

# Nuclear beta decay and electron capture in the origin of heavy elements

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# Outline

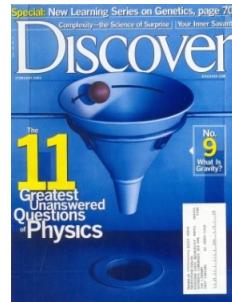
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- How were heavy elements from iron to uranium made?
- Accurate Nuclear Physics Inputs for r-process
  - $\beta$ -decay half-lives
- Where does r-process happen?
  - Electron capture rates in core-collapse supernova
- Summary and Perspectives

# How Were the Heavy Elements Made?

How were the heavy elements from iron to uranium made?

The 11 greatest unanswered questions of physics



## ● r-process

### ➤ Where does r-process happen?

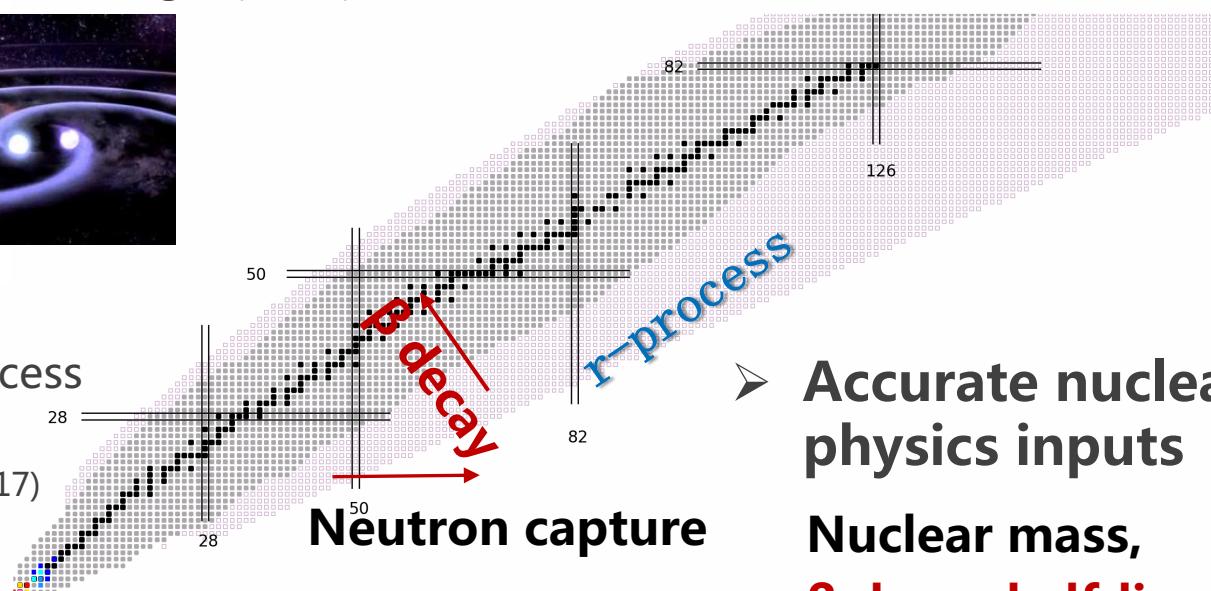
**Supernova** Neutron star merger (NSM)



GW170817 NSM :

One of the main r-process sites

Nature 551, 64; 67; 75; 80 (2017)  
Science 358, 1559 (2017)  
ApJL 848, L17; L19 (2017)



NSM:  
minimal contribution?

Nature 574, 497; 569, 241 (2019)  
arXiv:2102.05891v1

R-process path: far from stability, relies on theory!

- Accurate nuclear physics inputs
- Nuclear mass,
- β decay half-lives,
- Neutron-capture cross section,
- ...

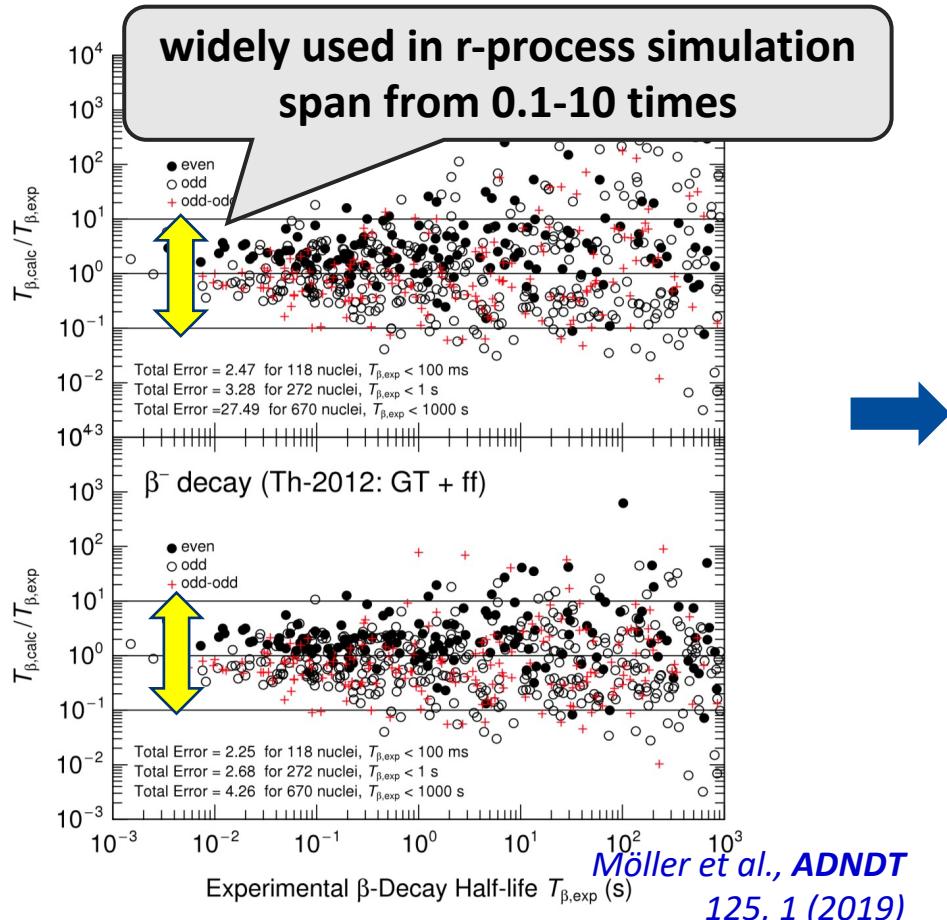
# Outline

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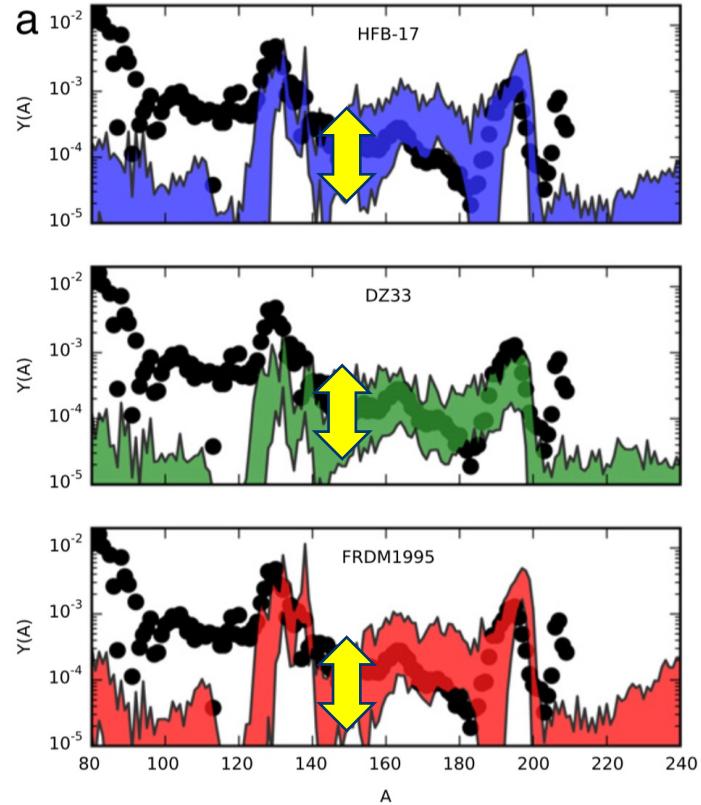
- How were heavy elements from iron to uranium made?
- Accurate Nuclear Physics Inputs for r-process
  - $\beta$ -decay half-lives
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# Consequences of uncertainties in $\beta$ -decay half-lives

- $\beta$ -decay half-lives by finite-range droplet model (FRDM) + quasiparticle random phase approximation (QRPA)
- Uncertainties caused by varying half-lives between  $10^{-1}$  - 10 times randomly



R-process abundance

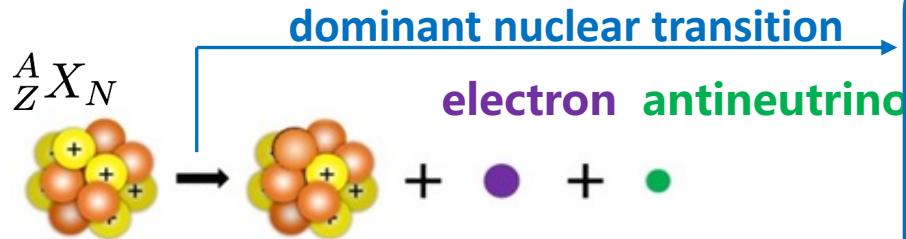


Mumpower et al., PPNP 86, 86 (2016)

Accurate description of nuclear  $\beta$ -decay half-lives is important for r-process study

# $\beta$ -decay Half-life Is a Hard Problem

- $\beta$ -decay



Gamow-Teller transition

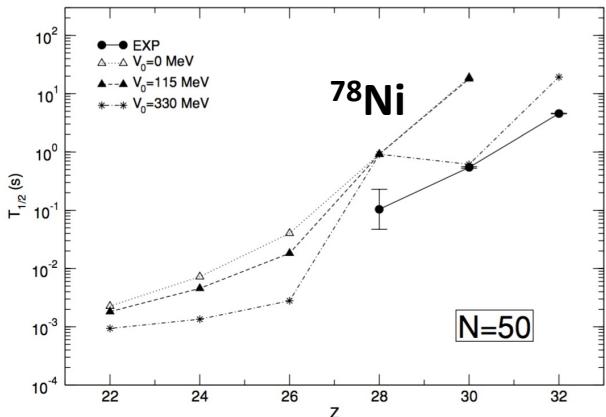
$$T^- \quad \Delta S=1 \quad \Delta L=0 \quad \Delta T=1$$

$$\hat{O}_{GT^-} = \sum_{i=1}^A \vec{\sigma}(i) \cdot \tau_-(i)$$

$Z+1, N-1$        $Z, N$

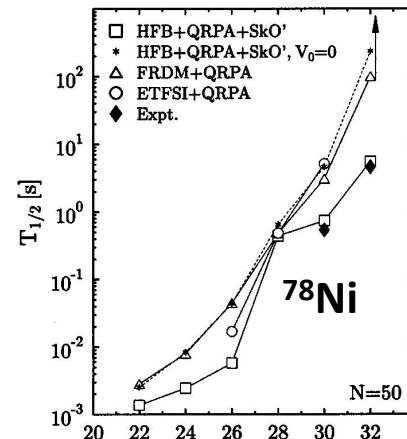
- RHB+QRPA

(quasiparticle random phase approximation)



Niksic, et al., PRC 71, 014308 (2005)

- Skyrme HFB+QRPA



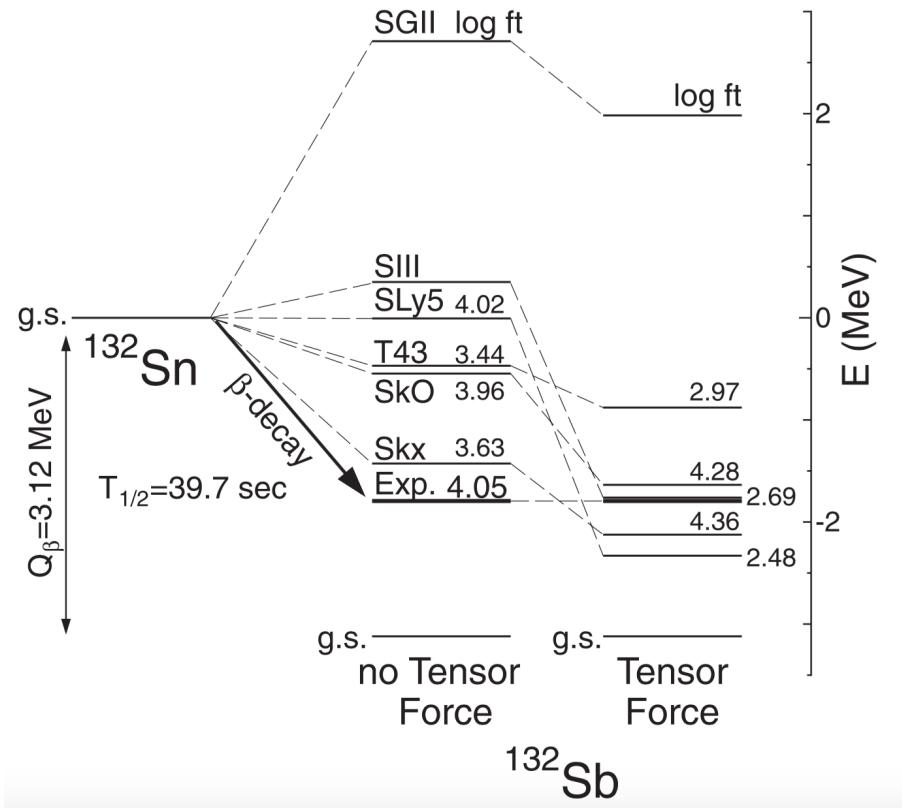
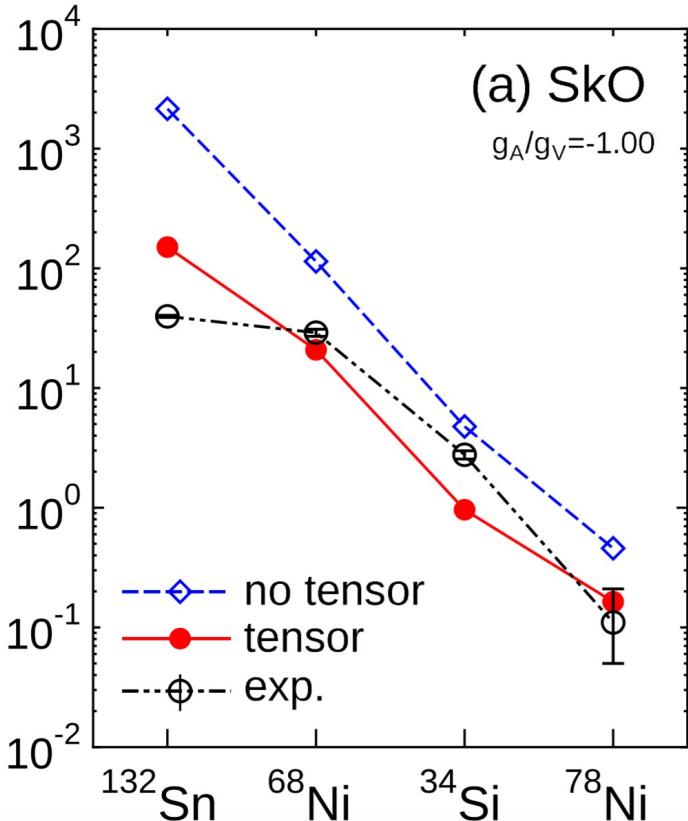
Engel, et al., PRC 60, 014302 (1999)

- Half-lives are **overestimated**
- Due to the nuclear structure part – Gamow-Teller transition

