



Chapter-4

Induction machine

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Overview

- 1 Overview
- 2 Introduction
 - Basic Concepts of measurement
- 3 Constructional features
- 4 Rotating Magnetic Field
- 5 Principle of operation
- 6 Torand Thevenin's Theorem
 - Torque-speed characteristics
 - Maximum Torque

Lecture Objectives

The objectives of this lecture are:

- To discuss the operating principle of Induction Motors(IM),
- To discuss the different variables that involve in IM,
- To discuss equivalent circuit and power relationships in IM,
- To discuss the torque of IM, and
- To discuss the different setups used to determine IM parameters.

Introduction

Three-phase induction motors are the most common and frequently encountered machines in industry

- Simple design, rugged, low-price, easy maintenance
- Wide range of power ratings: fractional horsepower to 10 MW
- Run essentially as constant speed from no-load to full load
- Its speed depends on the frequency of the power source
 - Not easy to have variable speed control
 - Requires a variable-frequency power-electronic drive for optimal speed control

Constructional features

An induction motor has two main parts

- A stationary stator
 - Consisting of a steel frame that supports a hollow, cylindrical core
 - Core, constructed from stacked laminations (why?), having a number of evenly spaced slots, providing the space for the stator winding

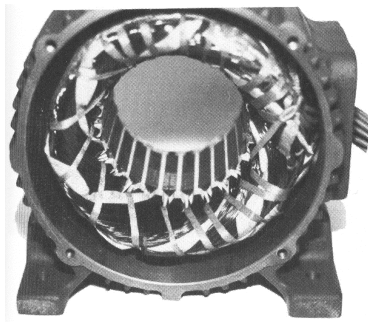
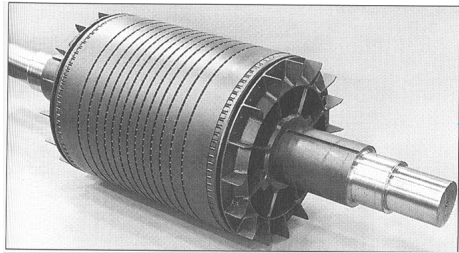


Figure 1: Stator of Induction Motor.

- A revolving rotor
 - composed of punched laminations, stacked to create a series of rotor slots, providing space for the rotor winding
 - one of two types of rotor windings
 - conventional 3-phase windings made of insulated wire (wound-rotor)
⇒ similar to the winding on the stator
 - aluminum bus bars shorted together at the ends by two aluminum rings, forming a squirrel-cage shaped circuit (squirrel-cage)
- Two basic design types depending on the rotor design
 - **Squirrel-cage:** conducting bars laid into slots and shorted at both ends by shorting rings.
 - **Wound-rotor:** complete set of three-phase windings exactly as the stator. Usually Y-connected, the ends of the three rotor wires are connected to 3 slip rings on the rotor shaft. In this way, the rotor circuit is accessible.



Squirrel cage rotor

Wound rotor

Notice the
slip rings

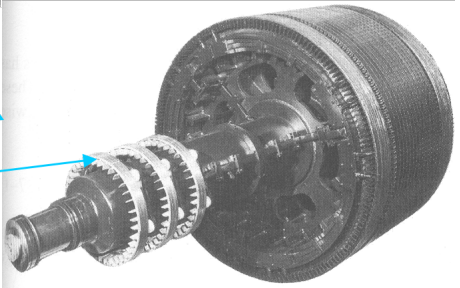


Figure 2. Different types of Induction Motors

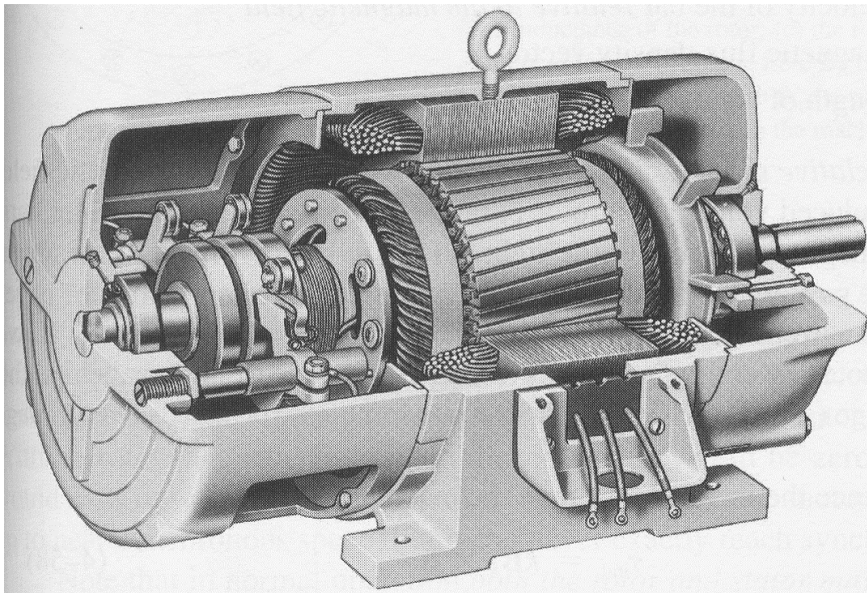


Figure 3: Cutaway in a typical wound-rotor IM. Notice the brushes and the slip rings.

