Addis Ababa University
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School of Mechanical & Industrial Engineering

Engineering Mechanics II (Dynamics)
MEng 2052

Chapter Three

Kinetics of Particles

Kinetics of Particles

- It is the study of the relations existing between the forces acting on body, the mass of the body, and the motion of the body.
- It is the study of the relation between unbalanced forces and the resulting motion.

- Newton 's first law and third law are sufficient for studying bodies at rest (statics) or bodies in motion with no acceleration.

- When a body accelerates (change in velocity magnitude or direction) Newton 's second law is required to relate the motion of the body to the forces acting on it.

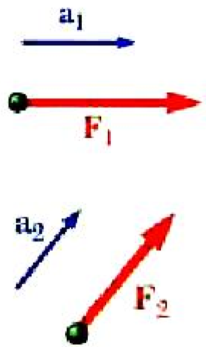
Kinetics of Force, Mass and Acceleration

- Newton 's Second Law: If the resultant force acting on a particle is not zero the particle will have an acceleration proportional to the magnitude of resultant and in the direction of the resultant.

- The basic relation between force and acceleration is found in Newton's second law of motion and its verification is entirely experimental.

- Consider a particle subjected to constant forces

$$\frac{F_1}{a_1} = \frac{F_2}{a_2} = \ldots = \frac{F}{a} = const$$



- We conclude that the constant is a measure of some property of the particle that does not change.

- This property is the inertia of the particle which is its resistance to rate of change of velocity.
- The mass m is used as a quantitative measure of inertia, and therefore the experimental relation becomes,

$$F=ma$$

- This relation provides a complete formulation of Newton's second law; it expresses not only that the magnitude **F** and **a** are proportional but also that the vector **F** and **a** have the same direction.

Equation of Motion and Solution of Problems

- When a particle of mass m acted upon by several forces. The Newton's second law can be expressed by the equation

$$\sum F = ma$$

- To determine the acceleration we must use the analysis used in kinematics, i.e
  - *Rectilinear motion*
  - *Curvilinear motion*

Rectilinear Motion

- If we choose the x-direction, as the direction of the rectilinear motion of a particle of mass m, the acceleration in the y and z direction will be zero, i.e

$$\sum F_x = ma_x$$
$$\sum F_y = 0$$
$$\sum F_z = 0$$

- Generally,

$$\sum F_x = ma_x$$
$$\sum F_y = ma_y$$
$$\sum F_z = ma_z$$

- Where the acceleration and resultant force are given by

$$a = a_x i + a_y j + a_z k$$
$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$\sum F = F_x i + F_y j + F_z k$$
$$\left|\sum F\right| = \sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2 + \left(\sum F_z\right)^2}$$

Curvilinear Motion

- In applying Newton's second law, we shall make use of the three coordinate descriptions of acceleration in curvilinear motion.

Rectangular Coordinates

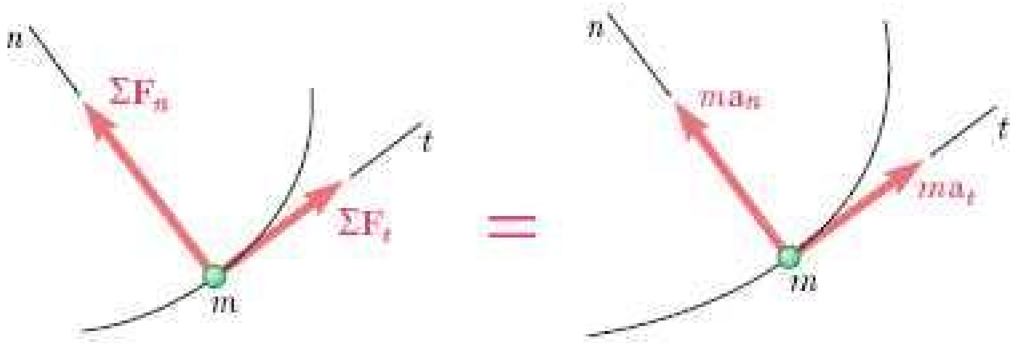
$$\sum F_x = ma_x$$
$$\sum F_y = ma_y$$

Where $a_x = \ddot{x}$ and $a_y = \ddot{y}$

Normal and Tangential Coordinates

$$\sum F_n = ma_n$$
$$\sum F_t = ma_t$$

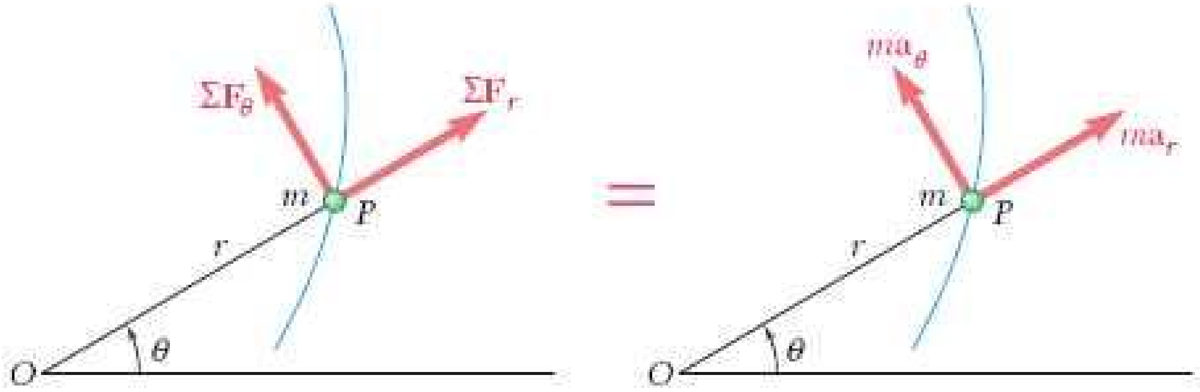
- Where $a_n = \rho \dot{\beta}^2 = \dfrac{v^2}{\rho}, a_t = \dot{v}$