# What are missing in the calculation?

- Nuclear Force: tensor force      skyrme HF + RPA + tensor

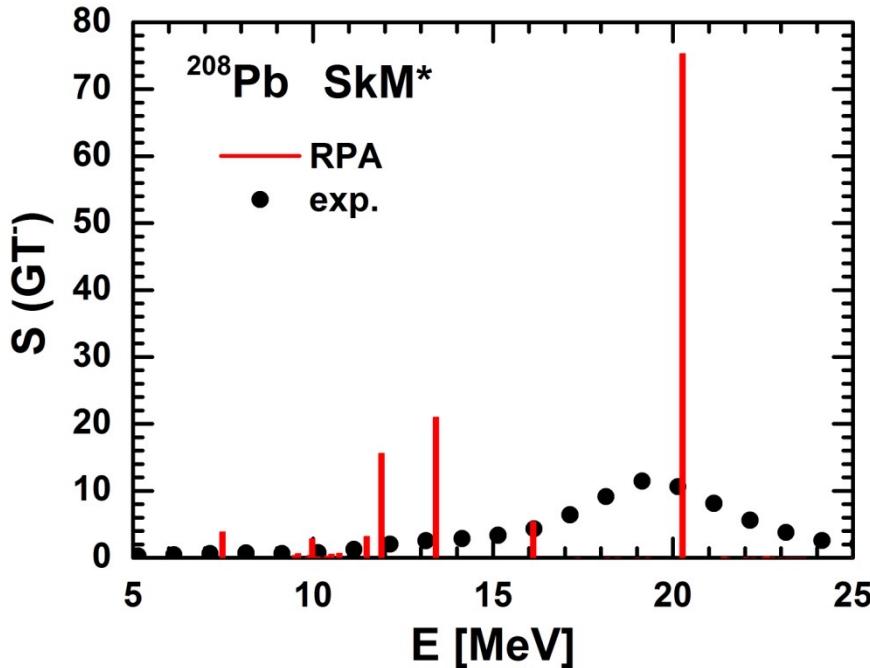
Minato and Bai, PRL 116, 089902 (2016)



- Nuclear Model: more correlations beyond RPA model

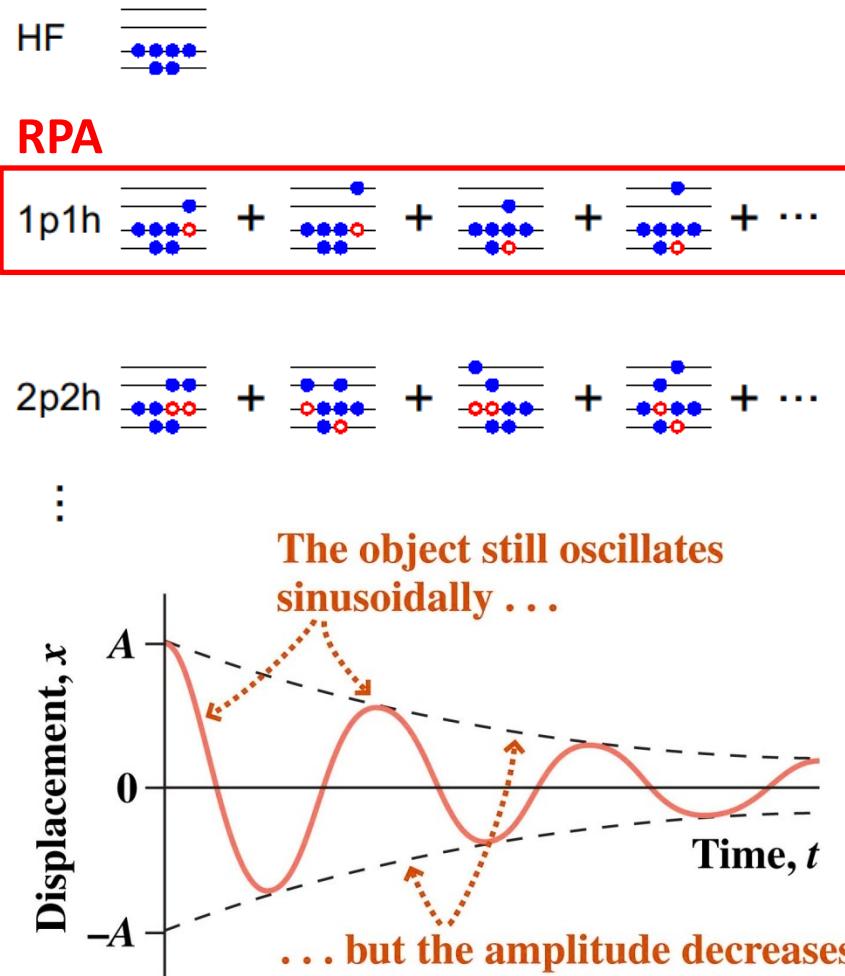
# Limits of (Q)RPA Description

- (Q)RPA cannot describe the spreading width

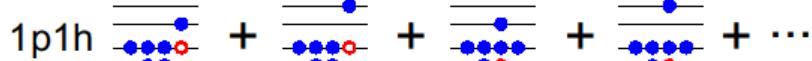


- Spreading Width (Damping Width)
  - energy and angular momentum of coherent vibrations
  - more complicated states of 2p-2h, 3p-3h, ... character

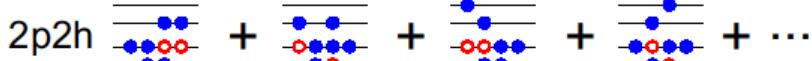
- Correlations beyond RPA



# Solution: RPA + PVC



RPA

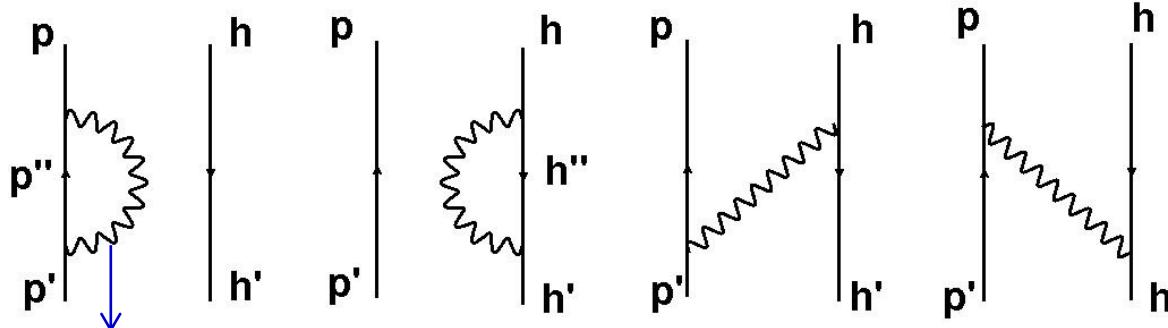


- Second RPA drozdz et al., PR 197, 1 (1990)

Gambacurta et al., PRL 125, 212501 (2020)

- RPA + PVC (particle vibration coupling)

:



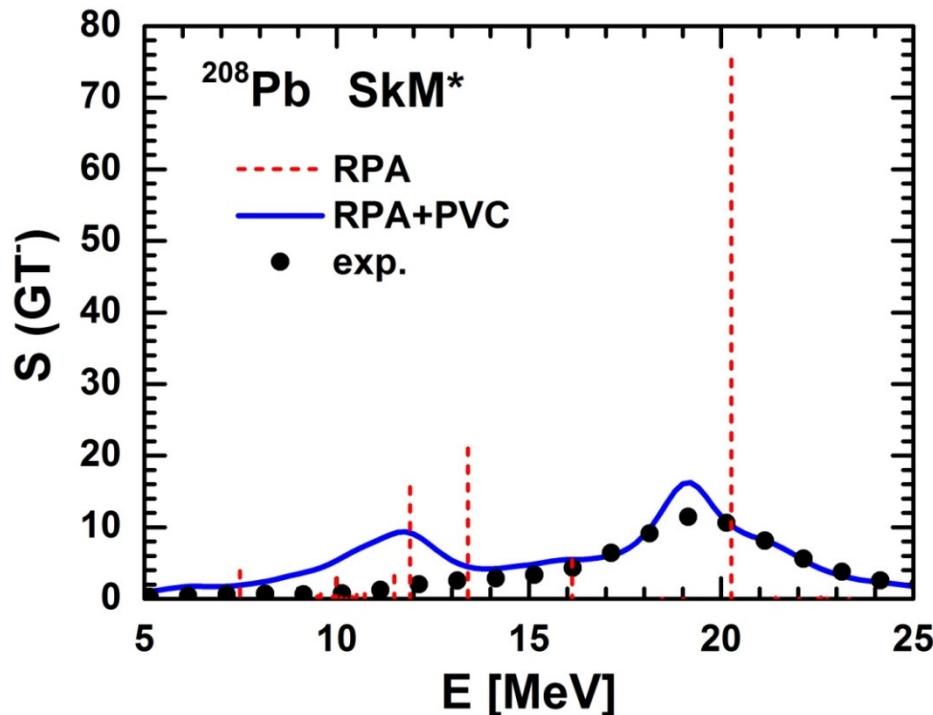
Low-lying vibration  
phonons  $|N\rangle$

$$W_{ph,p'h'}^{\downarrow}(\omega) = \sum_N \frac{\langle ph| V| N \rangle \langle N| V| p'h' \rangle}{\omega - \omega_N}$$

- RPA+PVC model based on Skyrme DFT  
Colo et al., PRC 50, 1496 (1994); Niu et al., PRC 85, 034314 (2012)
- RPA+PVC model based on relativistic DFT Litvinova et al., PRC 75,064308 (2007)

# Gamow-Teller Resonance

- Improved description of GT resonance in  $^{208}\text{Pb}$

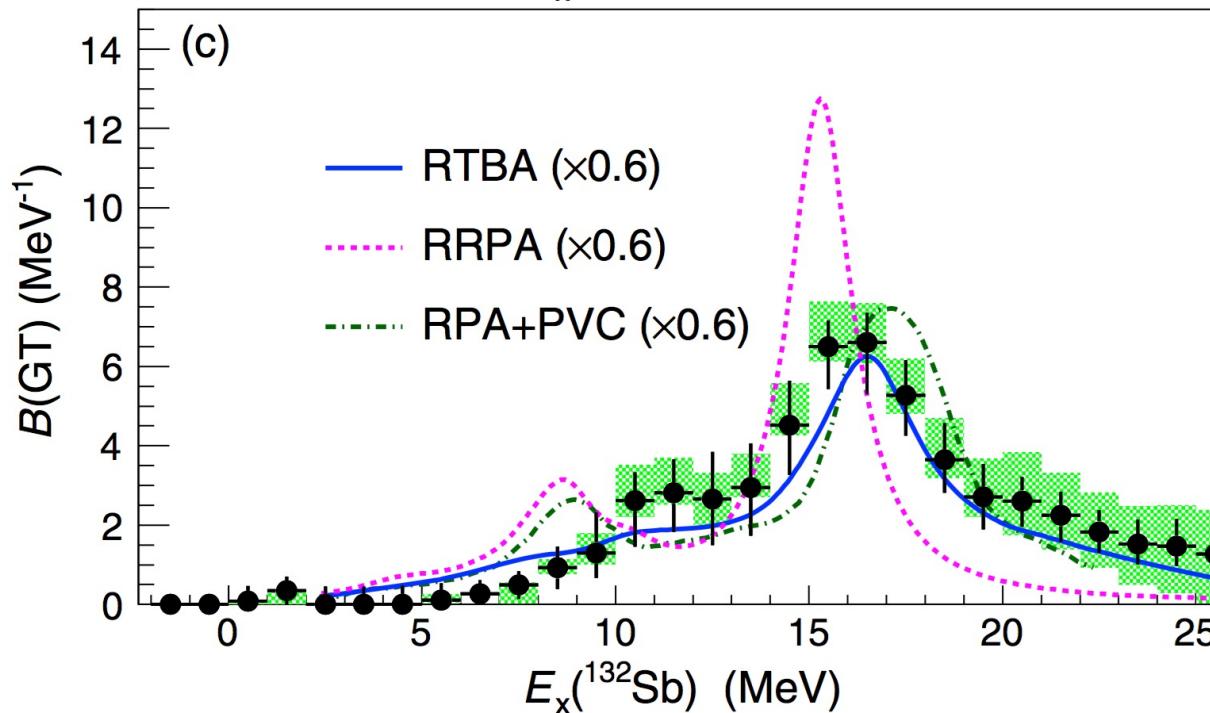


- ✓ Develop a spreading width
- ✓ Reproduce resonance lineshape

Y. F. Niu, G. Colo, and E. Vigezzi, PRC 90, 054328 (2014)

# Gamow-Teller Resonance

- Improved description of GT resonance in unstable nucleus  $^{132}\text{Sn}$



Yasuda, Sasano, et al., PRL 121, 132501 (2018)

Exp: (p,n) reaction @ RIBF, RIKEN

Yasuda, Sasano, et al., PRL 121, 132501 (2018)

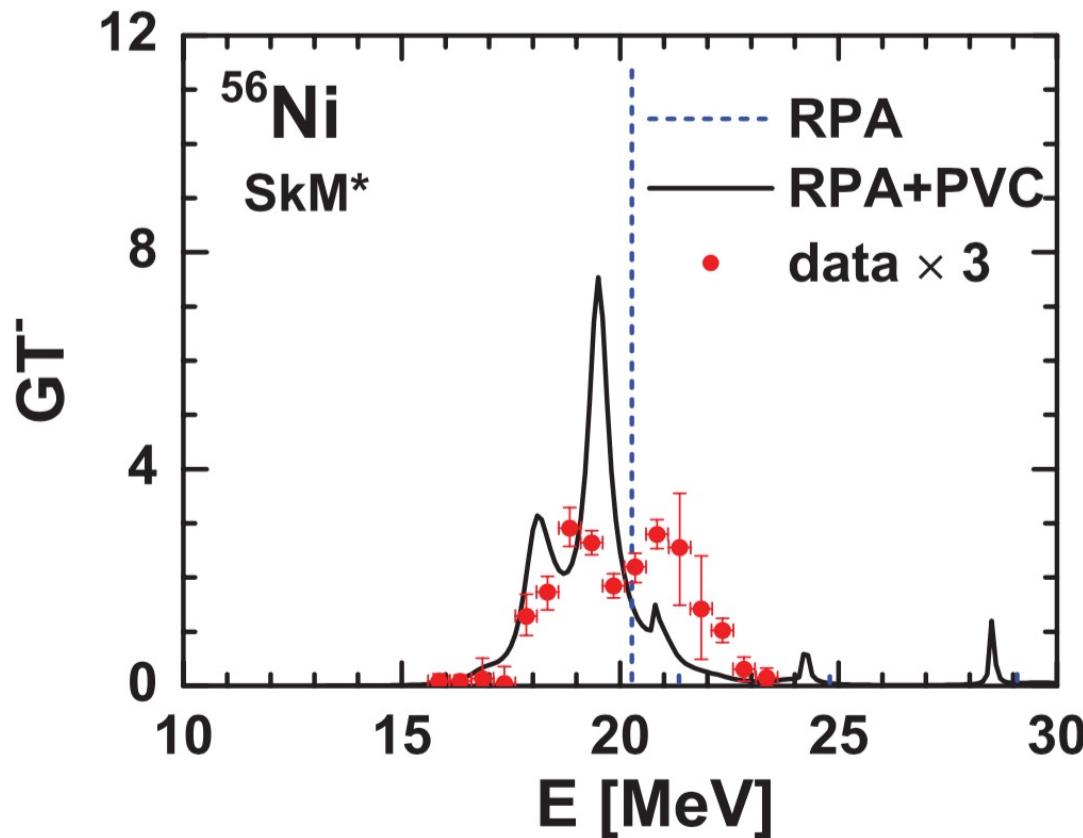
RPA+PVC: Y. F. Niu, G. Colo and E. Vigezzi, PRC 90, 054328 (2014)

RTBA: E. Litvinova et al., PLB 730, 307 (2014)

RRPA: H. Z. Liang, and Z. M. Niu private communication

# Gamow-Teller Resonance

- Reproduction of double-peak structure of GT resonance in  $^{56}\text{Ni}$

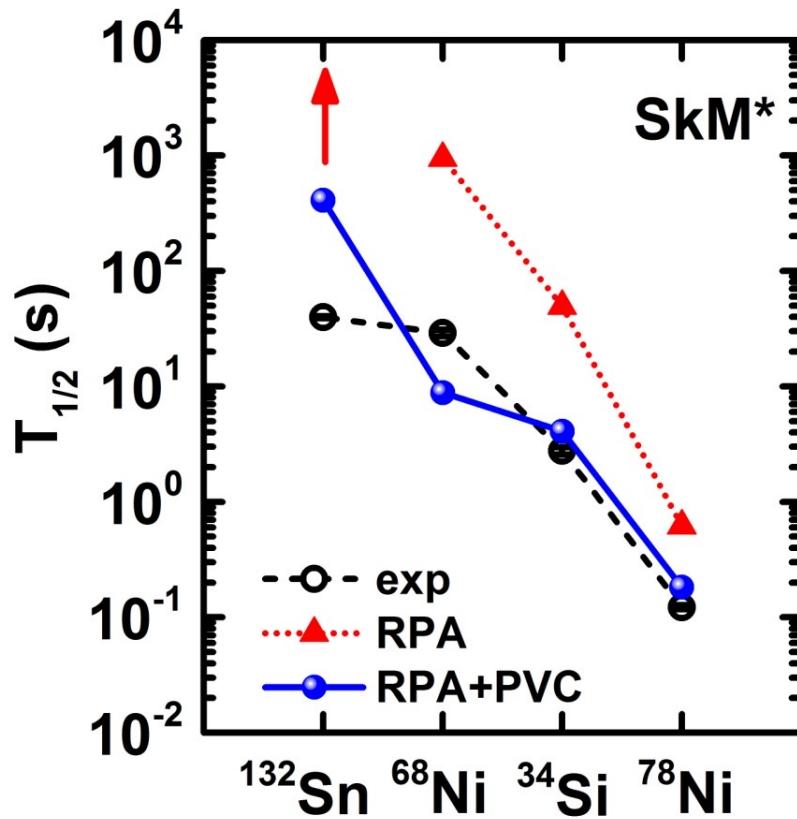


Y. F. Niu, G. Colo, M. Brenna, P.F. Bortignon, and J. Meng, **PRC** 85, 034314 (2012)

Exp: (p,n) reaction with  $T_p=296$  MeV @ NSCL, MSU  
Sasano et al., **PRL** 107, 202501 (2011)

