Chapter 2

The Particle Properties of Waves

2.1 Introduction

Classical physics, which mirrors the "physical reality" of our sense impressions, treats particles and waves as separate components of that reality. Traditionally, the mechanics of particles and the optics of waves are independent disciplines. But the physical reality which arises from the phenomena that occur in the microscopic world of atoms and molecules, electrons and nuclei there are neither particles nor waves in the classical sense.

The remarkable puzzles which gradually emerged from experimental physics as more and more physical phenomena were explored was that it appeared that light, which was normally regarded as a wave, sometimes behaved as if the ray of light were a stream of particles. Similarly, experimental evidence emerged suggesting that the electron, which has been treated as particle, behaves like waves. Finally, the wave-particle dualities lead to the emergence of Quantum Mechanics.

***Chapter Objective*:** to explore some of the experiments that manifest the particle nature of waves.

2.2 The Photoelectric Effect

It is known that in metals, the valence electrons are "free"- they are able to move easily from atom to atom, but are not able to leave the surface of the metal. If the material is irradiated with light, it can give electrons enough extra kinetic energy to allow them to escape the material. The ejected electrons are called *photoelectrons*. Consider the arrangement *fig*. 2.1.

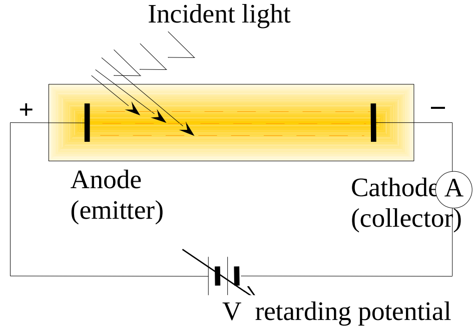

fig. 2.1

To remove an electron from a solid, it is necessary to give the electron a minimum energy W_o which will represent the "potential energy barrier" that the electron has to surmount in order to leave the solid. This minimum kinetic energy is called the *work function*, W_o. Energy in excess of this amount supplied by the light beam must reappear as kinetic energy, E_k, of the electron, i.e.,

Total energy supplied by light, $E_L = W_o + E_k$ 2.1

Experimental Results

1. Classically, if E_L is reduced by reducing the amount of light supplied, E_k must also decrease. So reducing the light *intensity* (the total power radiated per unit area per unit wavelength at a given temperature) of the light should reduce E_k. But this does not happen! It is observed that the *number* of electrons released reduce proportionally with light intensity but the *energy* of the electron remains the same. (The kinetic energy of the photoelectrons is independent of the light intensity.)

2. The maximum kinetic energy of the photoelectrons, for a given emitting material, depends only on the frequency of the light. (*fig*. 2.2)

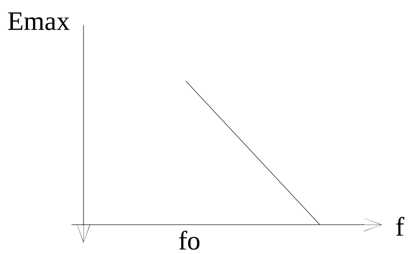

fig 2.2

$$E_{max} = \text{maximum electron energy}$$
$$¿ h(f - f_o)$$
$$¿ hf - hf_o$$
2.2

Where h = 6.63 x 10^{-34}J-s and f_o is the threshold frequency below which no photoemission occurs. f_o varies with the particular metal being illuminated.

3. The photoelectrons are emitted almost instantly ($\leq$ 3 x 10^{-9}s) following illumination of the emitter, independent of the intensity of the light.

Quantum Interpretation of the Photoelectric Effect

The photoelectric effect can only be explained by supposing light beam to consist *particles* of fixed energy. Reducing the intensity would reduce the number of particles flowing per unit time. If we suppose that when a particle "collides" with an electron, it can give the electron its whole energy, then reducing the rate of flow of particles (intensity) would reduce the number of collisions but the energy transferred at each collision would remain the same. Thus we could reduce the rate of emission of electrons without changing their kinetic energy.

According to Einstein, electromagnetic energy is *quantized*; in his words “the energy of a light ray spreading out from a point source is not continuously distributed over an increasing space but consists of a finite number of energy quanta which are localized at points in space, which move without dividing, and which can only be produced and absorbed as complete units.” These energy quanta of light are called *photons.*

Each photon has energy quantum,

$$E = hf \tag{2.3}$$

The empirical formula Eq.2.2 may be rewritten as:

$$hf = E_{\max} + hf_o$$

Einstein interpreted the three terms as:

hf - energy of each quantum of incident light.

$E_{\max}$ - the maximum photoelectron energy

hf_o - work function.

Example 1

An ultraviolet light of wavelength 3500 Å falls on a potassium surface of work function 2.0 eV. Calculate the maximum energy of the photoelectrons.

Example 2

The photoelectric work function of a metal is 2 eV. If the wavelength of the light incident on it is 3600 Å, find the kinetic energy of the fastest electrons emitted and the stopping potential. (Stopping or retarding potential is the opposing potential needed to stop the most energetic electrons.)

Exercise 1

Light of wavelength 400 nm is incident upon lithium (W_o = 2.9 eV.) Calculate:

a. The photon energy
b. The retarding potential.