Polar Coordinates

$$\sum F_r = ma_r$$
$$\sum F_\theta = ma_\theta$$

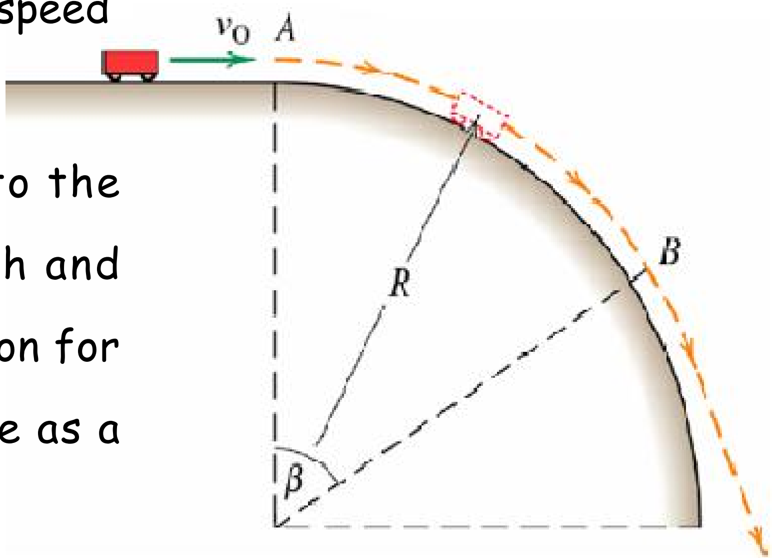
- Where $a_r = \ddot{r} - r\dot{\theta}^2$ and $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$



Example 1

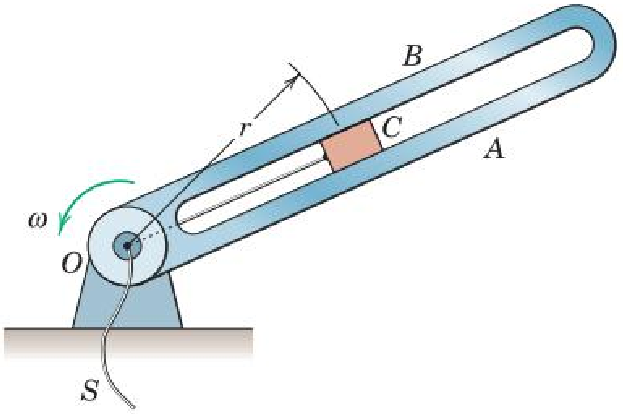
A small vehicle enters the top A of the circular path with a horizontal velocity v_o and gathers speed as it moves down the path.

Determine an expression for the angle β to the position where the vehicle leaves the path and becomes a projectile. Evaluate your expression for $v_o=0$. Neglect friction and treat the vehicle as a particle

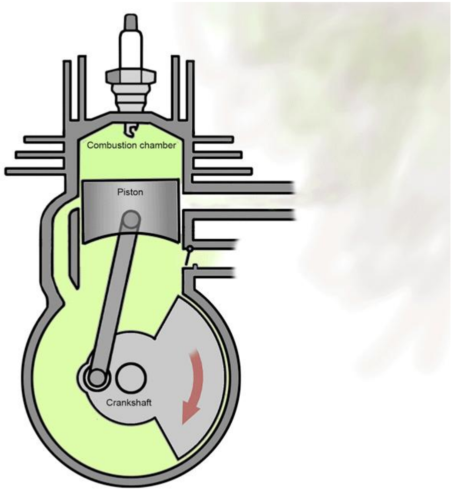


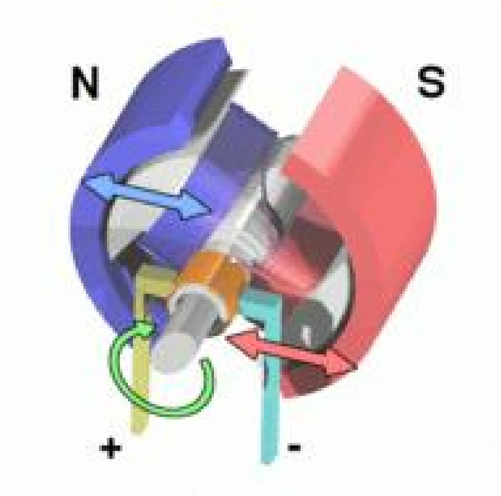
Example 2

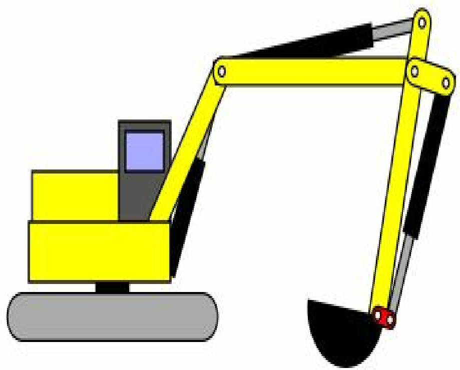
- The slotted arm revolves in the horizontal about the fixed vertical axis through point O. the 2 Kg slider C is drawn towards O at the constant rate of 50mm/s by pulling the cord S. At the instant for which r=225mm, the arm has a counterclockwise angular velocity ω=6rad/s and is slowing down at the rate of 2rad/s^2 .for this instant determine the tension T in the cord and the magnitude N of the force exerted on the slider by the sides of the smooth radial slot. Indicate which side, A or B, of the slot contacts the slider.



Work and Kinetic Energy







- The method of work and energy directly relates force, mass, velocity, and displacement.
- We apply this method:
  - When intervals of motion are involved where the change in velocity or the corresponding displacement of the particle is required.
- Integration of the forces with respect to the displacement of the particle leads to the equation of work and energy.

