Group Meeting 07.05

Three-body reaction theory in a model space

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Overview

The Schrodinger equation of three-body System

$$\left[E - K - V(r) - U_1\left(r_1\right) - U_2\left(r_2\right)\right]\psi = 0,$$

The "Distorted-wave" Faddeev differential equations

$$\left[E - K - V(r) - \mathscr{P}_\lambda\left(U_1 + U_2\right)\mathscr{P}_\lambda\right]\psi_\lambda = V\left[\psi_{1\lambda} + \psi_{2\lambda}\right]$$

$$\left[E - K - U_1\left(r_1\right)\right]\psi_{1\lambda} = \mathscr{P}_\lambda\left[U_1 - \mathscr{P}_\lambda U_1 \mathscr{P}_\lambda\right]\psi_\lambda + \mathscr{P}_\lambda U_1 \psi_{2\lambda}$$

$$\left[E - K - U_2\left(r_2\right)\right]\psi_{2\lambda} = \mathscr{P}_\lambda\left[U_2 - \mathscr{P}_\lambda U_2 \mathscr{P}_\lambda\right]\psi_\lambda + \mathscr{P}_\lambda U_2 \psi_{1\lambda},$$

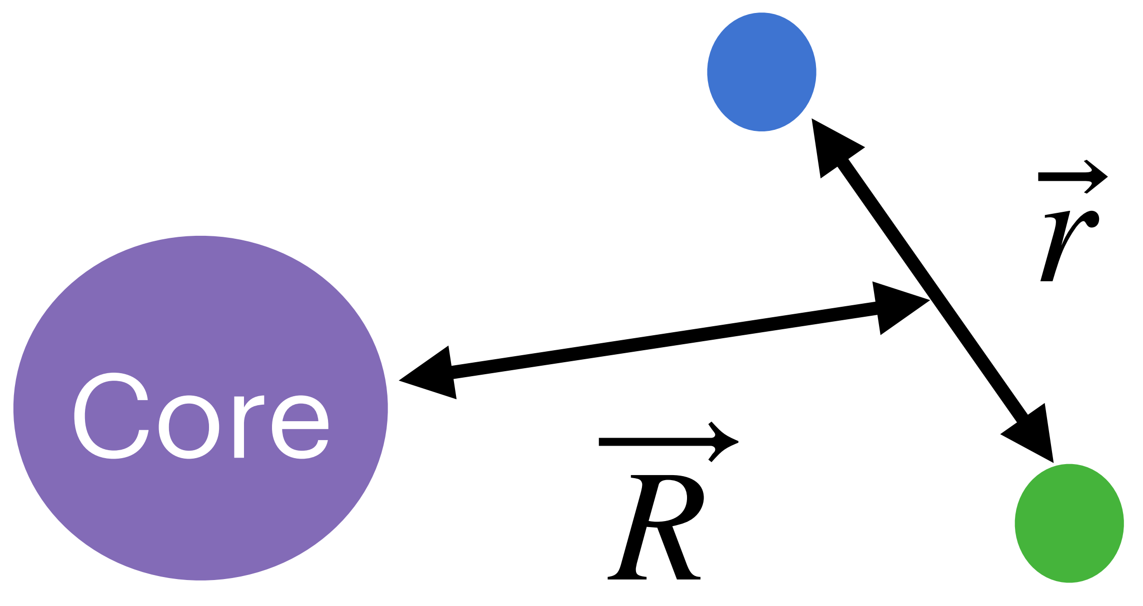
where $\psi = \psi_\lambda + \psi_{1\lambda} + \psi_{2\lambda}$

