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# Benchmark between CDCC and Faddeev method

Phys. Rev. C **76**, 064602 (2007)

Phys. Rev. C **85**, 054621 (2012)

Phys. Rev. C **94**, 051603 (2016)

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# $d + {}^{12}C$ elastic scattering

- 1, good overall agreement
- 2, Faddeev and CDCC  
can not reproduce  
backward angles data
- 3, coupling is important

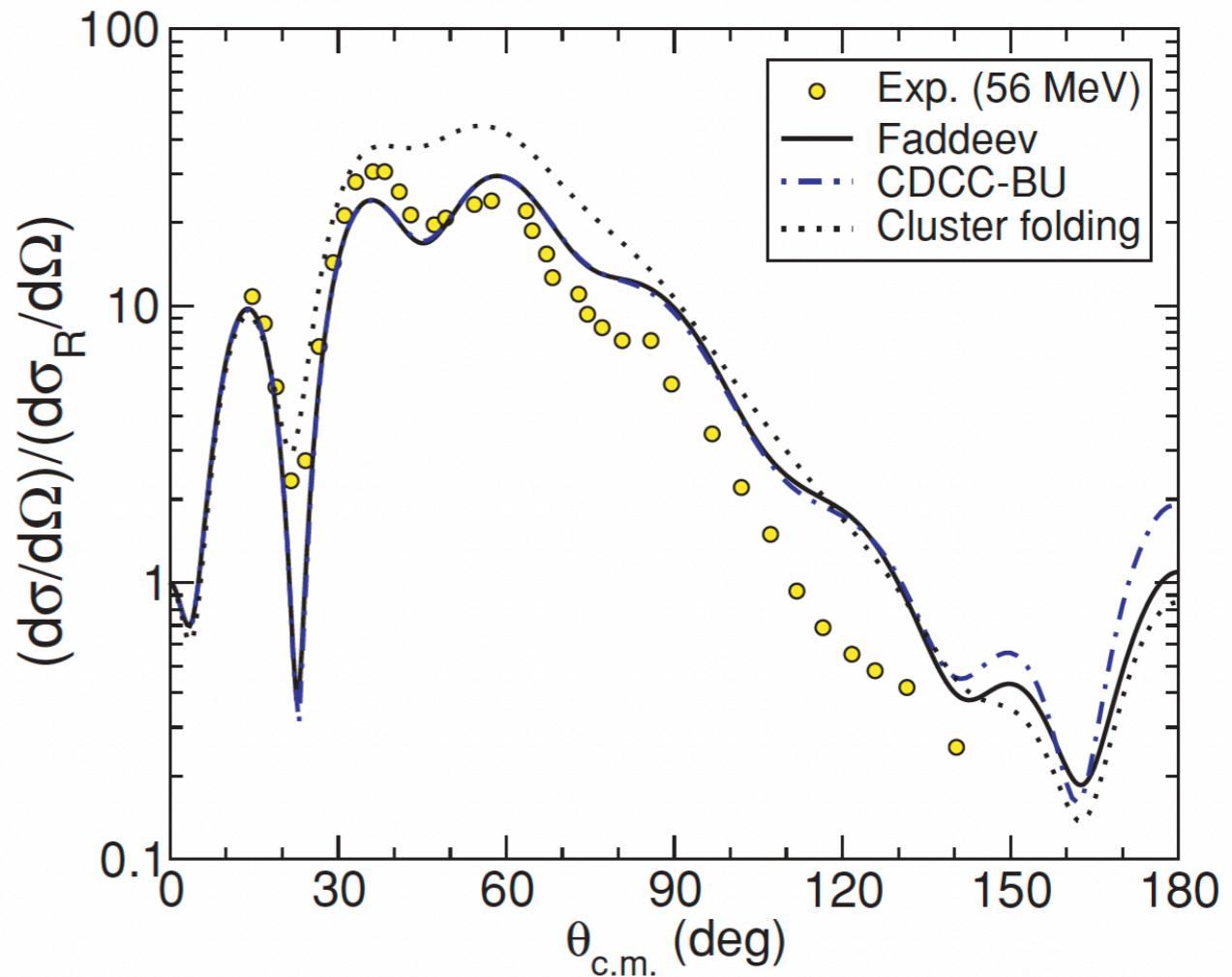


FIG. 2. (Color online) Elastic cross section for deuterons on  ${}^{12}\text{C}$  at  $E_d = 56$  MeV: the solid line corresponds to exact three-body results and the dash-dotted line to CDCC. The result of a single channel cluster folding (dotted) is also shown. The experimental data are from Ref. [22].

# $d + {}^{12}C$ elastic scattering

1, good overall agreement

2, Coulomb interaction

Can not be neglected

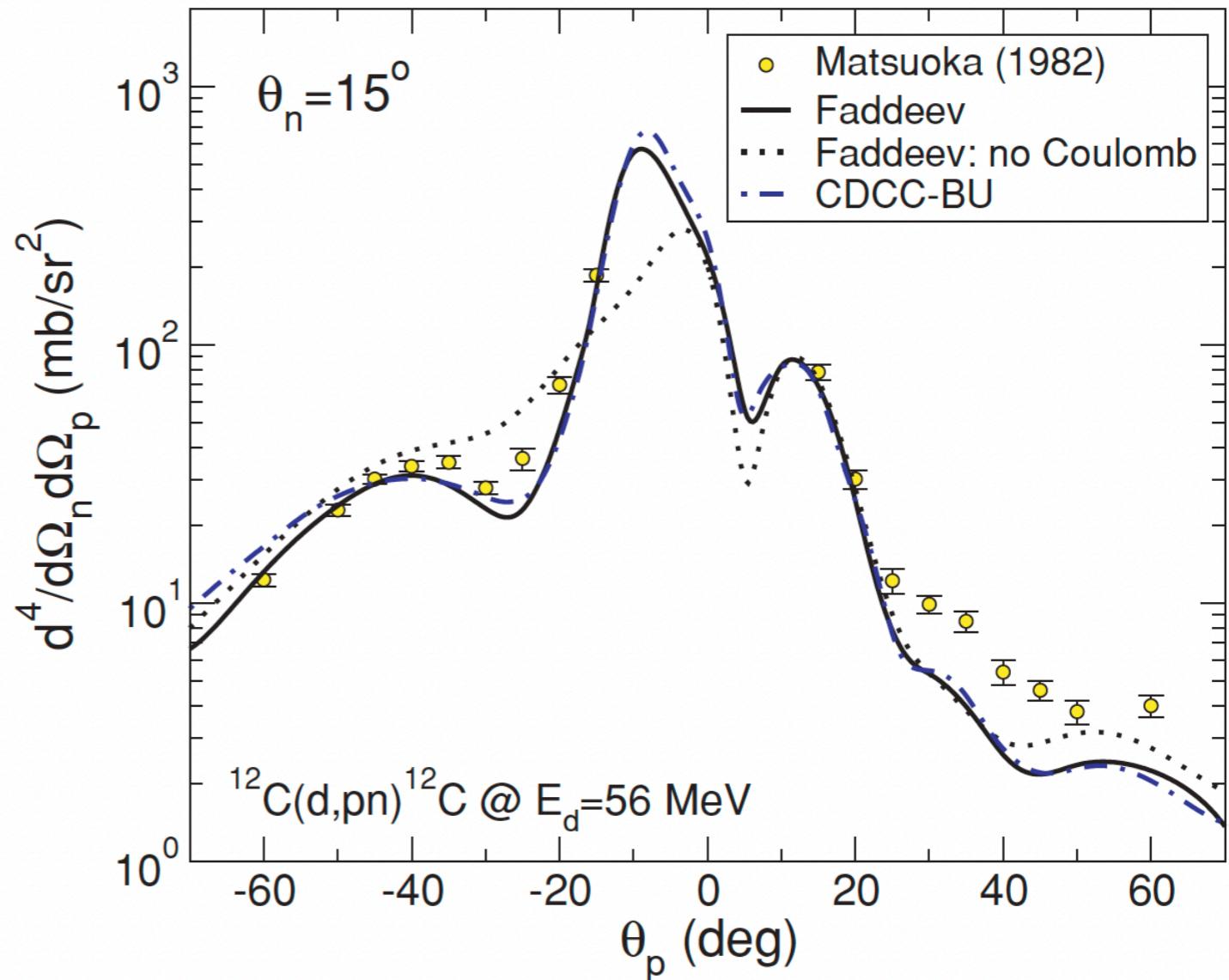


FIG. 3. (Color online) Semi-inclusive differential cross section vs proton scattering angle for the breakup of deuterons on  ${}^{12}\text{C}$  at  $E_d = 56$  MeV: the solid line corresponds to the exact three-body results and the dash-dotted line to CDCC. The dotted line corresponds to the three-body exact result in the absence of the Coulomb force. The experimental data are from Ref. [36].