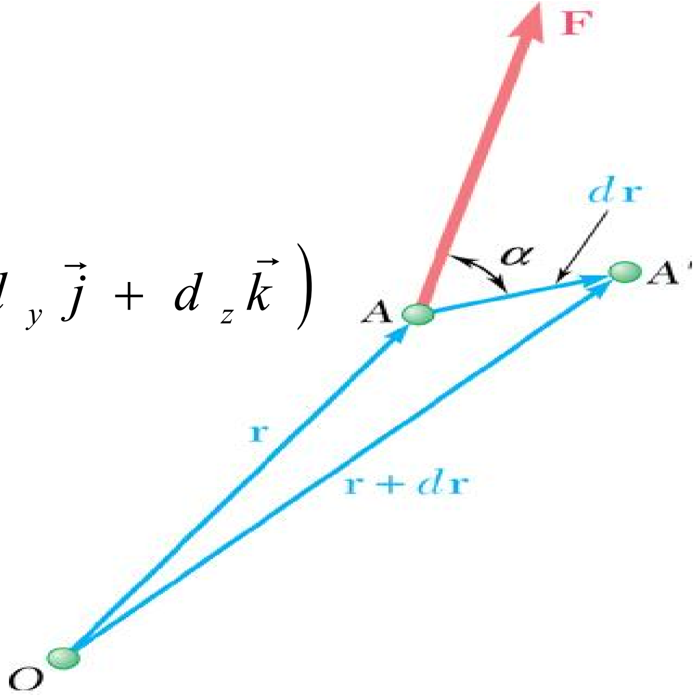
Work of Force

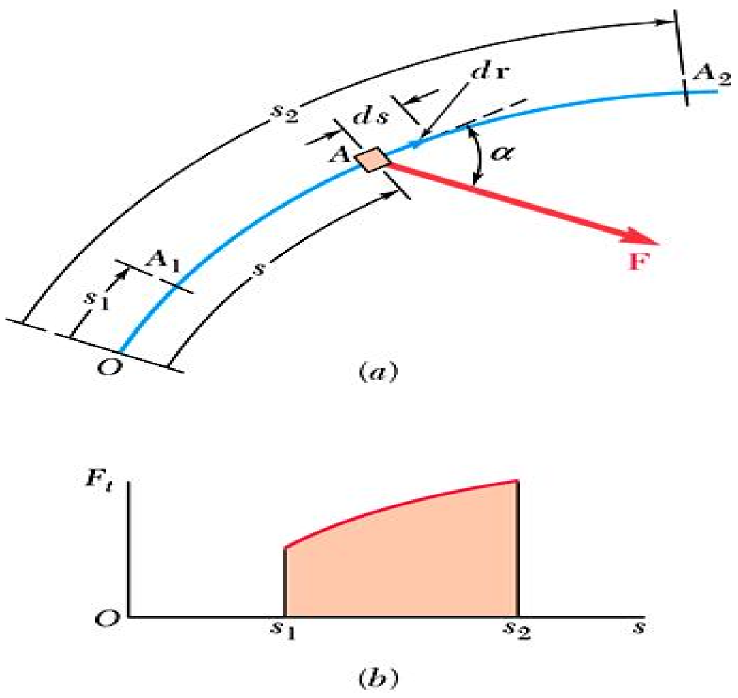
$$dU = \vec{F} \bullet d\vec{r}$$

$$dU = Fds \cos \alpha$$

$$dU = \left(F_x\vec{i} + F_y\vec{j} + F_z\vec{k}\right)\bullet\left(d_x\vec{i} + d_y\vec{j} + d_z\vec{k}\right)$$

$$= F_x dx + F_y dy + F_z dz$$



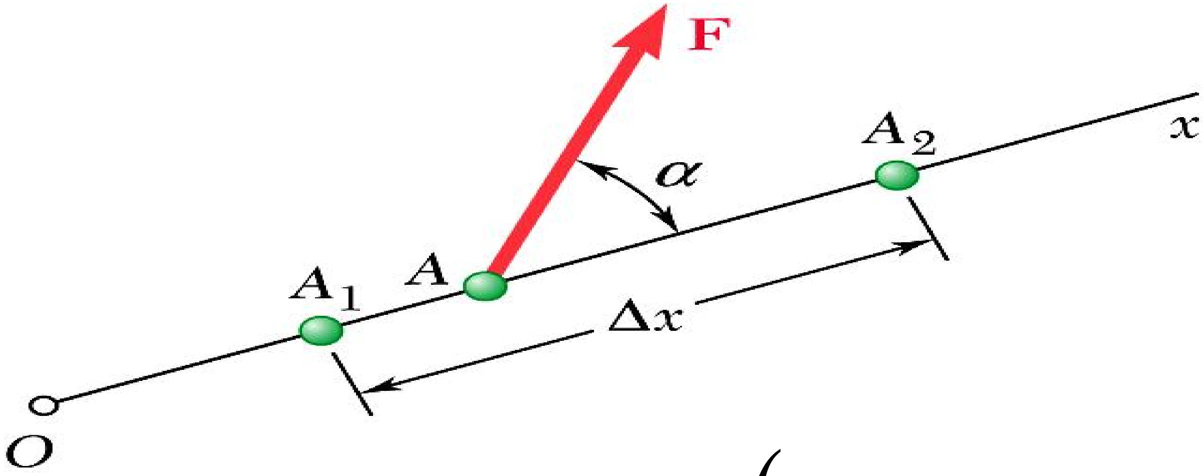


$$U_{1\to 2} = \int_{A_1}^{A_2} \vec{F} \bullet d\vec{r}$$

$$U_{1\to 2} = \int_{s_1}^{s_2} (F \cos \alpha)ds = \int_{s_1}^{s_2} F_t ds$$

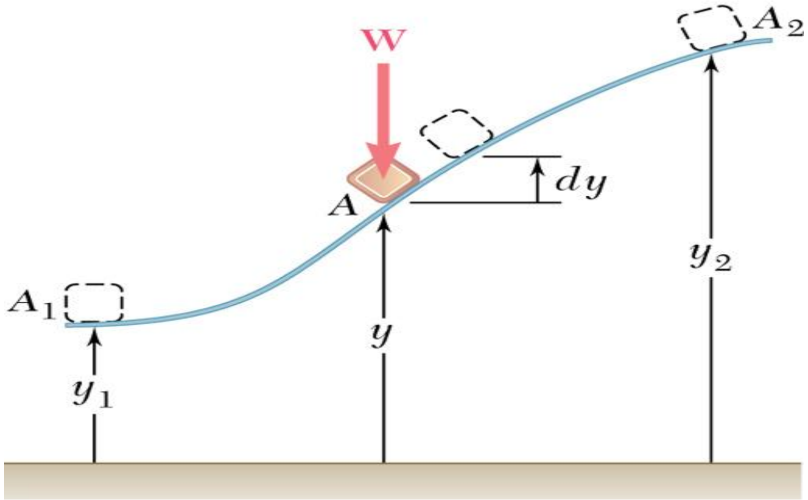
$$U_{1\to 2} = \int_{A_1}^{A_2} \left(F_x dx + F_y dy + F_z dz\right)$$

