Lecture-5

Introduction to Transformers

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Overview

1. Overview
2. Objective
3. Introduction
4. Transformer Variables and Their Relationships
   - Voltage relationships
   - Sinusoidal operation
   - Current relationships
   - Transformer rating
5. Types of Power Transformer
   - Auto-wound transformers
   - 3-phase transformers
6. Ideal Transformer Properties
   - Impedance transformation

Lecture Objectives

The objectives of this lecture are:

- To discuss the operating principle of transformers,
- To discuss the different variables that involve in transformers,
- To discuss the different types of power transformers, and
- To develop a model of ideal transformer and discuss the related impedance transformation.

Introduction

Shown below is a schematic representation of a *single-phase transformer* with two coils on a magnetic core, where the magnetic coupling is assumed to be perfect:

- the same flux ϕ passes through each turn of each coil.

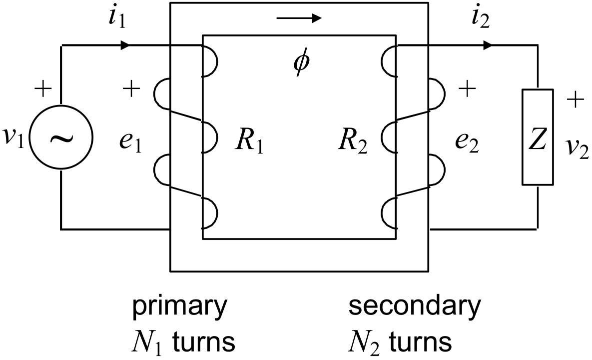

Figure 1: Transformer with source and load.

Transformer Variables and Their Relationships

Voltage relationships

- *Kirchhoff's voltage law* applied to the two windings gives:

$$v_1 = e_1 + R_1 i_1 = N_1 \frac{d\phi}{dt} + R_1 i_1 \tag{1}$$

$$v_2 = e_2 + R_2 i_2 = N_2 \frac{d\phi}{dt} + R_2 i_2 \tag{2}$$

- If the resistances R_1 and R_2 are negligible, then equations 1 and 2 become:

$$v_1 = N_1 \frac{d\phi}{dt} \tag{3}$$

$$v_2 = N_2 \frac{d\phi}{dt} \tag{4}$$

- Dividing these equations gives the important result:

$$\frac{v_1}{v_2} \approx \frac{N_1}{N_2} \tag{5}$$

Sinusoidal operation

- If the voltage source is sinusoidal, then the core flux will also be sinusoidal, hence:

$$\phi = \Phi_m \sin \omega t \tag{6}$$

- Substituting this expression in *equation-3* gives:

$$v_1 = N_1 \frac{d\phi}{dt} = N_1 \omega \Phi_m \cos \omega t = v_{1m} \cos \omega t \tag{7}$$

- Thus the maximum primary voltage is:

$$v_{1m} = N_1 \omega \Phi_m = 2\pi f N_1 \Phi_m = 2\pi f N_1 A B_m \tag{8}$$

where A is the cross-sectional area of the core and B_m is the maximum flux density in the core.

- A typical value for $B_m = 1.4T$ for the silicon steel characteristic in *figure-2*.

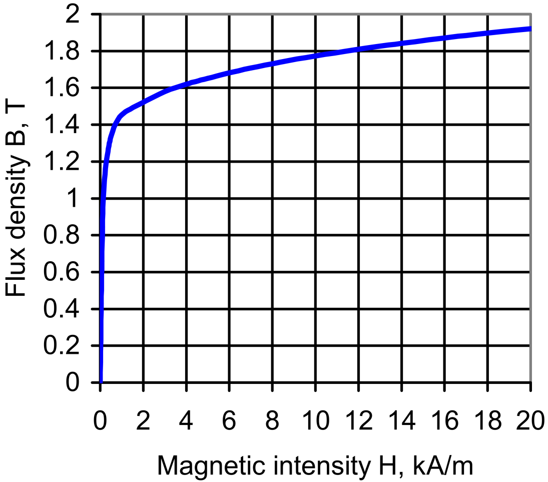

Figure 2: Silicon transformer steel.