# $^{12}\text{C}(d, pn)$ breakup

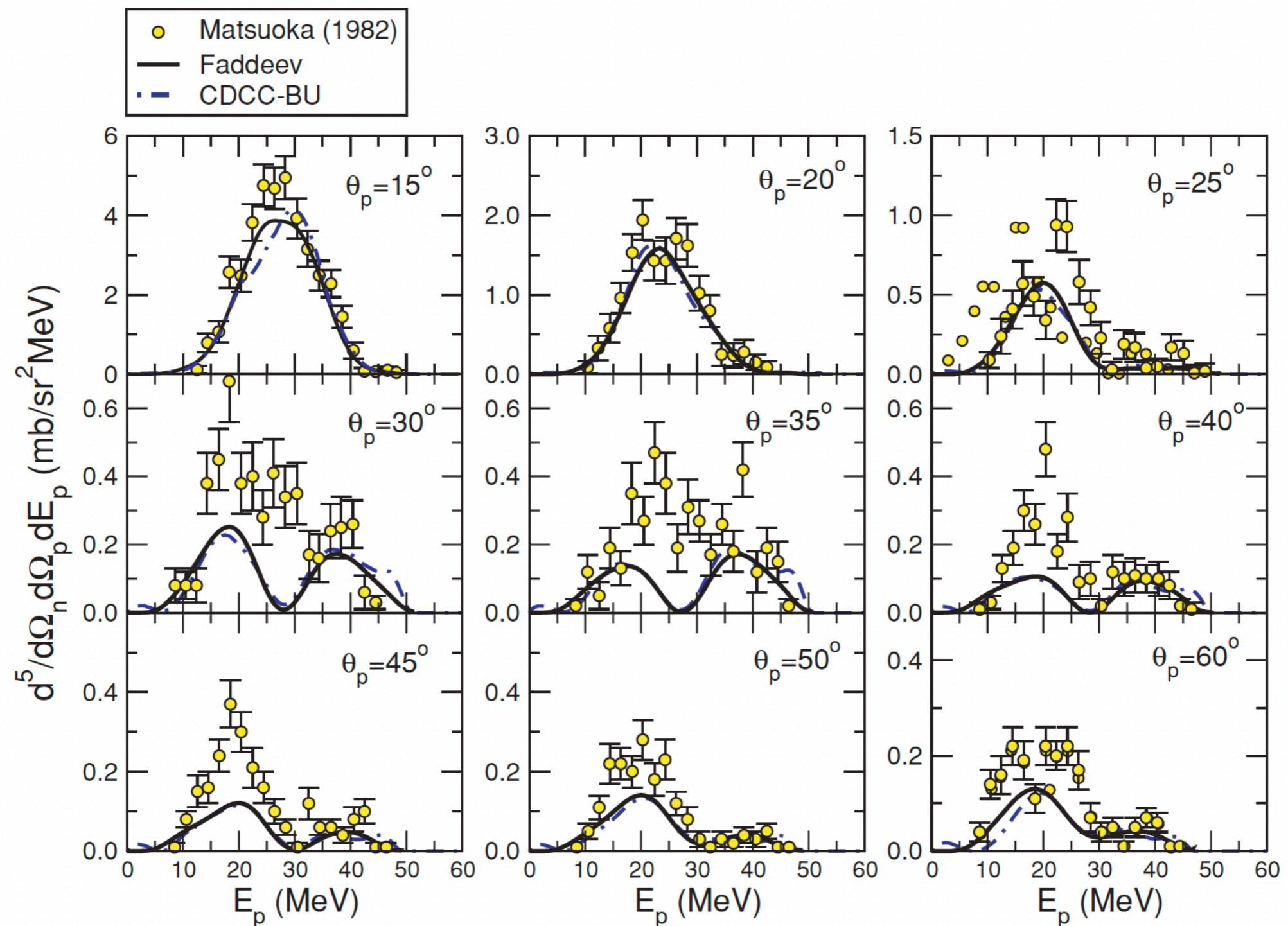
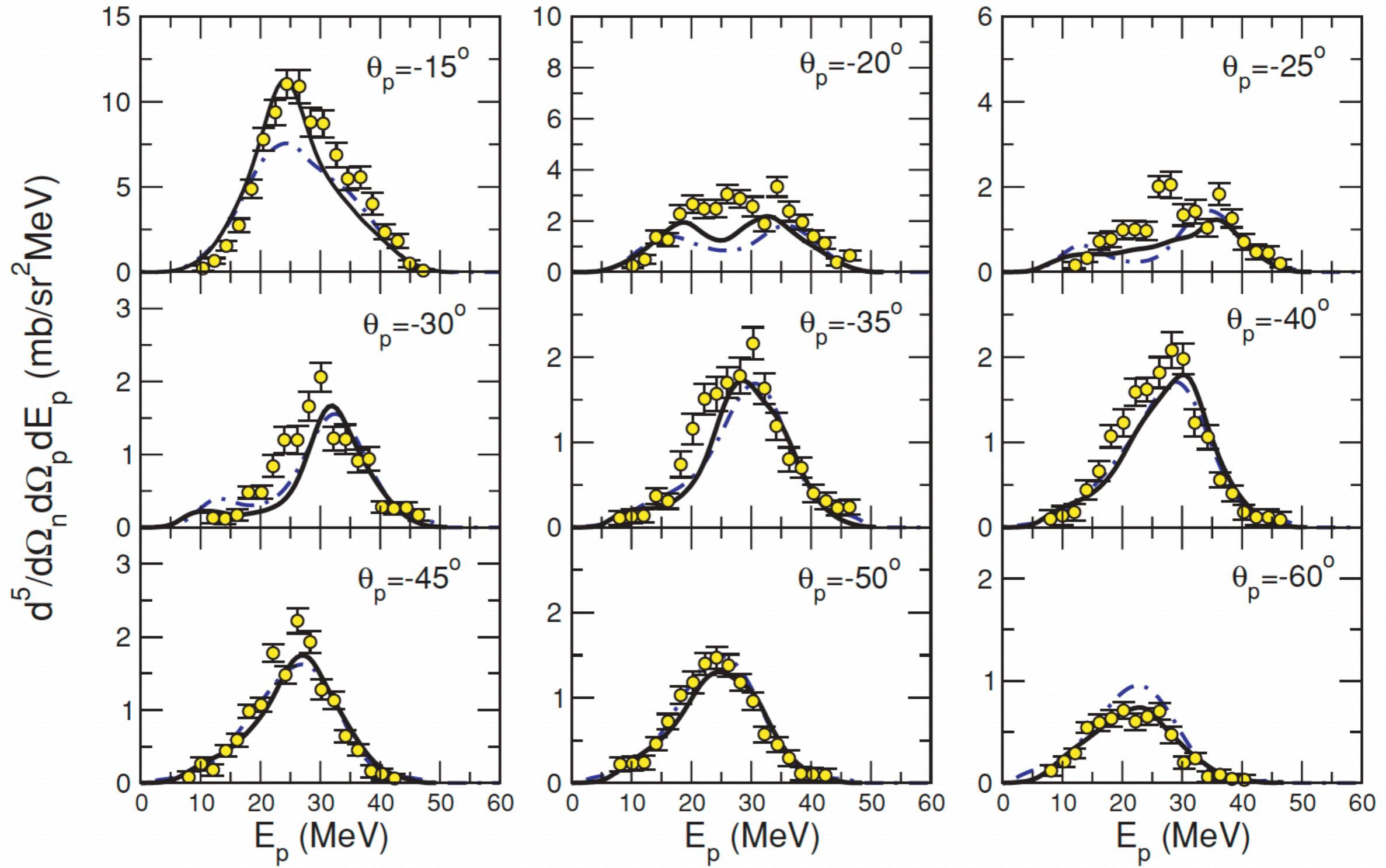


FIG. 4. (Color online) Exclusive differential cross section vs proton energy for the breakup of deuterons on  $^{12}\text{C}$  at  $E_d = 56$  MeV,  $\theta_n = 15^\circ$ , and  $\theta_p > 0$ : the solid line corresponds to exact three-body results and the dash-dotted line to CDCC. The experimental data are from Ref. [36].

# $^{12}C(d, pn)$ breakup



# $d + {}^{58}\text{Ni}$ elastic scattering

- 1, good overall agreement
- 2, Coupling effect is important

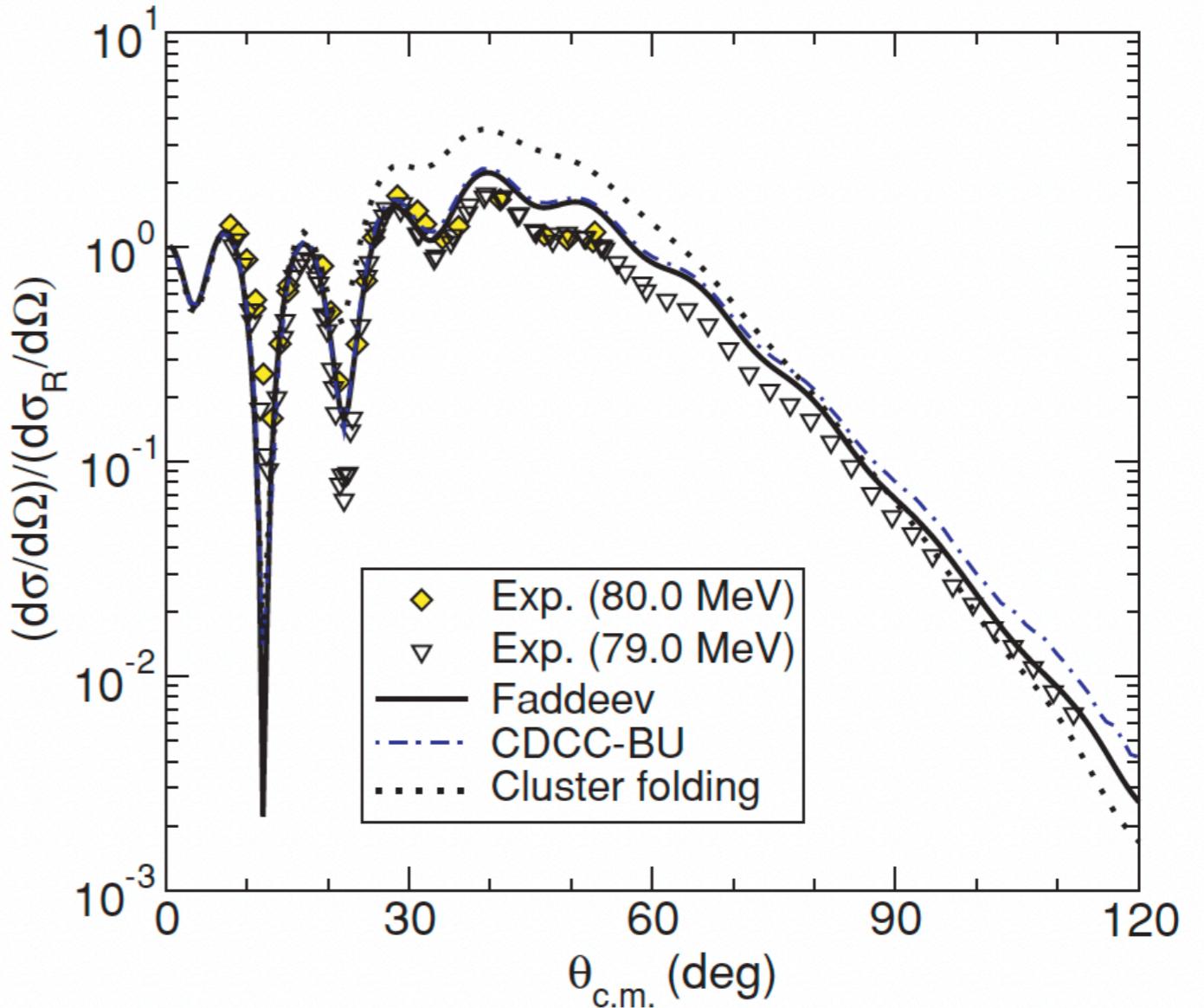


FIG. 6. (Color online) Elastic cross section for deuterons on  ${}^{58}\text{Ni}$  at  $E_d = 80$  MeV: the solid line corresponds to the exact three-body results and the dash-dotted line to CDCC results. The experimental data at 80 MeV (diamonds) are from Ref. [23], and those at 79 MeV (triangles) are from Ref. [24]. The dotted line is the CDCC calculation suppressing the coupling to the deuteron continuum (see text).