Work of a constant force in rectilinear motion



$$U_{1\to 2} = (F\cos\alpha)\Delta x$$

Work of the Force of Gravity



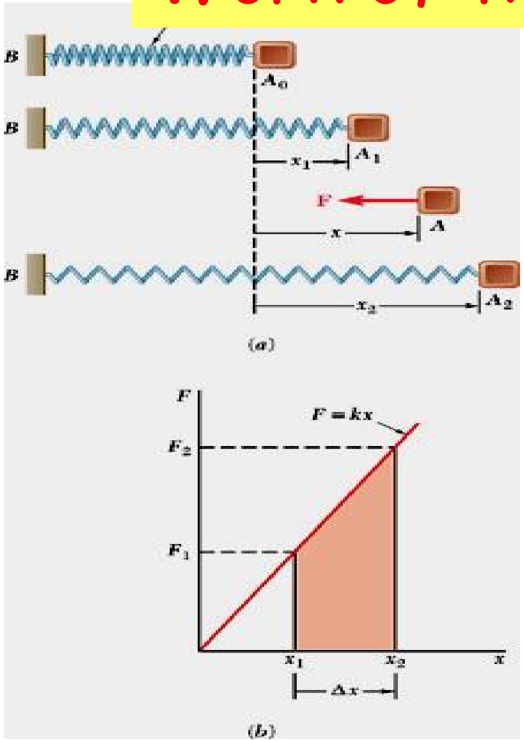
$$dU = -W\,dy$$

$$U_{1\to 2} = -\int_{y_1}^{y_2} W\,dy = Wy_1 - Wy_2$$

$$U_{1\to 2} = -W(y_2 - y_1) = -W\Delta y$$

When Δy is negative (moves down), the work is positive

Work of the Force Exerted by a Spring



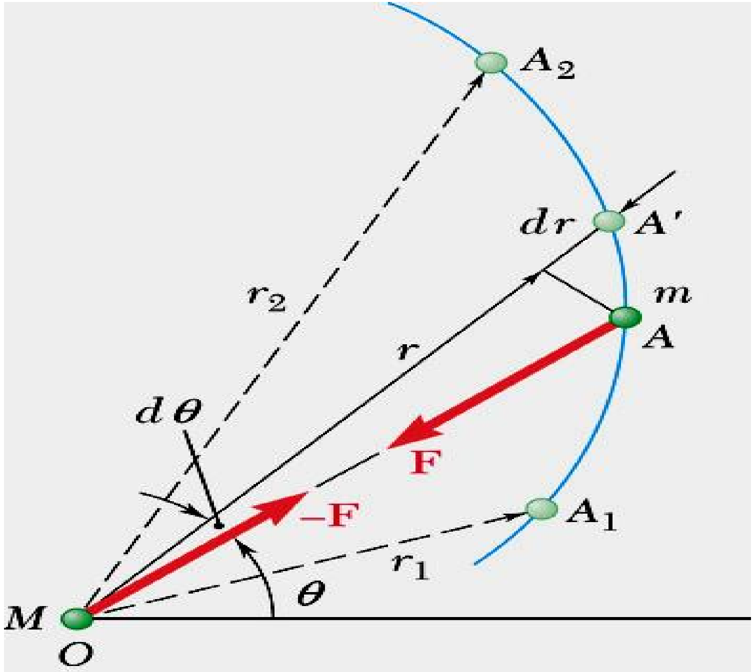
$$F = kx$$

$$dU = -F\,dx = -kx\,dx$$

$$U_{1\rightarrow 2} = -\int_{x_1}^{x_2} kx\,dx = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$$

If the spring returning to the undeformed position, then positive energy

Work of a gravitational Force



$$F = G\frac{Mm}{r^2}$$

$$dU = -Fdr = -G\frac{Mm}{r^2}dr$$

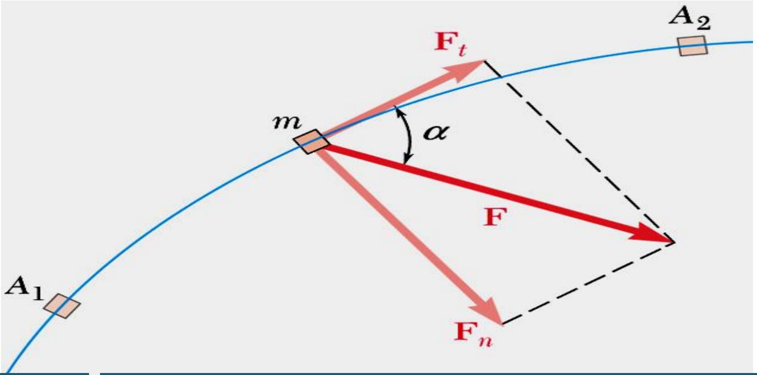
$$U_{1\rightarrow 2} = -\int_{r_1}^{r_2}\frac{GMm}{r^2}dr = \frac{GMm}{r_2} - \frac{GMm}{r_1}$$

Kinetic Energy of a Particle

$$F_t = ma_t = m\frac{dv}{dt} = m\frac{dv}{ds}\frac{ds}{dt} = mv\frac{dv}{ds}$$

$$F_t ds = mvdv$$

$$\int_{s_1}^{s_2} F_t ds = m\int_{v_1}^{v_2} vdv = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$



$$U_{1\to 2} = T_2 - T_1$$

$$T_1 + U_{1\to 2} = T_2$$

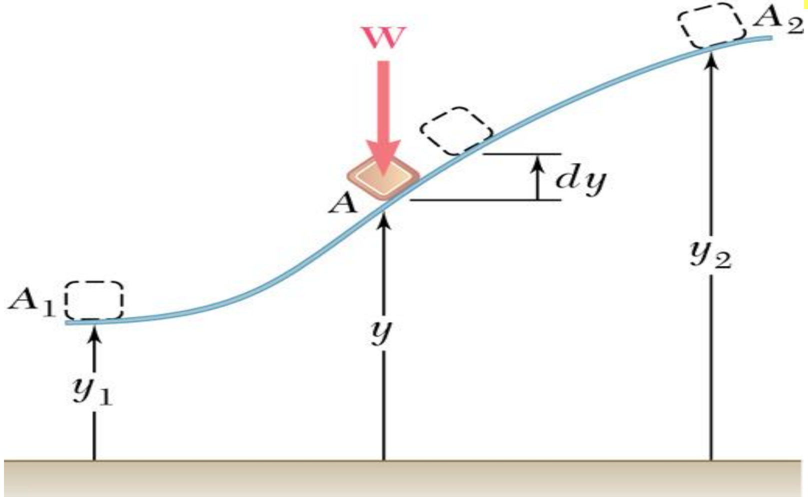
Power and Efficiency

- Friction energy dissipated by heat and reduce kinetic energy

$$power = \frac{dU}{dt} = \frac{\overline{F} \bullet d\vec{r}}{dt} = \overline{F} \bullet \vec{v}$$