Current relationships

- The relationship between the primary and secondary currents can be found by considering the magnetic circuit of the transformer.
- From the basic magnetic circuit equation, we have:

$$F = N_1 i_1 - N_2 i_2 = R\phi \tag{9}$$

- In a well-designed transformer, the reluctance R is small, so *equation-9* becomes:

$$N_1 i_1 - N_2 i_2 \approx 0 \tag{10}$$

- This gives the counterpart of *equation-5* for voltage:

$$\frac{i_1}{i_2} \approx \frac{N_2}{N_1} \tag{11}$$

- If sinusoidal voltages and currents are represented by phasors, the corresponding forms for the basic voltage and current equations are:

$$\frac{V_1}{V_2} \approx \frac{N_1}{N_2} \tag{12}$$

$$\frac{I_1}{I_2} \approx \frac{N_2}{N_1} \tag{13}$$

Transformer rating

- The maximum voltage at the primary terminals of a transformer is determined by *equation-8*, and is independent of the current.
- The maximum primary current is determined by the I^2R power loss in the resistance of the transformer windings, which generates heat in the transformer.
  - This power loss is independent of the applied voltage.
- For a given design of transformer, there is a maximum value for the product V_1I_1 at the primary terminals.
  - To a first approximation, this is also equal to the product V_2I_2 at the secondary terminals.
  - This maximum value does not depend on the phase angle between the voltage and the current.
- Transformer ratings, therefore, specify the apparent power VI (volt-amperes, VA) rather than the real power $VIcos\theta$ (watts, W).

Types of Power Transformer

Auto-wound transformers

In addition to the ordinary single-phase power transformer, two other types are in common use: auto-wound transformers, and 3-phase transformers.

1) ***Auto-wound transformers***
   - A transformer can have a single coil with an output taken from a portion of the coil, as shown in *figure-3*.
   - This is known as an *auto-wound transformer* or *auto-transformer*.

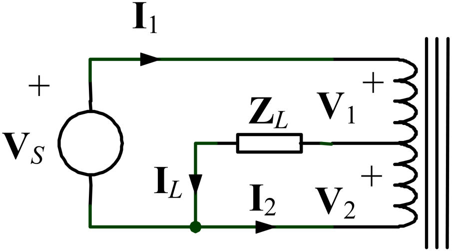

Figure 3: Auto-wound transformer.

- Unlike the normal transformer with two windings, known as a *double-wound transformer*, the *auto- wound transformer* does not provide electrical isolation between the primary and the secondary.
- Let N_1 be the number of turns on the upper part of the winding in *figure-3*, and N_2 the number of turns on the lower part.
- The conventional transformer *equations-12* and *13* apply to these parts of the winding, since they are equivalent to two separate windings with a common connection.

Figure 4: Variable transformers.

- Applying *Kirchhoff's* law to this circuit gives:

$$V_S = V_1 + V_2 \tag{14}$$

$$I_L = I_1 + I_2 \tag{15}$$

- As an example, suppose that $N_1 = N_2$. If the transformer is regarded as *ideal*, then $I_1 = I_2$ and $V_1 = V_2$.
- Equations *14* and *15* give:

$$V_S = V_1 + V_2 = 2V_L \tag{16}$$

$$I_L = I_1 + I_2 = 2I_1 = 2I_S \tag{17}$$

  where V_L is the voltage across the load and I_S is the current supplied by the source.
- This auto-wound transformer behaves as a step-down transformer with a ratio of $2:1$, and the current in each winding is equal to half of the load current.
- One example of auto transformer is variable transformer (shown in *figure-4* on previous page)

2) 3-phase transformers

- In 3-phase systems, it is common practice to use sets of three single-phase transformers.
- It is also possible, however, to make 3-phase transformers with three sets of windings on three limbs of a core, as shown in *figure-5*.

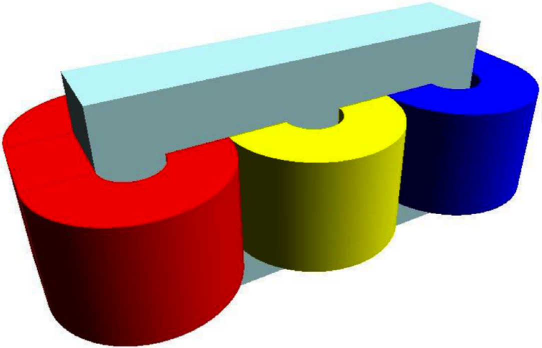

Figure 5: 3-phase transformer model.