# $p + {}^{11}\text{Be}$ elastic scattering

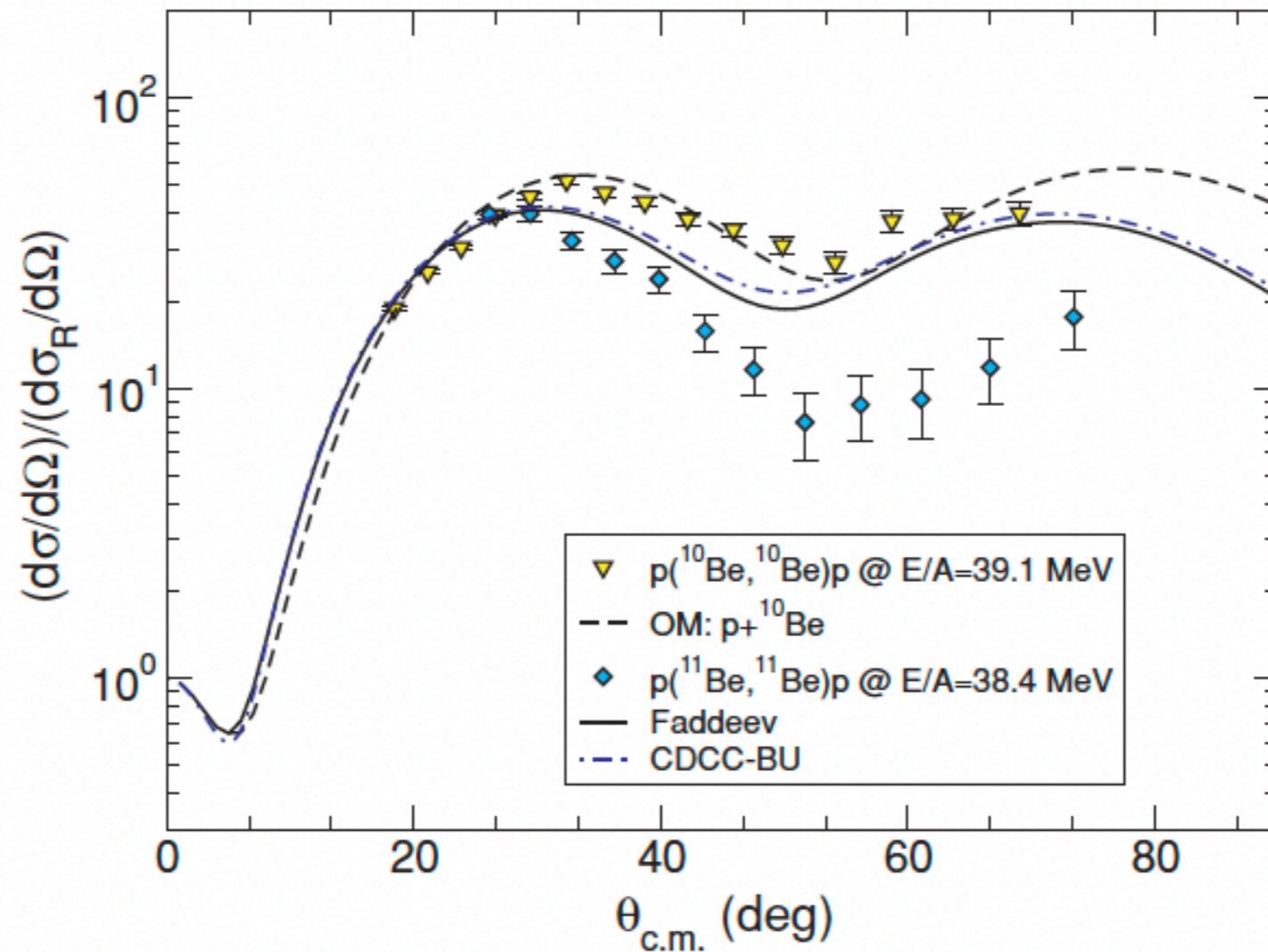


FIG. 7. (Color online)  ${}^1\text{H}({}^{11}\text{Be}, {}^{11}\text{Be})p$  elastic cross section at  $E_{\text{Lab}}/A = 38.4 \text{ MeV}$ . The solid line corresponds to exact three-body results while the dash-dotted line to CDCC. The dashed line corresponds to an optical potential fit to the corresponding  ${}^{10}\text{Be}-p$  data of Ref. [25] shown by the triangles. The diamonds correspond to  ${}^{11}\text{Be}-p$  elastic data of Ref. [25].

# $p(^{11}\text{Be}, ^{10}\text{Be})d$ transfer reaction

CDCC-TR\*

$$T_{\text{prior}} = \langle \Psi_f^{(-)} | U_{xt} + U_{ct} - U_{pt} | \phi_p \chi_p \rangle$$

Disagreement may  
Indicate that here  
CDCC method can  
Not reproduce the  
3B wave function

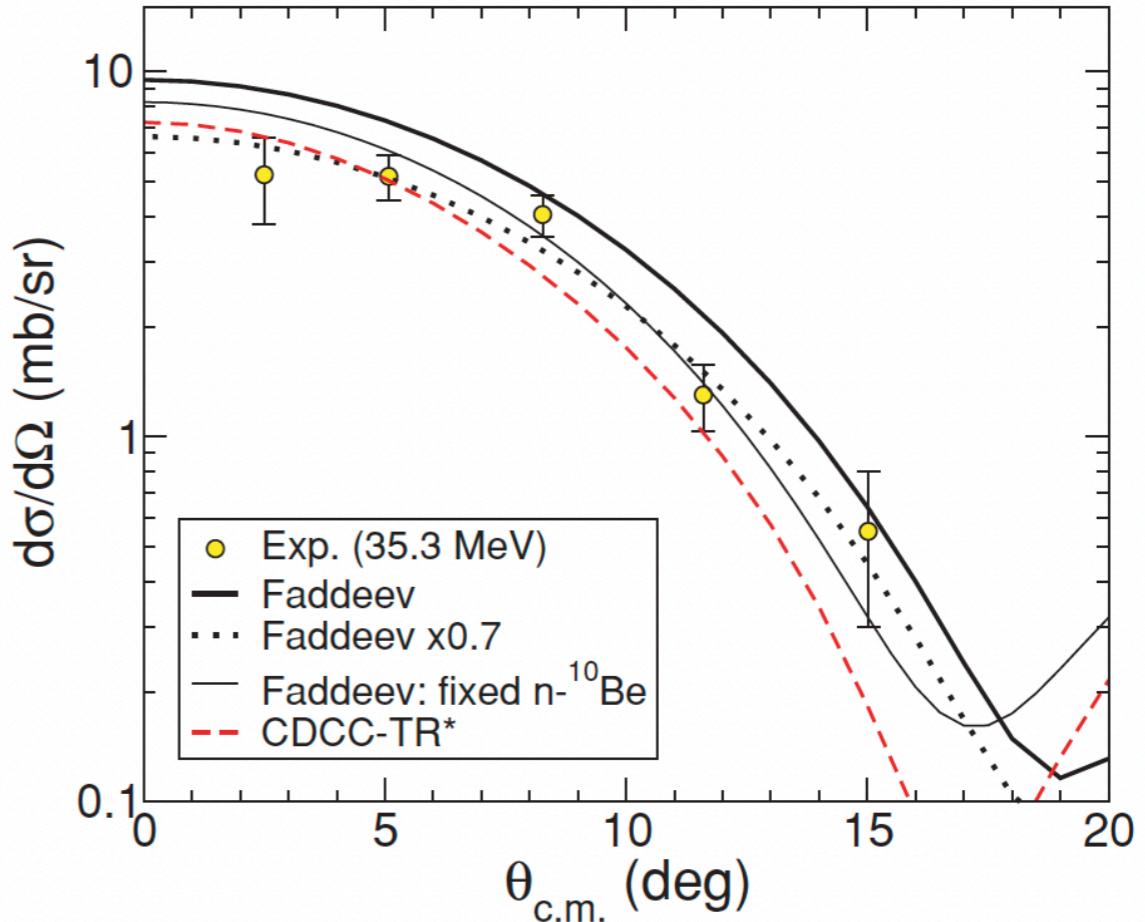


FIG. 8. (Color online) Transfer reaction  $^1\text{H}(^{11}\text{Be}, ^{10}\text{Be})d$  cross section at  $E_{\text{Lab}}/A = 38.4$  MeV. The thick solid line corresponds to the exact three-body result, while the dotted line corresponds to the same calculation multiplied by 0.7. The thin solid line is the exact calculation with a partial-wave independent  $n-^{10}\text{Be}$  interaction. The latter is to be compared with the CDCC-TR\* calculation (dashed line), as explained in the text. The experimental data are from Ref. [39] at  $E_p = 35.3$  MeV.

# $p(^{11}\text{Be}, ^{10}\text{Be})pn$ breakup reaction

- 1, CDCC is not well converged
- 2, CDCC can only Reproduce the Shape

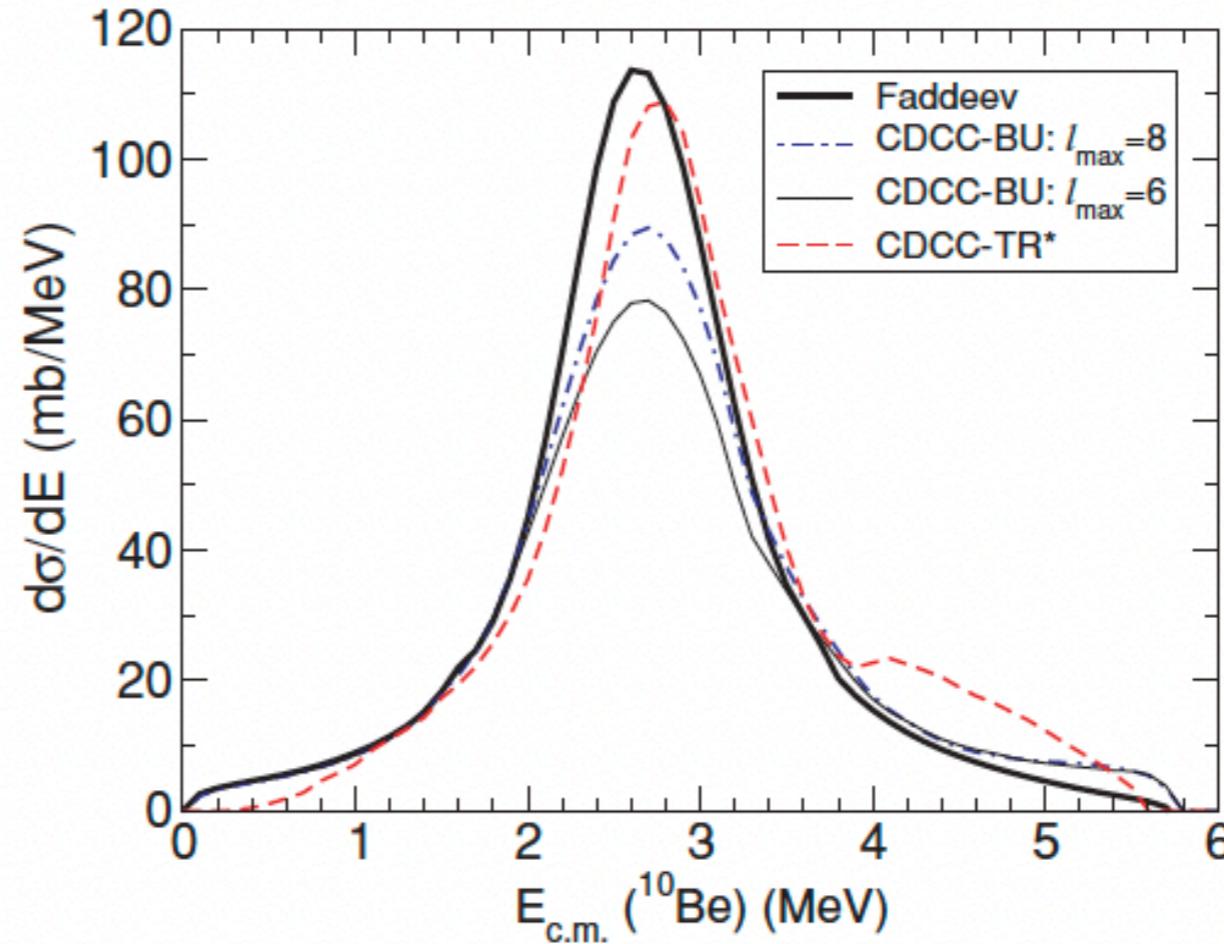


FIG. 9. (Color online) Semi-inclusive differential cross section for the reaction  $^1\text{H}(^{11}\text{Be}, ^{10}\text{Be})pn$ , at  $E_{\text{Lab}}/A = 38.4$  MeV, vs  $^{10}\text{Be}$  center of mass energy. The thick solid line corresponds to exact three-body results, the dashed line to CDCC-TR\*, the dash-dotted line and the thin solid line to CDCC-BU with  $l_{\max} = 8$  and  $l_{\max} = 6$ , respectively.