$$\eta = \frac{power \quad output}{power \quad input}$$

Potential Energy



$$U_{1\to 2} = -\int_{y_1}^{y_2} W dy = Wy_1 - Wy_2$$

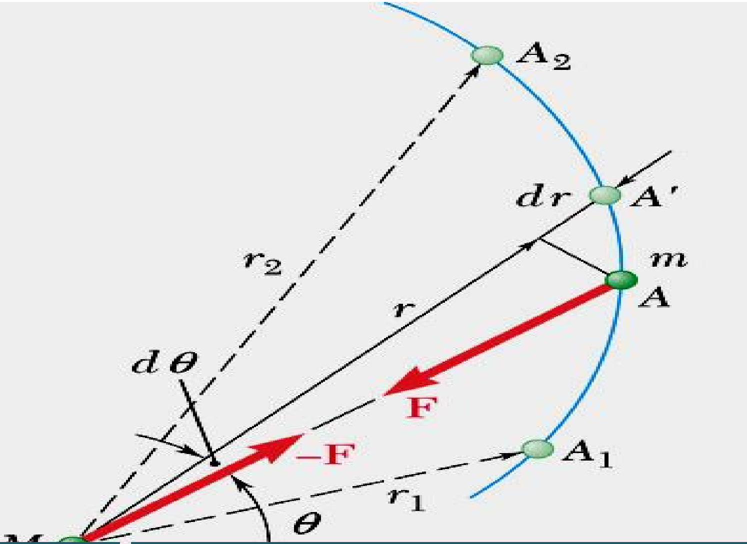
$$\boxed{U_{1\to 2} = (V_g)_1 - (V_g)_2}$$

$$V_g = Wy$$

Potential Energy of the body with respect to the force of gravity

When $V_{g2} > V_{g1}$, potential energy increases, and U_{1-2} is negative

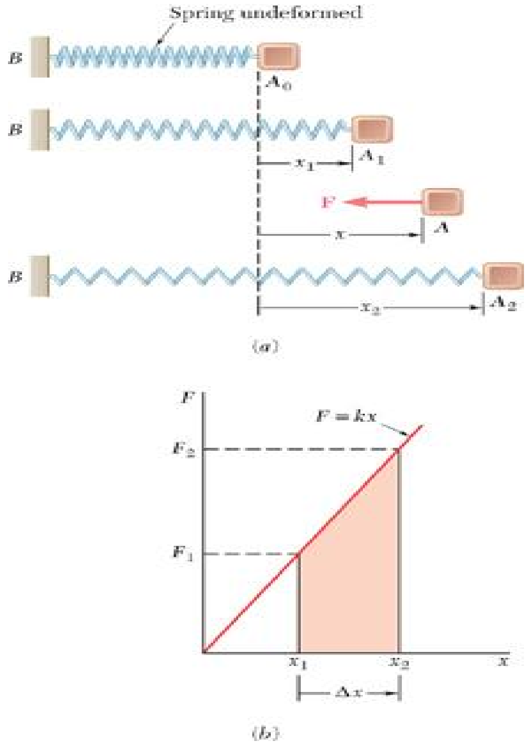
It should be noted that the expression just obtained for the potential energy of a body with respect to gravity is valid only as long as the weight of body can be assumed to remain constant, i.e., as long as the displacements of the body are small compared with the radius of the earth.



$$U_{1\to 2} = -\int_{r_1}^{r_2} \frac{GMm}{r^2}\,dr = \frac{GMm}{r_2} - \frac{GMm}{r_1}$$

$$V_g = G\frac{Mm}{r} = \frac{WR^2}{r}$$

R is from the center of the earth

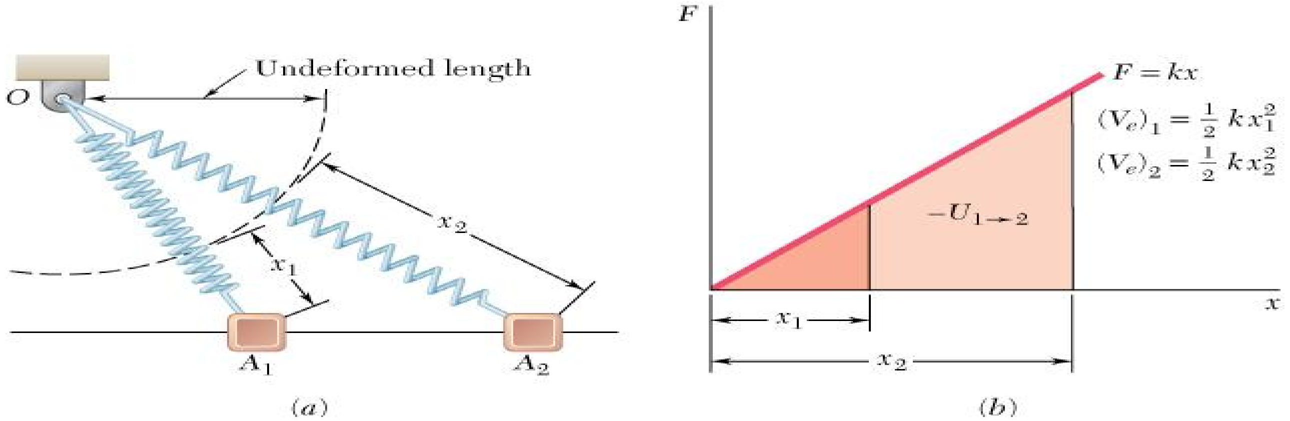


$$U_{1\to 2} = -\int_{x_1}^{x_2} kx\,dx = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$$

