MB}}{EI} dx = -\frac{53000 \text{ k-ft}^2}{EI}$$

$$f_{BX,BX} = \sum \int \frac{m_{BX}^2}{EI} dx = \frac{36000 \text{ k-ft}^3/\text{k}}{EI}$$

13.45 (Contd.) $f_{BY,BY} = \sum \int \frac{m_{BY}^2}{EI} dx = \frac{58666.67}{EI}$

$$f_{MB,MB} = \sum \int \frac{m_{MB}^2}{EI} dx = \frac{80}{EI}$$

$$f_{BX,BY} = f_{BY,BX} = \sum \int \frac{m_{BX} m_{BY}}{EI} dx = -\frac{30000}{EI}$$

$$f_{BX,MB} = f_{MB,BX} = \sum \int \frac{m_{BX} m_{MB}}{EI} dx = -\frac{1500}{EI}$$

$$f_{BY,MB} = f_{MB,BY} = \sum \int \frac{m_{BY} m_{MB}}{EI} dx = \frac{1600}{EI}$$

Compatibility Equations:

$$\Delta_{BXO} + f_{BX,BX} B_X + f_{BX,BY} B_Y + f_{BX,MB} M_B = 0$$

$$\Delta_{BYO} + f_{BY,BX} B_X + f_{BY,BY} B_Y + f_{BY,MB} M_B = 0$$

$$\theta_{BO} + f_{MB,BX} B_X + f_{MB,BY} B_Y + f_{MB,MB} M_B = 0$$

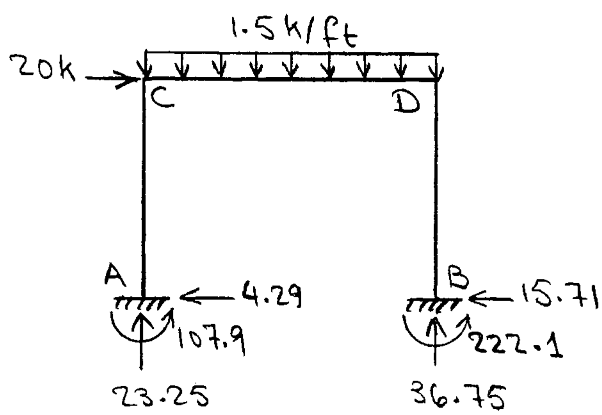
$$870000 + 36000 B_X - 30000 B_Y - 1500 M_B = 0$$

$$-2040000 - 30000 B_X + 58666.67 B_Y + 1600 M_B = 0$$

$$-53000 - 1500 B_X + 1600 B_Y + 80 M_B = 0$$

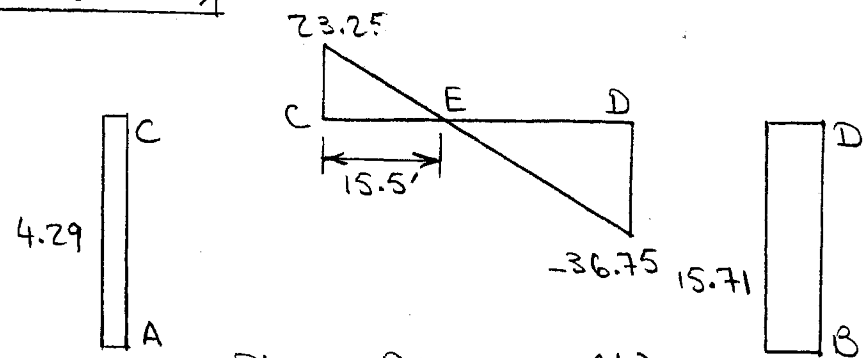
Solving these equations, we obtain:

$\underline{B_X = 15.71 \text{ k} \leftarrow}$ $\underline{B_Y = 36.75 \text{ k} \uparrow}$ $\underline{M_B = 222.1 \text{ k-ft} \circlearrowleft}$

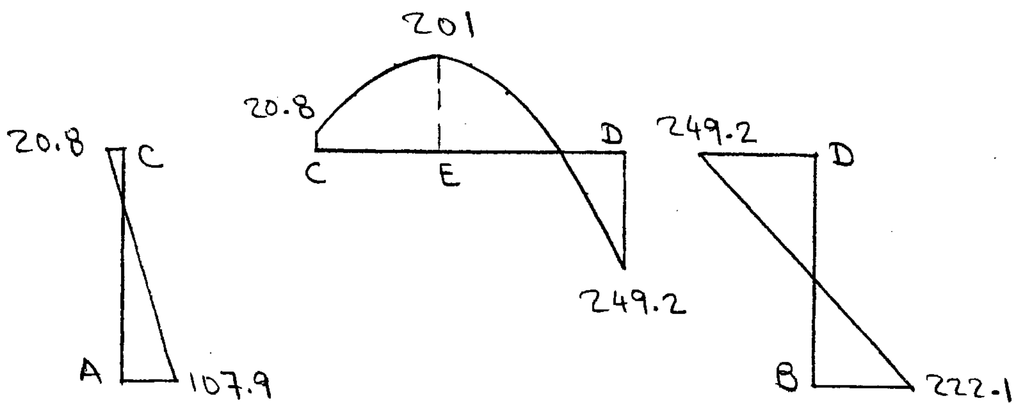


Reactions

13.45 (contd.)

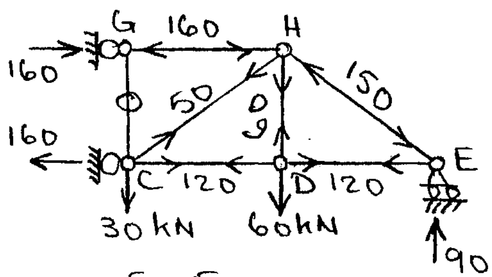


Shear Diagrams (k)

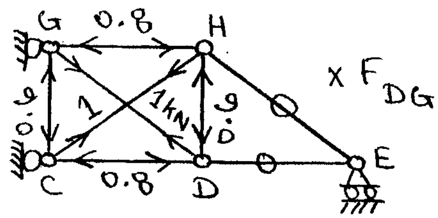


Bending Moment Diagrams (k-ft)

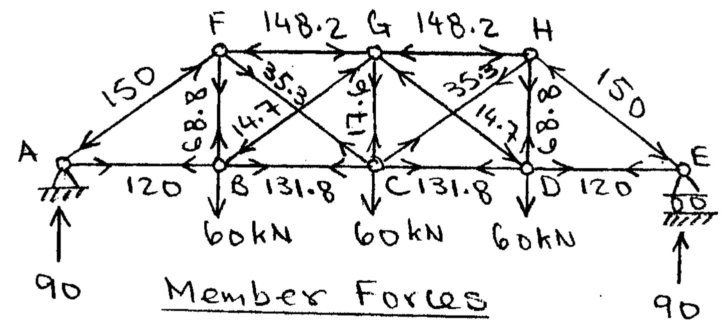
13.46 As the truss and the loading are symmetric, we will analyze only the right half CEGH of the truss.



F_0 Forces



u_{DG} Forces

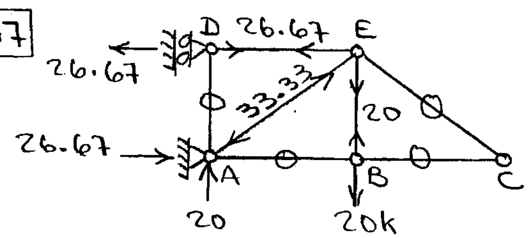


Member Forces

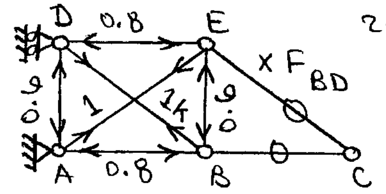
Member	L (m)	Relative A	F_0 (kN)	u_{DG} (kN/kN)	$\frac{F_0 u_{DG} L}{A}$	$\frac{u_{DG}^2 L}{A}$	$F = F_0 + u_{DG} F_{DG}$ (kN)
CD	8	1	120	-0.8	-768	5.12	131.8
DE	8	1	120	0	0	0	120
GH	8	1	-160	-0.8	1024	5.12	-148.2
CG	6	1/2	0	-0.6	0	4.32	8.8
DH	6	1	60	-0.6	-216	2.16	68.8
CH	10	1	50	1	500	10	35.3
EH	10	1	-150	0	0	0	-150
DG	10	1	0	1	0	10	-14.7
				Σ	540	36.72	

$$F_{DG} = -\frac{540}{36.72} = -14.7 \text{ kN} = \underline{14.7 \text{ kN (C)}}$$

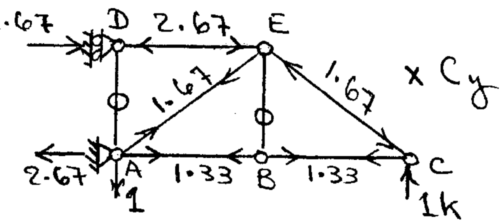
13.47



F_0 Forces



u_{BD} Forces



u_C Forces

Member	L (in.)	A (in.²)	F_0 (k)	u_{BD} (k/k)	u_C (k/k)	$\frac{F_0 u_{BD} L}{A}$	$\frac{F_0 u_C L}{A}$	$\frac{u_{BD}^2 L}{A}$	$\frac{u_C^2 L}{A}$	$\frac{u_{BD} u_C L}{A}$	$F = F_0 + u_{BD} F_{BD} + u_C C_y$ (k)
AB	192	8	0	−0.8	1.33	0	0	15.36	42.45	−25.54	−1.44
BC	192	8	0	0	1.33	0	0	0	42.45	0	8.29
DE	192	8	26.67	−0.8	−2.67	−512.06	−1709.01	15.36	171.09	51.26	0.31
AD	144	6	0	−0.6	0	0	0	8.64	0	0	−7.3
BE	144	6	20	−0.6	0	−288	0	8.64	0	0	12.7
AE	240	6	−33.33	1	1.67	−1333.2	−2226.44	40	111.56	66.8	−10.77
CE	240	8	0	0	−1.67	0	0	0	83.67	0	−10.4
BD	240	6	0	1	0	0	0	40	0	0	12.16
					Σ	−2133.26	−3935.45	128	451.22	92.52	

$\Delta_{BDO} = \frac{-2133.26}{E}$; $\Delta_{CO} = -\frac{3935.45}{E}$; $f_{BD,BD} = \frac{128}{E}$; $f_{CC} = \frac{451.22}{E}$

$f_{BD,C} = f_{C,BD} = \frac{92.52}{E}$

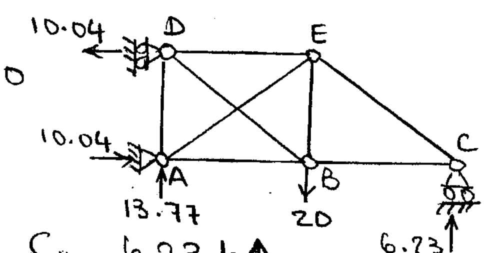
Compatibility Equations: $\Delta_{BDO} + f_{BD,BD} F_{BD} + f_{BD,C} C_y = 0$

$\Delta_{CO} + f_{C,BD} F_{BD} + f_{CC} C_y = 0$

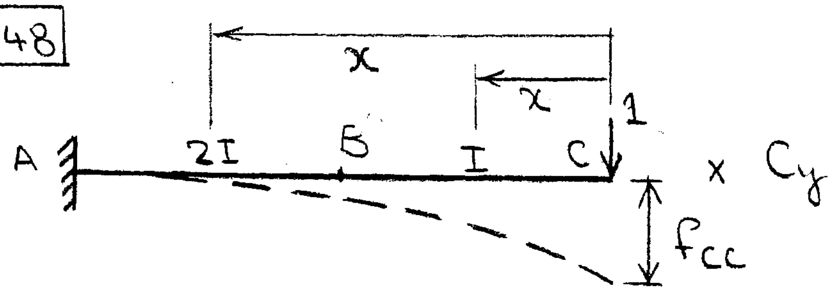
$-2133.26 + 128 F_{BD} + 92.52 C_y = 0$

$-3935.45 + 92.52 F_{BD} + 451.22 C_y = 0$

Solving these equations, we obtain: $\underline{F_{BD} = 12.16\text{ k (T)}}$; $\underline{C_y = 6.23\text{ k}\uparrow}$



13.48



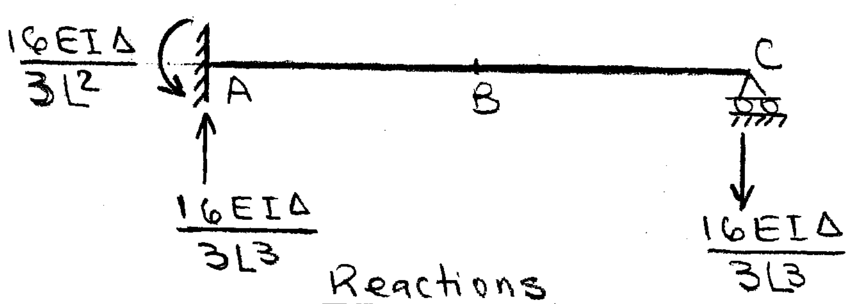
Using the virtual work method:

$$f_{CC} = \frac{1}{EI}\left[\int_0^{L/2} (-1x)^2\,dx + \frac{1}{2}\int_{L/2}^{L} (-1x)^2\,dx\right]$$

$$= \frac{3L^3}{16EI}$$

Compatibility Equation: $f_{CC}\,C_y = \Delta$

$$\underline{C_y = \frac{16EI\Delta}{3L^3}\downarrow}$$



Reactions

$\boxed{13.49}$ From the solution of Problem 13.1

$$\Delta_{DO} = -\frac{14760 \text{ kN.m}^3}{EI} = -\frac{14760}{200(3250)} = -0.0227 \text{ m}$$

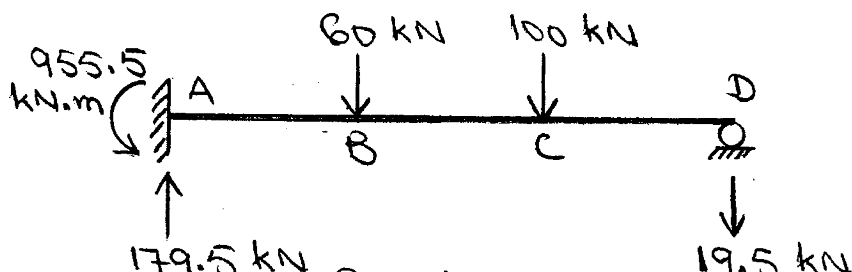
$$f_{DD} = \frac{243 \text{ kN.m}^3/\text{kN}}{EI} = \frac{243}{200(3250)} = 0.000374 \text{ m/kN}$$

Compatibility Equation:

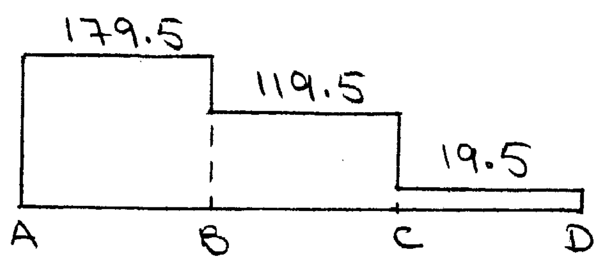
$$\Delta_{DO} + f_{DD} D_y = \Delta_D$$

$$-0.0227 + (0.000374) D_y = -0.03$$

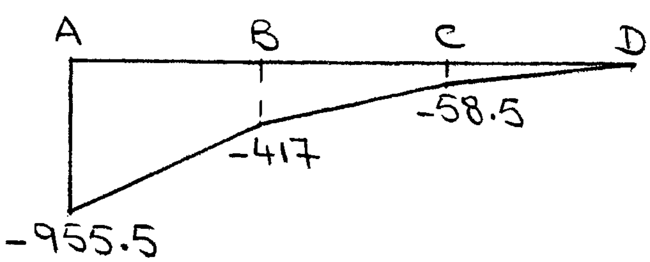
$$D_y = -\frac{0.0073}{0.000374} = -19.5 \text{ kN} = \underline{19.5 \text{ kN} \downarrow}$$



Reactions



Shear Diagram (kN)



Bending Moment Diagram (kN.m)

$\boxed{13.50}$ From the solution of Problem 13.9:

$$\Delta_{CO} = -\frac{158400\text{ k-ft}^3}{EI} = -\frac{158400(12)^3}{29000(1500)} = -6.292\text{ in.}$$

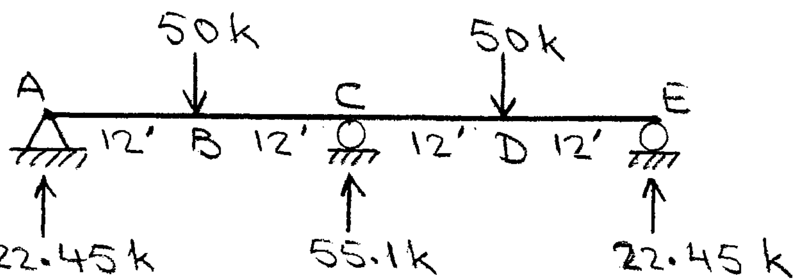
$$f_{CC} = \frac{2304\text{ k-ft}^3/\text{k}}{EI} = \frac{2304(12)^3}{29000(1500)} = 0.0915\text{ in./k}$$

Compatibility Equation:

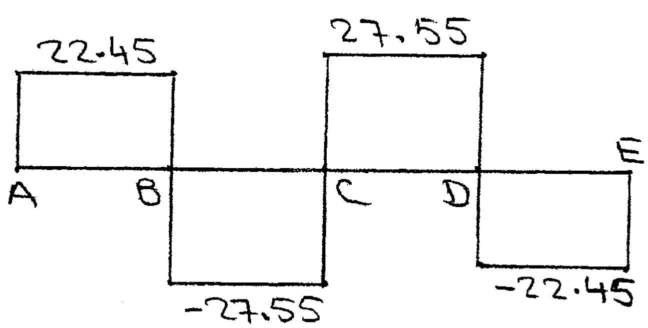
$$\Delta_{CO} + f_{CC}C_y = \Delta_C$$

$$-6.292 + (0.0915)C_y = -1.25$$

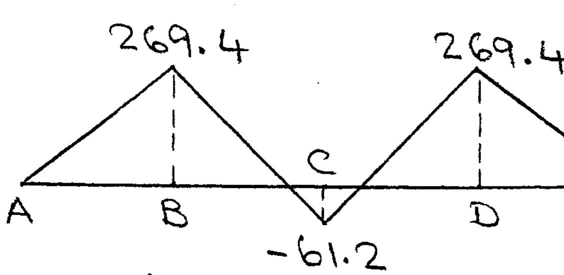
$$C_y = \frac{5.042}{0.0915} = \underline{55.1\text{ k}\uparrow}$$



Reactions



Shear Diagram (k)

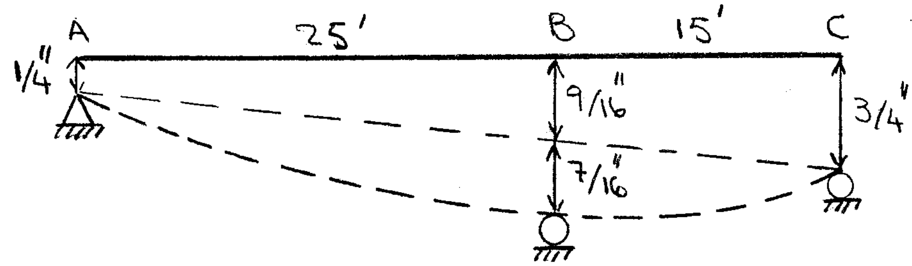


Bending Moment Diagram (k-ft)

13.51 From the solution of Problem 13.12:

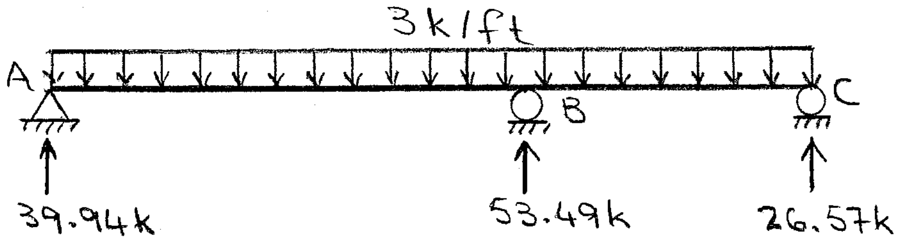
$$\Delta_{BO} = -\frac{61450.2\ \text{k-ft}^3}{EI} = -\frac{61450.2(12)^3}{29000(2500)} = -1.465\ \text{in.}$$

$$f_{BB} = \frac{805.66\ \text{k-ft}^3/\text{k}}{EI} = \frac{805.66(12)^3}{29000(2500)} = 0.0192\ \text{in./k}$$

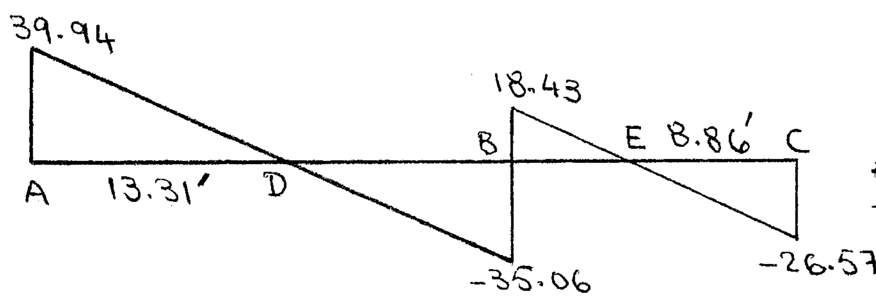


Compatibility Equation:

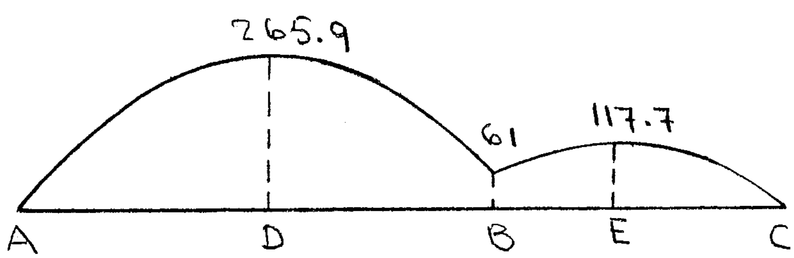
$$-1.465 + 0.0192\,B_y = -\frac{7}{16} \qquad \underline{B_y = 53.49\,\text{k}\uparrow}$$



Reactions



Shear Diagram (k)

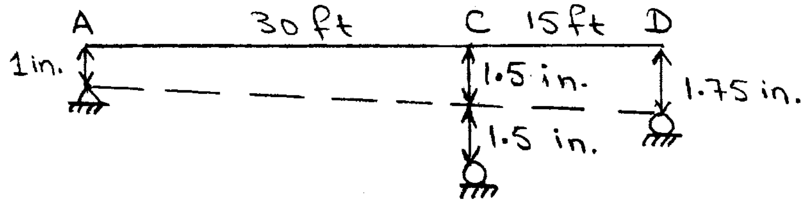


Bending Moment Diagram (k-ft)

$\boxed{13.52}$ From the solution of Problem 13.27:

$$\Delta_{CO} = -\frac{22046.28\ (\text{k-in})}{EA} = -\frac{22046.28}{29000(6)} = -0.127\ \text{in.}$$

$$f_{CC} = \frac{531.78\ (\text{in.})}{EA} = \frac{531.78}{29000(6)} = 0.00306\ \text{in./k}$$

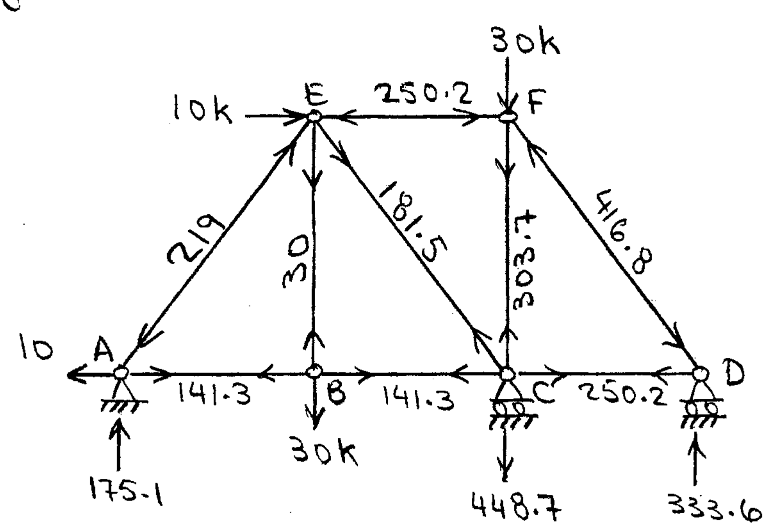


Compatibility Equation:

$$-0.127 + (0.00306)\,C_y = -1.5$$

$$C_y = -448.7\ \text{k} = \underline{448.7\ \text{k}\downarrow}$$

By using the F_O and u_C forces computed in the solution of Problem 13.17, we obtain the following member forces.



Member Forces and Reactions (k)

13.53 From the solution of Problem 13.37:

$$\Delta_{BO} = -\frac{72533.33\ \text{kN}\cdot\text{m}^3}{EI} = -\frac{72533.33}{70(1300)} = -0.797\ \text{m}$$

$$\Delta_{CO} = -\frac{204800\ \text{kN}\cdot\text{m}^3}{EI} = -\frac{204800}{70(1300)} = -2.251\ \text{m}$$

$$f_{BB} = \frac{170.67\ \text{kN}\cdot\text{m}^3/\text{kN}}{EI} = \frac{170.67}{70(1300)} = 0.00188\ \text{m/kN}$$

$$f_{BC} = f_{CB} = \frac{426.67\ \text{kN}\cdot\text{m}^3/\text{kN}}{EI} = \frac{426.67}{70(1300)} = 0.00469\ \text{m/kN}$$

$$f_{CC} = \frac{1365.33\ \text{kN}\cdot\text{m}^3/\text{kN}}{EI} = \frac{1365.33}{70(1300)} = 0.015\ \text{m/kN}$$

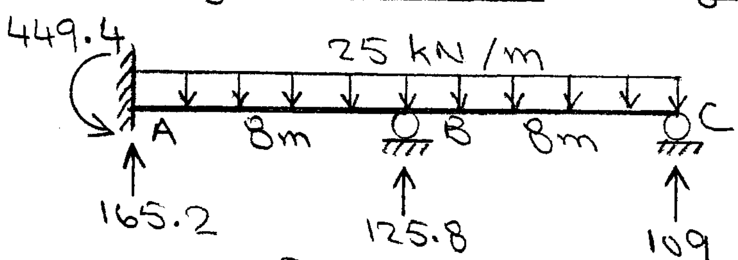
Compatibility Equations:

$$-0.797 + (0.00188)B_y + (0.00469)C_y = -0.05$$

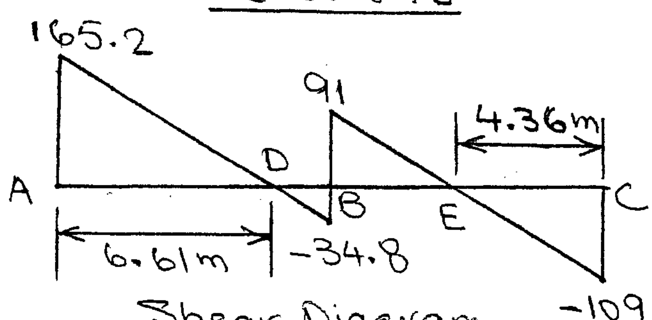
$$-2.251 + (0.00469)B_y + (0.015)C_y = -0.025$$

Solving these equations, we obtain

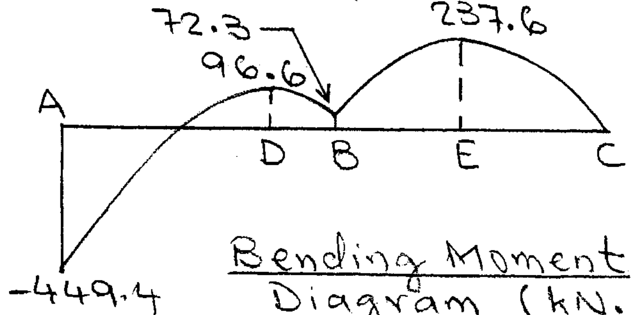
$B_y = 125.8\ \text{kN}\uparrow$ $\qquad$ $C_y = 109\ \text{kN}\uparrow$



Reactions



Shear Diagram (kN)



Bending Moment Diagram (kN.m)

13.54 From the solution of Problem 13.39:

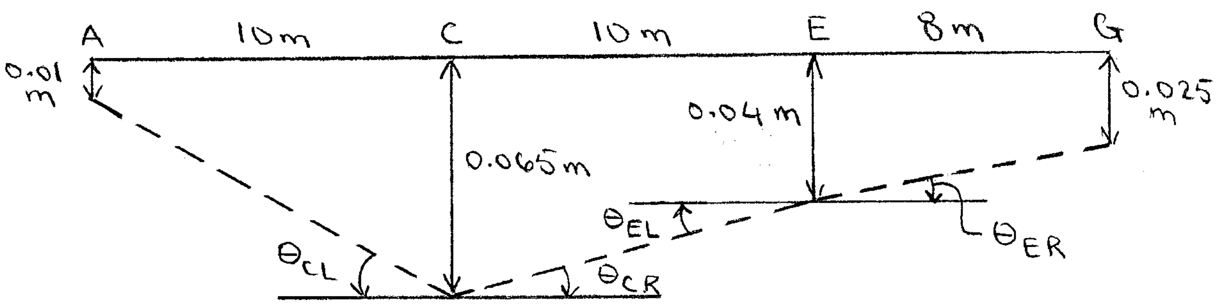
$$\theta_{COrel.} = \frac{1104 \text{ kN.m}^2}{EI} = \frac{1104}{200(500)} = 0.01104 \text{ rad.}$$

$$\theta_{EOrel.} = \frac{984 \text{ kN.m}^2}{EI} = 0.00984 \text{ rad.}$$

$$f_{CCrel.} = \frac{5 \text{ kN.m}^2/\text{kN.m}}{EI} = 0.00005 \text{ rad./kN.m}$$

$$f_{EErel.} = \frac{4.33 \text{ kN.m}^2/\text{kN.m}}{EI} = 0.0000433 \text{ rad./kN.m}$$

$$f_{CE} = f_{EC} = \frac{0.833 \text{ kN.m}^2/\text{kN.m}}{EI} = 0.00000833 \text{ rad./kN.m}$$



$$\theta_{CL} = \frac{0.065 - 0.01}{10} = 0.0055 \text{ rad.}$$

$$\theta_{CR} = \frac{0.065 - 0.04}{10} = 0.0025 \text{ rad.}$$

$$\theta_C = \theta_{CL} + \theta_{CR} = 0.008 \text{ rad.}$$

$$\theta_{EL} = \frac{0.04 - 0.065}{10} = -0.0025 \text{ rad.}$$

$$\theta_{ER} = \frac{0.04 - 0.025}{8} = 0.001875 \text{ rad.}$$

$$\theta_E = \theta_{EL} + \theta_{ER} = -0.000625 \text{ rad.}$$

Compatibility Equations:

$$\theta_{COrel.} + f_{CCrel.} M_C + f_{CE} M_E = \theta_C$$

$$\theta_{EOrel.} + f_{EC} M_C + f_{EErel.} M_E = \theta_E$$

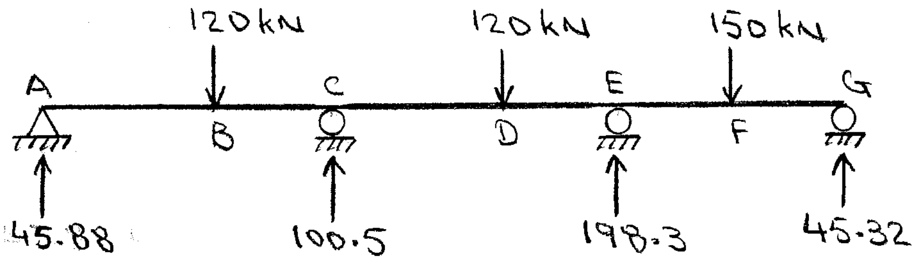
$$1104 + 5M_C + 0.833 M_E = 800$$

$$984 + 0.833 M_C + 4.33 M_E = -62.5$$

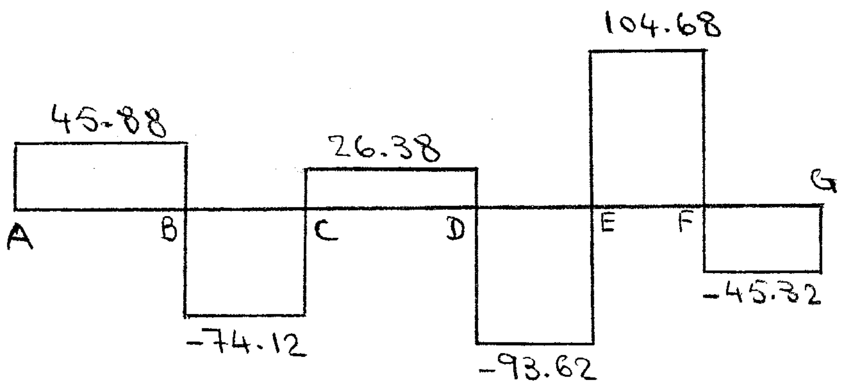
13.54 (contd.)

Solving these equations, we obtain

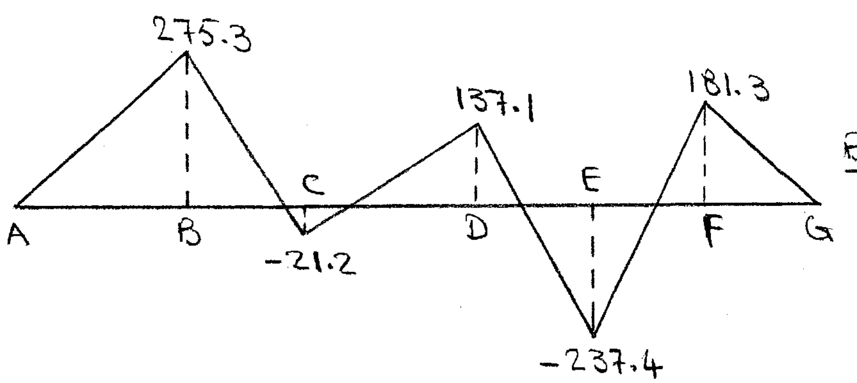
$M_C = -21.23$ kN.m $\quad M_E = -237.42$ kN.m



Reactions

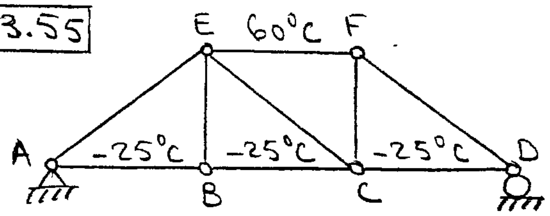


Shear Diagram (kN)

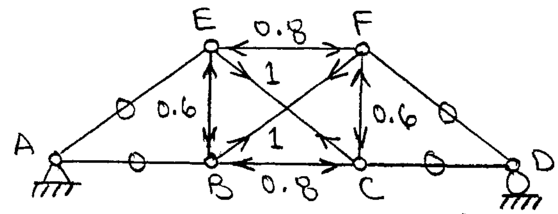


Bending Moment Diagram (kN.m)

13.55



Temperature Changes



u_{BF} Forces (kN)

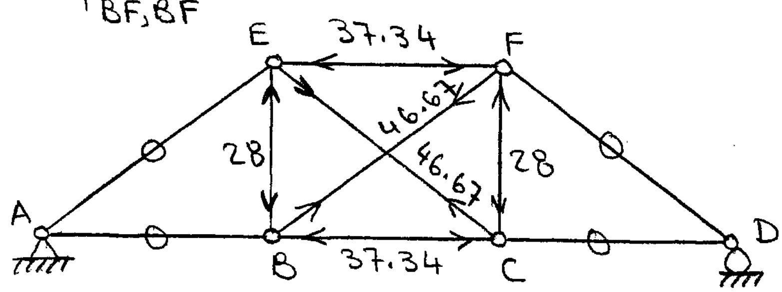
$$\Delta_{BFO} = \alpha \sum (\Delta T) L u_{BF} = 1.2 \times 10^{-5} (8) \left[-25(-0.8) + 60(-0.8) \right] = -0.002688 \text{ m} = -2.688 \text{ mm}$$

$$f_{BF,BF} = \frac{1}{EA} \sum u_{BF}^2 L$$

$$= \frac{1}{200(10^6)(0.003)} \left[2(1)^2(10) + 2(-0.8)^2(8) + 2(-0.6)^2(6) \right] = 57.6 (10^{-6}) \text{ m/kN}$$

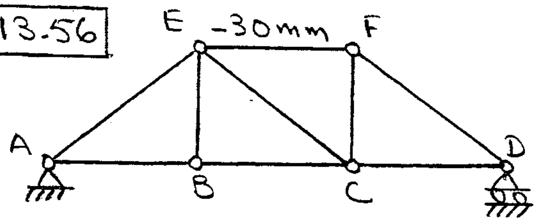
$$= 0.0576 \text{ mm/kN}$$

$$F_{BF} = -\frac{\Delta_{BFO}}{f_{BF,BF}} = \underline{46.67 \text{ kN (T)}}$$

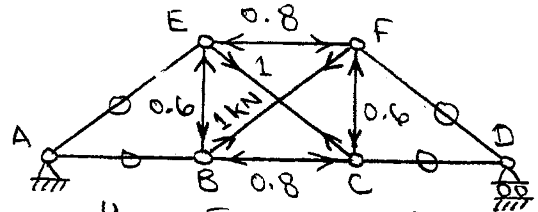


Member Forces (kN)

13.56



Fabrication Errors



u_{BF} Forces (kN)

$$\Delta_{BFO} = \Sigma \delta u_{BF} = -30(-0.8) = 24 \text{ mm}$$

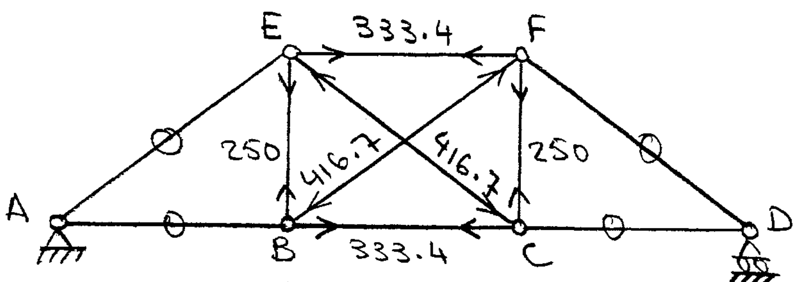
$$f_{BF,BF} = \frac{1}{EA} \Sigma u_{BF}^2 L$$

$$= \frac{1}{200(10^6)(0.003)} \left[2(1)^2 10 + 2(-0.8)^2 8 + 2(-0.6)^2 6 \right]$$

$$= 57.6(10^{-6}) \text{ m/kN} = 0.0576 \text{ mm/kN}$$

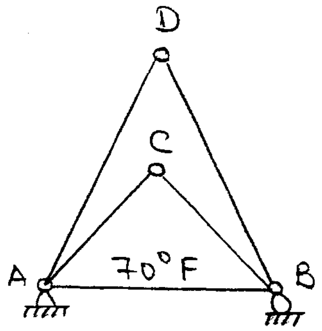
$$F_{BF} = -\frac{\Delta_{BFO}}{f_{BF,BF}} = -\frac{24}{0.0576} = -416.7 \text{ kN}$$

$$= \underline{416.7 \text{ kN (C)}}$$

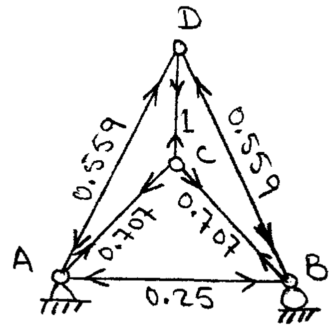


Member Forces (kN)

13.57



Temperature Change



u_{CD} Forces (k)

$$\Delta_{CDO} = \Sigma\alpha(\Delta T)L u_{CD} = 6.5\times10^{-6}(70)(20)(-0.25)$$

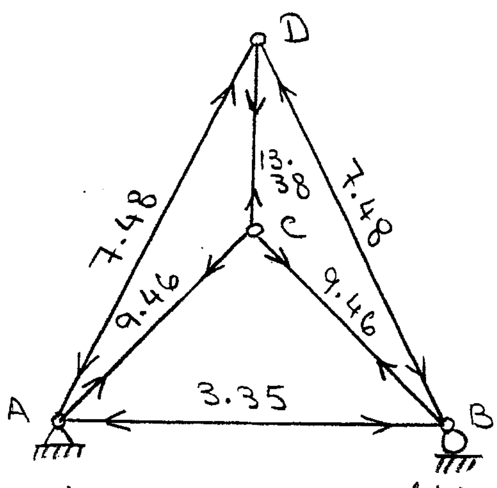
$$= -0.002275 \text{ ft} = -0.0273 \text{ in.}$$

$$f_{CD,CD} = \frac{1}{EA}\Sigma u_{CD}^2 L$$

$$= \frac{1}{29000(8)}\left[(1)^2(10) + (-0.25)^2(20) + 2(-0.559)^2 \times (22.36) + 2(0.707)^2(14.14)\right] = 0.00017 \text{ ft/k}$$

$$= 0.00204 \text{ in./k.}$$

$$F_{CD} = -\frac{\Delta_{CDO}}{f_{CD,CD}} = \underline{13.38 \text{ k (T)}}$$



Member Forces (k)

Chapter Fourteen
Three Moment Equation and the Method of Least Work

CHAPTER 14

14.1

Three-Moment Equation at Joint C:

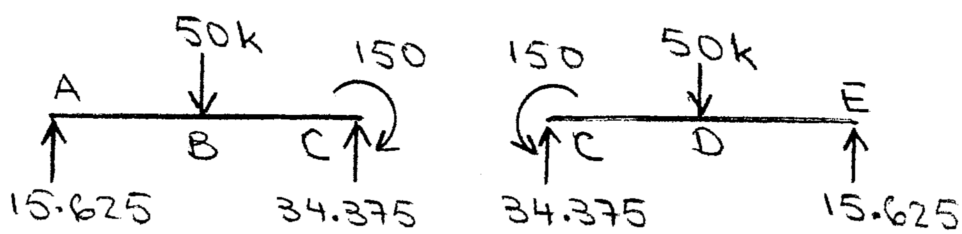
$$M_A = M_E = 0$$

$$2M_C\left(\frac{16}{2I}+\frac{16}{I}\right) = -\frac{50(16)^2(0.5)(1-0.25)}{2I}$$

$$-\frac{50(16)^2(0.5)(1-0.25)}{I}$$

$$48M_C = -7200$$

$$\underline{M_C = -150 \text{ k-ft}}$$



Span End Moments and Shears

For reactions, and shear and bending moment diagrams, see solution of Problem 13.11.

$\boxed{14.2}$

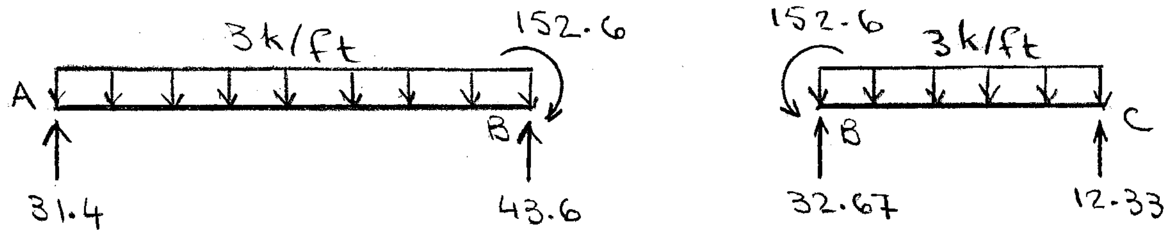
Three - Moment Equation at Joint B:

$$M_A = M_C = 0$$

$$2M_B\left(\frac{25}{2I} + \frac{15}{I}\right) = -\frac{3(25)^3}{4(2I)} - \frac{3(15)^3}{4I}$$

$$55M_B = -8390.625$$

$$\underline{M_B = -152.6 \text{ k-ft}}$$



Span End Moments and Shears

For reactions, and shear and bending moment diagrams, see solution of Problem 13.12.

$\boxed{14.3}$

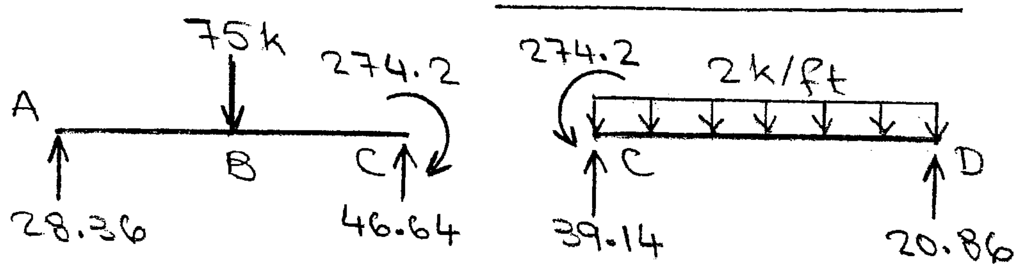
Three-Moment Equation at Joint C:

$$M_A = M_D = 0$$

$$2M_C\left(\frac{30}{3I}+\frac{30}{I}\right) = -\frac{75(30)^2(0.5)(1-0.25)}{3I} - \frac{2(30)^3}{4I}$$

$$80M_C = -21937.5$$

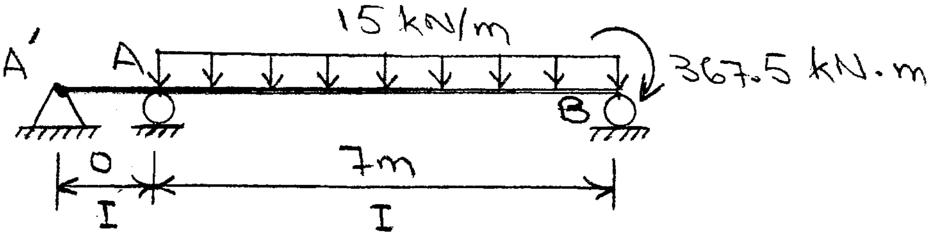
$$\underline{M_C = -274.2 \text{ k-ft}}$$



Span End Moments and Shears

For reactions, and shear and bending moment diagrams, see solution of Problem 13.13.

14.4



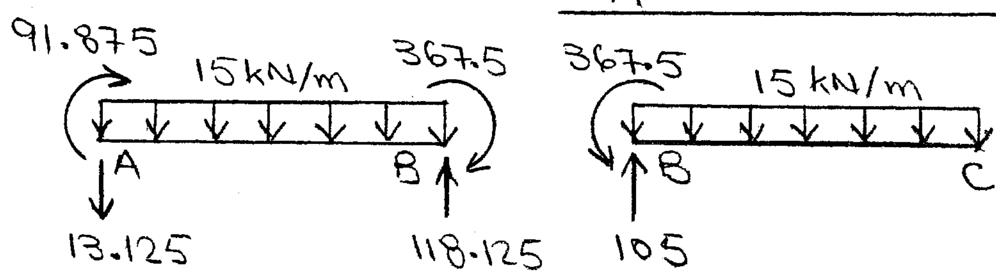
Three-Moment Equation at Joint A:

$$M_{A'} = 0 \quad , \quad M_B = -367.5 \text{ kN.m}$$

$$2M_A(0+7) - 367.5(7) = -\frac{1}{4}(15)(7)^3$$

$$14M_A = 1286.25$$

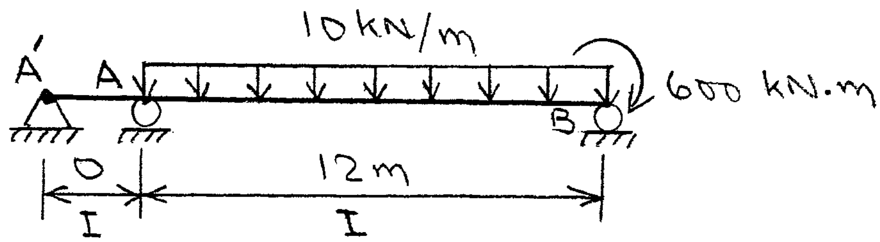
$$\underline{M_A = 91.875 \text{ kN.m}}$$



Span End Moments and Shears

For reactions, and shear and bending moment diagrams, see solution of Problem 13.15.

14.5



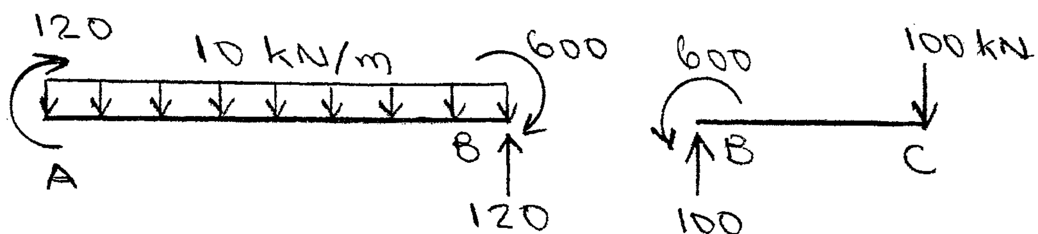
Three-Moment Equation at Joint A:

$$M_{A'} = 0, \qquad M_B = -600 \text{ kN.m}$$

$$2M_A(0+12) - 600(12) = -\frac{1}{4}(10)(12)^3$$

$$24 M_A = 2880$$

$$\underline{M_A = 120 \text{ kN.m}}$$



Span End Moments and Shears

For reactions, and shear and bending moment diagrams, see solution of Problem 13.16.

