

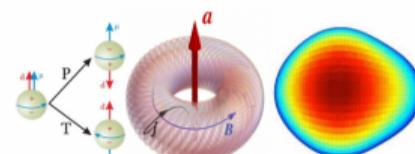
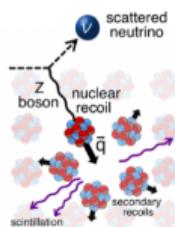
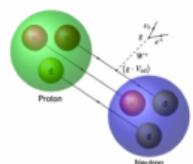
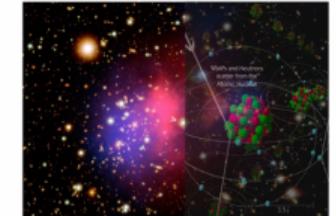
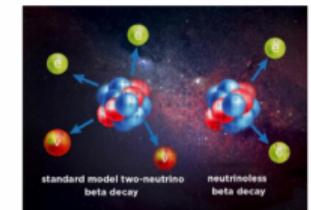
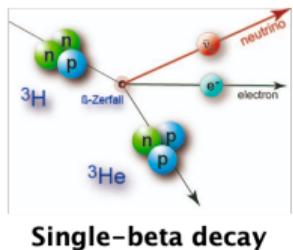
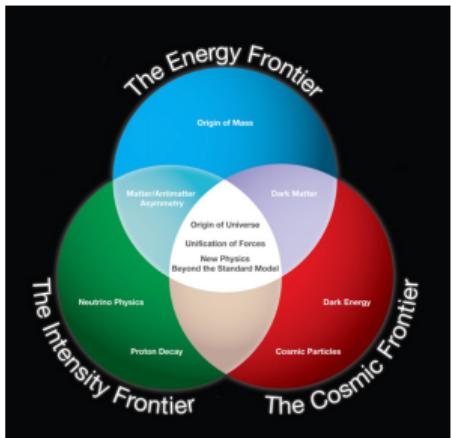
Modeling neutrinoless double-beta decay with operators from chiral effective field theory

Jiangming Yao (尧江明)

中山大学物理与天文学院

同济大学物理科学与工程学院, 上海, 2023年10月25日

- 1 Introduction to neutrinoless double-beta($0\nu\beta\beta$) decay
- 2 Modeling nuclear matrix elements of $0\nu\beta\beta$ decay
- 3 Recent studies with operators from (chiral) effective field theory
- 4 Summary and perspectives



- Three frontiers: for new physics
- Atomic nuclei: low-energy probes
- Fundamental interactions and symmetries.
- All about Nuclear Matrix Elements (NME)

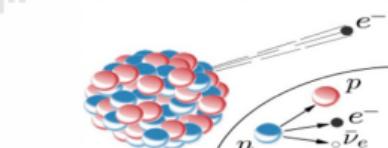
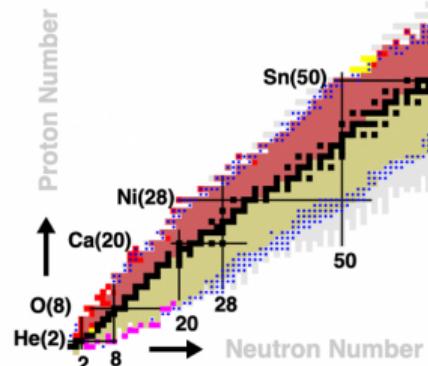
Stability of atomic nuclei against single- β decay

Nuclear Chart: decay mode of the ground state nuclide(NUBASE2020)

- Stable
- β^- decay
- β^+ or EC decay
- α decay
- Spontaneous fission
- Neutron decay
- Proton decay
- Decay mode: ?

■ Observed nuclide with unknown mass (809)

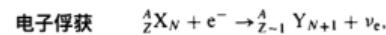
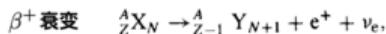
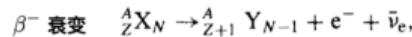
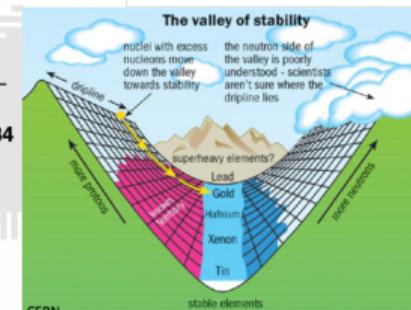
Stable nuclei: ~300
Known nuclei: 3000+
Possible existing nuclei: 8000+



[1] F.G. Kondev *et al.*, Chinese Phys. C. 45 (2021) 030001.

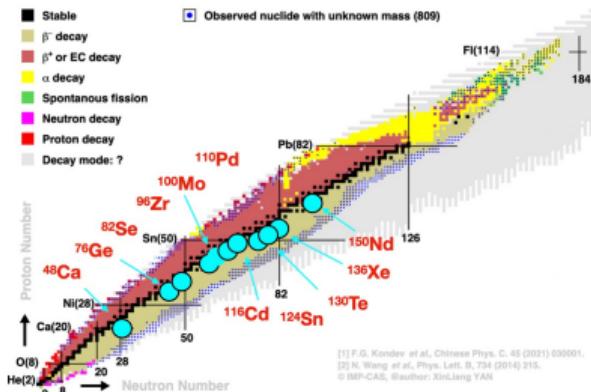
[2] N. Wang *et al.*, Phys. Lett. B, 734 (2014) 215.

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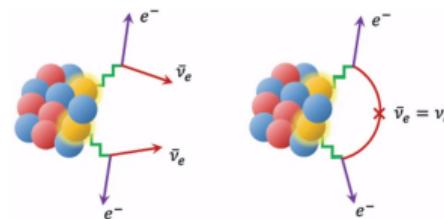
A special decay mode: $0\nu\beta\beta$ decay

Nuclear Chart: decay mode of the ground state nuclide(NUBASE2020)

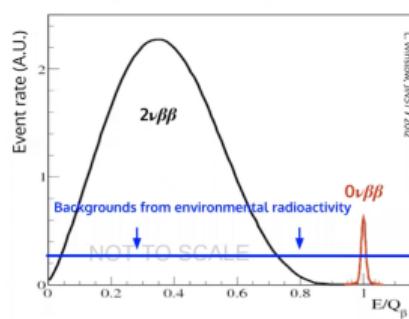
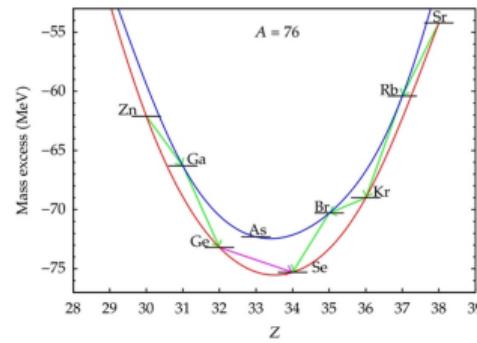


- The two modes of $\beta^- \beta^-$ decay:

$$(A, Z) \rightarrow (A, Z + 2) + 2e^- + (2\bar{\nu}_e)$$



- Kinetic energy spectrum of electrons



Neutrino oscillation

- From mass to flavor states

$$|\nu_\alpha\rangle = \sum_{j=1}^{N=3} U_{\alpha j}^* |\nu_j\rangle.$$

- $\Delta m_{ij}^2 (\neq 0)$, and $\theta_{ij} (\neq 0)$.

Open questions

- The nature of neutrinos.
- Neutrino mass m_j and its origin.

The observation of $0\nu\beta\beta$ decay would provide answers.

If $0\nu\beta\beta$ decay is driven by exchanging light massive Majorana neutrinos:

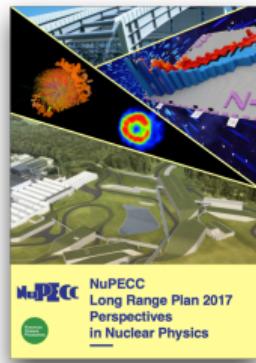
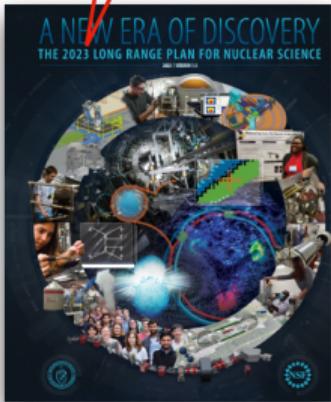
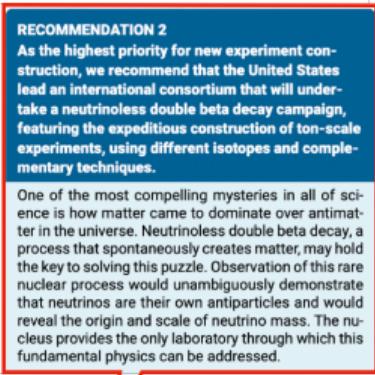
$$\langle m_{\beta\beta} \rangle \equiv \left| \sum_{j=1}^3 U_{ej}^2 m_j \right| = \left[\frac{m_e^2}{g_A^4 G_{0\nu} T_{1/2}^{0\nu} |M^{0\nu}|^2} \right]^{1/2}$$

- U_{ej} : elements of the PMNS matrix
- $G_{0\nu}$: phase-space factor
- $M^{0\nu}$: the nuclear matrix element

$$M^{0\nu} = \langle \Psi_F | \hat{O}^{0\nu} | \Psi_I \rangle$$

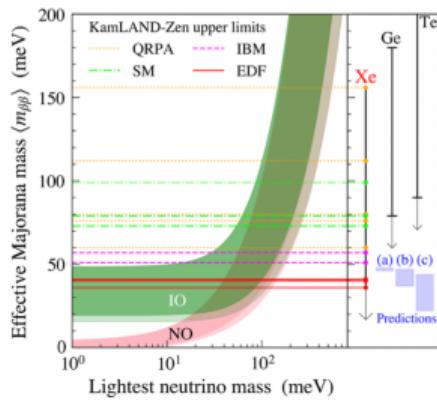
- Transition operator: $\hat{O}^{0\nu}$
- Nuclear many-body wfs: $|\Psi_{I/F}\rangle$

Current and next-generation of experiments



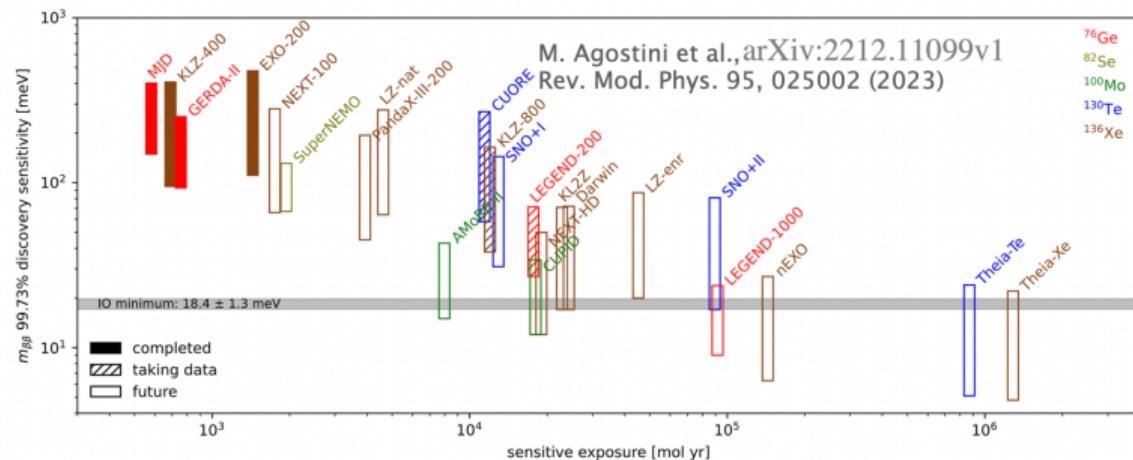
Constraints on neutrino mass from $0\nu\beta\beta$ decay

Isotope	$G_{0\nu}$ [10^{-14} yr^{-1}]	$M^{0\nu}$ [min, max]	$T_{1/2}^{0\nu}$ [yr]	$\langle m_{\beta\beta} \rangle$ [meV]	Experiments References
⁴⁸ Ca	2.48	[0.85, 2.94]	$> 5.8 \cdot 10^{22}$	[2841, 9828]	PRC78, 058501 (2008)
⁷⁶ Ge	0.24	[2.38, 6.64]	$> 1.8 \cdot 10^{26}$	[73, 180]	GERDA : PRL125, 252502(2020)
⁸² Se	1.01	[2.72, 5.30]	$> 4.6 \cdot 10^{24}$	[277, 540]	CUPID-0 : PRL129, 111801 (2023)
⁹⁶ Zr	2.06	[2.86, 6.47]	$> 9.2 \cdot 10^{21}$	[3557, 8047]	NPA847, 168 (2010)
¹⁰⁰ Mo	1.59	[3.84, 6.59]	$> 1.5 \cdot 10^{24}$	[310, 540]	CUPID-Mo : PRL126, 181802(2021)
¹¹⁶ Cd	0.48	[3.29, 5.52]	$> 2.2 \cdot 10^{23}$	[1766, 2963]	PRD 98, 092007 (2018)
¹³⁰ Te	1.42	[1.37, 6.41]	$> 2.2 \cdot 10^{25}$	[90, 305]	CUORE : Nature 604, 53(2022)
¹³⁶ Xe	1.46	[1.11, 4.77]	$> 2.3 \cdot 10^{26}$	[36, 156]	KamLAND-Zen : PRL130, 051801(2023)
¹⁵⁰ Nd	6.30	[1.71, 5.60]	$> 2.0 \cdot 10^{22}$	[1593, 5219]	NEMO-3: PRD 94, 072003 (2016)