Rotating Magnetic Field

- Balanced three phase windings, i.e. mechanically displaced 120 degrees from each other, fed by balanced three phase source
- A rotating magnetic field with constant magnitude is produced, rotating with a speed

$$n_{sync} = \frac{120f}{p} \text{ rpm} \quad (1)$$

where f is the supply frequency and p is the no. of poles and n_{sync} is called the synchronous speed in rpm (revolutions per minute).

Synchronous Speed

| P | 50 Hz | 60 Hz |
|----|-------|-------|
| 2 | 3000 | 3600 |
| 4 | 1500 | 1800 |
| 6 | 1000 | 1200 |
| 8 | 750 | 900 |
| 10 | 600 | 720 |
| 12 | 500 | 600 |

Rotating Magnetic Field

$$B_{net}(t) = B_a(t) + B_b(t) + B_c(t)$$

Principle of operation

- This rotating magnetic field cuts the rotor windings and produces an induced voltage in the rotor windings
- Due to the fact that the rotor windings are short circuited, for both squirrel cage and wound-rotor, and induced current flows in the rotor windings
- The rotor current produces another magnetic field
- A torque is produced as a result of the interaction of those two magnetic fields

$$\tau_m = k B_R \times B_S \quad (2)$$

τ_m is the induced torque and B_R and B_S are the magnetic flux densities of the rotor and the stator respectively.

Induction motor speed

At what speed will the IM run?

- Can the IM run at the synchronous speed, why?
- If rotor runs at the synchronous speed, which is the same speed of the rotating magnetic field, then the rotor will appear stationary to the rotating magnetic field and the rotating magnetic field will not cut the rotor.
- So, no induced current will flow in the rotor and no rotor magnetic flux will be produced so no torque is generated and the rotor speed will fall below the synchronous speed
- When the speed falls, the rotating magnetic field will cut the rotor windings and a torque is produced

- The induction motor will always run at a speed lower than the synchronous speed
- The difference between the motor speed and the synchronous speed is called the *Slip*

$$n_{slip} = n_{sync} - n_m \quad (3)$$

where

$$\begin{aligned} n_{slip} &= \text{slip speed} \\ n_{sync} &= \text{speed of the magnetic field} \\ n_m &= \text{mechanical shaft speed of the motor} \end{aligned}$$

The Slip

- The slip is given as:

$$S = \frac{n_{sync} - n_m}{n_{sync}} \quad (4)$$

where S is the *Slip*.

Notice that : if the rotor runs at synchronous speed

$$S = 0$$

if the rotor is stationary

$$S = 1$$

- Slip may be expressed as a percentage by multiplying the previous equation by 100.

Notice that the slip is a ratio and doesn't have units.

Induction Motors and Transformers

Both IM and transformer works on the principle of induced voltage

- **Transformer:** voltage applied to the *primary* windings produce an induced voltage in the *secondary* windings
- **Induction motor:** voltage applied to the *stator* windings produce an induced voltage in the rotor windings
- The difference is that, in the case of the induction motor, the secondary windings can *move*
- Due to the rotation of the rotor (the secondary winding of the IM), the induced voltage in the rotor does not have the same frequency of the stator (the primary) voltage

Rotor Frequency

- The frequency of the voltage induced in the rotor is given by:

$$f_r = \frac{P \times n}{120} \quad (5)$$

where

f_r = the rotor frequency (Hz)

P = number of stator poles

n = slip speed (rpm)

$$\begin{aligned} f_r &= \frac{P \times (n_s - n_m)}{120} \\ &= \frac{P \times s n_s}{120} = s f \end{aligned}$$

- What would be the frequency of the rotor's induced voltage at any speed n_m ?

$$f_r = sf \quad (6)$$

- When the rotor is blocked ($s = 1$) , the frequency of the induced voltage is equal to the supply frequency.
- On the other hand, if the rotor runs at synchronous speed ($s = 0$), the frequency will be *zero*.

- While the input to the induction motor is electrical power, its output is mechanical power.
- Any mechanical load applied to the motor shaft will introduce a Torque on the motor shaft.
- This torque is related to the motor output power and the rotor speed as:

$$\tau_{load} = \frac{P_{out}}{\omega_m} \quad Nm \quad (7)$$

where

$$\omega_m = \frac{2\pi n_m}{60} \quad rad/s$$

Example (1)

A 208-V, 10-hp, *four pole*, 60 Hz, Y-connected induction motor has a full-load slip of 5%

- (a) What is the synchronous speed of this motor?
- (b) What is the rotor speed of this motor at rated load?
- (c) What is the rotor frequency of this motor at rated load?
- (d) What is the shaft torque of this motor at rated load?