# $\beta$ -Decay Half-Lives in Magic Nuclei

- Improved description of  $\beta$ -decay half-lives



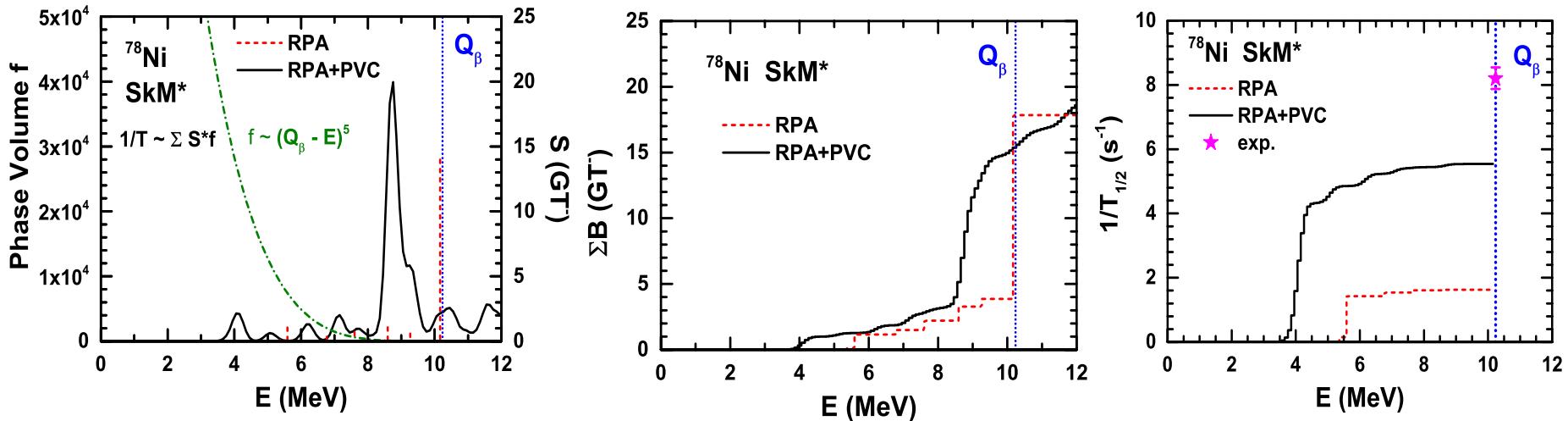
✓ Reduce half-lives systematically

✓ Reproduce  $\beta$ -decay half-lives

Y.F. Niu, Z. M. Niu, G. Colo, and E. Vigezzi, PRL 114, 142501 (2015)

Exp: G. Audi, F. G. Kondev, M. Wang, W. J. Huang, and S. Naimi, CPC 41, 030001 (2017)

# How PVC reduces half-lives?



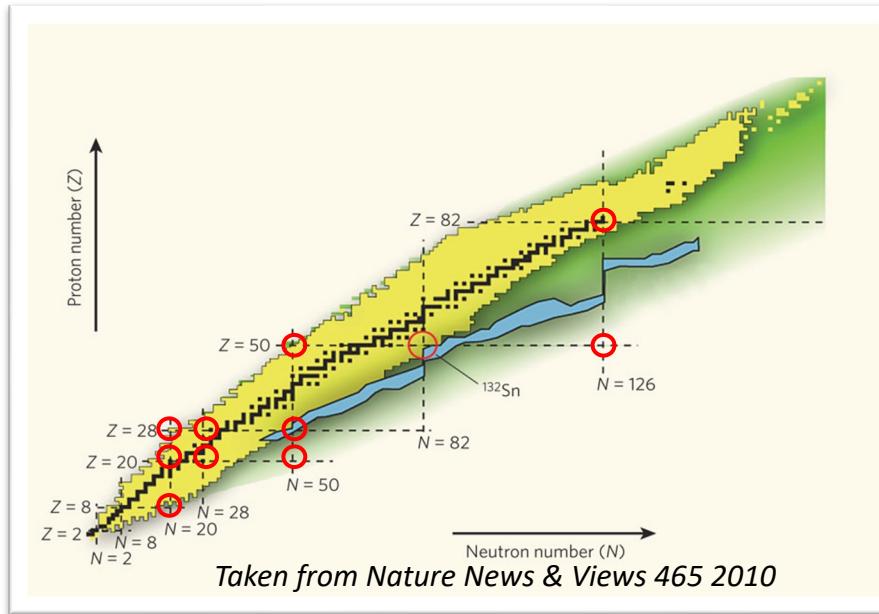
Exp.: Xu, et al., PRL 113, 032505, 2014

- Half-life
- Phase Volume

$$T_{1/2} = \frac{D}{g_A^2 \int^{Q_\beta} S(E) f(Z, \omega) dE},$$

$$f(Z, \omega_0) = \frac{1}{(m_e c^2)^5} \int_{m_e c^2}^{\omega_0} p_e E_e (\omega_0 - E_e)^2 F_0(Z + 1, E_e) dE_e.$$

# RPA+PVC: only for magic nuclei...

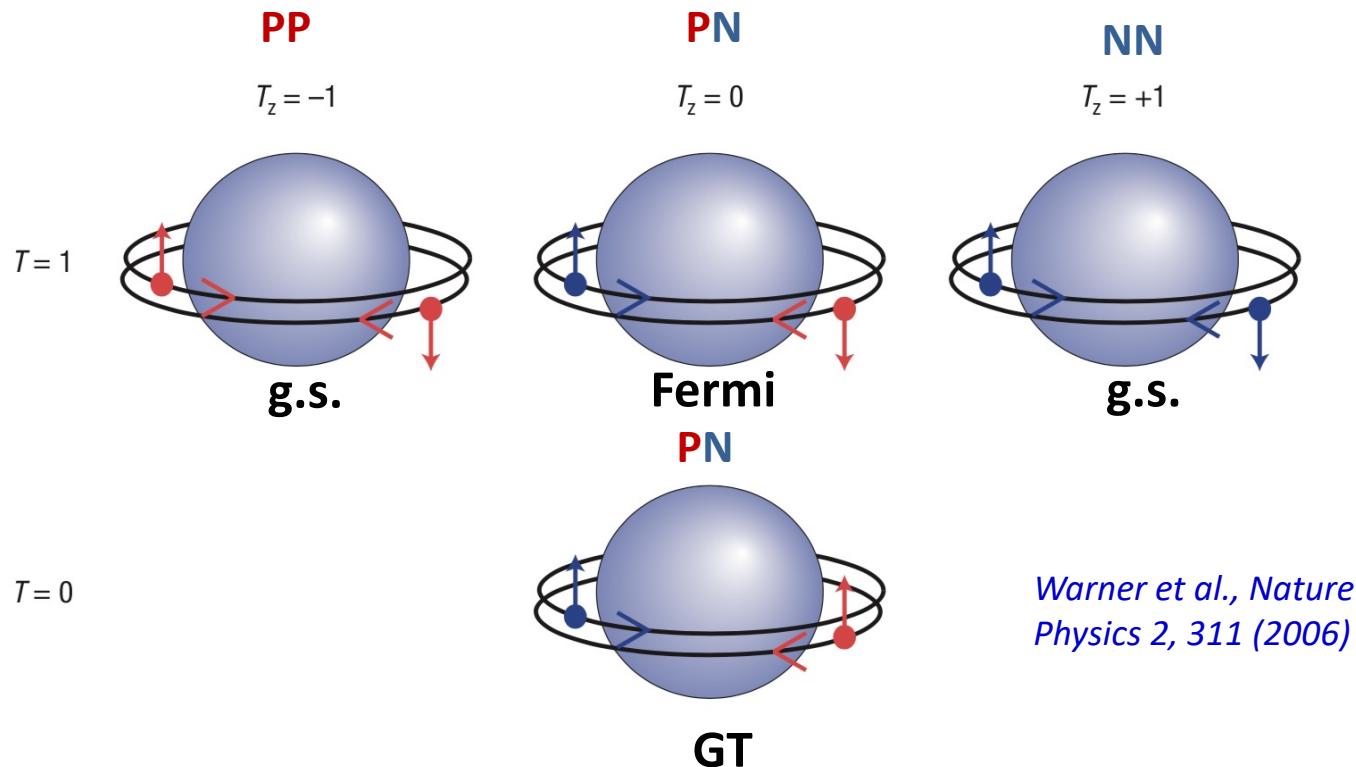


➤ To include pairing correlations for superfluid nuclei

Quasiparticle RPA + quasiparticle vibration coupling  
**(QRPA)** + **(QPVC)**

- ✓ for the study of Gamow-Teller resonance in superfluid nuclei
- ✓ for the study of  $\beta$ -decay half-lives in the whole isotopic chain

# Isovector and isoscalar pairing



- **Isovector Pairing**

$$V_{T=1}(\mathbf{r}_1, \mathbf{r}_2) = V_0 \frac{1 - P_\sigma}{2} \left(1 - \frac{\rho(\mathbf{r})}{\rho_0}\right) \delta(\mathbf{r}_1 - \mathbf{r}_2),$$

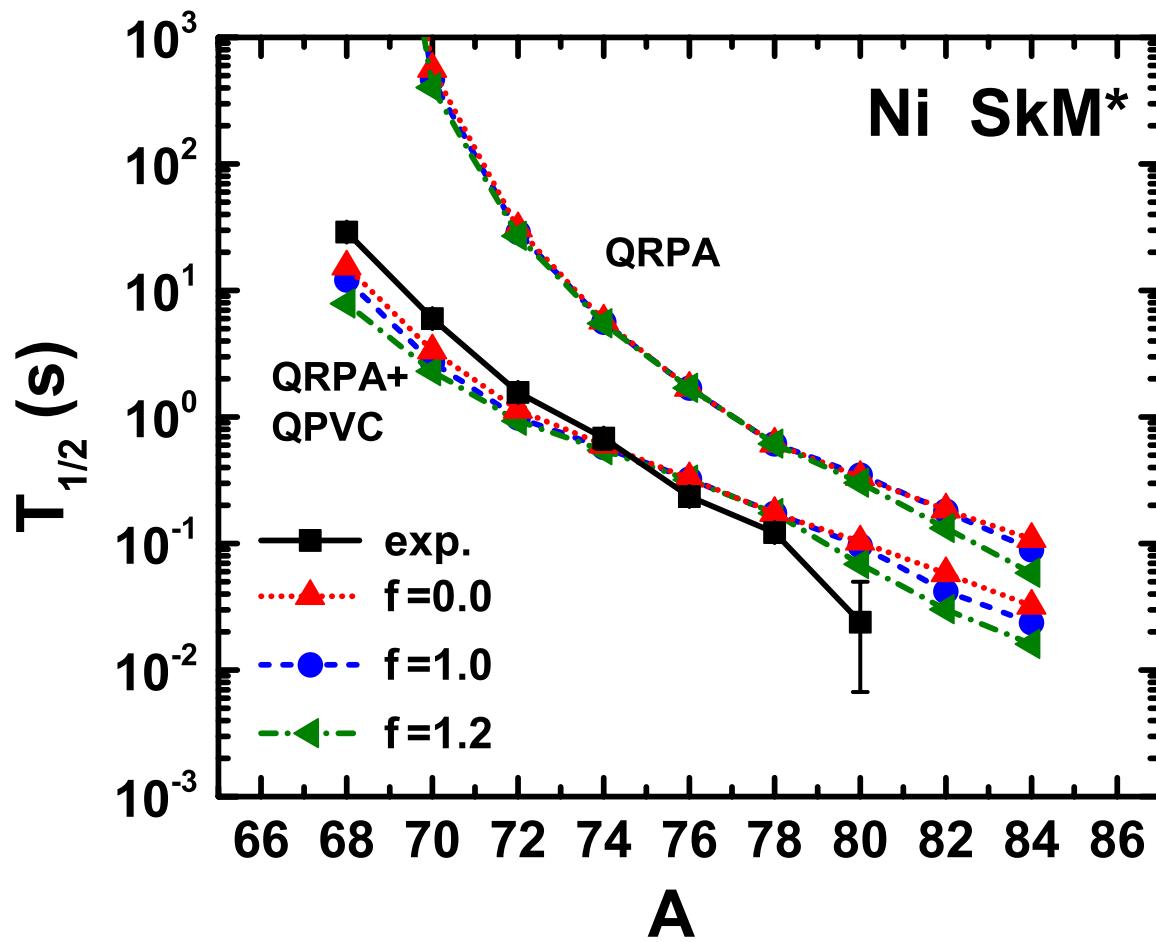
For ground state: pairing strength adjusted to reproduce empirical pairing gap

- **Isoscalar Pairing**

$$V_{T=0}(\mathbf{r}_1, \mathbf{r}_2) = f V_0 \frac{1 + P_\sigma}{2} \left(1 - \frac{\rho(\mathbf{r})}{\rho_0}\right) \delta(\mathbf{r}_1 - \mathbf{r}_2)$$

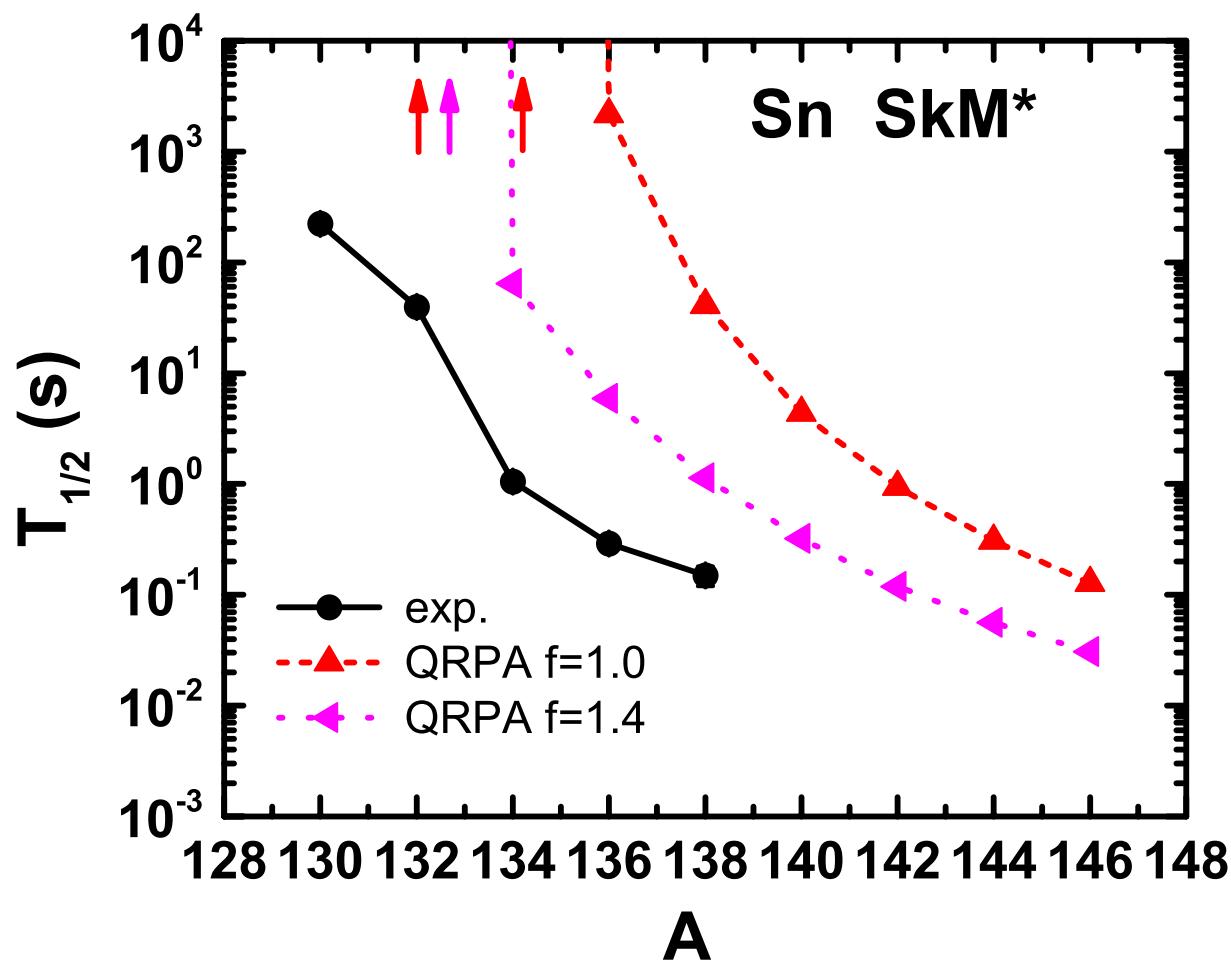
For GT: the same form as IV pairing, but with an adjustable pairing strength  $f$