Answer: a). 3.1 eV b) 0.2 V

Exercise 2

How many photons/second are contained in a beam of electromagnetic radiation of total power 100 W if the source is:

a. An AM radio station of 1200 kHz
b. 10 nm X-rays
c. 2 MeV gamma rays?

Answer: a). 1.26 x 10^{29} photons/s b) 5.03 x 10^{18} photons/s c) 3.12 x 10^{14} photons/s

2.3 The Compton Effect

In 1923 Compton, while studying monochromatic X-rays scattered by carbon atoms, found that the scattered beam contains two wavelengths, one the same as that of the incident beam (unmodified beam) and the other somewhat larger (modified beam.) the change of wavelength is due to loss of energy of the incident X-rays. This inelastic interaction is known as *Compton effect*. Classical electromagnetism cannot explain the modified wave. Einstein's photon particle concept must be correct.

Consider the collision between a photon and an electron shown below:

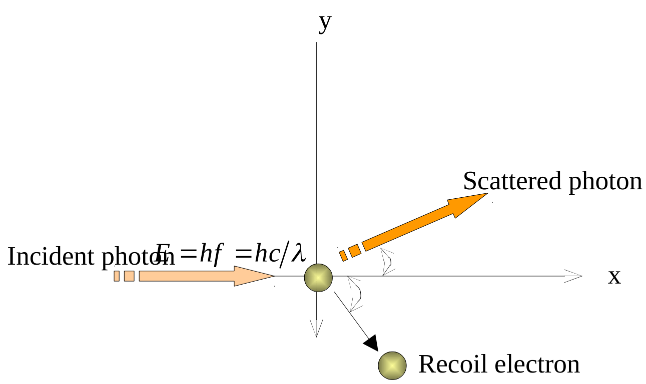

fig. 2.3

Conservation of energy:

$$\frac{hc}{\lambda} + m_o c^2 = \frac{hc}{\lambda'} + mc^2$$

$$\Rightarrow \quad m = m_o + \frac{h}{c}\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) \qquad 2.4$$

Conservation of momentum in *x*- and *y*- directions:

$$\frac{h}{\lambda} + 0 = \frac{h}{\lambda'}\cos\varphi + mv\cos\theta$$

$$0 = \frac{h}{\lambda'}\sin\varphi - mv\sin\theta$$

Then,

$$(mv\cos\theta)^2 + (mv\sin\theta)^2 = \left(\frac{h}{\lambda'}\sin\varphi\right)^2 + \left(\frac{h}{\lambda} - \frac{h}{\lambda'}\cos\varphi\right)^2$$

$$\Rightarrow \quad m^2 v^2 = \frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} - \frac{2h^2}{\lambda\lambda^2}\cos\varphi$$

but $$m=\frac{m_o}{\sqrt{1-v^2/c^2}} \Rightarrow m^2v^2=\left(m^2-m_{o^2}\right)c^2$$

$\Rightarrow$ $$\left(m^2-m_{o^2}\right)c^2=\frac{h^2}{\lambda^2}+\frac{h^2}{\lambda'^2}-\frac{2h^2}{\lambda\lambda^2}\cos\varphi \qquad 2.5$$

From Eq.2.4,

$$m^2=m_{o^2}+\frac{h^2}{c^2}\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\right)^2+\frac{2m_oh}{c}\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\right)$$

$\therefore$ $$\left(m^2-m_{o^2}\right)c^2=h^2\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\right)^2+2m_ohc\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\right) \qquad 2.6$$

Equating Eqs. 2.5 and 2.6,

$$\frac{h^2}{\lambda^2}+\frac{h^2}{\lambda'^2}-\frac{2h^2}{\lambda\lambda^2}\cos\varphi=h^2\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\right)^2+2m_ohc\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\right)$$, i.e.,

$$\lambda'-\lambda=\frac{h}{m_oc}\left(1-\cos\varphi\right) \qquad 2.7$$

Eq. 2.7 gives the change in wavelength ($\Delta\lambda$) as a function of the scattering angle ϕ.

$$\Delta\lambda=0.02426\left(1-\cos\varphi\right)\overset{o}{A}=0.04852\sin^2\left(\tfrac{1}{2}\varphi\right)\overset{o}{A}$$

Example 3

X-rays of wavelength 0.7080 Å are scattered from a carbon block through an angle of 90°. What is the modified wavelength and the percentage change $\Delta\lambda/\lambda$?

Exercise 3

Show that the recoil kinetic energy of an electron and its recoil angle θ in Compton scattering is:

$$E_k\left(\text{electron}\right)=\frac{\Delta\lambda/\lambda}{1+\Delta\lambda/\lambda}hf$$

$$\cot\theta=\left(1+\frac{hf}{m_oc^2}\right)\tan\tfrac{1}{2}\varphi$$

Exercise 4

Show that the maximum kinetic energy of the recoil electron in Compton scattering is given by:

$$E_{\max}\left(\text{electron}\right)=\frac{\frac{2hf}{m_oc^2}}{1+\frac{2hf}{m_oc^2}}hf$$

At what angles ϕ and θ does this occur?