Overview

Expend it in the set of basis $|\, k[lL]JM\big) \equiv |\, \phi_l(k,r)\left[Y_l(\hat{r}), Y_L(\hat{R})\right]_{JM}\Big)$,

$$\left[E - K - V(r)\right]\psi_\lambda = \mathscr{P}_\lambda\left(U_1 + U_2\right)\mathscr{P}_\lambda\psi_\lambda + V\left[\psi_{1\lambda} + \psi_{2\lambda}\right]$$

$$\left[E - \epsilon(k) - K_R\right] g^J_{\lambda LL}[P(k), \vec{R}] = \int d^3r d\hat{R} \left\{\phi_l\left(k', r\right)\left[Y_1(r), Y_L(R)\right]_{JM}\right\}^* \left\{\mathscr{P}_\lambda\left(U_1 + U_2\right)\mathscr{P}'_\lambda\psi' + V\left[\psi_{1\lambda} + \psi_{2\lambda}\right]\right\}$$



$$U_\lambda \equiv \mathscr{P}_\lambda\left(U_1 + U_2\right)\mathscr{P}'_\lambda \qquad \xi_\lambda \equiv \psi_{1\lambda} + \psi_{2\lambda}$$

Overview

$$\left[E-K-U_1\left(r_1\right)\right]\psi_{1\lambda}=\mathscr{P}_\lambda\left[U_1-\mathscr{P}_\lambda U_1\mathscr{P}_\lambda\right]\psi_\lambda+\mathscr{P}_\lambda U_1\psi_{2\lambda}$$
$$\left[E-K-U_2\left(r_2\right)\right]\psi_{2\lambda}=\mathscr{P}_\lambda\left[U_2-\mathscr{P}_\lambda U_2\mathscr{P}_\lambda\right]\psi_\lambda+\mathscr{P}_\lambda U_2\psi_{1\lambda},$$

With "Distorted-wave" Faddeev differential equations , they get

$$\left[E-K-U_1-U_2\right]\xi_\lambda=\left[U_1+U_2-U_\lambda\right]\psi_\lambda,$$
$$\left[E-K-V-U_\lambda\right]\psi_\lambda=V\xi_\lambda,$$

$$U\equiv U_1+U_2$$

$$\left[E-K-U\right]\xi_\lambda=\left[U-U_\lambda\right]\psi_\lambda,$$
$$\left[E-K-V-U_\lambda\right]\psi_\lambda=V\xi_\lambda,$$

Overview

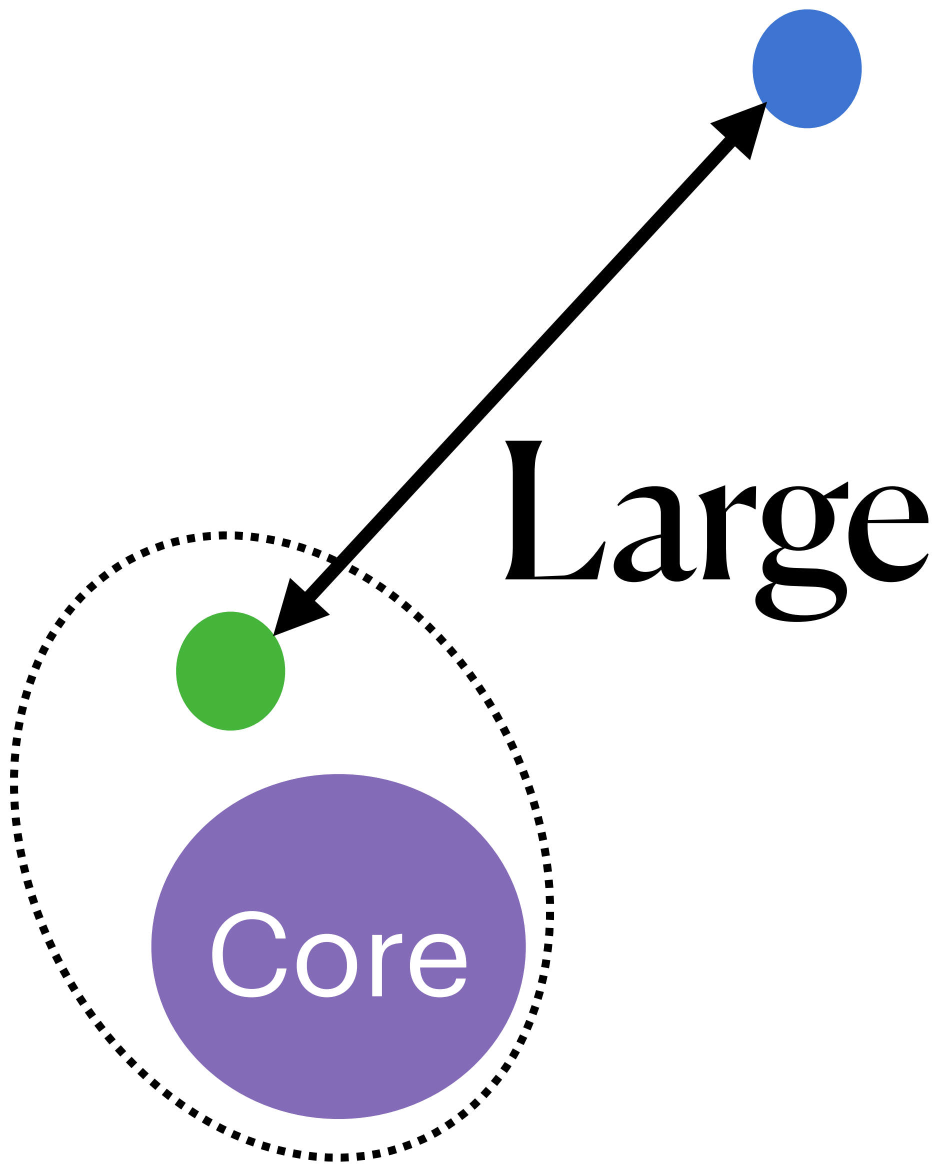
$\xi_\lambda \equiv \psi_{1\lambda} + \psi_{2\lambda}$ just has outgoing wave,