# $\beta$ -Decay Half-Lives in Ni isotopes



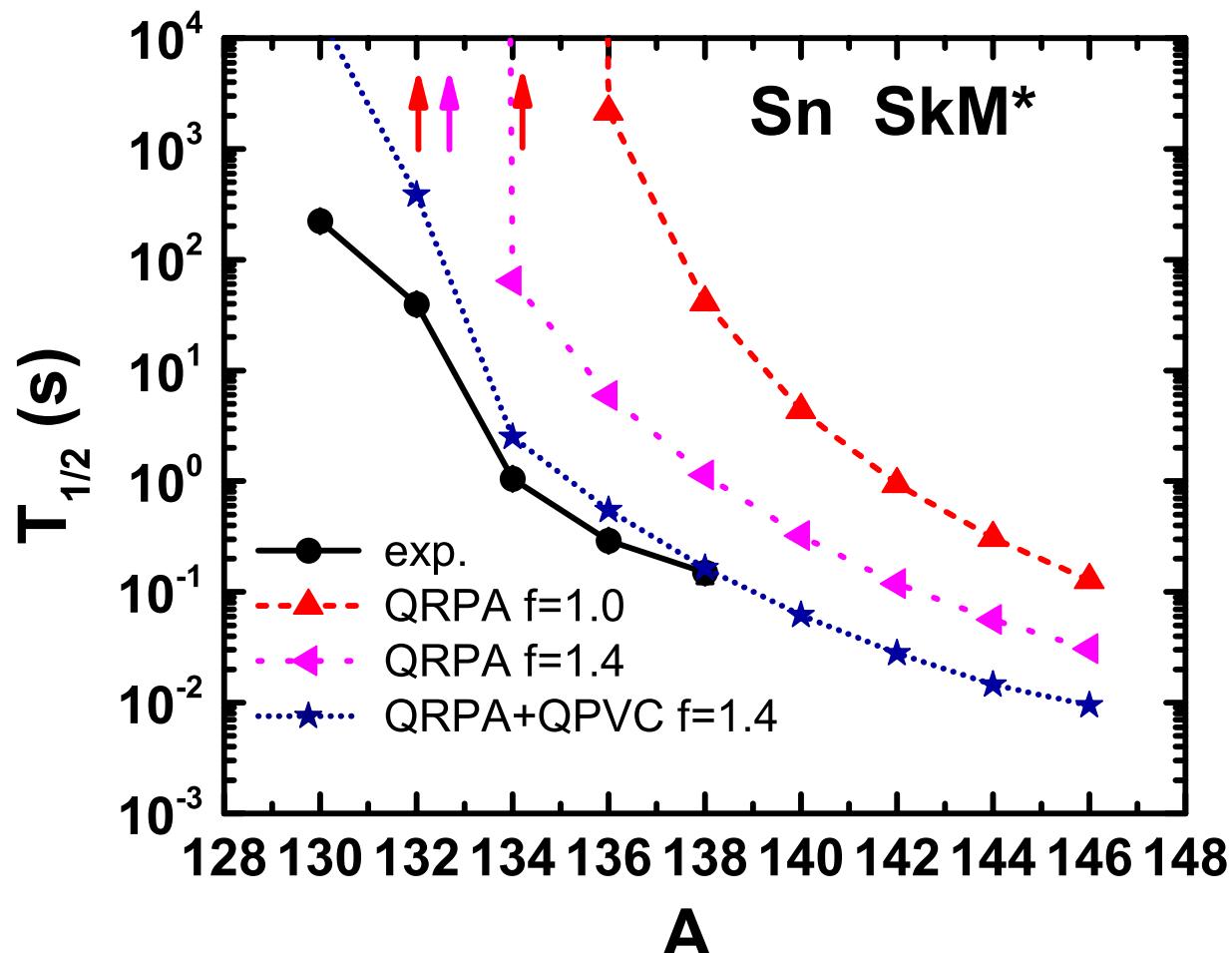
- Isoscalar Pairing:
  - similar at QRPA and QRPA+QPVC level
  - not so effective for Ni isotopes  
(nuclei before N=50 closed shell)
- QPVC:
  - reduce the half-lives

# $\beta$ -Decay Half-Lives in Sn isotopes



- Isoscalar Pairing:  
effective for Sn isotopes  
(nuclei above N=82 closed shell)

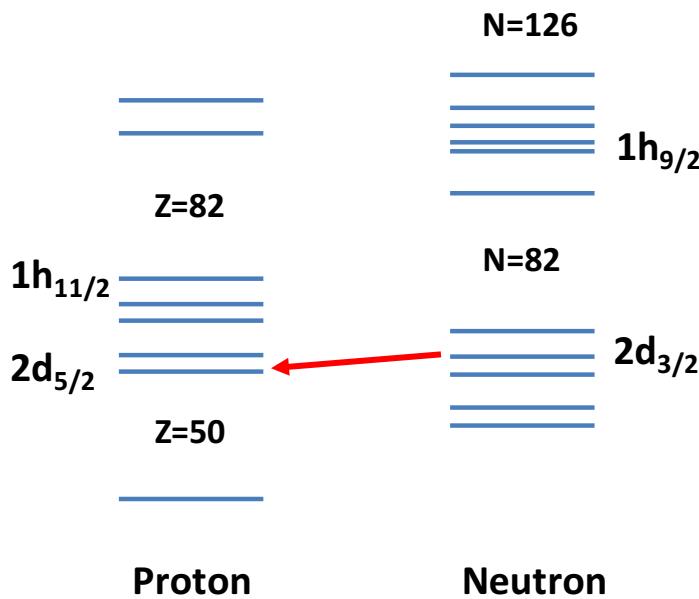
# $\beta$ -Decay Half-Lives in Sn isotopes



- Isoscalar Pairing:  
effective for Sn isotopes  
(nuclei above N=82 closed shell)
- Isoscalar Pairing + QPVC:  
reduce the half-lives

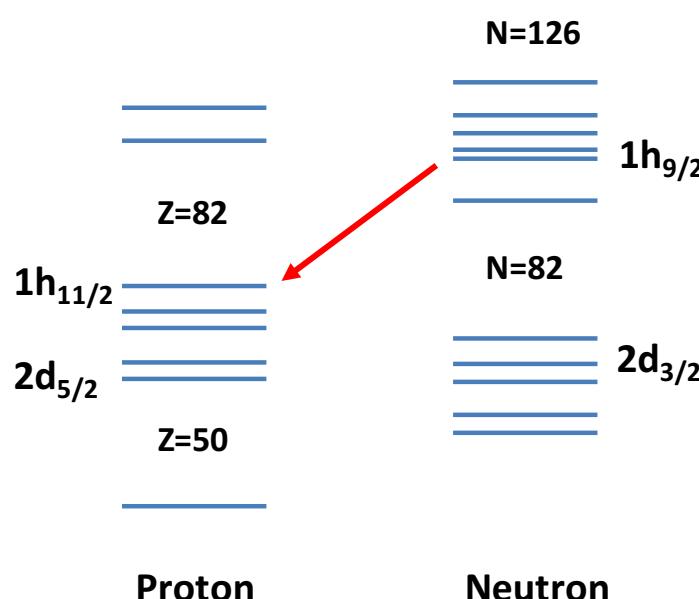
# The role of IS pairing

- $N < 82$



$$\begin{aligned}
 & V^{\text{pp}}(2\text{d}_{3/2} 2\text{d}_{5/2}, 2\text{d}_{3/2} 2\text{d}_{5/2}) \\
 & -0.50 \text{ (} ^{130}\text{Sn)} \\
 & u^2(2\text{d}_{3/2})u^2(2\text{d}_{5/2}) + v^2(2\text{d}_{3/2})v^2(2\text{d}_{5/2}) \\
 & 0.069 \text{ (} ^{130}\text{Sn)} \\
 & \text{ph type}
 \end{aligned}$$

- $N > 82$



$$\begin{aligned}
 & V^{\text{pp}}(1\text{h}_{9/2} 1\text{h}_{11/2}, 1\text{h}_{9/2} 1\text{h}_{11/2}) \\
 & -1.68 \text{ (} ^{144}\text{Sn)} \\
 & u^2(1\text{h}_{9/2})u^2(1\text{h}_{11/2}) + v^2(1\text{h}_{9/2})v^2(1\text{h}_{11/2}) \\
 & 0.89 \text{ (} ^{144}\text{Sn)} \\
 & \text{pp type}
 \end{aligned}$$

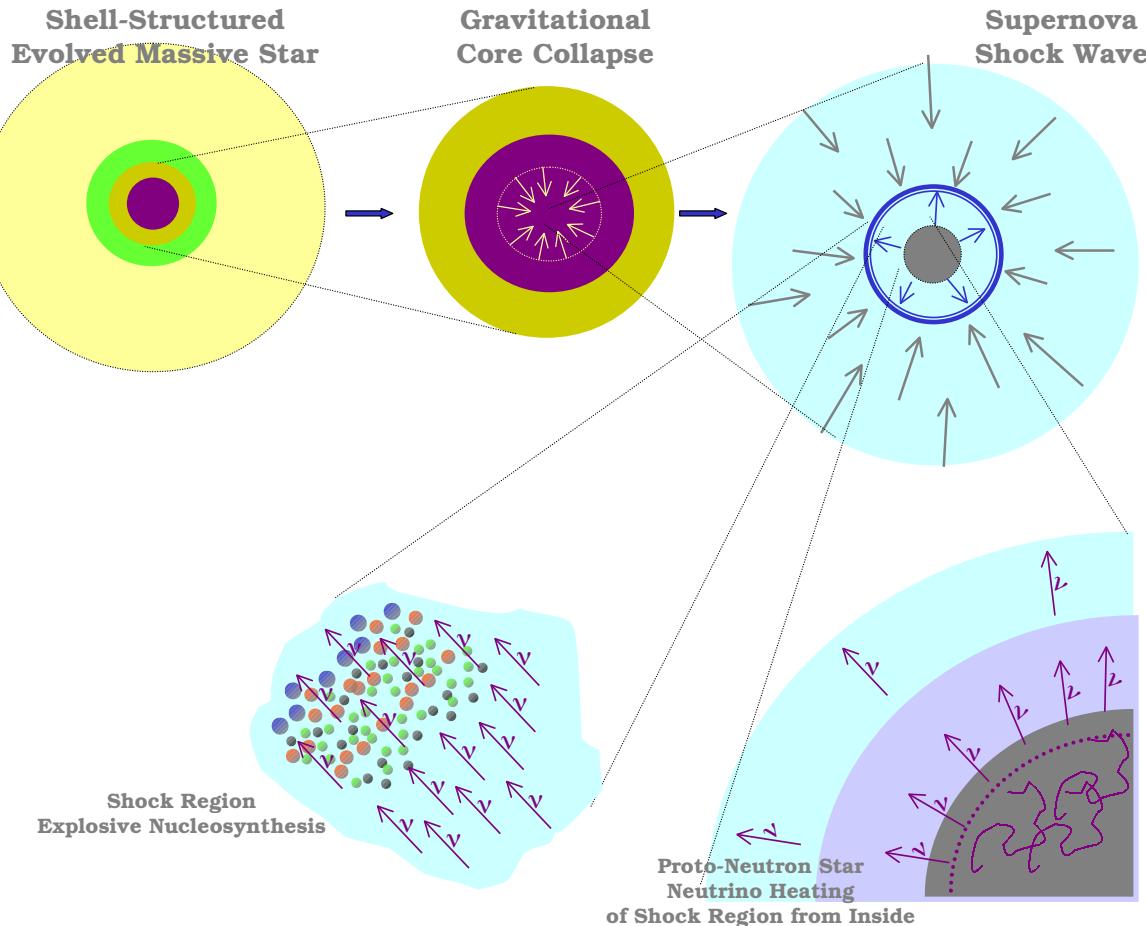
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- Accurate Nuclear Physics Inputs
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- Where does r-process happen?
  - Electron capture rates in core-collapse supernova
- Summary and Perspectives

# Electron capture in core-collapse supernova

## Collapse of a massive star and a supernova explosion



**Electron capture (EC)  
on nucleus**



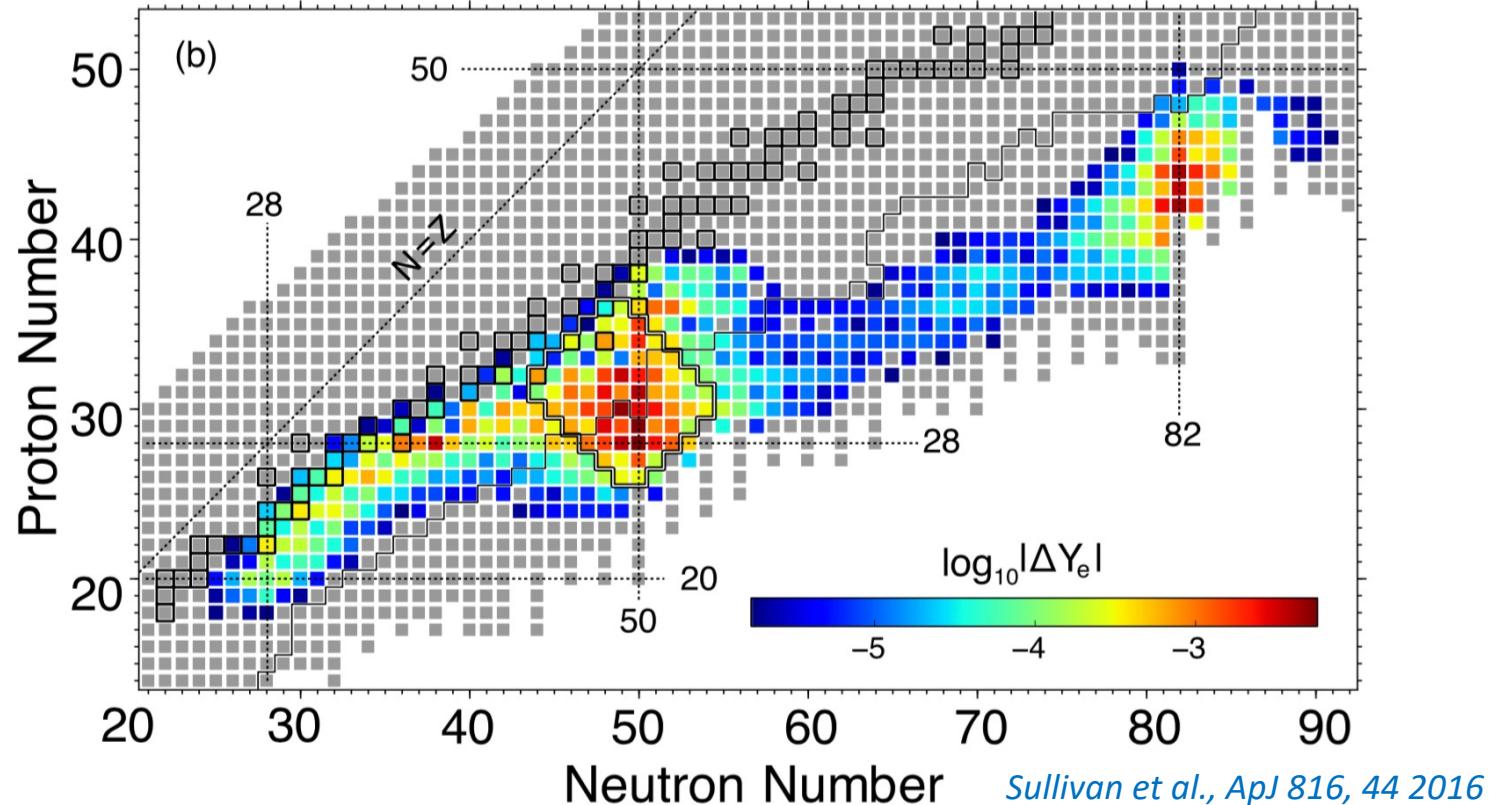
**change  $Y_e$   
change entropy**



**affect the strength  
of bounce shock  
and supernova  
evolution**

# Important electron-capturing nuclei

Top 500 electron-capturing nuclei with the largest absolute change to the electron fraction ( $Y_e$ ) up to neutrino trapping