# $p + {}^{11}\text{Be}$ elastic scattering

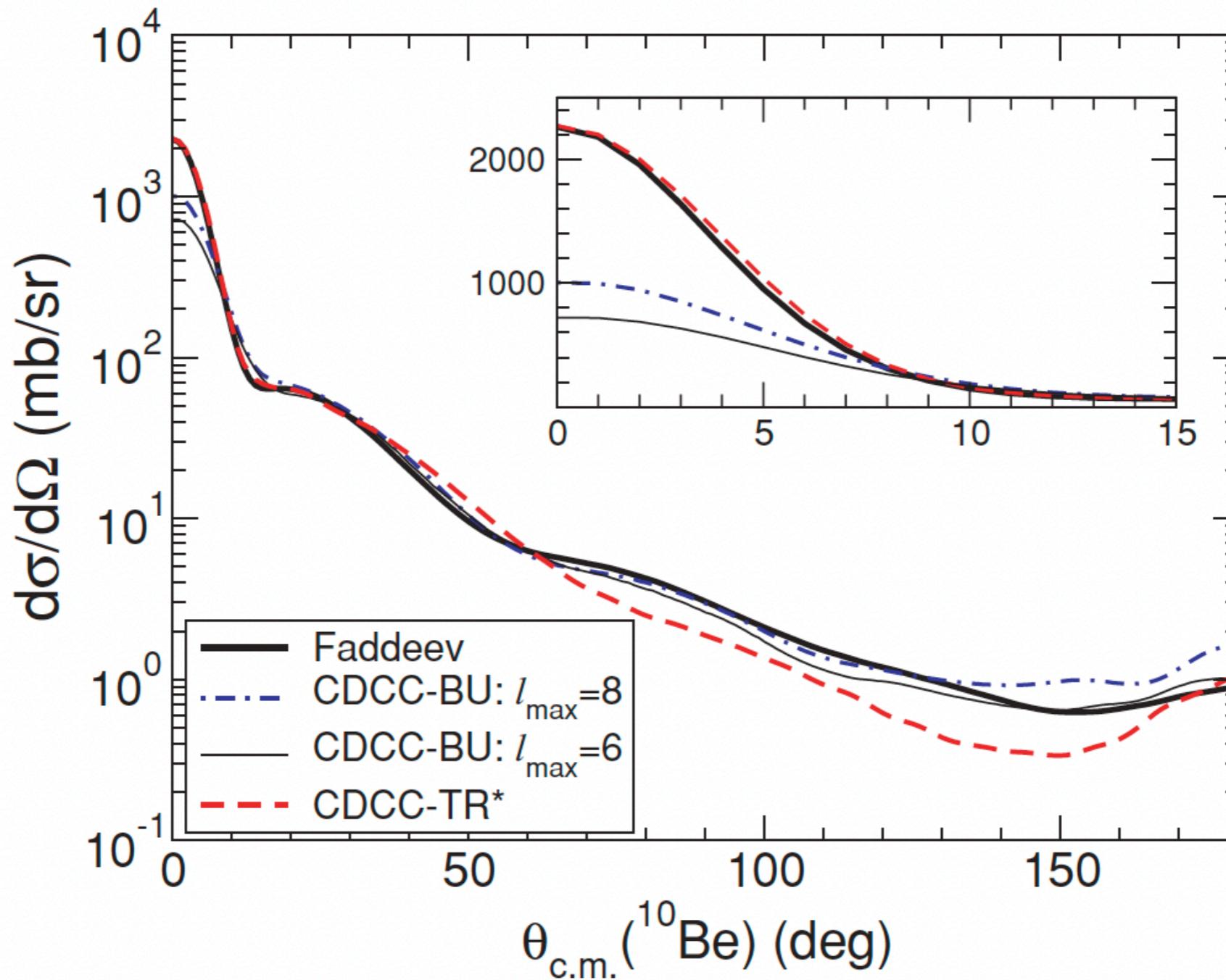


FIG. 10. (Color online) Same as Fig. 9, but showing  ${}^{10}\text{Be}$  angular distribution after energy integration.

# elastic scattering involving deuteron

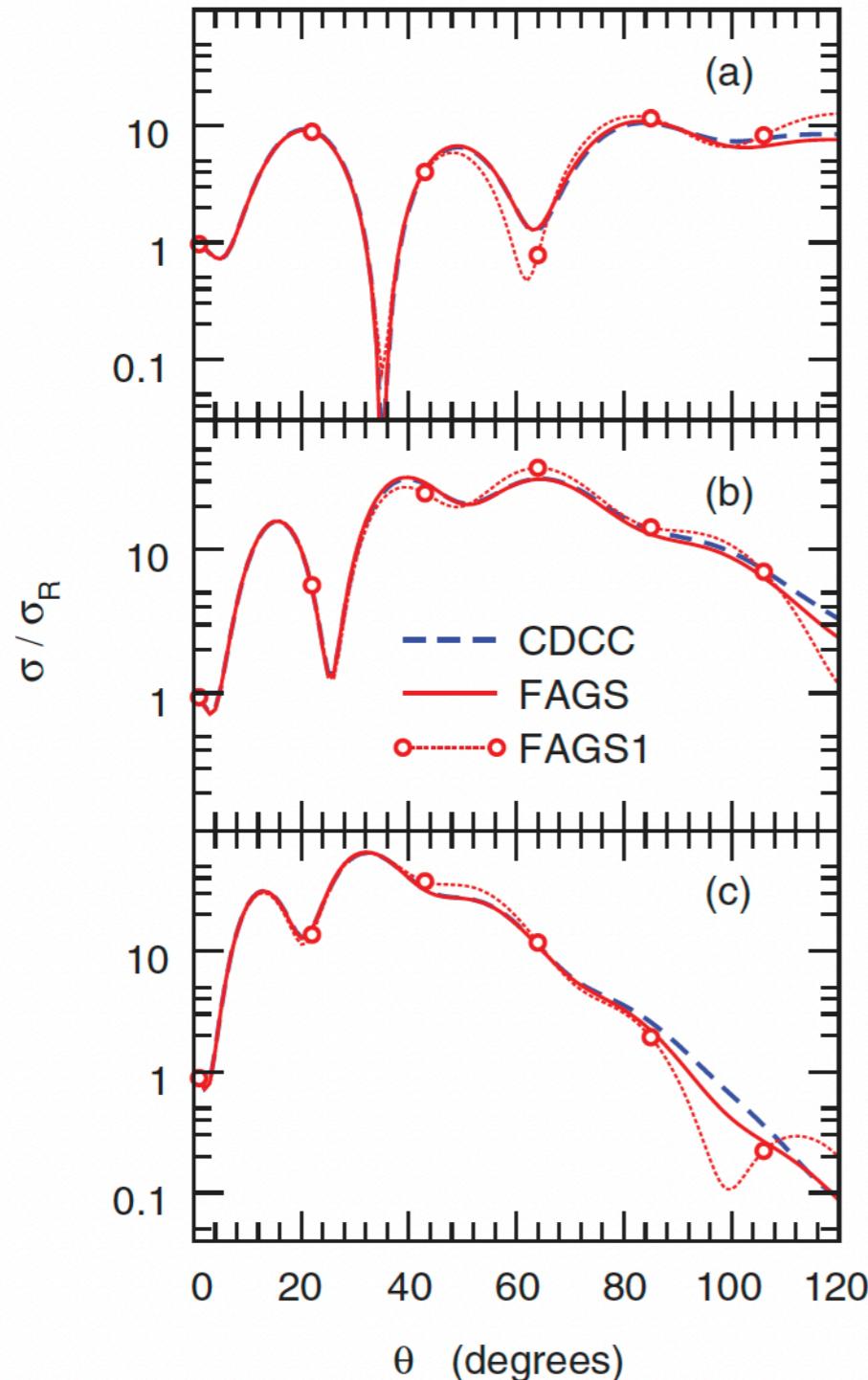


FIG. 2. (Color online) Elastic cross section for  $d + ^{10}\text{Be}$ :  
(a)  $E_d = 21.4$  MeV, (b)  $E_d = 40.9$  MeV, and (c)  $E_d = 71$  MeV.