(Hint: 1hp = 746 watts. hp = Horse Power)

Example (1)

Solution:

$$(a) \ n_{sync} = \frac{120f}{P} = \frac{120(60)}{4} = 1800 \text{ rpm}$$

$$(b) \ n_m = (1 - S)n_s = (1 - 0.05) \times 1800 = 1710 \text{ rpm}$$

$$(c) \ f_r = sf = 0.05 \times 60 = 3 \text{ Hz}$$

(d)

$$\begin{aligned} \tau_{load} &= \frac{P_{out}}{\omega_m} = \frac{P_{out}}{2\pi \frac{n_m}{60}} \\ &= \frac{10 \text{ hp} \times 746 \text{ watt/hp}}{1710 \times 2\pi \times (1/60)} = 41.7 \text{ Nm} \end{aligned}$$

Equivalent Circuit of IM

- The induction motor is similar to the transformer with the exception that its secondary windings are free to rotate.

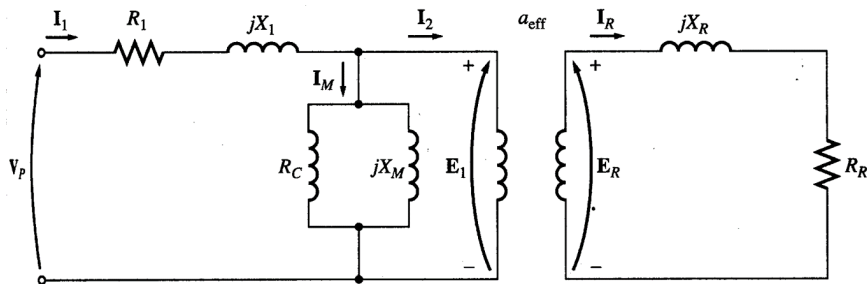


Figure 4: Equivalent circuit of an Induction Motor

- As noticed in the transformer, it is easier if we can combine these two circuits in one circuit but there are some difficulties

- When the rotor is *locked* (or *blocked*), i.e. $s = 1$, the largest voltage and rotor frequency are induced in the rotor, Why?
- On the other side, if the rotor rotates at synchronous speed, i.e. $s = 0$, the induced voltage and frequency in the rotor will be equal to zero, Why?

$$E_R = sE_{R0} \quad (8)$$

Where E_{R0} is the largest value of the rotor's induced voltage obtained at $s = 1$ (locked rotor)

- Since

$$X = \omega L = 2\pi f L$$

- as the frequency of the induced voltage in the rotor changes, the reactance of the rotor circuit also changes:

$$X_r = \omega_r L_r = 2\pi f_r L_r = 2\pi s f L_r = s X_{r0} \quad (9)$$

Where X_{r0} is the rotor reactance at the supply frequency (at blocked rotor)

- Then, the rotor equivalent circuit can be redrawn as follows:

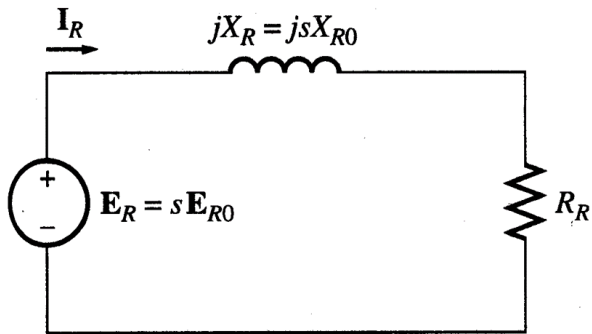


Figure 5: Rotor equivalent circuit

Where \mathbf{E}_R is the induced voltage in the rotor and R_R is the rotor resistance.

- The rotor current can be calculated as:

$$\begin{aligned}
 I_R &= \frac{E_R}{(R_R + jX_R)} \\
 &= \frac{sE_{R0}}{(R_R + jsX_{R0})}
 \end{aligned}$$

- Dividing both the numerator and denominator by s the previous equation takes the form:

$$I_R = \frac{E_{R0}}{\left(\frac{R_R}{s} + jX_{R0}\right)}$$

Where E_{R0} is the induced voltage and X_{R0} is the rotor reactance at blocked rotor condition ($s = 1$)

- From the previous equation the equivalent circuit can be redrawn as follows:

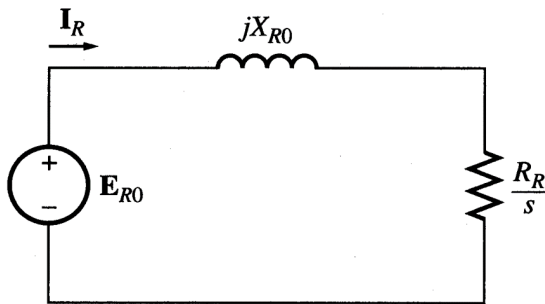


Figure 6: Rotor equivalent circuit (modified)

- The stator and rotor equivalent circuit can be combined and the combined equivalent circuit becomes:

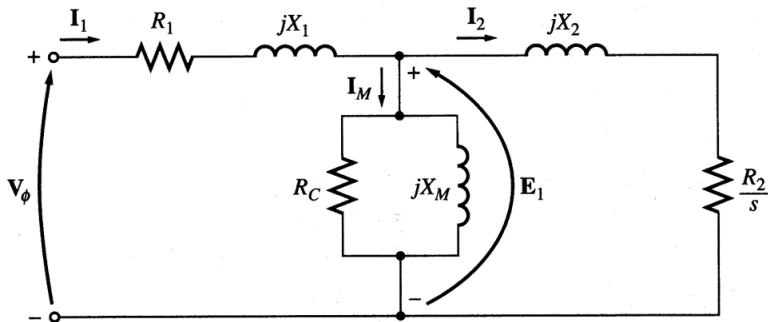


Figure 7: Rotor equivalent circuit (modified)

where $X_2 = a^2 X_{RO}$, $R_2 = a^2 R_R$, $I_2 = I_R/a$, $E_1 = aE_{R0}$ and $a = N_S/N_R$

Power losses in Induction machines

- Copper losses
 - Copper loss in the stator ($P_{SCL} = I_1^2 R_1$)
 - Copper loss in the rotor ($P_{RCL} = I_2^2 R_2$)
- Core loss (P_{core})
- Mechanical power loss due to friction and windage

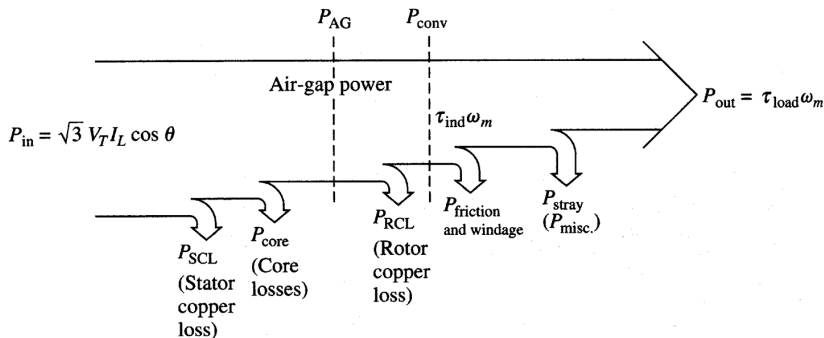


Figure 8: Power flow in induction motor

Power relations

$$P_{in} = \sqrt{3}V_L I_L \cos \theta = 3V_{ph} I_{ph} \cos \theta$$

$$P_{SCL} = 3I_1^2 R_1$$

$$P_{AG} = P_{in} - (P_{SCL} + P_{core})$$

$$P_{RCL} = 3I_2^2 R_2$$

$$P_{conv} = P_{AG} - P_{RCL}$$

$$P_{out} = P_{conv} - (P_{f+w} + P_{stary})$$

$$\tau_{ind} = \frac{P_{conv}}{\omega_m}$$

Equivalent Circuit

- The equivalent circuit can be rearranged as:

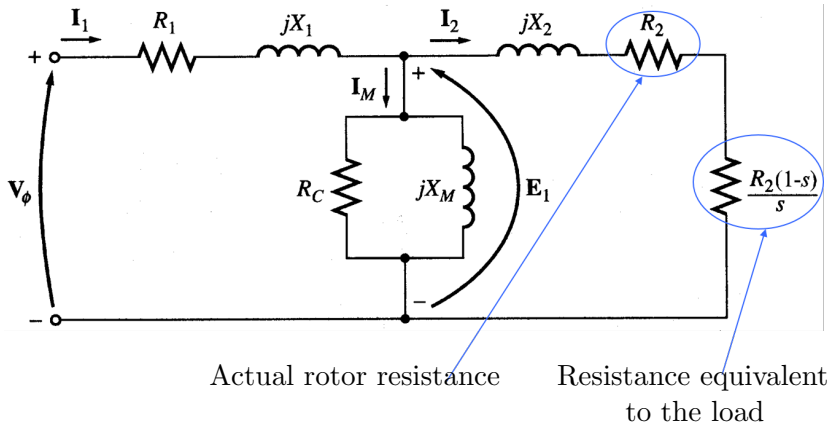


Figure 9: Rearranged equivalent circuit

Power relations

$$P_{in} = \sqrt{3}V_L I_L \cos \theta = 3V_{ph} I_{ph} \cos \theta$$

$$P_{SCL} = 3I_1^2 R_1$$

$$P_{AG} = P_{in} - (P_{SCL} + P_{core}) = P_{conv} + P_{RCL} = 3I_2^2 \frac{R_2}{s} = \frac{P_{RCL}}{s}$$