- The integrated contribution to core deleptonization up to neutrino trapping
$$Y_e(t = t_{\text{trapping}}) \simeq Y_e(t = 0) - \sum_i \Delta Y_e^i$$
- Primary contributors: neutron rich nuclei near **N=50** and N=82 closed neutron shells

# Theoretical study of electron-capture rates

Electron Capture Rate:

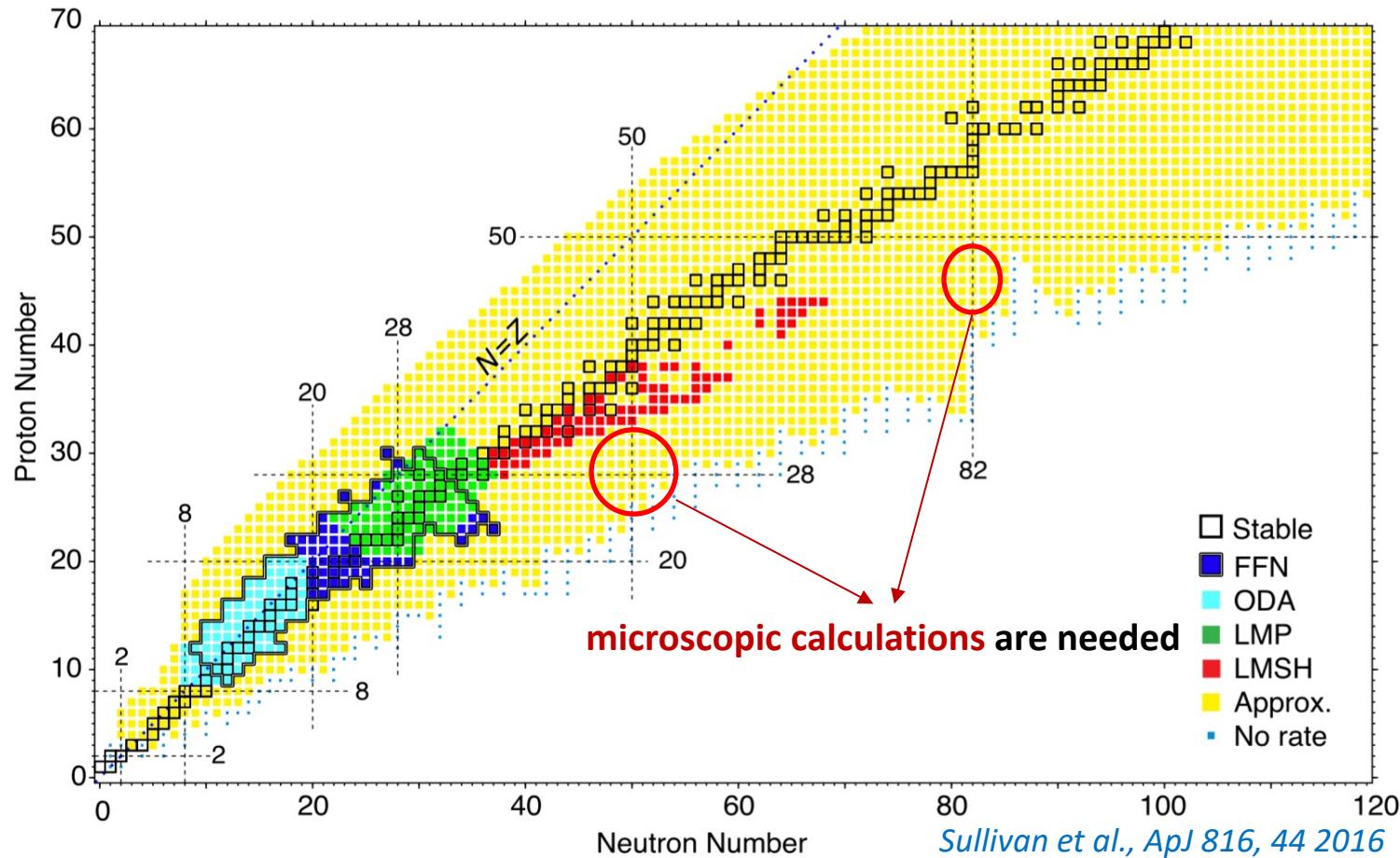
$$\lambda^{\text{ec}} = \frac{\ln 2}{6150 \text{ s}} \sum_J \sum_i \Phi_{Ji}^{(+)} F_i^{\text{ec}} = \sum_J \sum_i \lambda_{Ji}^{\text{ec}}$$

phase space factor

transition strength of **spin-isospin excitations** in T<sup>+</sup> direction:  
Fermi, Gamow-Teller, Spin-Dipole transitions ...

- **Independent Particle Model (IPM)**
  - ✓ first tabulation of weak interaction rates  $21 \leq A \leq 60$   
*Fuller, Fowler, Newman ApJ 252, 715, 1982; ApJ 293, 1, 1985* **FFN**
- **Large Scale Shell Model**
  - sd shell nuclei  $17 \leq A \leq 39$   $^{16}\text{O}$  core + effective interaction of Wildenthal  
*Oda et al., ADNDT 56, 231, 1994* **ODA**
  - pf shell nuclei  $45 \leq A \leq 65$  modified KB3 interaction  
*Langanke and Martinez-Pinedo, NPA 673, 481 (2000); ADNDT 79, 1 (2001)* **LMP**
- **Hybrid Model**
  - Shell Model Monte Carlo (SMMC) + Random Phase Approximation(RPA)  
pfg/sdg shell nuclei  $65 \leq A \leq 112$   
*Langanke et al., PRL 90, 241102 (2003)* **LMSH**

# Theoretical study of electron-capture rates



- Approx. - Approximate Rates estimated by  $\lambda = \frac{(\ln 2)B}{K} \left( \frac{T}{m_e c^2} \right)^5 [F_4(\eta) - 2\chi F_3(\eta) + \chi^2 F_2(\eta)]$
- Fitted by shell model calculation for nuclei not far from stability line  
⇒ For neutron rich nuclei, the formulas is not a good approximation

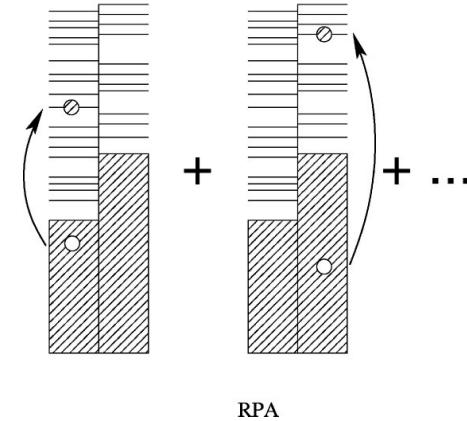
# Random Phase Approximation (RPA)

- **RPA:** widely used for the description of spin-isospin excitations

- The RPA excited state is generated by

$$Q_\nu^\dagger = \sum_{mi} X_{mi}^\nu a_m^\dagger a_i - \sum_{mi} Y_{mi}^\nu a_i^\dagger a_m$$

- Full 1p1h configuration space  $\Rightarrow$  almost whole nuclear chart



To study the electron capture in core-collapse supernova, inclusion of temperature effect is necessary! ( $T \sim 0 - 2$  MeV)

- **Finite Temperature RPA (FTRPA):** takes into account temperature self-consistently both in Hartree and RPA level

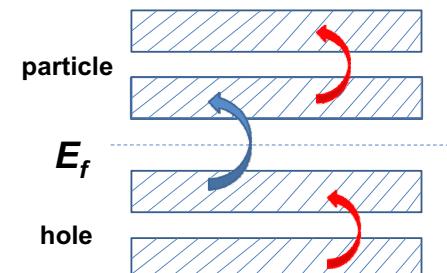
- Temperature is introduced by thermal occupation of each nucleon

$$f_{p(n)} = \frac{1}{1 + \exp\left(\frac{\epsilon_{p(n)} - \mu_{p(n)}}{kT}\right)}$$

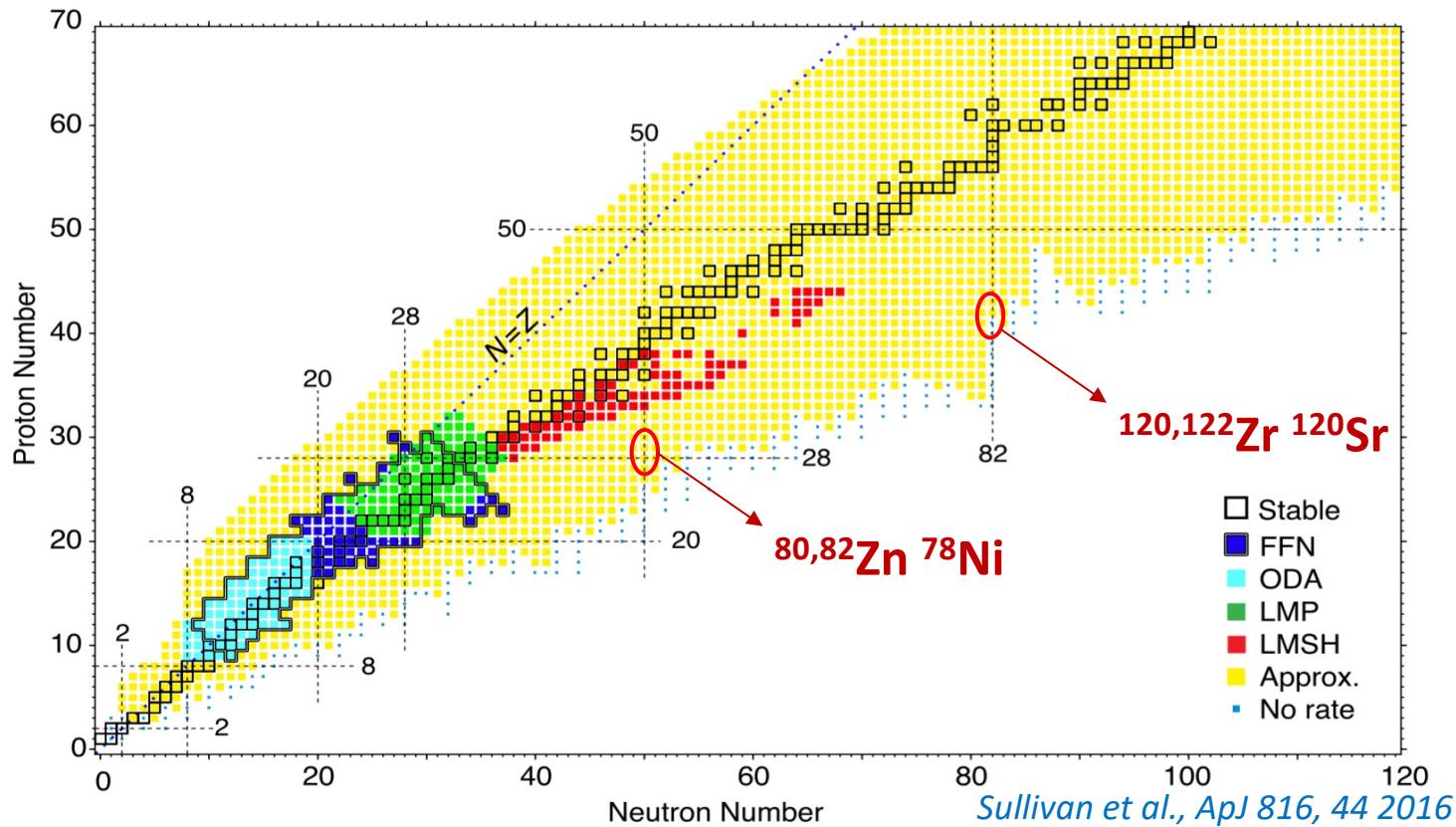
- Configuration space: p-h, p-p, and h-h pairs

N. Paar et al., PRC 80, 055801 (2009)

Y. F. Niu et al., PLB 681, 315 (2009)



# Electron capture study for important nuclei



Finite temperature RPA (FTRPA) can provide a universal tool to study the electron capture for almost the whole nuclear chart, so the important nuclei for supernova explosion will be studied, including

N~50:  $^{78}\text{Ni}$   $^{80}\text{Zn}$   $^{82}\text{Zn}$

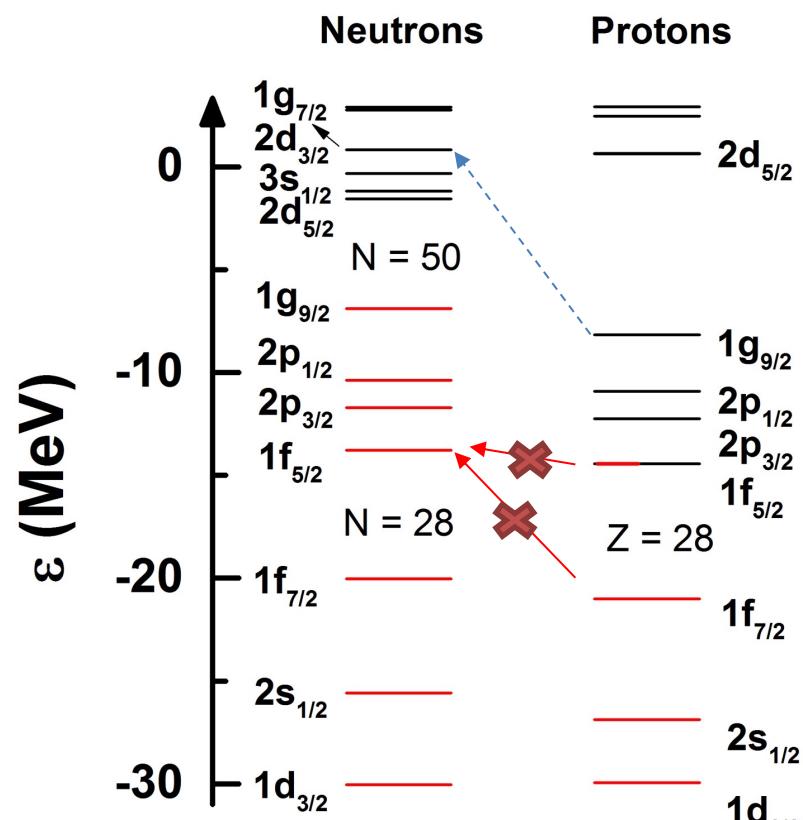
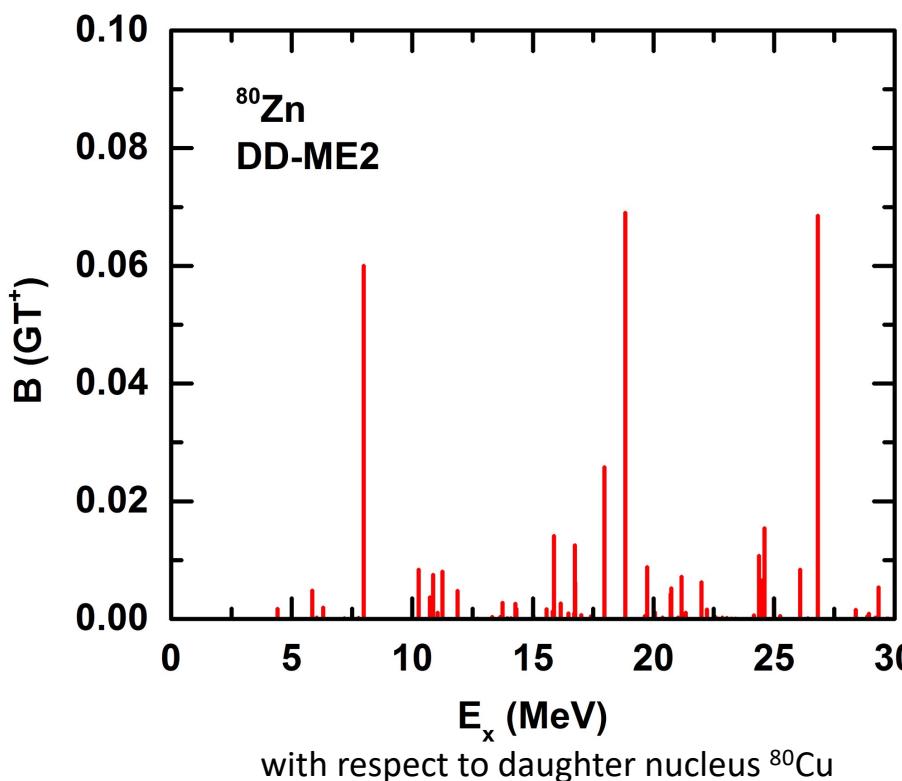
N~82:  $^{120}\text{Sr}$   $^{120}\text{Zr}$   $^{122}\text{Zr}$