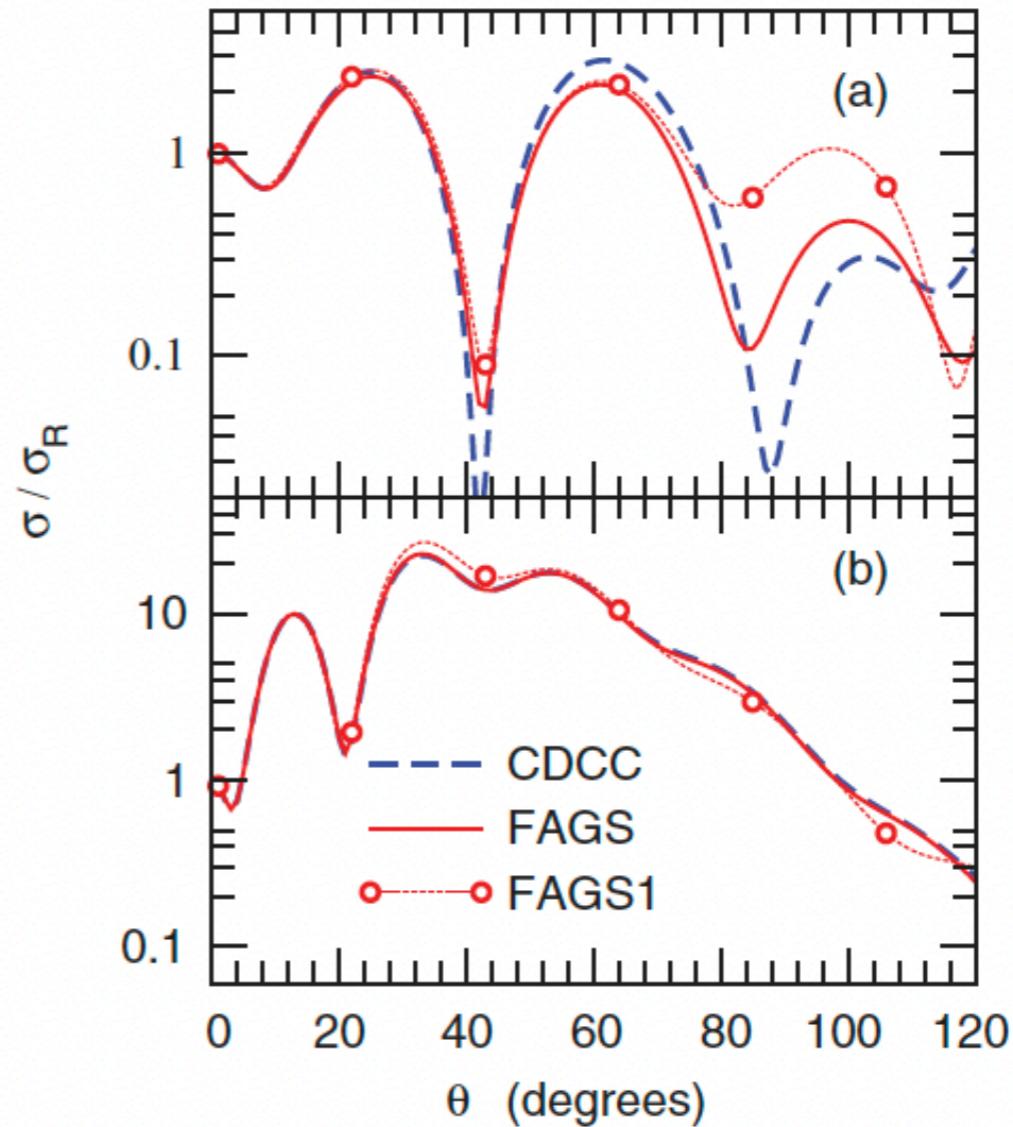


FIG. 3. (Color online) Elastic cross section for  $d + ^{12}\text{C}$ :  
(a)  $E_d = 12$  MeV and (b)  $E_d = 56$  MeV.

# elastic scattering involving deuteron

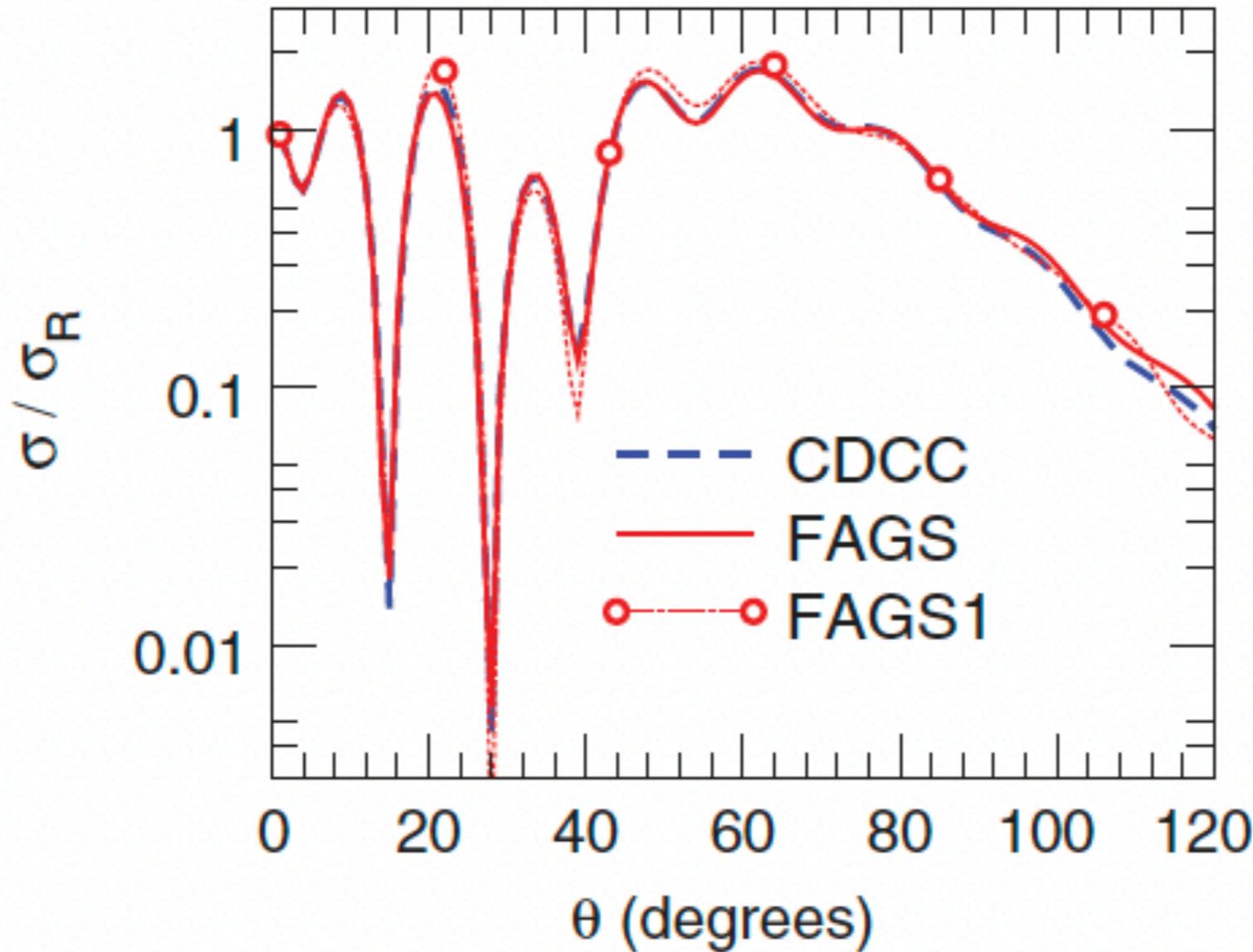


FIG. 4. (Color online) Elastic cross section for  $d + {}^{48}\text{Ca}$  at  $E_d = 56$  MeV.

# transfer scattering

Significant discrepancies  
Appear at higher energy

Two sets of auxiliary potential are included  
To test how much of  
The discrepancy can be  
Attributed to the choice of  
Optical potential

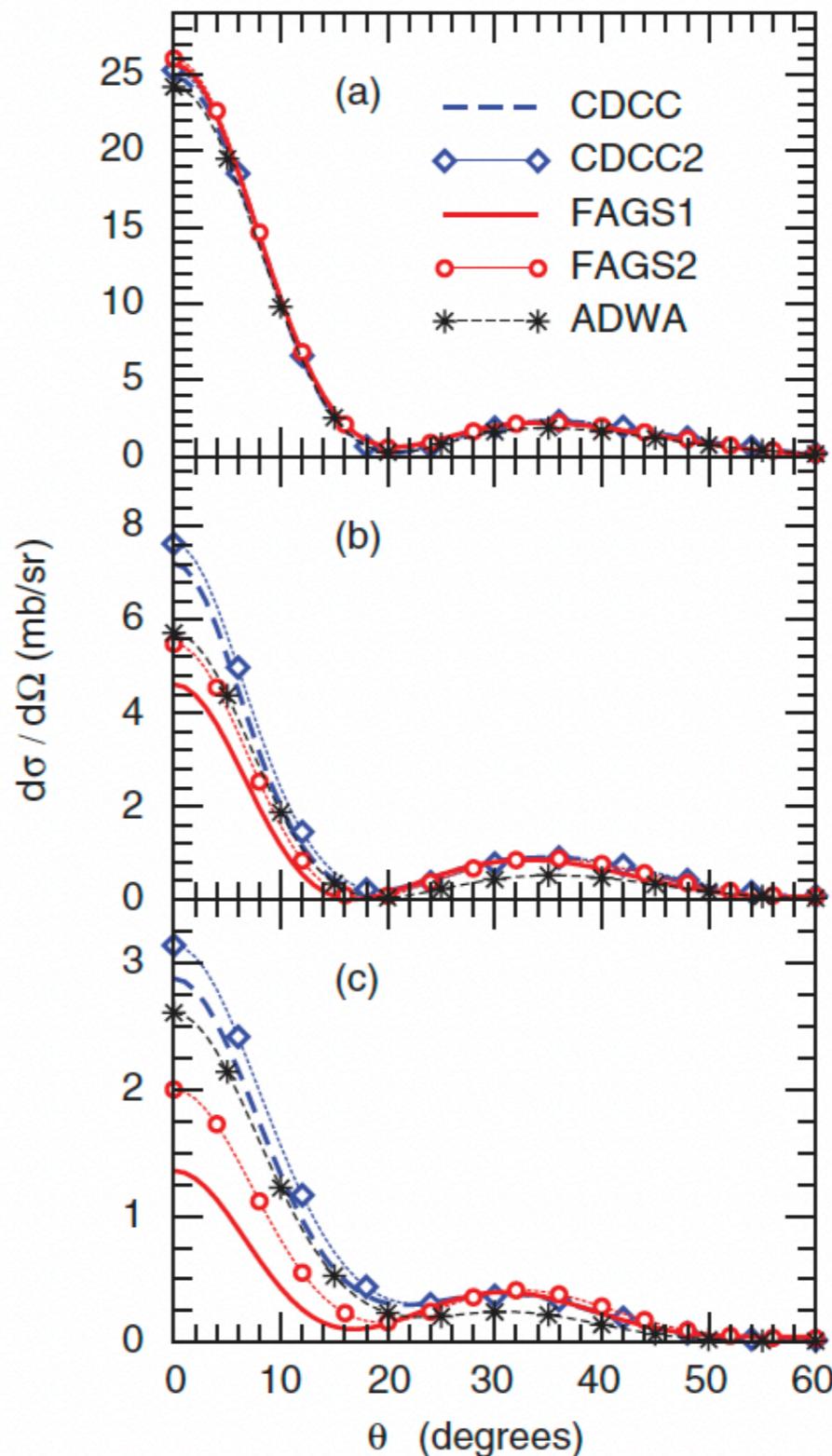


FIG. 5. (Color online) Angular distribution for  $^{10}\text{Be}(d, p)^{11}\text{Be}$ :  
(a)  $E_d = 21.4 \text{ MeV}$ , (b)  $E_d = 40.9 \text{ MeV}$ , and (c)  $E_d = 71 \text{ MeV}$ .