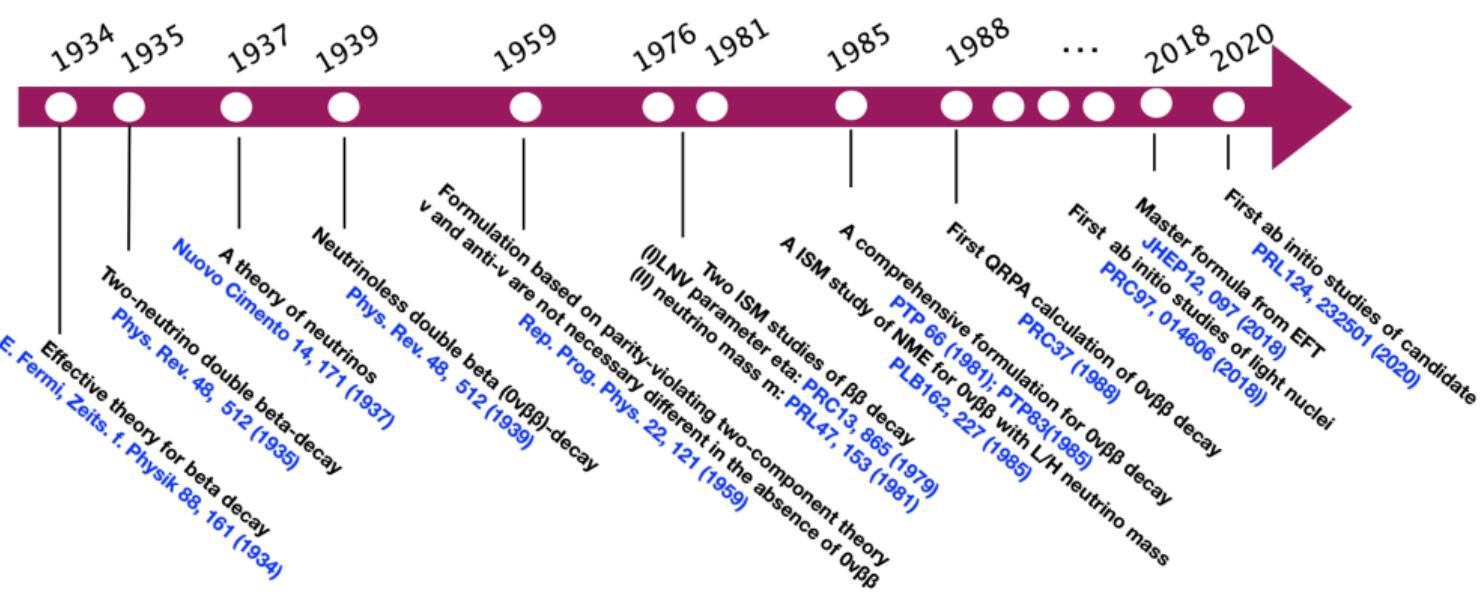
- The neutrino oscillation measurements: $\langle m_{\beta\beta} \rangle \in [20, 50] \text{ meV}$ for the inverted-ordering (IO) case.
- An uncertainty of a factor of about 3 or even more (originated from the NMEs) in the $\langle m_{\beta\beta} \rangle$ determined by $0\nu\beta\beta$ -decay.

Next-generation of experiments

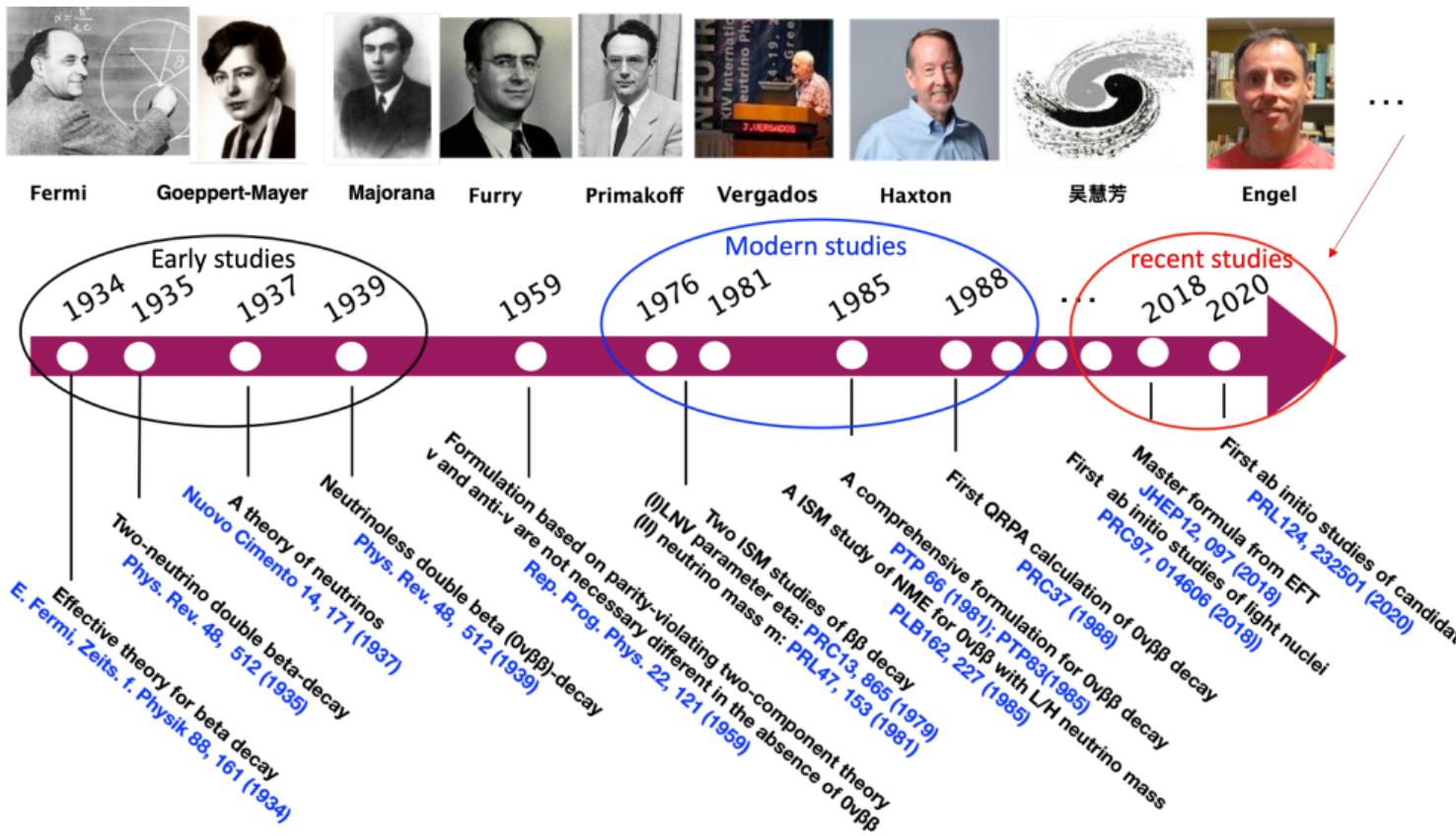


- Lifetime sensitivity of the ton-scale experiments: $> 10^{28}$ yr.
- Covering the entire parameter space for the IO neutrino masses **depending strongly on the employed NME**.

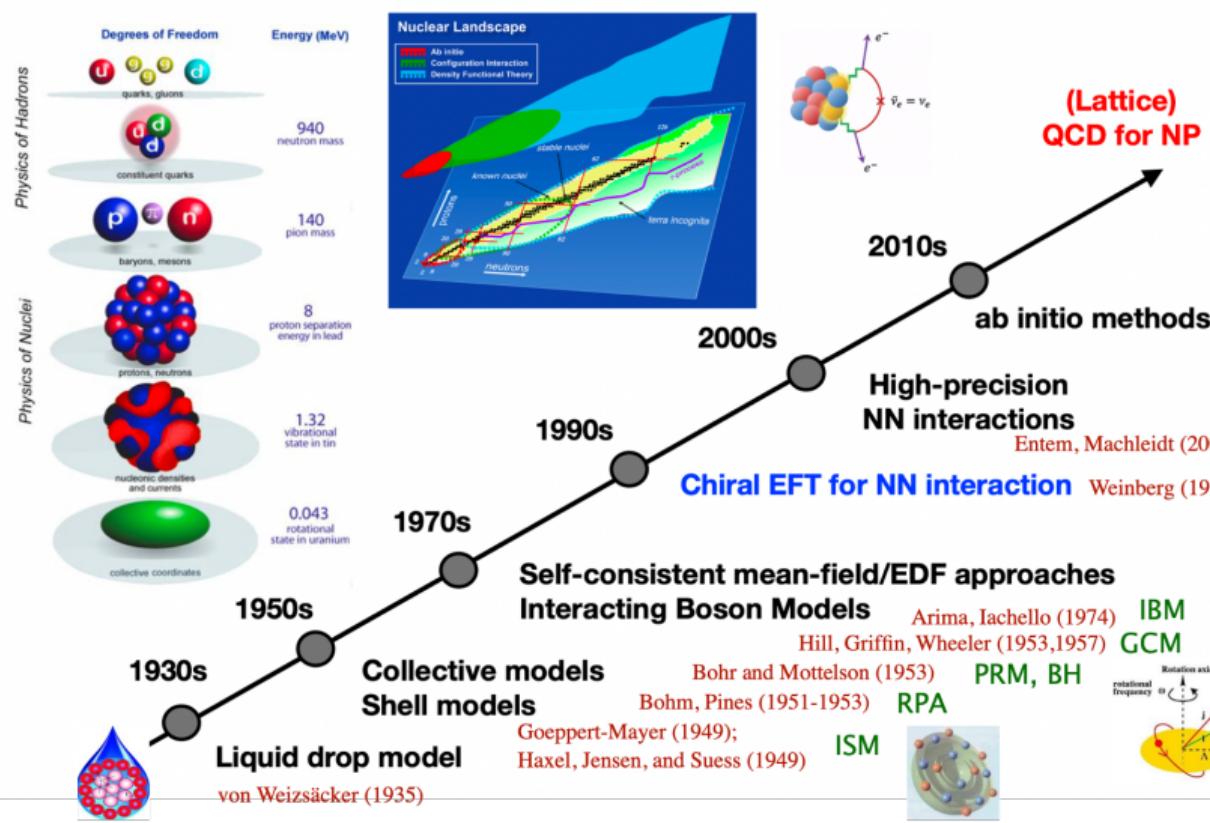
Brief history on modeling the $\beta(\beta\beta)$ decay rate



Brief history on modeling the $\beta(\beta\beta)$ decay rate



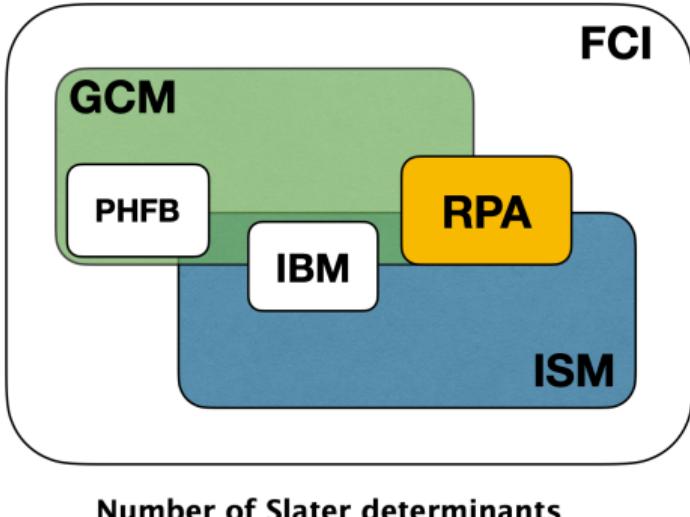
Development of nuclear models



Modern studies with phenom. nuclear forces

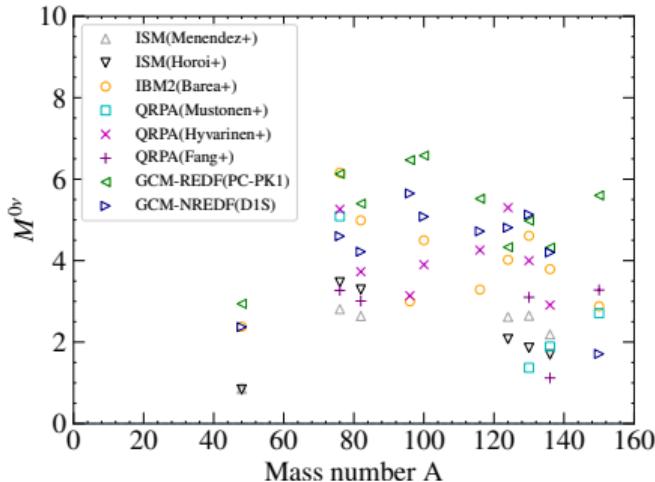
- Interacting shell models (ISM) Vergados (1976), Haxton (1981), H.F.Wu (1985, 1993), Caurier (2008), Menéndez (2009), Horoi (2010), Coraggio (2020)
- Particle-number (and angular-momentum) projected BCS (HFB) with a schematic (PP+QQ) hamiltonian Grotz, Klapdor (1985), Chandra (2008), Rath (2010), Hinohara (2014)
- Quasi-particle random-phase approx. (QRPA) with a G-matrix residual interaction Vogel- 2ν (1986), Engel (1988), Rodin (2003), Faessler (1998), Simkovic (1999), Fang (2010) or EDF Mustonen (2013), Terasaki (2015), Lv(2023), Bai (2023?)
- Interacting Boson Models (IBM) Barea (2009, 2012)
- GCM+EDFs Rodríguez (2010), Song (2014, 2017), Yao (2015)
- Angular momentum projected interacting shell model based on an effective interaction Iwata, Shimizu (2016), Jiao (2017, 2019) or REDF Wang (2021, 2023)
- Others: Generalized-seniority scheme Engel, Vogel, Ji, Pittel (1989)