# Gamow-Teller strength distribution ( $T^+$ )

- GT operator

$$\hat{F}_{\text{GT}}^\pm = \sum_{i=1}^A \sigma(i) \tau_\pm(i)$$

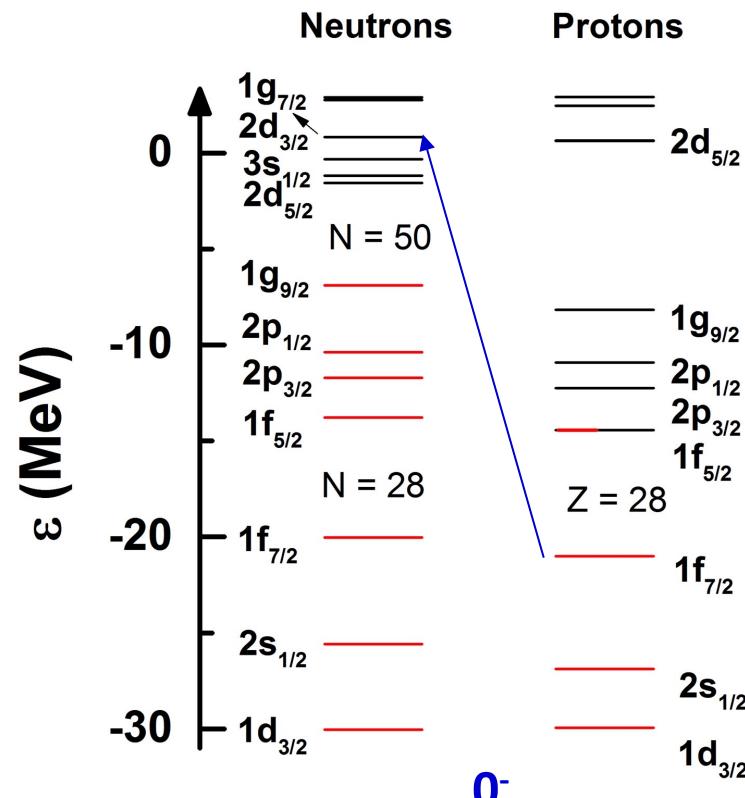
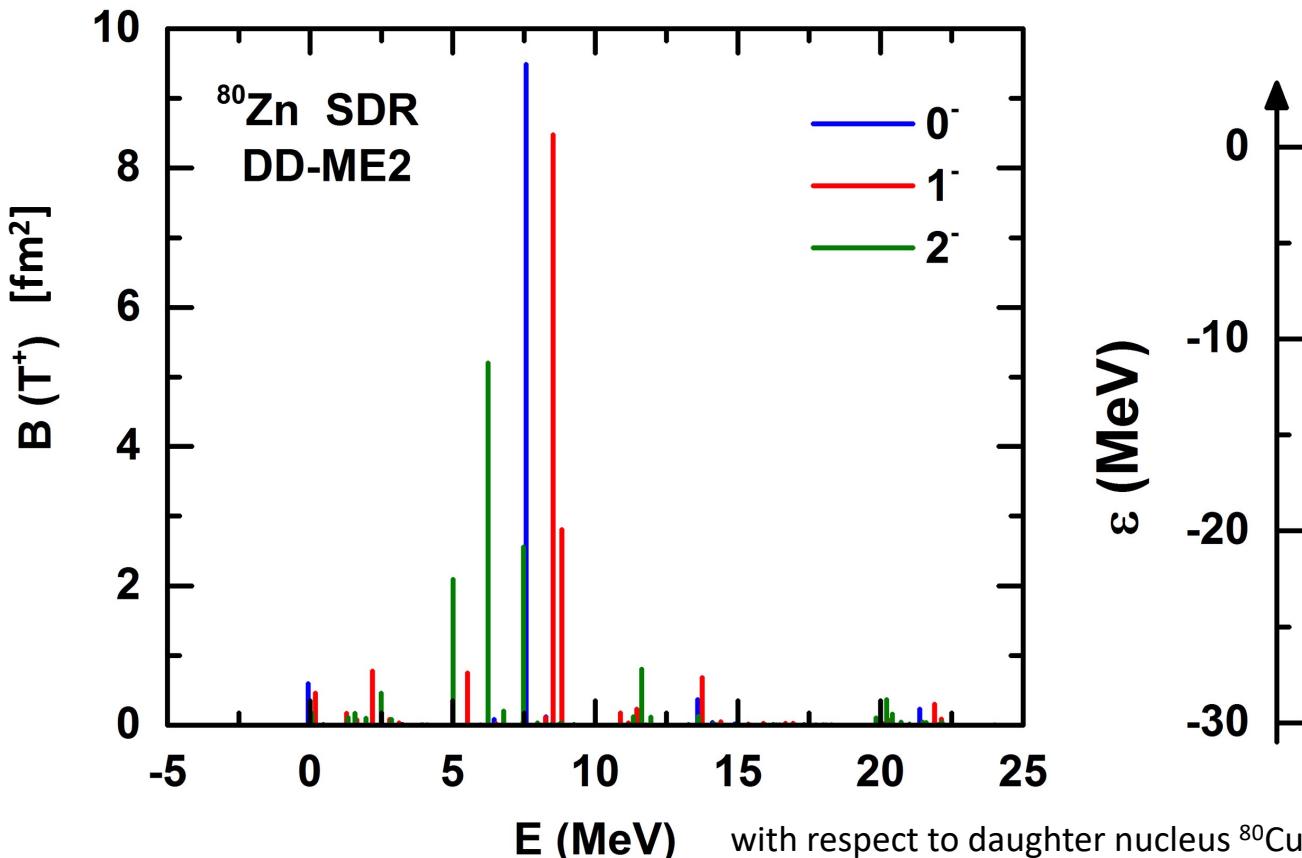
$$J^\pi = 1^+$$



- ✓ GT<sup>+</sup> transitions are almost blocked (Ikeda sum rule = 60)
- ✓ Pairing correlations or transitions across major shells make little transition strength possible

# Spin-Dipole strength distribution ( $T^+$ )

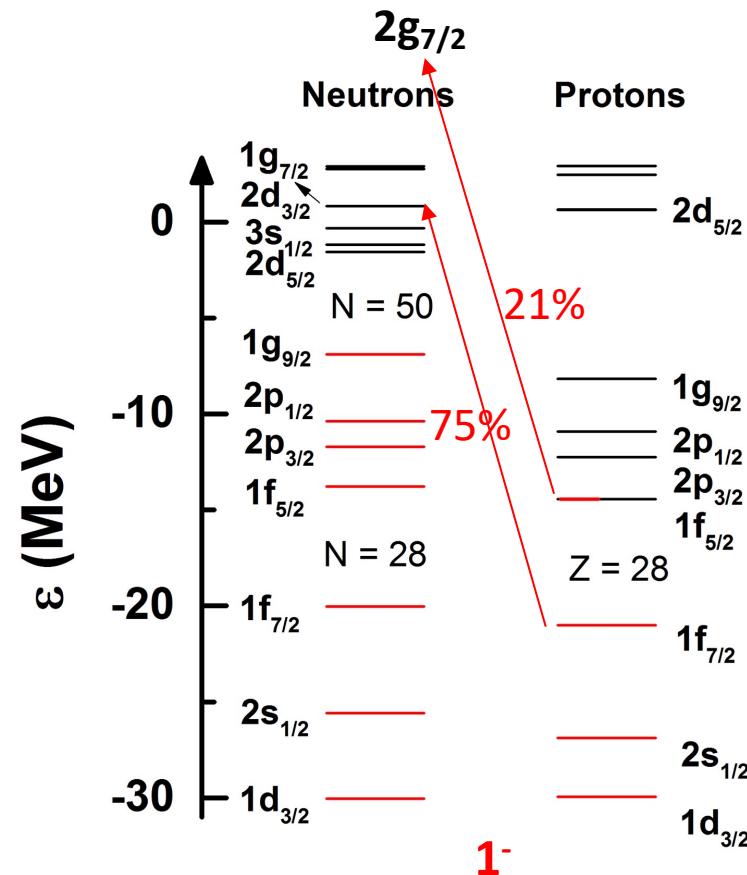
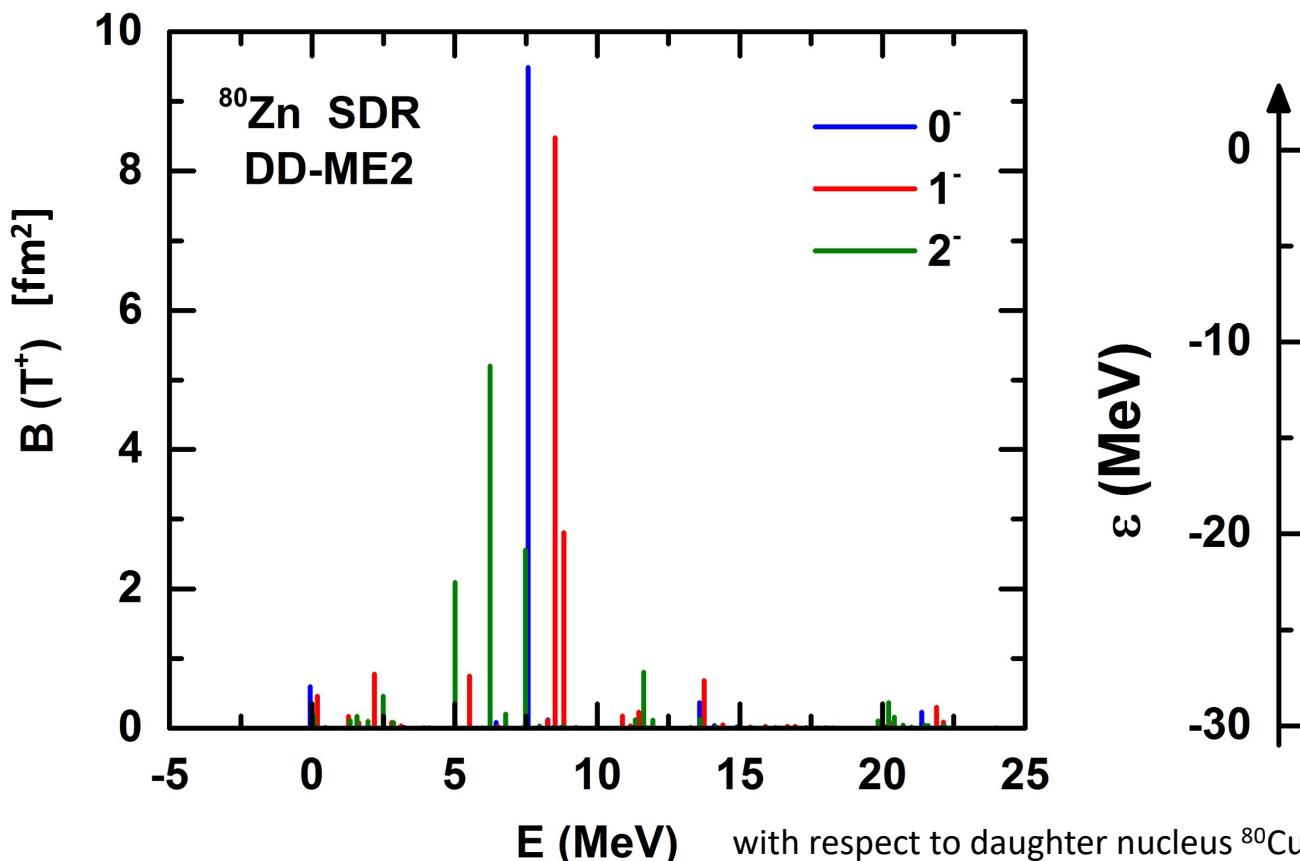
- SD operator**  $\hat{F}_{\text{SDR}}^{\pm} = \sum_{i=1}^A [r_i Y_1(i) \otimes \sigma(i)]_{J=0,1,2} \tau_{\pm}(i)$   $J^\pi = 0^-, 1^-, 2^- \quad \Delta S=1, \Delta L=1$



- ✓ Spin-Dipole transitions have significant strength
- ✓ SD transitions will dominate EC cross section of  $^{80}\text{Zn}$