- The corresponding fluxes are shown below.

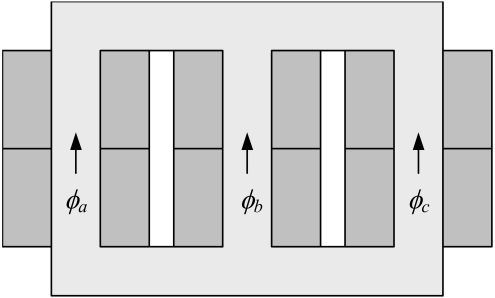

Figure 6: 3-phase transformer flux..

- In a balanced system with sinusoidal phase voltages, the fluxes are given by:

$$\begin{aligned} \phi_a &= \Phi_m \cos \omega t \\ \phi_b &= \Phi_m \cos(\omega t - 120^\circ) \\ \phi_c &= \Phi_m \cos(\omega t - 240^\circ) \end{aligned} \tag{18}$$

- There is no requirement for another limb to form a flux return path, because the fluxes ϕ_a, ϕ_b and ϕ_c sum to zero in a balanced 3-phase system.

Proof

$$\frac{\phi_a + \phi_b + \phi_c}{\Phi m}$$

$$\begin{aligned} &= \cos\omega t + cos(\omega t - 120^\circ) + \cos(\omega t - 240^\circ) \\ &= \cos\omega t + 2\cos\omega t \cos 120^\circ \\ &= \cos\omega t - \cos\omega t = 0 \end{aligned}$$

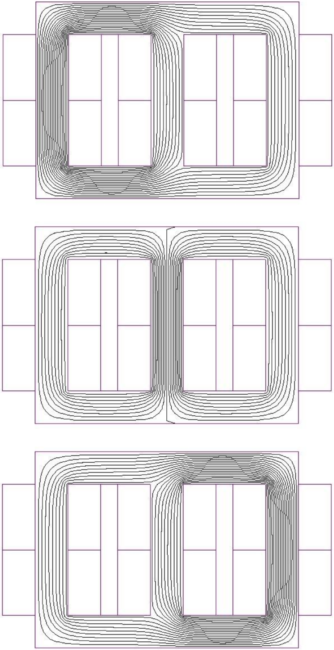

Figure: 3-phase transformer flux plots. (a) 0°, (b) 120°, (c) 240°

Ideal Transformer Properties

- If the primary and secondary windings have zero resistance, and the magnetic core has zero reluctance, then the approximate equalities in *equations-12* and *13* become exact equalities.
- This leads to the concept of an ideal transformer element, to accompany the other ideal elements of circuit theory.
- *Figure-7* shows a circuit symbol for the ideal transformer element.

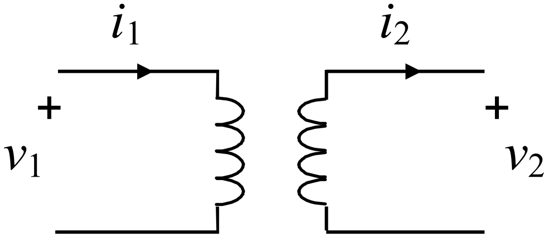

Figure 7: Ideal transformer element.

- The voltage and current relationships in the time and frequency domains are given below:

Time domain	**Frequency domain**
$\dfrac{v_2}{v_1} = \dfrac{N_2}{N_1} = n$	$\dfrac{V_2}{V_1} = \dfrac{N_2}{N_1} = n$
$\dfrac{i_1}{i_2} = \dfrac{N_2}{N_1} = n$	$\dfrac{I_1}{I_2} = \dfrac{N_2}{N_1} = n$

- The following properties of the ideal transformer can be deduced from the previous equations:
  - The voltage transformation is independent of the current, and *vice versa*.
  - If the secondary is short-circuited, so that $v_2 = 0$,the primary terminals appear to be short-circuited since $v_1 = 0$.
  - If the secondary is open-circuited, so that $i_2 = 0$,the primary terminals appear to be open-circuited since $i_1 = 0$.
  - The output power is equal to the input power, so there is no power loss in the element.

Impedance transformation

- The ideal transformer has the important property of transforming impedance values in a circuit.
- Consider an ideal transformer with an impedance Z_L connected to its secondary terminals, as shown in *figure-8*.

