Vertex Correction

In[27]:=
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Γ[q_, M_, μ_] = (-e^3)/(16 π^4) TID[GAD[ρ].(GSD[p2 + q + k] + M).GAD[μ].(GSD[p2 + k] + M).GAD[ρ] ×
    FAD[{k, 0}, {p2 + q + k, M}, {p2 + k, M}], k, ToPaVe → True, UsePaVeBasis → True]
```

Out[27]=
$$-\frac{1}{16\pi^4}e^3\left(-i(2-D)\pi^2\,\mathrm{B}_0\left(q^2, M^2, M^2\right)\gamma^\mu + 2i(2-D)\pi^2\,\mathrm{C}_{00}\left(M^2, M^2+2(\mathrm{p2}\cdot q)+q^2, q^2, M^2, 0, M^2\right)\gamma^\mu - i\pi^2\,\mathrm{C}_0\left(M^2, q^2, M^2+2(\mathrm{p2}\cdot q)+q^2, 0, M^2, M^2\right)\left(-D\,M\,(\gamma\cdot q).\gamma^\mu + 4\,M\,(\gamma\cdot q).\gamma^\mu - 2\,D\,(\gamma\cdot \mathrm{p2}).\gamma^\mu.(\gamma\cdot \mathrm{p2}) + 4\,(\gamma\cdot \mathrm{p2}).\gamma^\mu.(\gamma\cdot \mathrm{p2}) + 4\,D\,\gamma\cdot \mathrm{p2}\,\mathrm{p2}^\mu - 8\,\gamma\cdot \mathrm{p2}\,\mathrm{p2}^\mu - 4\,M\,q^\mu\right) + i\pi^2\left(-D\,M\,\gamma^\mu.(\gamma\cdot \mathrm{p2}) + 4\,M\,\gamma^\mu.(\gamma\cdot \mathrm{p2}) - D\,M\,(\gamma\cdot \mathrm{p2}).\gamma^\mu + 4\,M\,(\gamma\cdot \mathrm{p2}).\gamma^\mu + 2\,D\,(\gamma\cdot \mathrm{p2}).\gamma^\mu.(\gamma\cdot \mathrm{p2}) - 4\,(\gamma\cdot \mathrm{p2}).\gamma^\mu.(\gamma\cdot \mathrm{p2}) + 2\,(\gamma\cdot \mathrm{p2}).\gamma^\mu.(\gamma\cdot q) + D\,(\gamma\cdot q).\gamma^\mu.(\gamma\cdot \mathrm{p2}) - 4\,(\gamma\cdot q).\gamma^\mu.(\gamma\cdot \mathrm{p2}) - 8\,M\,\mathrm{p2}^\mu - 4\,D\,\gamma\cdot \mathrm{p2}\,\mathrm{p2}^\mu + 8\,\gamma\cdot \mathrm{p2}\,\mathrm{p2}^\mu\right)\mathrm{C}_1\left(M^2, M^2+2(\mathrm{p2}\cdot q)+q^2, q^2, M^2, 0, M^2\right) - i\pi^2\left(-D\,M\,\gamma^\mu.(\gamma\cdot q) + 4\,M\,\gamma^\mu.(\gamma\cdot q) - D\,M\,(\gamma\cdot q).\gamma^\mu + 4\,M\,(\gamma\cdot q).\gamma^\mu + D\,(\gamma\cdot \mathrm{p2}).\gamma^\mu.(\gamma\cdot q) - 2\,(\gamma\cdot \mathrm{p2}).\gamma^\mu.(\gamma\cdot q) + D\,(\gamma\cdot q).\gamma^\mu.(\gamma\cdot \mathrm{p2}) - 2\,(\gamma\cdot q).\gamma^\mu.(\gamma\cdot \mathrm{p2}) + D\,(\gamma\cdot q).\gamma^\mu.