Size of Single-particle basis



- ISM predicts small NMEs, while IBM and EDF predict large NMEs. Discrepancy is about a factor of THREE or even larger.
- Different models are not equivalent! Different schemes (model spaces and interactions): compare apples to oranges?
- Efforts in resolving the discrepancy: Challenging or even impossible?

JMY, J. Meng, Y.F. Niu, P. Ring, *PPNP* 126, 103965 (2022)

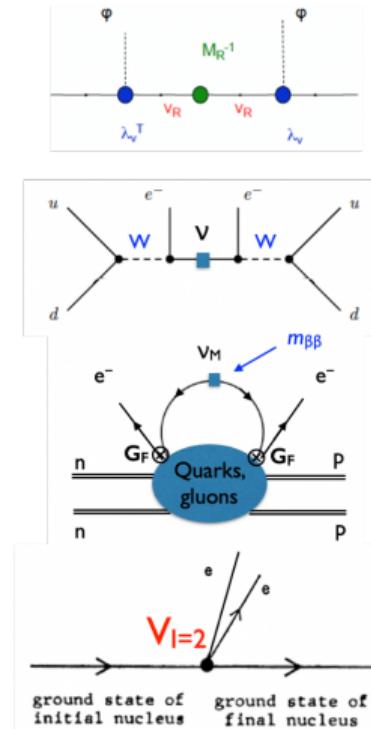
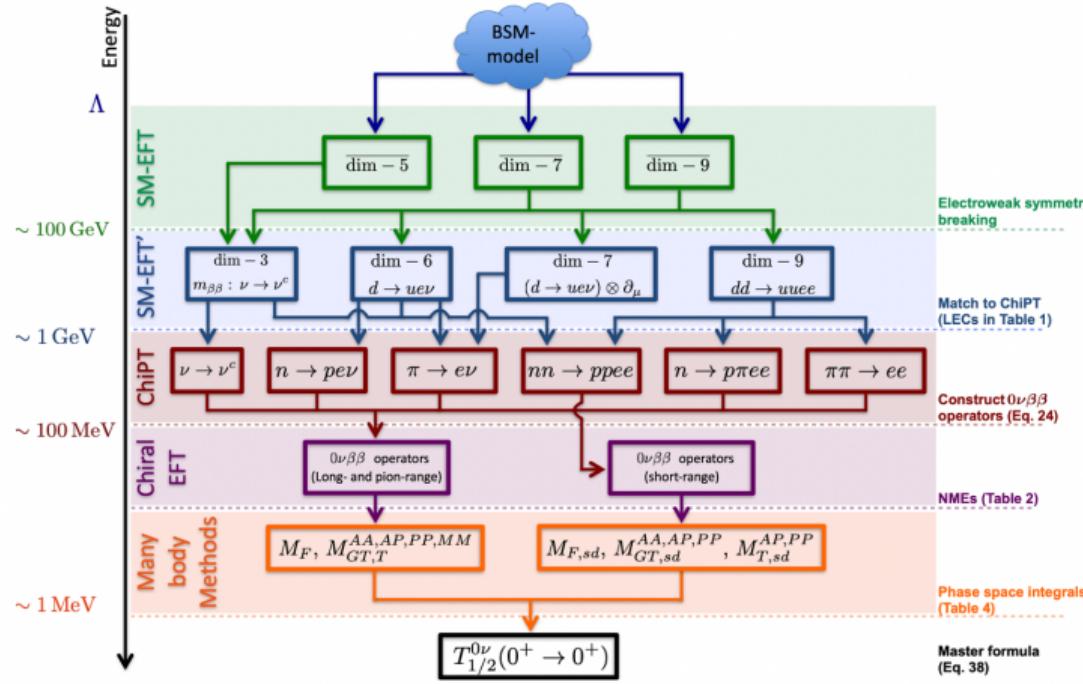


Strategy

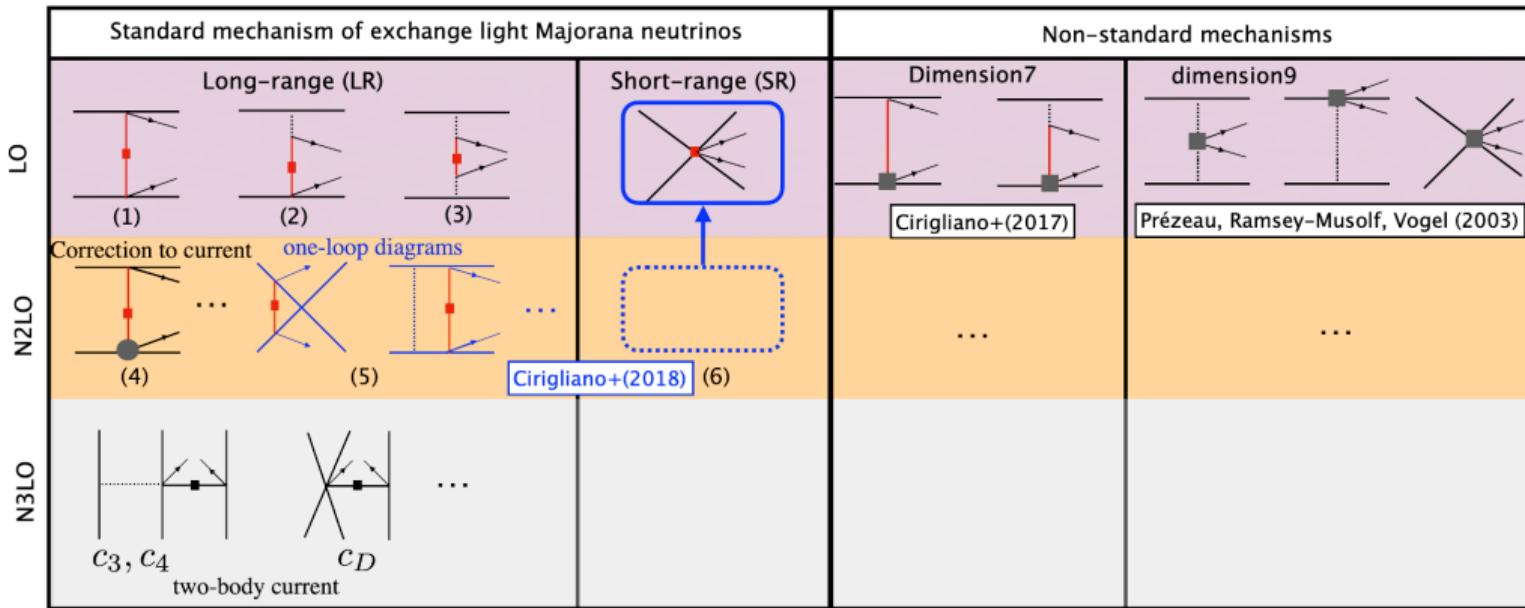
- **Operator forms:** (Chiral) effective field theory (EFT) to specify the forms of nuclear forces and weak transition operators (at different expansion orders of Q/Λ_χ).
- **Low-energy constants (LECs):** data on NN scattering and few-body system or Lattice QCD calculations.
- **Many-body solvers:** A systematically improvable nuclear model to solve the quantum many-body problem.

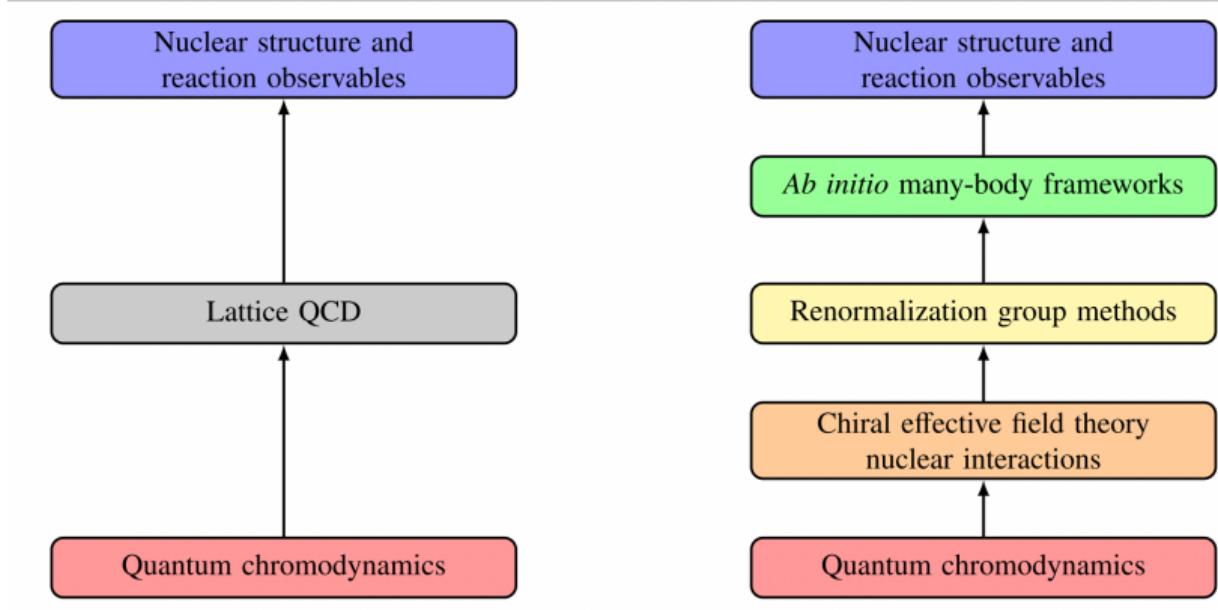
EFT: a model-independent analysis of operators at different energy scales

Cirigliano (2018)



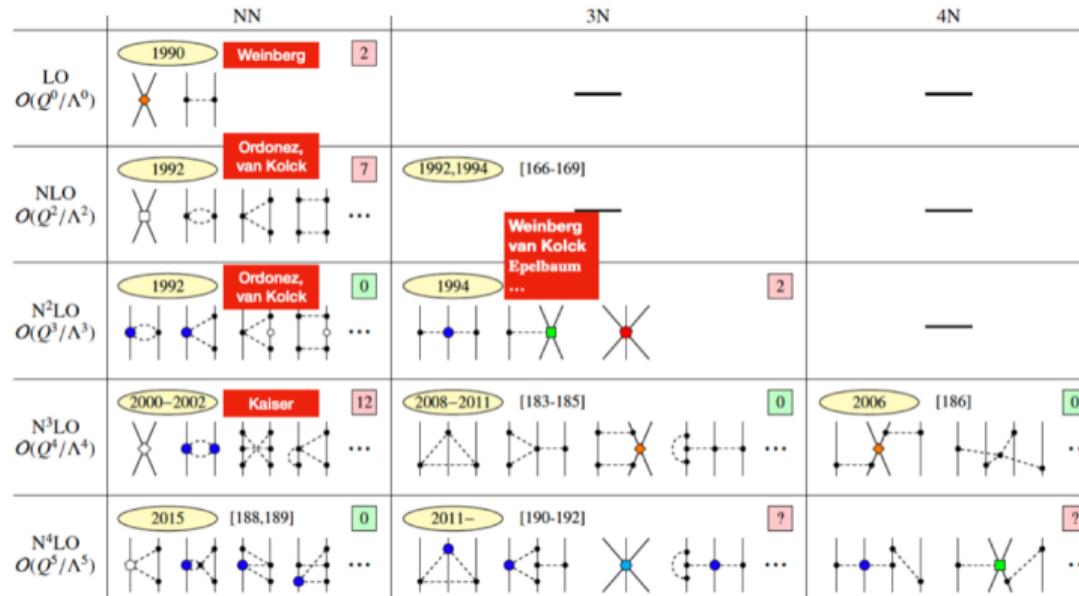
- At $E \sim 100$ MeV: operators are expressed in terms of nucleons, pions, and leptons.