14.6

Three-Moment Equation at Joint C:

$$M_A = 0$$

$$2M_C\left(\frac{10}{I}+\frac{10}{2I}\right)+M_E\left(\frac{10}{2I}\right) = -\frac{120(10)^2(0.6)(1-0.36)}{I} - \frac{120(10)^2(0.4)(1-0.16)}{2I}$$

$$30M_C + 5M_E = -6624 \qquad (1)$$

Three-Moment Equation at Joint E:

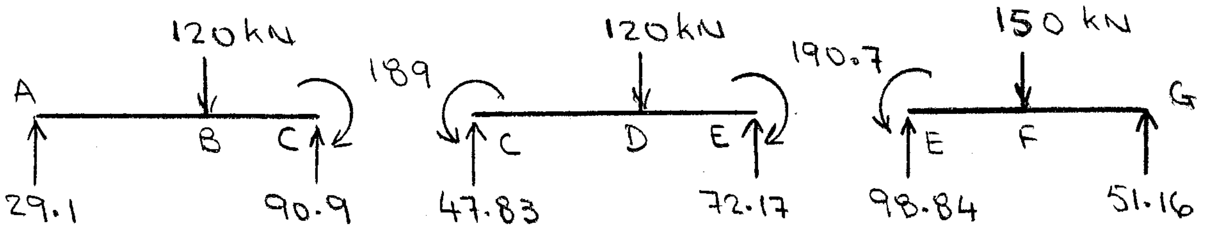
$$M_G = 0$$

$$M_C\left(\frac{10}{2I}\right)+2M_E\left(\frac{10}{2I}+\frac{8}{I}\right) = -\frac{120(10)^2(0.6)(1-0.36)}{2I} - \frac{150(8)^2(0.5)(1-0.25)}{I}$$

$$5M_C + 26M_E = -5904 \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously, we obtain:

$M_C = -189$ kN.m $\qquad$ $M_E = -190.7$ kN.m



Span End Moments and Shears

For reactions, and shear and bending moment diagrams, see solution of Problem 13.39.

$\boxed{14.7}$

Three-Moment Equation at Joint B:

$$M_A = 0$$

$$2M_B\left(\frac{24}{I}+\frac{24}{2I}\right)+M_C\left(\frac{24}{2I}\right)=-\frac{2.5(24)^3}{4I}-\frac{2.5(24)^3}{4(2I)}$$

$$72M_B + 12M_C = -12960 \qquad (1)$$

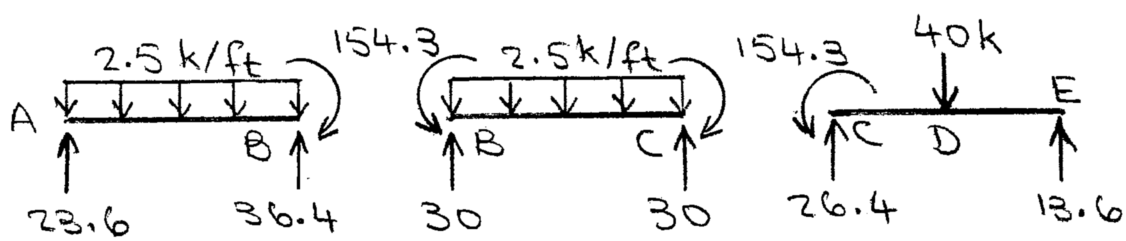
Three-Moment Equation at Joint C: $M_E = 0$

$$M_B\left(\frac{24}{2I}\right)+2\left(\frac{24}{2I}+\frac{24}{I}\right)=-\frac{2.5(24)^3}{4(2I)}-\frac{40(24)^2(0.5)(1-0.25)}{I}$$

$$12M_B + 72M_C = -12960 \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously, we obtain:

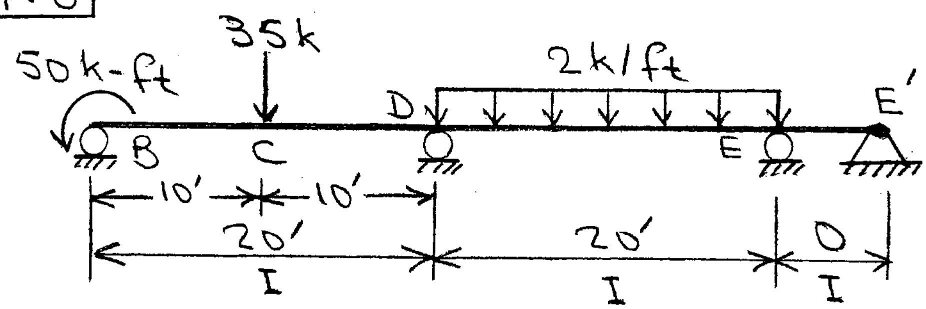
$$\underline{M_B = M_C = -154.3 \text{ k-ft}}$$



Span End Moments and Shears

For reactions, and shear and bending moment diagrams, see solution of Problem 13.40.

14.8



Three-Moment Equation at Joint D:

$$M_B = -50 \text{ k-ft}$$

$$-50(20) + 2M_D(20+20) + M_E(20) = -35(20)^2(0.5)(0.75) - \frac{1}{4}(2)(20)^3$$

$$80M_D + 20M_E = -8250 \qquad (1)$$

Three-Moment Equation at Joint E:

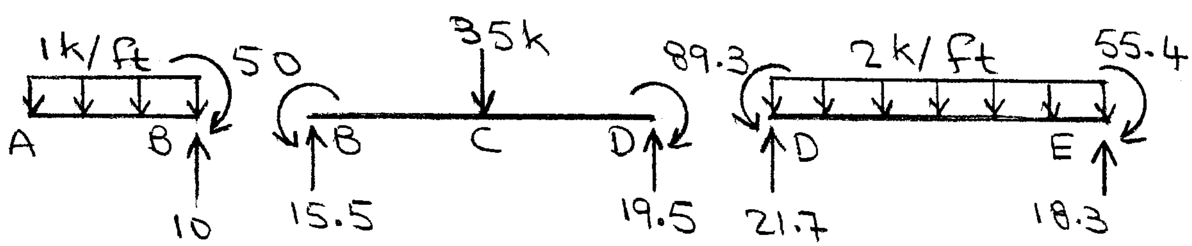
$$M_{E'} = 0$$

$$M_D(20) + 2M_E(20+0) = -\frac{1}{4}(2)(20)^3$$

$$20M_D + 40M_E = -4000 \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously, we obtain:

$\underline{M_D = -89.3 \text{ k-ft}}$ $\quad$ $\underline{M_E = -55.4 \text{ k-ft}}$



Span End Moments and Shears

For reactions, and shear and bending moment diagrams, see solution of Problem 13.38.

14.9

Three-Moment Equation at Joint B:

$$M_A = M_C = 0$$

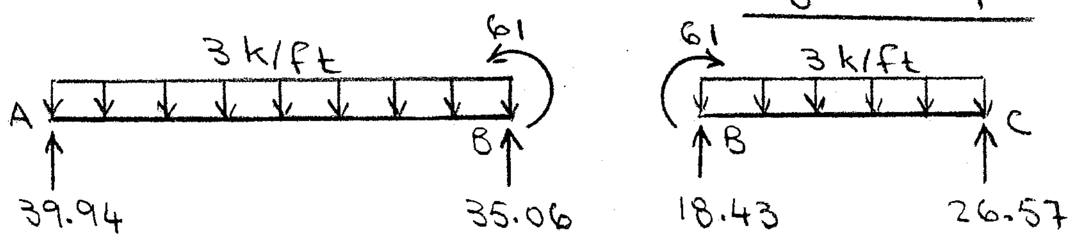
$$2M_B\left(\frac{25}{2I} + \frac{15}{I}\right) = -\frac{3(25)^3}{4(2I)} - \frac{3(15)^3}{4I} - 6E\left[\frac{0.25-1}{25(12)} + \frac{0.75-1}{15(12)}\right]$$

$$55M_B = -8390.625 + 0.0233EI$$

Substituting $EI = \frac{29000(2500)}{(12)^2} = 503472.2$ k-ft^2, we obtain:

$$55M_B = 3357$$

$$\underline{M_B = 61 \text{ k-ft}}$$



Span End Moments and Shears

For reactions, and shear and bending moment diagrams, see solution of Problem 13.51.

$\boxed{14.10}$

Three-Moment Equation at Joint C: $M_A = 0$

$$2M_C\left(\frac{10}{I}+\frac{10}{2I}\right)+M_E\left(\frac{10}{2I}\right)=\frac{-120(10)^2(0.6)(1-0.36)}{I}$$

$$-\frac{120(10)^2(0.4)(1-0.16)}{2I}-6E\left[\frac{0.01-0.065}{10}+\frac{0.04-0.065}{10}\right]$$

Substituting $EI = 200(500) = 100000\ kN.m^2$, we obtain:

$$30M_C + 5M_E = -1824 \qquad (1)$$

Three-Moment Equation at Joint E: $M_G = 0$

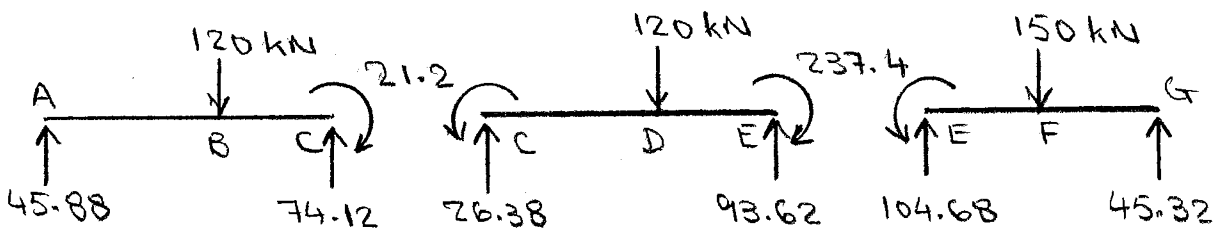
$$M_C\left(\frac{10}{2I}\right)+2M_E\left(\frac{10}{2I}+\frac{8}{I}\right)=-\frac{120(10)^2(0.6)(1-0.36)}{2I}$$

$$-\frac{150(8)^2(0.5)(1-0.25)}{I}-6E\left[\frac{0.065-0.04}{10}+\frac{0.025-0.04}{8}\right]$$

$$5M_C + 26M_E = -6279 \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously, we obtain

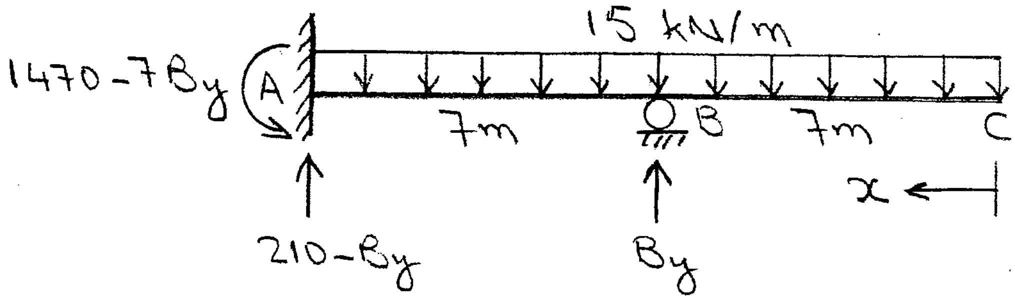
$\underline{M_C = -21.2\ kN.m}$ $\qquad$ $\underline{M_E = -237.4\ kN.m}$



Span End Moments and Shears

For reactions, and shear and bending moment diagrams, see solution of Problem 13.54.

$\boxed{14.11}$



Segment	x Coordinate Origin	Limits (m)	M	$\frac{\partial M}{\partial B_y}$
CB	C	0 − 7	$-7.5x^2$	0
BA	C	7 − 14	$-7.5x^2+B_y(x-7)$	$x-7$

$$\frac{\partial U}{\partial B_y} = \int \left(\frac{\partial M}{\partial B_y}\right)\frac{M}{EI}\,dx = 0$$

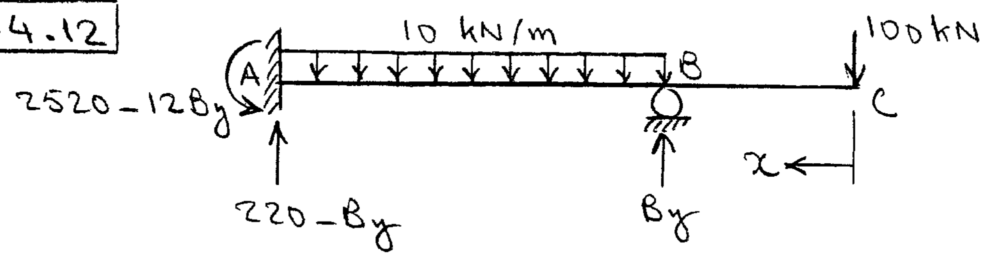
$$\frac{1}{EI}\int_7^{14} (x-7)\left[-7.5x^2 + B_y(x-7)\right] dx = 0$$

$$-25510.625 + (114.33)\,B_y = 0$$

$$\underline{B_y = 223.125 \text{ kN} \uparrow}$$

For reactions, and shear and bending moment diagrams, see solution of Problem 13.15.

$\boxed{14.12}$



Segment	x Coordinate		M	$\frac{\partial M}{\partial B_y}$
	Origin	Limits (m)		
CB	C	0 – 6	$-100x$	0
BA	C	6 – 18	$-100x - 5(x-6)^2 + B_y(x-6)$	$x-6$

$$\frac{\partial U}{\partial B_y} = \int \left(\frac{\partial M}{\partial B_y}\right)\frac{M}{EI}\,dx = 0$$

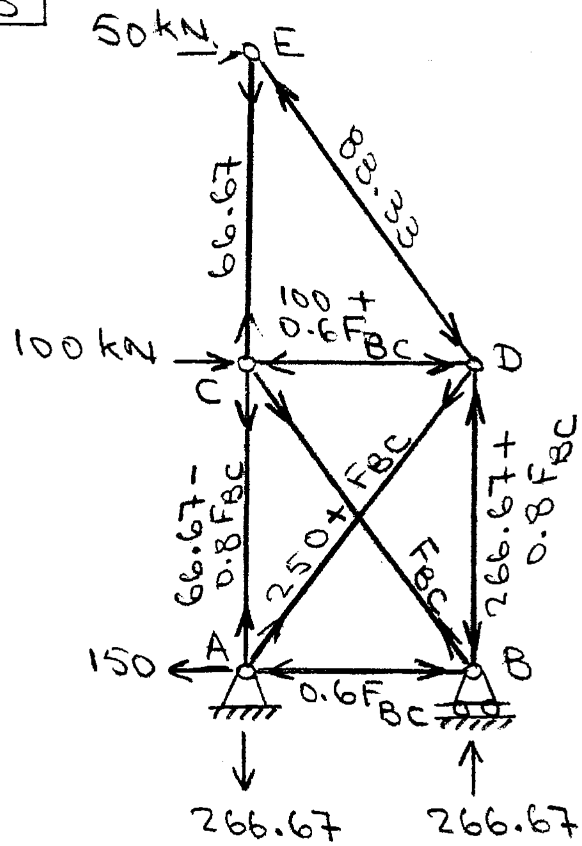
$$\frac{1}{EI}\int_6^{18}(x-6)\left[-100x - 5(x-6)^2 + B_y(x-6)\right]dx = 0$$

$$-126720 + 576 B_y = 0$$

$$B_y = \underline{220 \text{ kN} \uparrow}$$

For reactions, and shear and bending moment diagrams, see solution of Problem 13.16.

14.13



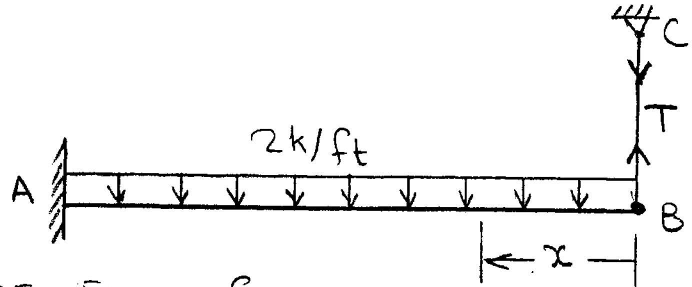
Member	L (m)	F	$\frac{\partial F}{\partial F_{BC}}$	$\left(\frac{\partial F}{\partial F_{BC}}\right)FL$	F (kN)
AB	3	$-0.6F_{BC}$	-0.6	$1.08F_{BC}$	71.9
CD	3	$-100-0.6F_{BC}$	-0.6	$180+1.08F_{BC}$	-28.1
AC	4	$66.67-0.8F_{BC}$	-0.8	$-213.33+2.56F_{BC}$	162.5
BD	4	$-266.67-0.8F_{BC}$	-0.8	$853.33+2.56F_{BC}$	-170.8
CE	4	66.67	0	0	66.7
AD	5	$250+F_{BC}$	1	$1250+5F_{BC}$	130.2
DE	5	-83.33	0	0	-83.3
BC	5	F_{BC}	1	$5F_{BC}$	-119.8
			Σ	$2070+17.28F_{BC}$	

$$\frac{\partial U}{\partial F_{BC}} = \frac{1}{EA}\sum\left(\frac{\partial F}{\partial F_{BC}}\right)FL = 0$$

$$\frac{1}{EA}(2070+17.28F_{BC}) = 0$$

$$F_{BC} = -119.8 \text{ kN} = \underline{119.8 \text{ kN (C)}}$$

14-14



$$\frac{\partial U}{\partial T} = \Sigma\left(\frac{\partial F}{\partial T}\right)\frac{FL}{AE} + \int\left(\frac{\partial M}{\partial T}\right)\frac{M}{EI}\,dx = 0$$

Cable BC: $F = T$; $\frac{\partial F}{\partial T} = 1$

Beam AB: $M = Tx - x^2$; $\frac{\partial M}{\partial T} = x$

$$\frac{\partial U}{\partial T} = \frac{1}{E}\left[\frac{T(5)(12)^2}{0.5} + \frac{(12)^4}{500}\int_0^{15} x\,(Tx - x^2)\,dx\right]$$

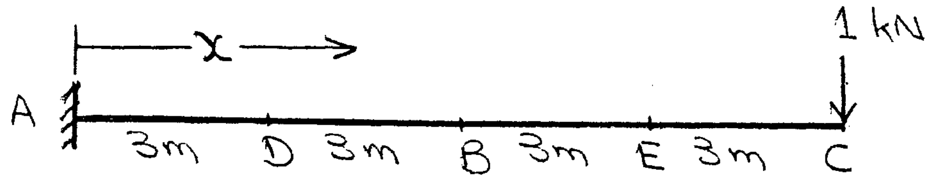
$$= 48096T - 524880 = 0$$

$\underline{T = 10.9\text{ k (T)}}$

Chapter Fifteen
Influence Lines for Statically Indeterminate Structures

CHAPTER 15

15.1 Select C_y ($\uparrow +$) as the redundant.



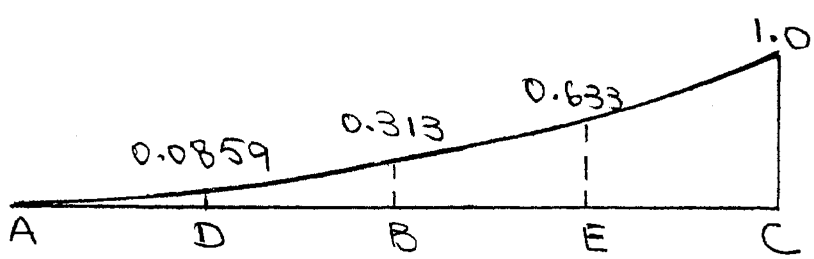
$$C_y = -\frac{f_{Cx}}{\bar{f}_{CC}} = -\frac{f_{xC}}{\bar{f}_{CC}}$$

Using beam deflection formulas:

$$f_{xC} = -\frac{1}{6EI}(x^3 - 36x^2)$$

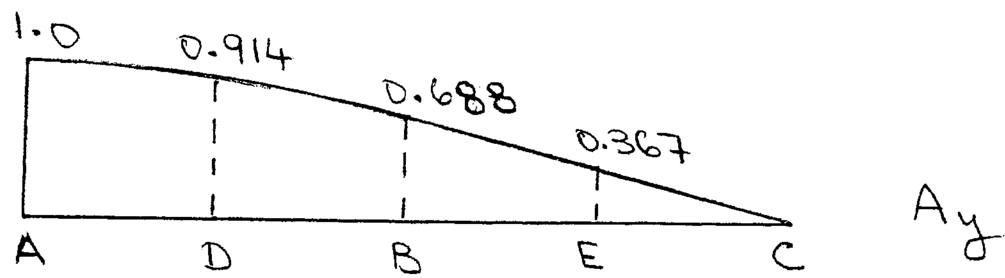
$$\bar{f}_{CC} = -f_{CC} = +\frac{576\ \text{kN.m}^3/\text{kN}}{EI}$$

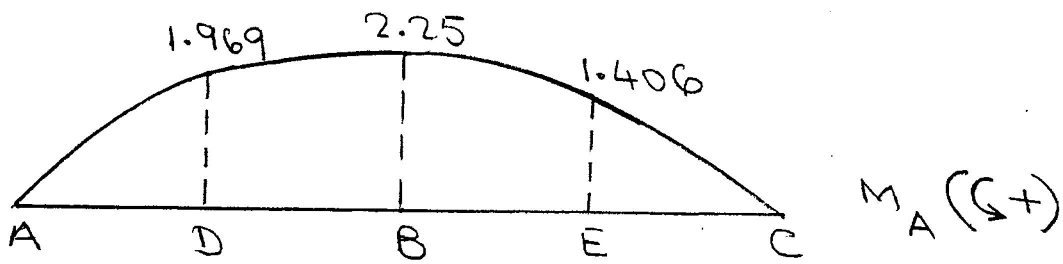
x (m)	$EI f_{xC}$	C_y (kN/kN)	A_y (kN/kN)	M_A (kN.m/kN)	S_B (kN/kN)	M_B (kN.m/kN)
0	0	0	1	0	0	0
3	−49.5	0.0859	0.914	1.969	−0.0859	0.516
6	−180	0.313	0.688	2.25	−0.313(L) 0.687(R)	1.875
9	−364.5	0.633	0.367	1.406	0.367	0.797
12	−576	1	0	0	0	0

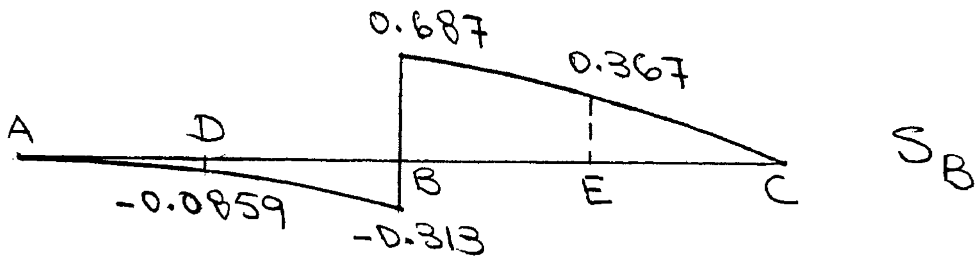


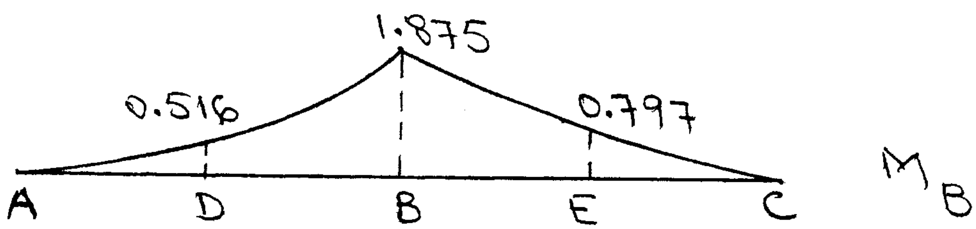
C_y

15.1 (Contd.)

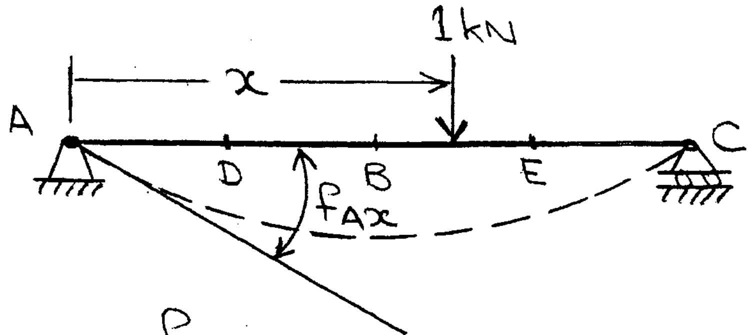

1.0
0.914
0.688
0.367
A
D
B
E
C
A_y


1.969
2.25
1.406
A
D
B
E
C
M_A (↻+)


0.687
0.367
A
D
B
E
C
-0.0859
-0.313
S_B


1.875
0.516
0.797
A
D
B
E
C
M_B

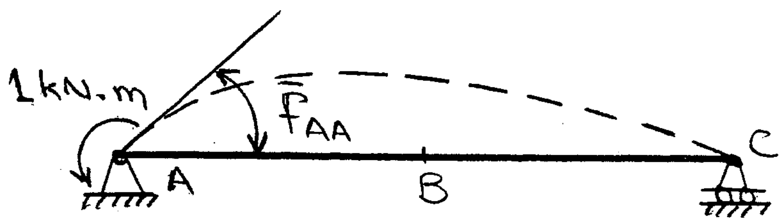
$\boxed{15.2}$ Select M_A (↻+) as the redundant.



$$M_A = -\frac{f_{Ax}}{\bar{f}_{AA}}$$

Using beam deflection formulas:

$$f_{Ax} = -\frac{1}{72EI}(x^3 - 36x^2 + 288x)$$

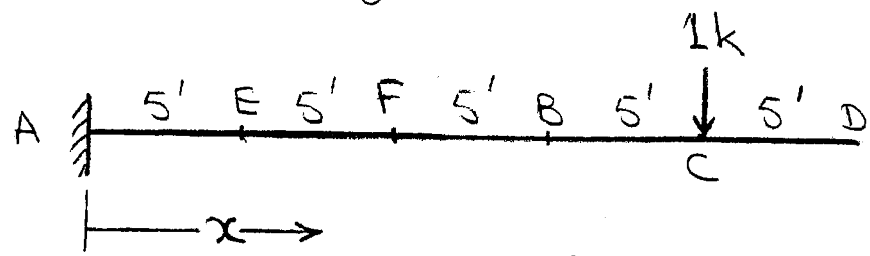


$$\bar{f}_{AA} = +\frac{4\text{ kN.m}^2/\text{kN.m}}{EI}$$

x (m)	$EI\, f_{Ax}$	M_A
0	0	0
3	−7.875	1.969
6	−9	2.25
9	−5.625	1.406
12	0	0

For influence lines, see solution of Problem 15.1.

15.3 Select C_y (↑+) as the redundant.



$$C_y = -\frac{f_{Cx}}{\bar{f}_{CC}} = -\frac{f_{xC}}{\bar{f}_{CC}}$$

Using beam deflection formulas:

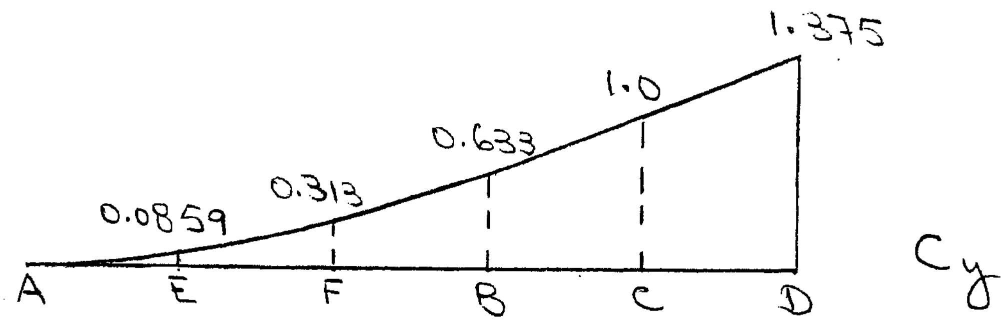
for $0 \leqslant x \leqslant 20'$ $\quad f_{xC} = \frac{1}{6EI}(x^3 - 60x^2)$

for $20' \leqslant x \leqslant 25'$ $\quad f_{xC} = \frac{200}{3EI}(20 - 3x)$

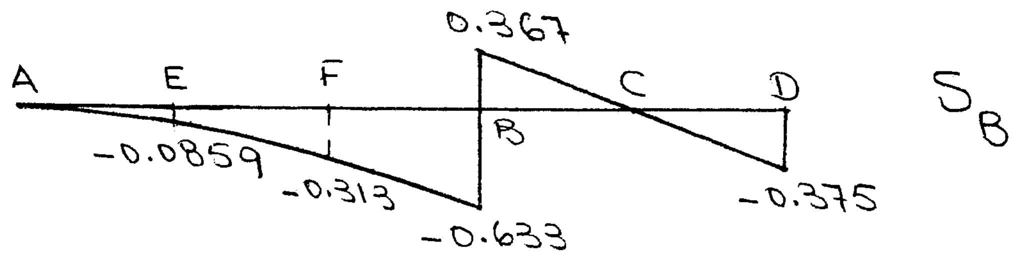
$$\bar{f}_{CC} = -f_{CC} = +\frac{2666.67 \text{ k-ft}^3/\text{k}}{EI}$$

x (ft)	$EI f_{xC}$	C_y (k/k)	S_B (k/k)	M_B (k-ft/k)
0	0	0	0	0
5	−229.17	0.0859	−0.0859	0.43
10	−833.33	0.313	−0.313	1.563
15	−1687.5	0.633	−0.633(L) 0.367(R)	3.164
20	−2666.67	1	0	0
25	−3666.67	1.375	−0.375	−3.125

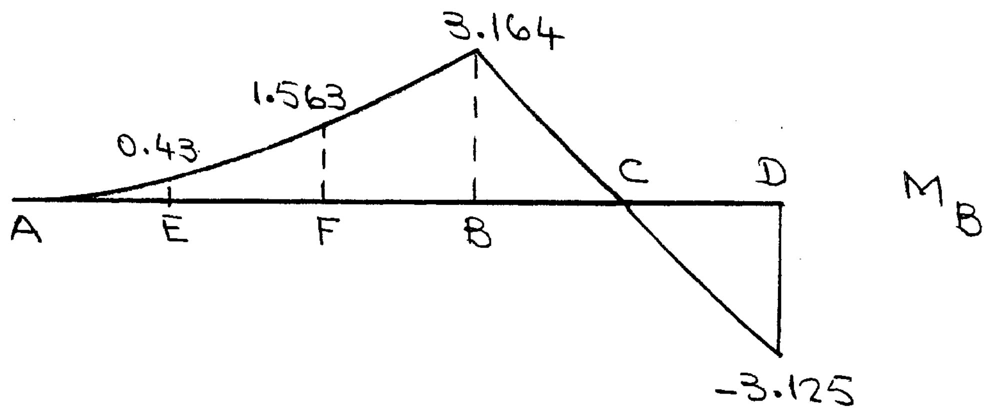
15.3 (Cont'd.)



1.375
1.0
0.633
0.313
0.0859
A
E
F
B
C
D
C_y

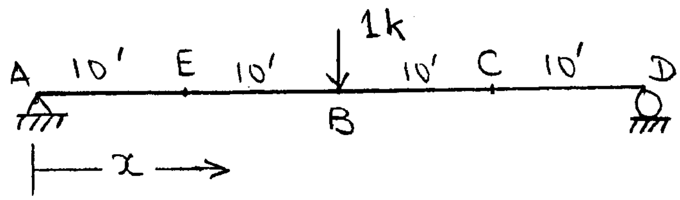


0.367
A
E
F
C
D
B
-0.0859
-0.313
-0.633
-0.375
S_B



3.164
1.563
0.43
C
D
A
E
F
B
-3.125
M_B

15.4 Select B_y (↑+) as the redundant.



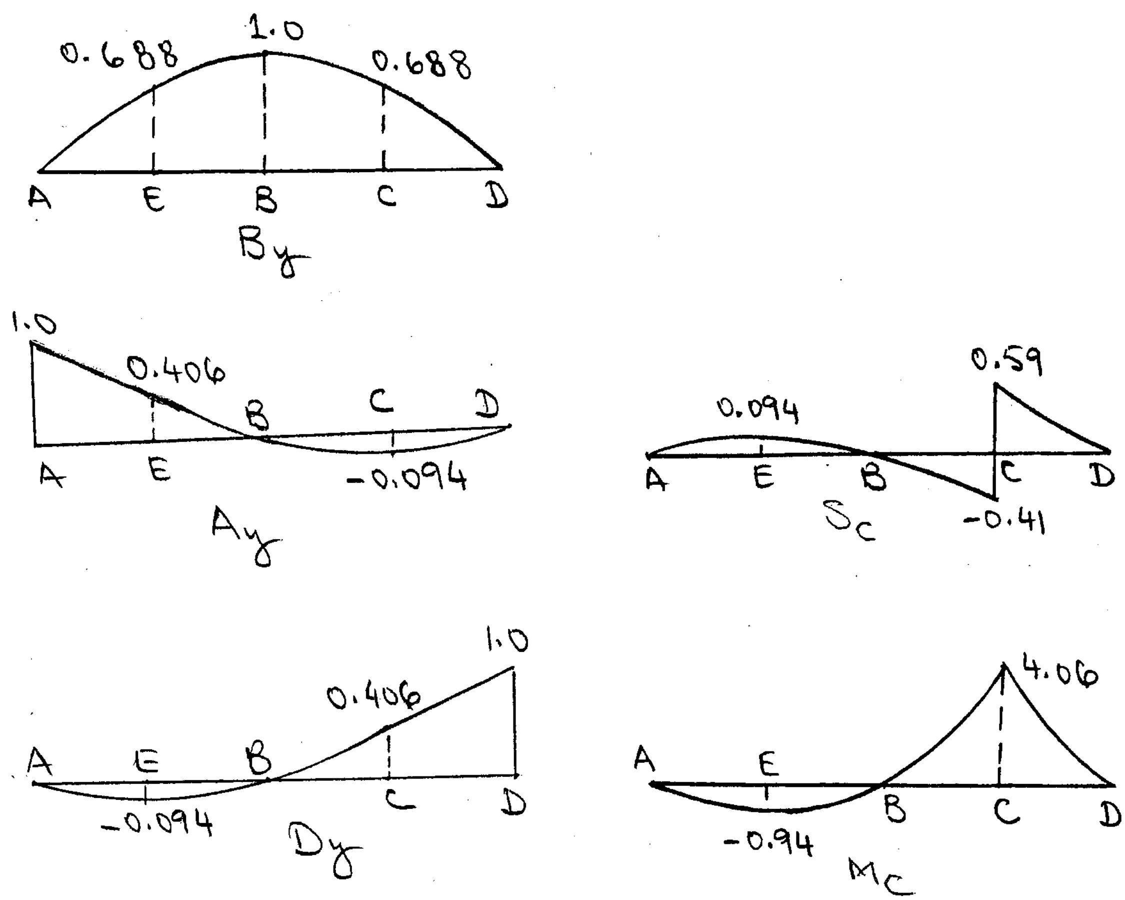
$$B_y = -\frac{f_{Bx}}{\bar{f}_{BB}} = -\frac{f_{xB}}{\bar{f}_{BB}}$$

Using beam-deflection formulas:

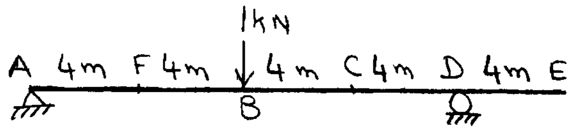
$$f_{EB} = f_{CB} = -\frac{916.7 \text{ k-ft}^3/\text{k}}{EI}$$

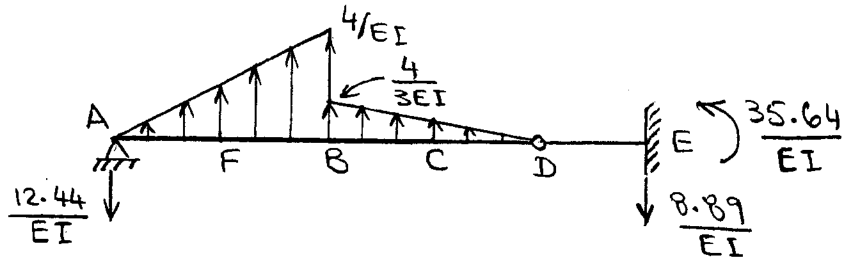
$$f_{BB} = -\frac{1333.3 \text{ k-ft}^3/\text{k}}{EI}$$

$$\bar{f}_{BB} = -f_{BB} = +\frac{1333.3 \text{ k-ft}^3/\text{k}}{EI}$$



$\boxed{15.5}$ Select B_y (↑+) as the redundant.

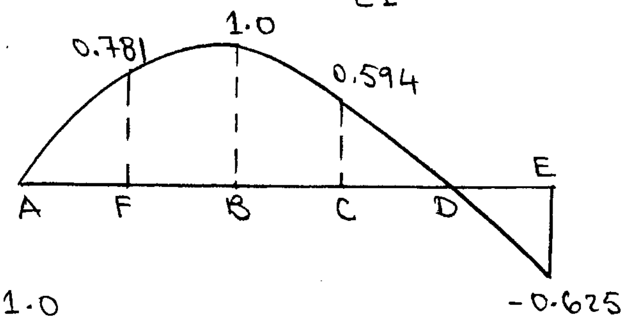




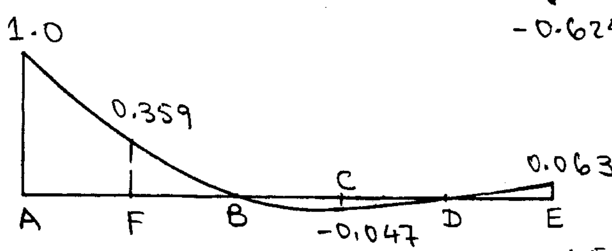
$$f_{BF} = f_{FB} = -\frac{44.43}{EI} \; ; \quad f_{BB} = -\frac{56.85}{EI}$$

$$f_{BC} = f_{CB} = -\frac{33.72}{EI} \; ; \quad f_{BE} = f_{EB} = +\frac{35.64}{EI}$$

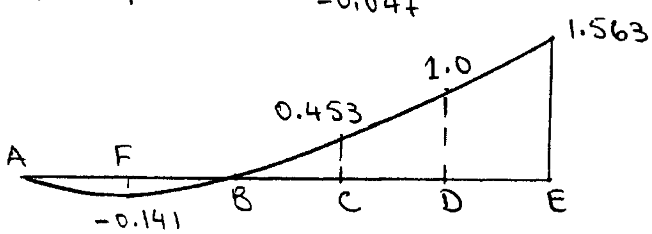
$$\bar{f}_{BB} = -f_{BB} = \frac{56.85}{EI}$$



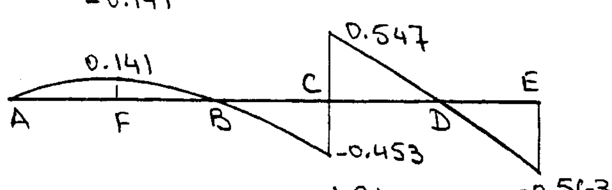
B_y



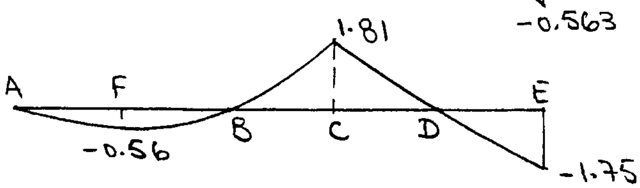
A_y



D_y

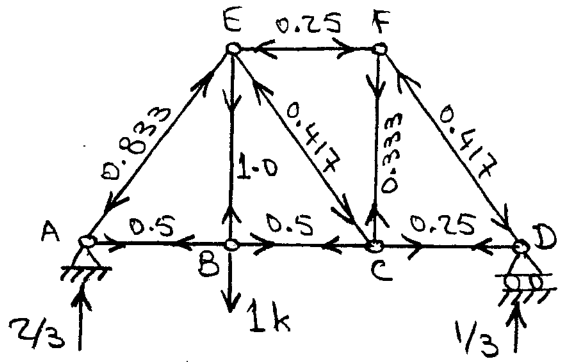


S_C

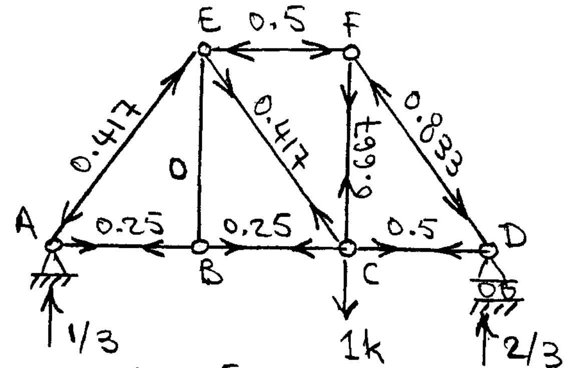


M_C

15.6 Select C_y (↑+) as the redundant.



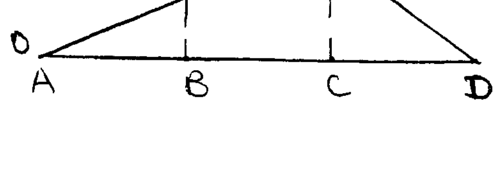
u_B Forces



u_C Forces

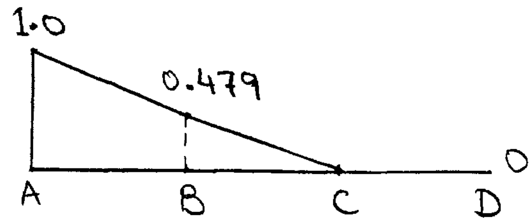
Member	L (ft)	u_B	u_C	$u_B u_C L$	$u_C^2 L$
AB	15	0.5	0.25	1.875	0.938
BC	15	0.5	0.25	1.875	0.938
CD	15	0.25	0.5	1.875	3.75
EF	15	−0.25	−0.5	1.875	3.75
BE	20	1.0	0	0	0
CF	20	0.333	0.667	4.442	8.898
AE	25	−0.833	−0.417	8.684	4.347
CE	25	−0.417	0.417	−4.347	4.347
DF	25	−0.417	−0.833	8.684	17.347
			Σ	24.963	44.315

Unit load at B: $C_y = -\dfrac{24.963}{44.315} = -0.563\text{k}$ or $0.563\text{k}\uparrow$

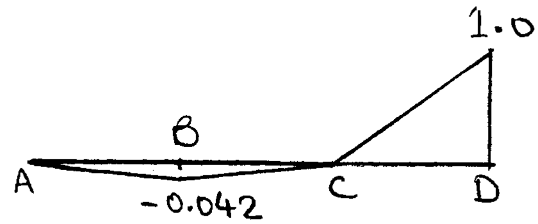


IL for C_y

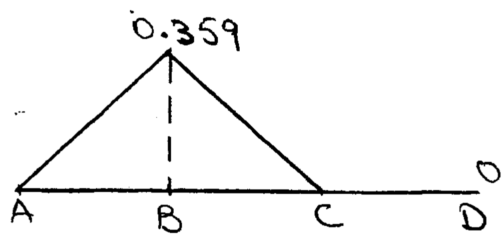
15.6 (cont'd.)



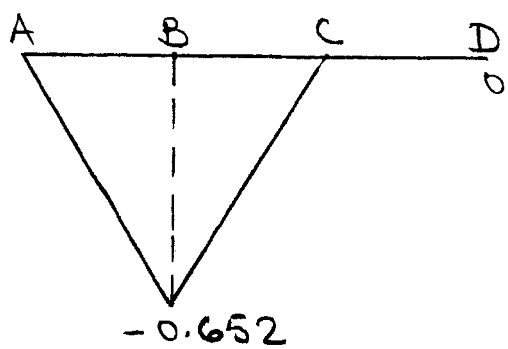
IL for A_y



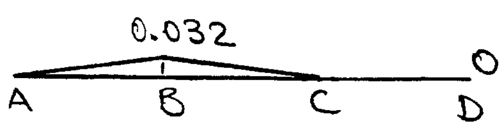
IL for D_y



IL for F_{BC}

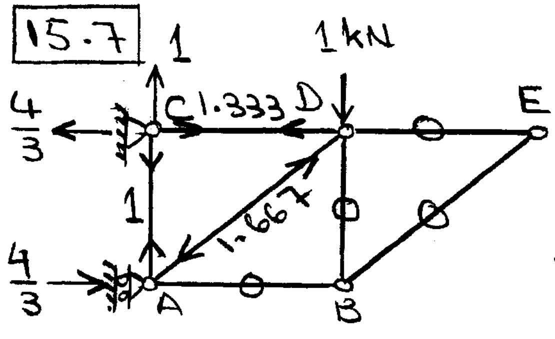


IL for F_{CE}

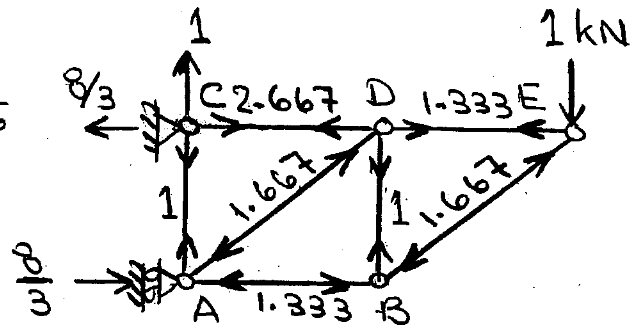


IL for F_{EF}

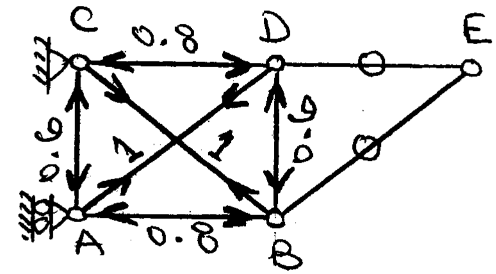
15.7



u_D Forces



u_E Forces



u_{BC} Forces

Member	L (m)	u_D	u_E	u_{BC}	$u_D u_{BC} L$	$u_E u_{BC} L$	$u_{BC}^2 L$
AB	8	0	−1.333	−0.8	0	8.533	5.12
CD	8	1.333	2.667	−0.8	−8.533	−17.067	5.12
DE	8	0	1.333	0	0	0	0
AC	6	1	1	−0.6	−3.6	−3.6	2.16
BD	6	0	1	−0.6	0	−3.6	2.16
AD	10	−1.667	−1.667	1	−16.667	−16.667	10
BE	10	0	−1.667	0	0	0	0
BC	10	0	0	1	0	0	10
				Σ	−28.8	−32.4	34.56

15.7 (contd.)

Unit load at D: $f_{BC,D} + f_{BC,BC} F_{BC} = 0$

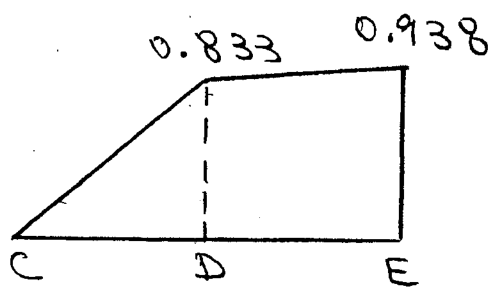
$$F_{BC} = \frac{28.8}{34.56} = \underline{0.833 \text{ kN/kN (T)}}$$

$$F_{CD} = u_D + u_{BC} F_{BC} = 1.333 - 0.8(0.833)$$
$$= \underline{0.667 \text{ kN/kN (T)}}$$

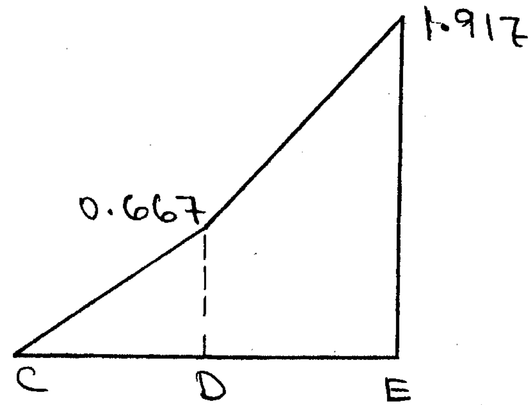
Unit load at E: $f_{BC,E} + f_{BC,BC} F_{BC} = 0$

$$F_{BC} = \frac{32.4}{34.56} = \underline{0.938 \text{ kN/kN (T)}}$$

$$F_{CD} = u_E + u_{BC} F_{BC} = 2.667 - 0.8(0.938)$$
$$= \underline{1.917 \text{ kN/kN (T)}}$$

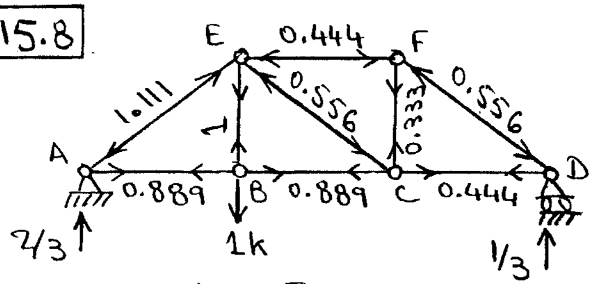


F_{BC}

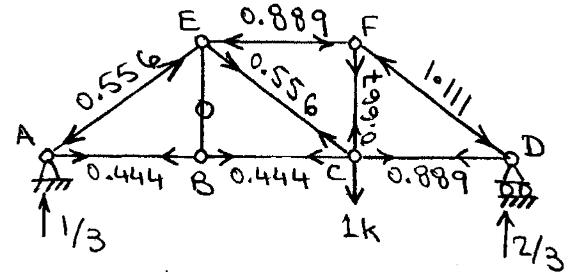


F_{CD}

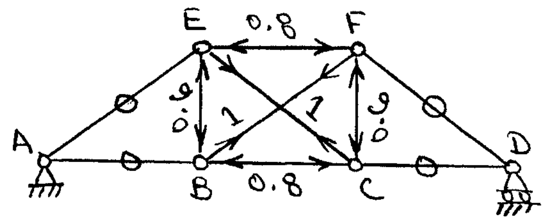
15.8



u_B Forces



u_C Forces



$\times F_{BF}$

u_{BF} Forces

Member	L (in.)	A (in.²)	u_B	u_C	u_{BF}	$\frac{u_B u_{BF} L}{A}$	$\frac{u_C u_{BF} L}{A}$	$\frac{u_{BF}^2 L}{A}$
AB	192	8	0.889	0.444	0	0	0	0
BC	192	8	0.889	0.444	−0.8	−17.07	−8.52	15.36
CD	192	8	0.444	0.889	0	0	0	0
EF	192	8	−0.444	−0.889	−0.8	8.52	17.07	15.36
BE	144	6	1	0	−0.6	−14.4	0	8.64
CF	144	6	0.333	0.667	−0.6	−4.8	−9.6	8.64
AE	240	8	−1.111	−0.556	0	0	0	0
DF	240	8	−0.556	−1.111	0	0	0	0
CE	240	6	−0.556	0.556	1	−22.24	22.24	40
BF	240	6	0	0	1	0	0	40
					Σ	−49.99	21.19	128

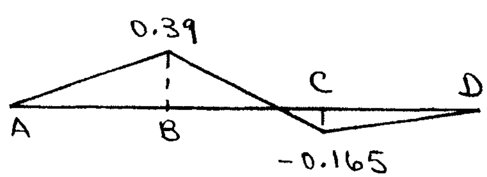
$\boxed{15.8\ (\text{contd.})}$

Unit load at B:

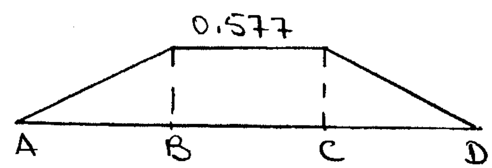
$$F_{BF} = \frac{49.99}{128} = 0.39\ k$$
$$F_{BC} = 0.889 - 0.8(0.39) = 0.577\ k$$
$$F_{CF} = 0.333 - 0.6(0.39) = 0.099\ k$$

Unit load at C:

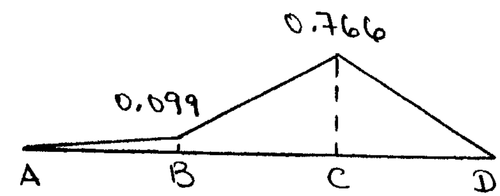
$$F_{BF} = -\frac{21.19}{128} = -0.165\ k$$
$$F_{BC} = 0.444 - 0.8(-0.165) = 0.577\ k$$
$$F_{CF} = 0.667 - 0.6(-0.165) = 0.766\ k$$



F_{BF}

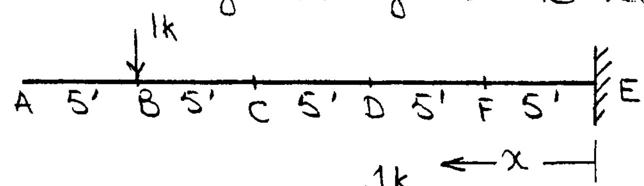


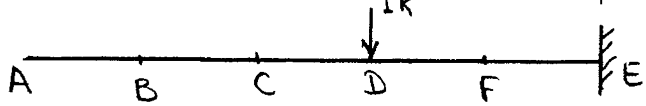
F_{BC}



F_{CF}

$\boxed{15.9}$ Select B_y and D_y as the redundants.