# transfer scattering

Significant discrepancies  
Appear at higher energy

No dependency of  
The selection of  
The optical potential

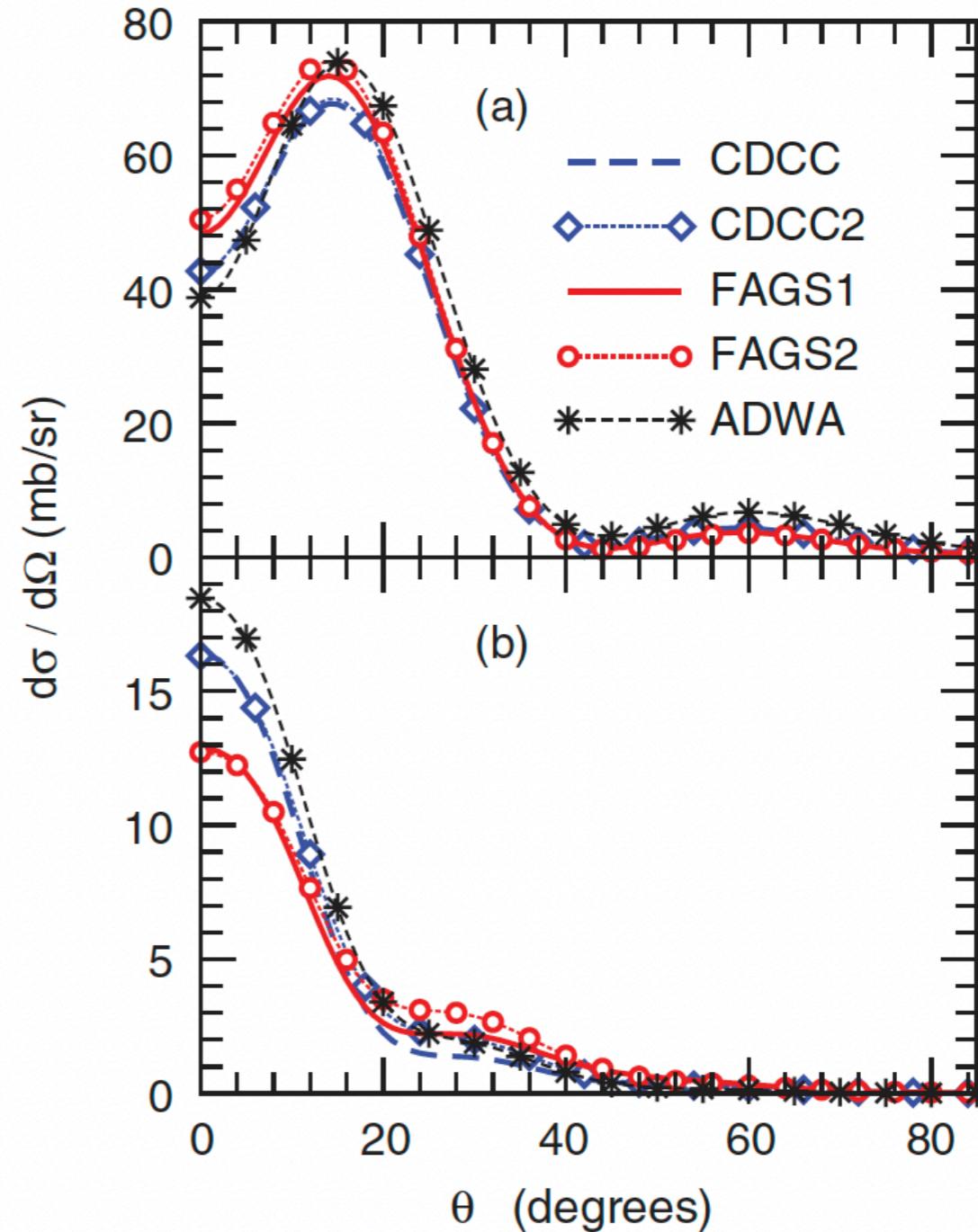


FIG. 6. (Color online) Angular distribution for  $^{12}\text{C}(d, p)^{13}\text{C}$ :  
(a)  $E_d = 12 \text{ MeV}$  and (b)  $E_d = 56 \text{ MeV}$ .

# transfer scattering

Discrepancies between  
Optical potentials  
And Faddeev/CDCC

Hard to make conclusion

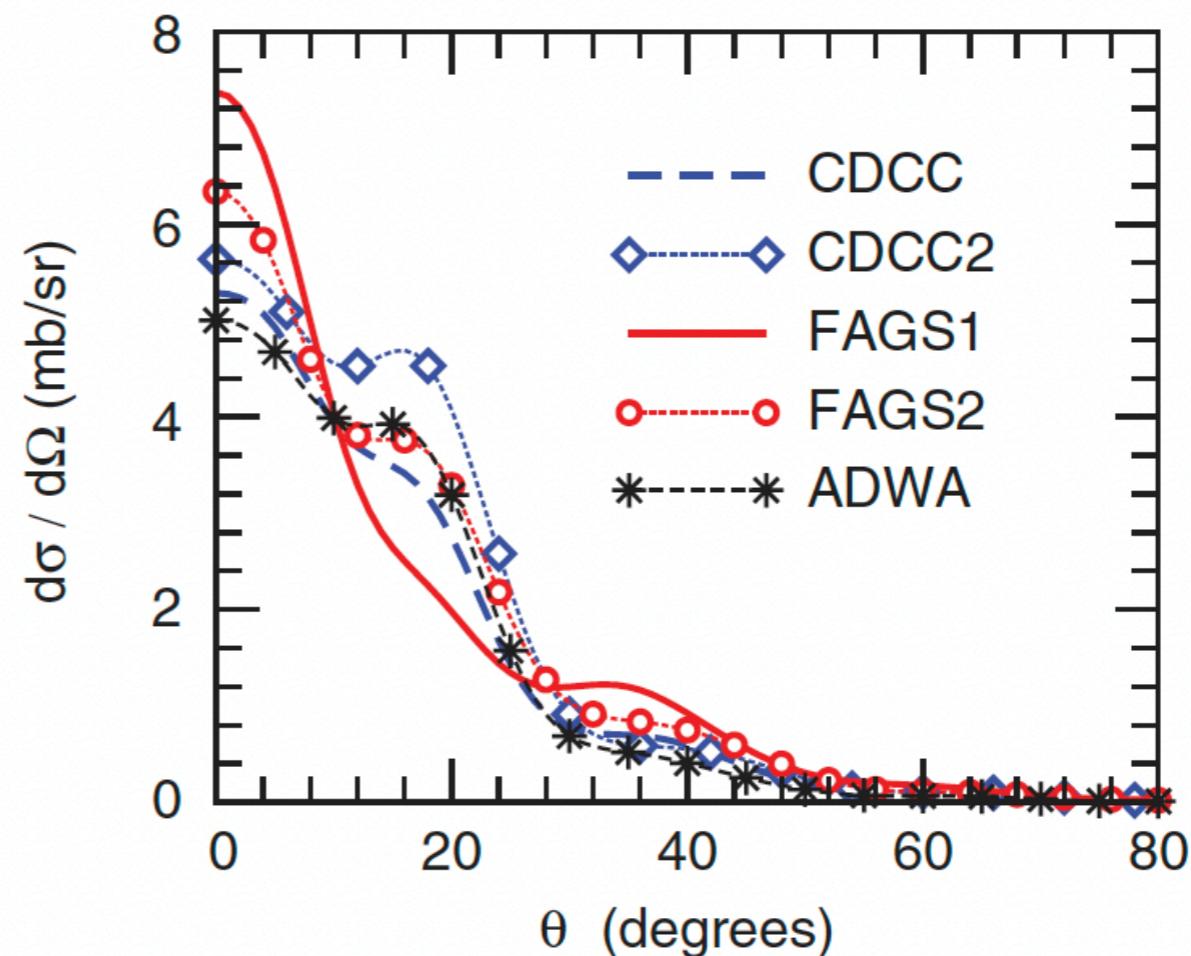


FIG. 7. (Color online) Angular distribution for  $^{48}\text{Ca}(d, p)^{49}\text{Ca}$  at  $E_d = 56$  MeV.

# deuteron breakup

Significant discrepancies  
Appears  
Even taking into  
Account the error estimated  
With model space truncation

This discrepancy  
Is removed when  
Energy goes up

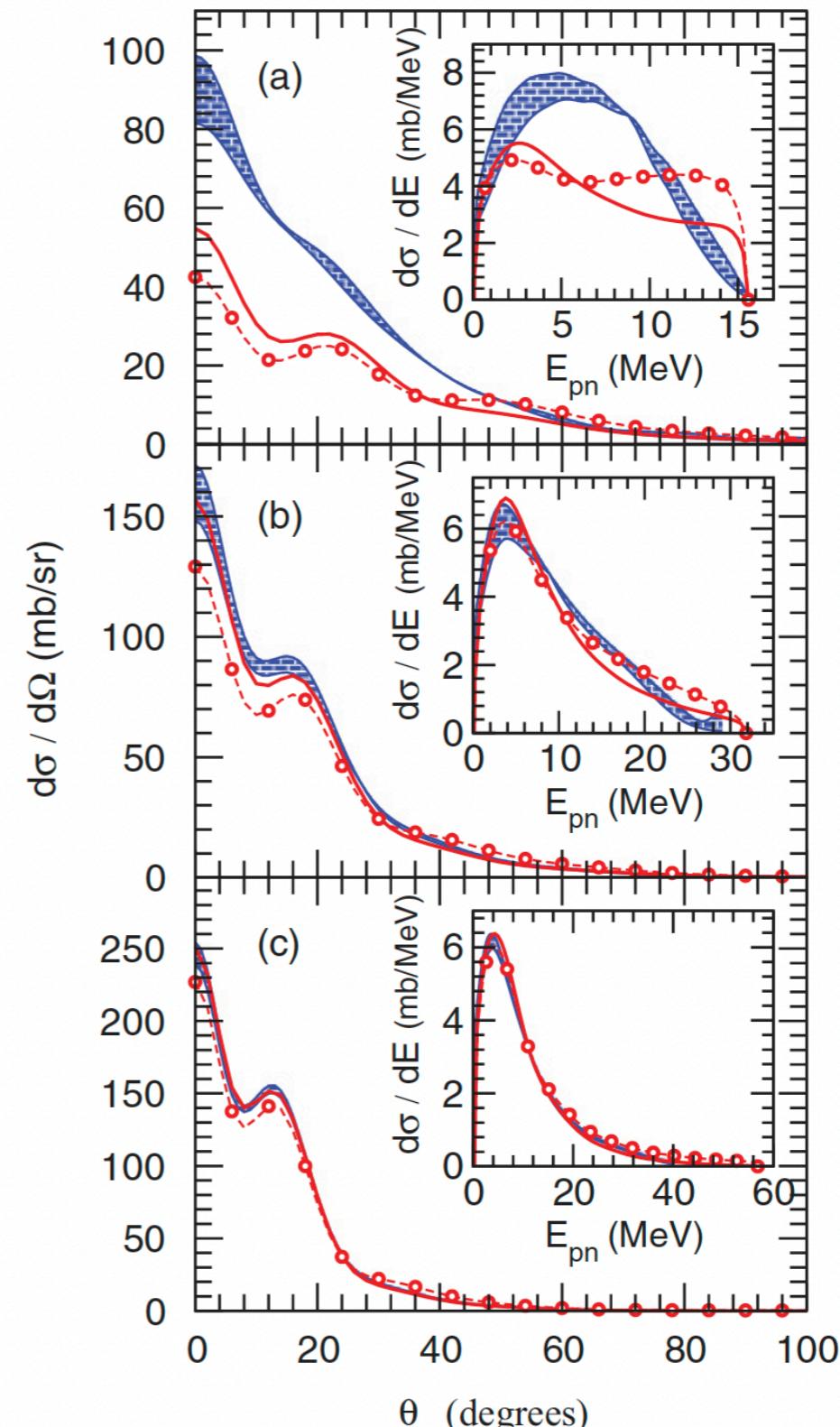


FIG. 8. (Color online) Breakup distributions for the  $^{10}\text{Be}(d, pn)^{10}\text{Be}$  reaction at (a)  $E_d = 21$  MeV, (b)  $E_d = 40.9$  MeV, and (c)  $E_d = 71$  MeV. Results for CDCC (hatched band), FAGS (solid), and FAGS1 (circles).