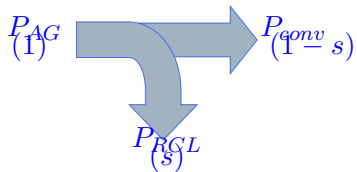
$$P_{RCL} = 3I_2^2 R_2$$

$$P_{conv} = P_{AG} - P_{RCL} = P_{RCL} \left(\frac{1}{s} - s \right) = \frac{P_{RCL}(1-s)}{s}$$

$$= (1-s)P_{AG}$$

$$P_{out} = P_{conv} - (P_{f+w} + P_{stray})$$

$$\tau_{ind} = \frac{P_{conv}}{\omega_m} = \frac{(1-s)P_{RCL}}{(1-s)\omega_s} = \frac{P_{AG}}{\omega_s}$$



$$P_{AG} : P_{RCL} : P_{conv}$$

$$1 : s : 1 - s$$

Example (2)

A $480 - V$, $60 - Hz$, $50 - hp$, three phase induction motor is drawing $60 - A$ at $0.85 - PF$ lagging. The *stator copper losses* are $2 - kW$, and the *rotor copper losses* are $700 - W$. The *friction and windage losses* are $600 - W$, the *core losses* are $1800 - W$, and the *stray losses* are negligible. Find the following quantities:

- (a) The air-gap power
- (b) The power converted
- (c) The output power
- (d) The efficiency of the motor.

Example (2)

Solution:

(a)

$$\begin{aligned}
 P_{in} &= \sqrt{3}V_L I_L \cos \theta \\
 &= \sqrt{3} \times 480 \times 60 \times 0.85 = 42.4 \text{ kW} \\
 P_{AG} &= P_{in} - P_{SCL} - P_{core} \\
 &= 42.4 - 2 - 1.8 = 38.6 \text{ kW}
 \end{aligned}$$

(b)

$$\begin{aligned}
 P_{conv} &= P_{AG} - P_{RCL} \\
 &= 38.6 - \frac{700}{1000} = 37.9 \text{ kW}
 \end{aligned}$$

Example (2)

Solution:

(c)

$$\begin{aligned}
 P_{out} &= P_{conv} - P_{(f+W)} \\
 &= 39.9 - \frac{600}{1000} = 37.3 \text{ kW} \quad (\cong 50 \text{ hp})
 \end{aligned}$$

(d)

$$\begin{aligned}
 \eta &= \frac{P_{out}}{P_{in}} \times 100\% \\
 &= \frac{37.3}{42.4} \times 100 = 88\%
 \end{aligned}$$

Example (3)

A 460 – V, 60 Hz, **four-pole**, *Y-connected* three-phase induction motor has the following impedances in ohms per phase referred to the stator circuit:

$$R_1 = 0.641 \, \Omega \quad R_2 = 0.332 \, \Omega$$

$$X_1 = 1.106 \, \Omega \quad X_2 = 0.464 \, \Omega \quad X_M = 26.3 \, \Omega$$

The total rotational losses are 1100 W and are assumed to be constant. The core loss is lumped in with the rotational losses. For a rotor slip of 2.2% at the *rated voltage* and *rated frequency*, find the motor's

- | | |
|--------------------|-----------------------------------|
| (a) Speed | (d) P_{conv} and P_{out} |
| (b) Stator Current | (e) τ_{ind} and τ_{out} |
| (c) Power Factor | (f) The efficiency of the motor. |

Example (3)

Solution:

(a)

$$n_{sync} = \frac{120f}{P} = \frac{120 \times 60}{4} = 1800 \text{ rpm}$$

$$n_m = (1 - 2)n_{sync} = (1 - 0.022) \times 1800 = 1760 \text{ rpm}$$

(b)

$$\begin{aligned} Z_2 &= \frac{R_2}{s} + jX_2 = \frac{0.332}{0.022} + j0.464 \\ &= 15.09 + j0.464 = 15.1 \angle 1.766^\circ \Omega \end{aligned}$$

$$\begin{aligned} Z_f &= \frac{1}{\frac{1}{jX_m} + \frac{1}{Z_2}} = \frac{1}{-j0.038 + 0.0662 \angle -1.766^\circ} \\ &= \frac{1}{0.0773 \angle -31.1^\circ} = 12.94 \angle 31.1^\circ \Omega \end{aligned}$$

Example (3)