Source term

$$\xi_\lambda = \left[E^+ - K - U\right]^{-1} \left[U - U_\lambda\right] \psi_\lambda,$$

This term will be small in the range of V.

Source term show there will be a 'hole' near the origin, where the low multipoles of U cancel with those of U_λ.



CDCC

$$(E - K - V - U)\psi = 0$$

$$(E - K - V - \mathscr{P}U)\mathscr{P}\psi = \mathscr{P}U\mathscr{Q}\psi,$$

$$(E - K - V - \mathscr{P}U)\mathscr{P}\psi = \mathscr{P}U(1 - \mathscr{P})\psi,$$

In the CDCC calculation, they assume

$$(E - K - V - \mathscr{P}U\mathscr{P})\psi^{CDCC} = 0,$$

CDCC

$$(E - K - V - \mathscr{P}U)\mathscr{P}\psi = \mathscr{P}U(1 - \mathscr{P})\psi,$$

Now, however, by the analysis given before, we recognize that $\mathscr{P}_\lambda U(1-\mathscr{P}_\lambda)\psi$ must be negligible in a domain $\mathscr{D}$, defined by $R<R_c$. We assume that we can find R_c in such a way that it is small enough to ensure that $\mathscr{P}_\lambda U(1-\mathscr{P}_\lambda)\psi$ is small in $\mathscr{D}$, so that partial waves with $l<\lambda$ are not much affected in $\mathscr{D}$ by those with $l>\lambda$. At the same time, R_c is large enough to ensure that the truncated tail of $\mathscr{P}_\lambda U \mathscr{P}_\lambda$ beyond R_c is small and smooth, so that the reflection of $\mathscr{P}_\lambda \psi_\lambda$ from outside is negligible. Then $\mathscr{P}_\lambda \psi$ satisfies approximately the same equation in $\mathscr{D}$ and approximately the same boundary conditions at $R=R_c$ as ψ_λ, so that $\psi_\lambda \approx \mathscr{P}_\lambda \psi$ in $\mathscr{D}$. The larger λ is, the larger R_c can be and the closer ψ_λ is to ψ.

$$(E - K - V - \mathscr{P}U\mathscr{P})\psi^{CDCC} = 0,$$

This term must be small.

$$\left[E - K - V - U_\lambda\right]\psi_\lambda = V\xi_\lambda,$$

$$\xi_\lambda = [E - K - U]^{-1}\left[U - U_\lambda\right]\psi_\lambda$$