Compatibility Eqs.: $f_{Bx} + \bar{f}_{BB} B_y + \bar{f}_{BD} D_y = 0$

$$f_{Dx} + \bar{f}_{DB} B_y + \bar{f}_{DD} D_y = 0$$

Using beam deflection formulas:

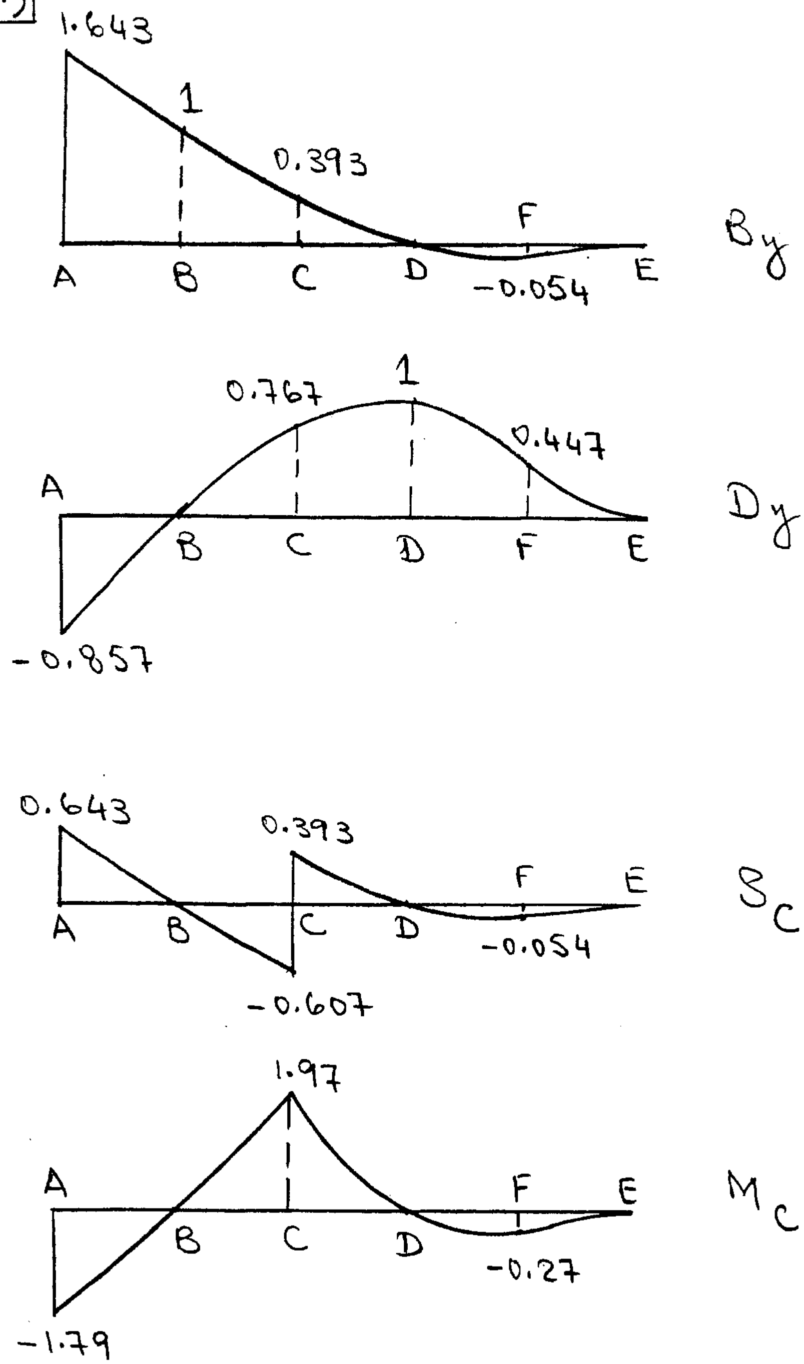
$$f_{Bx} = f_{xB} = \begin{cases} \dfrac{1}{6EI}(x^3 - 60x^2) & \text{for } 0 \leqslant x \leqslant 20' \\ \dfrac{200}{3EI}(20 - 3x) & \text{for } 20' \leqslant x \leqslant 25' \end{cases}$$

$$f_{Dx} = f_{xD} = \begin{cases} \dfrac{1}{6EI}(x^3 - 30x^2) & \text{for } 0 \leqslant x \leqslant 10' \\ \dfrac{50}{3EI}(10 - 3x) & \text{for } 10' \leqslant x \leqslant 25' \end{cases}$$

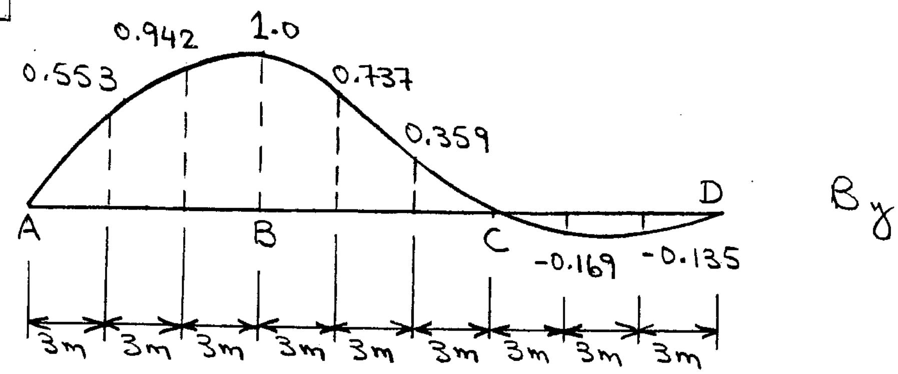
$$\bar{f}_{BB} = \frac{2666.7}{EI}; \quad \bar{f}_{BD} = \bar{f}_{DB} = \frac{833.3}{EI}; \quad \bar{f}_{DD} = \frac{333.3}{EI}$$

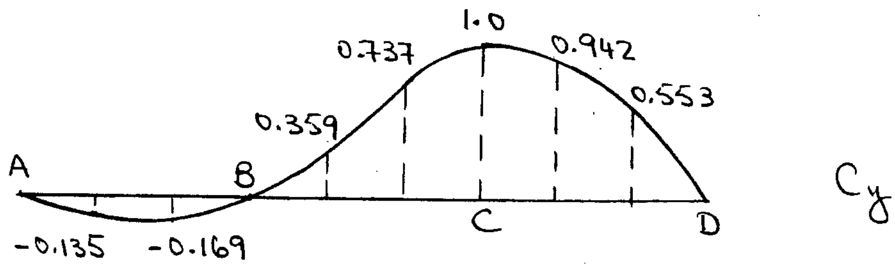
x (ft.)	$EI f_{Bx}$	$EI f_{Dx}$	B_y	D_y	S_C	M_C
0	0	0	0	0	0	0
5	-229.2	-104.2	-0.054	0.447	-0.054	-0.27
10	-833.3	-333.3	0	1	0	0
15	-1687.5	-583.3	0.393	0.767	-0.607(L) 0.393(R)	1.97
20	-2666.7	-833.3	1	0	0	0
25	-3666.7	-1083.3	1.643	-0.857	0.643	-1.79

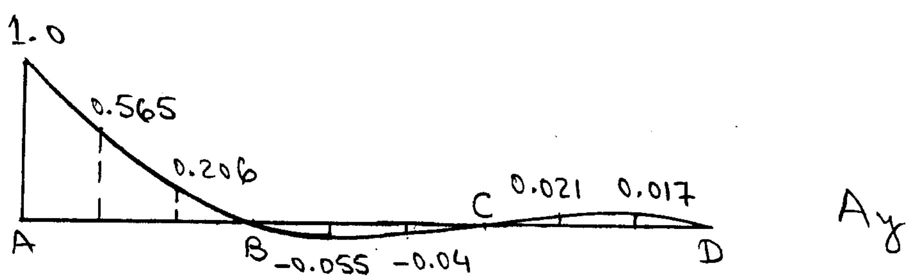
15.9 (Contd.)

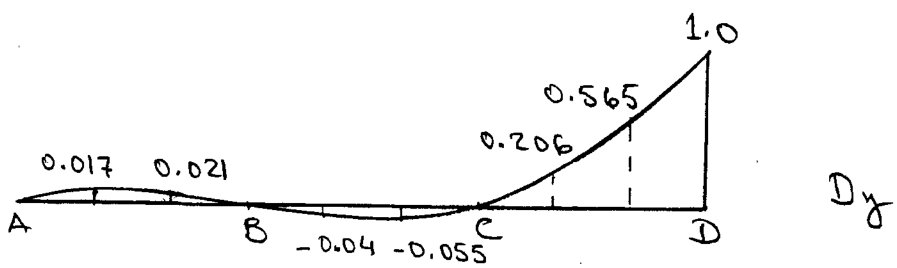

1.643
1
0.393
F
A
B
C
D
-0.054
E
By
0.767
1
0.447
A
B
C
D
F
E
Dy
-0.857
0.643
0.393
F
E
A
B
C
D
-0.054
Sc
-0.607
1.97
A
F
E
B
C
D
-0.27
Mc
-1.79

15.10

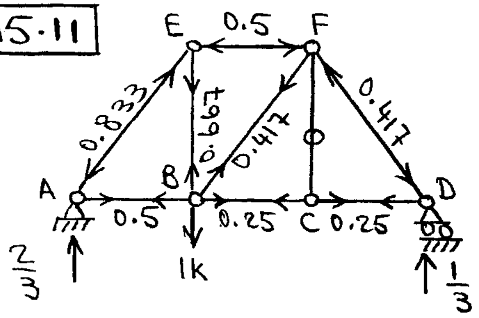

0.553
0.942
1.0
0.737
0.359
A
B
C
D
-0.169
-0.135
3m
3m
3m
3m
3m
3m
3m
3m
3m
By


1.0
0.737
0.942
0.553
0.359
A
B
C
D
-0.135
-0.169
Cy

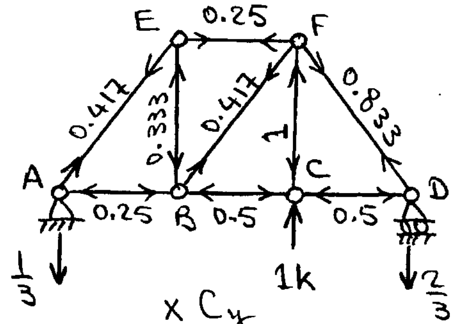

1.0
0.565
0.206
C
0.021
0.017
A
B
-0.055
-0.04
D
Ay


1.0
0.565
0.206
0.017
0.021
A
B
C
D
-0.04
-0.055
Dy

15-11

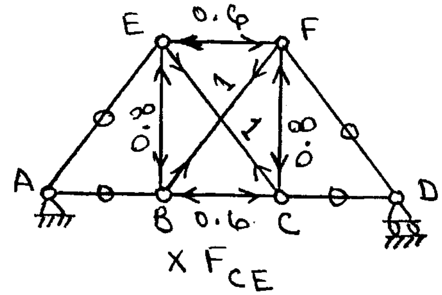


u_B Forces



$\times C_y$

u_C Forces



$\times F_{CE}$

u_{CE} Forces

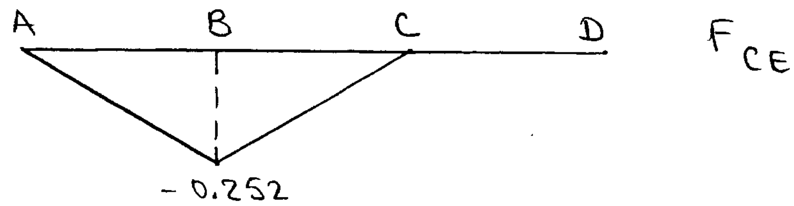
Member	L (ft)	u_B	u_C	u_{CE}	$u_B u_C L$	$u_B u_{CE} L$	$u_C^2 L$	$u_{CE}^2 L$	$u_C u_{CE} L$
AB	15	0.5	-0.25	0	-1.875	0	0.938	0	0
BC	15	0.25	-0.5	-0.6	-1.875	-2.25	3.75	5.4	4.5
CD	15	0.25	-0.5	0	-1.875	0	3.75	0	0
EF	15	-0.5	0.25	-0.6	-1.875	4.5	0.938	5.4	-2.25
BE	20	0.667	-0.333	-0.8	-4.442	-10.672	2.218	12.8	5.328
CF	20	0	-1	-0.8	0	0	20	12.8	16
AE	25	-0.833	0.417	0	-8.684	0	4.347	0	0
DF	25	-0.417	0.833	0	-8.684	0	17.347	0	0
BF	25	0.417	0.417	1	4.347	10.425	4.347	25	10.425
CE	25	0	0	1	0	0	0	25	0
				Σ	-24.963	2.003	57.635	86.4	34.003

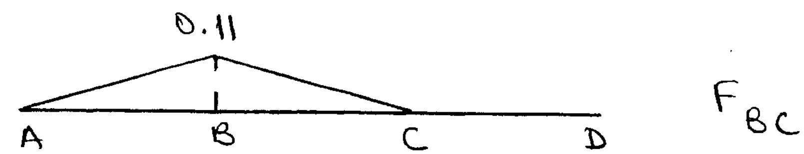
Unit load at B: $-24.963 + 57.635\,C_y + 34.003\,F_{CE} = 0$

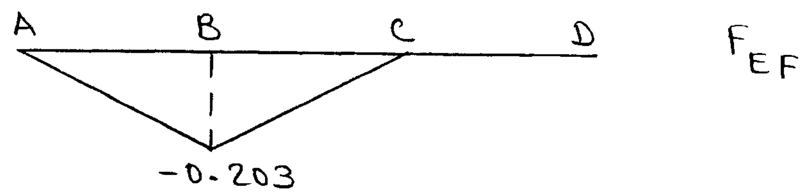
$2.003 + 34.003\,C_y + 86.4\,F_{CE} = 0$

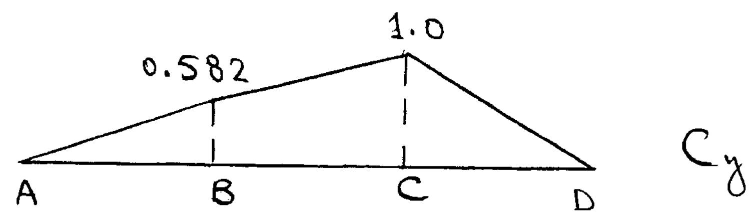
Solving these equations, we obtain: $C_y = 0.582\text{ k}$ $\quad F_{CE} = -0.252\text{ k}$

15-11 (Contd.)

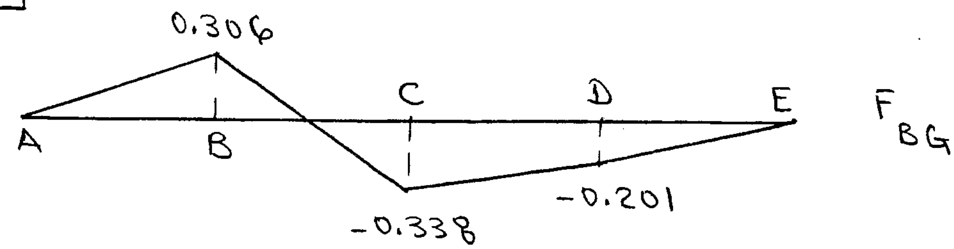




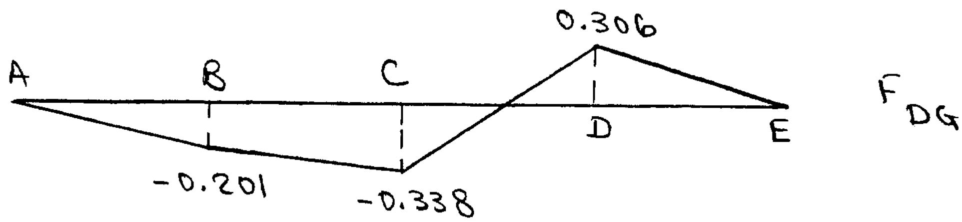




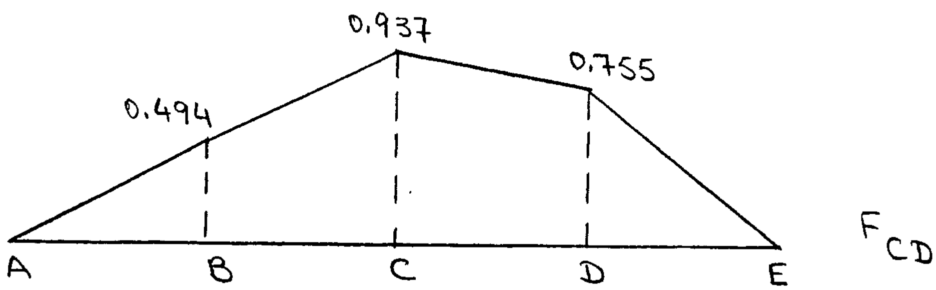
15.12



0.306
A
B
C
D
E
-0.338
-0.201
F_{BG}

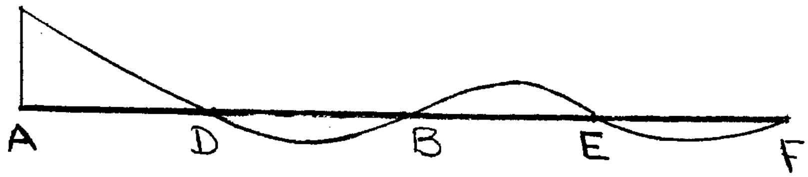


0.306
A
B
C
D
E
-0.201
-0.338
F_{DG}

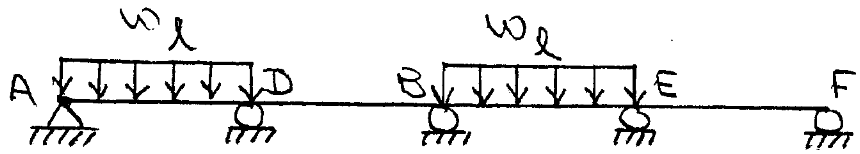


0.937
0.755
0.494
A
B
C
D
E
F_{CD}

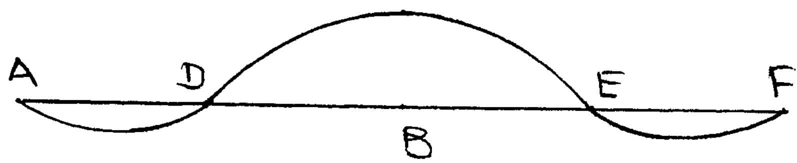
15.13



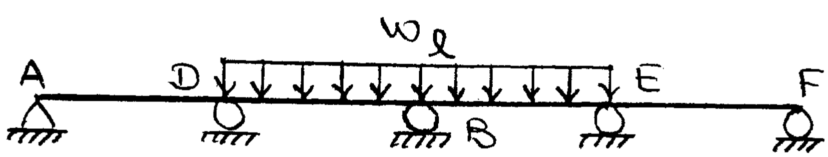
A_y



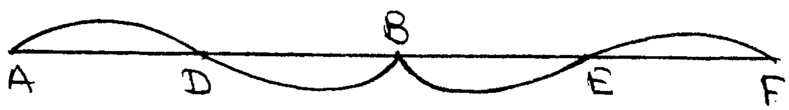
Live Load Arrangement for Maximum Positive A_y



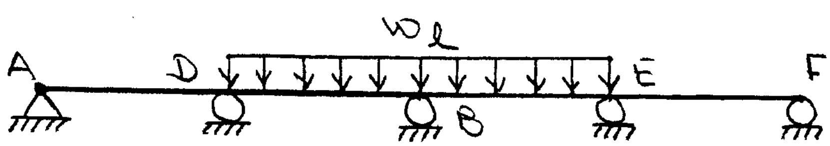
B_y



Live Load Arrangement for Maximum Positive B_y.

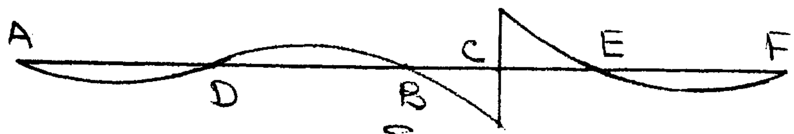


M_B

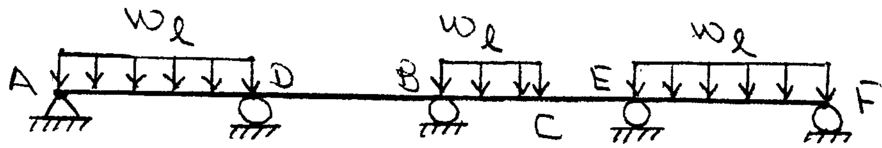


Live Load Arrangement for Maximum Negative M_B

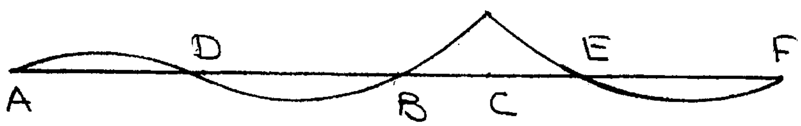
15.13 (Contd.)



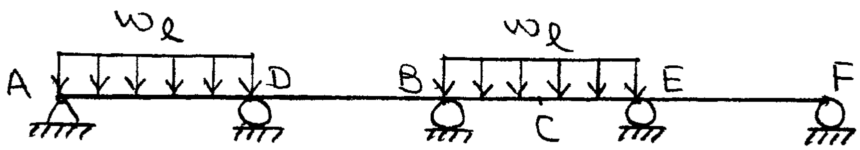
$\underline{S_C}$



Live Load Arrangement for Maximum Negative S_C

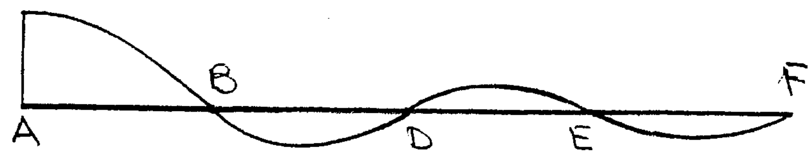


$\underline{M_C}$

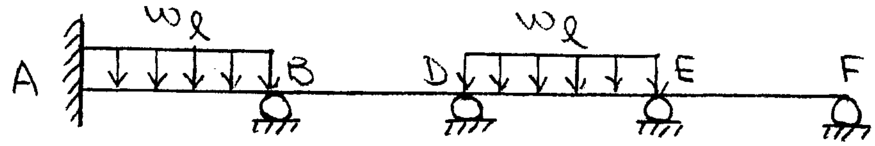


Live Load Arrangement for Maximum Positive M_C

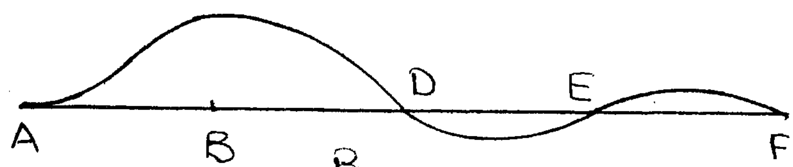
15.14



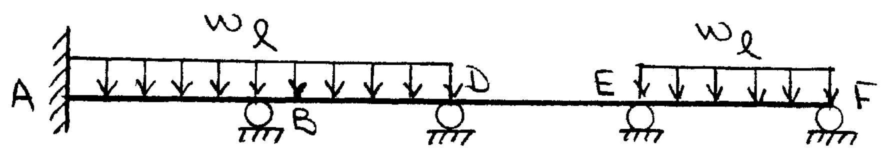
A_y



Live Load Arrangement for Maximum Positive A_y

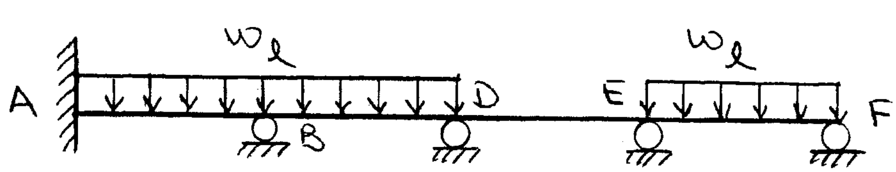


B_y



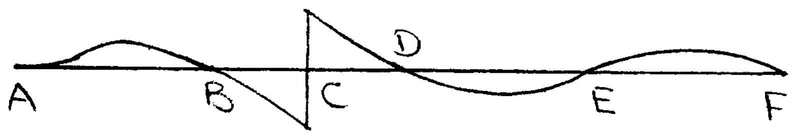
Live Load Arrangement for Maximum Positive B_y

M_B

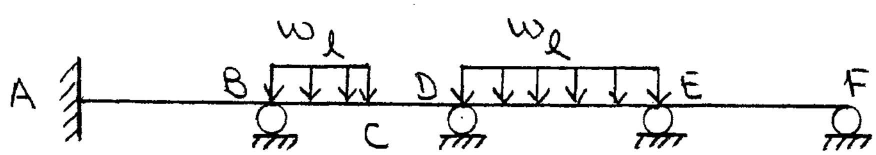


Live Load Arrangement for Maximum Negative M_B

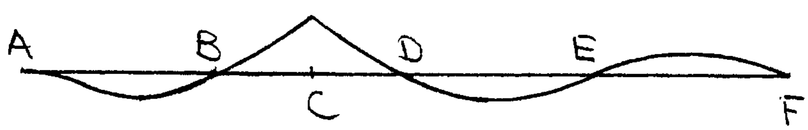
15.14 (Contd.)



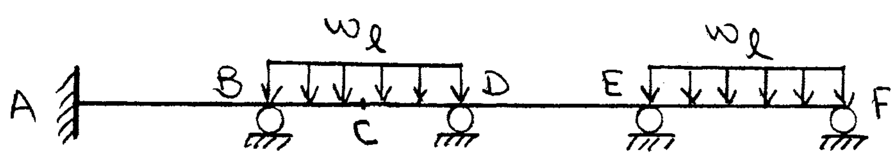
$\underline{S_C}$



Live Load Arrangement for Maximum Negative S_C



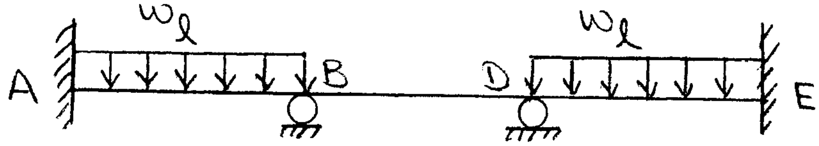
$\underline{M_C}$



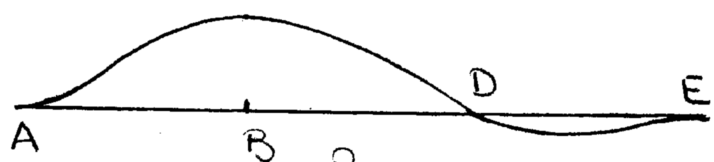
Live Load Arrangement for Maximum Positive M_C

15.15

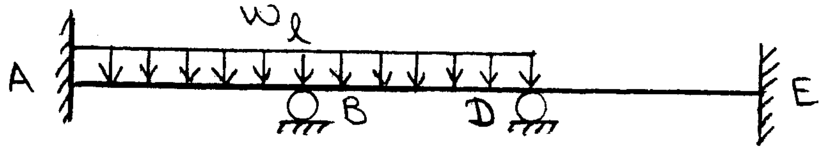
$\underline{A_y}$



Live Load Arrangement for Maximum Positive A_y

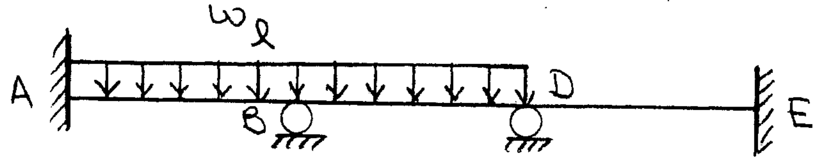


$\underline{B_y}$



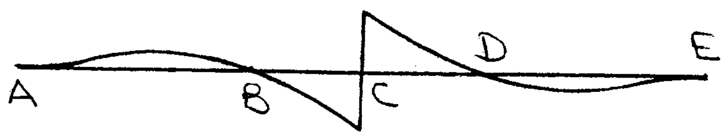
Live Load Arrangement for Maximum Positive B_y

$\underline{M_B}$

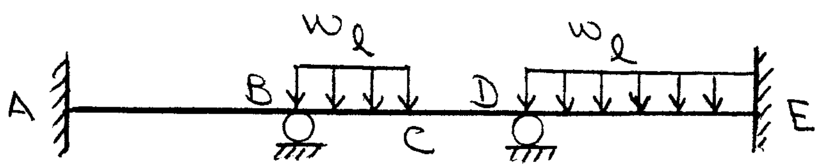


Live Load Arrangement for Maximum Negative M_B

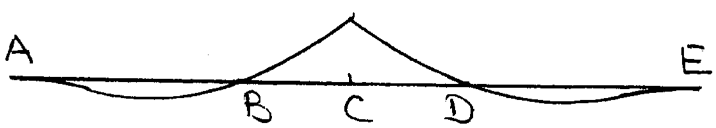
15.15 (Contd.)



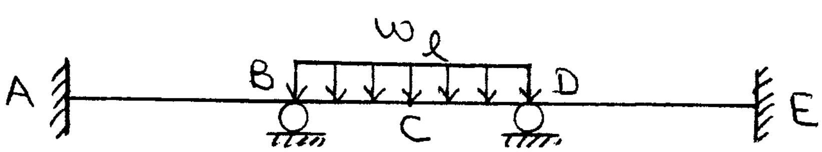
S_C



Live Load Arrangement for Maximum Negative S_C

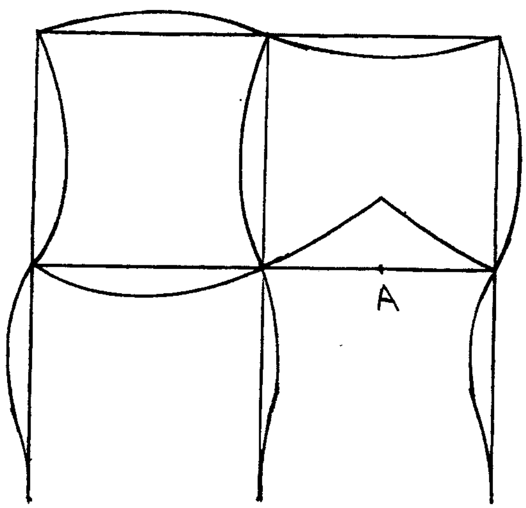


M_C

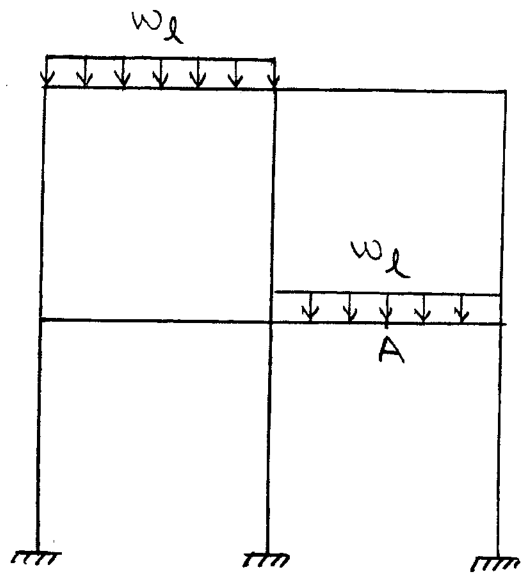


Live Load Arrangement for Maximum Positive M_C

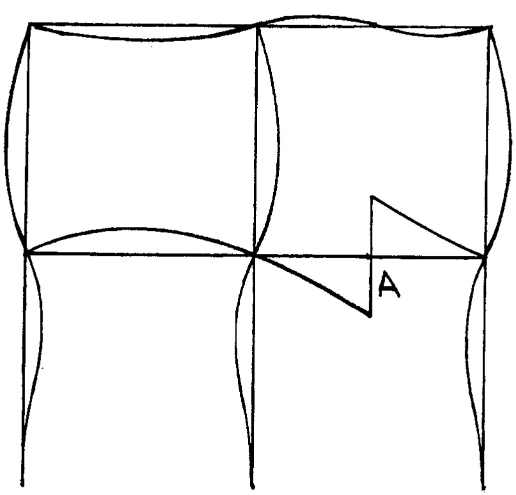
15.16



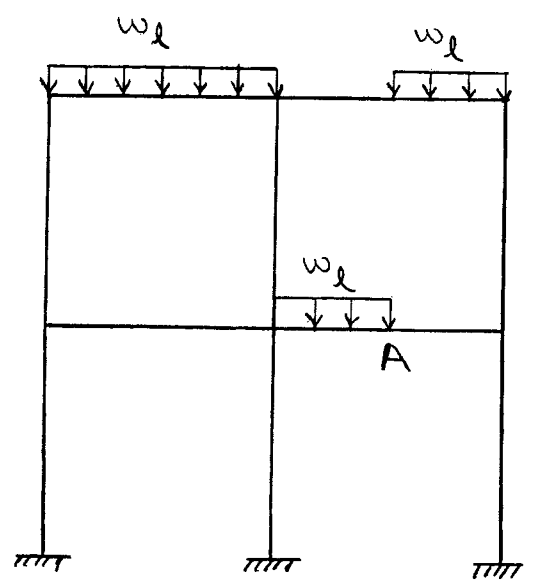
M_A



Live Load Arrangement for Max. Positive M_A

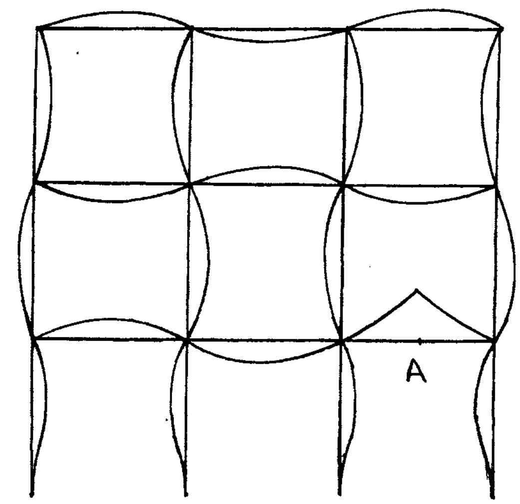


S_A

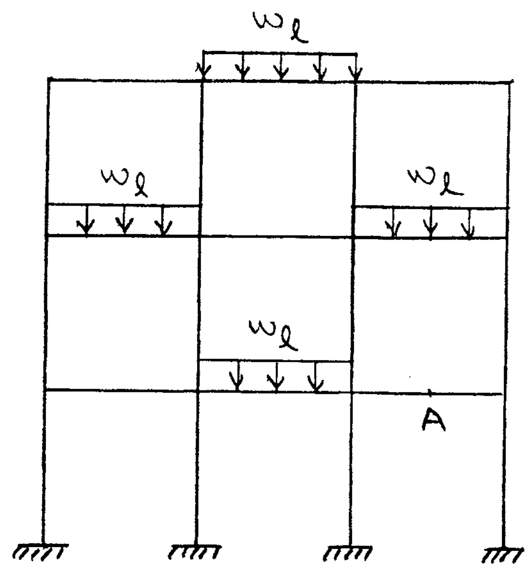


Live Load Arrangement for Max. Negative S_A

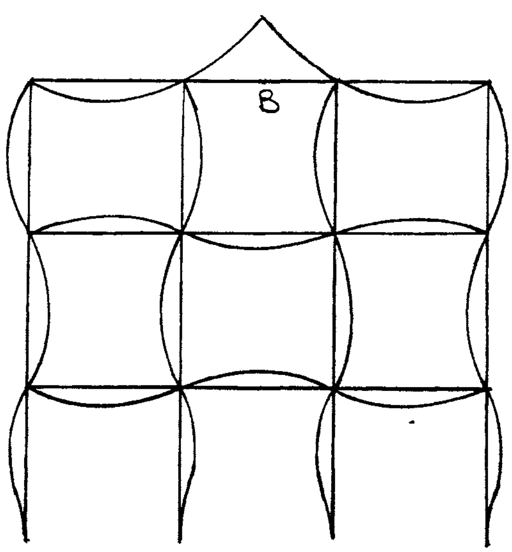
15.17



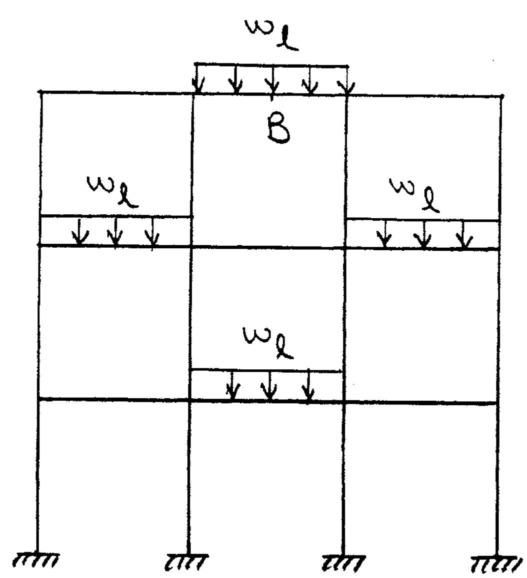
Qualitative I.L. for M_A



Live Load Arrangement for Max. Negative M_A



Qualitative I.L. for M_B



Live Load Arrangement for Max. Positive M_B

Chapter Sixteen
Slope-Deflection Method

16-1 Fixed end moments:

$FEM_{AC} = +40$ k-ft ; $FEM_{CA} = -80$ k-ft

$FEM_{CE} = +37.5$ k-ft ; $FEM_{EC} = -37.5$ k-ft

Slope-deflection equations:

$M_{AC} = 0.0667 EI\theta_C + 40$; $M_{CA} = 0.133 EI\theta_C - 80$

$M_{CE} = 0.133 EI\theta_C + 37.5$; $M_{EC} = 0.0667 EI\theta_C - 37.5$

Equilibrium equation: $M_{CA} + M_{CE} = 0$

$$(0.133 EI\theta_C - 80) + (0.133 EI\theta_C + 37.5) = 0$$

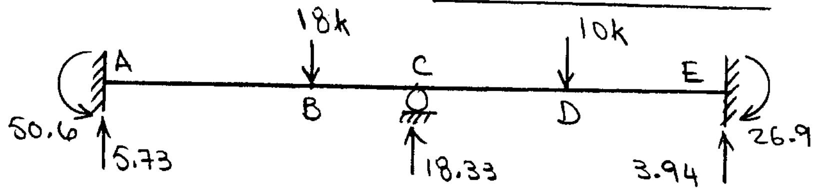
$$0.267 EI\theta_C - 42.5 = 0$$

$$EI\theta_C = 159.2 \text{ k-ft}^2$$

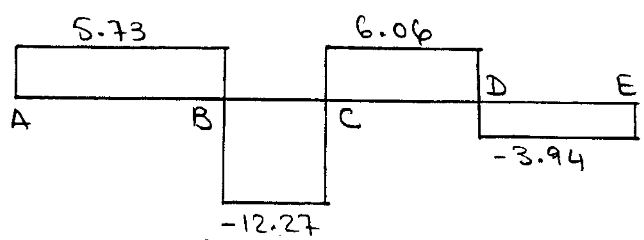
Member end moments. Substituting the numerical value of $EI\theta_C$ into the slope-deflection equations, we obtain:

$M_{AC} = 50.6$ k-ft ; $M_{CA} = -58.8$ k-ft

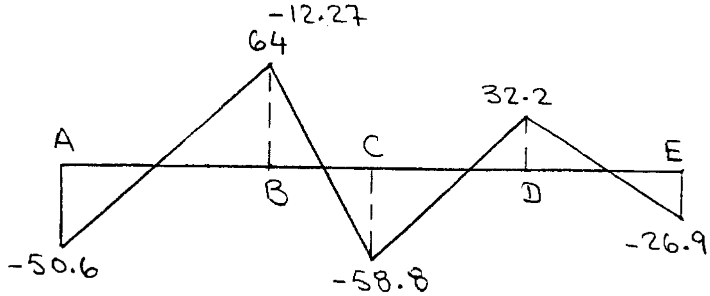
$M_{CE} = 58.8$ k-ft ; $M_{EC} = -26.9$ k-ft



Reactions



Shear Diagram (k)



Bending Moment Diagram (k-ft)

16.2 Fixed-end Moments:

$$FEM_{AB} = \frac{1.5(30)^2}{12} + \frac{20(30)}{8} = 187.5 \text{ k-ft}$$

$$FEM_{BA} = -187.5 \text{ k-ft}; \quad FEM_{BC} = \frac{3(20)^2}{12} = 100 \text{ k-ft}$$

$$FEM_{CB} = -100 \text{ k-ft}.$$

Slope-deflection equations:

$$M_{AB} = 0.0667EI\theta_B + 187.5; \quad M_{BA} = 0.133EI\theta_B - 187.5$$

$$M_{BC} = 0.2EI\theta_B + 100; \quad M_{CB} = 0.1EI\theta_B - 100$$

Equilibrium equation: $M_{BA} + M_{BC} = 0$

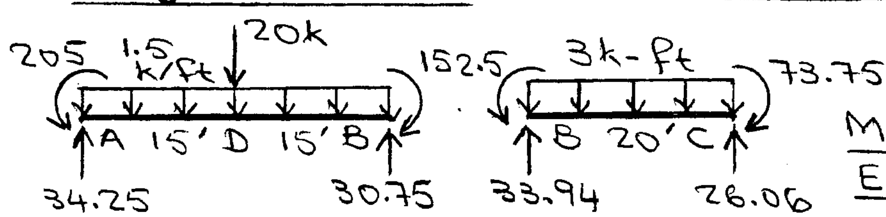
$$(0.133EI\theta_B - 187.5) + (0.2EI\theta_B + 100) = 0$$

$$EI\theta_B = 262.5 \text{ k-ft}^2$$

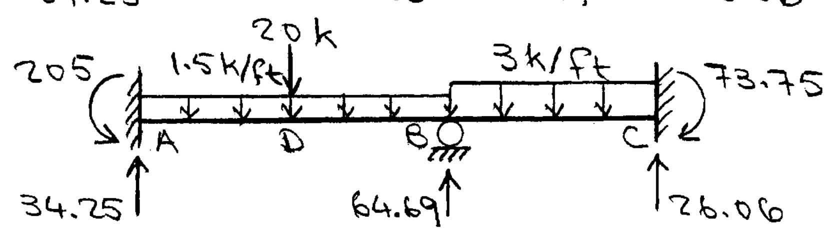
Member end moments: Substituting the numerical value of $EI\theta_B$ into the slope-deflection equations, we obtain:

$M_{AB} = 205$ k-ft; $M_{BA} = -152.5$ k-ft

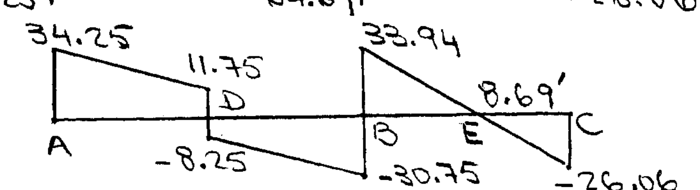
$M_{BC} = 152.5$ k-ft; $M_{CB} = -73.75$ k-ft



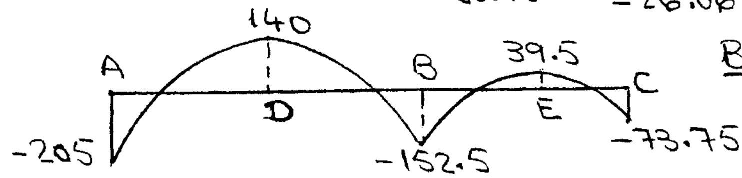
Member End Forces



Reactions



Shear Diagram (k)



Bending Moment Diagram (k-ft)

16.3 Fixed end moments:

$$FEM_{BE} = +400 \text{ kN.m}; \quad FEM_{EB} = -400 \text{ kN.m}$$

Slope-deflection equations:

$$M_{AB} = 0.222\,EI\theta_B; \quad M_{BA} = 0.444\,EI\theta_B$$

$$M_{BE} = 0.444\,EI\theta_B + 400; \quad M_{EB} = 0.222\,EI\theta_B - 400$$

Equilibrium equation: $M_{BA} + M_{BE} = 0$

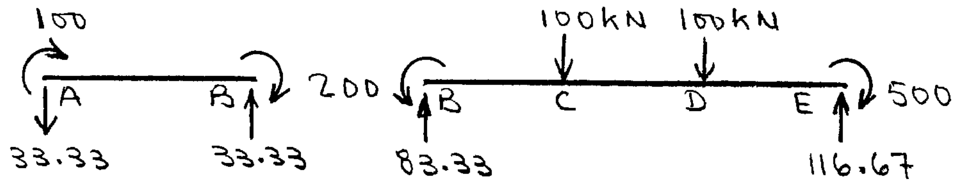
$$0.444\,EI\theta_B + (0.444\,EI\theta_B + 400) = 0$$

$$EI\theta_B = -450 \text{ kN.m}^2$$

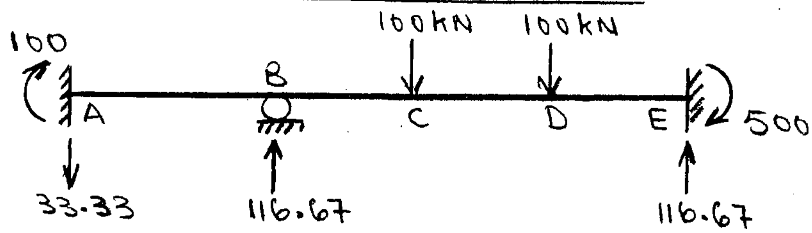
Member end moments: Substituting the numerical value of $EI\theta_C$ into the slope-deflection equations, we obtain

$$\underline{M_{AB} = -100 \text{ kN.m}}; \quad \underline{M_{BA} = -200 \text{ kN.m}}$$

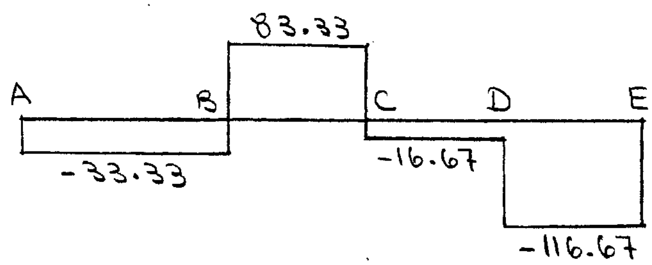
$$\underline{M_{BE} = 200 \text{ kN.m}}; \quad \underline{M_{EB} = -500 \text{ kN.m}}$$



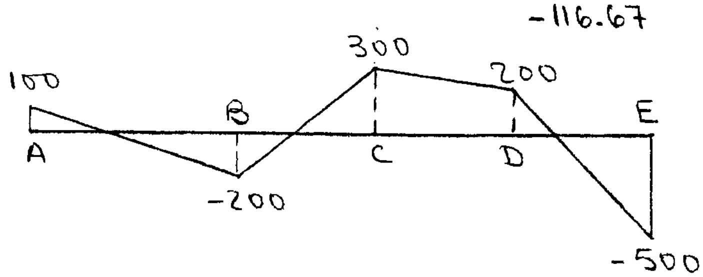
Member End Forces



Reactions



Shear Diagram (kN)



Bending Moment Diagram (kN.m)

16.4 <u>Fixed-end moments:</u>

$FEM_{AB} = \frac{25(8)^2}{12} = 133.33$ kN.m; $FEM_{BA} = -133.33$ kN.m

$FEM_{BC} = 133.33$ kN.m; $FEM_{CB} = -133.33$ kN.m

<u>Slope-deflection equations:</u> $\underline{M_{CB} = 0}$

$M_{AB} = 0.25 EI\theta_B + 133.33$; $M_{BA} = 0.5 EI\theta_B - 133.33$

$M_{BC} = 0.375 EI\theta_B + 200$

<u>Equilibrium equation:</u> $M_{BA} + M_{BC} = 0$

$$(0.5 EI\theta_B - 133.33) + (0.375 EI\theta_B + 200) = 0$$

$$EI\theta_B = -76.19 \text{ kN.m}^2$$

<u>Member end moments:</u> Substituting the numerical value of $EI\theta_B$ into the slope-deflection equations, we obtain:

$\underline{M_{AB} = 114.3 \text{ kN.m}}$; $\underline{M_{BA} = -171.4 \text{ kN.m}}$

$\underline{M_{BC} = 171.4 \text{ kN.m}}$

For reactions, and shear and bending moment diagrams, see solution of Problem 13.37.