Solution:

$$\begin{aligned}
 Z_{tot} &= Z_{stat} + Z_f \\
 &= 0.641 + j1.106 + 12.94/\underline{31.1^\circ} \\
 &= 11.72 + j7.79 = 14.07/\underline{33.6^\circ} \Omega
 \end{aligned}$$

$$I_1 = \frac{\frac{460/0^\circ}{\sqrt{3}}}{14.07/\underline{33.6^\circ}} = 18.88/\underline{-33.6^\circ} \text{ A}$$

(c)

$$PF = \cos 33.6^\circ = 0.833 \quad \textit{lagging}$$

(d)

$$\begin{aligned}
 P_{in} &= \sqrt{3}V_L I_L \cos \theta = \sqrt{3} \times 460 \times 18.88 \times 0.833 = 12530 \text{ W} \\
 P_{SCL} &= 3I_1^2 R_1 = 3(18.88)^2 \times 0.641 = 685 \text{ W} \\
 P_{AG} &= P_{in} - P_{SCL} = 12530 - 685 = 11845 \text{ W}
 \end{aligned}$$

Example (3)

Solution:

$$P_{conv} = (1 - s)P_{AG} = (1 - 0.022)(11845) = 11585 \text{ W}$$

$$P_{out} = P_{conv} - P_{f\&w} = 11585 - 1100 = 10485 \text{ W} \cong 14.1 \text{ hp}$$

(e)

$$\tau_m = \tau_{ind} = \frac{P_{AG}}{\omega_{sync}} = \frac{11845}{2\pi \times 1800/60} = 62.8 \text{ Nm}$$

$$\tau_{load} = \tau_{out} = \frac{P_{out}}{\omega_m} = \frac{10485}{2\pi \times 1760/60} = 56.9 \text{ Nm}$$

(f)

$$\eta = \frac{P_{out}}{P_{in}} = \frac{10485}{12530} \times 100\% = 83.7\%$$

Torque, power and Thevenin's Theorem

- *Thevenin's* theorem can be used to transform the network to the left of points 'a' and 'b' into an equivalent voltage source V_{TH} in series with equivalent impedance $R_{TH} + jX_{TH}$

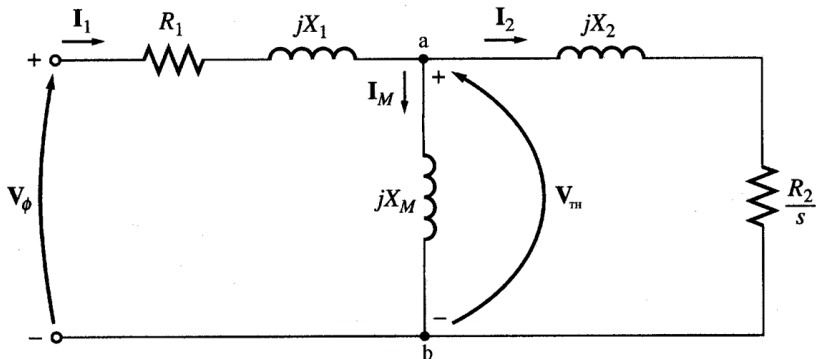


Figure 10: Rearranged equivalent circuit

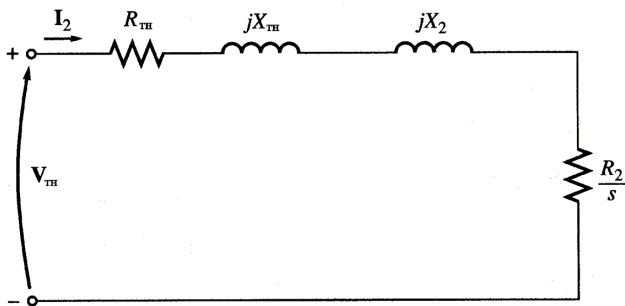


Figure 11: Thevenin equivalent circuit of IM.

$$V_{TH} = V_{\phi} \frac{jX_M}{R_1 + j(X_1 + X_M)} \quad |V_{TH}| = |V_{\phi}| \frac{X_M}{\sqrt{R_1^2 + (X_1 + X_M)^2}}$$

$$R_{TH} + jX_{TH} = (R_1 + jX_1) \parallel jX_M$$

- Since $X_M \gg X_1$ and $X_M \gg R_1$:

$$V_{TH} = V_\phi \frac{X_M}{X_1 + X_M} \quad (10)$$

- Because $X_M \gg X_1$ and $X_M + X_1 \gg R_1$

$$R_{TH} = R_1 \left(\frac{X_M}{X_1 + X_M} \right)^2 \quad (11)$$

and

$$X_{TH} \approx X_1 \quad (12)$$

$$I_2 = \frac{V_{TH}}{Z_T} = \frac{V_{TH}}{\sqrt{\left(R_{TH} + \frac{R_2}{s}\right)^2 + (X_{TH} + X_2)^2}} \quad (13)$$