K. Hebeler, Phys. Rep. 890, 1 (2021)

- Non-relativistic chiral 2N+3N interactions (Weinberg power counting and others)

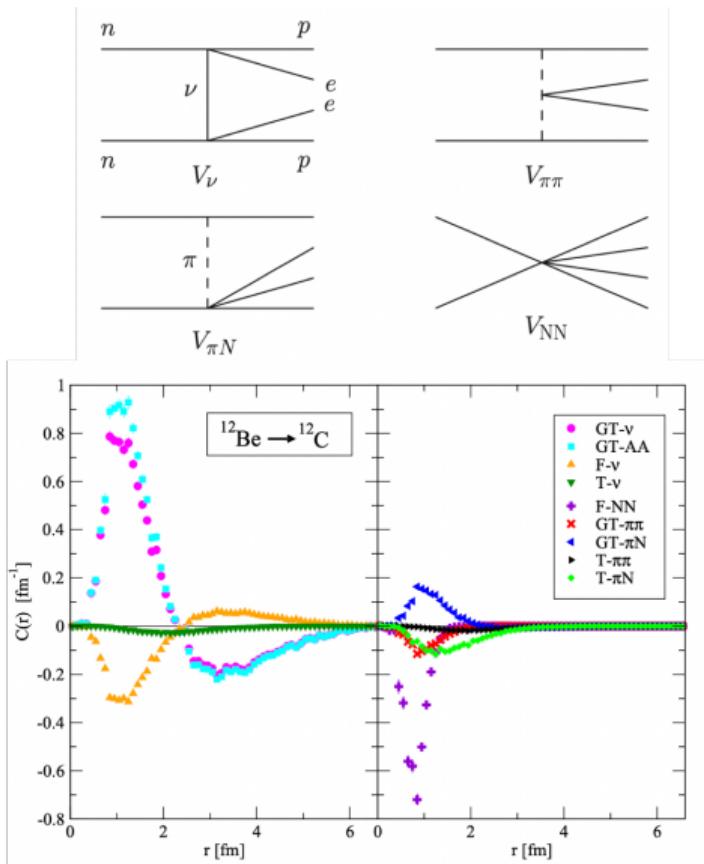


K. Hebeler, Phys. Rep. 890, 1 (2020)

- Relativistic chiral 2N interaction (up to $N^2\text{LO}$, different PC from the NR case)

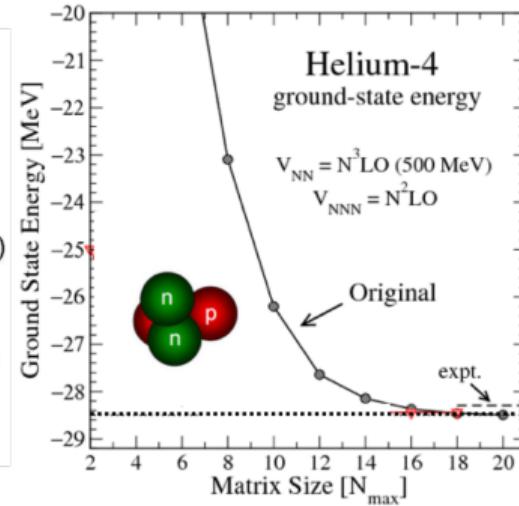
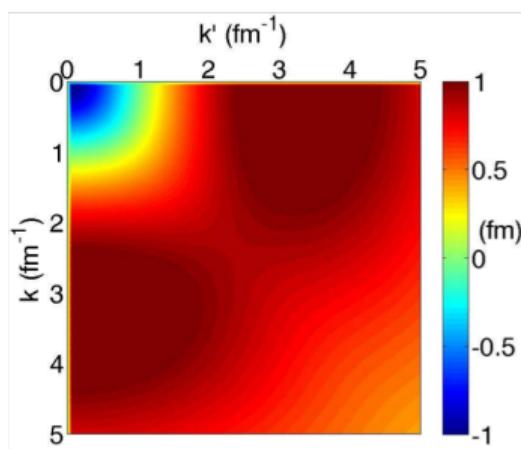
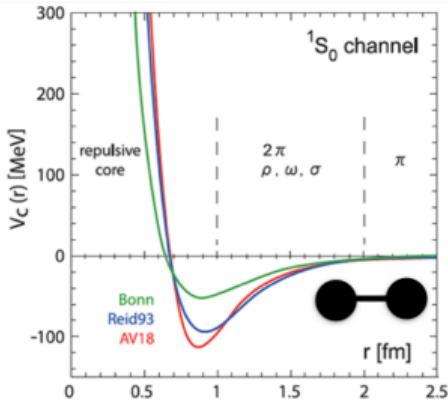
J.-X. Lu et al., PRL128, 142002 (2022)

The Monte-Carlo studies of $0\nu\beta\beta$ decay in light nuclei



- The variational Monte Carlo (VMC) with the NN(AV18) + 3N(Illinois-7).
- Light Majorana neutrino exchange + multi-TeV (dim9) mechanisms of LNV.
- The N²LO effects captured by nucleon form factors impact the matrix elements at 10% level.
- The non-factorizable terms at N2LO may lead to O(10%) corrections.
- indicating that the NME converges with the chiral expansion order for the weak operators.
- Difficult to extend to the candidate nuclei of $0\nu\beta\beta$ decay.

Challenges of basis-expansion methods



- Repulsive core & strong tensor force: low and high k modes strongly coupled.
- non-perturbative, poorly convergence in basis expansion methods.

S. Bogner et al., PPNP (2010)

- Apply unitary transformations to Hamiltonian

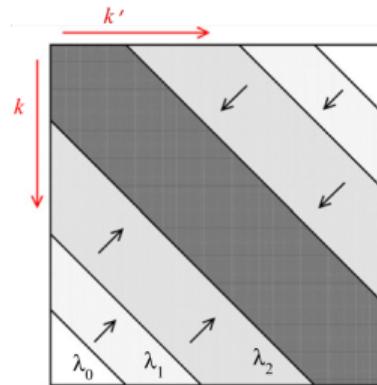
$$H_s = U_s H U_s^\dagger \equiv T_{\text{rel}} + V_s$$

from which one finds the flow equation

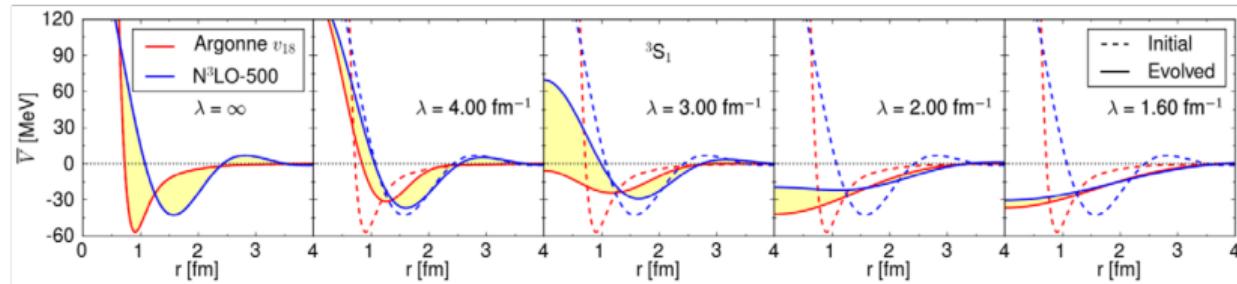
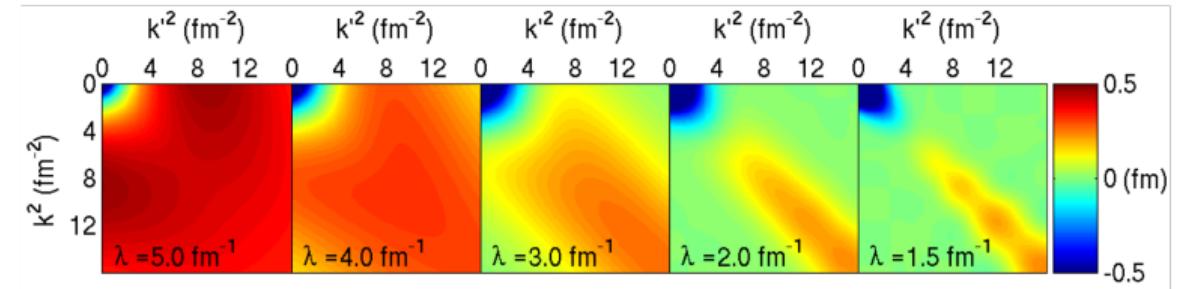
$$\frac{dH_s}{ds} = [\eta_s, H_s], \quad \eta_s = [T_{\text{rel}}, H_s]$$

Evolution of the potential

$$\frac{dV_s(k, k')}{ds} = -(k^2 - k'^2) V_s(k, k') + \frac{2}{\pi} \int_0^\infty q^2 dq (k^2 + k'^2 - 2q^2) V_s(k, q) V_s(q, k')$$



The flow parameter s is usually replaced with $\lambda = s^{-1/4}$ in units of fm^{-1} . [S. K. Bogner et al. \(2007\)](#)



Local projection of AV18 and $N^3\text{LO}(500 \text{ MeV})$ potentials $V(r)$.

- The hard core "disappears" in the softened interactions

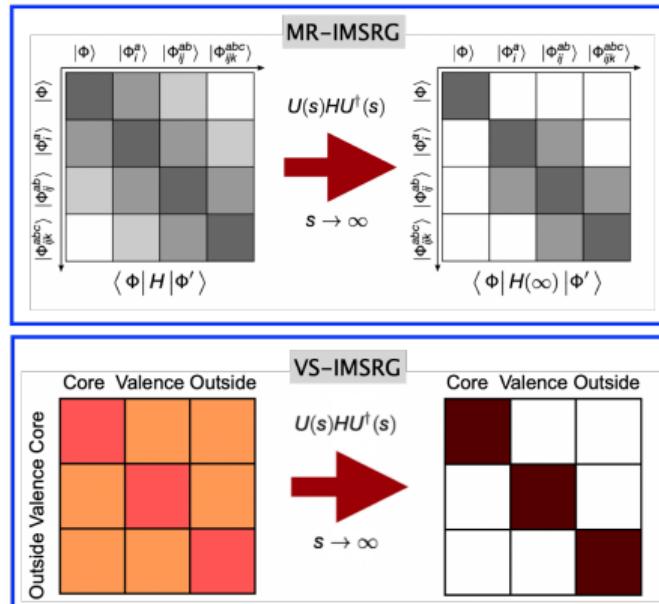
- Unitary transformations

$$H(s) = U(s)H_0U^\dagger(s)$$

Flow equation

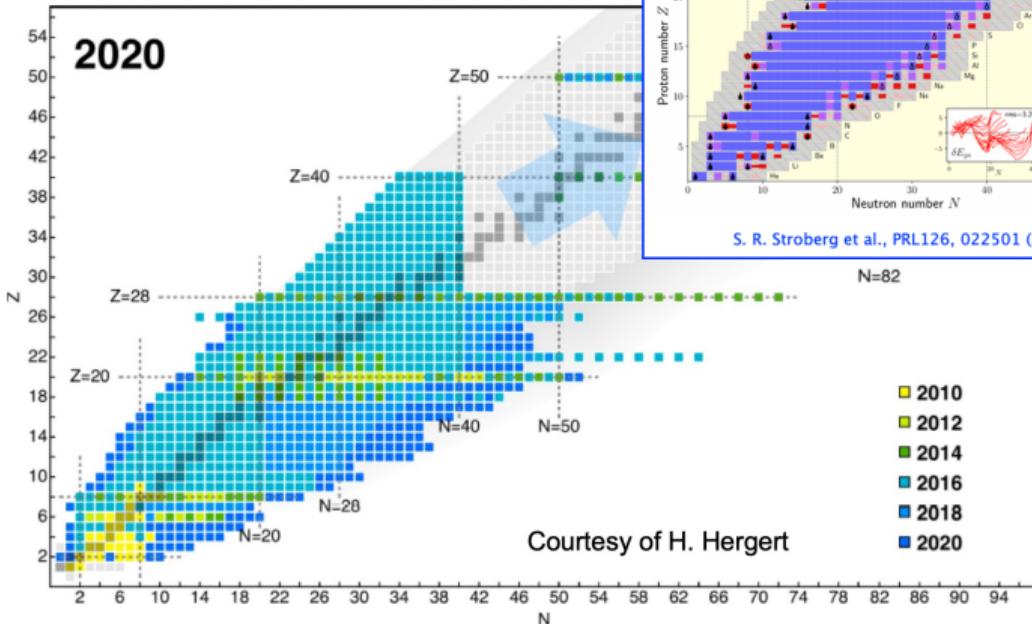
$$\frac{dH(s)}{ds} = [\eta(s), H(s)]$$

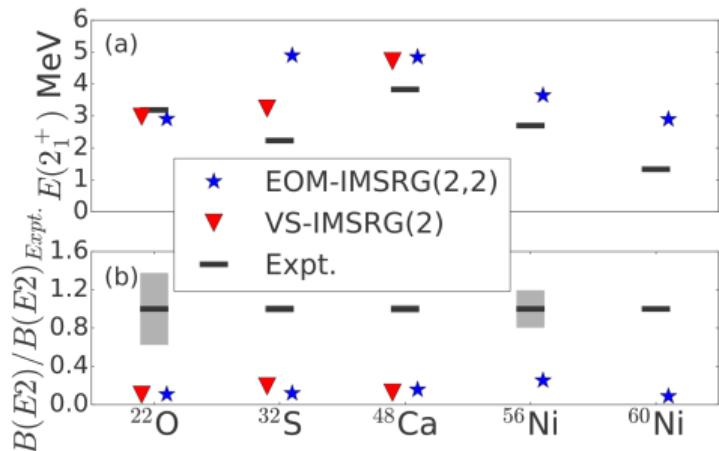
- Generator $\eta(s)$: chosen either to decouple a given **reference state** from its excitations or to decouple the valence space from the excluded spaces.
- Not necessary to construct the whole H matrix, computation complexity scales **polynomially** with nuclear size.