$$\boxed{U_{1\to 2} = (V_e)_1 - (V_e)_2}$$

$$V_e = \frac{1}{2}kx^2$$

Potential Energy of the body with respect to the elastic force F



Only the initial and final deflection of the spring are needed

Deflection of the spring is measured from its undeformed position

Equation of Work and Energy

$$U_{1\to 2} = \Delta T$$

where

$$U_{1\to 2} = U_{1\to 2}{}^{w} + U_{1\to 2}{}^{s} + U_{1\to 2}{}^{of}$$

Where $U_{1\to 2}{}^{of}$ =work done by other (non conservative/ path dependant) forces

$$-\Delta V_g - \Delta V_e + U_{1\to 2}{}^{of} = \Delta T$$

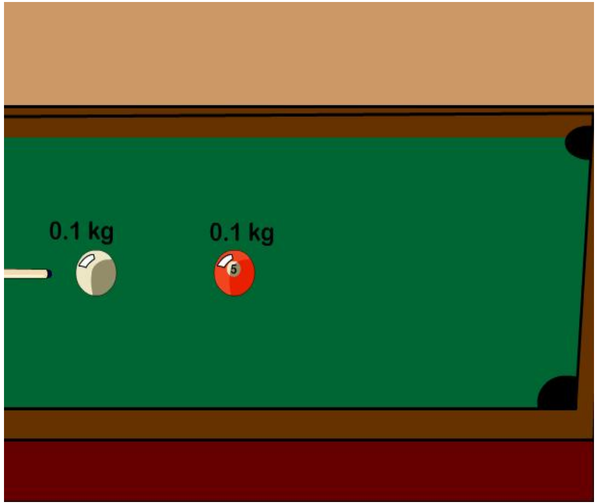
$$U_{1\to 2}{}^{of} = \Delta T + \Delta V_e + \Delta V_g$$

$$U_{1\to 2}{}^{of} = \Delta E$$

Advantages and Disadvantages of Work and Energy Method

- Advantage; 1. No need to calculate values between A_1 and A_2. Only final stages are counted 2. All scalars so can be added easily 3. Forces that do no work are ignored
- Disadvantage; can not determine accelerations, can not determine accelerations and forces that do no work

Impulse and Momentum

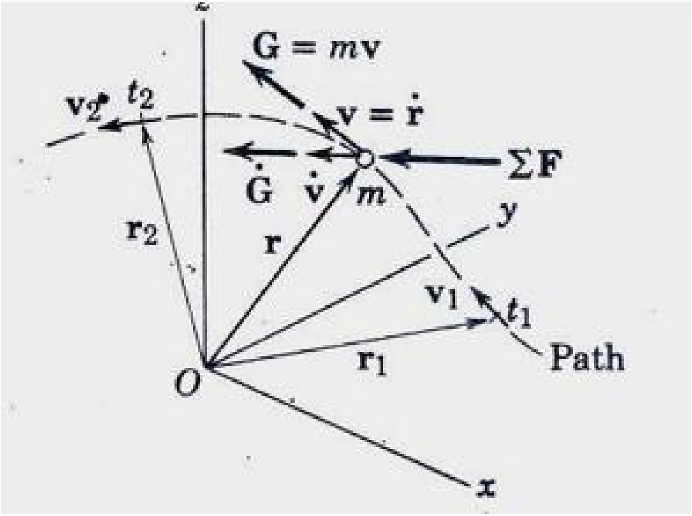


- Work and energy is obtained by integrating the equation of F=ma with respect to the displacement of the particle.
- Impulse and momentum can be generated by integrating the equation of motion (F=ma) with respect to time.

Linear Impulse and Momentum

- Consider a particle of a mass m which is subjected to several forces in space.

$$\sum \vec{F} = m\dot{\vec{v}} = \frac{d}{dt}(m\vec{v}) \quad or \quad \sum \vec{F} = \dot{G}\text{...........}(1)$$



- The product of the mass & the velocity is defined as the linear momentum.

$$G = m\vec{v}$$

- Equation (1) states that the resultant of all forces acting on a particle equals its time rate of change of linear momentum.
- It is valid as long as the mass m of the particle is not changing with time.
- The scalar components of equation (1) are:

$$\sum F_x = \dot{G}_x, \quad \sum F_y = \dot{G}_y, \quad \sum F_z = \dot{G}_z$$

- The effect of the resultant force $\sum F$ on the linear momentum of the particle over a finite period of time simply by integrating with respect to time t.

$$\sum F dt = dG$$

$$\int_{t_1}^{t_2} \sum F dt = G_2 - G_1 = \Delta G$$

- The linear momentum at time t_2 is $G_2 = mV_2$ and the linear momentum at time t_1 is $G_1 = mV_1$.
- The product of force and time is called linear impulse.

- The total linear impulse on a mass m equals the corresponding change in linear momentum of m.

$$G_1 + \int_{t_1}^{t_2} \sum F dt = G_2$$

The impulse integral is a vector

Scalar impulse momentum eqns.

$$\begin{cases} \int_{t_1}^{t_2} \sum F_x dt = (mV_x)_2 - (mV_x)_1 \\ \int_{t_1}^{t_2} \sum F_y dt = (mV_y)_2 - (mV_y)_1 \\ \int_{t_1}^{t_2} \sum F_z dt = (mV_z)_2 - (mV_z)_1 \end{cases}$$

Conservation of Linear Momentum

- If the resultant force on a particle is zero during an interval of time, its linear momentum G remains constant. In this case the linear momentum of a particle is said to be conserved.