$\boxed{16.5}$

Fixed-end moments:

$$FEM_{AB} = \frac{3(25)^2}{12} = 156.25 \text{ k-ft}; \quad FEM_{BA} = -156.25 \text{ k-ft}$$

$$FEM_{BC} = \frac{3(15)^2}{12} = 56.25 \text{ k-ft}; \quad FEM_{CB} = -56.25 \text{ k-ft}$$

Slope-deflection equations: $M_{AB} = M_{CB} = 0$

$$M_{BA} = \frac{3E(2I)}{25}\theta_B - 156.25 - \frac{156.25}{2} = 0.24\,EI\theta_B - 234.38$$

$$M_{BC} = \frac{3EI}{15}\theta_B + 56.25 + \frac{56.25}{2} = 0.2\,EI\theta_B + 84.38$$

Equilibrium equation: $M_{BA} + M_{BC} = 0$

$$(0.24EI\theta_B - 234.38) + (0.2EI\theta_B + 84.38) = 0$$

$$EI\theta_B = 340.91 \text{ k-ft}^2$$

Member end moments. Substituting the numerical value of $EI\theta_B$ into the slope-deflection equations, we obtain

$$M_{BA} = -152.6 \text{ k-ft}; \quad M_{BC} = 152.6 \text{ k-ft}$$

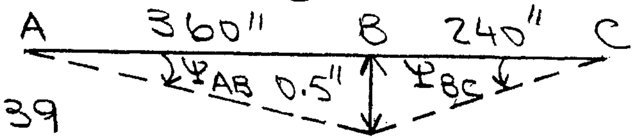
For reactions, and shear and bending moment diagrams, see solution of Problem 13.12.

16.6 Fixed-end moments:

$FEM_{AB} = 187.5$ k-ft; $FEM_{BA} = -187.5$ k-ft

$FEM_{BC} = 100$ k-ft; $FEM_{CB} = -100$ k-ft

Chord rotations:



$$\Psi_{AB} = -\frac{0.5}{360} = -0.00139$$

$$\Psi_{BC} = +\frac{0.5}{240} = +0.00208$$

Slope-deflection equations:

Using $EI = \dfrac{29000(1650)}{(12)^2}$ k-ft^2, we write

$$M_{AB} = 0.0667\,EI\theta_B + 92.3 + 187.5 = 0.0667\,EI\theta_B + 279.8$$

$$M_{BA} = 0.133\,EI\theta_B + 92.3 - 187.5 = 0.133\,EI\theta_B - 95.2$$

$$M_{BC} = 0.2\,EI\theta_B - 207.7 + 100 = 0.2\,EI\theta_B - 107.7$$

$$M_{CB} = 0.1\,EI\theta_B - 207.7 - 100 = 0.1\,EI\theta_B - 307.7$$

Equilibrium equation: $M_{BA} + M_{BC} = 0$

$$(0.133\,EI\theta_B - 95.2) + (0.2\,EI\theta_B - 107.7) = 0$$

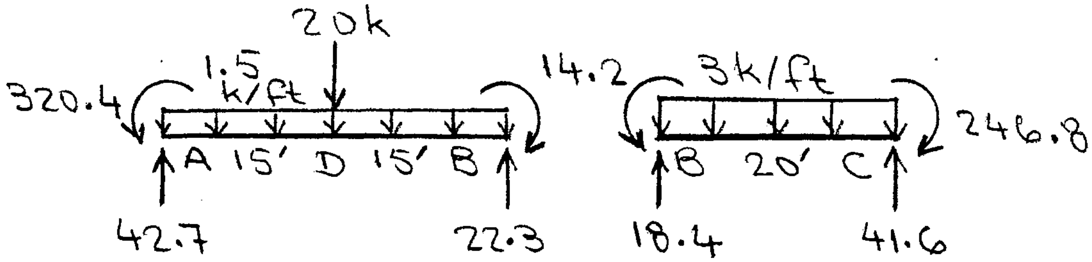
$$EI\theta_B = 609.31 \text{ k-ft}^2$$

Member end moments: Substituting the numerical value of $EI\theta_B$ into the slope-deflection equations, we obtain:

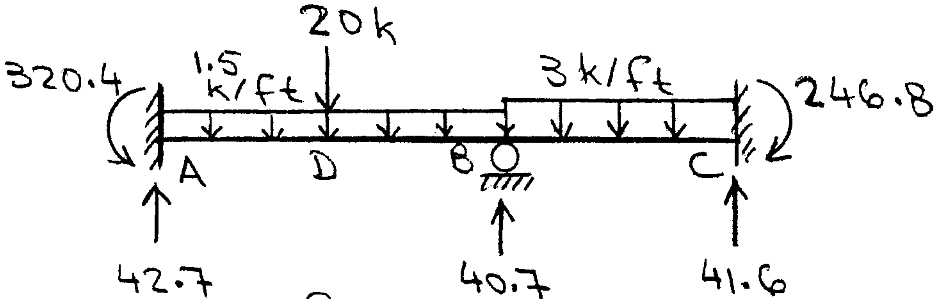
$M_{AB} = 320.4$ k-ft; $M_{BA} = -14.2$ k-ft

$M_{BC} = 14.2$ k-ft; $M_{CB} = -246.8$ k-ft

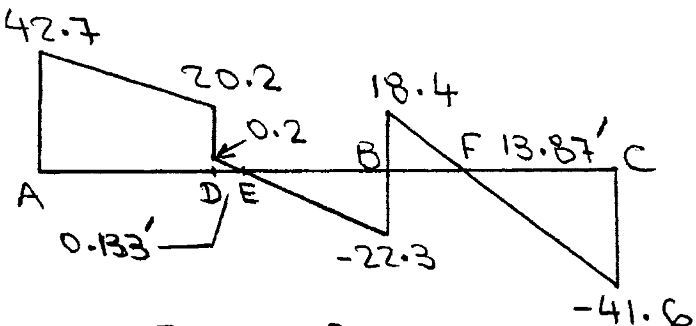
16.6 (Contd.)



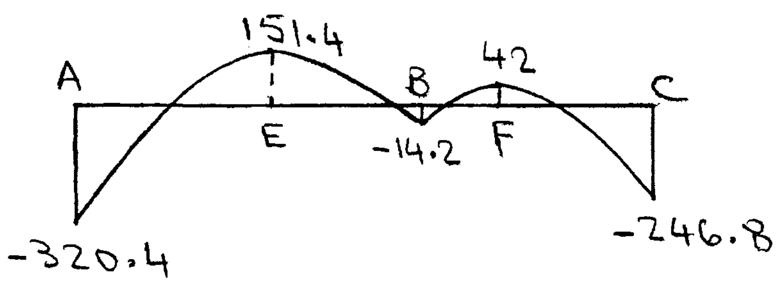
Member End Forces



Reactions



Shear Diagram (k)



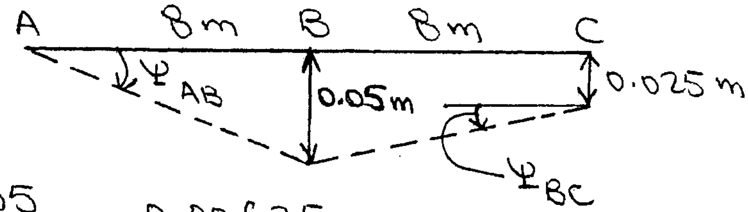
Bending Moment Diagram (k-ft)

$\boxed{16.7}$ Fixed-end moments:

$FEM_{AB} = 133.33$ kN.m ; $FEM_{BA} = -133.33$ kN.m

$FEM_{BC} = 133.33$ kN.m ; $FEM_{CB} = -133.33$ kN.m

Chord rotations:



$$\psi_{AB} = -\frac{0.05}{8} = -0.00625$$

$$\psi_{BC} = +\frac{0.025}{8} = +0.003125$$

Slope-deflection equations: $M_{CB} = 0$

Using $EI = 70(1300) = 91000$ kN.m^2, we write

$$M_{AB} = 0.25EI\theta_B + 426.56 + 133.33 = 0.25EI\theta_B + 559.89$$

$$M_{BA} = 0.5EI\theta_B + 426.56 - 133.33 = 0.5EI\theta_B + 293.23$$

$$M_{BC} = 0.375EI\theta_B - 106.64 + 133.33 + \frac{133.33}{2}$$
$$= 0.375EI\theta_B + 93.36$$

Equilibrium equation: $M_{BA} + M_{BC} = 0$

$$(0.5EI\theta_B + 293.23) + (0.375EI\theta_B + 93.36) = 0$$

$$EI\theta_B = -441.82 \text{ kN.m}^2$$

Member end moments: Substituting the numerical value of $EI\theta_B$ into the slope-deflection equations, we obtain:

$M_{AB} = 449.4$ kN.m ; $M_{BA} = 72.3$ kN.m

$M_{BC} = -72.3$ kN.m

For reactions, and shear and bending moment diagrams, see solution of Problem 13.53.

16.8 Fixed-end moments:

$$FEM_{AB} = \frac{1.5(25)^2}{12} = 78.125 \text{ k-ft}; \; FEM_{BA} = -78.125 \text{ k-ft}$$

$$FEM_{BC} = \frac{1.5(20)^2}{12} = 50 \text{ k-ft}; \; FEM_{CB} = -50 \text{ k-ft}.$$

$$FEM_{CD} = 78.125 \text{ k-ft}; \; FEM_{DC} = -78.125 \text{ k-ft}.$$

Slope-deflection equations:

$$M_{AB} = 0.08EI\theta_B + 78.125; \; M_{BA} = 0.16EI\theta_B - 78.125$$

$$M_{BC} = 0.2EI\theta_B + 0.1EI\theta_C + 50; \; M_{CB} = 0.1EI\theta_B + 0.2EI\theta_C - 50$$

$$M_{CD} = 0.16EI\theta_C + 78.125; \; M_{DC} = 0.08EI\theta_C - 78.125$$

Equilibrium equations: $M_{BA} + M_{BC} = 0$

$M_{CB} + M_{CD} = 0$

$$0.36EI\theta_B + 0.1EI\theta_C = 28.125$$

$$0.1EI\theta_B + 0.36EI\theta_C = -28.125$$

By solving these equations, we obtain:

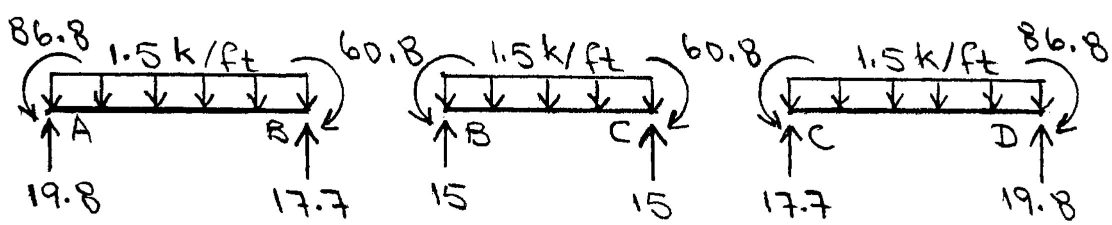
$$EI\theta_B = 108.173 \text{ k-ft}^2; \; EI\theta_C = -108.173 \text{ k-ft}^2$$

Member end moments: Substituting the numerical values of $EI\theta_B$ and $EI\theta_C$ into the slope-deflection equations, we obtain:

$M_{AB} = 86.8$ k-ft; $M_{BA} = -60.8$ k-ft

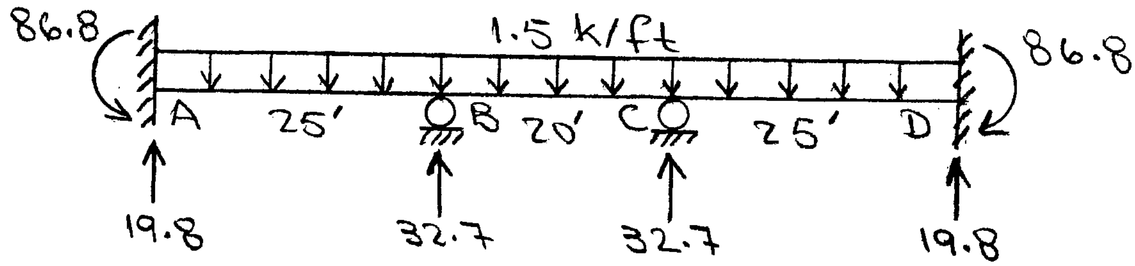
$M_{BC} = 60.8$ k-ft; $M_{CB} = -60.8$ k-ft

$M_{CD} = 60.8$ k-ft; $M_{DC} = -86.8$ k-ft

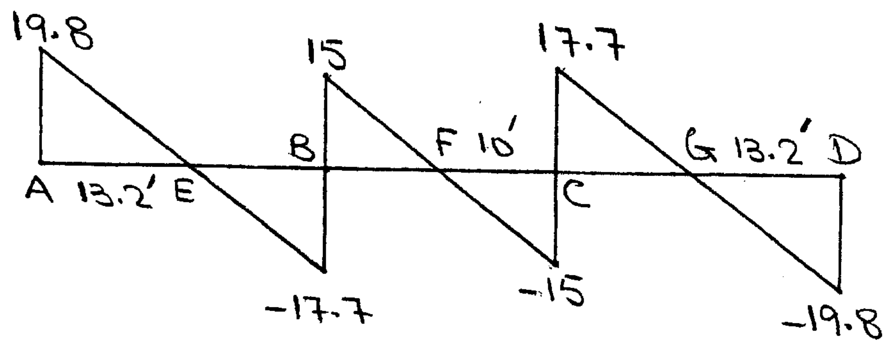


Member End Moments and Shears

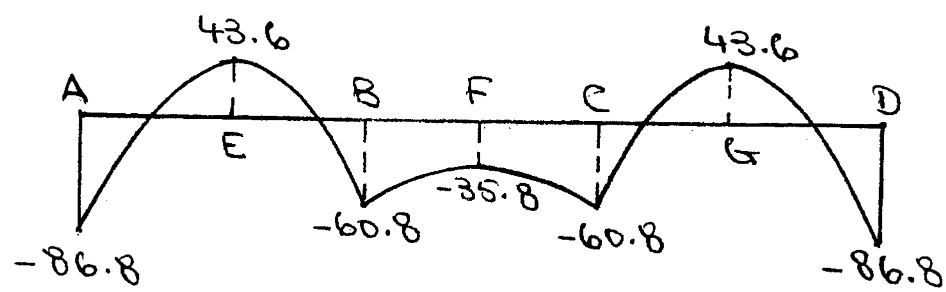
16.8 (Contd.)



Reactions



Shear Diagram (k)



Bending Moment Diagram (k-ft)

16.9 Fixed end moments:

$$FEM_{AB} = \frac{20(8)^2}{12} = 106.7 \text{ kN.m}; \quad FEM_{BA} = -106.7 \text{ kN.m}$$

$$FEM_{BC} = 106.7 \text{ kN.m}; \quad FEM_{CB} = -106.7 \text{ kN.m}$$

$$FEM_{CE} = \frac{60(8)}{8} = 60 \text{ kN.m}; \quad FEM_{EC} = -60 \text{ kN.m}$$

Slope-deflection equations:

$$M_{AB} = 0.25EI\theta_B + 106.7; \quad M_{BA} = 0.5EI\theta_B - 106.7$$

$$M_{BC} = 0.5EI\theta_B + 0.25EI\theta_C + 106.7$$

$$M_{CB} = 0.25EI\theta_B + 0.5EI\theta_C - 106.7$$

$$M_{CE} = 0.5EI\theta_C + 60; \quad M_{EC} = 0.25EI\theta_C - 60$$

Equilibrium equations: $M_{BA} + M_{BC} = 0$

$$M_{CB} + M_{CE} = 0$$

$$EI\theta_B + 0.25EI\theta_C = 0$$

$$0.25EI\theta_B + EI\theta_C = 46.7$$

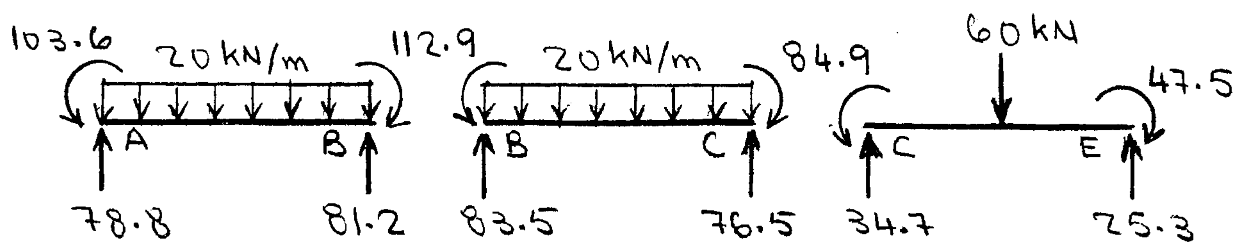
By solving these equations, we obtain

$$EI\theta_B = -12.45 \text{ kN.m}^2; \quad EI\theta_C = 49.81 \text{ kN.m}^2$$

Member end moments: Substituting the numerical values of $EI\theta_B$ and $EI\theta_C$ into the slope-deflection equations, we obtain

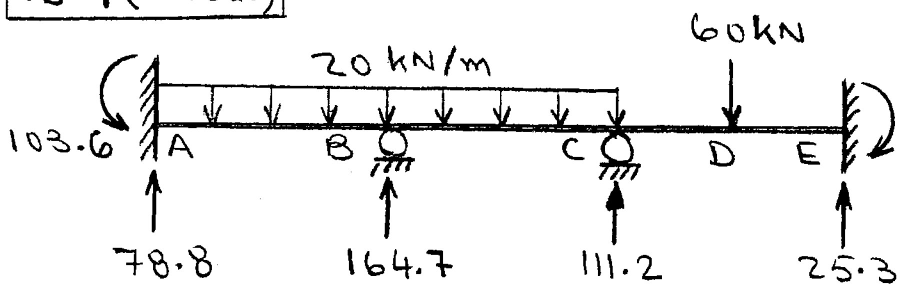
$M_{AB} = 103.6$ kN.m; $M_{BA} = -112.9$ kN.m; $M_{BC} = 112.9$ kN.m

$M_{CB} = -84.9$ kN.m; $M_{CE} = 84.9$ kN.m; $M_{EC} = -47.5$ kN.m

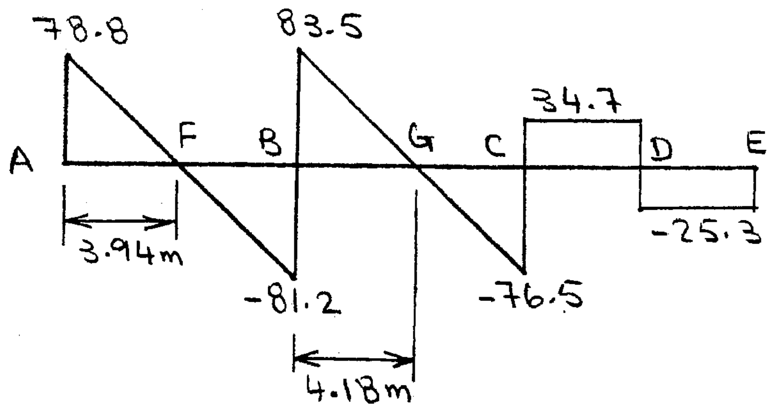


Member End Moments and Shears

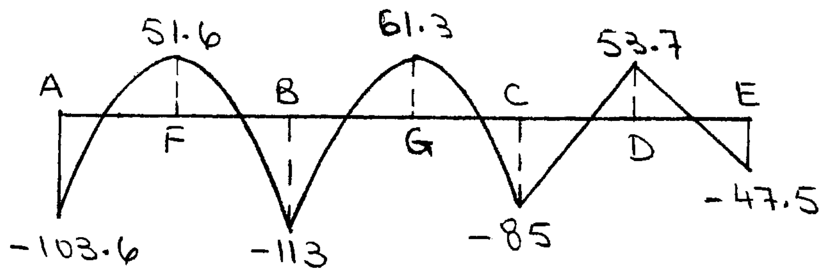
16.9 (contd.)



Reactions



Shear Diagram (kN)



Bending Moment Diagram (kN.m)

16.10 Fixed-end moments:

$FEM_{AB} = \frac{2(15)^2}{12} = 37.5$ k-ft; $FEM_{BA} = -37.5$ k-ft

$FEM_{BC} = \frac{3(10)^2}{30} = 10$ k-ft; $FEM_{CB} = -\frac{3(10)^2}{20} = -15$ k-ft

$FEM_{CD} = 15$ k-ft; $FEM_{DC} = -10$ k-ft

Slope-deflection equations:

$M_{AB} = 0.133 EI\theta_B + 37.5$; $M_{BA} = 0.267 EI\theta_B - 37.5$

$M_{BC} = 0.4 EI\theta_B + 0.2 EI\theta_C + 10$

$M_{CB} = 0.2 EI\theta_B + 0.4 EI\theta_C - 15$

$M_{CD} = 0.4 EI\theta_C + 15$; $M_{DC} = 0.2 EI\theta_C - 10$

Equilibrium equations: $M_{BA} + M_{BC} = 0$

$M_{CB} + M_{CD} = 0$

$$0.667 EI\theta_B + 0.2 EI\theta_C = 27.5$$
$$0.2 EI\theta_B + 0.8 EI\theta_C = 0$$

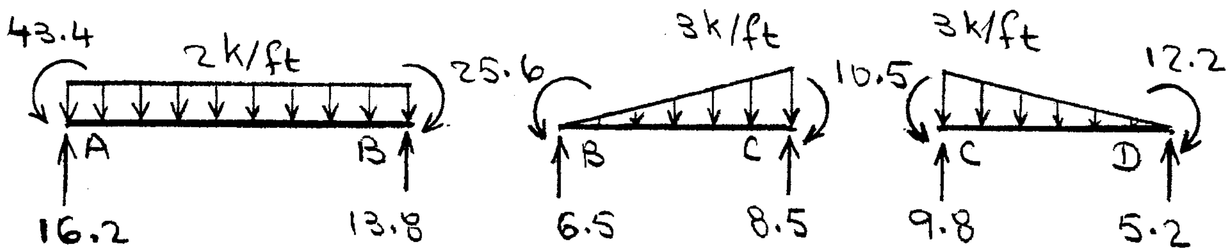
By solving these equations, we obtain

$$EI\theta_B = 44.59 \text{ k-ft}^2; \quad EI\theta_C = -11.15 \text{ k-ft}^2$$

Member end moments: Substituting the numerical values of $EI\theta_B$ and $EI\theta_C$ into the slope-deflection equations, we obtain

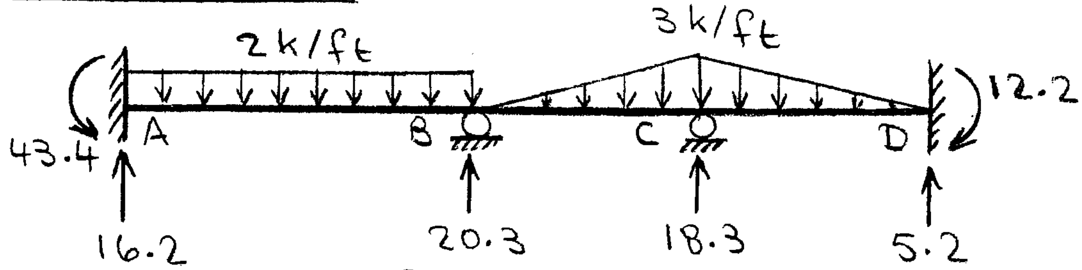
$M_{AB} = 43.4$ k-ft; $M_{BA} = -25.6$ k-ft; $M_{BC} = 25.6$ k-ft

$M_{CB} = -10.5$ k-ft; $M_{CD} = 10.5$ k-ft; $M_{DC} = -12.2$ k-ft

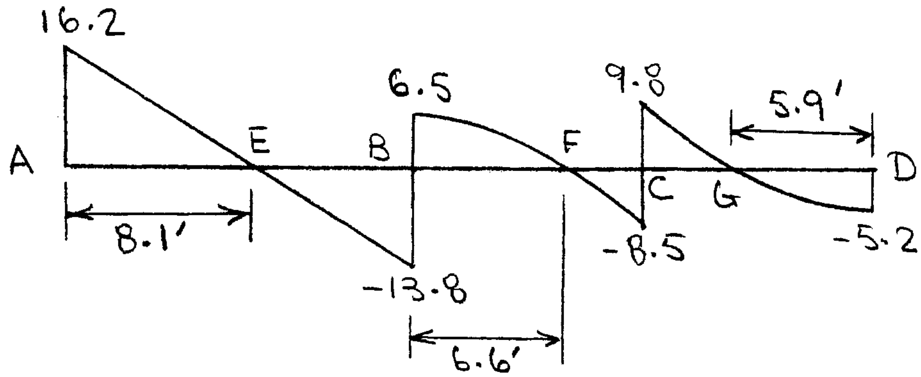


Member End Moments and Shears

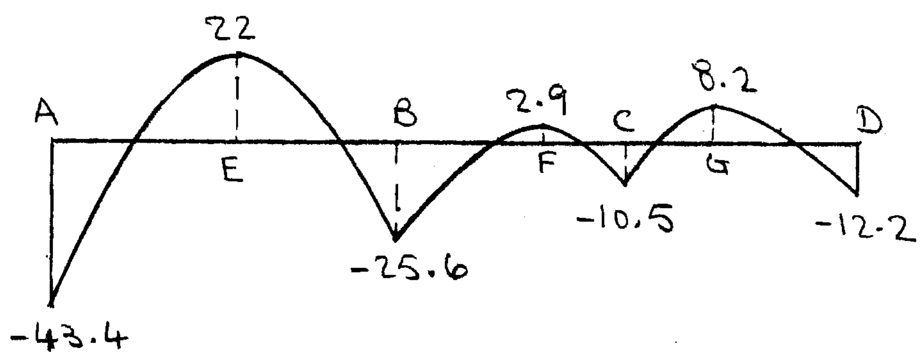
16.10 (contd.)



Reactions

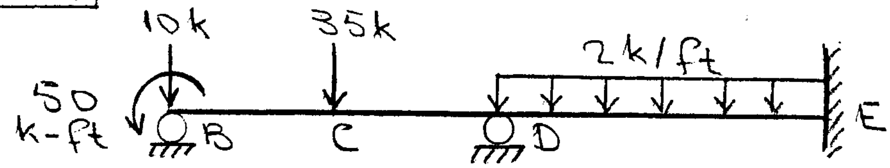


Shear Diagram (k)



Bending Moment Diagram (k-ft)

16.11



Fixed-end moments:

$$FEM_{BD} = \frac{35(20)}{8} = 87.5 \text{ k-ft}; \quad FEM_{DB} = -87.5 \text{ k-ft}$$

$$FEM_{DE} = \frac{2(20)^2}{12} = 66.67 \text{ k-ft}; \quad FEM_{ED} = -66.67 \text{ k-ft}$$

Slope-deflection equations:

$$M_{BD} = 0.2EI\theta_B + 0.1EI\theta_D + 87.5$$

$$M_{DB} = 0.1EI\theta_B + 0.2EI\theta_D - 87.5$$

$$M_{DE} = 0.2EI\theta_D + 66.67; \quad M_{ED} = 0.1EI\theta_D - 66.67$$

Equilibrium equations: $M_{BD} - 50 = 0$

$$M_{DB} + M_{DE} = 0$$

$$0.2EI\theta_B + 0.1EI\theta_D = -37.5$$

$$0.1EI\theta_B + 0.4EI\theta_D = 20.83$$

By solving these equations, we obtain:

$$EI\theta_B = -244.05 \text{ k-ft}^2; \quad EI\theta_D = 113.1 \text{ k-ft}^2$$

Member end moments: Substituting the numerical values of $EI\theta_B$ and $EI\theta_D$ into the slope-deflection equations, we obtain:

$M_{BD} = 50$ k-ft; $M_{DB} = -89.3$ k-ft

$M_{DE} = 89.3$ k-ft; $M_{ED} = -55.4$ k-ft

For reactions, and shear and bending moment diagrams, see solution of Problem 13.38.

$\boxed{16.12}$ Fixed-end moments:

$$FEM_{AC} = FEM_{CE} = \frac{120(6)(4)^2}{(10)^2} = 115.2 \text{ kN.m};$$

$$FEM_{CA} = FEM_{EC} = -\frac{120(4)(6)^2}{(10)^2} = -172.8 \text{ kN.m}$$

$$FEM_{EG} = \frac{150(8)}{8} = 150 \text{ kN.m}; \quad FEM_{GE} = -150 \text{ kN.m}$$

Slope-deflection equations: $M_{AC} = M_{GE} = 0$

$$M_{CA} = \frac{3EI}{10}\theta_C - 172.8 - \frac{115.2}{2} = 0.3EI\theta_C - 230.4$$

$$M_{CE} = \frac{2E(2I)}{10}(2\theta_C + \theta_E) + 115.2 = 0.4EI(2\theta_C + \theta_E) + 115.2$$

$$M_{EC} = 0.4EI(\theta_C + 2\theta_E) - 172.8$$

$$M_{EG} = \frac{3EI}{8}\theta_E + 150 + \frac{150}{2} = 0.375EI\theta_E + 225$$

Equilibrium equations: $M_{CA} + M_{CE} = 0$

$M_{EC} + M_{EG} = 0$

$$1.1EI\theta_C + 0.4EI\theta_E = 115.2$$

$$0.4EI\theta_C + 1.175EI\theta_E = -52.2$$

By solving these equations, we obtain

$$EI\theta_C = 137.96 \text{ kN.m}^2; \quad EI\theta_E = -91.39 \text{ kN.m}^2$$

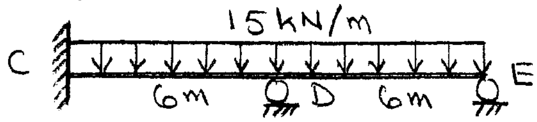
Member end moments. Substituting the numerical values of $EI\theta_C$ and $EI\theta_E$ into the slope-deflection equations, we obtain

$M_{CA} = -189$ kN.m ; $M_{CE} = 189$ kN.m

$M_{EC} = -190.7$ kN.m; $M_{EG} = 190.7$ kN.m

For reactions, and shear and bending moment diagrams, see solution of Problem 13.39.

16.13 As the beam and the loading are symmetric, we will analyze only the right half CE of the beam.



Fixed-end moments: $FEM_{CD} = 45$ kN.m; $FEM_{DC} = -45$ kN.m

$FEM_{DE} = 45$ kN.m; $FEM_{ED} = -45$ kN.m

Slope-deflection equations: $M_{ED} = 0$

$M_{CD} = 0.333\,EI\theta_D + 45$; $M_{DC} = 0.667\,EI\theta_D - 45$

$M_{DE} = \frac{3EI}{6}(\theta_D) + 45 + \frac{45}{2} = 0.5\,EI\theta_D + 67.5$

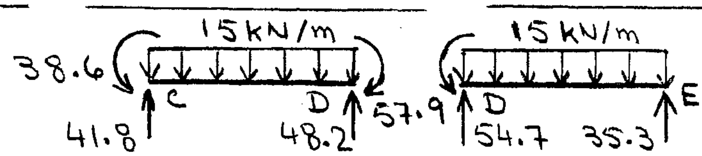
Equilibrium equation: $M_{DC} + M_{DE} = 0$

$1.167\,EI\theta_D + 22.5 = 0$; $EI\theta_D = -19.28$ kN.m^2

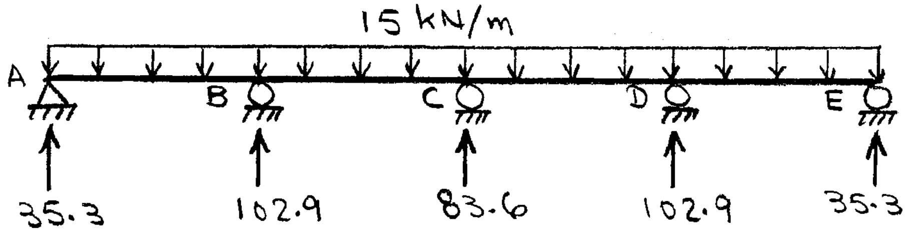
Member end moments: Substituting the numerical value of $EI\theta_D$ into the slope-deflection equations, we obtain

$M_{CD} = 38.6$ kN.m; $M_{DC} = -57.9$ kN.m; $M_{DE} = 57.9$ kN.m

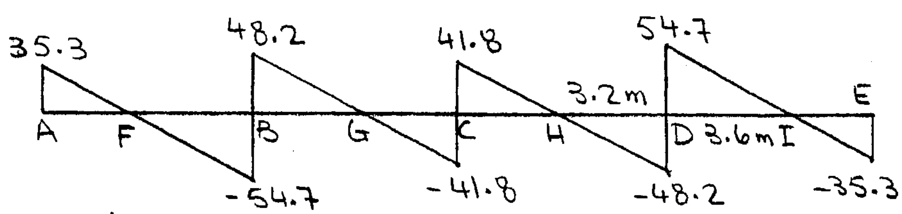
Member End Moments and Shears



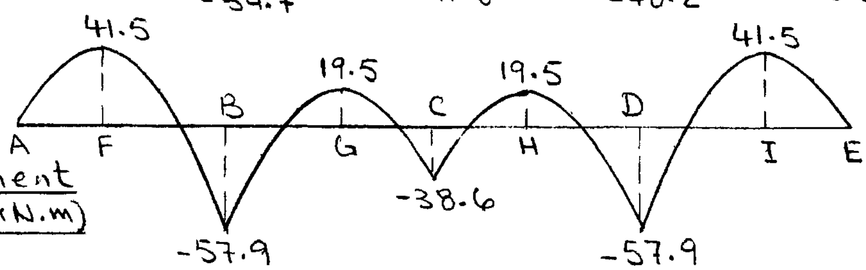
Reactions



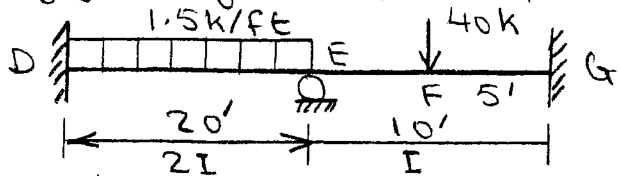
Shear Diagram (kN)



Bending Moment Diagram (kN.m)



$\boxed{16.14}$ As the beam and loading are symmetric, we will analyze only the right half DG of the beam.



Fixed end moments: $FEM_{DE} = 50$ k-ft; $FEM_{ED} = -50$ k-ft

$FEM_{EG} = 100$ k-ft; $FEM_{GE} = -100$ k-ft

Slope deflection equations:

$M_{DE} = 0.2EI\theta_E + 50$; $M_{ED} = 0.4EI\theta_E - 50$

$M_{EG} = 0.2EI\theta_E + 100$; $M_{GE} = 0.1EI\theta_E - 100$

Equilibrium equation: $M_{ED} + M_{EG} = 0$

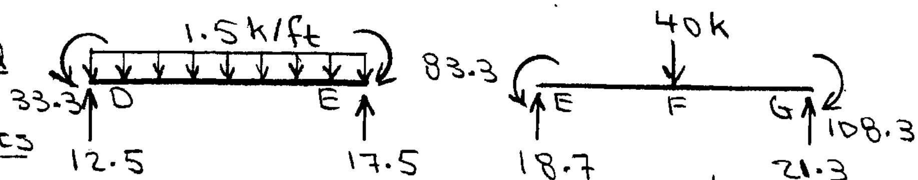
$0.6EI\theta_E + 50 = 0$ $EI\theta_E = -83.33$ k-ft²

Member end moments: Substituting the numerical value of $EI\theta_E$ into the slope-deflection equations, we obtain

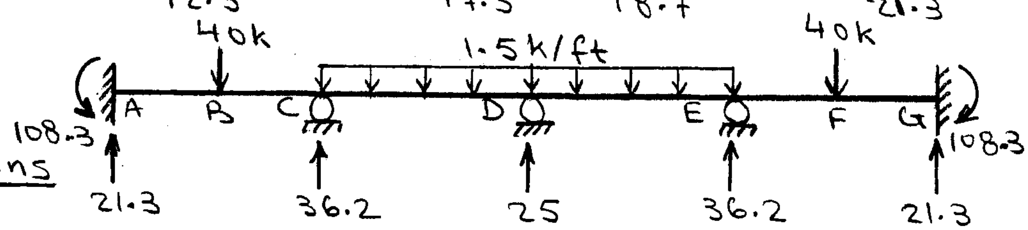
$M_{DE} = 33.3$ k-ft; $M_{ED} = -83.3$ k-ft

$M_{EG} = 83.3$ k-ft; $M_{GE} = -108.3$ k-ft

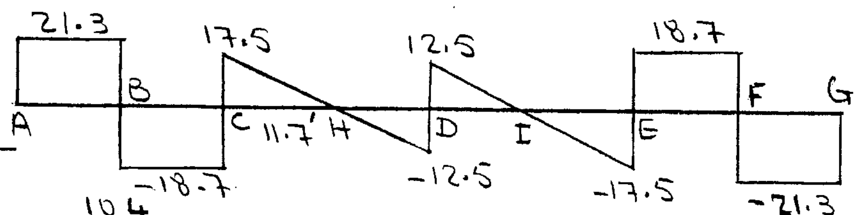
Member End Shears and Moments



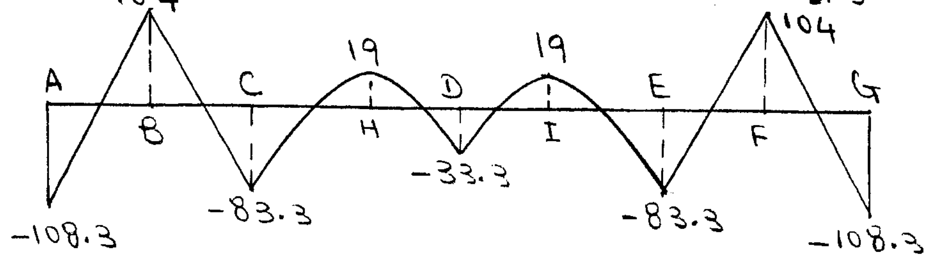
Reactions



Shear Diagram (k)



Bending Moment Diagram (k-ft)



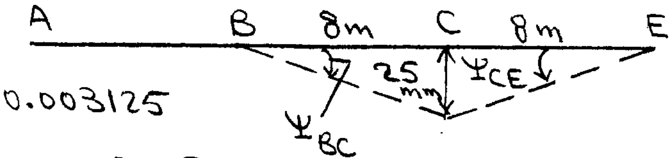
16.15 Fixed end moments:

$FEM_{AB} = 106.7$ kN.m; $FEM_{BA} = -106.7$ kN.m

$FEM_{BC} = 106.7$ kN.m; $FEM_{CB} = -106.7$ kN.m

$FEM_{CE} = 60$ kN.m; $FEM_{EC} = -60$ kN.m

Chord rotations:



$$\Psi_{BC} = -\frac{0.025}{8} = -0.003125$$

$$\Psi_{CE} = +\frac{0.025}{8} = +0.003125$$

Slope-deflection equations:

$M_{AB} = 0.25EI\theta_B + 106.7$; $M_{BA} = 0.5EI\theta_B - 106.7$

$M_{BC} = 0.5EI\theta_B + 0.25EI\theta_C + 238$

$M_{CB} = 0.25EI\theta_B + 0.5EI\theta_C + 24.6$

$M_{CE} = 0.5EI\theta_C - 71.3$; $M_{EC} = 0.25EI\theta_C - 191.3$

Equilibrium equations: $M_{BA} + M_{BC} = 0$

$M_{CB} + M_{CE} = 0$

$$EI\theta_B + 0.25EI\theta_C = -131.3$$

$$0.25EI\theta_B + EI\theta_C = 46.7$$

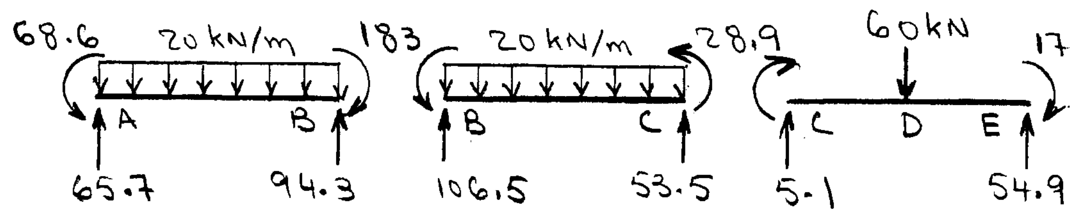
By solving these equations, we obtain

$$EI\theta_B = -152.51 \text{ kN.m}^2; \quad EI\theta_C = 84.83 \text{ kN.m}^2$$

Member end moments: Substituting the numerical values of $EI\theta_B$ and $EI\theta_C$ into the slope-deflection equations, we obtain

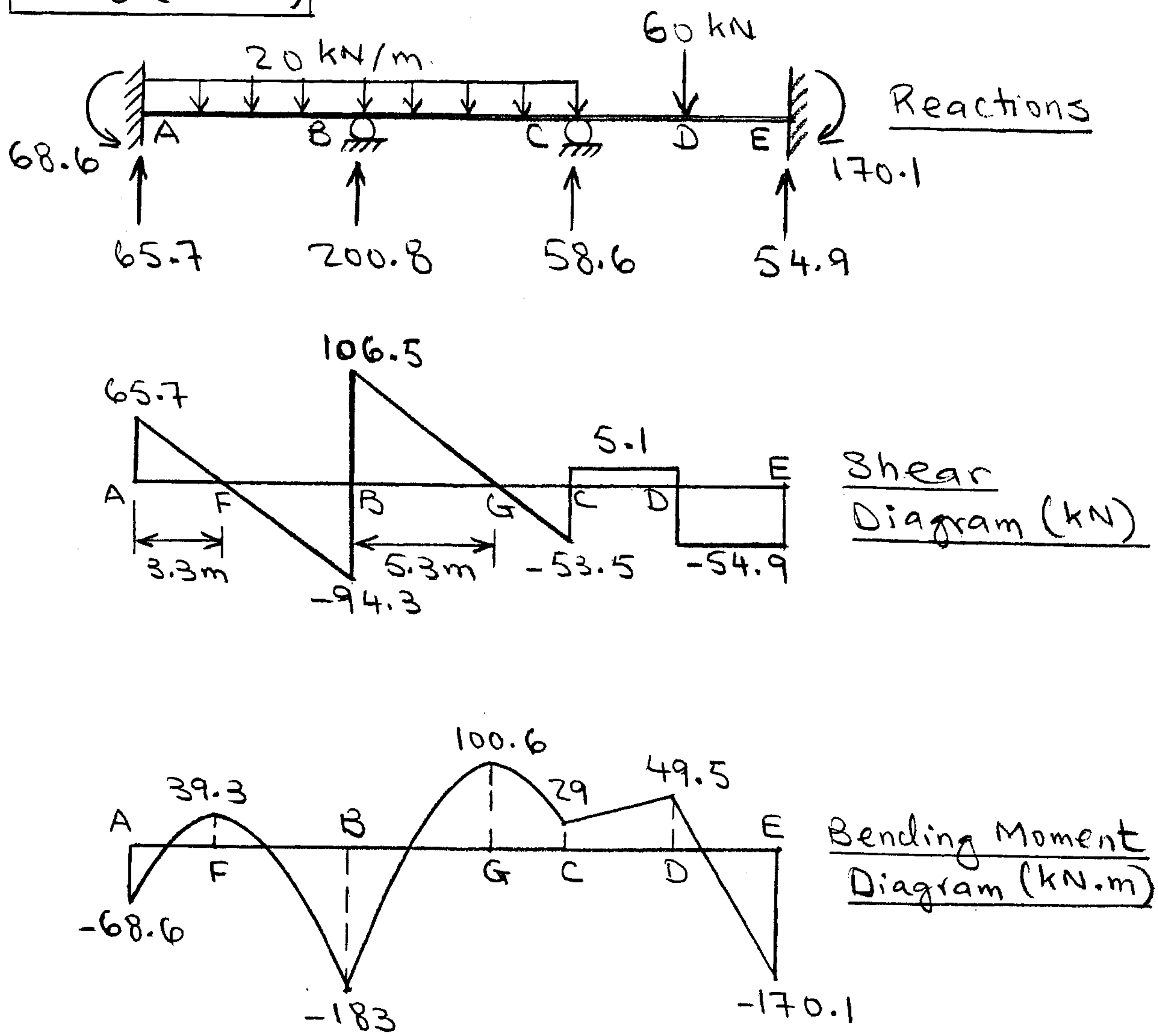
$M_{AB} = 68.6$ kN.m; $M_{BA} = -183$ kN.m; $M_{BC} = 183$ kN.m

$M_{CB} = 28.9$ kN.m; $M_{CE} = -28.9$ kN.m; $M_{EC} = -170.1$ kN.m



Member End Moments and Shears

16.15 (contd.)



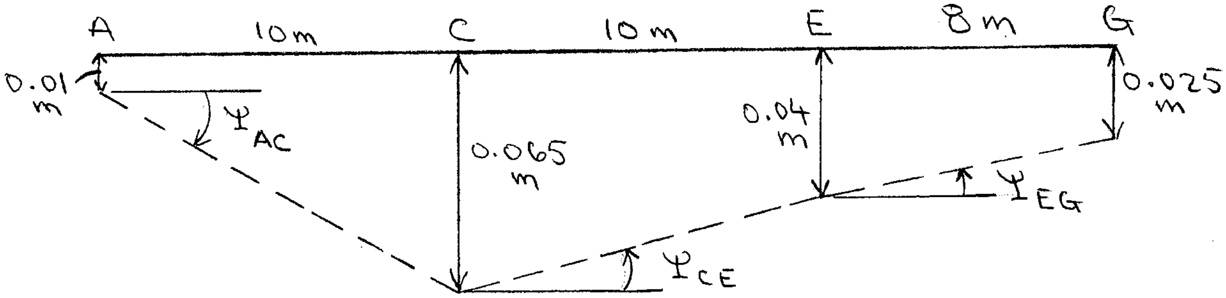
60 kN
20 kN/m
Reactions
A
B
C
D
E
68.6
170.1
65.7
200.8
58.6
54.9
106.5
65.7
5.1
E
Shear
Diagram (kN)
A
F
B
G
C
D
3.3m
5.3m
-53.5
-54.9
-94.3
100.6
49.5
39.3
29
A
B
E
Bending Moment
Diagram (kN.m)
F
G
C
D
-68.6
-183
-170.1

16.16 Fixed-end moments: $FEM_{AC} = FEM_{CE} = 115.2$ kN·m

$FEM_{CA} = FEM_{EC} = -172.8$ kN·m; $FEM_{EG} = 150$ kN·m

$FEM_{GE} = -150$ kN·m

Chord rotations:



$\Psi_{AC} = -\frac{0.055}{10} = -0.0055$; $\Psi_{CE} = \frac{0.025}{10} = 0.0025$

$\Psi_{EG} = \frac{0.015}{8} = 0.001875$

Slope-deflection equations: $M_{AC} = M_{GE} = 0$

Using $EI = 200(500) = 100000$ kN·m², we write:

$M_{CA} = 0.3EI\theta_C + 165 - 230.4 = 0.3EI\theta_C - 65.4$

$M_{CE} = 0.4EI(2\theta_C + \theta_E) - 300 + 115.2 = 0.4EI(2\theta_C + \theta_E) - 184.8$

$M_{EC} = 0.4EI(\theta_C + 2\theta_E) - 300 - 172.8 = 0.4EI(\theta_C + 2\theta_E) - 472.8$

$M_{EG} = 0.375EI\theta_E - 70.31 + 225 = 0.375EI\theta_E + 154.69$

Equilibrium equations:

$M_{CA} + M_{CE} = 0 \Rightarrow 1.1EI\theta_C + 0.4EI\theta_E = 250.2$

$M_{EC} + M_{EG} = 0 \Rightarrow 0.4EI\theta_C + 1.175EI\theta_E = 318.11$

By solving these equations, we obtain

$EI\theta_C = 147.23$ kN·m²; $EI\theta_E = 220.61$ kN·m²

Member end moments:

$M_{CA} = -21.2$ kN·m; $M_{CE} = 21.2$ kN·m

$M_{EC} = -237.4$ kN·m; $M_{EG} = 237.4$ kN·m

For reactions, and shear and bending moment diagrams, see solution of Problem 13.54.