H. Hergert et al., Phys. Rep. 621, 165 (2016); S. R. Stroberg et al., Annu. Rev. Nucl. Part. Sci. 69, 307 (2019)

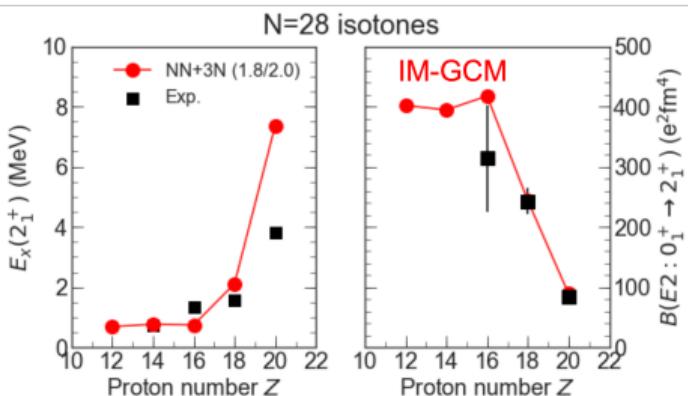
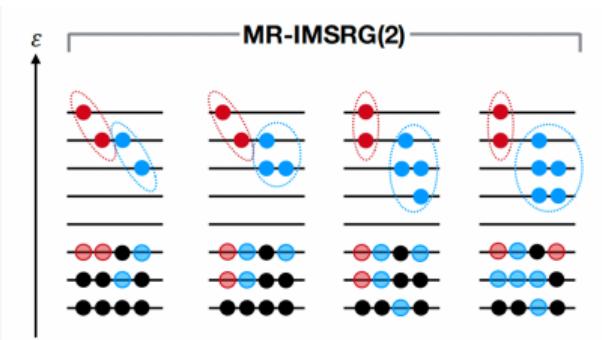
Progress in the ab initio studies of atomic nuclei





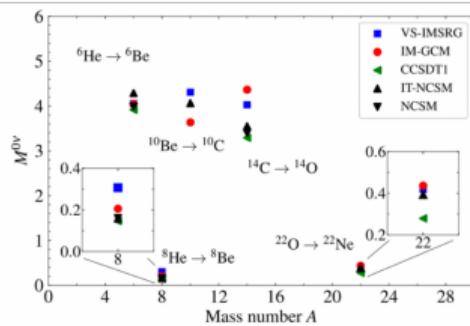
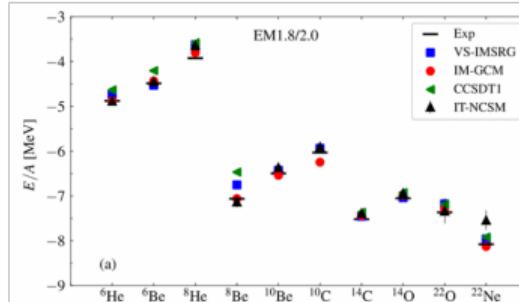
N. M. Parzuchowski et al., PRC(2017)

- The single-reference VS-IMSRG(2) is difficult to capture collective correlations.
- The IM-GCM (multi-reference IMSRG+GCM) is capable to describe deformed nuclei.

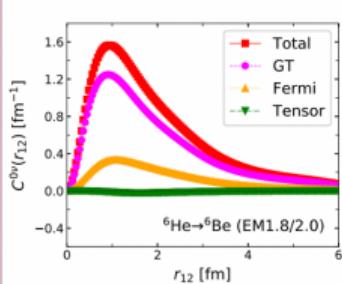


Benchmark studies of $0\nu\beta\beta$ decay in light nuclei

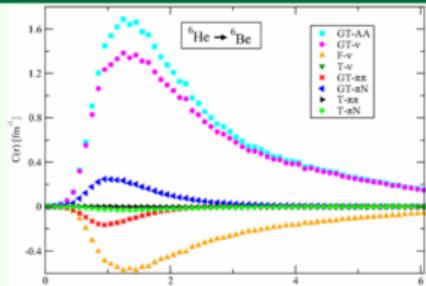
JMY et al., PRC103, 014315 (2021)



IM-GCM (EM1.8/2.0)



S. Pastore et al (2018)

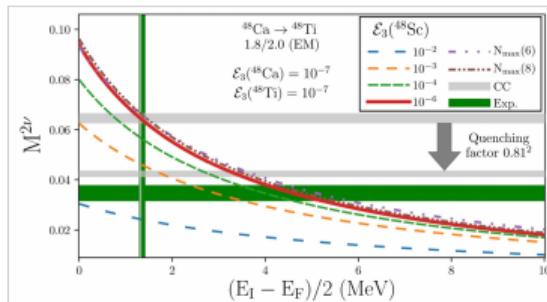
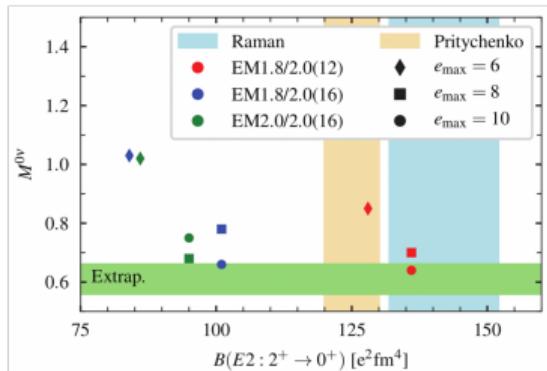
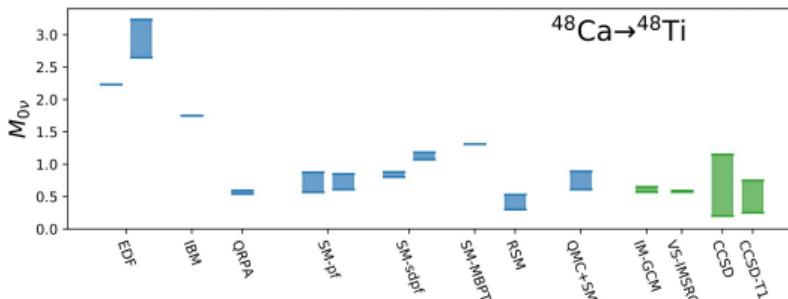


Note: A factor of $-g_A^2$ has been multiplied into the Fermi part.

- Using different ab initio methods but the same input to estimate of the truncation errors of many-body methods.

Ab initio methods for the lightest candidate ^{48}Ca

- Multi-reference in-medium generator coordinate method (IM-GCM)
JMY et al., PRL124, 232501 (2020)
- IMSRG+ISM (VS-IMSRG)
A. Belley et al., PRL126, 042502 (2021)
- Coupled-cluster with singlets, doublets, and partial triplets (CCSDT1) .
S. Novario et al., PRL126, 182502 (2021)



The missing piece in the LO transition operators

Featured in Physics Editors' Suggestion Open Access

New Leading Contribution to Neutrinoless Double- β Decay

Vincenzo Cirigliano, Wouter Dekens, Jordy de Vries, Michael L. Graesser, Emanuele Mereghetti, Saori Pastore, and Ubirajara van Kolck

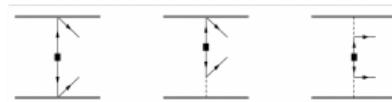
Phys. Rev. Lett. **120**, 202001 – Published 16 May 2018

Physics See Synopsis: A Missing Piece in the Neutrinoless Beta-Dec

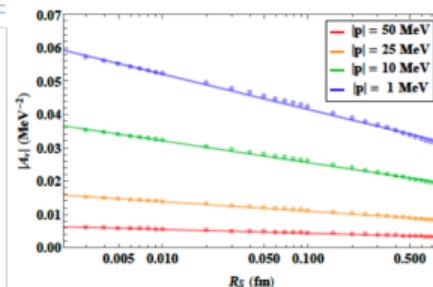
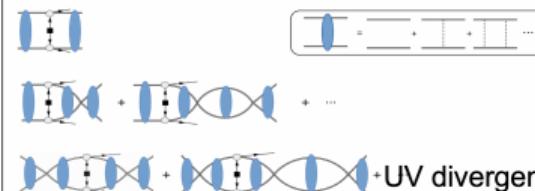
Nuclear force



Transition operator



LO

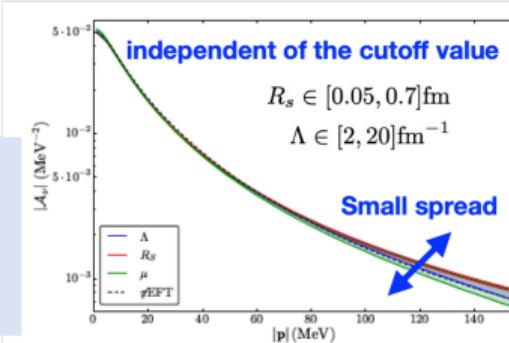
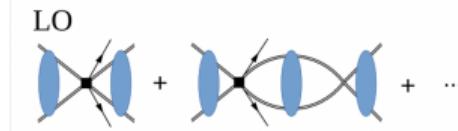


Lines fitted to $A_0 = a + b \ln R_S$
logarithmic dependence on R_S

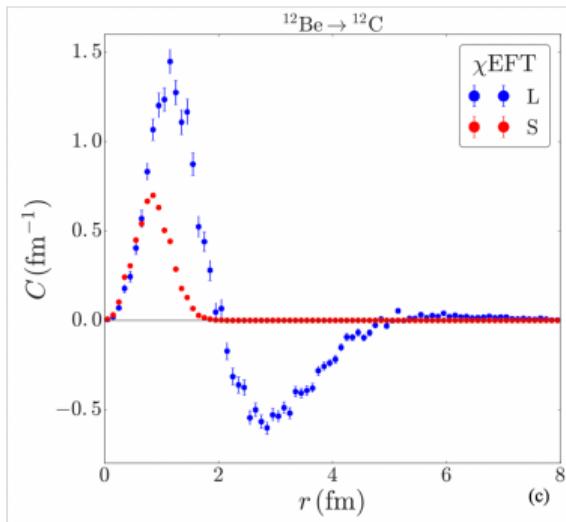
- The transition amplitude is regulator-dependent!
- Needs a counter term at LO in order to ensure renormalizability.

Introducing a contact transition operator

$$V_{\nu,S} = -2g_{\nu}^{NN}\tau^{(1)} + \tau^{(2)} + \dots$$



Contribution of the contact transition operator to the NME



A	Model	M_L	M_S	Uncertainty
6	AV18	7.45	0.48	$\pm 16\%$
	χ EFT	7.82	1.15	
12	AV18	0.653	0.518	$\pm 73\%$
	χ EFT	0.725	0.533	

According to the VMS calculation, the contribution of the contact transition operator

$$V_{\nu,S} = -2g_{\nu}^{NN}\tau^{(1)+}\tau^{(2)+}$$

to the NME of $0\nu\beta\beta$ decay of

- ^6He could be up to $\sim \pm 16\%$
- ^{12}Be could be up to $\sim \pm 73\%$

The actual contribution depends on the value of the LEC g_{ν}^{NN} , which should be determined by the data of the process or the calculation of a more fundamental theory for the process.