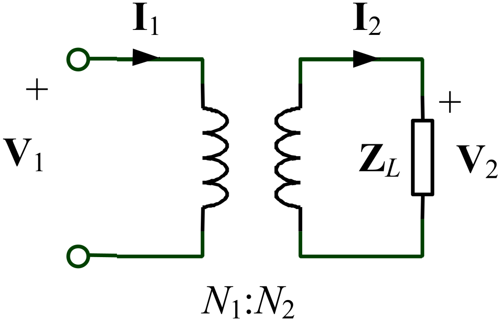

Figure 8: Ideal transformer with a load.

- The secondary impedance is given by:

$$Z_L = \frac{V_2}{I_2} \tag{19}$$

- At the primary terminals, the circuit presents an impedance given by:

$$\begin{aligned} Z_{Lin} &= \frac{V_1}{I_1} = \frac{\frac{N_1 V_2}{N_2}}{\frac{N_2 I_2}{N_1}} \\ &= \left(\frac{N_1}{N_2}\right)^2 \frac{V_2}{I_2} = \left(\frac{N_1}{N_2}\right)^2 Z_L = \frac{Z_L}{n^2} \end{aligned} \tag{20}$$

- Thus the combination of an ideal transformer of ratio n and an impedance Z_L can be replaced by an equivalent impedance Z_L/n^2.

Referred impedances

- *Figure-9(a)* shows an ideal transformer with a load impedance Z_L connected to the secondary.
- Another impedance Z_2 is in series with Z_L. The input impedance of this circuit is:

$$Z_{in} = \frac{Z_2 + Z_L}{n^2} \qquad (21)$$

- The input impedance of the circuit in *figure-9(b)* is:

$$Z_{in} = Z_2' + \frac{Z_L}{n^2} \qquad (22)$$

where $Z_2' = Z_2/n^2$

- The impedance Z_2' is termed the secondary impedance Z_2 *referred* to the primary.

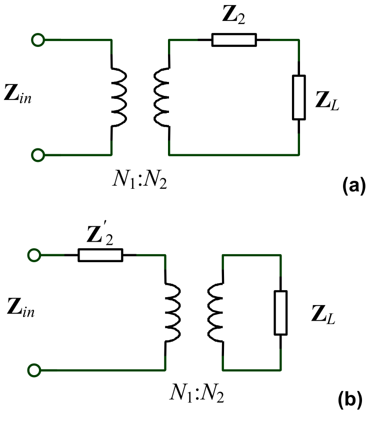

Figure 9: Referred Impedance.

- In a similar way, a primary impedance Z_1 can be referred to the secondary, as shown in *figure-10*.
- In this case, the referred impedance is given by:

$$Z_1'' = n^2 Z_1 \tag{23}$$

- The concept of referred impedance is often a useful device for simplifying circuits containing transformers.

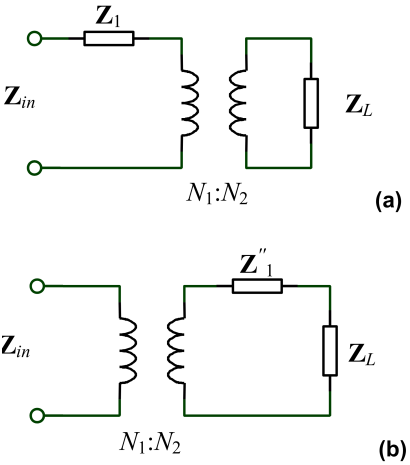

Figure 10: Referred Impedance.

- It is conventional to use a *single prime* (') to denote quantities referred to the primary side, and a *double prime* (") to denote quantities referred to the secondary side.

Example (1)

A single-phase transformer with a $2 - kVA$ rating has a $480 - V$ primary, and a $120 - V$ secondary. Determine the primary and secondary full-load currents of the transformer.

Solution:

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Solution:

- Primary full-load current, $I_{1,f}$

$$I_{1,f} = \frac{2\ VA \times 1000}{480\ V} = 4.17\ A$$

- Secondary full-load current, $I_{2,f}$

$$I_{2,f} = \frac{2\ VA \times 1000}{120\ V} = 16.67\ A$$

It may seem strange at first, but the transformer current will be higher in the winding which produces the lower voltage. This concept is important to understand in order to avoid transformer or conductor overloading.