# Spin-Dipole strength distribution ( $T^+$ )

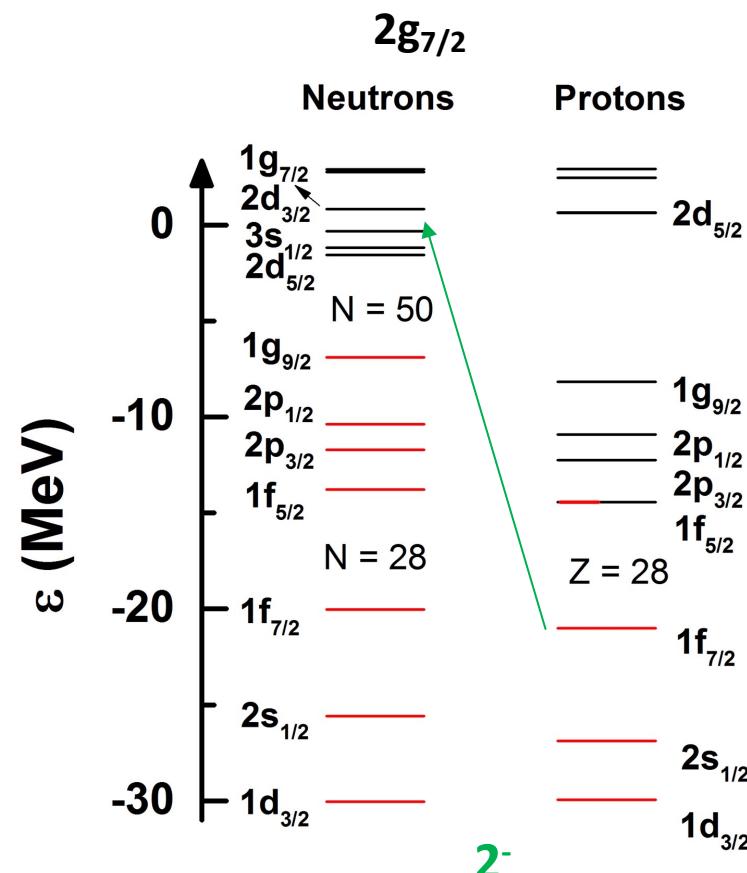
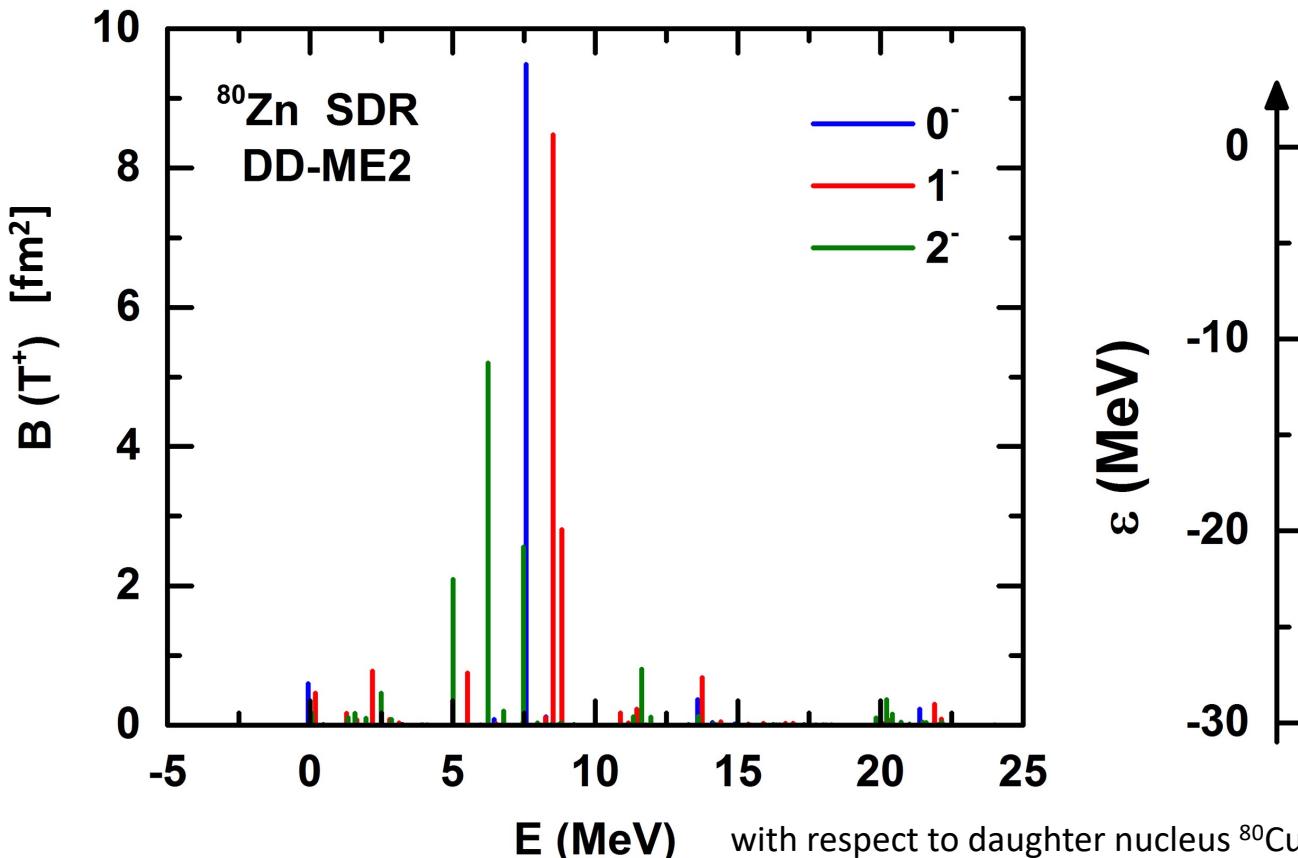
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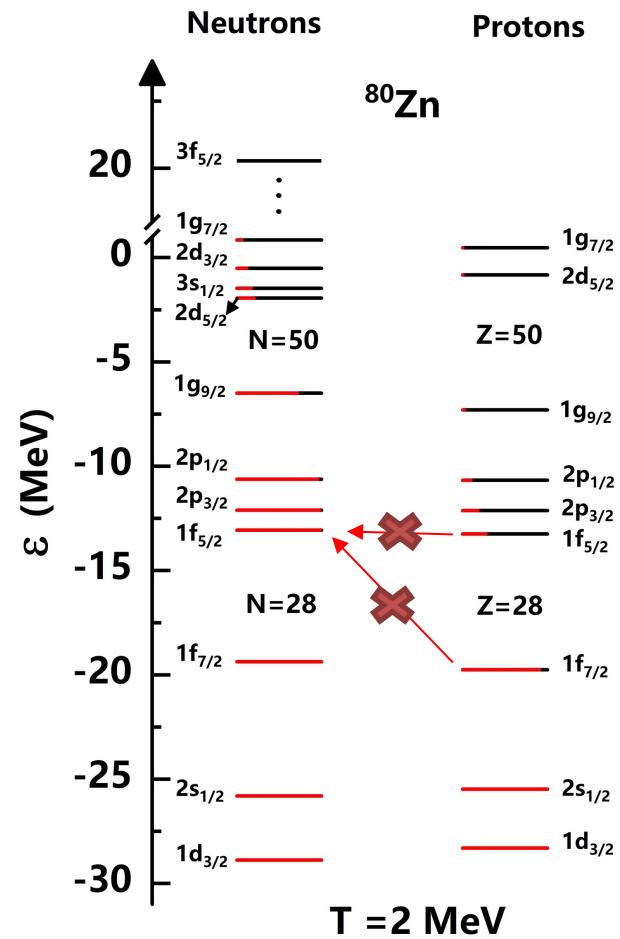
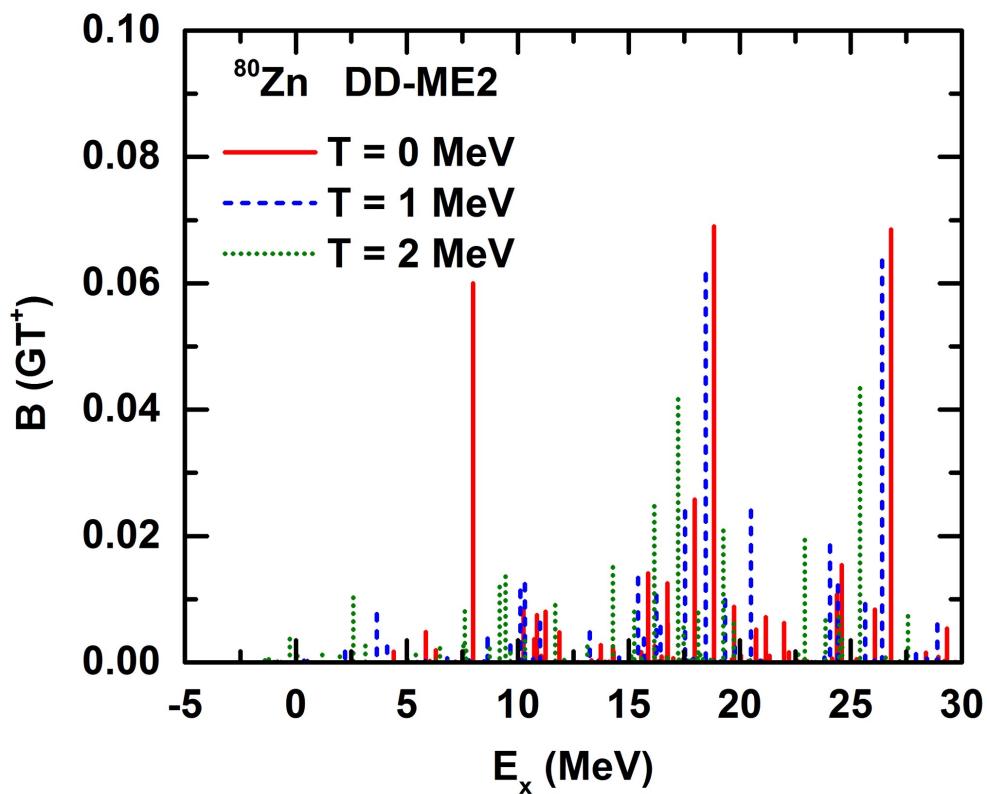
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# Temperature effects

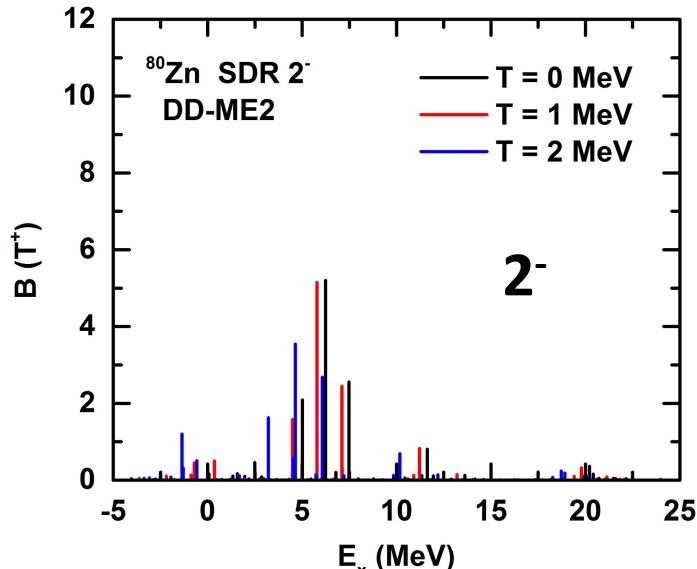
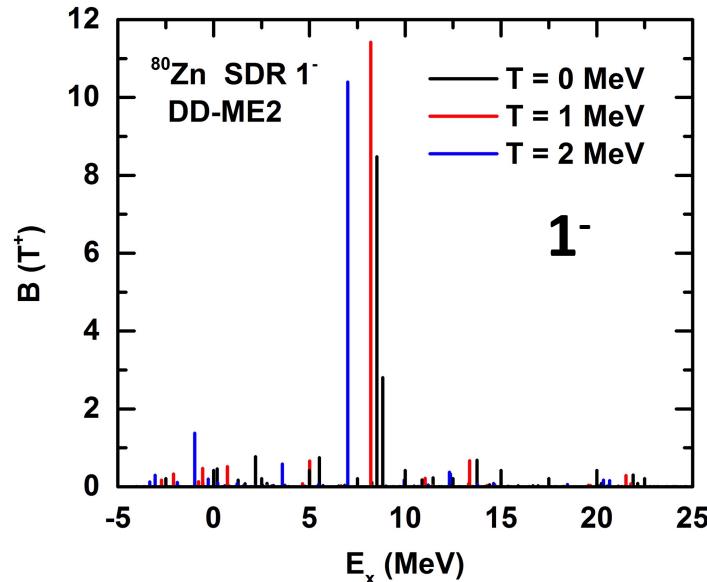
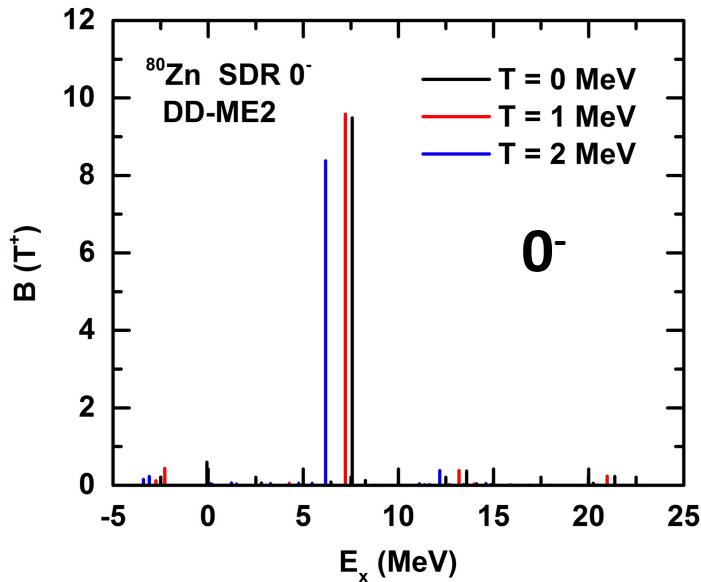
## GT transitions at finite temperature



- ✓ Even temperature cannot unblock the  $GT^+$  transition due to large neutron excess
- In stellar environment,  $GT^+$  still cannot contribute much to EC rates

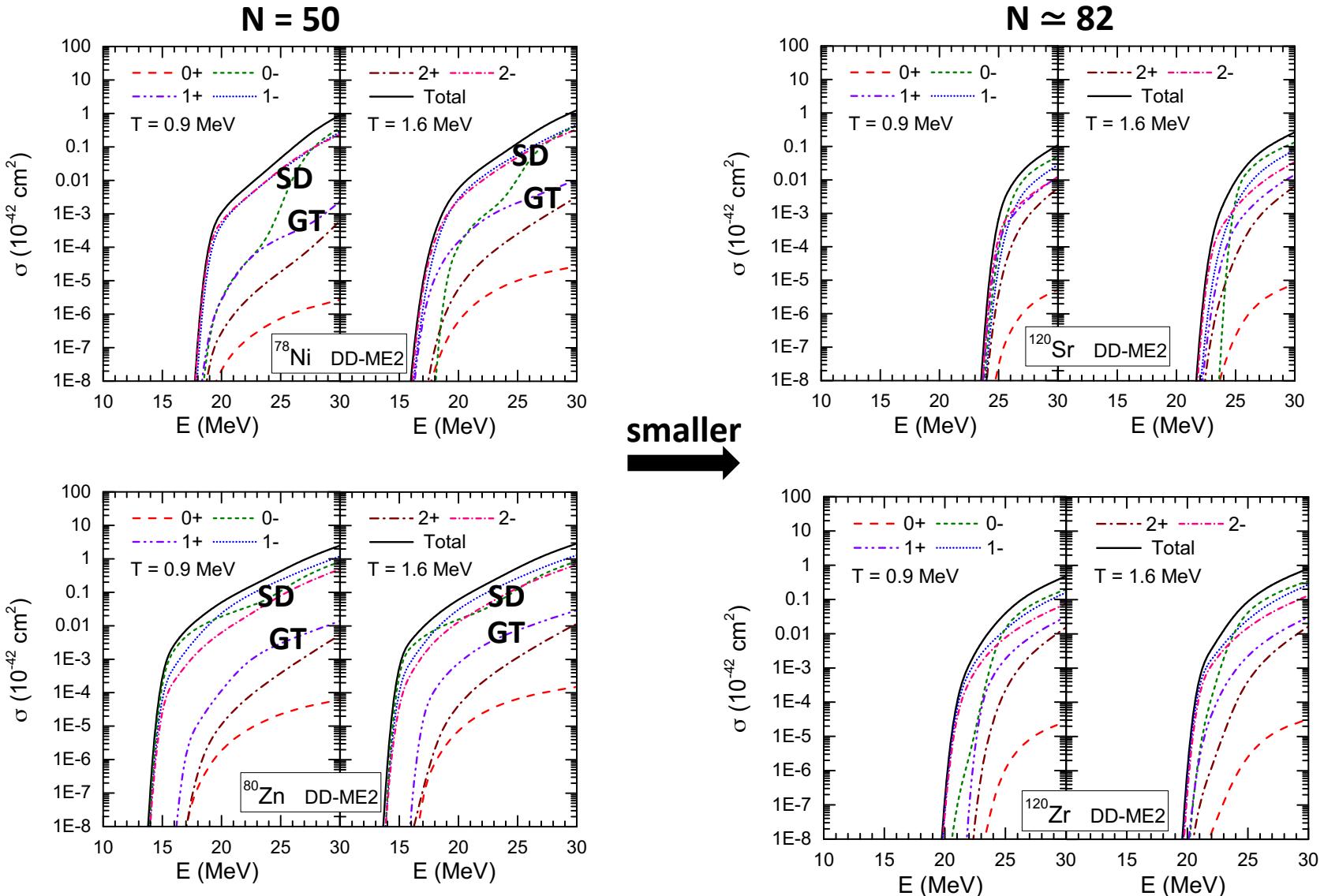
# Temperature effects

## Spin Dipole Transitions at finite temperature



- ✓ Temperature decreases energies, but changes are small.
- Spin-dipole transition data measured at Lab (zero temperature) can still be applied to EC study in supernova.