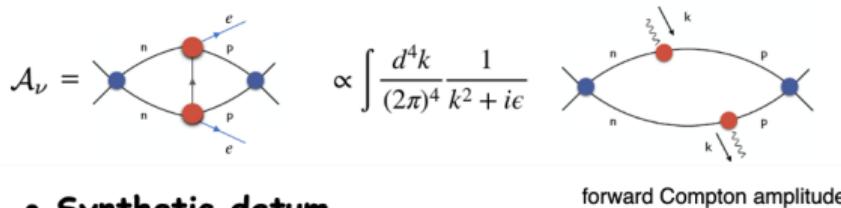
Toward Complete Leading-Order Predictions for Neutrinoless Double β Decay

Vincenzo Cirigliano, Wouter Dekens, Jordy de Vries, Martin Hoferichter, and Emar Mereghetti

Phys. Rev. Lett. **126**, 172002 (2021) – Published 30 April 2021

- **Cottingham formula** [W.N. Cottingham, Ann. Phys. 25, 424 \(1963\)](#)

$$\mathcal{A}_\nu \propto \int \frac{d^4 k}{(2\pi)^4} \frac{g_{\mu\nu}}{k^2 + i\epsilon} \int d^4 x e^{ik \cdot x} \langle pp | T\{ j_w^\mu(x) j_w^\nu(0) \} | nn \rangle$$

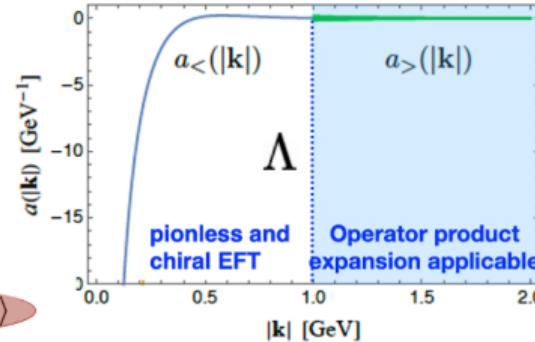


- **Synthetic datum**

$$\begin{aligned} \mathcal{A}_\nu(|\mathbf{p}|, |\mathbf{p}'|) \times e^{-i(\delta_{1S_0}(|\mathbf{p}|) + \delta_{1S_0}(|\mathbf{p}'|))} &= - \left(2.271 - 0.075 \tilde{\mathcal{C}}_1(4M_\pi) \right) \times 10^{-2} \text{ MeV}^{-2} \\ |\mathbf{p}| = 25 \text{ MeV } (|\mathbf{p}'| = 30 \text{ MeV}) &= -1.95(5) \tilde{\mathcal{C}}_1 \times 10^{-2} \text{ MeV}^{-2}, \end{aligned}$$

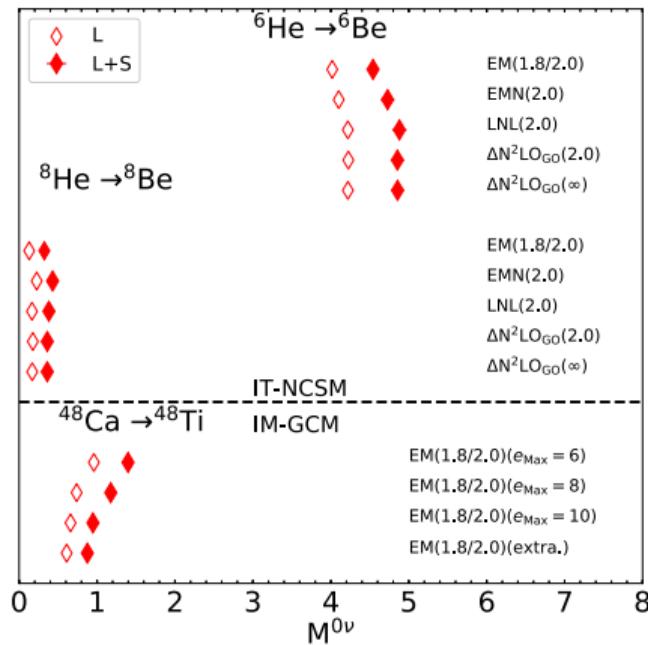
Uncertainty from the estimate of the **inelastic** contributions

The transition amplitude is observable and thus scheme independent.



$$\begin{aligned} \mathcal{A}_\nu^{\text{full}} &= \int_0^\infty d|\mathbf{k}| a^{\text{full}}(|\mathbf{k}|) = \mathcal{A}^< + \mathcal{A}^>, \\ \mathcal{A}^< &= \int_0^\Lambda d|\mathbf{k}| a_<(|\mathbf{k}|), \\ \mathcal{A}^> &= \int_\Lambda^\infty d|\mathbf{k}| a_>(|\mathbf{k}|), \end{aligned}$$

- The LEC g_ν^{NN} consistent with the employed chiral interaction (EM1.8/2.0) is determined based on the synthetic data.
- The contact term turns out to enhance (instead of quench) the NME for ^{48}Ca by 43(7)%, thus the half-life $T_{1/2}^{0\nu\beta\beta}$ is only half of the previously expected value.
- The uncertainty (7%) is due to the synthetic data which can be reduced by using an accurate value of the LEC (g_ν^{NN}).



R. Wirth, JMY, H. Hergert, PRL127, 242502 (2021)

A recent study in the relativistic chiral EFT shows that

- the $nn \rightarrow ppe^- e^-$ transition amplitude \mathcal{A}_ν is regulator-independent, thus no need to introduce the contact transition operator.
- The predicted $\mathcal{A}_\nu = 0.02085 \text{ MeV}^{-2}$, about 10% larger than the value by Cirigliano (2021).
- The discrepancy could be attributed to the different power counting:** the LO of relativistic chiral EFT contains partial N2LO contribution of non-relativistic EFT.

Y.L. Yang and P. W. Zhao, arXiv:2308.03356v1 (2023)

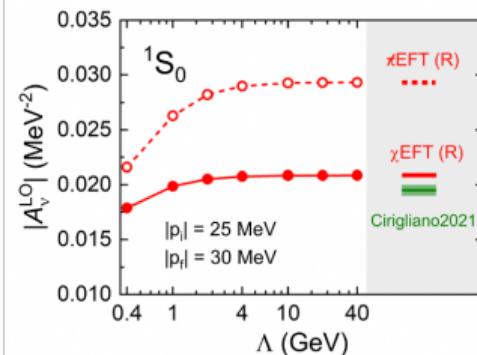
Bethe-Salpeter equation

$$T(\vec{p}', \vec{p}) = V(\vec{p}', \vec{p}) + \int \frac{d^3 p''}{(2\pi)^3} V(\vec{p}', \vec{p}'') \frac{M_N^2}{E_{p''}} \frac{1}{p^2 - p''^2 + i\epsilon} T(\vec{p}'', \vec{p})$$

$$E_{p''} \equiv \sqrt{M_N^2 + p''^2}.$$

Lippmann-Schwinger equation

$$\hat{T}(\vec{p}', \vec{p}) = \hat{V}(\vec{p}', \vec{p}) + \int d^3 p'' \hat{V}(\vec{p}', \vec{p}'') \frac{M_N}{p^2 - p''^2 + i\epsilon} \hat{T}(\vec{p}'', \vec{p})$$



With both the long- and short-range transition operators, the VS-IMSRG method is applied to study the NMEs of heavier candidates:

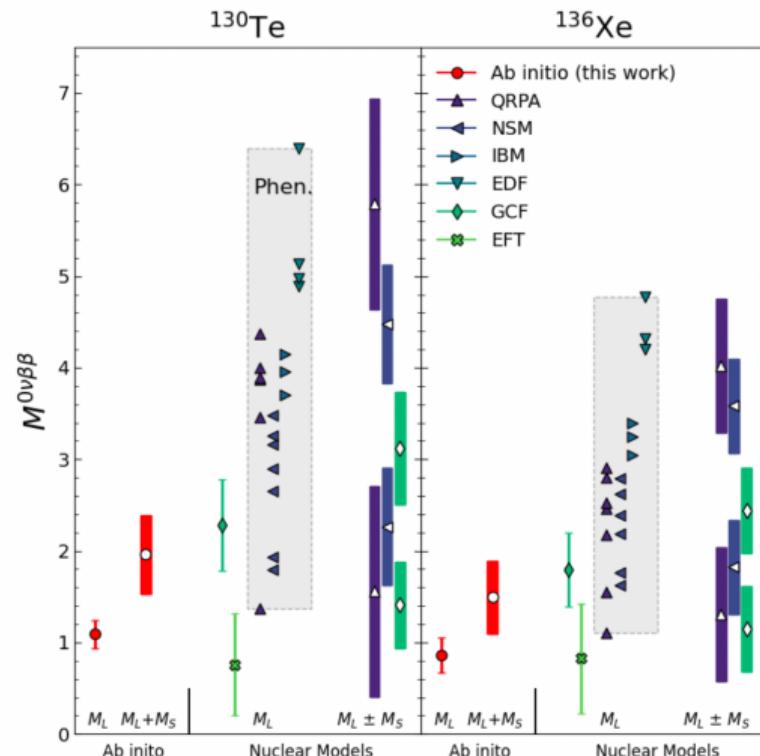
- For ^{130}Te , $M_{L+S}^{0\nu} \in [1.52, 2.40]$
- For ^{136}Xe , $M_{L+S}^{0\nu} \in [1.08, 1.90]$

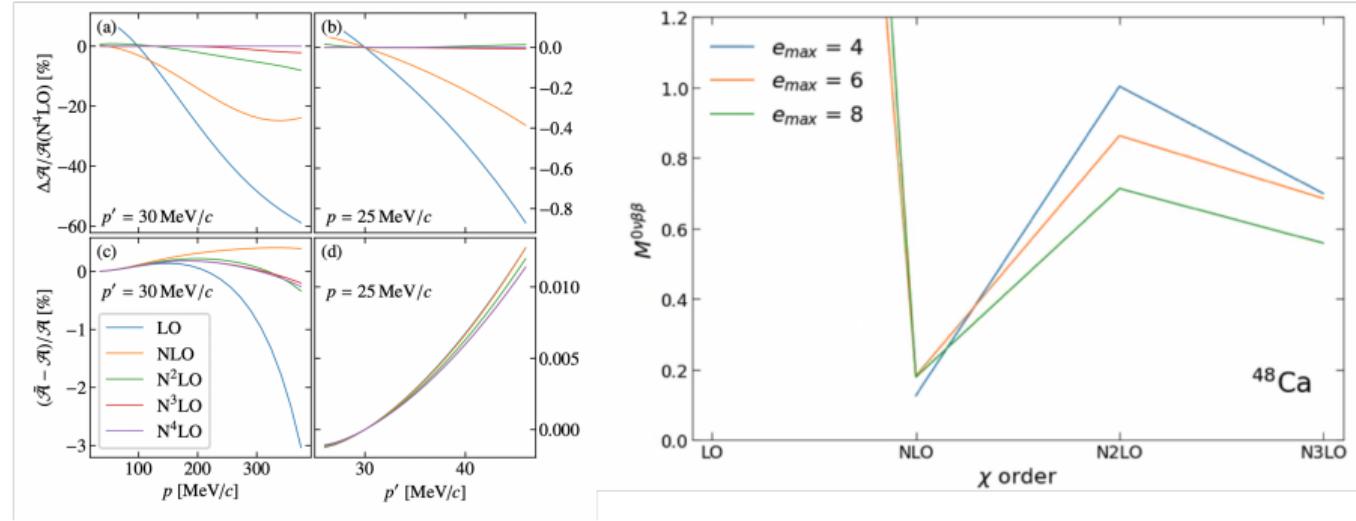
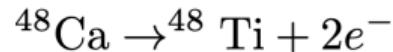
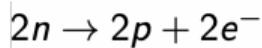
The uncertainty is composed of different sources: nuclear interaction, reference-state, basis extrapolation, closure approximation, and the LEC for the short-range transition operators.

The values are generally smaller than those from phenomenological nuclear models.

A more comprehensive quantification analysis
different nuclear many-body solvers, convergence
of NMEs with chiral expansion orders, etc.

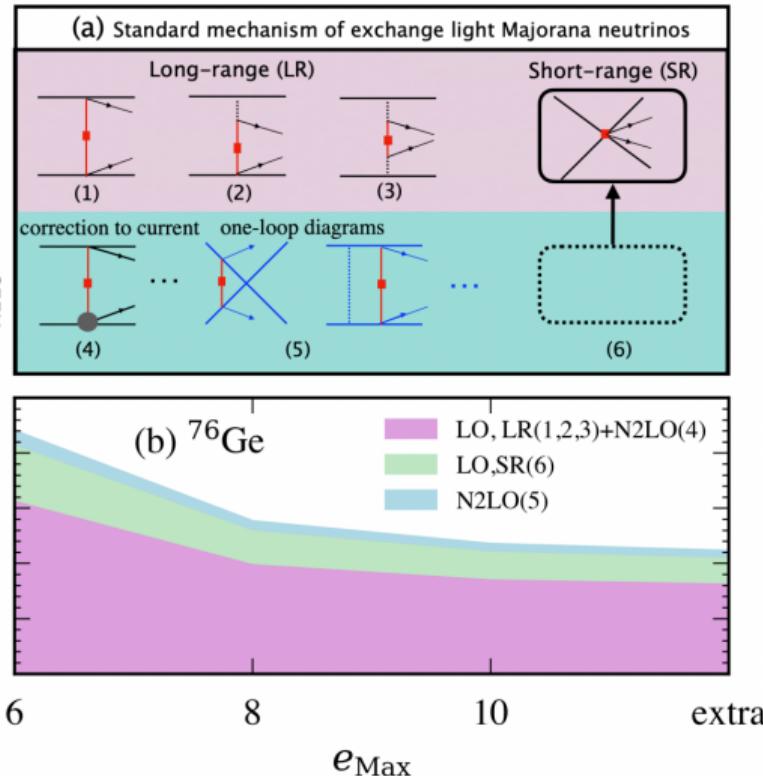
A. Belley et al, arXiv:2307.15156 (2023)





- The $\mathcal{A}_\nu(2n \rightarrow 2p + 2e^-)$ converges quickly w.r.t. the chiral expansion order of nuclear interactions. Negligible contribution beyond NLO, particular true for low momentum cases. [R. Wirth, JMY, H. Hergert, PRL127, 242502 \(2021\)](#)
- Convergence is slightly slower in candidate nucleus ^{48}Ca .