$\boxed{16.17}$ Fixed-end moments:

$$FEM_{AC} = \frac{75(6)}{8} = 56.25 \text{ kN.m}; \; FEM_{CA} = -56.25 \text{ kN.m}$$

$$FEM_{CD} = \frac{25(9)^2}{12} = 168.75 \text{ kN.m}; \; FEM_{DC} = -168.75 \text{ kN.m}$$

Slope-deflection equations: $M_{DC} = 0$

$$M_{AC} = 0.333 EI\theta_C + 56.25; \; FEM_{CA} = 0.667 EI\theta_C - 56.25$$

$$M_{CD} = \frac{3EI}{9}(\theta_C) + 168.75 + \frac{168.75}{2} = 0.333 EI\theta_C + 253.13$$

Equilibrium equation: $M_{CA} + M_{CD} = 0$

$$EI\theta_C = -196.88 \text{ kN.m}^2$$

Member end moments. Substituting the numerical value of $EI\theta_C$ into the slope-deflection equations, we obtain:

$M_{AC} = -9.4$ kN.m; $M_{CA} = -187.5$ kN.m

$M_{CD} = 187.5$ kN.m

For reactions, see solution of Problem 13.42.

$\boxed{16.18}$ Fixed-end moments:

$$FEM_{AC} = FEM_{CA} = FEM_{CD} = FEM_{DC} = 0$$

$$FEM_{BC} = \frac{2(15)^2}{12} = 37.5\text{ k-ft}; \quad FEM_{CB} = -37.5\text{ k-ft}$$

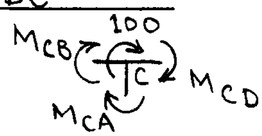
Slope-deflection equations:

$$M_{AC} = 0.133EI\theta_C; \quad M_{CA} = 0.267EI\theta_C$$

$$M_{BC} = 0.133EI\theta_C + 37.5; \quad M_{CB} = 0.267EI\theta_C - 37.5$$

$$M_{CD} = 0.2EI\theta_C; \quad M_{DC} = 0$$

Equilibrium equation:



$$M_{CA} + M_{CB} + M_{CD} + 100 = 0$$

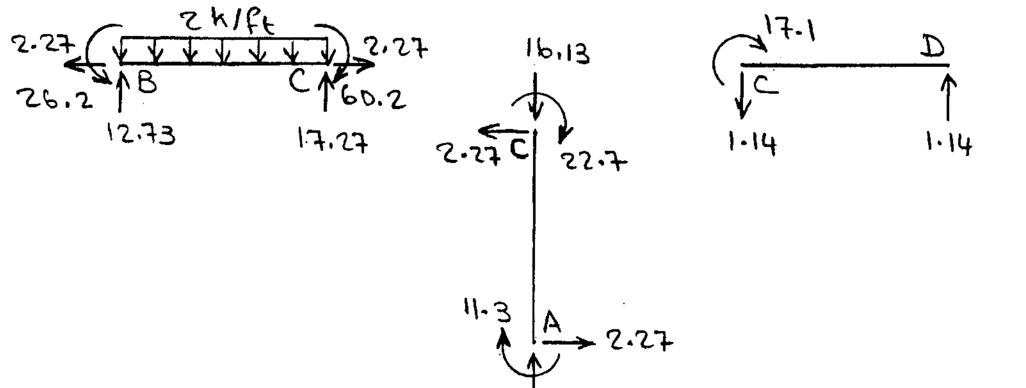
$$(0.267EI\theta_C) + (0.267EI\theta_C - 37.5) + (0.2EI\theta_C) + 100 = 0$$

$$0.734EI\theta_C = -62.5 \qquad EI\theta_C = -85.15\text{ k-ft}^2$$

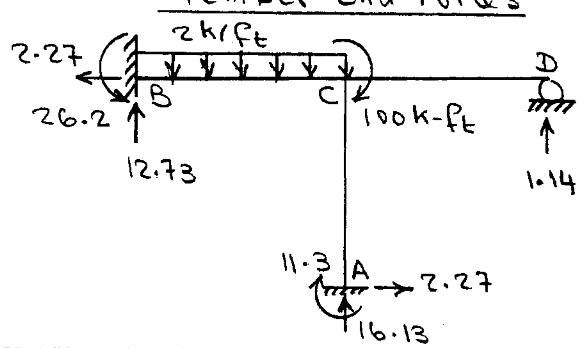
Member end moments: $M_{AC} = -11.3$ k-ft;

$M_{CA} = -22.7$ k-ft; $M_{BC} = 26.2$ k-ft; $M_{CB} = -60.2$ k-ft;

$M_{CD} = -17.1$ k-ft.

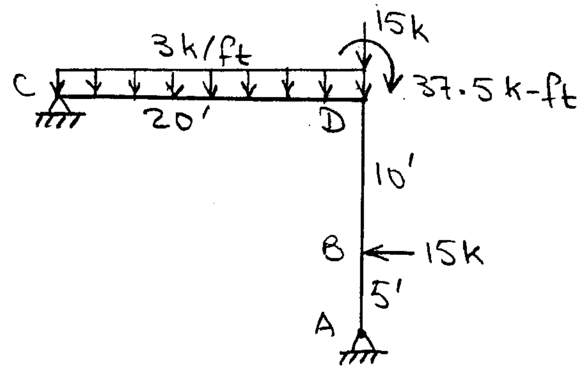


Member End Forces



Reactions

16.19



Fixed-end moments:

$$FEM_{AD} = -\frac{15(5)(10)^2}{(15)^2} = -33.33 \text{ k-ft}$$

$$FEM_{DA} = \frac{15(10)(5)^2}{(15)^2} = 16.67 \text{ k-ft}$$

$$FEM_{CD} = \frac{3(20)^2}{12} = 100 \text{ k-ft}; \quad FEM_{DC} = -100 \text{ k-ft.}$$

Slope-deflection equations: $M_{AD} = M_{CD} = 0$

$$M_{DA} = \frac{3EI}{15}(\theta_D) + 16.67 + \frac{33.33}{2} = 0.2EI\theta_D + 33.34$$

$$M_{DC} = \frac{3E(2I)}{20}(\theta_D) - 100 - \frac{100}{2} = 0.3EI\theta_D - 150$$

Equilibrium equation: $M_{DA} + M_{DC} + 37.5 = 0$

$$(0.2EI\theta_D + 33.34) + (0.3EI\theta_D - 150) + 37.5 = 0$$

$$0.5EI\theta_D = 79.16; \quad EI\theta_\theta = 158.32 \text{ k-ft}^2$$

Member end moments:

$M_{DA} = 65$ k-ft; $\quad M_{DC} = -102.5$ k-ft

For reactions, see solution of Problem 13.21.

16.20 Fixed-end moments: $FEM_{AC} = FEM_{CA} = 0$

$FEM_{BD} = FEM_{DB} = 0$

$FEM_{CD} = \frac{30(10)^2}{12} = 250$ kN.m ; $FEM_{DC} = -250$ kN.m

Slope-deflection equations:

$M_{AC} = 0.25\, EI\theta_C$; $M_{CA} = 0.5\, EI\theta_C$

$M_{BD} = 0.25\, EI\theta_D$; $M_{DB} = 0.5\, EI\theta_D$

$M_{CD} = 0.4EI\theta_C + 0.2\, EI\theta_D + 250$

$M_{DC} = 0.2EI\theta_C + 0.4\, EI\theta_D - 250$

Equilibrium equations: $M_{CA} + M_{CD} = 0$

$M_{DB} + M_{DC} = 0$

$0.9EI\theta_C + 0.2EI\theta_D = -250$

$0.2EI\theta_C + 0.9EI\theta_D = 250$

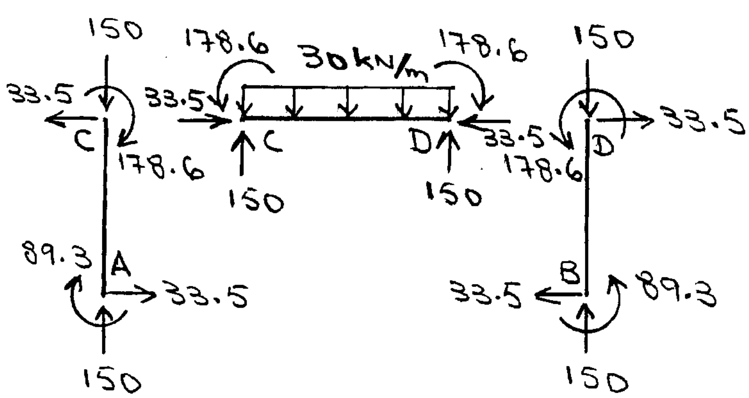
By solving these equations, we obtain:

$EI\theta_C = -357.14$ kN.m^2 ; $EI\theta_D = 357.14$ kN.m^2

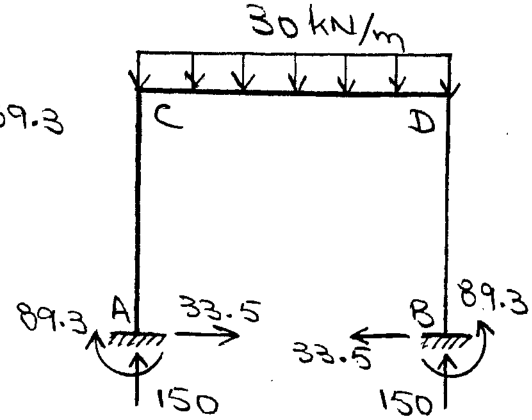
Member end moments: $M_{AC} = -89.3$ kN.m ;

$M_{CA} = -178.6$ kN.m ; $M_{BD} = 89.3$ kN.m ; $M_{DB} = 178.6$ kN.m ;

$M_{CD} = 178.6$ kN.m ; $M_{DC} = -178.6$ kN.m.



Member End Forces

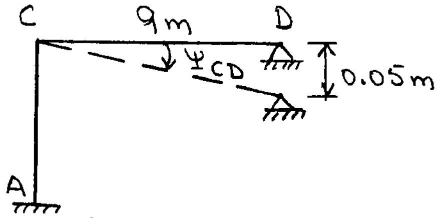


Reactions

16.21 Fixed-end moments:

$FEM_{AC} = 56.25$ kN.m ; $FEM_{CA} = -56.25$ kN.m

$FEM_{CD} = 168.75$ kN.m ; $FEM_{DC} = -168.75$ kN.m



Chord rotation:

$$\psi_{CD} = -\frac{0.05}{9} = -0.00556$$

Slope-deflection equations: $M_{DC} = 0$

$M_{AC} = 0.333\,EI\theta_C + 56.25$; $M_{CA} = 0.667\,EI\theta_C - 56.25$

Using $EI = 200(400)$ kN-m², we write

$$M_{CD} = 0.333\,EI\theta_C + 148.27 + 168.75 + \frac{168.75}{2}$$

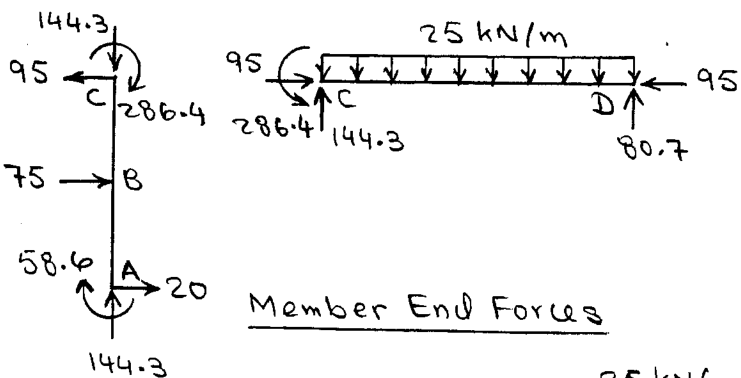
$$= 0.333\,EI\theta_C + 401.4$$

Equilibrium equation: $M_{CA} + M_{CD} = 0$

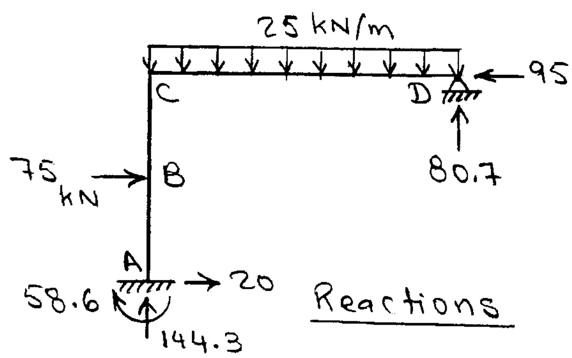
$$EI\theta_C = -345.15 \text{ kN-m}^2$$

Member end moments: $M_{AC} = -58.6$ kN.m

$M_{CA} = -286.4$ kN.m ; $M_{CD} = 286.4$ kN.m



Member End Forces



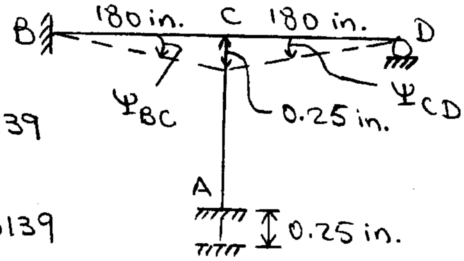
Reactions

16.22 Fixed-end moments:

$FEM_{AC} = FEM_{CA} = FEM_{CD} = FEM_{DC} = 0$

$FEM_{BC} = 37.5$ k-ft ; $FEM_{CB} = -37.5$ k-ft .

Chord rotations:



$\Psi_{BC} = -\frac{0.25}{180} = -0.00139$

$\Psi_{CD} = +\frac{0.25}{180} = +0.00139$

Slope-deflection equations. $M_{DC} = 0$

Using $EI = \frac{29000(3500)}{(12)^2}$ k-ft^2 , we write

$M_{AC} = 0.133\,EI\theta_C$; $M_{CA} = 0.267\,EI\theta_C$

$M_{BC} = 0.133\,EI\theta_C + 391.9 + 37.5 = 0.133\,EI\theta_C + 429.4$

$M_{CB} = 0.267\,EI\theta_C + 391.9 - 37.5 = 0.267\,EI\theta_C + 354.4$

$M_{CD} = 0.2\,EI\theta_C - 196$

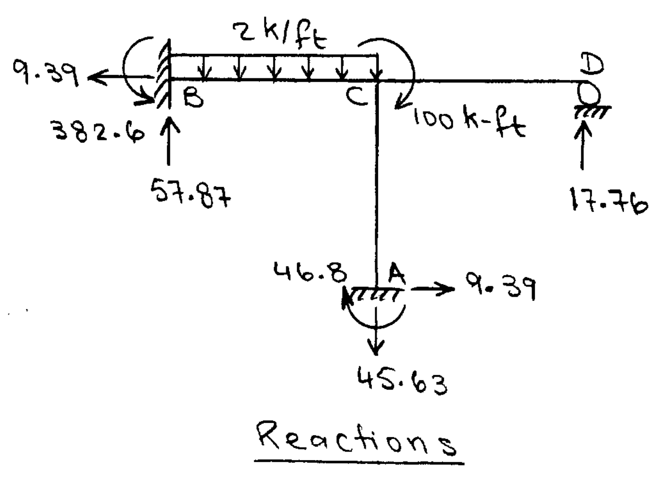
Equilibrium equation: $M_{CA} + M_{CB} + M_{CD} + 100 = 0$

$0.734\,EI\theta_C = -258.4 \qquad EI\theta_C = -352$ k-ft^2

Member end moments: $M_{AC} = -46.8$ k-ft ;

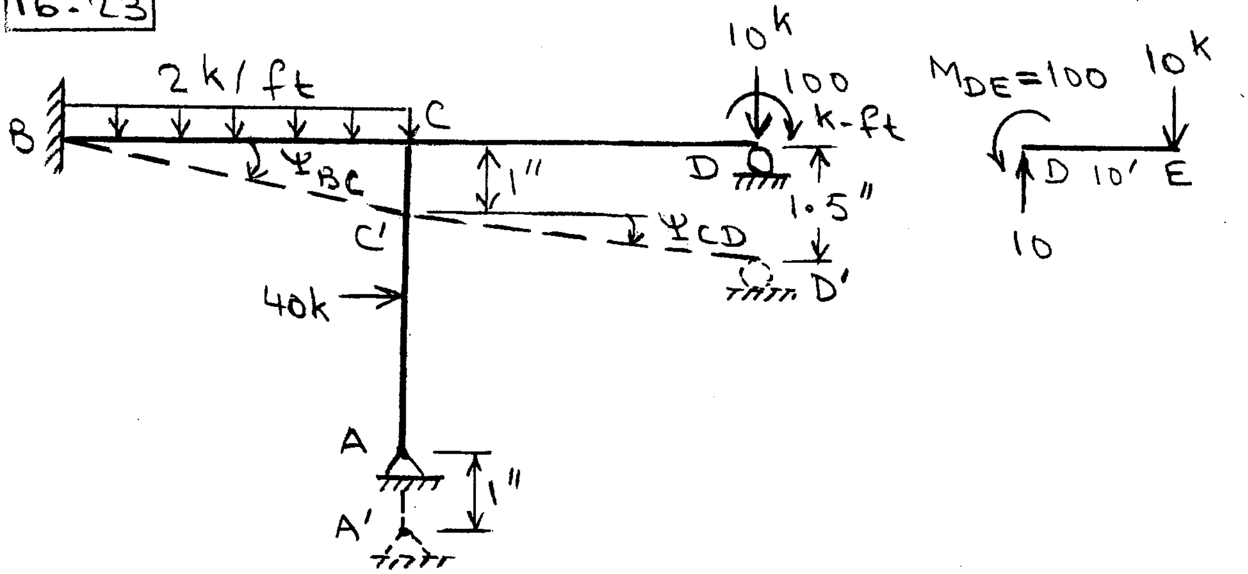
$M_{CA} = -94$ k-ft ; $M_{BC} = 382.6$ k-ft ;

$M_{CB} = 260.4$ k-ft ; $M_{CD} = -266.4$ k-ft.



Reactions

16-23



Fixed-end moments: $FEM_{CD} = FEM_{DC} = 0$

$FEM_{BC} = 150$ k-ft ; $FEM_{CB} = -150$ k-ft

$FEM_{AC} = 100$ k-ft ; $FEM_{CA} = -100$ k-ft

Chord rotations:

$$\Psi_{BC} = -\frac{1}{30(12)} = -0.00728\ ; \quad \Psi_{CD} = -\frac{0.5}{30(12)} = -0.00139$$

Slope-deflection equations: $M_{AC} = 0$; $M_{DE} = 100$ k-ft

$M_{CA} = 0.15EI\theta_C - 150$; $M_{BC} = 0.0667EI\theta_C + 265.7$

$M_{CB} = 0.133EI\theta_C - 34.3$

$M_{CD} = 0.133EI\theta_C + 0.0667EI\theta_D + 57.9$

$M_{DC} = 0.0667EI\theta_C + 0.133EI\theta_D + 57.9$

Equilibrium equations: $M_{CA} + M_{CB} + M_{CD} = 0$

$M_{DC} + 100 = 0$

$0.417EI\theta_C + 0.0667EI\theta_D = 126.4$

$0.0667EI\theta_C + 0.133EI\theta_D = -157.9$

By solving these equations, we obtain:

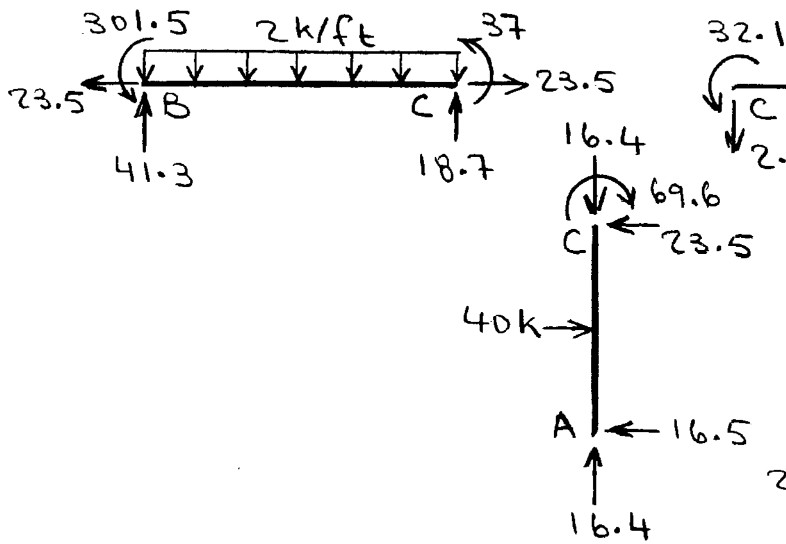
$EI\theta_C = 536$ k-ft^2; $EI\theta_D = -1456$ k-ft^2

16.23 (contd.)

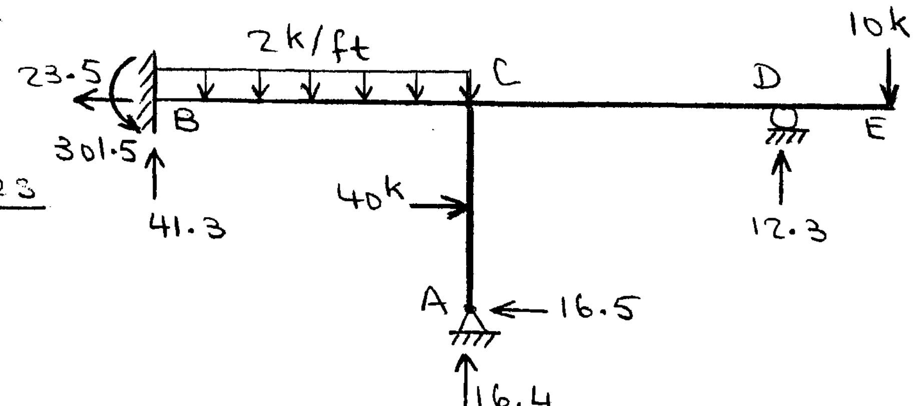
Member end moments: Substituting the numerical values of $EI\theta_C$ and $EI\theta_D$ into the slope-deflection equations, we obtain

$M_{CA} = -69.6$ k-ft; $M_{BC} = 301.5$ k-ft; $M_{CB} = 37$ k-ft; $M_{CD} = 32.1$ k-ft

$M_{DC} = -100$ k-ft.



Member End Forces



Reactions

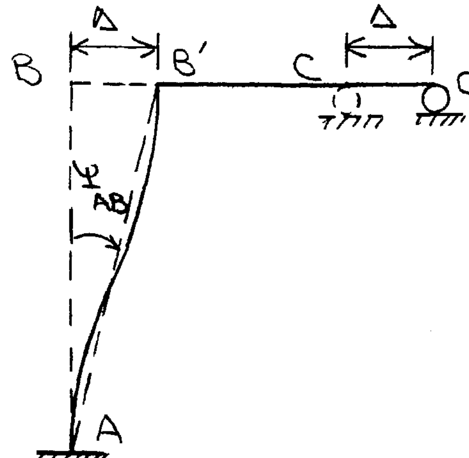
$\boxed{16.24}$ Fixed-end moments: $FEM_{AB} = FEM_{BA} = 0$

$FEM_{BC} = \frac{2(15)^2}{12} = 37.5$ k-ft; $FEM_{CB} = -37.5$ k-ft

Chord rotations:

$\Psi_{AB} = -\frac{\Delta}{20}$

$\Psi_{BC} = 0$



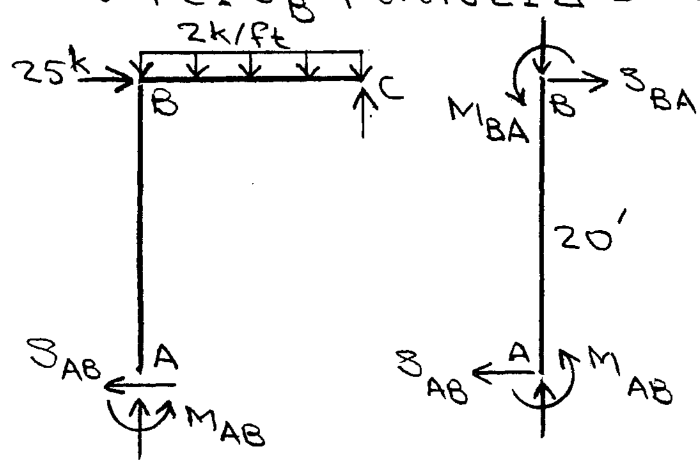
Slope-deflection equations:

$M_{AB} = 0.1EI\theta_B + 0.015EI\Delta$; $M_{BA} = 0.2EI\theta_B + 0.015EI\Delta$

$M_{BC} = 0.2EI\theta_B + 56.25$; $\underline{M_{CB} = 0}$

Equilibrium equations: $M_{BA} + M_{BC} = 0$

$$0.4EI\theta_B + 0.015EI\Delta = -56.25 \qquad (1)$$



$\Sigma F_x = 0 \qquad S_{AB} = 25 \qquad \frac{M_{AB} + M_{BA}}{20} = 25$

$$0.3EI\theta_B + 0.03EI\Delta = 500 \qquad (2)$$

By solving equations (1) and (2) simultaneously, we obtain:

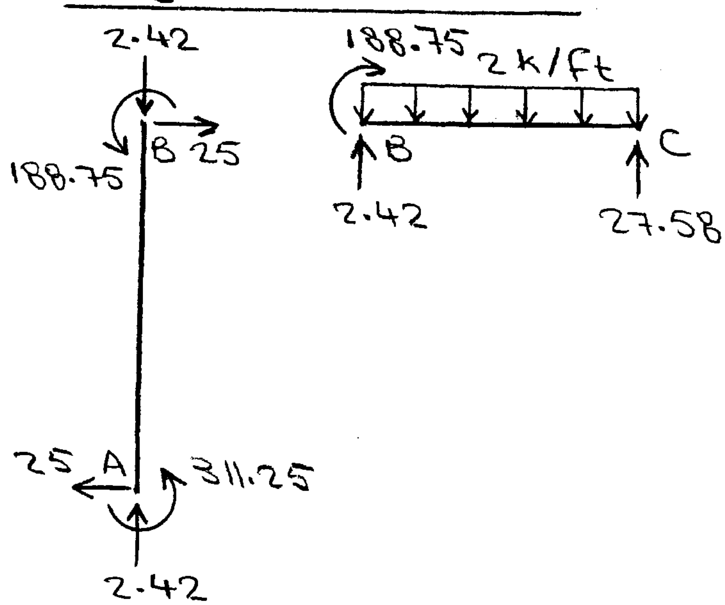
$EI\theta_B = -1225$ k-ft^2; $EI\Delta = 28916.67$ k-ft^3.

16.24 (contd.)

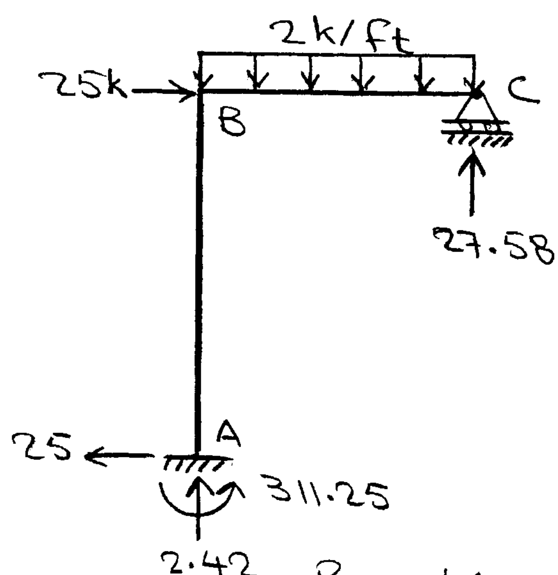
Member end moments: Substituting the numerical values of $EI\theta_B$ and $EI\Delta$ into the slope-deflection equations, we obtain:

$M_{AB} = 311.25$ k-ft; $M_{BA} = 188.75$ k-ft;

$M_{BC} = -188.75$ k-ft.



Member End Forces



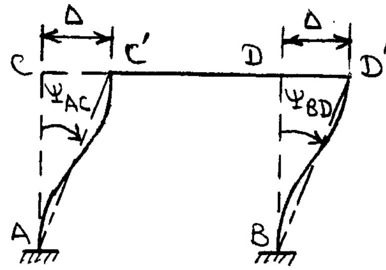
Reactions

16.25 Fixed-end moments. The non-zero fixed-end moments are: $FEM_{CD} = \frac{1.5(40)^2}{12} = 200$ k-ft; and $FEM_{DC} = -200$ k-ft.

Chord rotations:

$$\Psi_{AC} = \Psi_{BD} = -\frac{\Delta}{30}$$

$$\Psi_{CD} = 0$$



Slope-deflection equations:

$M_{AC} = 0.0667EI(\theta_C + 0.1\Delta)$; $M_{CA} = 0.0667EI(2\theta_C + 0.1\Delta)$

$M_{BD} = 0.0667EI(\theta_D + 0.1\Delta)$; $M_{DB} = 0.0667EI(2\theta_D + 0.1\Delta)$

$M_{CD} = 0.1EI(2\theta_C + \theta_D) + 200$

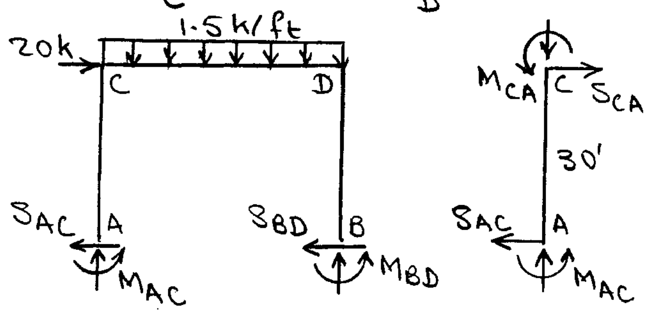
$M_{DC} = 0.1EI(\theta_C + 2\theta_D) - 200$

Equilibrium equations: $M_{CA} + M_{CD} = 0$

$$0.333EI\theta_C + 0.1EI\theta_D + 0.00667EI\Delta = -200 \quad (1)$$

$$M_{DC} + M_{DB} = 0$$

$$0.1EI\theta_C + 0.333EI\theta_D + 0.00667EI\Delta = 200 \quad (2)$$



$$\Sigma F_X = 0 \qquad S_{AC} + S_{BD} = 20$$

$$\frac{M_{AC} + M_{CA}}{30} + \frac{M_{BD} + M_{DB}}{30} = 20$$

$$0.00667EI\theta_C + 0.00667EI\theta_D + 0.000889EI\Delta = 20 \quad (3)$$

Solving Eqs. (1) thru (3) simultaneously, we obtain:

$EI\theta_C = -1308$ k-ft^2; $EI\theta_D = 408$ k-ft^2; $EI\Delta = 29242$ k-ft^3

Member end moments: $M_{AC} = 107.8$ k-ft; $M_{CA} = 20.8$ k-ft

$M_{BD} = 222.3$ k-ft; $M_{DB} = 249.2$ k-ft; $M_{CD} = -20.8$ k-ft

$M_{DC} = -249.2$ k-ft. For reactions, see solution of Problem 13.45.

16.26 Fixed-end moments: $FEM_{AC} = FEM_{CA} = 0$

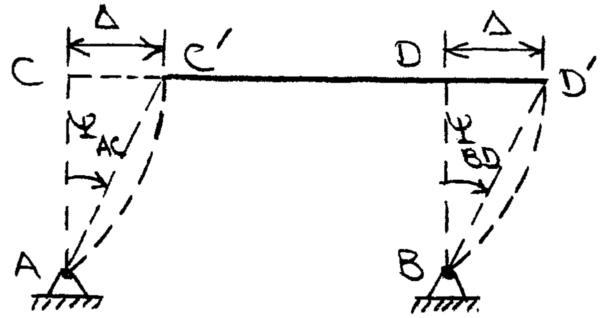
$FEM_{BD} = FEM_{DB} = 0$; $FEM_{CD} = \frac{3(30)^2}{12} = 225$ k-ft;

$FEM_{DC} = -225$ k-ft.

Chord rotations:

$\psi_{AC} = \psi_{BD} = -\frac{\Delta}{15}$

$\psi_{CD} = 0$



Slope-deflection equations: $M_{AC} = M_{BD} = 0$

$M_{CA} = 0.2EI\theta_C + 0.0133EI\Delta$

$M_{DB} = 0.2EI\theta_D + 0.0133EI\Delta$

$M_{CD} = 0.133EI\theta_C + 0.0667EI\theta_D + 225$

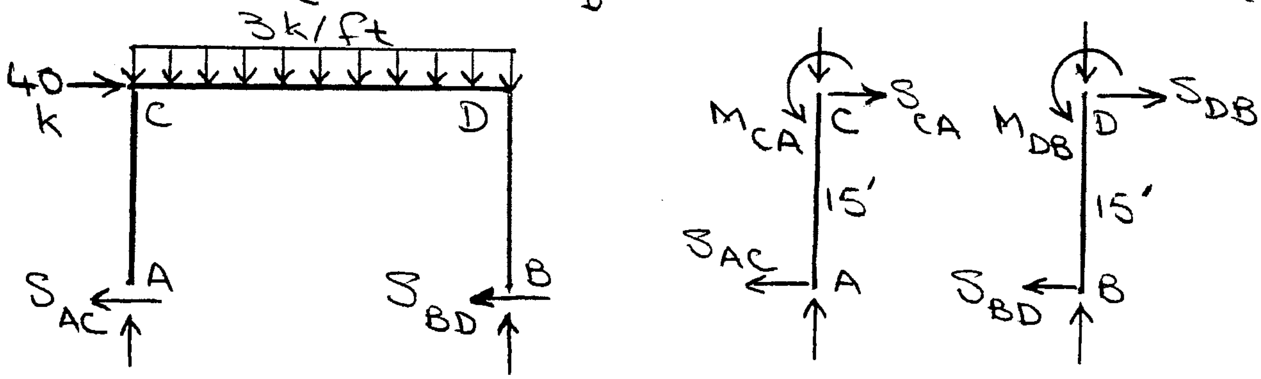
$M_{DC} = 0.0667EI\theta_C + 0.133EI\theta_D - 225$

Equilibrium equations: $M_{CA} + M_{CD} = 0$

$0.333EI\theta_C + 0.0667EI\theta_D + 0.0133EI\Delta = -225 \quad (1)$

$M_{DB} + M_{DC} = 0$

$0.0667EI\theta_C + 0.333EI\theta_D + 0.0133EI\Delta = 225 \quad (2)$



$\Sigma F_x = 0 \qquad S_{AC} + S_{BD} = 40 \qquad \frac{M_{CA}}{15} + \frac{M_{DB}}{15} = 40$

$0.0133EI\theta_C + 0.0133EI\theta_D + 0.00178\Delta = 40 \quad (3)$

16.26 (Contd.)

By solving Eqs. (1) thru (3) simultaneously, we obtain:

$EI\theta_C = -2343.75$ k-ft^2; $EI\theta_D = -656.25$ k-ft^2;

$EI\Delta = 45000$ k-ft^3.

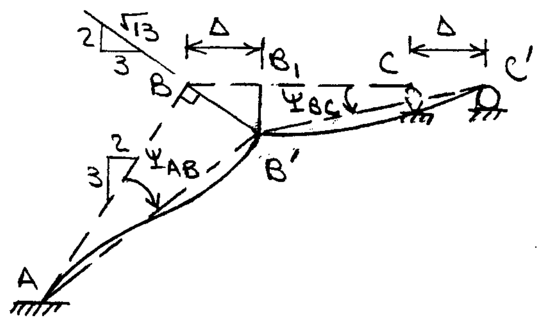
Member end moments:

$M_{CA} = 131.25$ k-ft; $M_{DB} = 468.75$ k-ft;

$M_{CD} = -131.25$ k-ft; $M_{DC} = -468.75$ k-ft.

For reactions, see solution of Problem 13.22.

15.27 Chord rotations:



$$\Psi_{AB} = -\frac{BB'}{L_{AB}} = -\frac{(\sqrt{13}/3)\Delta}{14.42} = -0.0833\,\Delta$$

$$\Psi_{BC} = \frac{B_1B'}{L_{BC}} = \frac{(2/3)\Delta}{12} = 0.0556\,\Delta$$

Slope-deflection equations: $M_{CB} = 0$

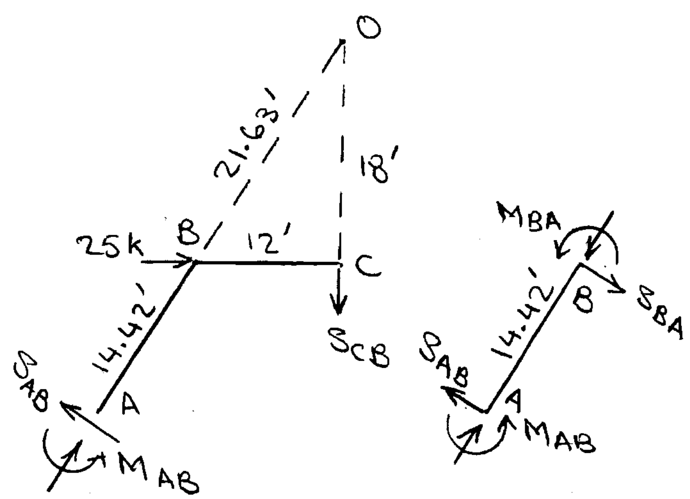
$$M_{AB} = 0.139 EI\,\theta_B + 0.0347\,EI\Delta$$

$$M_{BA} = 0.277 EI\,\theta_B + 0.0347\,EI\Delta$$

$$M_{BC} = 0.25 EI\,\theta_B - 0.0139\,EI\Delta$$

Equilibrium equations: $M_{BA} + M_{BC} = 0$

$$0.527 EI\theta_B + 0.0208 EI\Delta = 0 \qquad (1)$$



$$+\circlearrowleft \Sigma M_O = 0 \qquad M_{AB} - S_{AB}(36.05) + 25(18) = 0$$

$$M_{AB} - \left(\frac{M_{AB} + M_{BA}}{14.42}\right)(36.05) + 450 = 0$$

$$0.901 EI\theta_B + 0.139 EI\Delta = 450 \qquad (2)$$

16.27 (contd.)

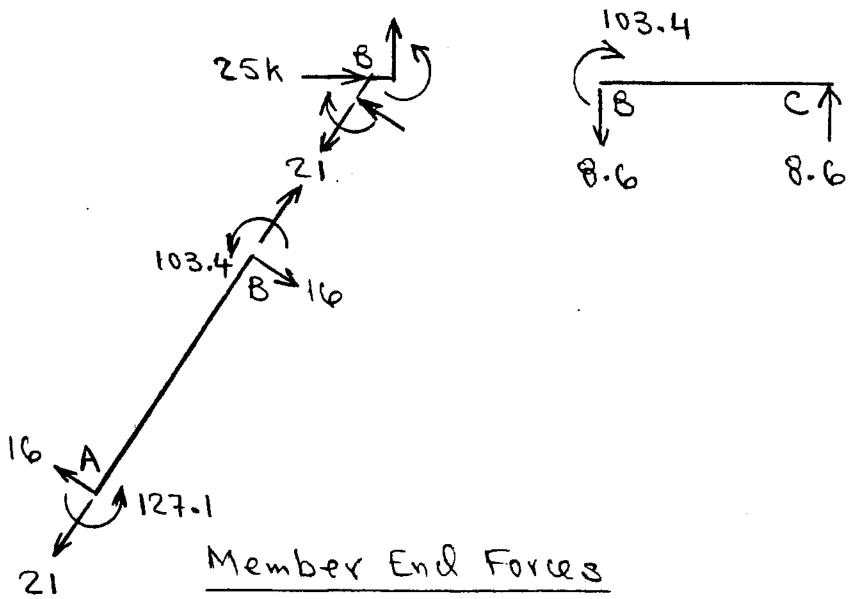
By solving Eqs. (1) and (2), we obtain:

$EI\theta_B = -171.7$ k-ft^2; $EI\Delta = 4350$ k-ft^3

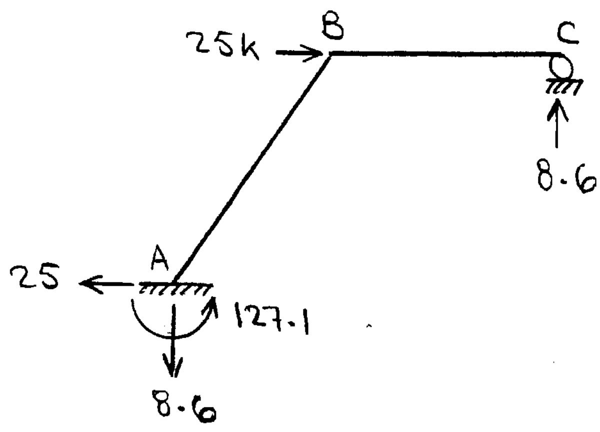
Member end moments. Substituting the numerical values of $EI\theta_B$ and $EI\Delta$ into the slope-deflection equations, we obtain:

$M_{AB} = 127.1$ k-ft; $M_{BA} = 103.4$ k-ft;

$M_{BC} = -103.4$ k-ft.



Member End Forces

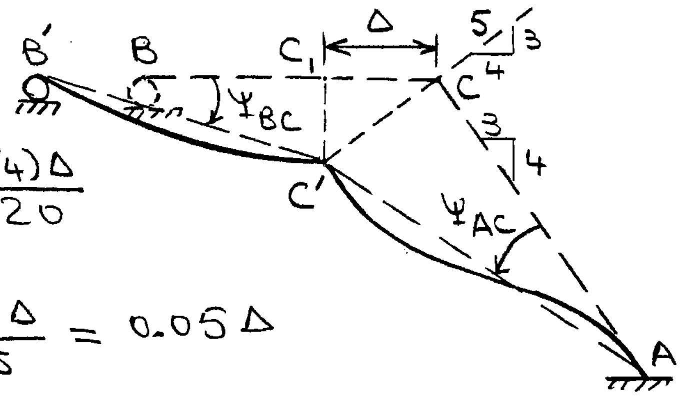


Reactions

16.28 Fixed end moments:

$$FEM_{BC} = \frac{1.5(20)^2}{12} = 50 \text{ k-ft}; \quad FEM_{CB} = -50 \text{ k-ft}$$

Chord rotations:



$$\Psi_{BC} = -\frac{C_1C'}{L_{BC}} = -\frac{(3/4)\Delta}{20}$$
$$= -0.0375\,\Delta$$

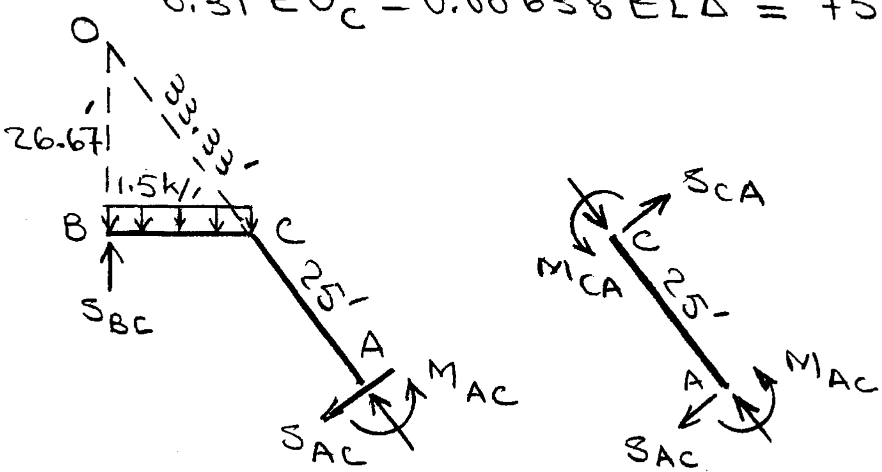
$$\Psi_{AC} = \frac{CC'}{L_{AC}} = \frac{(5/4)\Delta}{25} = 0.05\,\Delta$$

Slope-deflection equations: $\underline{M_{BC} = 0}$

$$M_{CB} = 0.15EI(\theta_C + 0.0375\,\Delta) - 75$$
$$M_{CA} = 0.08\,EI(2\theta_C - 0.15\,\Delta)$$
$$M_{AC} = 0.08\,EI(\theta_C - 0.15\Delta)$$

Equilibrium equations: $M_{CB} + M_{CA} = 0$

$$0.31\,EI\theta_C - 0.00638\,EI\Delta = 75 \qquad (1)$$



$$+\circlearrowright \Sigma M_O = 0 \qquad M_{AC} - S_{AC}(58.33) - 1.5(20)10 = 0$$

$$M_{AC} - \left(\frac{M_{AC} + M_{CA}}{25}\right)(58.33) - 300 = 0$$

$$-0.48\,EI\theta_C + 0.044\,EI\Delta = 300 \qquad (2)$$

By solving Eqs. (1) and (2), we obtain

$$EI\theta_C = 493 \text{ k-ft}^2; \quad EI\Delta = 12196 \text{ k-ft}^3.$$

16.28 (Contd.) <u>Member end moments:</u> Substituting the numerical values of $EI\theta_C$ and $EI\Delta$ into the slope-deflection equations, we obtain

$\underline{M_{CB} = 67.5 \text{ k-ft}}$; $\underline{M_{CA} = -67.5 \text{ k-ft}}$

$\underline{M_{AC} = -106.9 \text{ k-ft}}$

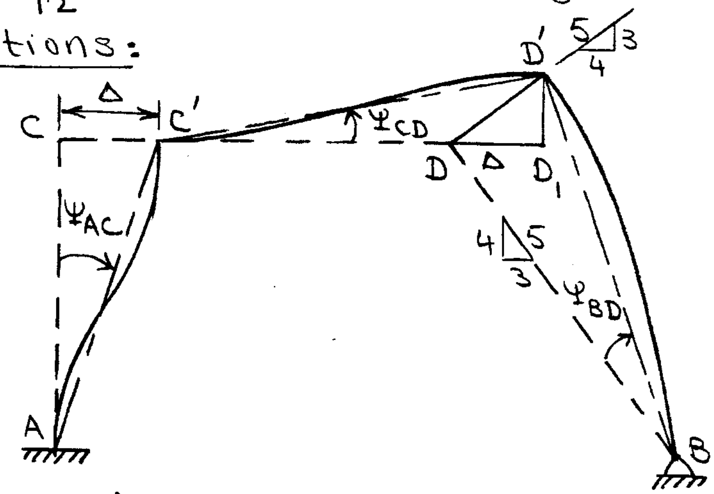
For reactions, see solution of Problem 13.24.

16-29 Fixed-end moments: $FEM_{BD} = FEM_{DB} = 0$

$$FEM_{AC} = \frac{50(4)}{8} = 25 \text{ kN.m}; \quad FEM_{CA} = -25 \text{ kN.m}$$

$$FEM_{CD} = \frac{18(5)^2}{12} = 37.5 \text{ kN.m}; \quad FEM_{DC} = -37.5 \text{ kN.m}$$

Chord rotations:



$$\Psi_{AC} = -\frac{CC'}{L_{AC}} = -\frac{\Delta}{4} = -0.25\Delta$$

$$\Psi_{BD} = -\frac{DD'}{L_{BD}} = -\frac{(5/4)\Delta}{5} = -0.25\Delta$$

$$\Psi_{CD} = \frac{D_1D'}{L_{CD}} = \frac{(3/4)\Delta}{5} = 0.15\Delta$$

Slope-deflection equations: $M_{BD} = 0$

$$M_{AC} = 0.5EI(\theta_C + 0.75\Delta) + 25$$

$$M_{CA} = 0.5EI(2\theta_C + 0.75\Delta) - 25$$

$$M_{CD} = 0.4EI(2\theta_C + \theta_D - 0.45\Delta) + 37.5$$

$$M_{DC} = 0.4EI(\theta_C + 2\theta_D - 0.45\Delta) - 37.5$$

$$M_{DB} = 0.6EI(\theta_D + 0.25\Delta)$$

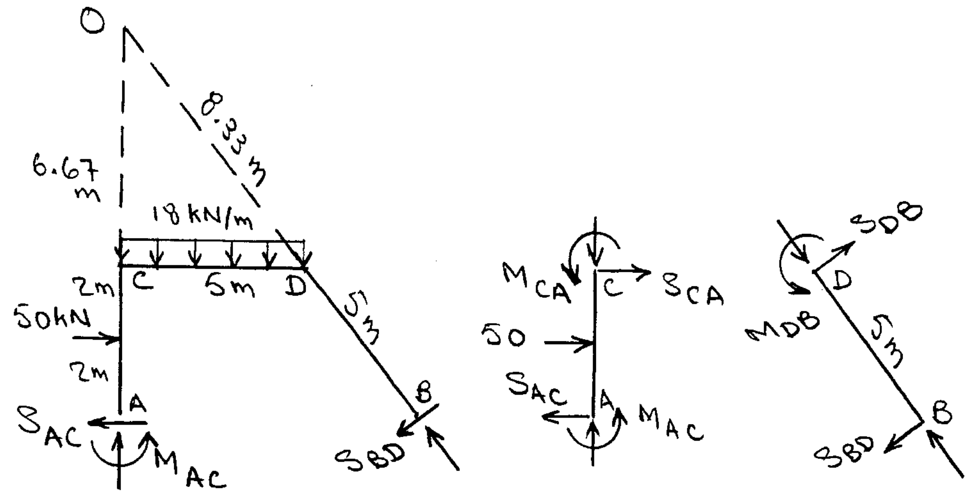
Equilibrium equations: $M_{CA} + M_{CD} = 0$

$$1.8EI\theta_C + 0.4EI\theta_D + 0.195EI\Delta = -12.5 \qquad (1)$$

$$M_{DC} + M_{DB} = 0$$

$$0.4EI\theta_C + 1.4EI\theta_D - 0.03EI\Delta = 37.5 \qquad (2)$$

16.29 (contd.)



