Chapter 4

STRUCTURE OF THE ATOM

4.1 The Atomic Models of Thomson and Rutherford

The first model of the atom was given by J.J. Thomson in 1898. He assumed that atoms are homogeneous uniform spheres of positively charged matter in which electrons are embedded, much like “raisins in plum pudding.” In 1911, Geiger and Marsden conducted an experiment in which they studied the scattering of α-particles (ionized helium atom) by a very thin foil of gold. A calumniated beam of α-particles hitting a gold foil showed that deflection through large angles. Most of these particles went undeflected while a few were turn back. Thomson’s model could not explain it. Rutherford suggested that the atom consists of a central massive nucleus where its entire mass and positive charge is concentrated and it is this nucleus which is responsible for repelling the alpha particles. Around the nucleus there is a mist of electrons whose total charge is equal in magnitude to the charge in the nucleus.

Joseph John Thomson was born in 1856 in England, of Scottish parentage. He studied engineering at Owens College, Manchester, and moved on to Trinity College, Cambridge. In 1884 he became Cavendish Professor of Physics. One of his students was Ernest Rutherford, who would later succeed him in the post. For his discovery of the electron, he was awarded a Nobel Prize in 1906. In 1918 he became Master of Trinity College, Cambridge, where he remained until his death. He died in 1940 and was buried in Westminster Abbey, close to Isaac Newton.

4.2 Rutherford Scattering

A charged particle of mass m, charge $Z_1 e$ and velocity v_o is incident on a target material or scatterer of charge $Z_2 e$. The *impact parameter*, b, is the closest distance of approach between the beam particle and scatterer if the projectile had continued in a straight line. Let us find the relation between b and the scattering angle θ.

$$\Delta \mathbf{p} = \int \mathbf{F}_{\Delta \mathbf{p}} dt$$

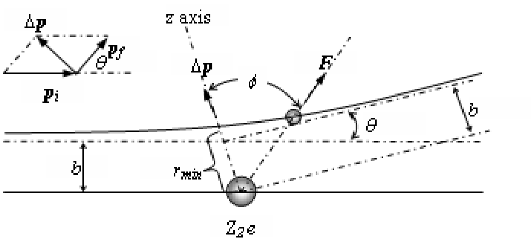

Figure 4.1: α-particle scattering

Ernest Rutherford (1871-1937) was the brilliant New Zealand physicist who explained natural radioactivity, determined the structure of the atom, and changed one element into another (nitrogen to oxygen) by splitting an atom's nucleus. A farm boy from New Zealand's South Island, he spent most of his professional career overseas at McGill University in Montreal, Canada (1895-98), and at Manchester University (1898-1907) and Cambridge University (1919-37) in the United Kingdom. Rutherford was an energetic pioneer in nuclear physics: he discovered (and named) alpha and beta radiation, named the nucleus and proton and won the 1908 Nobel prize in chemistry for explaining radioactivity as the disintegration of atoms. Rutherford's description of an atomic structure with orbital electrons became the accepted model (with further help provided by his student and colleague, Niels Bohr), and in 1920 he predicted the existence of the neutron, which was later discovered by James Chadwick.

where $\mathbf{F}_{\Delta\mathbf{p}}$ is the force along the direction of $\Delta\mathbf{p}$.

$$\begin{aligned}\Delta\mathbf{p} &= \mathbf{P}_f - \mathbf{P}_i \\ p_i &\approx p_f = mv_o \quad \text{(assume no recoil of the scatterer)}\end{aligned}$$

Then (isosceles triangles of $\mathbf{P}_i$, $\mathbf{P}_f$, and $\Delta\mathbf{p}$)

$$\frac{\Delta\mathbf{p}}{\sin\theta} = \frac{mv_o}{\sin(\frac{\pi-\theta}{2})}$$

Since

$$\begin{aligned}\sin\left(\frac{\pi-\theta}{2}\right) &= \cos\left(\frac{\theta}{2}\right) \\ \sin\theta &= 2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right) \\ \therefore \Delta\mathbf{p} &= 2mv_o\sin\left(\frac{\theta}{2}\right)\end{aligned}$$

The coulomb force $\mathbf{F}$ is along the instantaneous direction of the position vector $\mathbf{r}$ (unit vector $\hat{\mathbf{a}}_r$)

$$\mathbf{F} = \frac{1}{4\pi\varepsilon_o}\frac{Z_1Z_2e^2}{r^2}\hat{\mathbf{a}}_r = F\hat{\mathbf{a}}_r$$

and $F_{\Delta\mathbf{p}} = F\cos\phi$ since $\Delta\mathbf{p}$ is in $\phi = 0$ direction. Therefore,

$$\begin{aligned}\Delta\mathbf{p} = 2mv_o\sin(\theta/2) &= \int F\cos\phi dt \\ &= \frac{Z_1Z_2e^2}{4\pi\varepsilon_o}\int\frac{\cos\phi}{r^2}dt\end{aligned}$$

The instantaneous angular momentum must be conserved, so

$$\begin{aligned}
mr^2\frac{d\phi}{dt} &= mv_o b \\
\Rightarrow r^2 &= \frac{v_o b}{d\phi/dt} \\
\therefore 2mv_o\sin(\theta/2) &= \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_o}\int\frac{\cos\phi}{v_o b}\frac{d\phi}{dt} \\
&= \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_o v_o b}\int_{\phi_i}^{\phi_f}\cos\phi d\phi
\end{aligned}$$