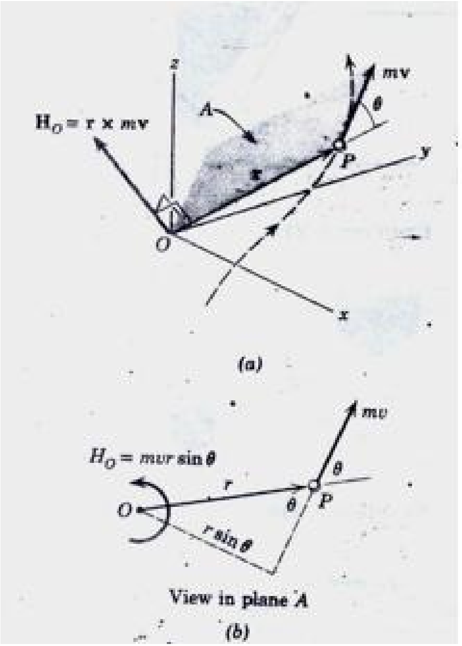
$$\Delta G = 0 \quad or \quad G_1 = G_2$$

Angular Impulse and Momentum

- The moment of a linear momentum vector $\vec{mv}$ about the origin O is defined as the angular momentum of P & is given by the product relation for $\vec{H}_o$ moment of vector:-

$$H_0 = \vec{r} \times m\vec{v}$$

- The angular momentum then is a vector perpendicular to the plane A defined by $\vec{r}$ & $\vec{v}$



- The scalar component of angular momentum is:-

$$H_o = \vec{r} \times m\vec{v} = m(v_z y - v_y z)\vec{i} + m(v_x z - v_z x)\vec{j} + m(v_y x - v_x y)\vec{k}$$

$$H_o = m\begin{vmatrix} i & j & k \\ x & y & z \\ v_x & v_y & v_z \end{vmatrix}$$

$$H_x = m(v_z y - v_y z), \quad H_y = m(v_x z - v_z x) \quad H_z = m(v_y x - v_x y)$$

- If $\sum F$ represents the resultant of all forces acting on the particles P, the moment M_o about the origin O is the vector cross product.

$$\sum M_o = \vec{r} \times \sum \vec{F} = \vec{r} \times m\dot{\vec{v}}$$

$$\dot{H}_o = \dot{\vec{r}} \times m\vec{v} + \vec{r} \times m\dot{\vec{v}} \Rightarrow \vec{v} \times m\vec{v} + \vec{r} \times m\dot{\vec{v}}$$

$$\sum M_o = \dot{H}_o ....(*)$$

- The moment about the fixed point O of all forces acting on m equals the time rate of change of angular momentum of m about O.

- To obtain the effect of moment $\sum M_o$ on the angular momentum of the particle over a finite period of time;

$$\int_{t_1}^{t_2} \sum M_o dt = H_{o_2} - H_{o_1} = \Delta H_o$$

$$where \quad H_{o_2} = \vec{r}_2 \times m\vec{v}_2 \ \& \ H_{o_1} = \vec{r}_1 \times m\vec{v}_1$$

- The product of moment & time is angular impulse the total angular impulse on M about a fixed point O equals the corresponding change in angular momentum of M about O.

$$H_{o_1} + \int_{t_1}^{t_2} \sum M_o dt = H_{o_2}$$

Conservation of Angular Momentum

- If the resultant moment about a fixed point O of all forces acting on a particle is zero during the interval of time, equation (*) requires that its angular momentum H_0 about that point remains constant.

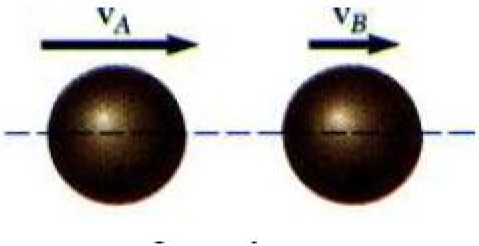
$$\Delta H_o = 0 \quad or \quad H_{o_1} = H_{o_2}$$

Impact

- Impact Refers to the collision b/n two bodies and is characterized by the generation of relatively large contact forces that act over a very short interval of time.

a) Direct Central Impact

- Consider the collinear motion of two spheres of masses m_1 and m_2 travelling with velocities V_1 & V_2. If V_1 is greater than V_2, collision occurs with the contact forces directed along the line of centers.

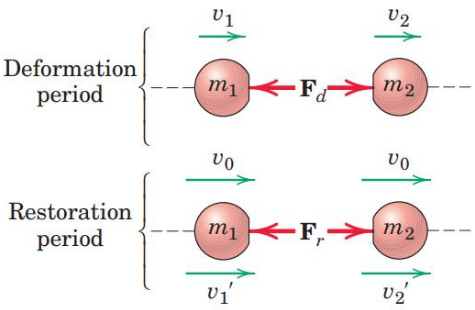


- In as much as the contact forces are equal & opposite during impact; the linear momentum of the system remains unchanged.

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

- For given masses & initial conditions, the momentum equation contains two unknowns, $\mathbf{v}_1'$ & $\mathbf{v}_2'$, an additional relationship is required.
- The relationship must reflect the capacity of the contacting bodies to recover from the impact & can be expressed by the ratio *e* of the magnitude of the restoration impulse to the magnitude of the deformation impulse. This ratio is called the coefficient of restitution.

- F_r - contact force during restoration period
- F_d - contact force during deformation period

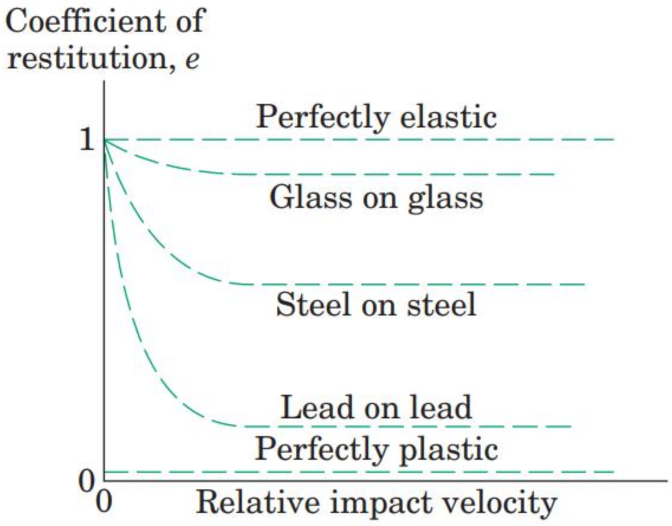


$$e=\frac{\int_{t_o}^{t} F_r dt}{\int_{0}^{t_o} F_d dt}=\frac{m_1[-v_1'-(-v_o)]}{m_1[-v_o-(-v_1)]}=\frac{v_o-v_1'}{v_1-v_o} ....for \quad particle \quad 1$$

$$e=\frac{\int_{t_o}^{t} F_r dt}{\int_{0}^{t_o} F_d dt}=\frac{m_2[v_2'-(v_o)]}{m_2[v_o-(v_2)]}=\frac{v_2'-v_o}{v_o-v_2} ....for \quad particle \quad 2$$