- Then the power converted to mechanical, P_{conv} , becomes:

$$P_{conv} = 3I_2^2 \frac{R_2(1-s)}{s} \quad (14)$$

- And the internal mechanical torque, τ_m , is:

$$\tau_{ind} = \frac{P_{conv}}{(1-s)\omega_s} = \frac{3I_2^2 \frac{R_2}{s}}{\omega_s} = \frac{P_{AG}}{\omega_s} \quad (15)$$

- Substituting P_{AG} in *equation-15*, the internal produced torque will have an expression of:

$$\begin{aligned}\tau_{ind} &= \frac{3}{\omega_s} \left(\frac{V_{TH}}{\sqrt{(R_{TH} + \frac{R_2}{s})^2 + (X_{TH} + X_2)^2}} \right)^2 \left(\frac{R_2}{s} \right) \\ &= \frac{1}{\omega_s} \frac{3V_{TH}^2 \left(\frac{R_2}{s} \right)}{\left(R_{TH} + \frac{R_2}{s} \right)^2 + (X_{TH} + X_2)^2}\end{aligned}\quad (16)$$

Torque-speed characteristics

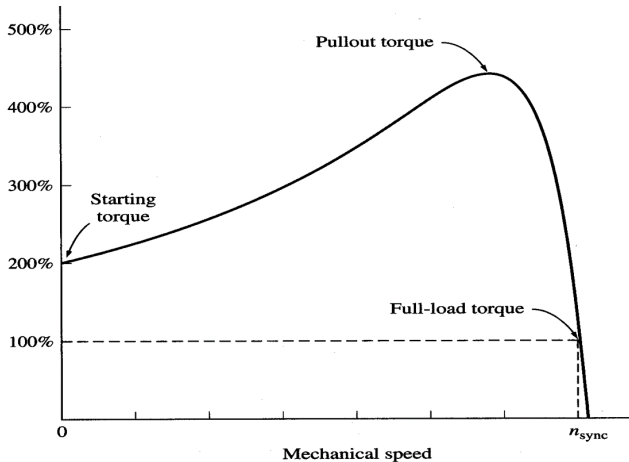


Figure 12: Typical torque-speed characteristics of induction motor.

Notes!

- The induced torque is zero at synchronous speed.
- The curve is nearly linear between no-load and full load. In this range, the rotor resistance is much greater than the reactance, so the rotor current, torque increase linearly with the slip.
- There is a maximum possible torque that can't be exceeded. This torque is called *pullout torque* and is **2** to **3** times the *rated full-load torque*.
- The starting torque of the motor is slightly higher than its full-load torque, so the motor will start carrying any load it can supply at full load.
- The torque of the motor for a given slip varies as the square of the applied voltage.
- If the rotor is driven faster than synchronous speed it will run as a generator, converting mechanical power to electric power.

Complete Speed-Torque Characteristics

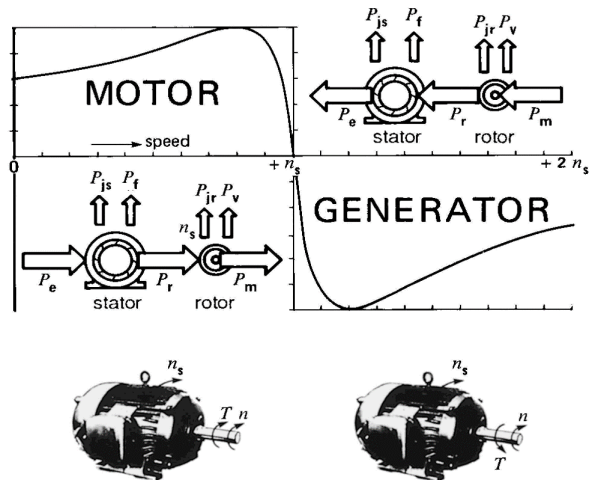


Figure 13: Complete Speed-Torque characteristics of induction motor.

Maximum Torque

- Maximum torque occurs when the power transferred to R_2/s is maximum.
- This condition occurs when R_2/s equals the magnitude of the impedance $R_{TH} + j(X_{TH} + X_2)$

$$\frac{R_2}{s\tau_{max}} = \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}$$

$$s\tau_{max} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}} \quad (17)$$

- The corresponding maximum torque of an induction motor equals

$$\tau_{max} = \frac{1}{2\omega_s} \left(\frac{3V_{TH}^2}{R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}} \right) \quad (18)$$

$$\tau_{max} = \frac{1}{2\omega_s} \left(\frac{3V_{TH}^2}{R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}} \right)$$

- The slip at maximum torque is directly proportional to the rotor resistance R_2
- The maximum torque is independent of R_2 Rotor resistance can be increased by inserting external resistance in the rotor of a *wound-rotor induction* motor.
 - The value of the maximum torque remains unaffected but the speed at which it occurs can be controlled.