# deuteron breakup

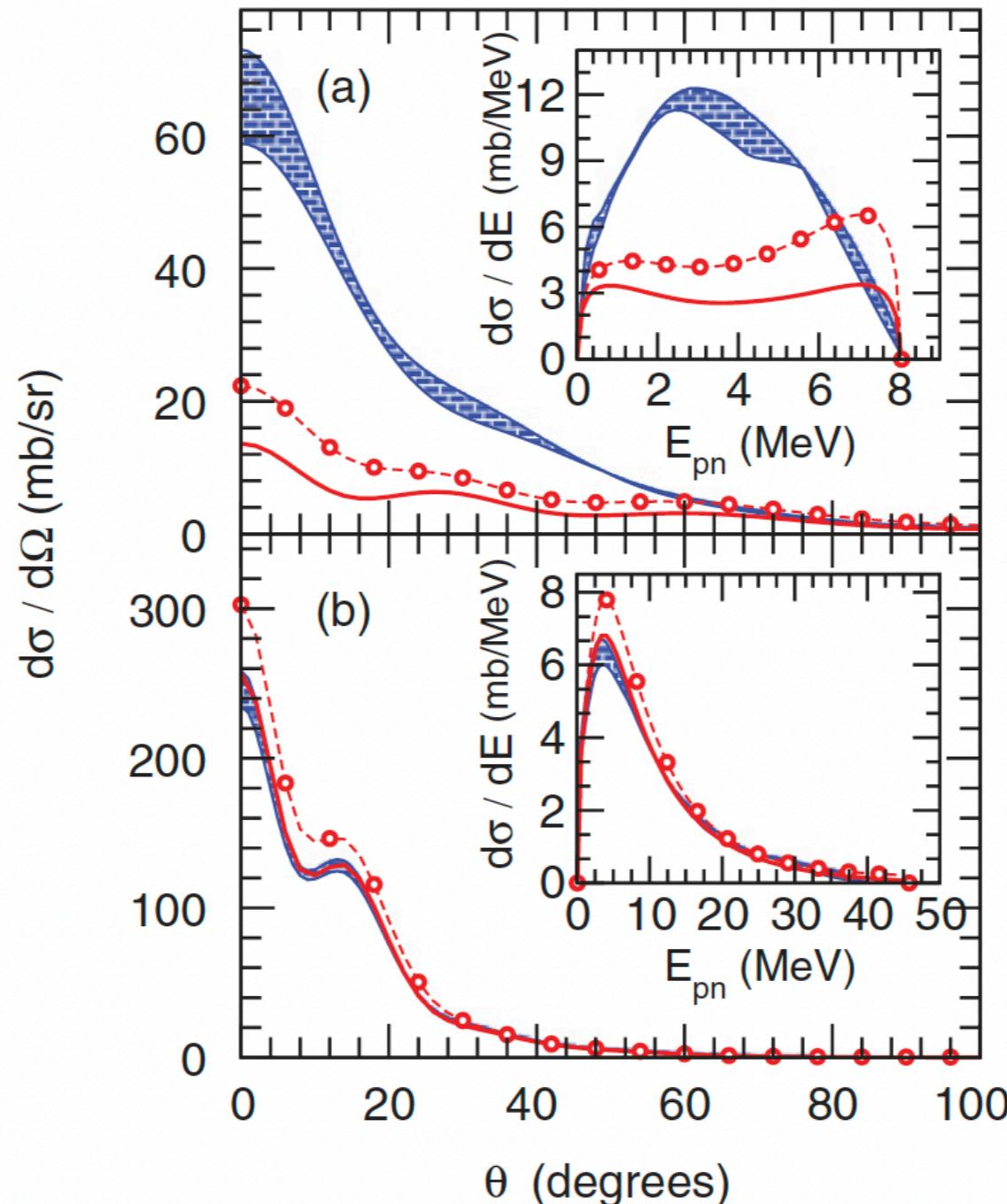


FIG. 9. (Color online) Breakup distributions for the  $^{12}\text{C}(d, pn)^{12}\text{C}$  reaction at (a)  $E_d = 12 \text{ MeV}$  and (b)  $E_d = 56 \text{ MeV}$ . Results for CDCC (hatched band), FAGS (solid), and FAGS1 (circles).

# deuteron breakup

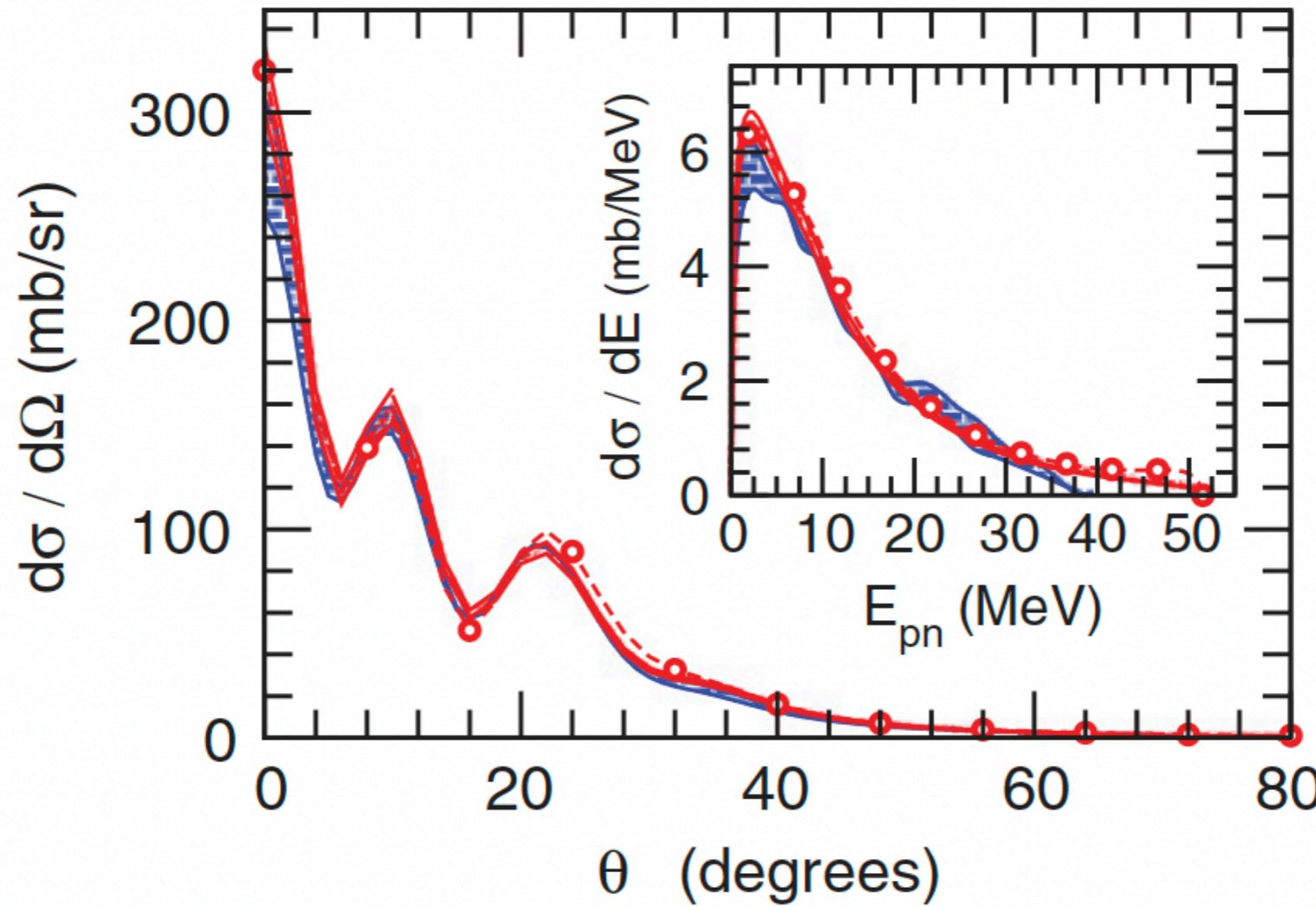


FIG. 10. (Color online) Breakup distributions for the  $^{48}\text{Ca}(d, pn)^{48}\text{Ca}$  reaction at  $E_d = 56$  MeV. Results for CDCC (hatched band), FAGS (solid), and FAGS1 (circles).