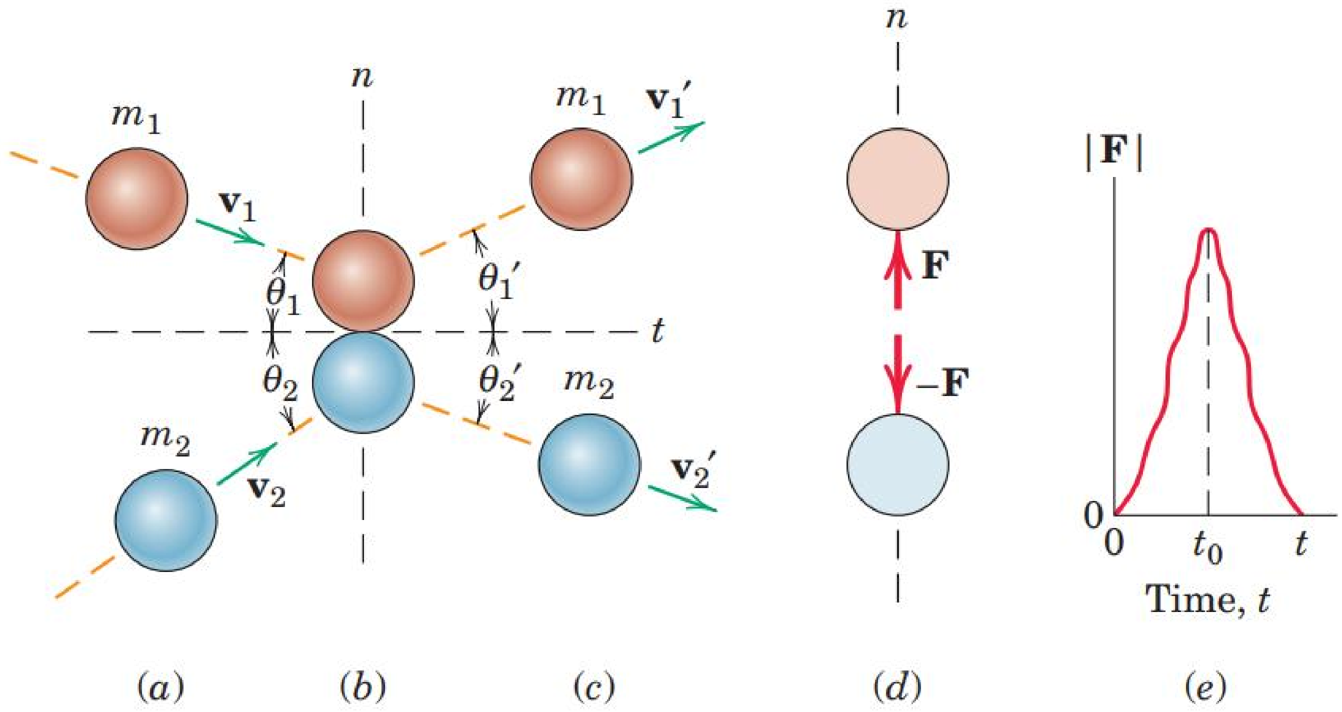
$$e=\frac{v_2'-v_1'}{v_1-v_2}=\frac{|\text{relative velocity of separation}|}{|\text{relative velocity of approach}|}$$

- According to classical theory of impact, the value e=1 means that the capacity of the two particles to recover equal their tendency to deform.
- The value e=0, on the other hand describes inelastic or plastic impact where the particles stick together after collision & the loss of energy is a maximum.



b) Oblique Direct Central Impact

- Here the initial and final velocities are not parallel.
- The spherical particles of mass m1 & m2 have initial velocities v1 & v2 in the same plane & approach each other on a collision course.
- The direction of the velocity vector are measured from the direction tangent to the contacting surfaces.



$$(v_1)_n = -v_1 \sin\theta_1, \;\; (v_1)_t = v_1 \cos\theta_1,$$

$$(v_2)_n = -v_2 \sin\theta_2, \;\; (v_2)_t = v_2 \cos\theta_2,$$

There will be four unknown namely, $(v_1')_n$, $(v_1')_t$, $(v_2')_n$, & $(v_2')_t$

1) Momentum of the system is conserved in the n-direction,

$$m_1(v_1)_n + m_2(v_2)_n = m_1(v_1')_n + m_2(v_2')_n$$

2) & 3) The momentum for each particle is conserved in the t-direction since there is no impulse on either particle in the t- direction.

$$m_1(v_1)_t = m_1(v_1')_t$$

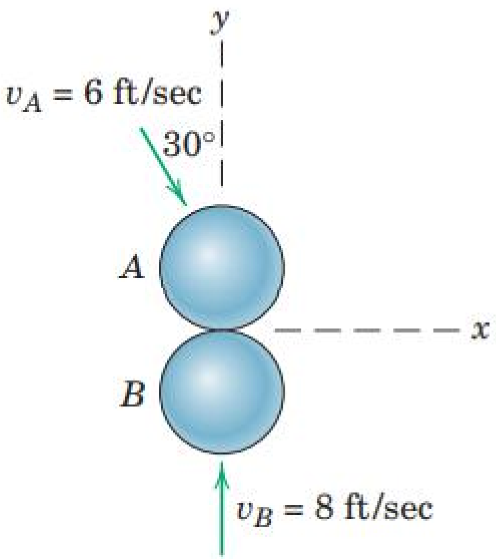
$$m_2(v_2)_t = m_2(v_2')_t$$

3) the coefficient of restitution, the velocity component in the n- direction,

$$e = \frac{(v_2')_n - (v_1')_n}{(v_1)_n - (v_2)_n}$$

AAiT

The two identical steel balls moving with initial velocities v_A and v_B collide as shown. If the coefficient of restitution is $e = 0.7$, determine the velocity of each ball just after impact and the percentage loss n of system kinetic energy.



Suggested Problems

From 7th Edition , Merriam Engineering Mechanics Dynamics

Chapter III Problem 3/.......

3,9,13,23,24,29,32,34,43,45,48,54,68,72,84,8996,99,109,112,113,117,127,132,137,140,144,

149,151,158,164,176,178,187,191,192,198,200,

227,226,235,241,249,253,258,259,262,and264