$+\circlearrowright \sum M_O = 0$

$$M_{AC} - S_{AC}(10.67) - S_{BD}(13.33) + 50(8.67) - 18(5)2.5 = 0$$

$$M_{AC} - \left(25 + \frac{M_{AC}+M_{CA}}{4}\right)10.67 - \left(\frac{M_{DB}}{5}\right)13.33 + 208.5 = 0$$

$$3.5EI\theta_C + 1.6EI\theta_D + 2.025EI\Delta = -33.25 \qquad (3)$$

By solving Eqs. (1) thru (3) simultaneously, we obtain:

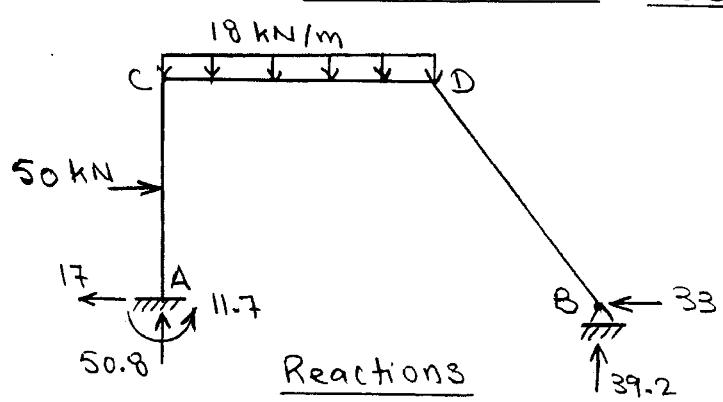
$EI\theta_C = -11.33$ kN-m^2; $EI\theta_D = 29.59$ kN-m^2

$EI\Delta = -20.22$ kN-m^3

Member end moments. Substituting the numerical values of $EI\theta_C$, $EI\theta_D$ and $EI\Delta$ into the slope-deflection equations, we obtain:

$M_{AC} = 11.7$ kN.m; $M_{CA} = -43.9$ kN.m;

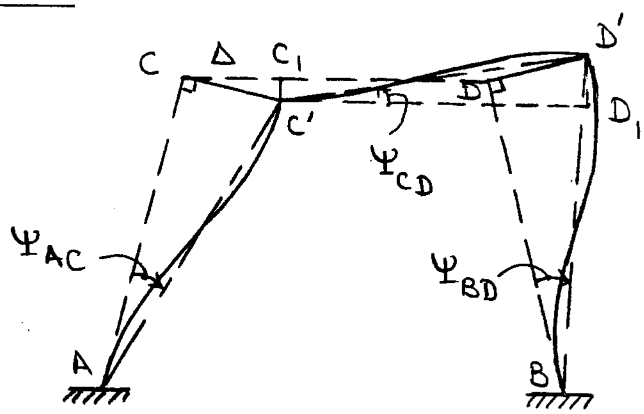
$M_{CD} = 43.9$ kN.m; $M_{DC} = -14.7$ kN.m; $M_{DB} = 14.7$ kN.m



Reactions

$\boxed{16.30}$ Fixed-end moments: The non-zero fixed-end moments are: $FEM_{CD} = \dfrac{3(16)^2}{12} = 64$ k-ft and $FEM_{DC} = -64$ k-ft.

Chord rotations:



$$\Psi_{AC} = -\frac{CC'}{L_{AC}} = -\frac{(\sqrt{17}/4)\Delta}{16.49} = -0.0625\Delta$$

$$\Psi_{BD} = -\frac{DD'}{L_{BD}} = -\frac{(\sqrt{17}/4)\Delta}{16.49} = -0.0625\Delta$$

$$\Psi_{CD} = \frac{D_1D'}{L_{CD}} = \frac{2(1/4)\Delta}{16} = 0.03125\Delta$$

Slope-deflection equations:

$M_{AC} = 0.121EI(\theta_C + 0.188\Delta)$; $M_{CA} = 0.121EI(2\theta_C + 0.188\Delta)$

$M_{BD} = 0.121EI(\theta_D + 0.188\Delta)$; $M_{DB} = 0.121EI(2\theta_D + 0.188\Delta)$

$M_{CD} = 0.125EI(2\theta_C + \theta_D - 0.0938\Delta) + 64$

$M_{DC} = 0.125EI(\theta_C + 2\theta_D - 0.0938\Delta) - 64$

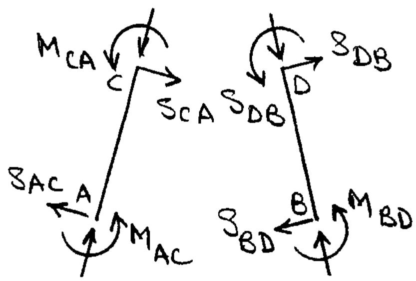
Equilibrium equations: $M_{CA} + M_{CD} = 0$

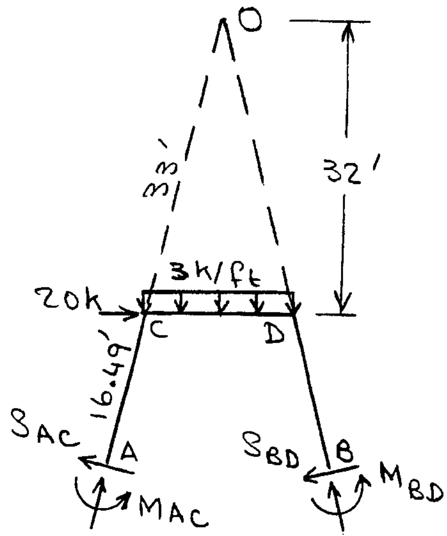
$$0.492EI\theta_C + 0.125E\theta_D + 0.011EI\Delta = -64 \qquad (1)$$

$M_{DC} + M_{DB} = 0$

$$0.125EI\theta_C + 0.492EI\theta_D + 0.011EI\Delta = 64 \qquad (2)$$

16.30 (Contd.)





$+\circlearrowleft \Sigma M_O = 0$

$$M_{AC} + M_{BD} - (S_{AC} + S_{BD})49.49 + 20(32) = 0$$

$$M_{AC} + M_{BD} - \frac{49.49}{16.49}(M_{AC} + M_{CA} + M_{BD} + M_{DB}) + 640 = 0$$

$$0.968\,EI\theta_C + 0.968\,EI\theta_D + 0.227\,EI\Delta = 640 \qquad (3)$$

By solving Eqs. (1) thru (3), we obtain:

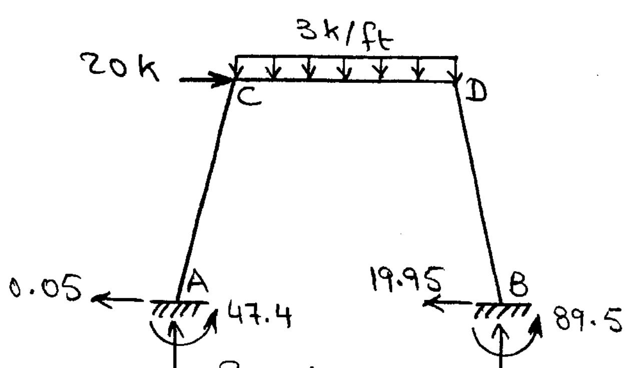
$EI\theta_C = -233.7$ k-ft², $EI\theta_D = 115.1$ k-ft²;

$EI\Delta = 3325$ k-ft³

Member end moments. Substituting the numerical values of $EI\theta_C$, $EI\theta_D$ and $EI\Delta$ into the slope-deflection equations, we obtain: $M_{AC} = 47.4$ k-ft;

$M_{CA} = 19$ k-ft; $M_{BD} = 89.5$ k-ft; $M_{DB} = 103.5$ k-ft;

$M_{CD} = -19$ k-ft; $M_{DC} = -103.5$ k-ft.



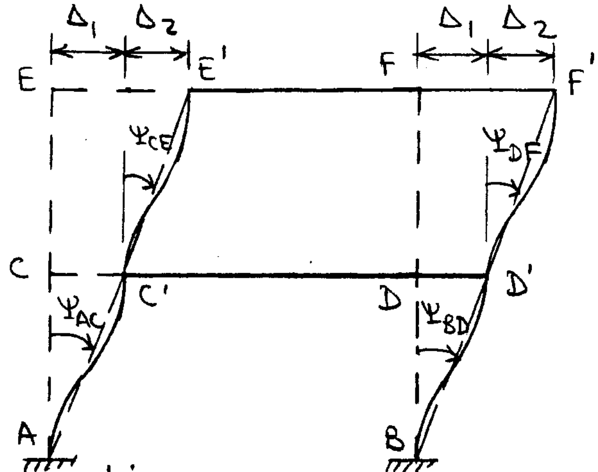
Reactions

16.31 Chord rotations:

$$\Psi_{AC} = \Psi_{BD} = -\frac{\Delta_1}{15}$$

$$\Psi_{CE} = \Psi_{DF} = -\frac{\Delta_2}{15}$$

$$\Psi_{CD} = \Psi_{EF} = 0$$



Slope-deflection equations:

$$M_{AC} = 0.133EI(\theta_C + 0.2\Delta_1);\ M_{CA} = 0.133EI(2\theta_C + 0.2\Delta_1)$$

$$M_{BD} = 0.133EI(\theta_D + 0.2\Delta_1);\ M_{DB} = 0.133EI(2\theta_D + 0.2\Delta_1)$$

$$M_{CE} = 0.133EI(2\theta_C + \theta_E + 0.2\Delta_2)$$

$$M_{EC} = 0.133EI(\theta_C + 2\theta_E + 0.2\Delta_2)$$

$$M_{DF} = 0.133EI(2\theta_D + \theta_F + 0.2\Delta_2)$$

$$M_{FD} = 0.133EI(\theta_D + 2\theta_F + 0.2\Delta_2)$$

$$M_{CD} = 0.133EI(2\theta_C + \theta_D);\ M_{DC} = 0.133EI(\theta_C + 2\theta_D)$$

$$M_{EF} = 0.133EI(2\theta_E + \theta_F);\ M_{FE} = 0.133EI(\theta_E + 2\theta_F)$$

Equilibrium equations: $M_{CA} + M_{CD} + M_{CE} = 0$

$$EI(6\theta_C + \theta_D + \theta_E + 0.2\Delta_1 + 0.2\Delta_2) = 0 \qquad (1)$$

$$M_{DB} + M_{DC} + M_{DF} = 0$$

$$EI(\theta_C + 6\theta_D + \theta_F + 0.2\Delta_1 + 0.2\Delta_2) = 0 \qquad (2)$$

$$M_{EC} + M_{EF} = 0$$

$$EI(\theta_C + 4\theta_E + \theta_F + 0.2\Delta_2) = 0 \qquad (3)$$

$$M_{FD} + M_{FE} = 0$$

$$EI(\theta_D + \theta_E + 4\theta_F + 0.2\Delta_2) = 0 \qquad (4)$$

$$S_{CE} + S_{DF} = 9$$

$$\frac{1}{15}\left[(M_{CE} + M_{EC}) + (M_{DF} + M_{FD})\right] = 9$$

16.31 (Contd.)

$$EI(3\theta_C + 3\theta_D + 3\theta_E + 3\theta_F + 0.8\Delta_2) = 1015 \quad (5)$$

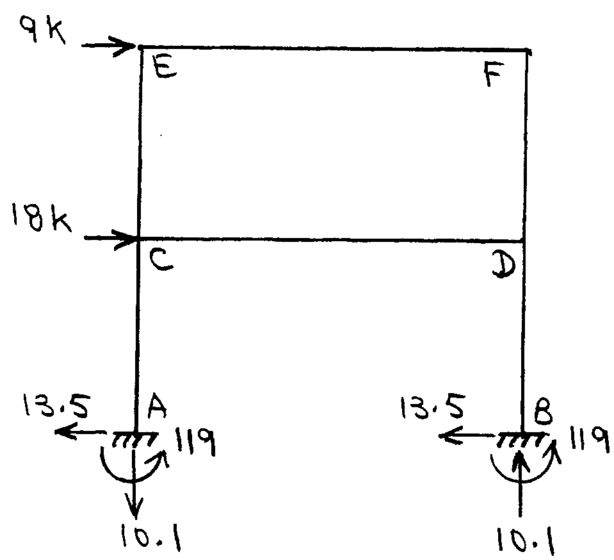
$$S_{AC} + S_{BD} = 27$$

$$\frac{1}{15}\left[(M_{AC} + M_{CA}) + (M_{BD} + M_{DB})\right] = 27$$

$$EI(3\theta_C + 3\theta_D + 0.8\Delta_1) = 3045 \quad (6)$$

By solving Eqs. (1) thru (6) simultaneously, we obtain: $EI\theta_C = EI\theta_D = -267.6$ k ft^2; $EI\theta_E = EI\theta_F = -110.7$ k-ft^2; $EI\Delta_1 = 5813$ k-ft^3; $EI\Delta_2 = 4106$ k-ft^3.

Member end moments: $M_{AC} = M_{BD} = 119$ k-ft; $M_{CA} = M_{DB} = 83.5$ k-ft; $M_{CE} = M_{DF} = 23.3$ k-ft; $M_{EC} = M_{FD} = 44.2$ k-ft; $M_{CD} = M_{DC} = -106.8$ k-ft; $M_{EF} = M_{FE} = -44.2$ k-ft.

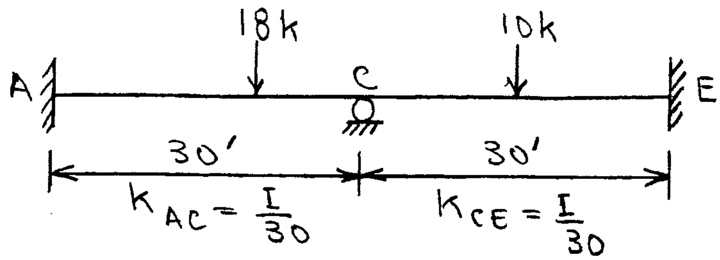


Reactions

Chapter Seventeen
Moment-Distribution Method

CHAPTER 17

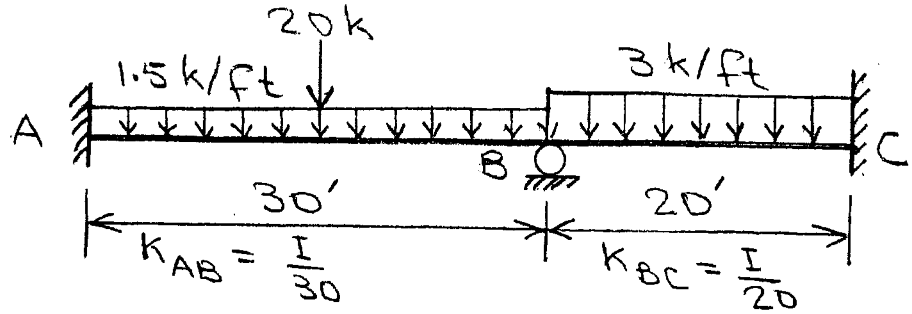
17-1



	A	CA	CE	E
DF		0.5	0.5	
FEM	40	−80	37.5	−37.5
		21.3	21.3	
	10.6			10.6
Final Moments	50.6	−58.7	58.8	−26.9

For reactions, and shear and bending moment diagrams, see solution of Problem 16.1.

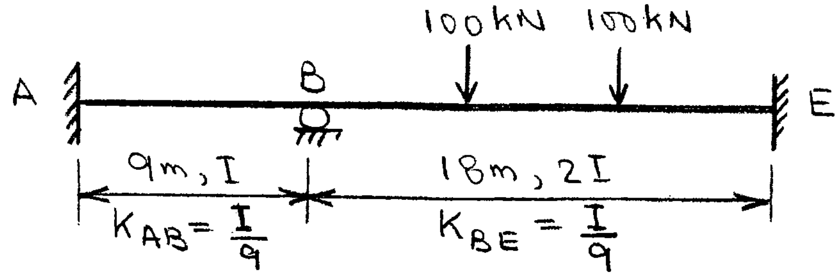
17.2



DF		0.4	0.6	
FEM	187.5	−187.5	100	−100
		35	52.5	
	17.5			26.25
Final Moments	205	−152.5	152.5	−73.75

For reactions, and shear and bending moment diagrams, see solution of Problem 16.2.

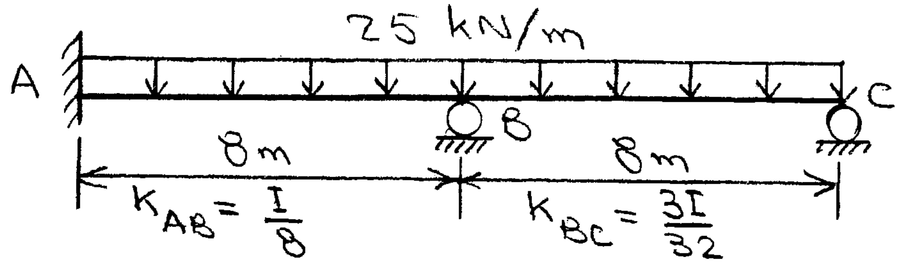
$\boxed{17.3}$



	A	B	B	E
DF		0.5	0.5	
FEM	0	0	400	-400
		-200	-200	
	-100			-100
Final Moments	-100	-200	200	-500

For reactions, and shear and bending moment diagrams, see solution of Problem 16.3.

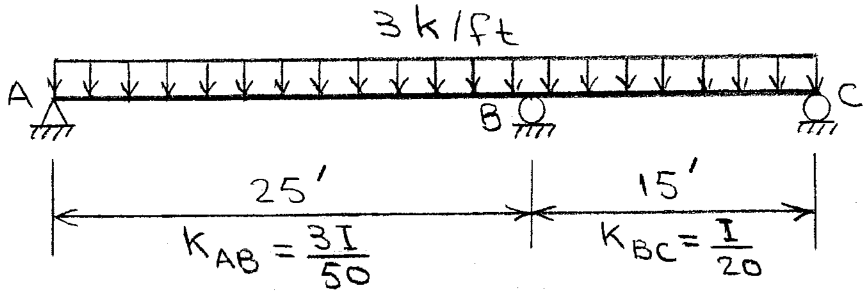
$\boxed{17.4}$



	A	B (AB)	B (BC)	C
DF		0.571	0.429	
FEM	133.3	−133.3	133.3	−133.3
				133.3
			66.7	
		−38.1	−28.6	
	−19			
Final Moments	114.3	−171.4	171.4	0

For reactions, and shear and bending moment diagrams, see solution of Problem 13.37.

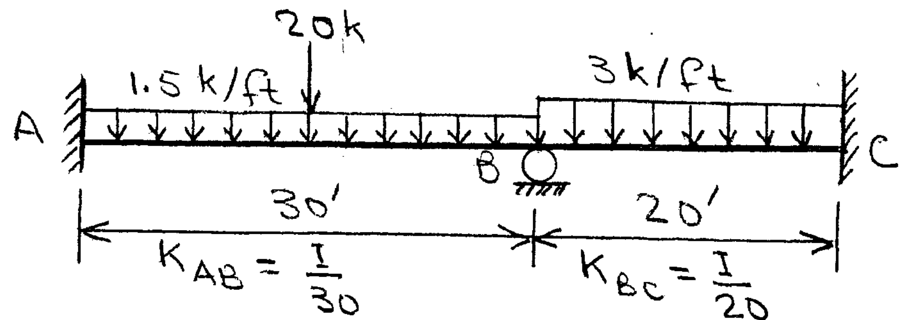
17-5



DF		0.545	0.455	
FEM	156.3	−156.3	56.3	−56.3
	−156.3	54.5	45.5	56.3
		−78.1	28.1	
		27.3	22.7	
Final Moments	0	−152.6	152.6	0

For reactions, and shear and bending moment diagrams, see solution of Problem 13.12.

$\boxed{17.6}$



$$FEM_{AB} = \frac{1.5(30)^2}{12} + \frac{20(30)}{8} + \frac{6(29000)(1650)(0.5)}{(30)^2(12)^3}$$

$$= 112.5 + 75 + 92.3 = 279.8 \text{ k-ft}$$

$$FEM_{BA} = -112.5 - 75 + 92.3 = -95.2 \text{ k-ft}$$

$$FEM_{BC} = \frac{3(20)^2}{12} - \frac{6(29000)(1650)(0.5)}{(20)^2(12)^3}$$

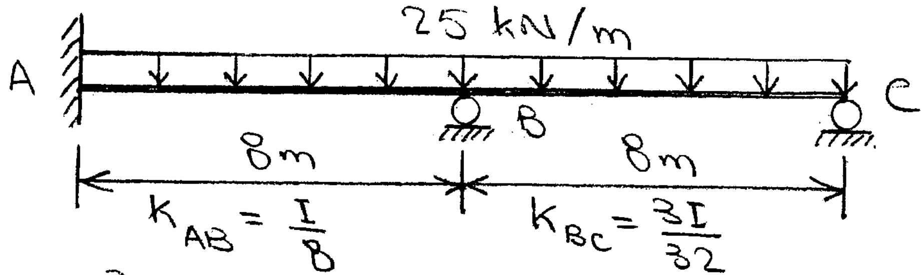
$$= 100 - 207.7 = -107.7 \text{ k-ft}$$

$$FEM_{CB} = -100 - 207.7 = -307.7 \text{ k-ft}$$

DF		0.4	0.6	
FEM	279.8	-95.2	-107.7	-307.7
		81.2	121.7	
	40.6			60.9
Final Moments	320.4	-14	14	-246.8

For reactions, and shear and bending moment diagrams, see solution of Problem 16.6.

$\boxed{17.7}$



$$FEM_{AB} = \frac{25(8)^2}{12} + \frac{6(70)(1300)(0.05)}{(8)^2}$$
$$= 133.3 + 426.6 = 559.9 \text{ kN.m}$$

$$FEM_{BA} = -133.3 + 426.6 = 293.3 \text{ kN.m}$$

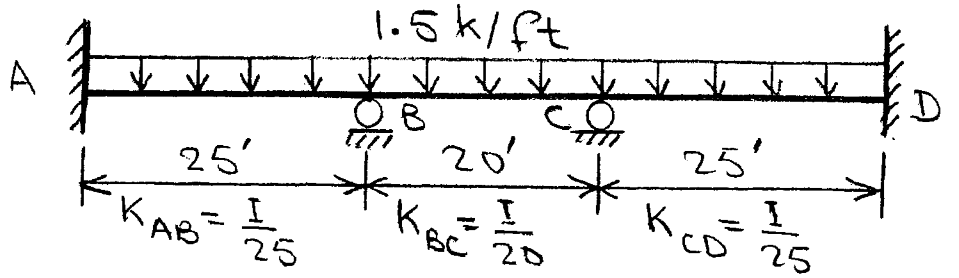
$$FEM_{BC} = 133.3 - \frac{6(70)(1300)(0.025)}{(8)^2}$$
$$= 133.3 - 213.3 = -80 \text{ kN.m}$$

$$FEM_{CB} = -133.3 - 213.3 = -346.6 \text{ kN.m}$$

DF		0.571	0.429	
FEM	559.9	293.3	-80	-346.6
		-121.8	-91.5	346.6
	-60.9		173.3	
		-99	-74.3	
	-49.5			
Final Moments	449.5	72.5	-72.5	0

For reactions, and shear and bending moment diagrams, see solution of Problem 13.53.

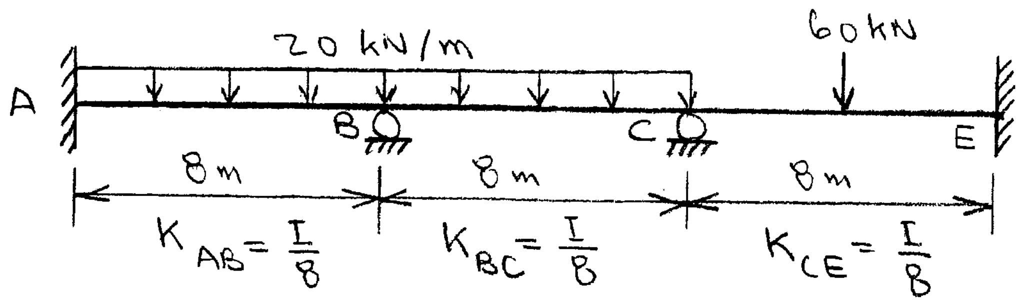
17.8



DF		4/9	5/9	5/9	4/9	
FEM	78.1	-78.1	50	-50	78.1	-78.1
		12.5	15.6	-15.6	-12.5	
	6.2		-7.8	7.8		-6.2
		3.5	4.3	-4.3	-3.5	
	1.7		-2.2	2.2		-1.7
		1	1.2	-1.2	-1	
	0.5		-0.6	0.6		-0.5
		0.3	0.3	-0.3	-0.3	
	0.1		-0.2	0.2		-0.1
		0.1	0.1	-0.1	-0.1	
Final Moments	86.6	-60.7	60.7	-60.7	60.7	-86.6

For reactions, and shear and bending moment diagrams, see solution of Problem 16.8.

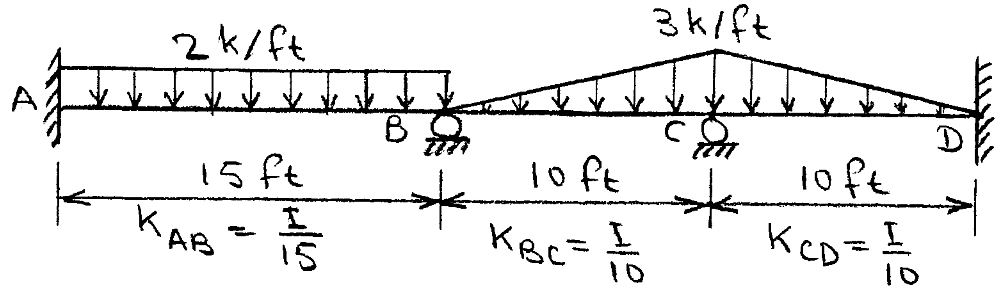
17.9



DF		0.5	0.5	0.5	0.5	
FEM	106.7	−106.7	106.7	−106.7	60	−60
				23.4	23.4	
			11.7			11.7
		−5.9	−5.9			
	−3			−3		
				1.5	1.5	
			0.8			0.8
		−0.4	−0.4			
	−0.2			−0.2		
				0.1	0.1	
Final Moments	103.5	−113	112.9	−84.9	85	−47.5

For reactions, and shear and bending moment diagrams, see solution of Problem 16.9.

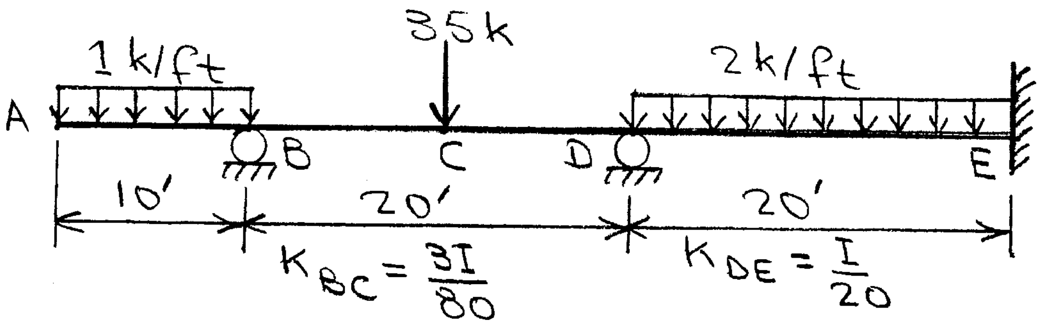
17.10



DF		0.4	0.6	0.5	0.5	
FEM	37.5	−37.5	10	−15	15	−10
		11	16.5			
	5.5			8.3		
				−4.2	−4.2	
			−2.1			−2.1
		0.8	1.3			
	0.4			0.7		
				−0.3	−0.3	
			−0.2			−0.2
		0.1	0.1			
Final Moments	43.4	−25.6	25.6	−10.5	10.5	−12.3

For reactions, and shear and bending moment diagrams, see solution of Problem 16.10.

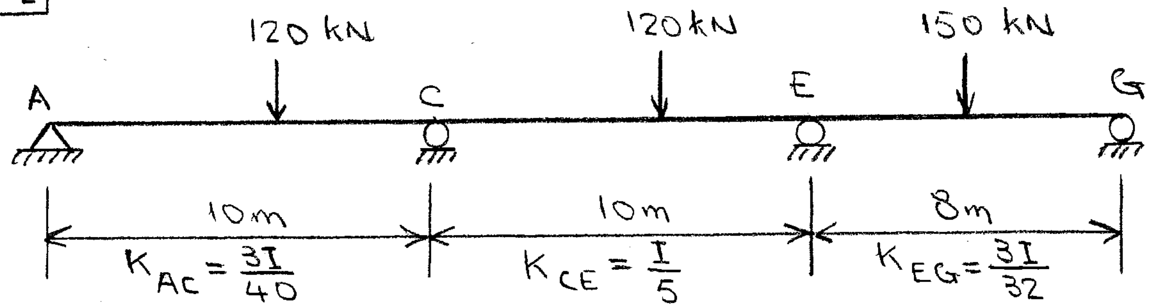
17.11



DF		1	0.429	0.571	
FEM	−50	87.5	−87.5	66.7	−66.7
		−37.5	8.9	11.9	
			−18.8		5.9
			8.1	10.7	
					5.4
Final Moments	−50	50	−89.3	89.3	−55.4

For reactions, and shear and bending moment diagrams, see solution of Problem 13.38.

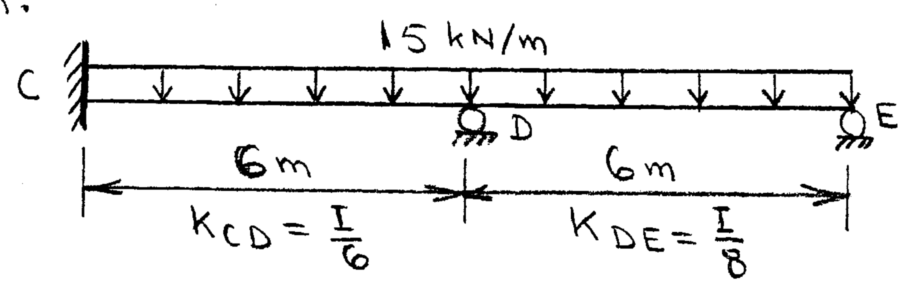
17.12



DF		0.273	0.727	0.681	0.319	
FEM	115.2	-172.8	115.2	-172.8	150	-150
	-115.2	15.7	41.9	15.5	7.3	150
		-57.6	7.8	20.9	75	
		13.6	36.2	-65.3	-30.6	
			-32.7	18.1		
		8.9	23.8	-12.3	-5.8	
			-6.2	11.9		
		1.7	4.5	-8.1	-3.8	
			-4.1	2.3		
		1.1	3	-1.6	-0.7	
			-0.8	1.5		
		0.2	0.6	-1	-0.5	
			-0.5	0.3		
		0.1	0.4	-0.2	-0.1	
Final Moments	0	-189.1	189.1	-190.8	190.8	0

For reactions, and shear and bending moment diagrams, see solution of Problem 13.39.

17.13 As the beam and loading are symmetric, we will analyze only the right half CE of the beam.



	CD	DC	DE	ED
DF		0.571	0.429	1
FEM	45	−45	45	−45
				45
			22.5	
		−12.8	−9.7	
	−6.4			
Final Moments	38.6	−57.8	57.8	0

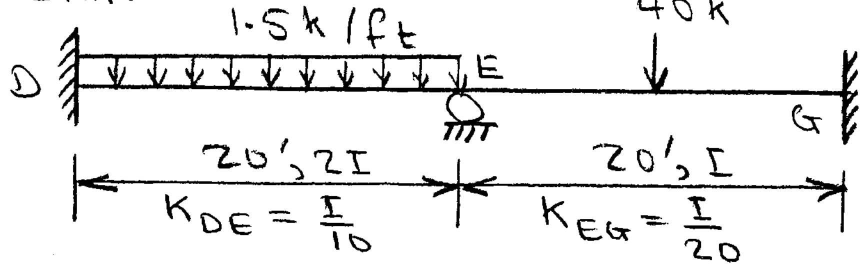
Because of symmetry, the member end moments for the left half of the beam are:

$\underline{M_{AB} = 0}$; $\underline{M_{BA} = -57.8 \text{ kN.m}}$

$\underline{M_{BC} = 57.8 \text{ kN.m}}$; $\underline{M_{CB} = -38.6 \text{ kN.m}}$

For reactions, and shear and bending moment diagrams, see solution of Problem 16.13.

$\boxed{17.14}$ As the beam and loading are symmetric, we will analyze only the right half DG of the beam.



DF		2/3	1/3	
FEM	50	−50	100	−100
		−33.3	−16.7	
	−16.7			−8.3
Final Moments	33.3	−83.3	83.3	−108.3

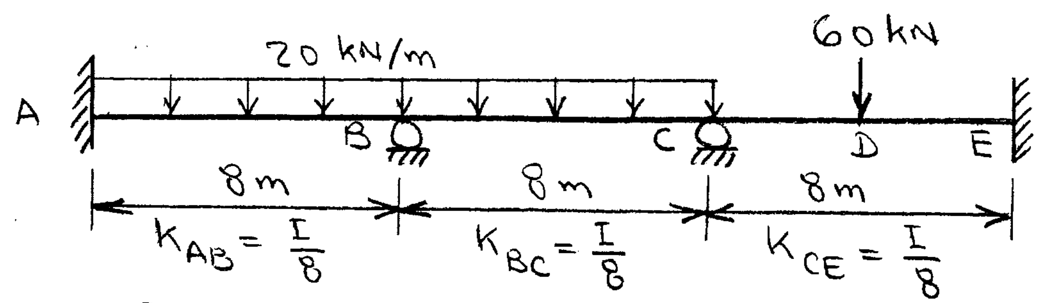
Because of symmetry, the member end moments for the left half of the beam are:

$\underline{M_{AC} = 108.3 \text{ k-ft}}$; $\underline{M_{CA} = -83.3 \text{ k-ft}}$

$\underline{M_{CD} = 83.3 \text{ k-ft}}$; $\underline{M_{DC} = -33.3 \text{ k-ft}}$

For reactions, and shear and bending moment diagrams, see solution of Problem 16.14.

17.15



$$FEM_{AB} = \frac{20(8)^2}{12} = 106.7 \text{ kN.m}; \quad FEM_{BA} = -106.7 \text{ kN.m}$$

$$FEM_{BC} = 106.7 + \frac{6(70)(800)(0.025)}{(8)^2} = 106.7 + 131.3$$

$$= 238 \text{ kN.m}$$

$$FEM_{CB} = -106.7 + 131.3 = 24.6 \text{ kN.m}$$

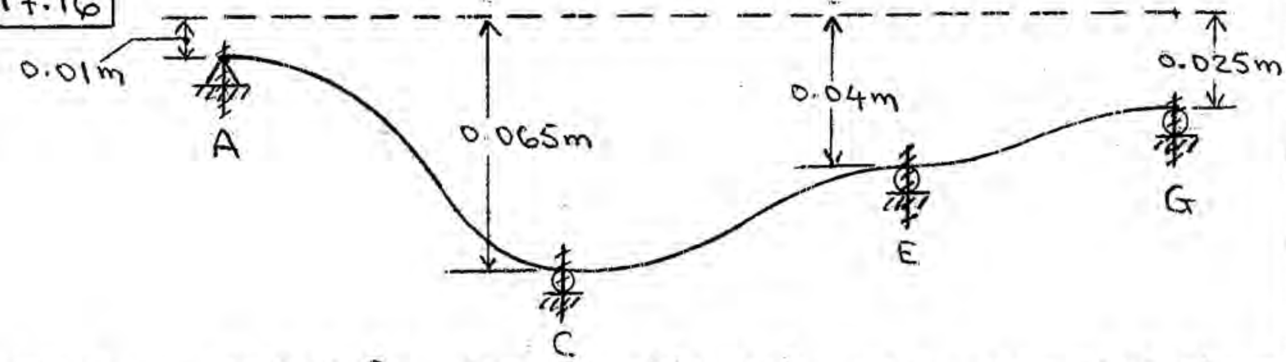
$$FEM_{CE} = \frac{60(8)}{8} - 131.3 = 60 - 131.3 = -71.3 \text{ kN.m}$$

$$FEM_{EC} = -60 - 131.3 = -191.3 \text{ kN.m}$$

DF		0.5	0.5	0.5	0.5	
FEM	106.7	-106.7	238	24.6	-71.3	-191.3
		-65.7	-65.7	23.4	23.4	
	-32.8		11.7	-32.8		11.7
		-5.8	-5.8	16.4	16.4	
	-2.9		8.2	-2.9		8.2
		-4.1	-4.1	1.5	1.5	
	-2.1		0.7	-2.1		0.7
		-0.4	-0.4	1	1	
	-0.2		0.5	-0.2		0.5
		-0.2	-0.2	0.1	0.1	
Final Moments	68.7	-182.9	182.9	29	-28.9	-170.2

For reactions, and shear and bending moment diagrams, see solution of Problem 16.15.

17.16



$$FEM_{AC} = \frac{120(6)(4)^2}{(10)^2} + \frac{6(200)(500)(0.055)}{(10)^2} = 115.2 + 330 = 445.2 \text{ kN.m}$$

$$FEM_{CA} = -\frac{120(6)^2(4)}{(10)^2} + 330 = -172.8 + 330 = 157.2 \text{ kN.m}$$

$$FEM_{CE} = 115.2 - \frac{6(200)(1000)(0.025)}{(10)^2} = 115.2 - 300 = -184.8 \text{ kN.m}$$

$$FEM_{EC} = -172.8 - 300 = -472.8 \text{ kN.m}$$

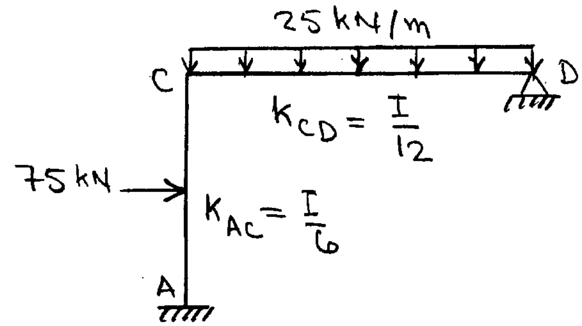
$$FEM_{EG} = \frac{150(8)}{8} - \frac{6(200)(500)(0.015)}{(8)^2} = 150 - 140.6 = 9.4 \text{ kN.m}$$

$$FEM_{GE} = -150 - 140.6 = -290.6 \text{ kN.m}$$

DF		0.273	0.727	0.681	0.319	
FEM	445.2	157.2	-184.8	-472.8	9.4	-290.6
	-445.2	7.5	20.1	315.6	147.8	290.6
		-222.6	157.8	10	145.3	
		17.7	47.1	-105.8	-49.5	
			-52.9	23.6		
		14.4	38.5	-16.1	-7.5	
			-8	19.2		
		2.2	5.8	-13.1	-6.1	
			-6.5	2.9		
		1.8	4.7	-2	-0.9	
			-1	2.4		
		0.3	0.7	-1.6	-0.8	
			-0.8	0.4		
		0.2	0.6	-0.3	-0.1	
Final Moments	0	-21.3	21.3	-237.6	237.6	0

For reactions, and shear and bending moment diagrams, see solution of Problem 13.54.

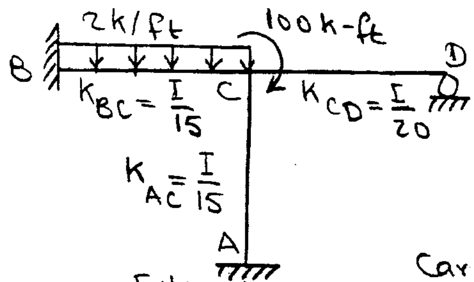
$\boxed{17.17}$



	AC	CA	CD	DC
DF		2/3	1/3	
FEM	56.3	-56.3	168.8	-168.8
		-75	-37.5	168.8
	-37.5		84.4	
		-56.3	-28.1	
	-28.2			
Final Moments	-9.4	-187.6	187.6	0

for reactions, see solution of Problem 13.42

17.18

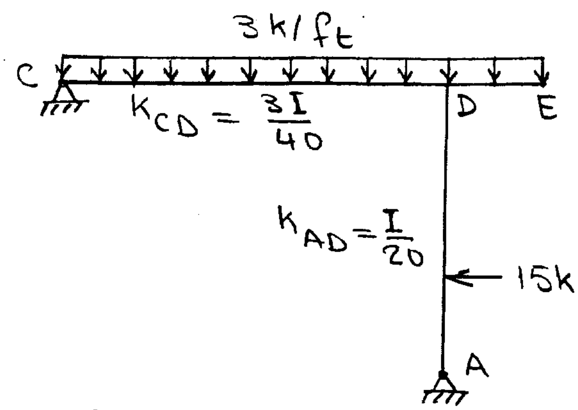


Carry Over

	BC	CB	External Moment at Joint C	CA	CD	DC	AC
DF		0.364	—	0.364	0.272		
FEM	37.5	−37.5	100	0	0	0	0
		−22.7	—	−22.7	−17.1		
	−11.3						−11.3
Final Moments	26.2	−60.2	100	−22.7	−17.1	0	−11.3

For reactions, see solution of Problem 16.18.

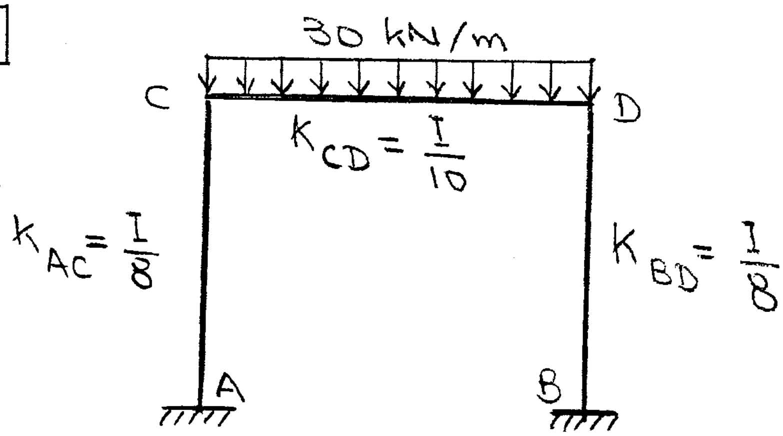
17.19



	CD	DC	DE	DA	AD
DF		0.6	–	0.4	
FEM	100	−100	37.5	16.7	−33.3
	−100	27.5		18.3	33.3
		−50		16.7	
		20		13.3	
Final Moments	0	−102.5	37.5	65	0

For reactions, see solution of Problem 13.21.

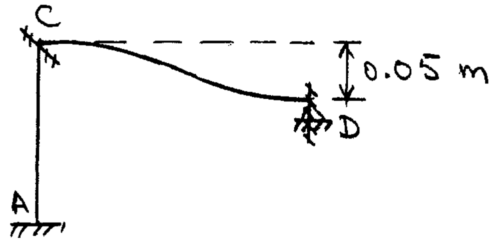
17.20



	AC	CA	CD	DC	DB	BD
DF		5/9	4/9	4/9	5/9	
FEM	0	0	250	-250	0	0
		-138.9	-111.1	111.1	138.9	
	-69.4		55.6	-55.6		69.4
		-30.9	-24.7	24.7	30.9	
	-15.4		12.3	-12.3		15.4
		-6.8	-5.5	5.5	6.8	
	-3.4		2.7	-2.7		3.4
		-1.5	-1.2	1.2	1.5	
	-0.8		0.6	-0.6		0.8
		-0.3	-0.3	0.3	0.3	
	-0.2		0.1	-0.1		0.2
Final Moments	-89.2	-178.4	178.5	-178.5	178.4	89.2

For reactions, see solution of problem 16.20.

17.21



$$FEM_{AC} = \frac{75(6)}{8} = 56.3 \text{ k-ft} \; ; \; FEM_{CA} = -56.3 \text{ k-ft}$$

$$FEM_{CD} = \frac{25(9)^2}{12} + \frac{6(200)400(0.05)}{(9)^2} = 168.8 + 296.3$$

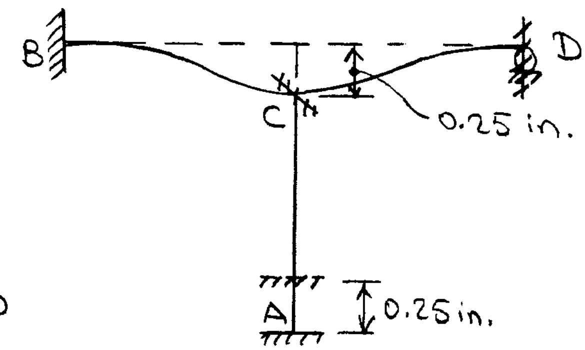
$$= 465.1 \text{ k-ft}$$

$$FEM_{DC} = -168.1 + 296.3 = 128.2 \text{ k-ft}$$

	AC	CA	CD	DC
DF		2/3	1/3	
FEM	56.3	-56.3	465.1	128.2
		-272.5	-136.3	-128.2
	-136.3		-64.1	
		42.7	21.4	
	21.4			
Final Moments	-58.6	-286.1	286.1	0

For reactions, see solution of Problem 16-21

17.22



$$FEM_{AC} = FEM_{CA} = 0$$

$$FEM_{BC} = \frac{2(15)^2}{12} + \frac{6(29000)(3500)(0.25)}{(15)^2(12)^3} = 37.5 + 391.6$$

$$= 429.1 \text{ k-ft}$$

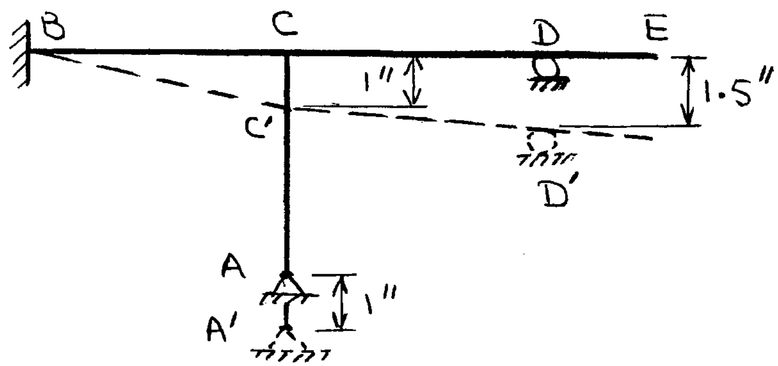
$$FEM_{CB} = -37.5 + 391.6 = 354.1 \text{ k-ft}$$

$$FEM_{CD} = FEM_{DC} = -391.6 \text{ k-ft}$$

	BC	CB	External Moment at Joint C	CA	CD	DC	AC
DF		0.364	–	0.364	0.272		
FEM	429.1	354.1	100	0	-391.6	-391.6	0
		-22.7		-22.7	-17.1	391.6	
	-11.3				195.8		-11.3
		-71.3		-71.3	-53.2		
	-35.7						-35.7
Final Moments	382.1	260.1	100	-94	-266.1	0	-47

For reactions, see solution of Problem 16.22.

17.23



$K_{AC} = \frac{3}{4}\left(\frac{I}{20}\right) = \frac{3I}{80}$

$K_{BC} = \frac{I}{30}$; $K_{CD} = \frac{3}{4}\left(\frac{I}{30}\right) = \frac{I}{40}$

$FEM_{AC} = \frac{40(20)}{8} = 100$ k-ft; $FEM_{CA} = -100$ k-ft

$FEM_{BC} = \frac{2(30)^2}{12} + \frac{6(10000)(3000)(1)}{(30)^2(12)^3} = 150 + 115.7 = 265.7$ k-ft

$FEM_{CB} = -150 + 115.7 = -34.3$ k-ft; $FEM_{CD} = \frac{6(10000)(3000)(0.5)}{(30)^2(12)^3} = 57.9$ k-ft

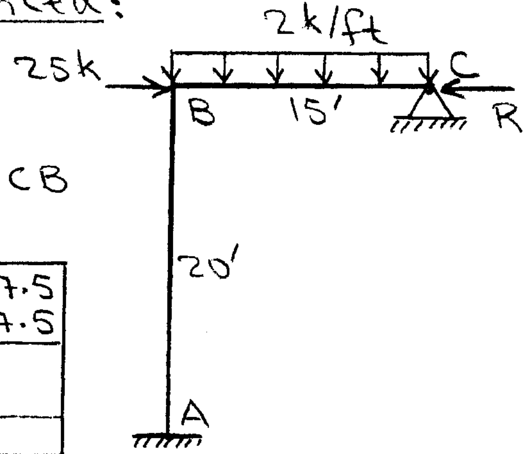
$FEM_{DC} = 57.9$ k-ft; $FEM_{DE} = 10(10) = 100$ k-ft.

BC	CB	CA	CD	DC	DE	AC	
	0.348	0.391	0.261	1	–	1	DF
265.7	−34.3	−100	57.9	57.9	100	100	FEM
	26.6	29.9	19.9	−157.9		−100	
13.3		−50	−79				
	44.9	50.4	33.7				
22.4							
301.4	37.2	−69.7	32.5	−100	100	0	Final Moments

For reactions, see solution of problem 16.23.