# closed channel

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open channel

$$\chi_c \rightarrow U_{L,\eta_i}^{(-)}(K_i R) \delta_{cc_0} - \sqrt{K_0/K_i} S_{cc_0} U_{L,\eta_i}^{(+)}(K_i R)$$

closed channel

$$\chi_c \rightarrow -S_{cc_0} W_{-\eta_i, L+1/2}(-2iK_i R)$$

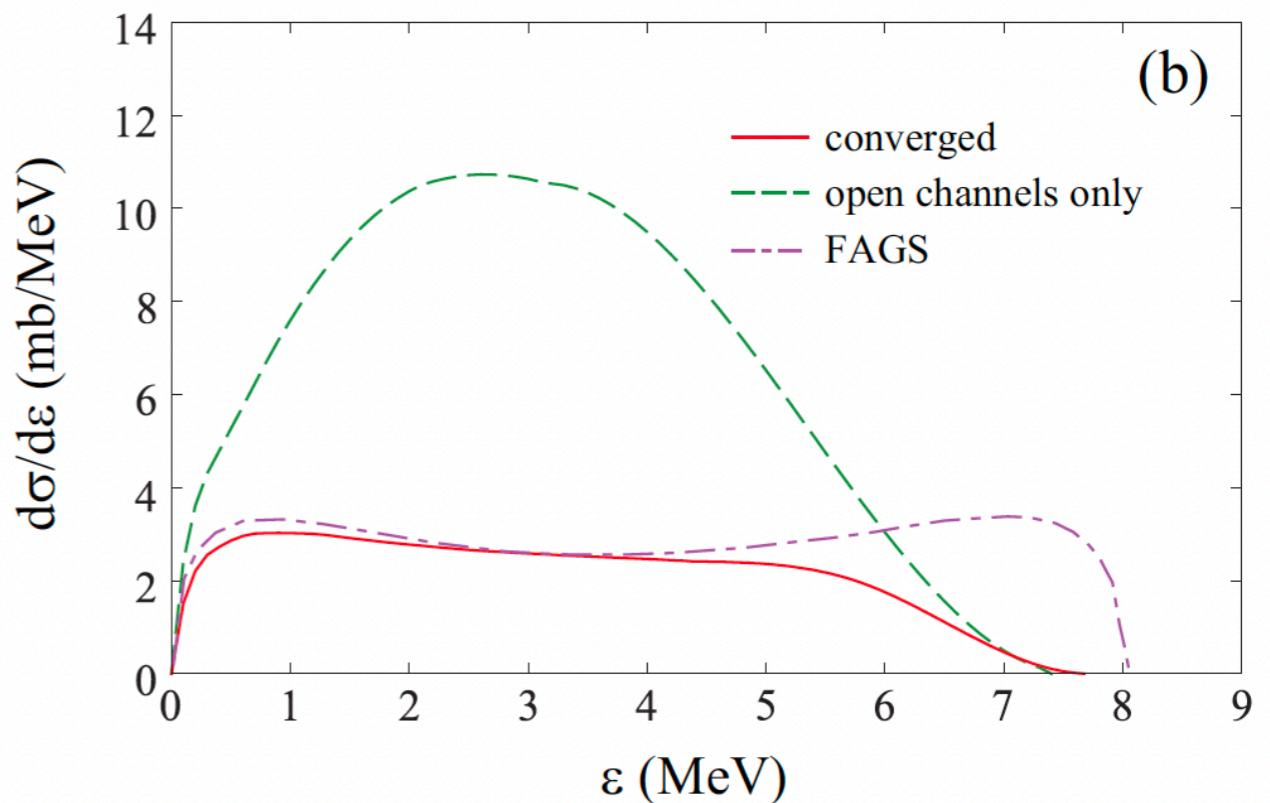
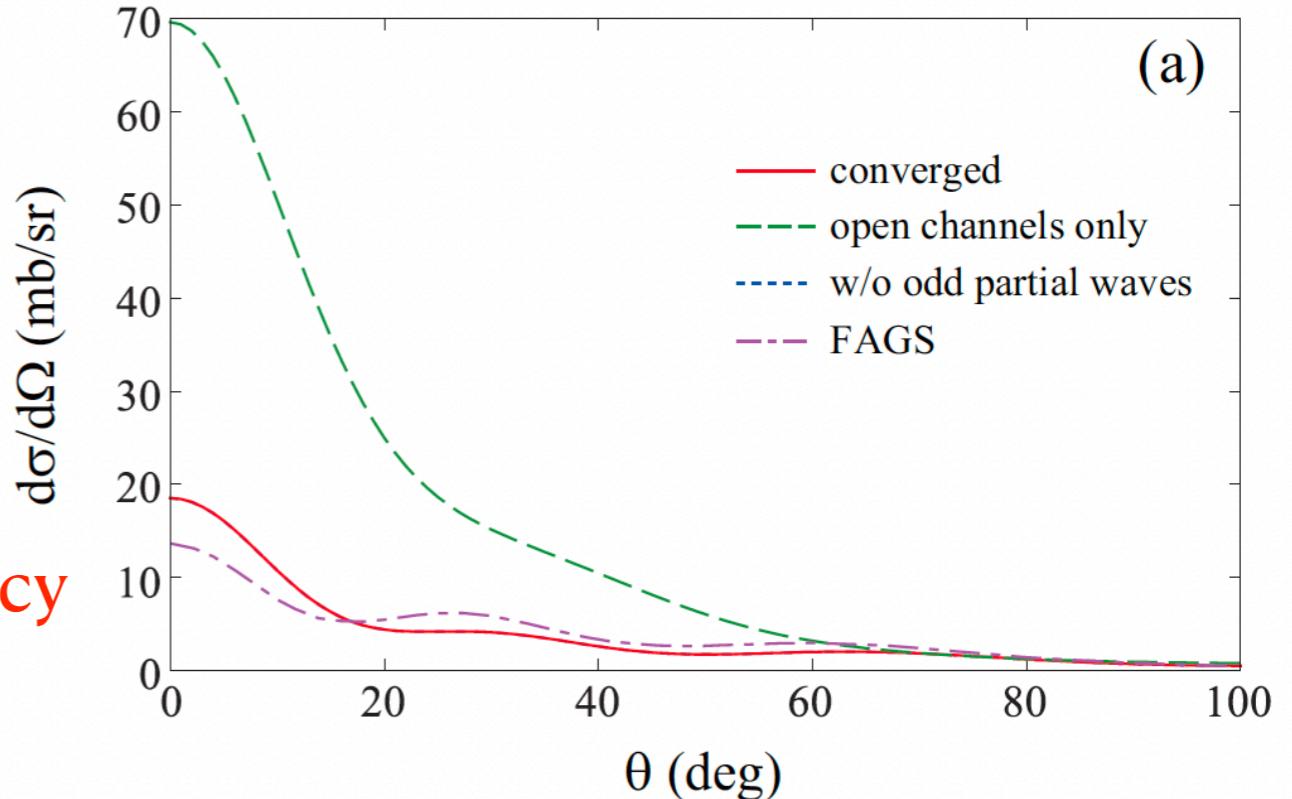
For closed channels  $S$ –matrix is not related to observables directly.

# closed channel

All channel: converged and match well

only open channel: significant discrepancy

FIG. 1. (a) Angular distribution and (b) breakup energy distribution of the elastic breakup cross section for  $^{12}\text{C}(d, pn)^{12}\text{C}$  at  $E_d = 12$  MeV. The solid, dashed, and dash-dotted lines in each panel show the converged CDCC result, the result of the CDCC method calculated with including only the open channels, and the result of FAGS theory taken from Ref. [11], respectively. The dotted line in (a) is the same as the solid line but omitting the odd partial waves between  $p$  and  $n$ .



# closed channel

Same calculation but for  
 $^{10}\text{Be}(d, pn)^{10}\text{Be}$  @ 21 MeV

Well converged when  
Including all channels

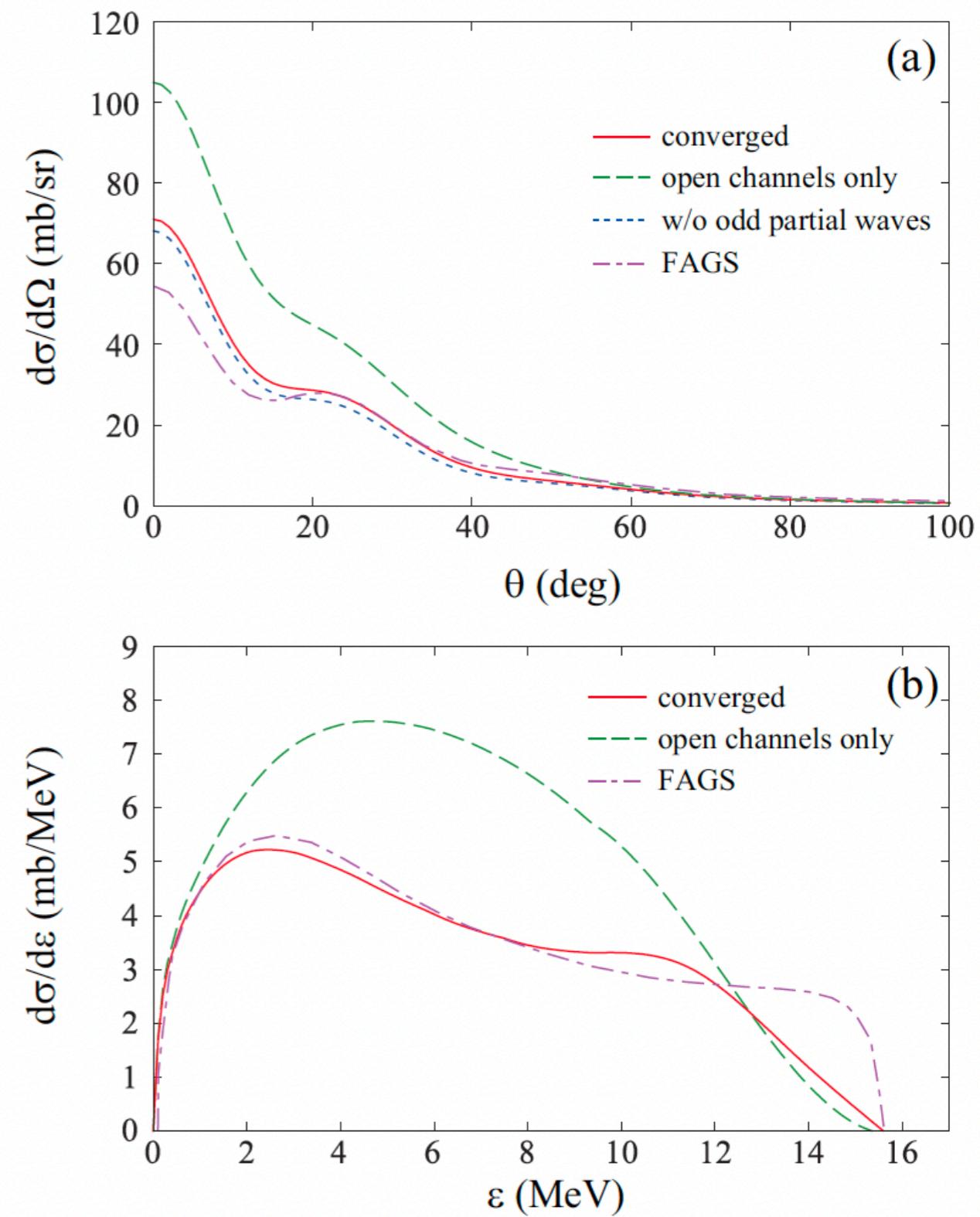


FIG. 3. Same as Fig. 1 but for  $^{10}\text{Be}(d, pn)^{10}\text{Be}$  at  $E_d = 21$  MeV.

# Summary

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1. CDCC is able to provide a good approximation to FAGS for elastic scattering.
2. CDCC is a very good approximation of FAGS at reactions around 10 MeV/u, but not so good for higher beam energies.
3. Significant discrepancy occurs when evaluating break up reactions. But when energy goes higher, CDCC reproduces the FAGS results well.
4. Inclusion of closed channels may solve the above problem.