Let ϕ_i be on the -ve side and ϕ_f on the +ve side of $z'(\phi = 0)$, then

$$\phi_i = -\phi_f, \quad -\phi_i + \phi_f + \theta = \pi \Rightarrow \phi_i = -(\pi-\theta)/2, \quad \phi_f = (\pi-\theta)/2$$

$$\begin{aligned}
\frac{8\pi\varepsilon_o m v_o^2 b}{Z_1 Z_2 e^2}\sin(\theta/2) &= \int_{-(\pi-\theta)/2}^{(\pi-\theta)/2}\cos\phi d\phi = 2\cos(\theta/2) \\
\therefore b &= \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_o m v_o^2}\cot(\theta/2)
\end{aligned}$$

Or with $E_k = \frac{1}{2}mv_o^2$ - kinetic energy of the particle:

$$b = \frac{Z_1 Z_2 e^2}{8\pi\varepsilon_o E_k}\cot(\theta/2) \tag{4.1}$$

4.3 Estimation of Nuclear Radius

For a given E_k and b there ia a distance of closest approach between a bombarding particle and target scatterer of like charges. The minimum separation occurs for a head-on collision. The particle turns around and scatters backward at 180^o. At the instant the particle turns around, the entire kinetic energy has been converted into coulomb potential energy.

$$E_k = \frac{(Z_1 e)(Z_2 e)}{4\pi\varepsilon_o r}$$

$$\therefore r_{min} = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_o E_k} \tag{4.2}$$

For α particles ($Z_1 = 2$) of 7.7 MeV scattering on aluminium ($Z_2 = 19$) or gold($Z_2 = 79$)

$$\begin{aligned}
r_{min}(\text{aluminium}) &= 5\times 10^{-15}\text{m} \\
r_{min}(\text{gold}) &= 3\times 10^{-14}\text{m}
\end{aligned}$$

We now know that nuclear radii vary from 1 to 10×10^{-15}m.

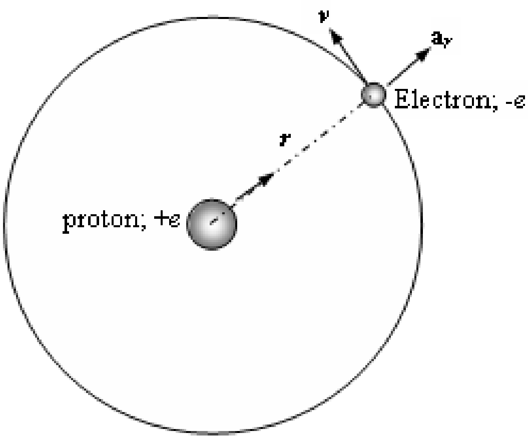

Figure 4.2: Electron orbit of hydrogen atom

4.4 Electron Orbit (Hydrogen Atom)

The coulomb attractive force between the nucleus and the orbiting electron is

$$\mathbf{F}_e = -\frac{1}{4\pi\varepsilon_o}\frac{e^2}{r^2}\hat{\mathbf{a}}_r$$

The electron's radial acceleration is

$$a_r = \frac{v^2}{r}, \quad v - \text{the tangential velocity}$$

From Newton's second law,

$$\frac{1}{4\pi\varepsilon_o}\frac{e^2}{r^2} = \frac{mv^2}{r}$$

Implying

$$v = \frac{e}{\sqrt{4\pi\varepsilon_o mr}} \tag{4.3}$$

The total energy E of the electron in a hydrogen atom is the sum of its kinetic energy ($\frac{1}{2}mv^2$) and potential energy ($-\frac{e^2}{4\pi\varepsilon_o r}$)

$$\begin{aligned} E &= E_k + E_p \\ &= \frac{mv^2}{2} - \frac{e^2}{4\pi\varepsilon_o r} \end{aligned}$$

Substituting v from Eq. 4.3

$$E = -\frac{e^2}{8\pi\varepsilon_o r} \tag{4.4}$$

The total energy is negative, indicating a bound, attractive system. Experimentally it is found that a binding energy of 13.6 eV is required to separate a hydrogen atom into a proton and an electron. Therefore,

$$r = -\frac{e}{\sqrt{4\pi\varepsilon_o mr}} = 5.23 \times 10^{-11}\text{m}$$

In summary, the classical atomic model assumes that the atom consists of a small, massive, positively charged nucleus surrounded by moving electrons, resembling the planetary model of the solar system.

This model fails to explain the origin of spectral lines. An orbiting electron has an acceleration directed towards the center. According to the classical electrodynamics, an accelerated charged particle emits radiation. This implies that with the loss of energy by radiation, the electron orbit should shrink and the electron should crash into the nucleus. This process would occur in about 10^{-9}s. Also, by this process, it would emit a *continuous* range of radiations. Actually, however, it is observed that atoms are stable and emit only *discrete* spectral lines when excited.

To overcome this difficulty, Bohr discarded the classical atomic model and put forward his *quantum model* of the atom. The main idea in it is that electrons can revolve in certain *stationary orbits* only, those in which no radiation of energy takes place.

***Exercise* 1**: Calculate the time, according to classical laws, it would take the electron of the hydrogen atom to radiate its energy and crash into the nucleus. (Hint: The radiated power P is given by $\left(\frac{1}{4\pi\varepsilon_o}\right)\left(\frac{2Q^2}{3c^3}\right)\left(\frac{d^2\mathbf{r}}{dt^2}\right)^2$ where Q is the charge, c the speed of light, and $\mathbf{r}$ the position vector of the electron from the center of the atom.)