Convergence w.r.t. chiral expansion order for ^{76}Ge



A. Belley, JMY et al, arXiv:2308.15634 (2023)

JMYao

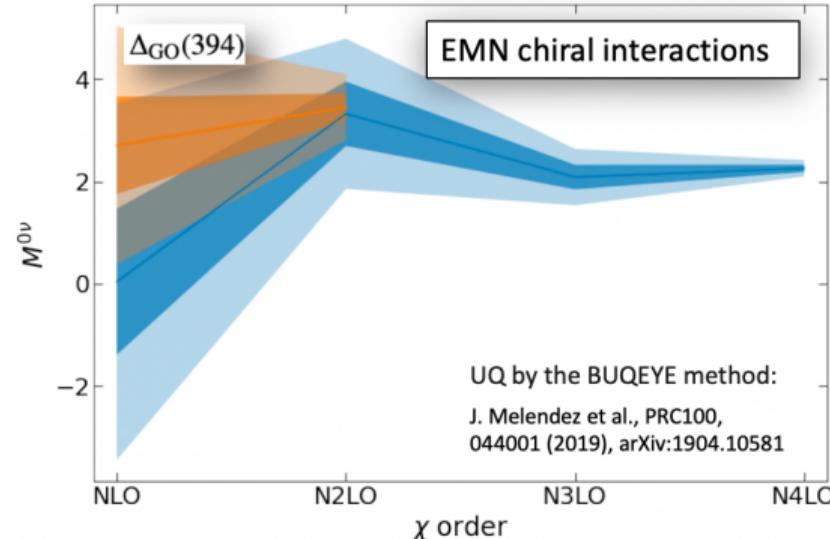
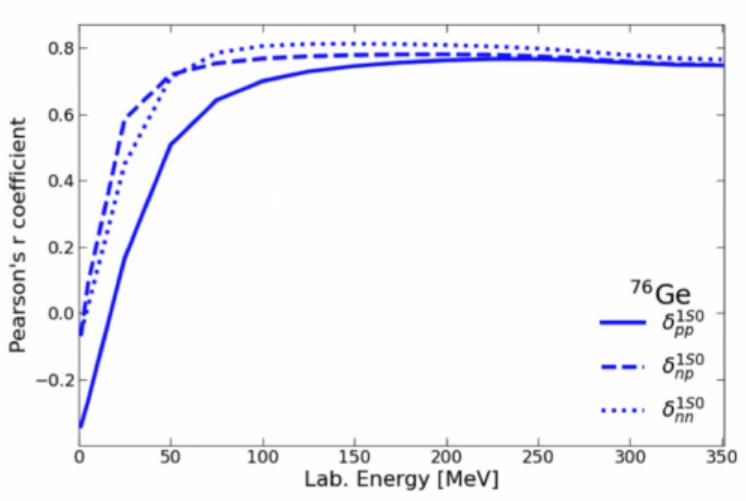
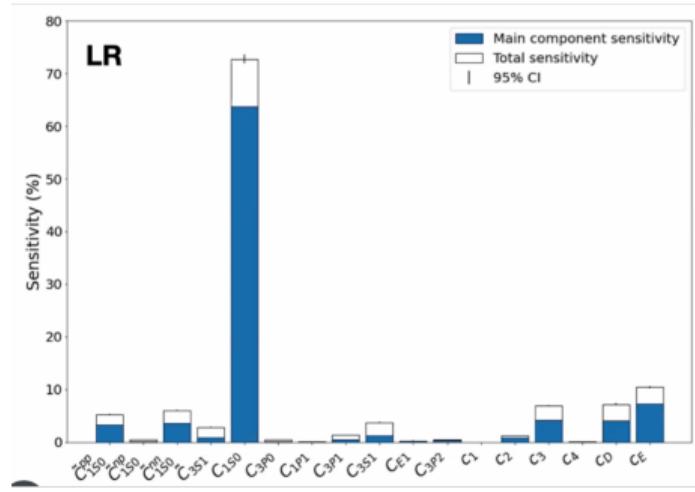


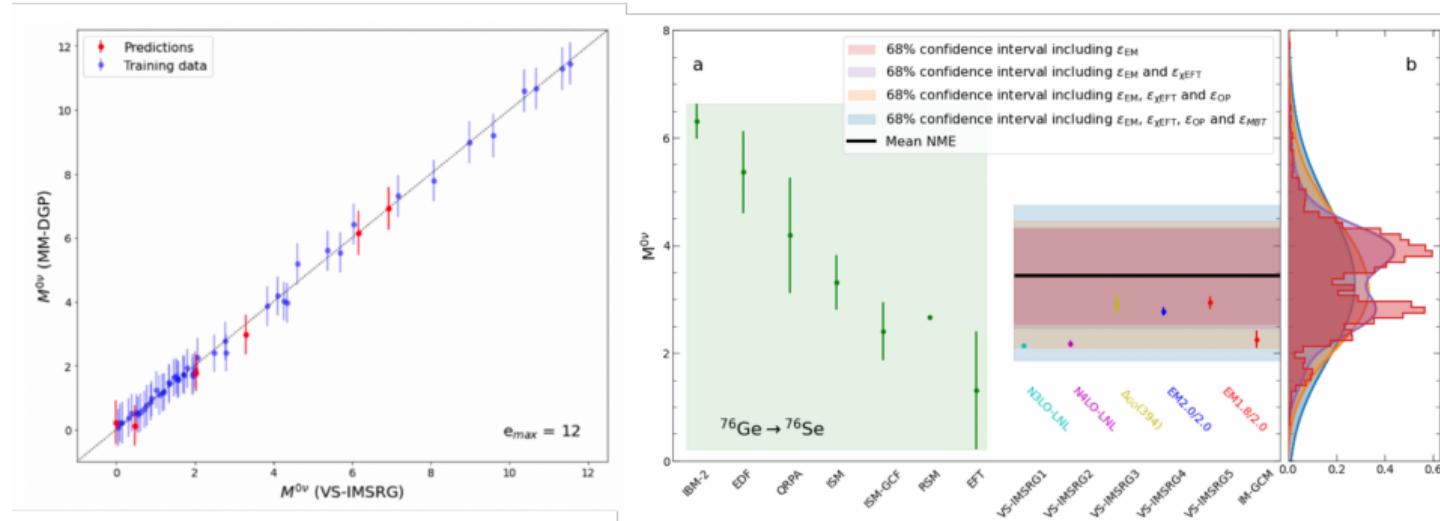
Table 1 | The recommended value for the total NME of $0\nu\beta\beta$ decay in ^{76}Ge , together with the uncertainties from different sources.

$M^{0\nu}$	ϵ_{LEC}	$\epsilon_{\chi\text{EFT}}$	ϵ_{MBT}	ϵ_{OP}	ϵ_{EM}
$3.44^{+1.33}_{-1.56}$	0.9	0.3	0.8	0.5	<0.06



- The long-range part of the NME is sensitive to the LEC C_{1S_0} .
 - The phase shift of the 1S_0 channel is linearly correlated to the NME.
 - The neutron-proton phase-shift $\delta_{np}^{^1S_0}$ at 50 MeV is used to weight the samples.

Uncertainty quantification of NME for ^{76}Ge



- Emulator, 8188 samples of chiral interactions, phase shift, $M^{0\nu} = 3.44^{+1.33}_{-1.56}$.
- Current upper limit for the effective neutrino mass $\langle m_{\beta\beta} \rangle = 141^{+117}_{-39}$ meV.
- The next-generation ton-scale Germanium experiment ($\sim 1.3 \times 10^{28}$ yr): $m_{\beta\beta} = 17^{+14}_{-5}$ meV, covering almost the entire range of IO hierarchy.

- **$0\nu\beta\beta$ decay:** lepton-number-violation process, a complementary way to determine the absolute mass scale of neutrinos.
- **Next-generation experiments:** tonne-scale detectors with a half-life sensitivity up to 10^{28} years.
- **Large uncertainty in NMEs:** systematical uncertainty, impacting extracted neutrino mass, attracting a lot of efforts from nuclear community.
- **Ab initio studies of NMEs:** remarkable progress, disclosing non-trivial contributions from high-energy light neutrinos. The NMEs for heavier candidate nuclei (^{48}Ca , ^{76}Ge , ^{82}Se , ^{130}Te , ^{136}Xe) have been computed. Convergence w.r.t. the chiral expansion order is rather rapid.

Next

- Considering higher-order nuclear interactions, reducing many-body truncation errors, and finding more constraints to shrink the uncertainty.
- Contributions from other mechanisms.

Collaborators

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- **SWU**: Longjun Wang

- **MSU**: Scott Bogner, Heiko Hergert, Roland Wirth

- **UNC**: Jonathan Engel, A. M. Romero

- **TRIUMF**: Antonie Belly, Jason Holt

- **TU Darmstadt**: Takayuki Miyagi

- **Notre-Dame U**: Ragnar Stroberg

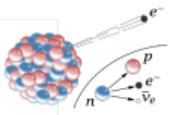
- **UAM**: Benjamin Bally, Tomas Rodriguez

Thank you for your attention!

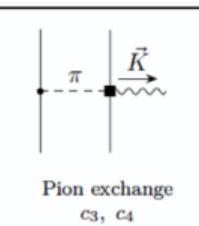
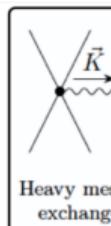
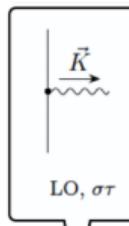
- The half-life of single-beta decay

$$t_{1/2} = \frac{\kappa}{f_0(B_F + B_{\text{GT}})},$$

$$B_F = \frac{g_V^2}{2J_i + 1} |M_F|^2, \quad B_{\text{GT}} = \frac{g_A^2}{2J_i + 1} |M_{\text{GT}}|^2$$

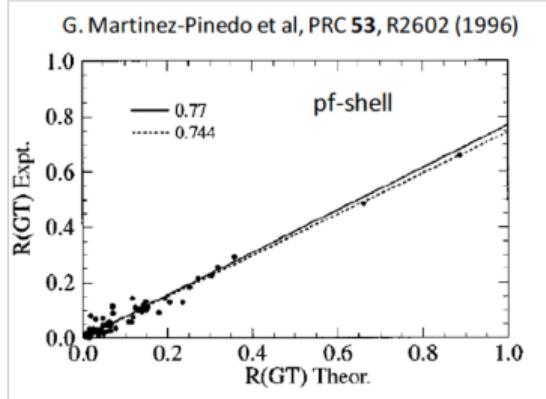


- The charge-changing axial-vector current



$$\vec{J}^A(\vec{K}) = \sum_j i g_A \sigma_j \tau_j^\pm e^{i \vec{K} \cdot \vec{r}_j}.$$

2B currents



Park, T.-S. et al. PRC67, 055206 (2003);
 M. Hoferichter et al., PRD102, 074018 (2020)



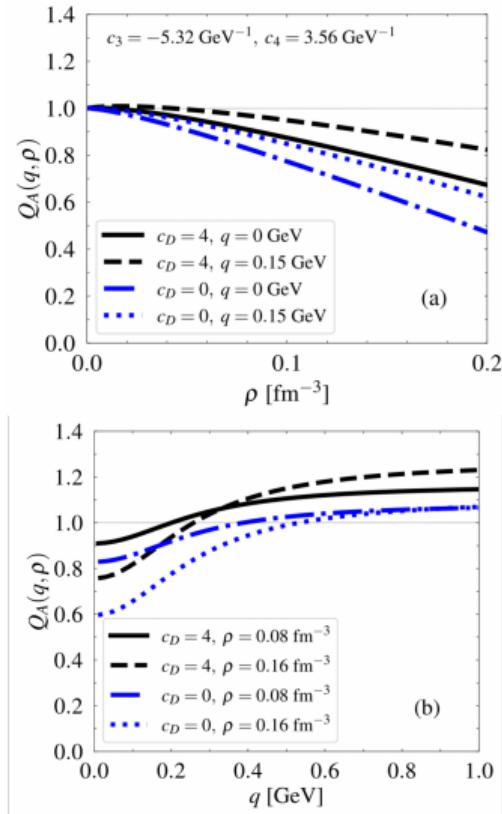
$$g_A^{\text{eff}}(q, 0, \rho) \equiv g_A \left\{ 1 - \frac{\rho}{f_\pi^2} \left[-\frac{c_D}{4} \frac{1}{g_A A_\chi} + \frac{2c_3}{3} \frac{q^2}{4m_\pi^2 + q^2} + \frac{I(\rho, 0)}{3} \left(2c_4 - c_3 + \frac{1}{2m_p} \right) \right] \right\}$$