Example (2)

A transformer has $500\ turns$ of the primary winding and $10\ turns$ of the secondary winding.

(a) Determine the secondary voltage if the secondary circuit is open and the primary voltage is $120\ V$.

(b) Determine the current in the primary and secondary winding, given that the secondary winding is connected to a resistance load $15\ \Omega$?

Solution:

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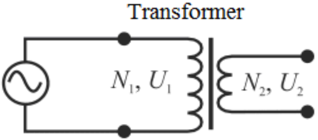
Solution:

(a) To solve this task we use the fact that the ratio of the voltage on the secondary and on the primary coil is the same as the ratio of the number of turns of both coils:



Example (2)

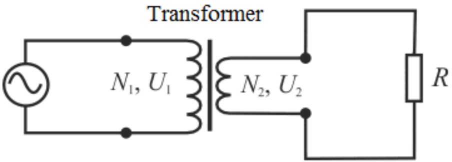
$$n = \frac{U_2}{U_1} = \frac{N_2}{N_1}$$

- Then the unknown secondary voltage is:

$$U_2 = \frac{N_2}{N_1} U_1 = \frac{10}{500} \cdot 120V = 2.4\ V$$

If the secondary circuit is open, the secondary voltage is 2.4 V.

(b) For this section the circuit takes the form:



Here $I_1 = U_1/R' = U_1/(R/n^2) = U_1 n^2/R \quad \Rightarrow \quad I_1 = 3.2\ mA$

Example (3)

An autotransformer has a coil with total number of turns $N_{CD} = 200$ between terminals C and D. It has got one tapping at A such that $N_{AC} = 100$ and another tapping at B such that $N_{BA} = 50$. Calculate currents in various parts of the circuit and show their directions when $400\ V$ supply is connected across AC and two resistive loads of $60\ \Omega$ & $40\ \Omega$ are connected across BC and DC respectively.

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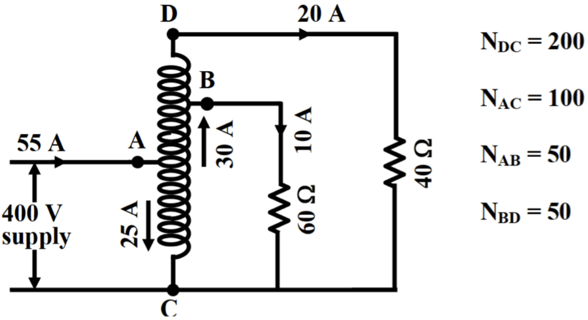

Example (3)

Solution:

First calculate the voltages applied across the loads remembering the fact that voltage per turn in a transformer remains constant.

$$
\begin{aligned}
\text{Supply voltage across } AC,\ V_{AC} &= 400 \text{ V} \\
\text{Number of turns between A \& C } N_{AC} &= 100 \\
\text{Voltage per turn} &= 400/100 = 4 \\
\text{Voltage across the } 40\Omega \text{ load} &= N_{DC} \times \text{ voltage per turn} \\
&= 200 \times 4 = 800\ V \\
\text{So, current through } 40\Omega &= 800/40 = 20 \text{ A} \\
\text{Voltage across the } 60\Omega \text{ load} &= N_{BC} \times \text{ voltage per turn} \\
&= 150 \times 4 = 600\ V \\
\text{So, current through } 60\Omega &= 600/60 = 10 \text{ A}
\end{aligned}
$$

Example (3)

- Total output kVA will be the simple addition of the kVAs supplied to the loads i,e.,

$$(600 \times 10 + 800 \times 20) = 22000\ VA = 22\ KVA$$

- Assuming the autotransformer to be ideal, input kVA must also be $22\ kVA$. We are therefore in a position to calculate the current drawn from the supply.
  Current drawn from the supply $= 22000/400 = 55\ A$
- Finally, currents in various parts of the circuit become:

$$\begin{aligned} \text{Current in DB part of the winding, } I_{BD} &= 20 \text{ A} \\ \text{Applying KCL at B, current in AB part, } I_{AB} &= 20 + 10 = 30 \text{ A} \\ \text{Applying KCL at A, current in AC part } I_{AC} &= 55\text{-}30 = 25 \text{ A} \end{aligned}$$

Questions?