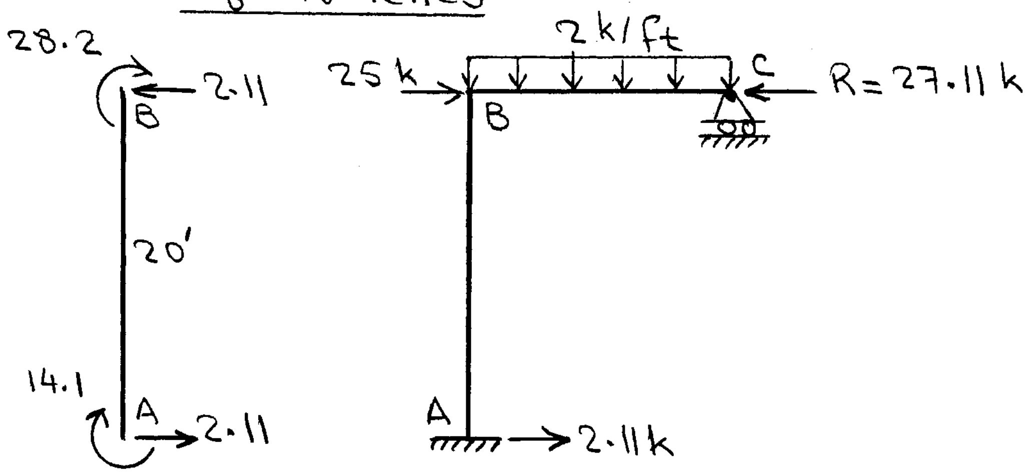
$\boxed{17.24}$ $K_{AB} = \dfrac{I}{20}$, $K_{BC} = \dfrac{3}{4}\left(\dfrac{I}{15}\right) = \dfrac{I}{20}$

Part I – Sidesway prevented:



AB	BA	BC	CB
	0.5	0.5	
		37.5	−37.5
	−18.8	−18.8	37.5
−9.4		18.8	
	−9.4	−9.4	
−4.7			
−14.1	−28.2	28.1	0

M_O Moments



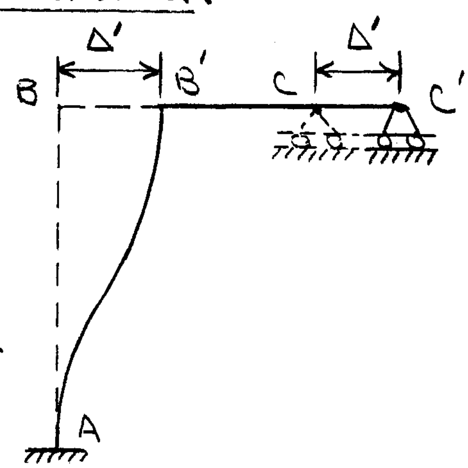
Part II – Sidesway permitted:

$FEM_{AB} = FEM_{BA}$

$= \dfrac{6EI\Delta'}{(20)^2}$

Let $FEM_{AB} = FEM_{BA}$

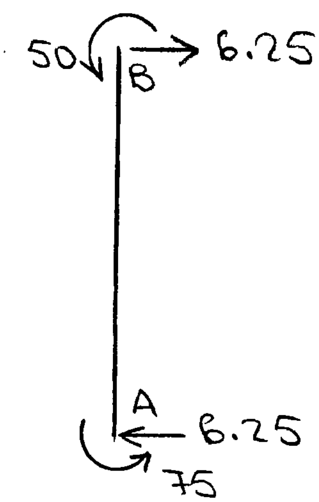
$= 100$ k-ft.

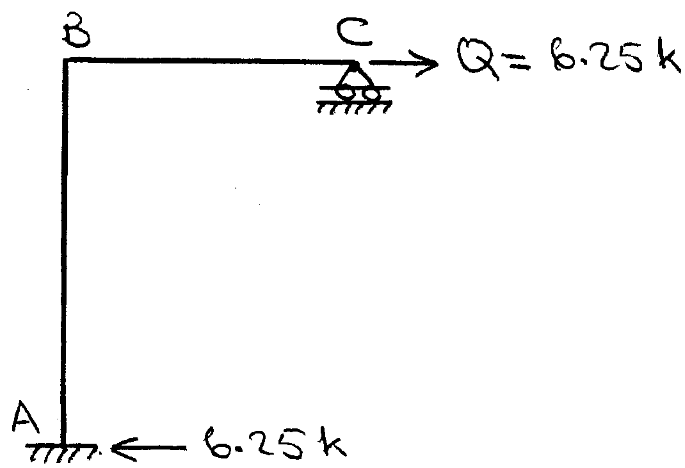


17.24 (Contd.)

AB	BA	BC	CB
	0.5	0.5	
100	100		
	-50	-50	
-25			
75	50	-50	0

M_Q Moments





Actual member end moments:

$$M_{AB} = -14.1 + \left(\frac{27.11}{6.25}\right)75 = \underline{311.2 \text{ k-ft}}$$

$$M_{BA} = -28.2 + \left(\frac{27.11}{6.25}\right)50 = \underline{188.7 \text{ k-ft}}$$

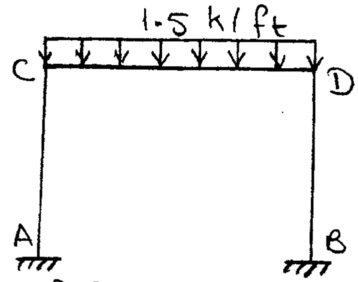
$$M_{BC} = 28.1 - \left(\frac{27.11}{6.25}\right)50 = \underline{-188.8 \text{ k-ft}}$$

$$\underline{M_{CB} = 0}$$

For reactions, see solution of Problem 16.24.

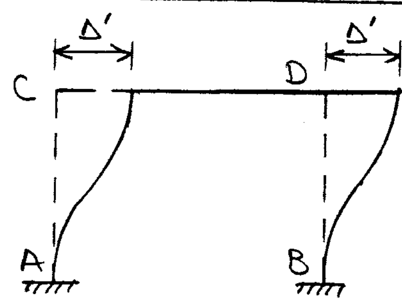
$\boxed{17.25}$ $K_{AC} = K_{BD} = \frac{I}{30}$; $K_{CD} = \frac{2I}{40} = \frac{I}{20}$

Part I - No Sidesway:



	AC	CA	CD	DC	DB	BD
DF		0.4	0.6	0.6	0.4	
FEM	0	0	200	-200	0	0
		-80	-120	120	80	
	-40		60	-60		40
		-24	-36	36	24	
	-12		18	-18		12
		-7.2	-10.8	10.8	7.2	
	-3.6		5.4	-5.4		3.6
		-2.2	-3.2	3.2	2.2	
	-1.1		1.6	-1.6		1.1
		-0.6	-1.0	1.0	0.6	
	-0.3		0.5	-0.5		0.3
		-0.2	-0.3	0.3	0.2	
	-0.1		0.2	-0.2		0.1
		-0.1	-0.1	0.1	0.1	
M_0 Moments	-57.1	-114.3	114.3	-114.3	114.3	57.1

Part II - Sidesway:



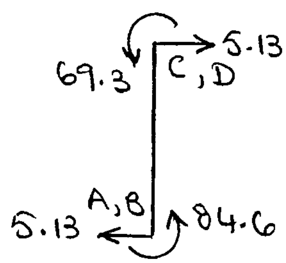
$FEM_{CD} = FEM_{DC} = 0$

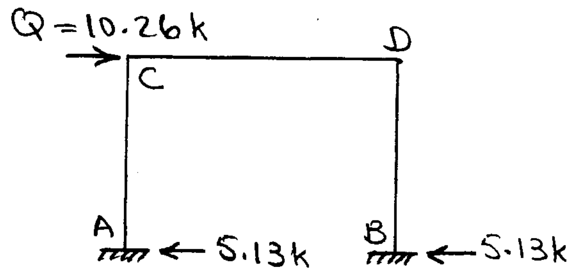
$FEM_{AC} = FEM_{CA} = FEM_{BD} = FEM_{DB} = \frac{6EI\Delta'}{(30)^2}$

let $\frac{6EI\Delta'}{(30)^2} = 100$ k-ft

17.25 (contd.)

	AC	CA	CD	DC	DB	BD
DF		0.4	0.6	0.6	0.4	
FEM	100	100	0	0	100	100
		-40	-60	-60	-40	
	-20		-30	-30		-20
		12	18	18	12	
	6		9	9		6
		-3.6	-5.4	-5.4	-3.6	
	-1.8		-2.7	-2.7		-1.8
		1.1	1.6	1.6	1.1	
	0.6		0.8	0.8		0.6
		-0.3	-0.5	-0.5	-0.3	
	-0.2		-0.3	-0.3		-0.2
		0.1	0.2	0.2	0.1	
M_Q Moments	84.6	69.3	-69.3	-69.3	69.3	84.6





Actual member end moments: $M = M_0 + \left(\frac{20}{Q}\right) M_Q$

$M_{AC} = 107.8$ k-ft; $M_{CA} = 20.8$ k-ft;

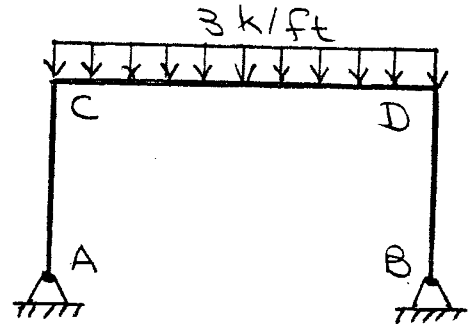
$M_{CD} = -20.8$ k-ft; $M_{DC} = -249.4$ k-ft;

$M_{DB} = 249.4$ k-ft; $M_{BD} = 222$ k-ft

For reactions, see solution of Problem 13.45

$\boxed{17.26}$ $K_{AC} = K_{BD} = \frac{3}{4}\left(\frac{I}{15}\right) = \frac{I}{20}$; $K_{CD} = \frac{I}{30}$

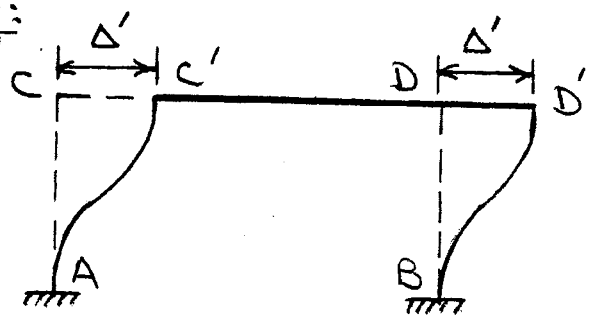
Part I – No Sidesway:



	AC	CA	CD	DC	DB	BD
DF		0.6	0.4	0.4	0.6	
FEM	0	0	225	−225	0	0
		−135	−90	90	135	
			45	−45		
		−27	−18	18	27	
			9	−9		
		−5.4	−3.6	3.6	5.4	
			1.8	−1.8		
		−1.1	−0.7	0.7	1.1	
			0.4	−0.4		
		−0.2	−0.2	0.2	0.2	
M_o Moments	0	−168.7	168.7	−168.7	168.7	0

Part II – Sidesway:

$FEM_{CD} = FEM_{DC} = 0$

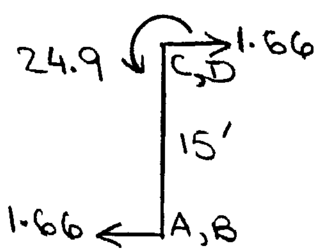


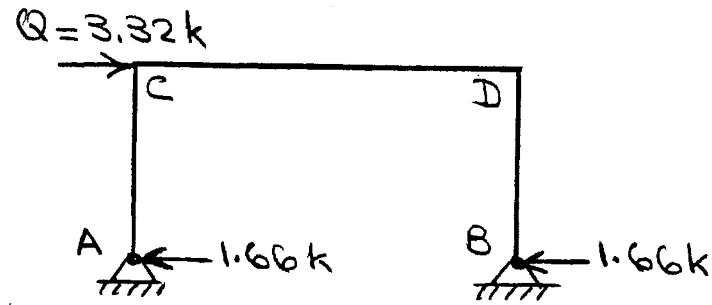
$$FEM_{AC} = FEM_{CA} = FEM_{BD} = FEM_{DB} = \frac{6EI\Delta'}{(15)^2}$$

Let $\frac{6EI\Delta'}{(15)^2} = 100$ k-ft.

17.26 (Contd.)

	AC	CA	CD	DC	DB	BD
DF		0.6	0.4	0.4	0.6	
FEM	100	100	0	0	100	100
	-100	-60	-40	-40	-60	-100
		-50	-20	-20	-50	
		42	28	28	42	
			14	14		
		-8.4	-5.6	-5.6	-8.4	
			-2.8	-2.8		
		1.7	1.1	1.1	1.7	
			0.6	0.6		
		-0.4	-0.2	-0.2	-0.4	
M_Q Moments	0	24.9	-24.9	-24.9	24.9	0





Actual member end moments: $M = M_O + \left(\frac{40}{Q}\right) M_Q$

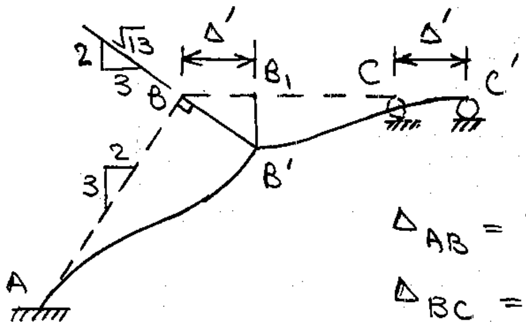
$M_{AC} = M_{BD} = 0$; $M_{CA} = 131.3$ k-ft

$M_{CD} = -131.3$ k-ft; $M_{DC} = -468.7$ k-ft

$M_{DB} = 468.7$ k-ft.

For reactions, see solution of Problem 13.22.

$\boxed{17.27}$ $K_{AB} = \frac{I}{14.42}$; $K_{BC} = \frac{3}{4}\left(\frac{I}{12}\right) = \frac{I}{16}$



$\Delta_{AB} = BB' = \frac{\sqrt{13}}{3}\Delta' = 1.202\Delta'$

$\Delta_{BC} = B_1B' = \frac{2}{3}\Delta' = 0.667\Delta'$

$$FEM_{AB} = FEM_{BA} = \frac{6EI(1.202\Delta')}{(14.42)^2}$$

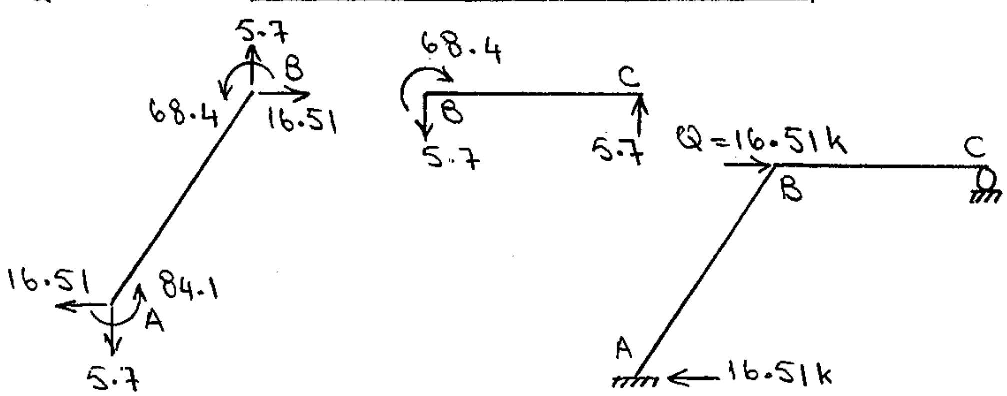
$$FEM_{BC} = FEM_{CB} = -\frac{6EI(0.667\Delta')}{(12)^2}$$

Let $FEM_{AB} = FEM_{BA} = \frac{6EI(1.202\Delta')}{(14.42)^2} = 100$ k-ft

$EI\Delta' = 2883.2$

$FEM_{BC} = FEM_{CB} = -80.1$ k-ft

	AB	BA	BC	CB
DF		0.526	0.474	
FEM	100	100	-80.1	-80.1
		-10.5	-9.4	80.1
	-5.3		40.1	
		-21.1	-19	
	-10.6			
M_Q Moments	84.1	68.4	-68.4	0



Actual member end moments: $M = \frac{25}{16.51} M_Q$

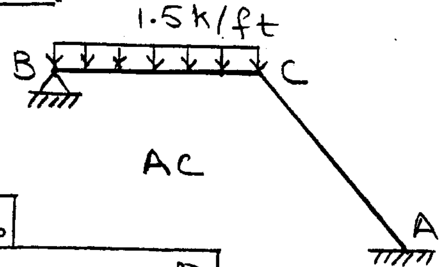
$M_{AB} = 127.3$ k-ft ; $M_{BA} = 103.6$ k-ft ;

$M_{BC} = -103.6$ k-ft , $M_{CB} = 0$

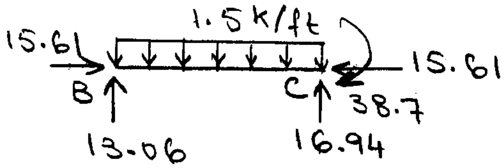
For reactions, see solution of Problem 16.27.

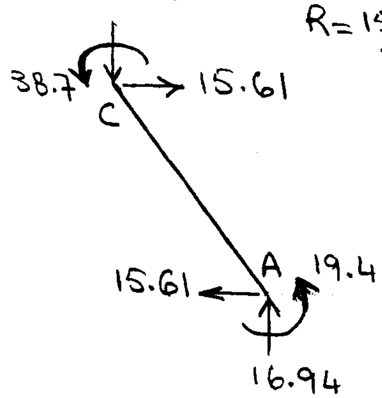
$\boxed{17.28}$ $K_{CB} = \frac{3}{4}\left(\frac{I}{20}\right) = \frac{3I}{80}$; $K_{CA} = \frac{I}{25}$

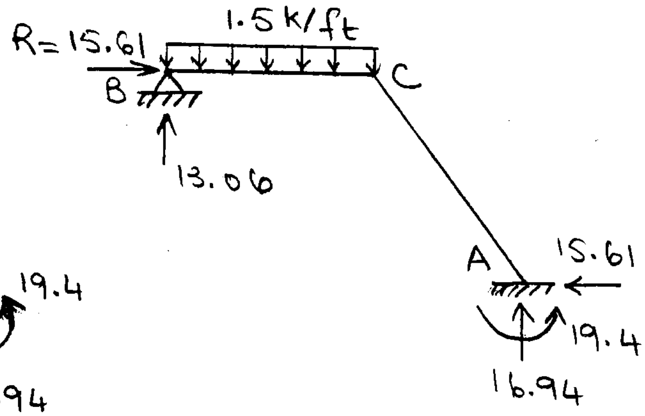
Part I – Sidesway prevented:



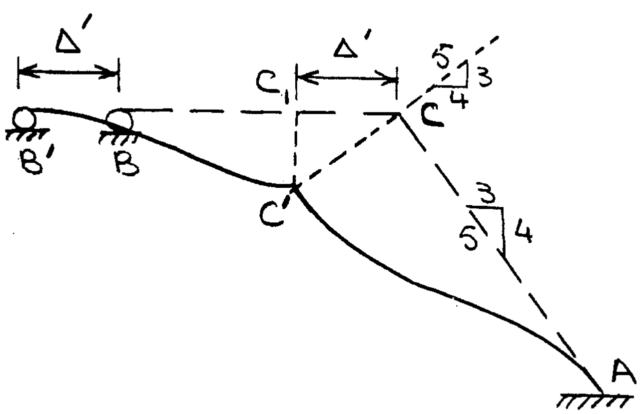
	BC	CB	CA	AC
DF		0.484	0.516	
FEM	50	−50	0	0
	−50	24.2	25.8	
		−25		12.9
		12.1	12.9	
				6.5
M_O Moments	0	−38.7	38.7	19.4







Part II – Sidesway permitted:



$\Delta_{BC} = \frac{3}{4}\Delta'$

$\Delta_{AC} = \frac{5}{4}\Delta'$

17.28 (contd.)

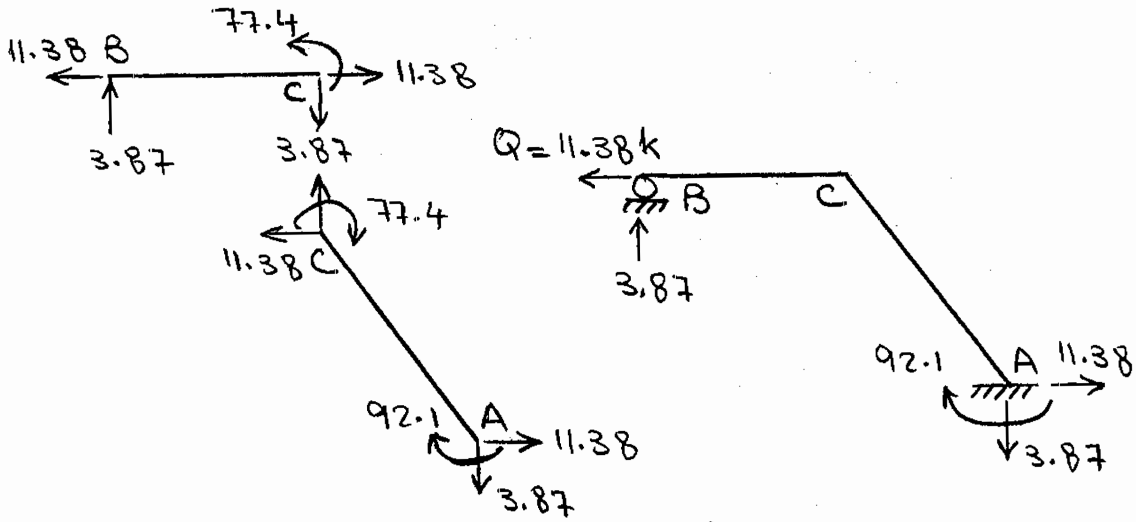
$$FEM_{BC} = FEM_{CB} = \frac{6EI(0.75\Delta')}{(20)^2}$$

Let $FEM_{BC} = FEM_{CB} = 100$ k-ft

$EI\Delta' = 8,888.89$

$$FEM_{CA} = FEM_{AC} = -\frac{6EI(1.25\Delta')}{(25)^2} = -106.7 \text{ k-ft}$$

	BC	CB	CA	AC
DF		0.484	0.516	
FEM	100	100	-106.7	-106.7
	-100	3.2	3.5	
		-50		1.7
		24.2	25.8	
				12.9
M_Q Moments	0	77.4	-77.4	-92.1

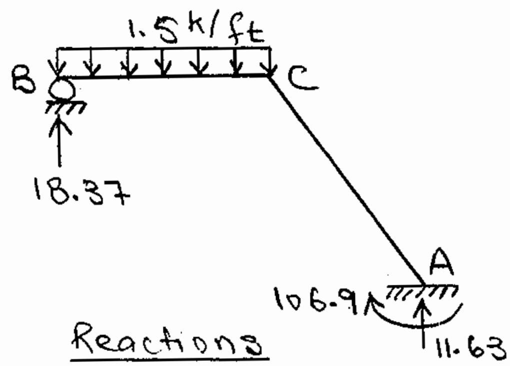


Actual Moments: $M = M_O + \left(\frac{R}{Q}\right)M_Q$

$M_{BC} = 0$; $M_{CB} = 67.5$ k-ft

$M_{CA} = -67.5$ k-ft

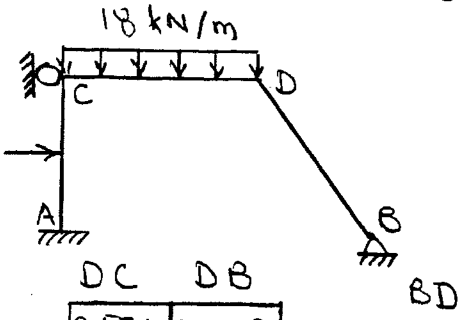
$M_{AC} = -106.9$ k-ft



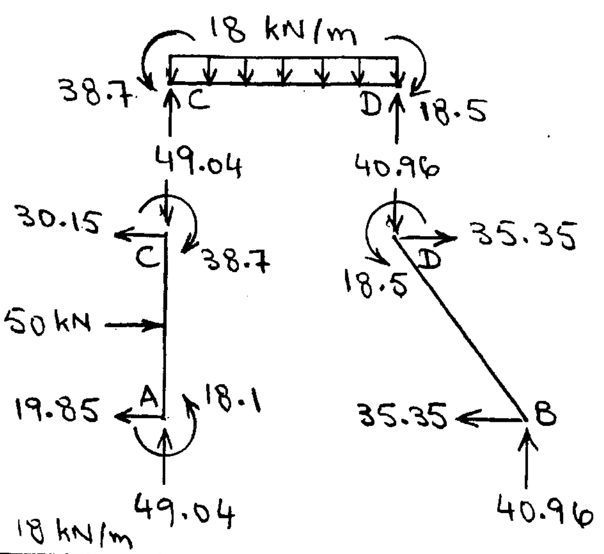
Reactions

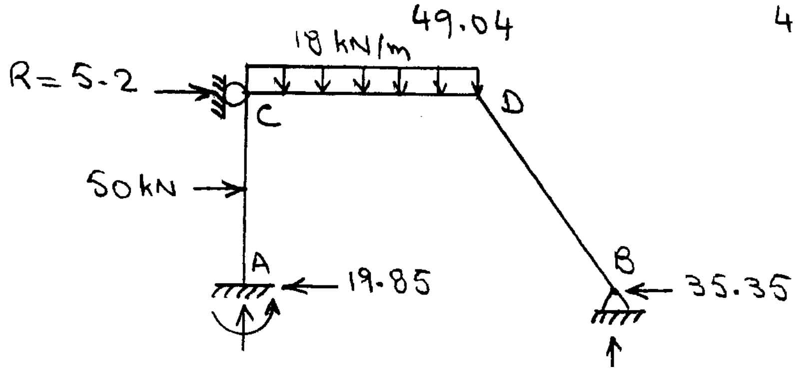
$\boxed{17.29}$ $K_{AC} = \frac{I}{4}$; $K_{CD} = \frac{I}{5}$; $K_{BD} = \frac{3}{4}\left(\frac{I}{5}\right) = \frac{3I}{20}$

Part I - Sidesway prevented:

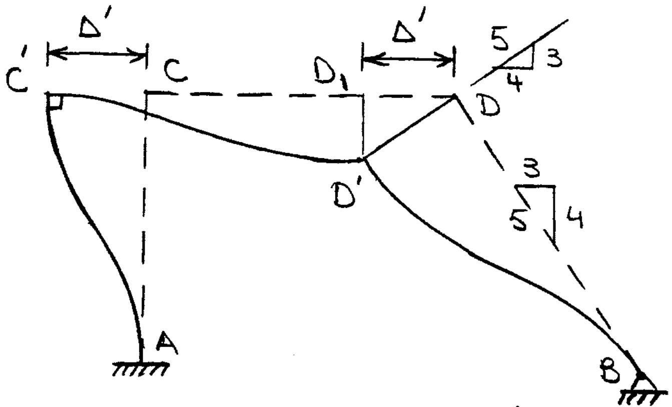


	AC	CA	CD	DC	DB	BD
DF		0.556	0.444	0.571	0.429	
FEM	25	-25	37.5	-37.5	0	0
		-7	-5.5	21.4	16.1	
	-3.5		10.7	-2.8		
		-5.9	-4.8	1.6	1.2	
	-3		0.8	-2.4		
		-0.4	-0.4	1.4	1.0	
	-0.2		0.7	-0.2		
		-0.4	-0.3	0.1	0.1	
	-0.2			-0.2		
				0.1	0.1	
M_0 Moments	18.1	-38.7	38.7	-18.5	18.5	0





17.29 (contd.) Part II – Sidesway permitted:



$\Delta_{AC} = \Delta'$

$\Delta_{CD} = \dfrac{3}{4}\Delta' = 0.75\,\Delta'$

$\Delta_{BD} = \dfrac{5}{4}\Delta' = 1.25\,\Delta'$

$FEM_{AC} = FEM_{CA} = -\dfrac{6EI\Delta'}{(4)^2}$

Let $FEM_{AC} = FEM_{CA} = -100$ kN.m

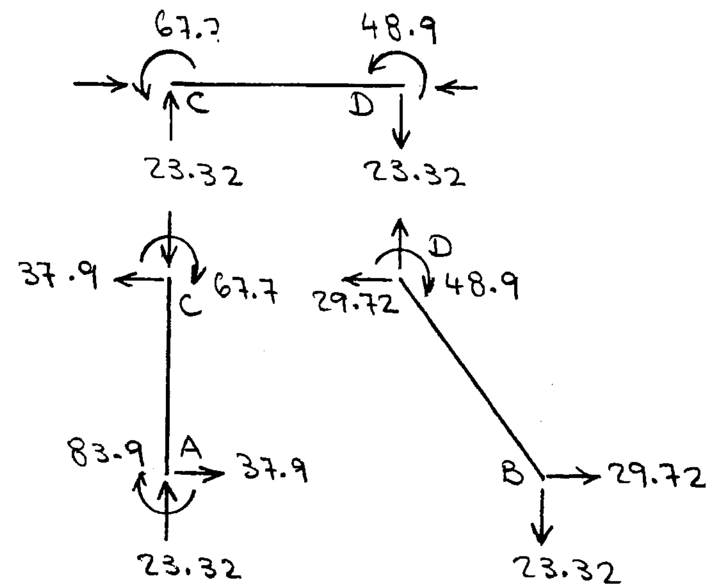
$EI\Delta' = 266.7$

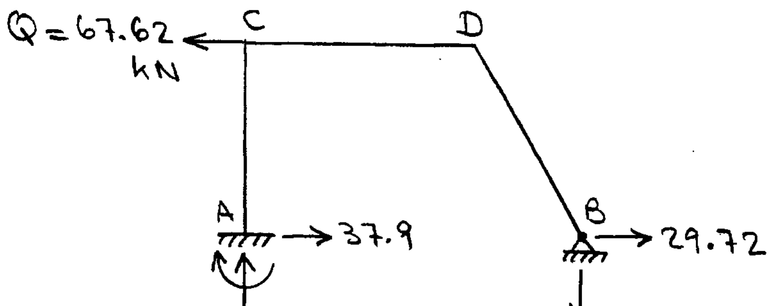
$FEM_{CD} = FEM_{DC} = \dfrac{6EI(0.75\Delta')}{(5)^2} = 48$ kN.m

$FEM_{BD} = FEM_{DB} = -\dfrac{6EI(1.25\Delta')}{(5)^2} = -80$ kN.m

	AC	CA	CD	DC	DB	BD
DF		0.556	0.444	0.571	0.429	
FEM	-100	-100	48	48	-80	-80
		28.9	23.1	18.3	13.7	80
	14.5		9.2	11.6	40	
		-5.1	-4.1	-29.5	-22.1	
	-2.6		-14.8	-2.1		
		8.2	6.6	1.2	0.9	
	4.1		0.6	3.3		
		-0.3	-0.3	-1.9	-1.4	
	-0.2		-1.0	-0.2		
		0.6	0.4	0.1	0.1	
	0.3			0.2		
				-0.1	-0.1	
M_Q Moments	-83.9	-67.7	67.7	48.9	-48.9	0

17.29 (contd.)



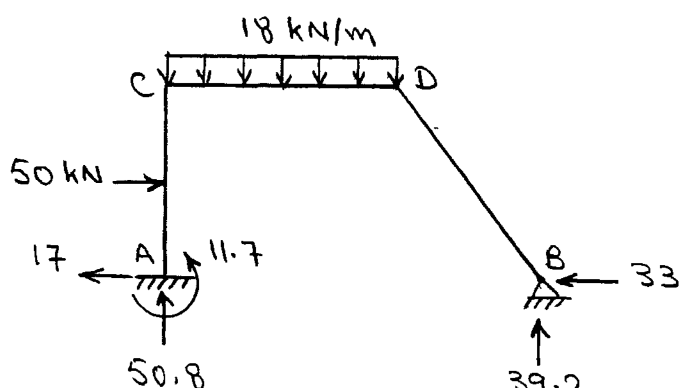


Actual member end moments: $M = M_0 + \left(\frac{R}{Q}\right) M_Q$

$M_{AC} = 11.7$ kN.m ; $M_{CA} = -43.9$ kN.m

$M_{CD} = 43.9$ kN.m ; $M_{DC} = -14.7$ kN.m

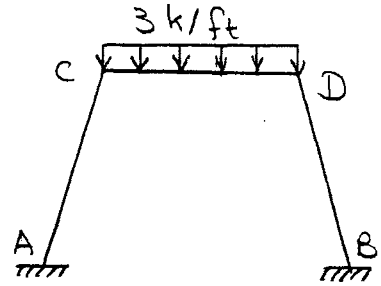
$M_{DB} = 14.7$ kN.m ; $M_{BD} = 0$



Reactions

$\boxed{17.30}$ $K_{AC} = K_{BD} = \frac{I}{16.49}$; $K_{CD} = \frac{I}{16}$

Part I – No Sidesway:



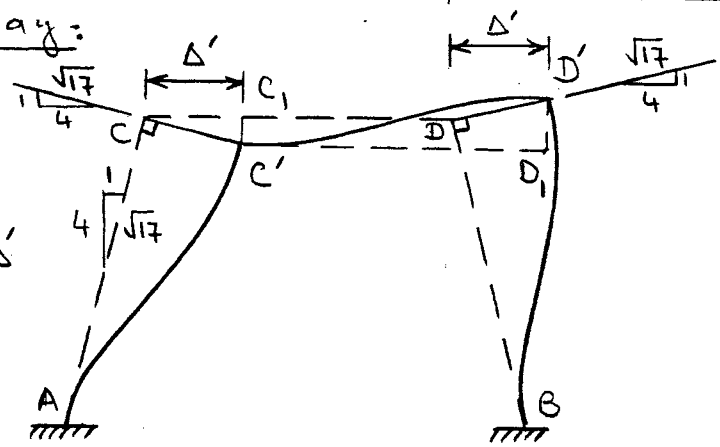
	AC	CA	CD	DC	DB	BD
DF		0.492	0.508	0.508	0.492	
FEM	0	0	64	-64	0	0
		-31.5	-32.5	32.5	31.5	
	-15.8		16.3	-16.3		15.8
		-8	-8.3	8.3	8	
	-4		4.2	-4.2		4
		-2.1	-2.1	2.1	2.1	
	-1.1		1.1	-1.1		1.1
		-0.5	-0.6	0.6	0.5	
	-0.3		0.3	-0.3		0.3
		-0.1	-0.2	0.2	0.1	
M_0 Moments	-21.2	-42.2	42.2	-42.2	42.2	21.2

Part II – Sidesway:

$\Delta_{AC} = CC' = \frac{\sqrt{17}}{4}\Delta'$
$= 1.031\,\Delta'$

$\Delta_{CD} = D_1D' = 2\left(\frac{1}{4}\right)\Delta'$
$= 0.5\,\Delta'$

$\Delta_{BD} = DD' = \frac{\sqrt{17}}{4}\Delta'$
$= 1.031\,\Delta'$



$$FEM_{AC} = FEM_{CA} = \frac{6EI(1.031\Delta')}{(16.49)^2}$$

Let $FEM_{AC} = FEM_{CA} = 100$ k-ft

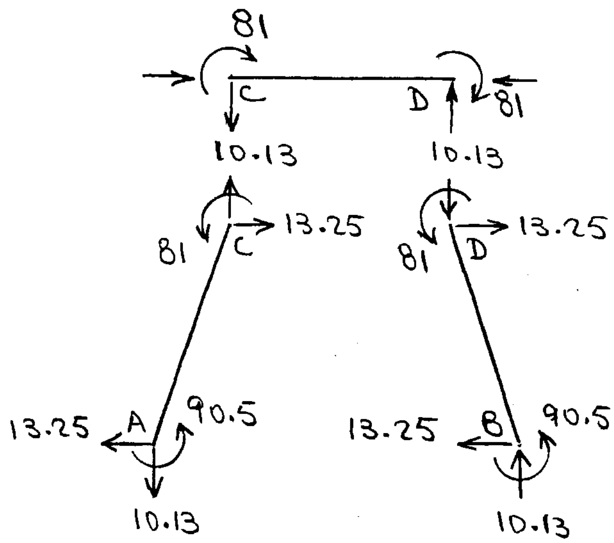
$EI\Delta' = 4395.7$

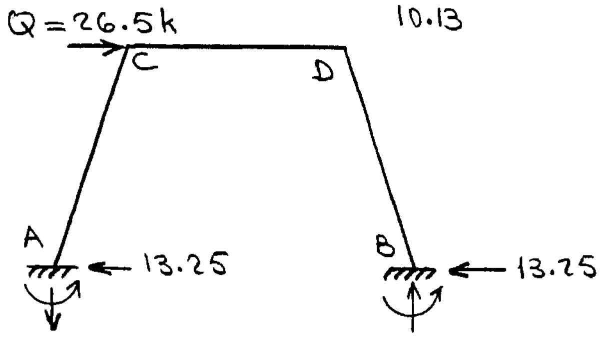
$$FEM_{BD} = FEM_{DB} = \frac{6EI(1.031\Delta')}{(16.49)^2} = 100 \text{ k-ft}$$

$$FEM_{CD} = FEM_{DC} = -\frac{6EI(0.5\Delta')}{(16)^2} = -51.5 \text{ k-ft.}$$

17.30 (contd.)

	AC	CA	CD	DC	DB	BD
DF		0.492	0.508	0.508	0.492	
FEM	100	100	-51.5	-51.5	100	100
		-23.9	-24.6	-24.6	-23.9	
	-12		-12.3	-12.3		-12
		6.1	6.2	6.2	6.1	
	3.1		3.1	3.1		3.1
		-1.5	-1.6	-1.6	-1.5	
	-0.8		-0.8	-0.8		-0.8
		0.4	0.4	0.4	0.4	
	0.2		0.2	0.2		0.2
		-0.1	-0.1	-0.1	-0.1	
M_Q Moments	90.5	81	-81	-81	81	90.5





Actual member end moments: $M = M_O + \left(\frac{20}{26.5}\right) M_Q$

$M_{AC} = 47.1$ k-ft; $M_{CA} = 19$ k-ft; $M_{CD} = -19$ k-ft;

$M_{DC} = -103.3$ k-ft; $M_{DB} = 103.3$ k-ft; $M_{BD} = 89.5$ k-ft.

For reactions, see solution of Problem 16.30.

17-31 $K_{AC} = K_{BD} = K_{CE} = K_{DF} = \frac{I}{15}$; $K_{CD} = K_{EF} = \frac{2I}{30} = \frac{I}{15}$

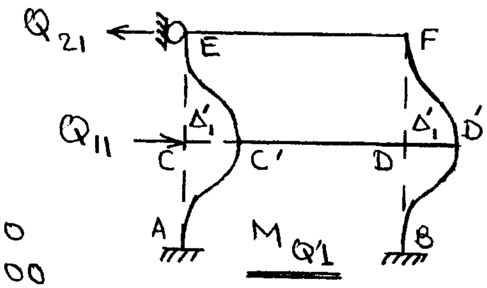
Let $\frac{6EI\Delta_1'}{(15)^2} = 100$ k-ft.

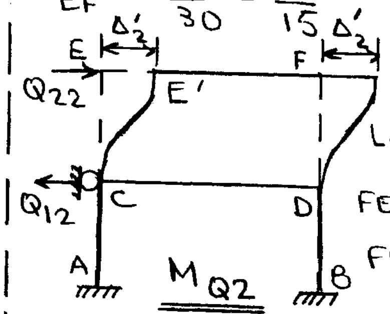
$FEM_{AC} = FEM_{CA} = 100$

$FEM_{BD} = FEM_{DB} = 100$

$FEM_{CE} = FEM_{EC} = -100$

$FEM_{DF} = FEM_{FD} = -100$





Let $\frac{6EI\Delta_2'}{(15)^2} = 100$

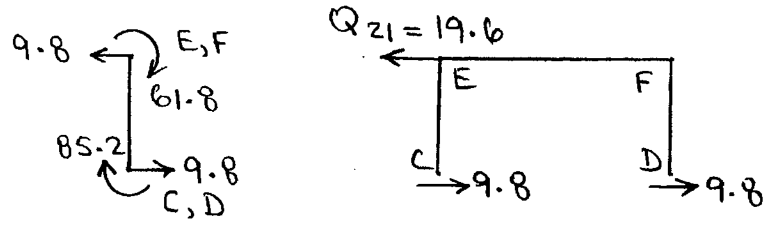
$FEM_{CE} = FEM_{EC} = 100$

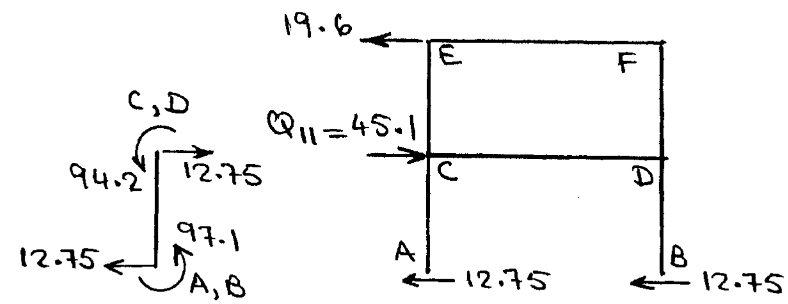
$FEM_{DF} = FEM_{FD} = 100$

Carry Over

	AC	CA	CD	CE	EC	EF	FE	FD	DF	DC	DB	BD
DF		1/3	1/3	1/3	1/2	1/2	1/2	1/2	1/3	1/3	1/3	
FEM	100	100	0	-100	-100	0	0	-100	-100	0	100	100
					50	50	50	50				
				25		25	25		25			
		-8.3	-8.4	-8.3	-12.5	-12.5	-12.5	-12.5	-8.3	-8.4	-8.3	
	-4.2		-4.2	-6.3	-4.2	-6.3	-6.3	-4.2	-6.3	-4.2		-4.2
		3.5	3.5	3.5	5.3	5.3	5.3	5.3	3.5	3.5	3.5	
	1.8		1.8	2.7	1.8	2.7	2.7	1.8	2.7	1.8		1.8
		-1.5	-1.5	-1.5	-2.3	-2.3	-2.3	-2.3	-1.5	-1.5	-1.5	
	-0.8		-0.8	-1.2	-0.8	-1.2	-1.2	-0.8	-1.2	-0.8		-0.8
		0.7	0.7	0.7	1	1	1	1	0.7	0.7	0.7	
	0.4		0.4	0.5	0.4	0.5	0.5	0.4	0.5	0.4		0.4
		-0.3	-0.3	-0.3	-0.5	-0.5	-0.5	-0.5	-0.3	-0.3	-0.3	
	-0.2		-0.2	-0.3	-0.2	-0.3	-0.3	-0.2	-0.3	-0.2		-0.2
		0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.2	0.2	0.2	
	0.1		0.1	0.2	0.1	0.2	0.2	0.1	0.2	0.1		0.1
		-0.1	-0.1	-0.1	-0.2	-0.2	-0.2	-0.2	-0.1	-0.1	-0.1	
MQ1 Moments	97.1	94.2	-8.8	-85.2	-61.8	61.7	61.7	-61.8	-85.2	-8.8	94.2	97.1

17-31 (contd.)

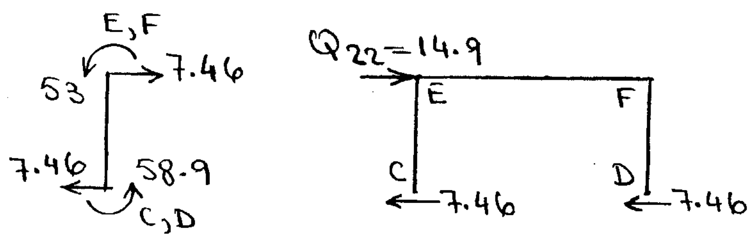


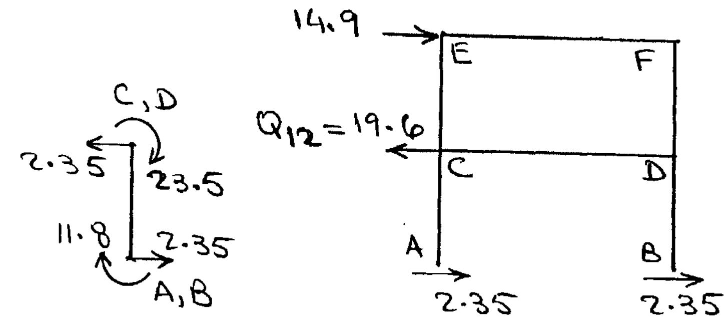


Carry Over

	AC	CA	CD	CE	EC	EF	FE	FD	DF	DC	DB	BD
DF		1/3	1/3	1/3	1/2	1/2	1/2	1/2	1/3	1/3	1/3	
FEM	0	0	0	100	100	0	0	100	100	0	0	0
		-33.3	-33.4	-33.3	-50	-50	-50	-50	-33.3	-33.4	-33.3	
	-16.7		-16.7	-25	-16.7	-25	-25	-16.7	-25	-16.7		-16.7
		13.9	13.9	13.9	20.9	20.9	20.9	20.9	13.9	13.9	13.9	
	7		7	10.5	7	10.5	10.5	7	10.5	7		7
		-5.8	-5.8	-5.8	-8.8	-8.8	-8.8	-8.8	-5.8	-5.8	-5.8	
	-2.9		-2.9	-4.4	-2.9	-4.4	-4.4	-2.9	-4.4	-2.9		-2.9
		2.4	2.4	2.4	3.7	3.7	3.7	3.7	2.4	2.4	2.4	
	1.2		1.2	1.9	1.2	1.9	1.9	1.2	1.9	1.2		1.2
		-1	-1	-1	-1.6	-1.6	-1.6	-1.6	-1	-1	-1	
	-0.5		-0.5	-0.8	-0.5	-0.8	-0.8	-0.5	-0.8	-0.5		-0.5
		0.4	0.4	0.4	0.7	0.7	0.7	0.7	0.4	0.4	0.4	
	0.2		0.2	0.4	0.2	0.4	0.4	0.2	0.4	0.2		0.2
		-0.2	-0.2	-0.2	-0.3	-0.3	-0.3	-0.3	-0.2	-0.2	-0.2	
	-0.1		-0.1	-0.2	-0.1	-0.2	-0.2	-0.1	-0.2	-0.1		-0.1
		0.1	0.1	0.1	0.2	0.2	0.2	0.2	0.1	0.1	0.1	
M_{Q2} Moments	-11.8	-23.5	-35.4	58.9	53	-52.8	-52.8	53	58.9	-35.4	-23.5	-11.8

17.31 (contd.)





By superimposing the horizontal forces at joints C and E, we write

$$45.1c_1 - 19.6c_2 = 18$$

$$-19.6c_1 + 14.9c_2 = 9$$

By solving these equations, we obtain $c_1 = 1.545$ and $c_2 = 2.636$.

Member end moments: $M = c_1 M_{Q1} + c_2 M_{Q2}$

$M_{AC} = M_{BD} = 119$ k-ft; $M_{CA} = M_{DB} = 83.5$ k-ft

$M_{CD} = M_{DC} = -106.8$ k-ft; $M_{CE} = M_{DF} = 23.5$ k-ft

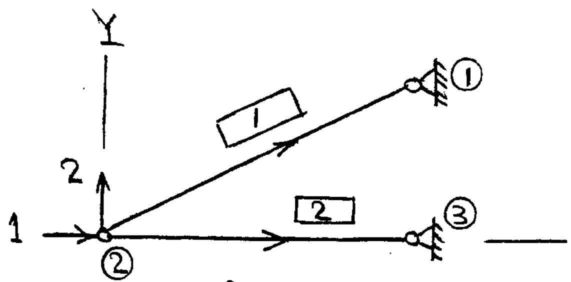
$M_{EC} = M_{FD} = 44$ k-ft; $M_{EF} = M_{FE} = -44$ k-ft.

For reactions, see solution of Problem 16.31.

Chapter Eighteen
Introduction to Matrix Structural Analysis

CHAPTER 18

$\boxed{18.1}$



Member 1: $L = 11.18$ ft; $\cos\theta = 0.894$; $\sin\theta = 0.447$

$$[K_1] = EA \begin{array}{c} \begin{array}{cccc} 1 & 2 & 0 & 0 \end{array} \\ \left[\begin{array}{cccc} 0.0715 & 0.0357 & -0.0715 & -0.0357 \\ & 0.0179 & -0.0357 & -0.0179 \\ & \text{sym.} & 0.0715 & 0.0357 \\ & & & 0.0179 \end{array}\right] \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 0 \\ 0 \end{array}$$

Member 2: $L = 10$ ft; $\cos\theta = 1$; $\sin\theta = 0$

$$[K_2] = EA \begin{array}{c} \begin{array}{cccc} 1 & 2 & 0 & 0 \end{array} \\ \left[\begin{array}{cccc} 0.1 & 0 & -0.1 & 0 \\ & 0 & 0 & 0 \\ & \text{sym.} & 0.1 & 0 \\ & & & 0 \end{array}\right] \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 0 \\ 0 \end{array}$$

Structure stiffness matrix:

$$[S] = EA \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \left[\begin{array}{cc} 0.1715 & 0.0357 \\ 0.0357 & 0.0179 \end{array}\right] \end{array} \begin{array}{c} \\ 1 \\ 2 \end{array}$$

Joint load vector: $\{P\} = \left[\begin{array}{c} 0 \\ -24 \end{array}\right] \begin{array}{c} 1 \\ 2 \end{array}$ k

Joint displacements: By solving the equations $\{P\} = [S]\{d\}$, we obtain: $\{d\} = \dfrac{1}{EA} \left[\begin{array}{c} 477.26 \\ -2292.3 \end{array}\right] \begin{array}{c} 1 \\ 2 \end{array}$

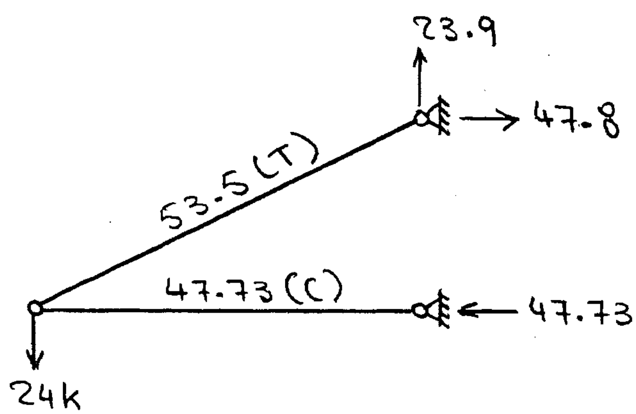
18.1 (contd.) **Member forces:**

$$\{v_1\} = \frac{1}{EA}\begin{array}{c}\begin{bmatrix} 477.26 \\ -2292.3 \\ 0 \\ 0 \end{bmatrix}\end{array}\begin{array}{c} 1 \\ 2 \\ 0 \\ 0 \end{array} \qquad \{u_1\} = [T_1]\{v_1\} = \frac{1}{EA}\begin{bmatrix} -598 \\ 0 \end{bmatrix}$$

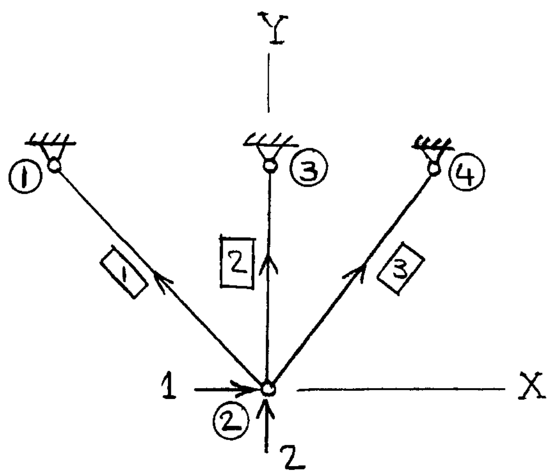
$$\{Q_1\} = [k_1]\{u_1\} = \begin{bmatrix} -53.5 \\ 53.5 \end{bmatrix} k \; ; \quad \{F_1\} = [T_1]^T\{Q_1\} = \begin{bmatrix} -47.8 \\ -23.9 \\ 47.8 \\ 23.9 \end{bmatrix} k$$

$$\{v_2\} = \frac{1}{EA}\begin{array}{c}\begin{bmatrix} 477.26 \\ -2292.3 \\ 0 \\ 0 \end{bmatrix}\end{array}\begin{array}{c} 1 \\ 2 \\ 0 \\ 0 \end{array} \qquad \{u_2\} = [T_2]\{v_2\} = \frac{1}{EA}\begin{bmatrix} 477.26 \\ 0 \end{bmatrix}$$

$$\{Q_2\} = [k_2]\{u_2\} = \begin{bmatrix} 47.73 \\ -47.73 \end{bmatrix} k \; ; \quad \{F_2\} = [T_2]^T\{Q_2\} = \begin{bmatrix} 47.73 \\ 0 \\ -47.73 \\ 0 \end{bmatrix} k$$



18.2.