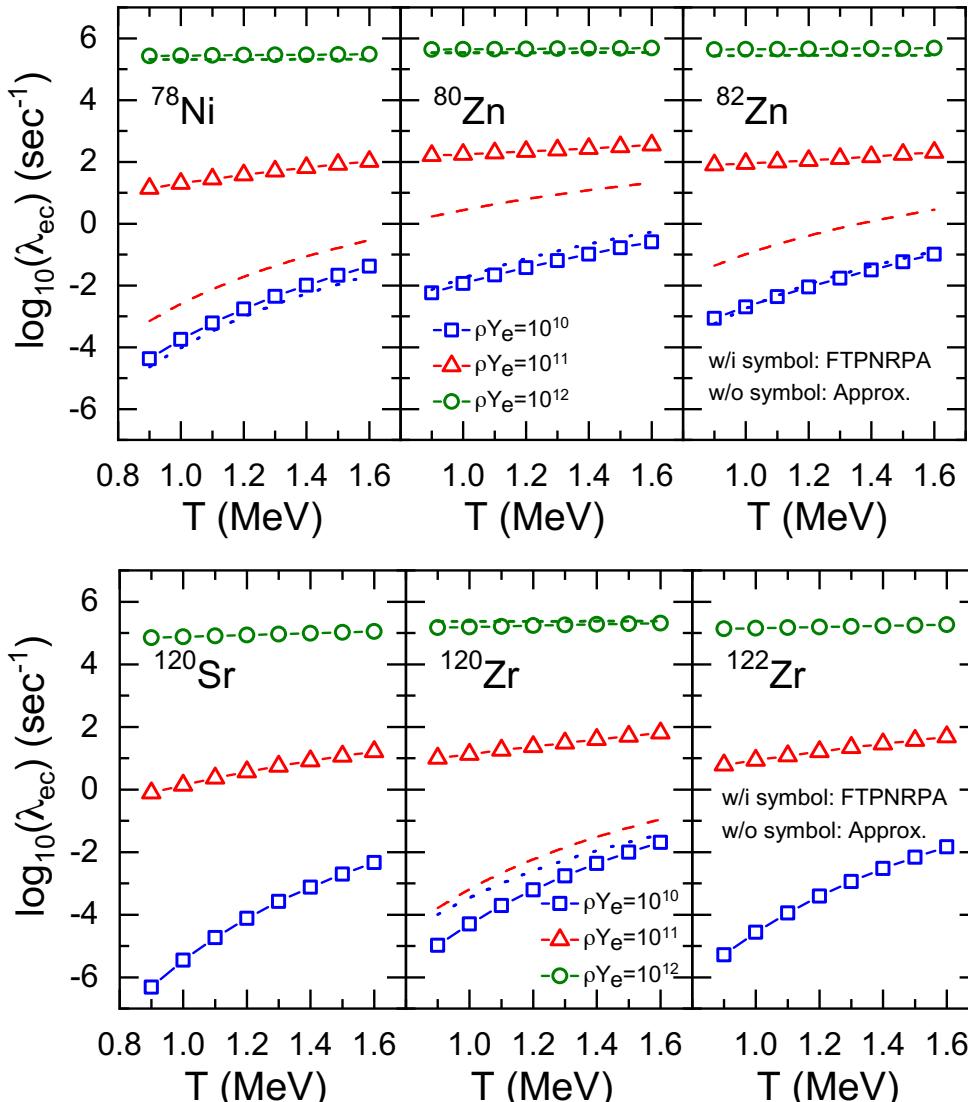
# Electron capture cross sections



- For these neutron rich nuclei, spin dipole transitions dominate the cross section
- Even at high temperatures, GT transitions are not considerably unblocked

# Electron capture rates

## Electron capture rates at different stellar environment

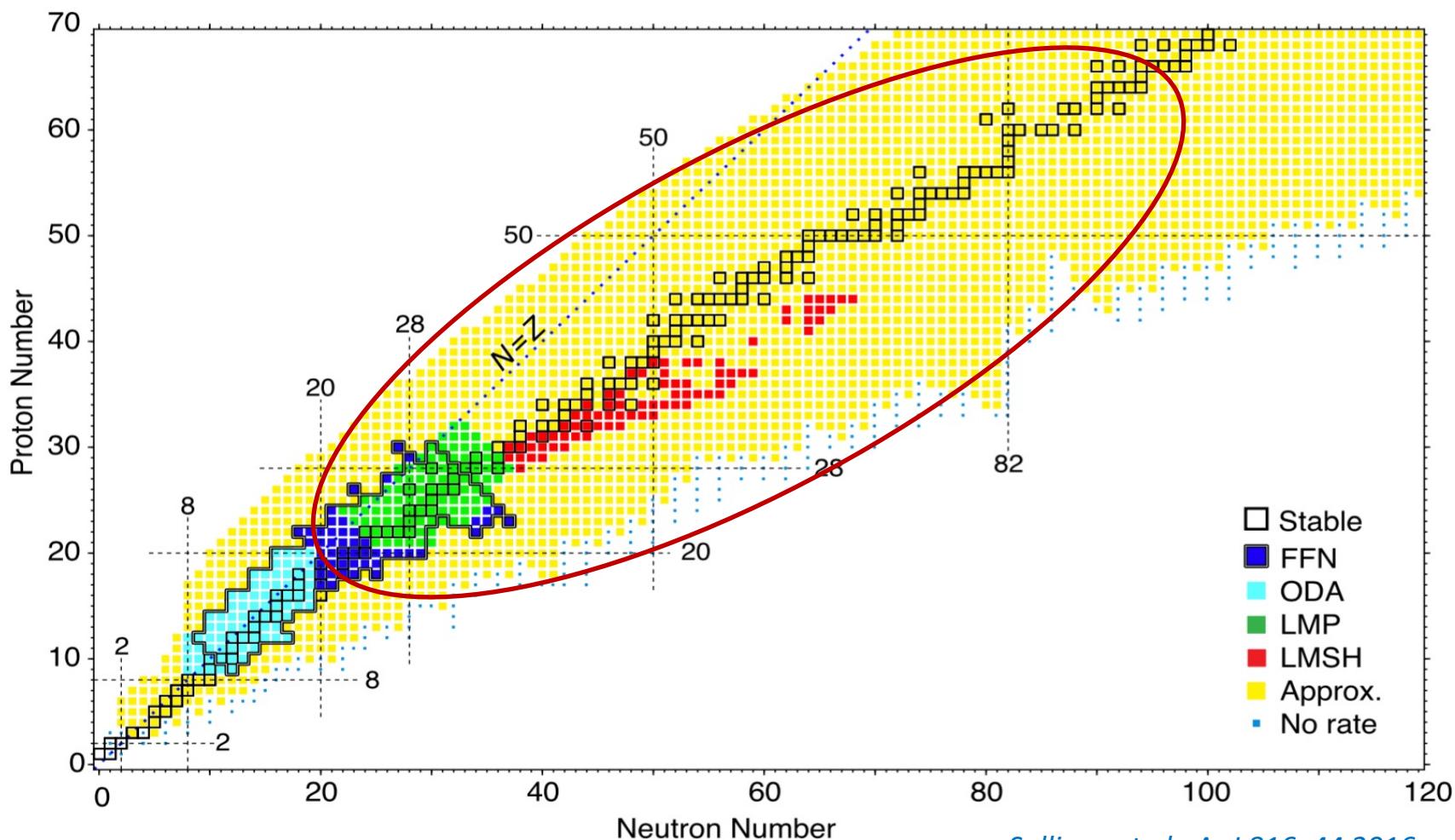


- With the increase of electron density, the EC rates are increased by several orders of magnitude.
- At lower electron densities, the EC rates have big increase with temperature, but at high densities, the rate is not sensitive to temperature.
- Approx.

$$\lambda = \frac{(\ln 2)B}{K} \left( \frac{T}{m_e c^2} \right)^5 [F_4(\eta) - 2\chi F_3(\eta) + \chi^2 F_2(\eta)]$$

Rates from approximation formula at  $10^{11} \text{ g/cm}^3$  is much underestimated compared to our results

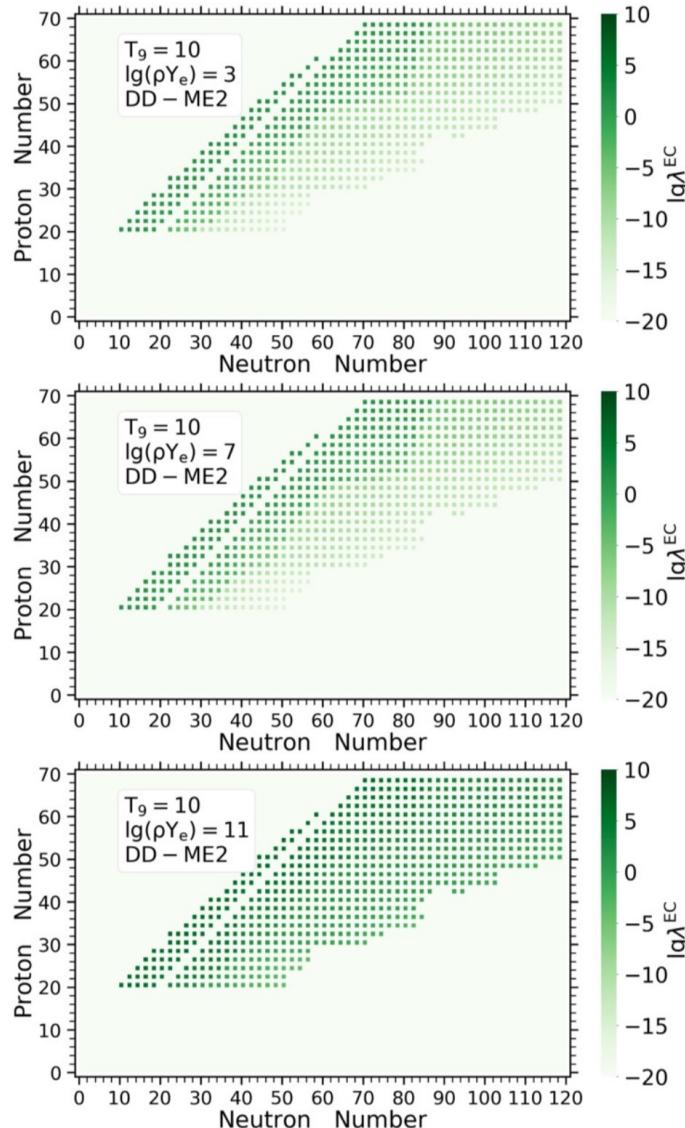
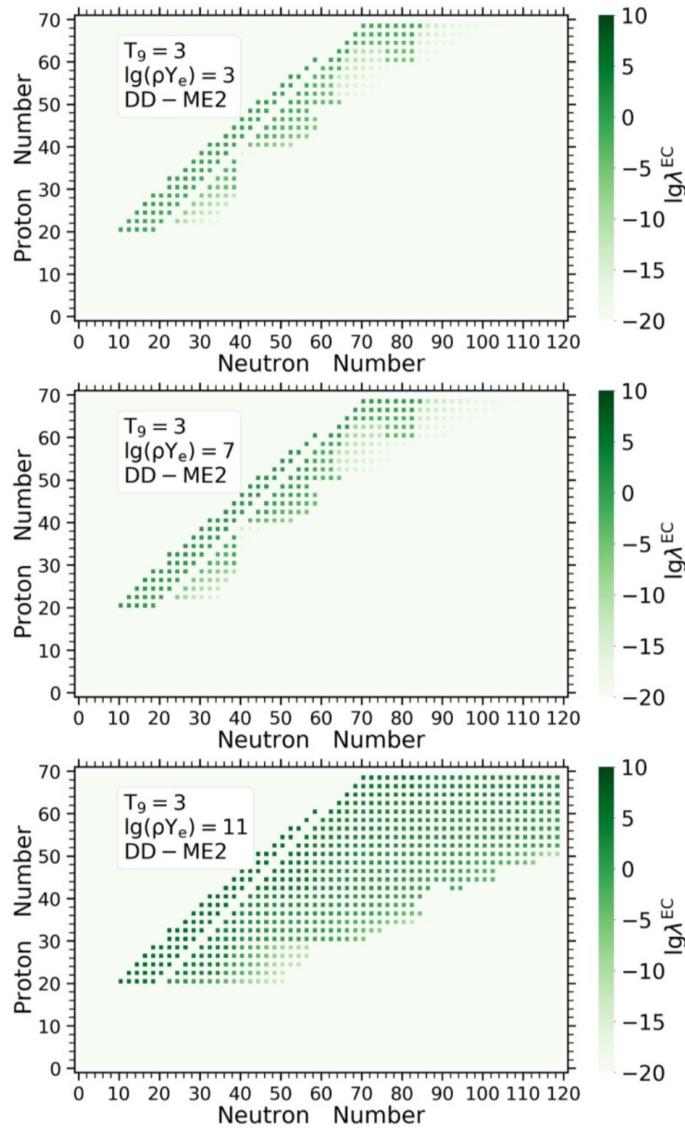
# Systematic calculations for Z=20-68 even-even nuclei



Sullivan et al., ApJ 816, 44 2016

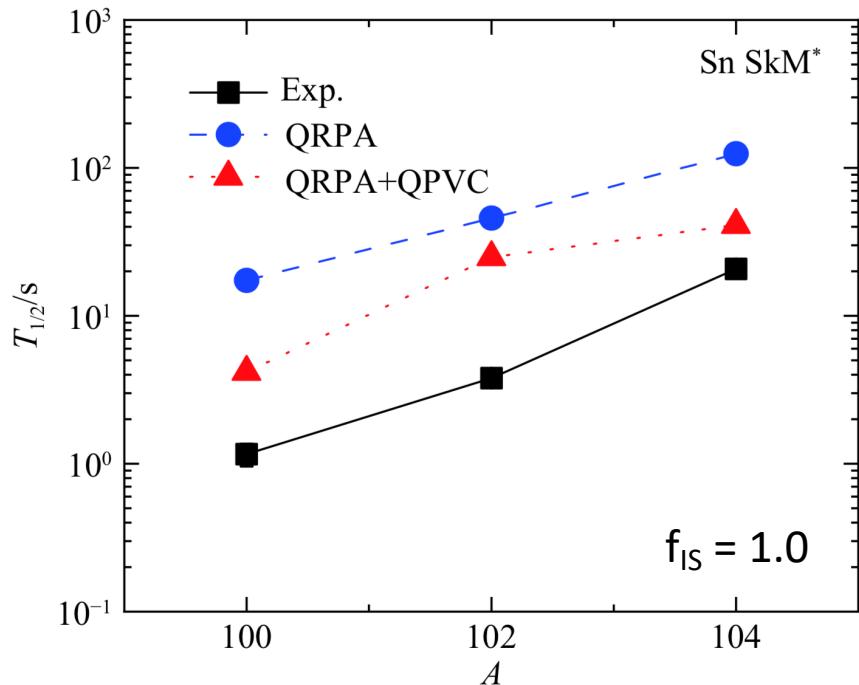
- FTRRPA model is used for systematic calculation of EC rates for Z=20-68 even-even nuclei
- Only GT transitions are considered for simplification.

# Systematic calculations for Z=20-68 even-even nuclei

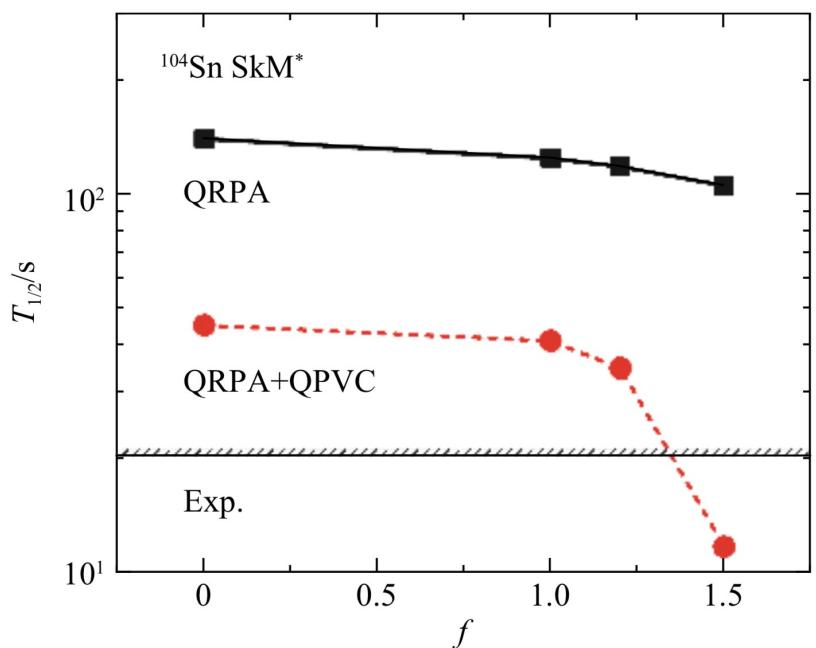


# $\beta^+$ / EC Half-lives by QRPA+QPVC

➤  $\beta^+$ / EC Half-lives of neutron-deficient nuclei



➤ The effect of isoscalar pairing



- The  $\beta^+$ / EC half-lives are overestimated by one order of magnitude.
- QPVC reduces the half-lives of these nuclei.

- With the increase of isoscalar pairing strength,  $\beta^+$ / EC half-lives decrease.
- QPVC results decrease faster than QRPA.
- QRPA results cannot reproduce exp. even at large  $f_{IS}$ , while QPVC reproduces exp. at  $f_{IS} \sim 1.25$ .

# Summary and Perspectives

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Towards the understanding of origin of heavy elements

- Accurate Nuclear Physics Inputs:  $\beta$ -decay
  - ✓ Go beyond RPA/QRPA: we developed self-consistent RPA+PVC / QRPA+QPVC model
  - ✓ Successfully describe the GT resonance and  $\beta$ -decay half-lives in doubly magic nuclei and superfluid nuclei using the same Skyrme interaction
- Electron Capture Rates in core-collapse supernova
  - ✓ FTRPA provides a universal tool for the calculation of EC rates for core-collapse supernova

Perspective:

- Extend QRPA+QPVC model to finite-temperature case, and apply it for EC study in core-collapse supernova

# Acknowledgement

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- Sichuan University: Bai Chunlin
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*Thank you!*