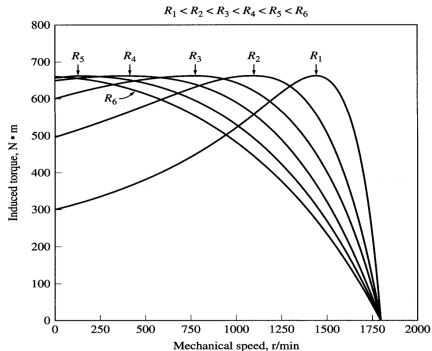


Figure 14: Effect of rotor resistance on torque-speed characteristic.

Example (4)

A *two-pole*, $50 - Hz$ induction motor supplies $15 kW$ to a load at a speed of $2950 rpm$.

- (a) What is the motor's slip?
- (b) What is the induced torque in the motor in Nm under these conditions?
- (c) What will be the operating speed of the motor if its torque is doubled?
- (d) How much power will be supplied by the motor when the torque is doubled?

Example (4)

Solution:

(a)

$$n_{sync} = \frac{120f}{P} = \frac{120 \times 50}{2} = 3000 \text{ rpm}$$
$$s = \frac{n_{sync} - n_m}{n_{sync}} = \frac{3000 - 2950}{300} = 0.0167 \text{ or } 1.67\%$$

(b) Since no P_{f+w} is given, assume: $P_{conv} = P_{load}$ and $\tau_{ind} = \tau_{load}$

$$\tau_{ind} = \frac{P_{conv}}{\omega_m} = \frac{15 \times 10^3}{2950 \times 2\pi/60} = 48.6 \text{ Nm}$$

Example (4)

Solution:

- (c) In the low-slip region, the torque-speed curve is linear and the induced torque is direct proportional to slip. So, if the torque is doubled the new slip will be **3.33%** and the motor speed will be

$$n_m = (1 - s)n_{sync} = (1 - 0.0333) \times 3000 = 2900 \text{ rpm}$$

(d)

$$\begin{aligned} P_{conv} &= \tau_{ind} \omega_m \\ &= (2 \times 48.6) \times (2900 \times \frac{2\pi}{60}) = 29.5 \text{ kW} \end{aligned}$$

Example (5)

A $460 - V$, $25 - hp$, $60 Hz$, **four-pole**, *Y-connected* three-phase induction motor has the following impedances in ohms per phase referred to the stator circuit:

$$\begin{aligned} R_1 &= 0.641 \, \Omega & R_2 &= 0.332 \, \Omega \\ X_1 &= 1.106 \, \Omega & X_2 &= 0.464 \, \Omega & X_M &= 26.3 \, \Omega \end{aligned}$$

- (a) What is the maximum torque of this motor? At what speed and slip does it occur?
- (b) What is the starting torque of this motor?
- (c) If the rotor resistance is doubled, what is the speed at which the maximum torque now occur? What is the new starting torque of the motor?

Example (5)

Solution: First lets calculate the important *Thevanen* values

$$|V_{TH}| = |V_\phi| \frac{X_M}{\sqrt{R_1^2 + (X_1 + X_M)^2}} = 255.2 \text{ V}$$

$$R_{TH} = R_1 \left(\frac{X_M}{X_1 + X_M} \right)^2 = 0.590 \text{ } \Omega$$

$$X_{TH} \approx X_1 = 1.106 \text{ } \Omega$$

(a)

$$s_{\tau_{max}} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}} = \frac{0.332}{\sqrt{(0.590)^2 + (1.106 + 0.464)^2}} = 0.198$$

the corresponding speed is:

$$(1 - s)n_{sync} = (1 - 0.198) \times 1800 = 1444 \text{ rpm}$$

Example (5)

Solution:

The torque at this speed is

$$\tau_{max} = \frac{1}{2\omega_s} \left(\frac{3V_{TH}^2}{R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}} \right) = 229 \text{ Nm}$$

- (b) The starting torque can be found from the torque eqn. by substituting $s = 1$. And *equation-16* takes the form:

$$\tau_{start} = \frac{3V_{TH}^2 R_2}{\omega_s [(R_{TH} + R_2)^2 + (X_{TH} + X_2)^2]} = 104 \text{ Nm}$$

Example (5)

Solution:

- (c) If the rotor resistance is doubled, then the slip at maximum torque doubles too.

$$s_{\tau_{max}} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}} = 0.369$$

The corresponding speed is:

$$n_m = (1 - s)n_{sync} = (1 - 0.0369) \times 1800 = 1087 \text{ rpm}$$

The maximum torque is still: $\tau_{max} = 229 \text{ Nm}$

The starting torque is now:

$$\tau_{start} = \frac{3V_{TH}^2 R_2}{\omega_s [(R_{TH} + R_2)^2 + (X_{TH} + X_2)^2]} = 170 \text{ Nm}$$

Questions?