Member 1: $L = 271.53$ in.; $\cos\theta = -0.707$; $\sin\theta = 0.707$

$$[K_1] = 147.3 \begin{array}{c} \begin{array}{cccc} 1 & 2 & 0 & 0 \end{array} \\ \begin{bmatrix} 1 & -1 & -1 & 1 \\ & 1 & 1 & -1 \\ \text{sym.} & & 1 & -1 \\ & & & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 0 \\ 0 \end{array}$$

Member 2: $L = 192$ in.; $\cos\theta = 0$; $\sin\theta = 1.$

$$[K_2] = 312.5 \begin{array}{c} \begin{array}{cccc} 1 & 2 & 0 & 0 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ & 1 & 0 & -1 \\ & & 0 & 0 \\ & & & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 0 \\ 0 \end{array}$$

Member 3: $L = 240$ in.; $\cos\theta = 0.6$; $\sin\theta = 0.8$

$$[K_3] = 333.3 \begin{array}{c} \begin{array}{cccc} 1 & 2 & 0 & 0 \end{array} \\ \begin{bmatrix} 0.36 & 0.48 & -0.36 & -0.48 \\ & 0.64 & -0.48 & -0.64 \\ & & 0.36 & 0.48 \\ & & & 0.64 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 0 \\ 0 \end{array}$$

Structure stiffness matrix:

$$[S] = \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 267.3 & 12.7 \\ 12.7 & 673.1 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \end{array}$$

Joint load vector: $\{P\} = \begin{bmatrix} 20 \\ -50 \end{bmatrix}$ k

18.2 (Contd.) <u>Joint displacements</u>: By solving the equations $\{P\} = [S]\{d\}$, we obtain:

$$\{d\} = \begin{bmatrix} 0.07842 \\ -0.07576 \end{bmatrix} \text{ in.}$$

<u>Member forces</u>:

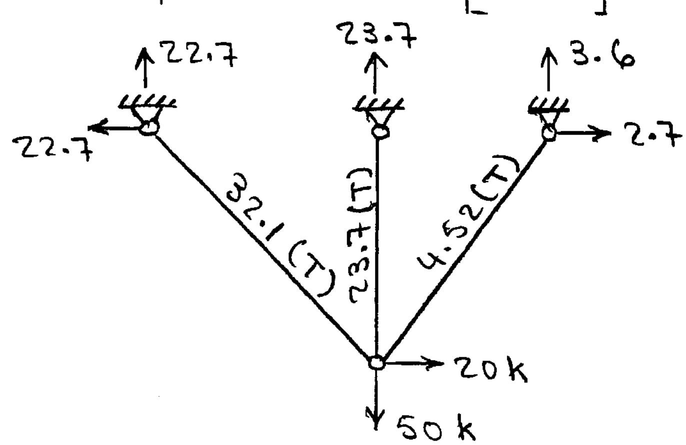
$$\{v_1\} = \{v_2\} = \{v_3\} = \begin{array}{c c} \begin{bmatrix} 0.07842 \\ -0.07576 \\ 0 \\ 0 \end{bmatrix} & \begin{array}{c} 1 \\ 2 \\ 0 \\ 0 \end{array} \end{array} \text{ in.}$$

$$\{u_1\} = [T_1]\{v_1\} = \begin{bmatrix} -0.109 \\ 0 \end{bmatrix} \text{in.}; \quad \{Q_1\} = [k_1]\{u_1\} = \begin{bmatrix} -32.1 \\ 32.1 \end{bmatrix} \text{k}$$

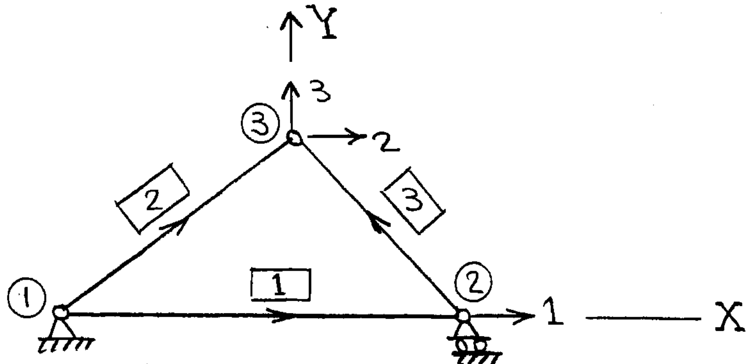
$$\{F_1\} = [T]^T\{Q_1\} = \begin{bmatrix} 22.7 \\ -22.7 \\ -22.7 \\ 22.7 \end{bmatrix} \text{k} \qquad \{u_2\} = \begin{bmatrix} -0.07576 \\ 0 \end{bmatrix} \text{in.}$$

$$\{Q_2\} = \begin{bmatrix} -23.7 \\ 23.7 \end{bmatrix} \text{k}; \qquad \{F_2\} = \begin{bmatrix} 0 \\ -23.7 \\ 0 \\ 23.7 \end{bmatrix} \text{k}$$

$$\{u_3\} = \begin{bmatrix} -0.01356 \\ 0 \end{bmatrix} \text{in.}; \quad \{Q_3\} = \begin{bmatrix} -4.52 \\ 4.52 \end{bmatrix} \text{k}; \quad \{F_3\} = \begin{bmatrix} -2.71 \\ -3.61 \\ 2.71 \\ 3.61 \end{bmatrix} \text{k}$$



$\boxed{18.3}$



Member 1: $L = 14\,m$; $\cos\theta = 1$; $\sin\theta = 0$

$$[K_1] = \frac{EA}{14}\begin{array}{c} \begin{array}{cccc} 0 & 0 & 1 & 0 \end{array} \\ \left[\begin{array}{cccc} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}\begin{array}{c} \\ 0 \\ 0 \\ 1 \\ 0 \end{array}$$

Member 2: $L = 10\,m$; $\cos\theta = 0.8$; $\sin\theta = 0.6$

$$[K_2] = \frac{EA}{10}\begin{array}{c} \begin{array}{cccc} 0 & 0 & 2 & 3 \end{array} \\ \left[\begin{array}{cccc} 0.64 & 0.48 & -0.64 & -0.48 \\ 0.48 & 0.36 & -0.48 & -0.36 \\ -0.64 & -0.48 & 0.64 & 0.48 \\ -0.48 & -0.36 & 0.48 & 0.36 \end{array}\right] \end{array}\begin{array}{c} \\ 0 \\ 0 \\ 2 \\ 3 \end{array}$$

Member 3: $L = 8.485\,m$; $\cos\theta = -0.707$; $\sin\theta = 0.707$

$$[K_3] = \frac{EA}{16.97}\begin{array}{c} \begin{array}{cccc} 1 & 0 & 2 & 3 \end{array} \\ \left[\begin{array}{cccc} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{array}\right] \end{array}\begin{array}{c} \\ 1 \\ 0 \\ 2 \\ 3 \end{array}$$

Structure stiffness matrix:

$$[S] = EA\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \left[\begin{array}{ccc} 0.1304 & -0.05893 & 0.05893 \\ -0.05893 & 0.1229 & -0.01093 \\ 0.05893 & -0.01093 & 0.09493 \end{array}\right] \end{array}\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}$$

Joint load vector:

$$\{P\} = \left[\begin{array}{c} 0 \\ 80 \\ -120 \end{array}\right]\begin{array}{c} 1 \\ 2 \\ 3 \end{array} \text{ kN}$$

18.3 (Contd.) <u>Joint displacements:</u> By solving the equations $\{P\} = [S]\{d\}$, we obtain:

$$\{d\} = \frac{1}{EA}\begin{bmatrix}1439\\1161\\-2024\end{bmatrix}\begin{matrix}1\\2\\3\end{matrix}$$

<u>Member forces:</u>

$$\{v_1\} = \frac{1}{EA}\begin{bmatrix}0\\0\\1439\\0\end{bmatrix}\begin{matrix}0\\0\\1\\0\end{matrix} \qquad \{u_1\} = [T_1]\{v_1\} = \frac{1}{EA}\begin{bmatrix}0\\1439\end{bmatrix}$$

$$\{Q_1\} = [k_1]\{u_1\} = \begin{bmatrix}-102.8\\102.8\end{bmatrix}\text{kN}$$

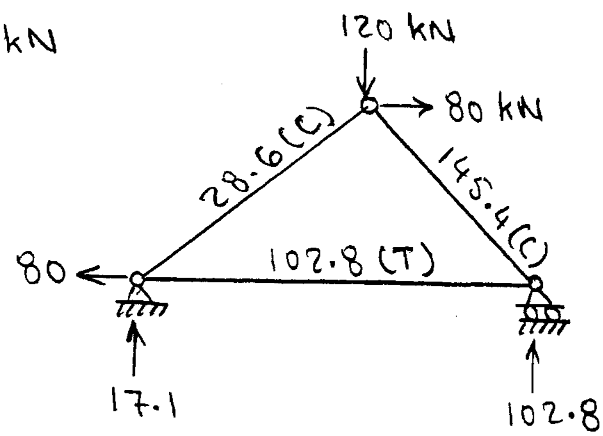
$$\{F_1\} = \begin{bmatrix}-102.8\\0\\102.8\\0\end{bmatrix}\text{kN}; \qquad \{v_2\} = \frac{1}{EA}\begin{bmatrix}0\\0\\1161\\-2024\end{bmatrix}\begin{matrix}0\\0\\2\\3\end{matrix}$$

$$\{u_2\} = \frac{1}{EA}\begin{bmatrix}0\\-285.6\end{bmatrix}; \qquad \{Q_2\} = \begin{bmatrix}28.56\\-28.56\end{bmatrix}\text{kN}$$

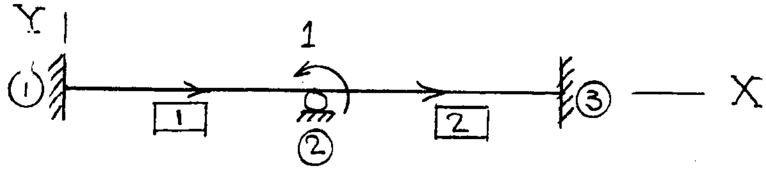
$$\{F_2\} = \begin{bmatrix}22.85\\17.14\\-22.85\\-17.14\end{bmatrix}\text{kN}; \qquad \{v_3\} = \frac{1}{EA}\begin{bmatrix}1439\\0\\1161\\-2024\end{bmatrix}\begin{matrix}1\\0\\2\\3\end{matrix}$$

$$\{u_3\} = \frac{1}{EA}\begin{bmatrix}-1017.4\\-2251.8\end{bmatrix}; \qquad \{Q_3\} = \begin{bmatrix}145.4\\-145.4\end{bmatrix}\text{kN}$$

$$\{F_3\} = \begin{bmatrix}-102.8\\102.8\\102.8\\-102.8\end{bmatrix}\text{kN}$$



$\boxed{18.4}$



$$\begin{array}{r|cccc}
\text{Member 2} \rightarrow & 0 & 1 & 0 & 0 \\
\text{Member 1} \rightarrow & 0 & 0 & 0 & 1
\end{array}$$

$$[K_1] = [K_2] = \frac{EI}{L^3}\begin{array}{c} \begin{bmatrix} 12 & 180 & -12 & 180 \\ & 3600 & -180 & 1800 \\ & \text{sym.} & 12 & -180 \\ & & & 3600 \end{bmatrix} \end{array}\begin{array}{cc} 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 1 & 0 \end{array}$$

$$\{F_{f1}\} = \{Q_{f1}\} = \begin{bmatrix} 4.67 \\ 40 \\ 13.33 \\ -80 \end{bmatrix}\begin{array}{c} 0 \\ 0 \\ 0 \\ 1 \end{array} \qquad \{F_{f2}\} = \{Q_{f2}\} = \begin{bmatrix} 5 \\ 37.5 \\ 5 \\ -37.5 \end{bmatrix}\begin{array}{c} 0 \\ 1 \\ 0 \\ 0 \end{array}$$

$$[S] = \frac{EI}{L^3}[7200]1 \; ; \qquad \{P_f\} = [-42.5]1$$

Joint displacements: $[S]\{d\} = -\{P_f\}$

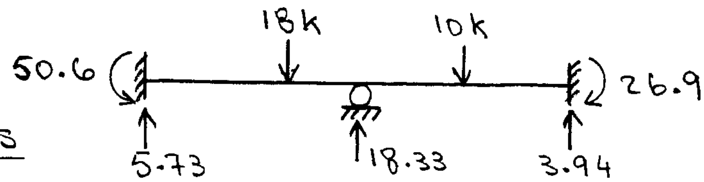
$$\{d\} = \frac{L^3}{EI}[0.005903]$$

Member forces:

$$\{u_1\} = \{v_1\} = \frac{L^3}{EI}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.005903 \end{bmatrix}\begin{array}{c} 0 \\ 0 \\ 0 \\ 1 \end{array} \; ; \quad \{u_2\} = \{v_2\} = \frac{L^3}{EI}\begin{bmatrix} 0 \\ 0.005903 \\ 0 \\ 0 \end{bmatrix}\begin{array}{c} 0 \\ 1 \\ 0 \\ 0 \end{array}$$

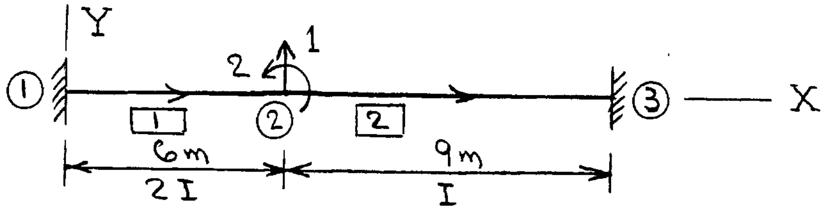
$$\{F_1\} = \{Q_1\} = [k_1]\{u_1\} + \{Q_{f1}\} = \begin{bmatrix} 5.73 \text{ k} \\ 50.6 \text{ k-ft} \\ 12.27 \text{ k} \\ -58.8 \text{ k-ft} \end{bmatrix}$$

$$\{F_2\} = \{Q_2\} = \begin{bmatrix} 6.06 \text{ k} \\ 58.8 \text{ k-ft} \\ 3.94 \text{ k} \\ -26.9 \text{ k-ft} \end{bmatrix}$$



Reactions

18.5



Member 1:

$$[K_1] = EI \begin{array}{cc} \begin{matrix} 0 & \quad 0 & \quad 1 & \quad 2 \end{matrix} & \\ \begin{bmatrix} 0.111 & 0.333 & -0.111 & 0.333 \\ & 1.333 & -0.333 & 0.667 \\ & \text{sym.} & 0.111 & -0.333 \\ & & & 1.333 \end{bmatrix} & \begin{matrix} 0 \\ 0 \\ 1 \\ 2 \end{matrix} \end{array}$$

Member 2:

$$[K_2] = EI \begin{array}{cc} \begin{matrix} 1 & \quad 2 & \quad 0 & \quad 0 \end{matrix} & \\ \begin{bmatrix} 0.0165 & 0.0741 & -0.0165 & 0.0741 \\ & 0.444 & -0.0741 & 0.222 \\ & \text{sym.} & 0.0165 & -0.0741 \\ & & & 0.444 \end{bmatrix} & \begin{matrix} 1 \\ 2 \\ 0 \\ 0 \end{matrix} \end{array}$$

Structure stiffness matrix:

$$[S] = EI \begin{array}{cc} \begin{matrix} 1 & \quad 2 \end{matrix} & \\ \begin{bmatrix} 0.1275 & -0.2589 \\ -0.2589 & 1.777 \end{bmatrix} & \begin{matrix} 1 \\ 2 \end{matrix} \end{array}$$

Joint load vector:

$$\{P\} = \begin{bmatrix} -150 \\ 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \end{matrix}$$

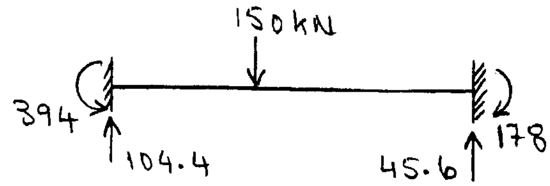
Joint displacements: By solving the equations $\{P\} = [S]\{d\}$, we obtain: $\{d\} = \frac{1}{EI}\begin{bmatrix} -1671 \\ -243.4 \end{bmatrix}\begin{matrix} 1 \\ 2 \end{matrix}$

Member forces:

$$\{u_1\} = \{v_1\} = \frac{1}{EI}\begin{bmatrix} 0 \\ 0 \\ -1671 \\ -243.4 \end{bmatrix}\begin{matrix} 0 \\ 0 \\ 1 \\ 2 \end{matrix} \; ; \quad \{u_2\} = \{v_2\} = \frac{1}{EI}\begin{bmatrix} -1671 \\ -243.4 \\ 0 \\ 0 \end{bmatrix}\begin{matrix} 1 \\ 2 \\ 0 \\ 0 \end{matrix}$$

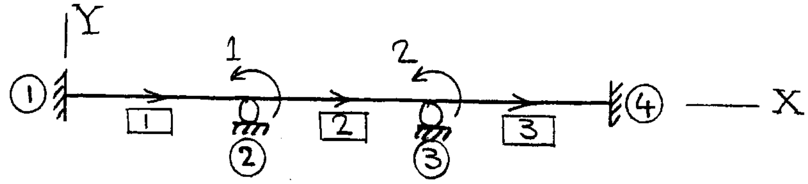
$$\{F_1\} = \{Q_1\} = [k_1]\{u_1\} = \begin{bmatrix} 104.4 \text{ kN} \\ 394 \text{ kN.m} \\ -104.4 \text{ kN} \\ 232 \text{ kN.m} \end{bmatrix}$$

$$\{F_2\} = \{Q_2\} = \begin{bmatrix} -45.6 \text{ kN} \\ -232 \text{ kN.m} \\ 45.6 \text{ kN} \\ -178 \text{ kN.m} \end{bmatrix}$$



Reactions

$\boxed{18.6}$



Member 3 → 0 2 0 0

Member 2 → 0 1 0 2

Member 1 → 0 0 0 1

$$[K_1]=[K_2]=[K_3]=\frac{EI}{L^3}\begin{array}{c}\begin{bmatrix}12 & 108 & -12 & 108\\ & 1296 & -108 & 648\\ & \text{sym.} & 12 & -108\\ & & & 1296\end{bmatrix}\end{array}\begin{array}{ccc}0 & 0 & 0\\ 0 & 1 & 2\\ 0 & 0 & 0\\ 1 & 2 & 0\end{array}$$

$$\{F_{f1}\}=\{Q_{f1}\}=\begin{bmatrix}27\\81\\27\\-81\end{bmatrix}\begin{array}{c}0\\0\\0\\1\end{array};\qquad \{F_{f3}\}=\{Q_{f3}\}=\begin{bmatrix}13.5\\40.5\\13.5\\-40.5\end{bmatrix}\begin{array}{c}0\\2\\0\\0\end{array}$$

Structure stiffness matrix:

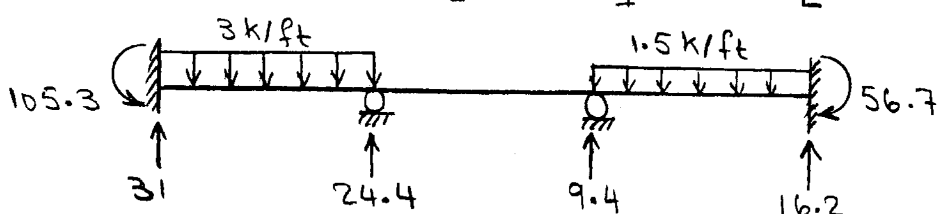
$$[S]=\frac{EI}{L^3}\begin{array}{c}\begin{array}{cc}1 & 2\end{array}\\ \begin{bmatrix}2592 & 648\\ 648 & 2592\end{bmatrix}\end{array}\begin{array}{c}\\1\\2\end{array}\qquad \{P_f\}=\begin{bmatrix}-81\\40.5\end{bmatrix}$$

Joint displacements: By solving the equations $-\{P_f\}=[S]\{d\}$, we obtain: $\{d\}=\frac{L^3}{EI}\begin{bmatrix}0.0375\\-0.025\end{bmatrix}\begin{array}{c}1\\2\end{array}$

Member forces:

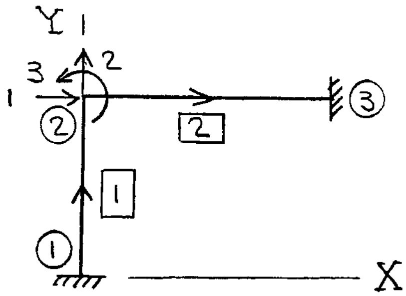
$$\{u_1\}=\frac{L^3}{EI}\begin{bmatrix}0\\0\\0\\0.0375\end{bmatrix};\quad \{u_2\}=\frac{L^3}{EI}\begin{bmatrix}0\\0.0375\\0\\-0.025\end{bmatrix};\quad \{u_3\}=\frac{L^3}{EI}\begin{bmatrix}0\\-0.025\\0\\0\end{bmatrix}$$

$$\{Q_1\}=\begin{bmatrix}31\text{ k}\\105.3\text{ k-ft}\\23\text{ k}\\-32.4\text{ k-ft}\end{bmatrix};\quad \{Q_2\}=\begin{bmatrix}1.4\text{ k}\\32.4\text{ k-ft}\\-1.4\text{ k}\\-8.1\text{ k-ft}\end{bmatrix};\quad \{Q_3\}=\begin{bmatrix}10.8\text{ k}\\8.1\text{ k-ft}\\16.2\text{ k}\\-56.7\text{ k-ft}\end{bmatrix}$$



Reactions

$\boxed{18.7}$



Member 1: $L = 15\text{ ft};\ \cos\theta = 0;\ \sin\theta = 1$

$$[k_1] = EI\times10^{-3}\begin{bmatrix} 115.2 & 0 & 0 & -115.2 & 0 & 0 \\ & 3.556 & 26.67 & 0 & -3.556 & 26.67 \\ & & 266.7 & 0 & -26.67 & 133.3 \\ & \text{Sym.} & & 115.2 & 0 & 0 \\ & & & & 3.556 & -26.67 \\ & & & & & 266.7 \end{bmatrix}$$

$$\{Q_{f1}\} = \begin{bmatrix} 0 \\ 15 \\ 37.5 \\ 0 \\ 15 \\ -37.5 \end{bmatrix};\ [T_1] = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix};\ \{F_{f1}\} = \begin{array}{c c} \begin{bmatrix} -15 \\ 0 \\ 37.5 \\ -15 \\ 0 \\ -37.5 \end{bmatrix} & \begin{matrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{matrix} \end{array}$$

$$[K_1] = [T_1]^T[k_1][T_1] = EI\times10^{-3}\begin{array}{c c} \begin{matrix} 0 & 0 & 0 & 1 & 2 & 3 \end{matrix} & \\ \begin{bmatrix} 3.556 & 0 & -26.67 & -3.556 & 0 & -26.67 \\ & 115.2 & 0 & 0 & -115.2 & 0 \\ & & 266.7 & 26.67 & 0 & 133.3 \\ & & & 3.556 & 0 & 26.67 \\ & \text{Sym.} & & & 115.2 & 0 \\ & & & & & 266.7 \end{bmatrix} & \begin{matrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{matrix} \end{array}$$

Member 2: $L = 20\text{ ft};\ \cos\theta = 1;\ \sin\theta = 0$

$$[k_2] = [K_2] = EI\times10^{-3}\begin{array}{c c} \begin{matrix} 1 & 2 & 3 & 0 & 0 & 0 \end{matrix} & \\ \begin{bmatrix} 86.4 & 0 & 0 & -86.4 & 0 & 0 \\ & 1.5 & 15 & 0 & -1.5 & 15 \\ & & 200 & 0 & -15 & 100 \\ & & & 86.4 & 0 & 0 \\ & \text{Sym.} & & & 1.5 & -15 \\ & & & & & 200 \end{bmatrix} & \begin{matrix} 1 \\ 2 \\ 3 \\ 0 \\ 0 \\ 0 \end{matrix} \end{array}$$

$$\{F_{f2}\} = \{Q_{f2}\} = \begin{array}{c c} \begin{bmatrix} 0 \\ 10 \\ 50 \\ 0 \\ 10 \\ -50 \end{bmatrix} & \begin{matrix} 1 \\ 2 \\ 3 \\ 0 \\ 0 \\ 0 \end{matrix} \end{array}$$

$\boxed{18.7 \text{ (Contd.)}}$ Structure matrices:

$$[S] = EI\times10^{-3}\begin{array}{c} \\ \\ \\ \\ \end{array}\begin{array}{ccc|c} 1 & 2 & 3 & \\ \end{array}$$

$$[S] = EI\times10^{-3}\begin{bmatrix} 89.96 & 0 & 26.67 \\ 0 & 116.7 & 15 \\ 26.67 & 15 & 466.7 \end{bmatrix}\begin{matrix}1\\2\\3\end{matrix} ; \quad \{P_f\} = \begin{bmatrix} -15 \\ 10 \\ 12.5 \end{bmatrix}\begin{matrix}1\\2\\3\end{matrix}$$

Joint displacements: By solving the equations $-\{P_f\} = [S]\{d\}$, we obtain:

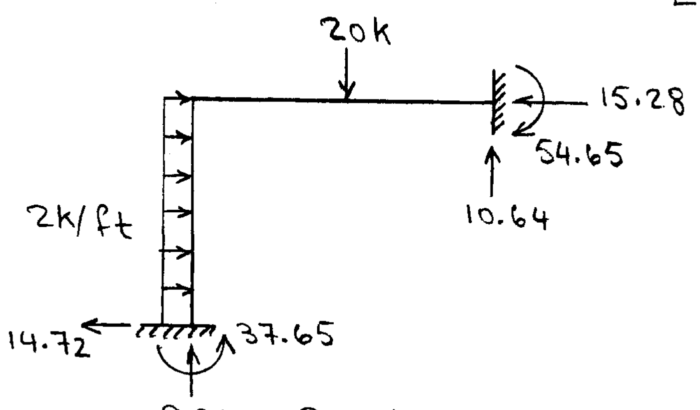
$$\{d\} = \frac{1}{EI\times10^{-3}}\begin{bmatrix} 0.1769 \\ -0.08129 \\ -0.03427 \end{bmatrix}$$

Member forces:

$$\{v_1\} = \frac{1}{EI\times10^{-3}}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.1769 \\ -0.08129 \\ -0.03427 \end{bmatrix}\begin{matrix}0\\0\\0\\1\\2\\3\end{matrix} ; \quad \{u_1\} = \frac{1}{EI\times10^{-3}}\begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.08129 \\ -0.1769 \\ -0.03427 \end{bmatrix}$$

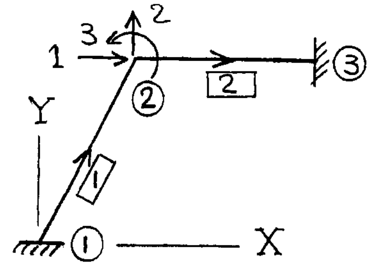
$$\{Q_1\} = \begin{bmatrix} 9.36 \text{ k} \\ 14.72 \text{ k} \\ 37.65 \text{ k-ft} \\ -9.36 \text{ k} \\ 15.29 \text{ k} \\ -41.92 \text{ k-ft} \end{bmatrix} ; \quad \{F_1\} = \begin{bmatrix} -14.72 \text{ k} \\ 9.36 \text{ k} \\ 37.65 \text{ k-ft} \\ -15.29 \text{ k} \\ -9.36 \text{ k} \\ -41.92 \text{ k-ft} \end{bmatrix}$$

$$\{u_2\} = \{v_2\} = \frac{1}{EI\times10^{-3}}\begin{bmatrix} 0.1769 \\ -0.08129 \\ -0.03427 \\ 0 \\ 0 \\ 0 \end{bmatrix}\begin{matrix}1\\2\\3\\0\\0\\0\end{matrix} ; \quad \{F_2\} = \{Q_2\} = \begin{bmatrix} 15.28 \text{ k} \\ 9.36 \text{ k} \\ 41.92 \text{ k-ft} \\ -15.28 \text{ k} \\ 10.64 \text{ k} \\ -54.65 \text{ k-ft} \end{bmatrix}$$



Reactions

$\boxed{18.8}$



Member 1: $L = 11.18\text{ m}$; $\cos\theta = 0.447$; $\sin\theta = 0.894$

$$[k_1] = \begin{bmatrix} 71556 & 0 & 0 & -71556 & 0 & 0 \\ & 687 & 3840 & 0 & -687 & 3840 \\ & & 28623 & 0 & -3840 & 14311 \\ & & & 71556 & 0 & 0 \\ & \text{sym.} & & & 687 & -3840 \\ & & & & & 28623 \end{bmatrix}$$

$$[T_1] = \begin{bmatrix} 0.447 & 0.894 & 0 & 0 & 0 & 0 \\ -0.894 & 0.447 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.447 & 0.894 & 0 \\ 0 & 0 & 0 & -0.894 & 0.447 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[K_1] = [T_1]^T[k_1][T_1] = \begin{array}{c} \begin{array}{cccccc} 0 & 0 & 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 14847 & 28320 & -3433 & -14847 & -28320 & -3433 \\ & 57327 & 1716 & -28320 & -57327 & 1716 \\ & & 28623 & 3433 & -1716 & 14311 \\ & & & 14847 & 28320 & 3433 \\ & \text{sym.} & & & 57327 & -1716 \\ & & & & & 28623 \end{bmatrix} \end{array} \begin{array}{c} \\ 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{array}$$

Member 2: $L = 10\text{ m}$; $\cos\theta = 1$; $\sin\theta = 0$

$$[k_2] = [K_2] = \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 0 & 0 & 0 \end{array} \\ \begin{bmatrix} 80000 & 0 & 0 & -80000 & 0 & 0 \\ & 960 & 4800 & 0 & -960 & 4800 \\ & & 32000 & 0 & -4800 & 16000 \\ & & & 80000 & 0 & 0 \\ & \text{sym.} & & & 960 & -4800 \\ & & & & & 32000 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 0 \\ 0 \\ 0 \end{array}$$

Structure matrices:

$$[S] = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 94847 & 28320 & 3433 \\ 28320 & 58287 & 3084 \\ 3433 & 3084 & 60623 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} ; \quad \{P\} = \begin{bmatrix} 0 \\ 0 \\ -150 \end{bmatrix}$$

18.8 (contd.) Joint displacements: By solving the equations $\{P\} = [S]\{d\}$, we obtain:

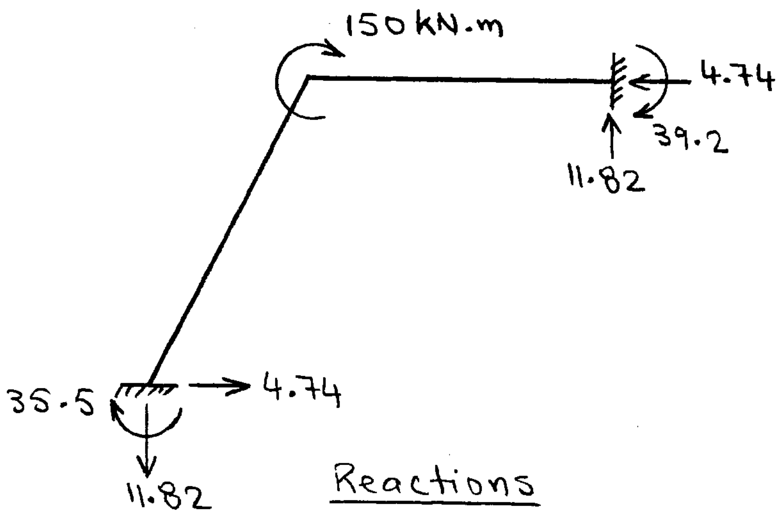
$$\{d\} = 10^{-4} \begin{array}{c} \begin{bmatrix} 0.5924 \text{ m} \\ 1.0259 \text{ m} \\ -24.829 \text{ rad.} \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \end{array}$$

Member forces:

$$\{v_1\} = 10^{-4} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.5924 \\ 1.0259 \\ -24.829 \end{bmatrix} \begin{array}{c} 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{array} ; \quad \{u_1\} = 10^{-4} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1.182 \\ -0.07103 \\ -24.829 \end{bmatrix}$$

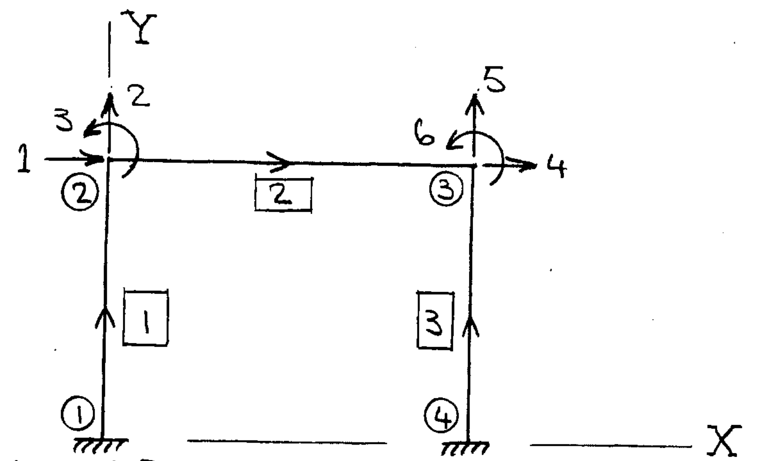
$$\{Q_1\} = \begin{bmatrix} -8.46 \text{ kN} \\ -9.53 \text{ kN} \\ -35.5 \text{ kN.m} \\ 8.46 \text{ kN} \\ 9.53 \text{ kN} \\ -71 \text{ kN.m} \end{bmatrix} ; \quad \{F_1\} = \begin{bmatrix} 4.74 \text{ kN} \\ -11.82 \text{ kN} \\ -35.5 \text{ kN.m} \\ -4.74 \text{ kN} \\ 11.82 \text{ kN} \\ -71 \text{ kN.m} \end{bmatrix}$$

$$\{u_2\} = \{v_2\} = 10^{-4} \begin{bmatrix} 0.5924 \\ 1.0259 \\ -24.829 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{array}{c} 1 \\ 2 \\ 3 \\ 0 \\ 0 \\ 0 \end{array} ; \quad \{F_2\} = \{Q_2\} = \begin{bmatrix} 4.74 \text{ kN} \\ -11.82 \text{ kN} \\ -79 \text{ kN.m} \\ -4.74 \text{ kN} \\ 11.82 \text{ kN} \\ -39.2 \text{ kN.m} \end{bmatrix}$$



Reactions

18.9



Members 1 and 3: $L = 30\text{ ft};\ \cos\theta = 0;\ \sin\theta = 1$

$$[k_1] = [k_3] = EI \times 10^{-3} \begin{bmatrix} 80 & 0 & 0 & -80 & 0 & 0 \\ & 0.444 & 6.67 & 0 & -0.444 & 6.67 \\ & & 133.3 & 0 & -6.67 & 66.67 \\ & \text{Sym.} & & 80 & 0 & 0 \\ & & & & 0.444 & -6.67 \\ & & & & & 133.3 \end{bmatrix}$$

$$[T_1] = [T_3] = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Member 3 → 0 0 0 4 5 6

Member 1 → 0 0 0 1 2 3

$$[K_1] = [K_3] = EI \times 10^{-3} \begin{array}{c} \begin{bmatrix} 0.444 & 0 & -6.67 & -0.444 & 0 & -6.67 \\ & 80 & 0 & 0 & -80 & 0 \\ & & 133.3 & 6.67 & 0 & 66.67 \\ & & & 0.444 & 0 & 6.67 \\ & \text{Sym.} & & & 80 & 0 \\ & & & & & 133.3 \end{bmatrix} \end{array} \begin{array}{cc} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{array}$$

Member 2: $L = 40\text{ ft};\ \cos\theta = 1;\ \sin\theta = 0$

$$[k_2] = [K_2] = EI \times 10^{-3} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 120 & 0 & 0 & -120 & 0 & 0 \\ & 0.375 & 7.5 & 0 & -0.375 & 7.5 \\ & & 200 & 0 & -7.5 & 100 \\ & & & 120 & 0 & 0 \\ & \text{Sym.} & & & 0.375 & -7.5 \\ & & & & & 200 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}$$

18.9 (Contd.)

$$\{F_{f2}\} = \{Q_{f2}\} = \begin{array}{c} \begin{bmatrix} 0 \\ 30 \\ 200 \\ 0 \\ 30 \\ -200 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}$$

Structure matrices:

$$[S] = EI \times 10^{-3} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 120.444 & 0 & 6.67 & -120 & 0 & 0 \\ & 80.375 & 7.5 & 0 & -0.375 & 7.5 \\ & & 333.3 & 0 & -7.5 & 100 \\ & & & 120.444 & 0 & 6.67 \\ & \text{Sym.} & & & 80.375 & -7.5 \\ & & & & & 333.33 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}$$

$$\{P\} = \begin{bmatrix} 20 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}; \quad \{P_f\} = \{F_{f2}\}; \quad \{P\} - \{P_f\} = \begin{bmatrix} 20 \\ -30 \\ -200 \\ 0 \\ -30 \\ 200 \end{bmatrix} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}$$

Joint displacements: By solving the equations $\{P\} - \{P_f\} = [S]\{d\}$, we obtain:

$$\{d\} = \frac{1}{EI \times 10^{-3}} \begin{bmatrix} 29.4204 \\ -0.2906 \\ -1.314 \\ 29.289 \\ -0.4594 \\ 0.4044 \end{bmatrix} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}$$

Member forces:

$$\{v_1\} = \frac{1}{EI \times 10^{-3}} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 29.4204 \\ -0.2906 \\ -1.314 \end{bmatrix} \begin{array}{c} 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \qquad \{u_1\} = \frac{1}{EI \times 10^{-3}} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.2906 \\ -29.4204 \\ -1.314 \end{bmatrix}$$

18.9 (contd.)

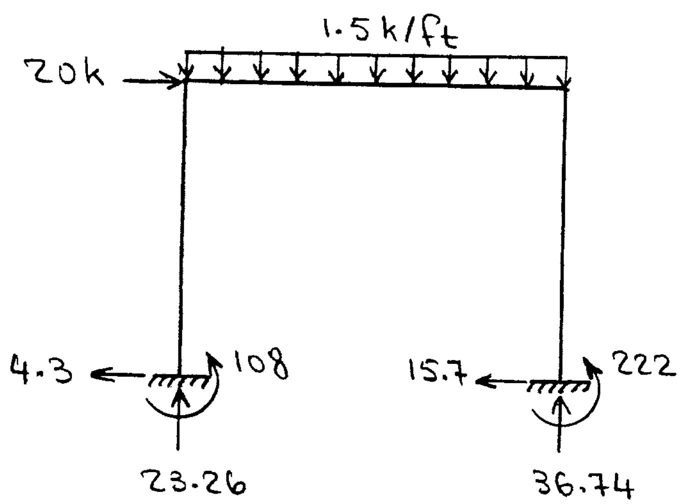
$$\{Q_1\} = \begin{bmatrix} 23.26 \text{ k} \\ 4.3 \text{ k} \\ 108 \text{ k-ft} \\ -23.26 \text{ k} \\ -4.3 \text{ k} \\ 21 \text{ k-ft} \end{bmatrix}$$

$\{u_2\} = \{v_2\} = \{d\}$;

$$\{Q_2\} = \begin{bmatrix} 15.7 \text{ k} \\ 23.26 \text{ k} \\ -21 \text{ k-ft} \\ -15.7 \text{ k} \\ 36.74 \text{ k} \\ -249 \text{ k-ft} \end{bmatrix}$$

$$\{v_3\} = \frac{1}{EI \times 10^{-3}} \begin{array}{c} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 29.289 \\ -0.4594 \\ 0.4044 \end{bmatrix} \end{array} \begin{array}{c} 0 \\ 0 \\ 0 \\ 4 \\ 5 \\ 6 \end{array} ; \quad \{u_3\} = \frac{1}{EI \times 10^{-3}} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.4594 \\ -29.289 \\ 0.4044 \end{bmatrix}$$

$$\{Q_3\} = \begin{bmatrix} 36.74 \text{ k} \\ 15.7 \text{ k} \\ 222 \text{ k-ft} \\ -36.74 \text{ k} \\ -15.7 \text{ k} \\ 249 \text{ k-ft} \end{bmatrix}$$



Reactions

Appendix B
Review of Matrix Algebra
&
Appendix C
Computer Software

APPENDIX B

B.1

$$[C] = [A] + 3[B] = \begin{bmatrix} 18 & -11 & 18 \\ -11 & 19 & 28 \\ 18 & 28 & 4 \end{bmatrix}$$

B.2

$$[C] = 2[A] - [B] = \begin{bmatrix} 7 & 8 \\ 11 & 7 \\ 1 & 0 \end{bmatrix}$$

$\boxed{B.3}$

$$[C] = \begin{bmatrix} -6 & 4 & -2 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix} = -12 - 4 - 10 = \underline{-26}$$

$$[D] = \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix} \begin{bmatrix} -6 & 4 & -2 \end{bmatrix} = \begin{bmatrix} -12 & 8 & -4 \\ 6 & -4 & 2 \\ -30 & 20 & -10 \end{bmatrix}$$

$\boxed{B.4}$

$$[C] = \begin{bmatrix} 2 & -5 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} -3 & 4 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} -26 & 3 \\ 27 & -17 \end{bmatrix}$$

$$[D] = \begin{bmatrix} -3 & 4 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & -5 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} -26 & 27 \\ 3 & -17 \end{bmatrix}$$

$\boxed{B.5}$

$$[A][B] = \begin{bmatrix} 8 & -2 & 5 \\ 1 & -4 & 3 \\ 2 & 0 & 6 \end{bmatrix} \begin{bmatrix} 1 & -5 \\ 7 & 0 \\ 0 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} (8-14) & (-40-15) \\ (1-28) & (-5-9) \\ (2) & (-10-18) \end{bmatrix} = \begin{bmatrix} -6 & -55 \\ -27 & -14 \\ 2 & -28 \end{bmatrix}$$

$$[AB]^T = \begin{bmatrix} -6 & -27 & 2 \\ -55 & -14 & -28 \end{bmatrix} \qquad (1)$$

$$[B]^T[A]^T = \begin{bmatrix} 1 & 7 & 0 \\ -5 & 0 & -3 \end{bmatrix} \begin{bmatrix} 8 & 1 & 2 \\ -2 & -4 & 0 \\ 5 & 3 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} (8-14) & (1-28) & (2) \\ (-40-15) & (-5-9) & (-10-18) \end{bmatrix}$$

$$= \begin{bmatrix} -6 & -27 & 2 \\ -55 & -14 & -28 \end{bmatrix} \qquad (2)$$

From Eqs. (1) and (2) we can see that:

$$\underline{[AB]^T = [B]^T[A]^T}$$

$\boxed{\text{B-6}}$

$$\left[\begin{array}{ccc|c} 2 & 5 & -1 & 15 \\ 5 & -1 & 3 & 27 \\ -1 & 3 & 4 & 14 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2.5 & -0.5 & 7.5 \\ 0 & -13.5 & 5.5 & -10.5 \\ 0 & 5.5 & 3.5 & 21.5 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0.518 & 5.555 \\ 0 & 1 & -0.407 & 0.778 \\ 0 & 0 & 5.739 & 17.221 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

Thus,

$$\underline{x_1 = 4 \qquad x_2 = 2 \qquad x_3 = 3}$$

B.7

$$\left[\begin{array}{ccc|c} -12 & -3 & 6 & 45 \\ 5 & 2 & -4 & -9 \\ 10 & 1 & -7 & -32 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0.25 & -0.5 & -3.75 \\ 0 & 0.75 & -1.5 & 9.75 \\ 0 & -1.5 & -2 & 5.5 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -7 \\ 0 & 1 & -2 & 13 \\ 0 & 0 & -5 & 25 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -7 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -5 \end{array}\right]$$

Thus,

$$\underline{x_1 = -7 \quad x_2 = 3 \quad x_3 = -5}$$

$\boxed{8.8}$

$$\left[\begin{array}{cccc|c} 5 & -2 & 6 & 0 & 0 \\ -2 & 4 & 1 & 3 & 18 \\ 6 & 1 & 6 & 8 & -29 \\ 0 & 3 & 8 & 7 & 11 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & -0.4 & 1.2 & 0 & 0 \\ 0 & 3.2 & 3.4 & 3 & 18 \\ 0 & 3.4 & -1.2 & 8 & -29 \\ 0 & 3 & 8 & 7 & 11 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1.625 & 0.375 & 2.25 \\ 0 & 1 & 1.063 & 0.938 & 5.625 \\ 0 & 0 & -4.814 & 4.811 & -48.125 \\ 0 & 0 & 4.811 & 4.186 & -5.875 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 2 & -14 \\ 0 & 1 & 0 & 2 & -5 \\ 0 & 0 & 1 & -1 & 10 \\ 0 & 0 & 0 & 9 & -54 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 & 7 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 & -6 \end{array}\right]$$

Thus,

$$\underline{x_1 = -2 \quad x_2 = 7 \quad x_3 = 4 \quad x_4 = -6}$$

$\boxed{B.9}$

$$\left[\begin{array}{ccc|ccc} 4 & -3 & -1 & 1 & 0 & 0 \\ -2 & 5 & 1 & 0 & 1 & 0 \\ 6 & -4 & -5 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -0.75 & -0.25 & 0.25 & 0 & 0 \\ 0 & 3.5 & 0.5 & 0.5 & 1 & 0 \\ 0 & 0.5 & -3.5 & -1.5 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -0.143 & 0.357 & 0.215 & 0 \\ 0 & 1 & 0.143 & 0.143 & 0.286 & 0 \\ 0 & 0 & -3.572 & -1.572 & -0.143 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0.42 & 0.22 & -0.04 \\ 0 & 1 & 0 & 0.08 & 0.28 & 0.04 \\ 0 & 0 & 1 & 0.44 & 0.04 & -0.28 \end{array}\right]$$

Thus,

$$[A]^{-1} = \begin{bmatrix} 0.42 & 0.22 & -0.04 \\ 0.08 & 0.28 & 0.04 \\ 0.44 & 0.04 & -0.28 \end{bmatrix}$$

B.10

$$\left[\begin{array}{cccc|cccc} 4 & 2 & 0 & -3 & 1 & 0 & 0 & 0 \\ 2 & 3 & -4 & 0 & 0 & 1 & 0 & 0 \\ 0 & -4 & 2 & -1 & 0 & 0 & 1 & 0 \\ -3 & 0 & -1 & 5 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0.5 & 0 & -0.75 & 0.25 & 0 & 0 & 0 \\ 0 & 2 & -4 & 1.5 & -0.5 & 1 & 0 & 0 \\ 0 & -4 & 2 & -1 & 0 & 0 & 1 & 0 \\ 0 & 1.5 & -1 & 2.75 & 0.75 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 1 & -1.125 & 0.375 & -0.25 & 0 & 0 \\ 0 & 1 & -2 & 0.75 & -0.25 & 0.5 & 0 & 0 \\ 0 & 0 & -6 & 2 & -1 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1.625 & 1.125 & -0.75 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & -0.7917 & 0.2083 & 0.0833 & 0.1667 & 0 \\ 0 & 1 & 0 & 0.0833 & 0.0833 & -0.1667 & -0.3333 & 0 \\ 0 & 0 & 1 & -0.3333 & 0.1667 & -0.3333 & -0.1667 & 0 \\ 0 & 0 & 0 & 2.2917 & 0.7917 & -0.0833 & 0.3333 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0.4818 & 0.0545 & 0.2818 & 0.3455 \\ 0 & 1 & 0 & 0 & 0.0545 & -0.1636 & -0.3455 & -0.0364 \\ 0 & 0 & 1 & 0 & 0.2818 & -0.3455 & -0.1182 & 0.1455 \\ 0 & 0 & 0 & 1 & 0.3455 & -0.0364 & 0.1455 & 0.4364 \end{array}\right]$$

Thus,

$$[A]^{-1} = \begin{bmatrix} 0.4818 & 0.0545 & 0.2818 & 0.3455 \\ 0.0545 & -0.1636 & -0.3455 & -0.0364 \\ 0.2818 & -0.3455 & -0.1182 & 0.1455 \\ 0.3455 & -0.0364 & 0.1455 & 0.4364 \end{bmatrix}$$

APPENDIX C

$\boxed{C.1}$ $\frac{L}{360} = \frac{80}{360} = 0.222 \text{ ft} = 2.67 \text{ in.}$

(a) $\underline{A = 9.12 \text{ in}^2}$

(b) $\underline{A = 6.33 \text{ in}^2}$

(c) $\underline{A = 8.66 \text{ in}^2}$

$\boxed{C.2}$ $\dfrac{L}{360} = \dfrac{24}{360} = 0.0667\ m$

(a) $\underline{A = 8470\ mm^2}$

(b) $\underline{A = 7620\ mm^2}$

(c) $\underline{A = 6860\ mm^2}$

$\boxed{C.3}$ $\underline{I = 1257 \text{ in}^4}$