The quenching factor:

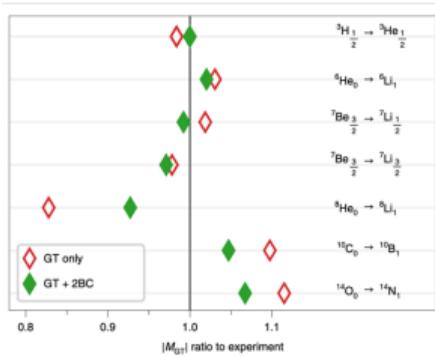
$$Q_A(q, \rho) \equiv g_A^{\text{eff}}(q, \rho)/g_A = 1 + A[q]\rho + B\rho^{1/3} + C$$

where the coefficients A , B and C are defined as

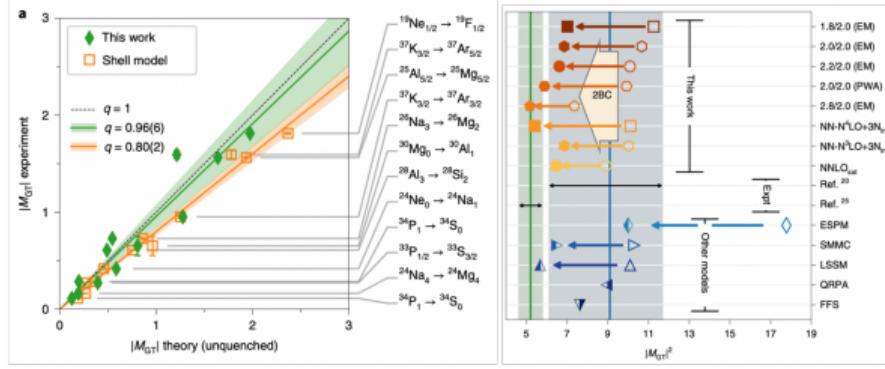
$$\begin{aligned} A[q] &= \frac{c_D}{4f_\pi^2} \frac{1}{g_A \Lambda_\chi} - \frac{1}{3f_\pi^2} \left[\left(2c_4 - c_3 + \frac{1}{2m_p} \right) + 2c_3 \frac{q^2}{4m_\pi^2 + q^2} \right] \\ B &= \frac{m_\pi^2}{f_\pi^2} \left(\frac{2}{3\pi^2} \right)^{2/3} \left(2c_4 - c_3 + \frac{1}{2m_p} \right), \\ C &= -\frac{2m_\pi^3}{3\pi^2 f_\pi^2} \left(2c_4 - c_3 + \frac{1}{2m_p} \right) \tan^{-1} \left(\frac{k_F}{m_\pi} \right). \end{aligned}$$



Ab initio methods for nuclear single- β decay



NCSM

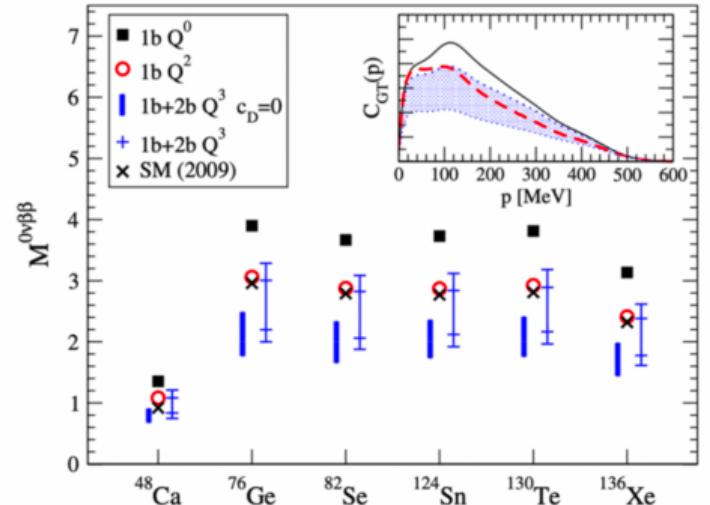
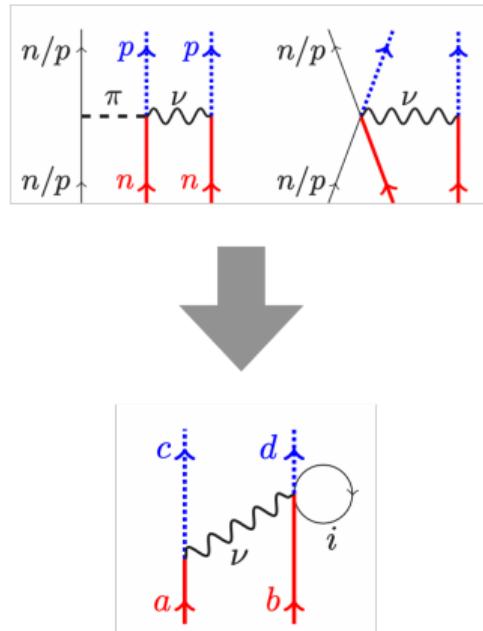


VS-IMSRG

CC

- **VS-IMSRG:** a unitary transformation is constructed to decouple a valence-space Hamiltonian H_{vs} . The eigenstates are obtained by a subsequent diagonalization of the H_{vs} .
- A proper treatment of **strong nuclear correlations** and the consistency between **2BCs and three-nucleon forces** explain the g_A -quenching puzzle in conventional valence-space shell-model calculations.

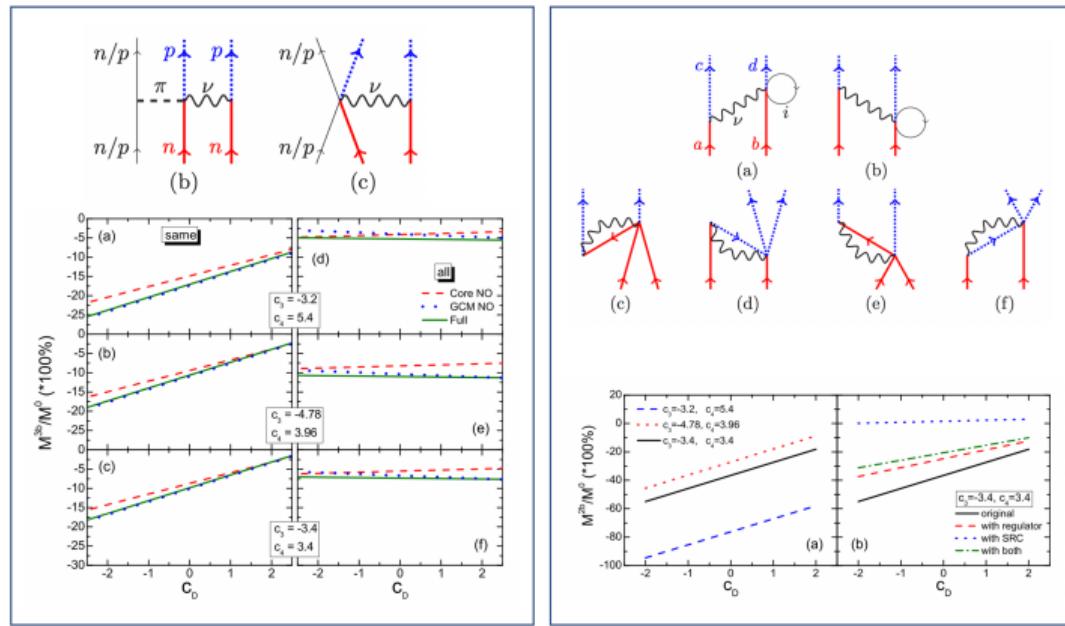
Two-body current effect



J. Menendez et al., PRL107, 062501 (2011)

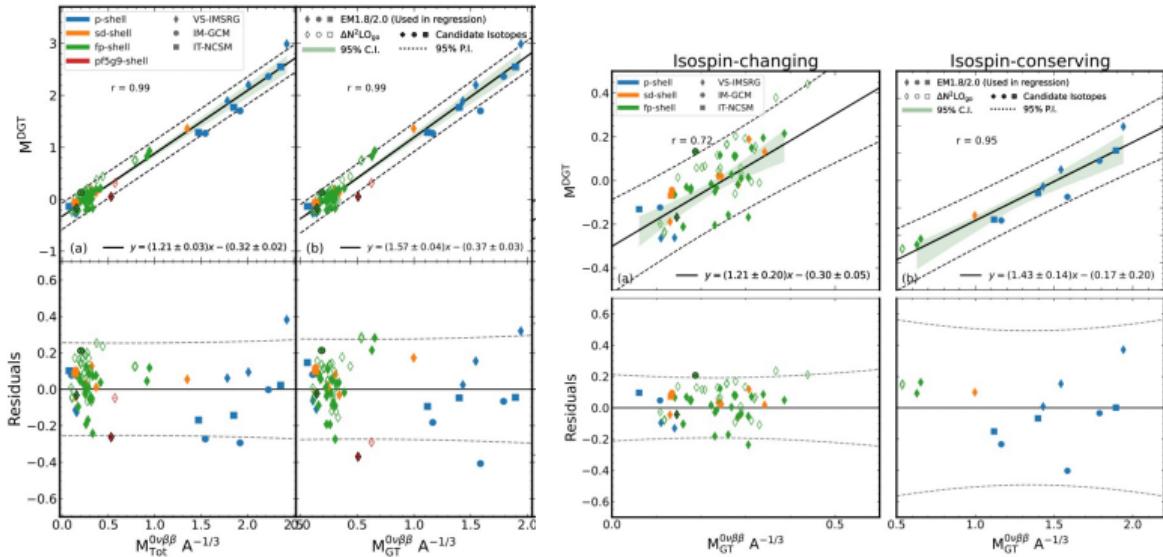
- The 2B current changes NMEs ranging from -35% to 10% .

Two-body current effect



- The 3B operators quench matrix elements by about 10%,
- The 2B operators can produce somewhat larger quenching.

Correlation relation between NLDBD and DGT



- Weak correlation between $M^{0\nu}$ and M^{DGT} .
- Other observables: $2\nu\beta\beta$ decay, excitation energies?

JMY et al., PRC106, 014315 (2022)

Correlation relation from VS-IMSRG for $^{76}\text{Ge-Se}$

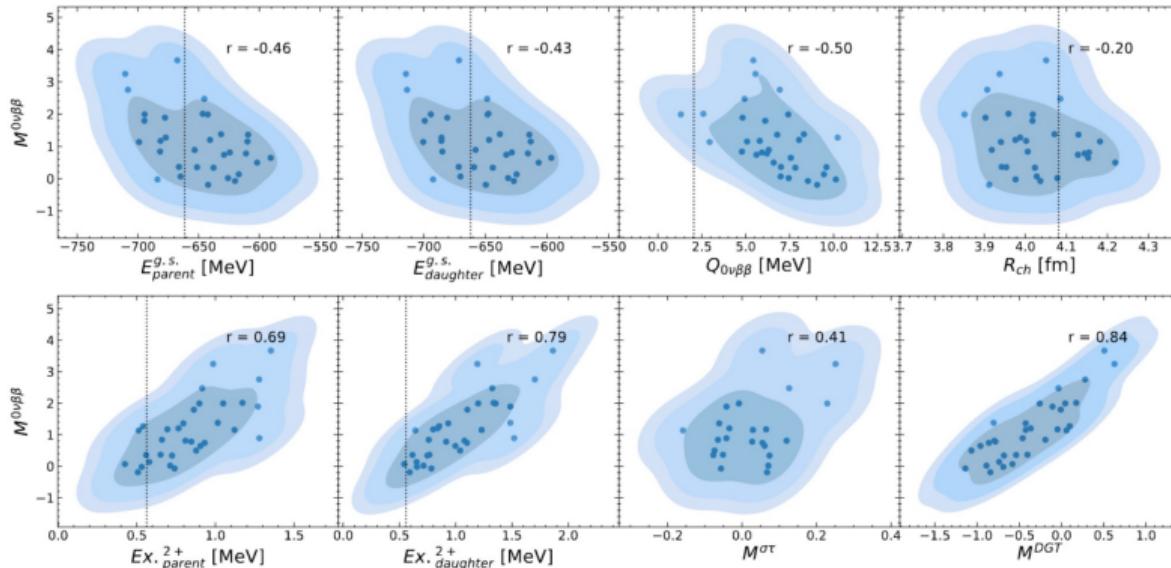


Figure: The correlation between different nuclear observables and $M^{0\nu}$ using 34 LECS samples of the Delta-full NNLO_{go}(394) interaction.

A. Belley et al., arXiv:2210.05809v1 [nucl-th]