(\gamma\cdot q) - 2\,(\gamma\cdot q).\gamma^\mu.(\gamma\cdot q) - 2\,D\,\gamma\cdot q\,\mathrm{p2}^\mu + 4\,\gamma\cdot q\,\mathrm{p2}^\mu - 8\,M\,q^\mu - 2\,D\,\gamma\cdot \mathrm{p2}\,q^\mu + 4\,\gamma\cdot \mathrm{p2}\,q^\mu\right)\mathrm{C}_2\left(M^2, M^2+2(\mathrm{p2}\cdot q)+q^2, q^2, M^2, 0, M^2\right) + 2i(2-D)\pi^2\,\gamma\cdot \mathrm{p2}\,\mathrm{p2}^\mu\,\mathrm{C}_{11}\left(M^2, M^2+2(\mathrm{p2}\cdot q)+q^2, q^2, M^2, 0, M^2\right) - 2i(2-D)\pi^2\left(\gamma\cdot q\,\mathrm{p2}^\mu + \gamma\cdot \mathrm{p2}\,q^\mu\right)\mathrm{C}_{12}\left(M^2, M^2+2(\mathrm{p2}\cdot q)+q^2, q^2, M^2, 0, M^2\right) + 2i(2-D)\pi^2\,\gamma\cdot q\,q^\mu\,\mathrm{C}_{22}\left(M^2, M^2+2(\mathrm{p2}\cdot q)+q^2, q^2, M^2, 0, M^2\right)\right)$$

In[28]:=
```
Γ0 = Γ[q, M, μ] /. q → 0 // Simplify
```

Out[28]=
$$\frac{1}{16\pi^2}ie^3\left(-\left((D-2)\,\gamma^\mu\,\mathrm{B}_0\left(0, M^2, M^2\right)\right) + \mathrm{C}_0\left(M^2, 0, M^2, 0, M^2, M^2\right)\left(-\left((D-4)\,M\,0.\gamma^\mu\right) - 2\,(D-2)\left((\gamma\cdot \mathrm{p2}).\gamma^\mu.(\gamma\cdot \mathrm{p2}) - 2\,\mathrm{p2}^\mu\,\gamma\cdot \mathrm{p2}\right)\right) + 2\,(D-2)\,\gamma^\mu\,\mathrm{C}_{00}\left(M^2, M^2, 0, M^2, 0, M^2\right) + \mathrm{C}_1\left(M^2, M^2, 0, M^2, 0, M^2\right)\left((D-4)\,M\,\gamma^\mu.(\gamma\cdot \mathrm{p2}) + (D-4)\,M\,(\gamma\cdot \mathrm{p2}).\gamma^\mu - D\,0.\gamma^\mu.(\gamma\cdot \mathrm{p2}) - 2\,D\,(\gamma\cdot \mathrm{p2}).\gamma^\mu.(\gamma\cdot \mathrm{p2}) + 4\,D\,\mathrm{p2}^\mu\,\gamma\cdot \mathrm{p2} + 8\,M\,\mathrm{p2}^\mu + 4\times 0.\gamma^\mu.(\gamma\cdot \mathrm{p2}) - 2\,(\gamma\cdot \mathrm{p2}).\gamma^\mu.0 + 4\,(\gamma\cdot \mathrm{p2}).\gamma^\mu.(\gamma\cdot \mathrm{p2}) - 8\,\mathrm{p2}^\mu\,\gamma\cdot \mathrm{p2}\right) + 2\,(D-2)\,\mathrm{p2}^\mu\,\gamma\cdot \mathrm{p2}\,\mathrm{C}_{11}\left(M^2, M^2, 0, M^2, 0, M^2\right) + \mathrm{C}_2\left(M^2, M^2, 0, M^2, 0, M^2\right)\left(-\left((D-4)\,M\,0.\gamma^\mu\right) - (D-4)\,M\,\gamma^\mu.0 + (D-2)\,(0.\gamma^\mu.0 + 0.\gamma^\mu.(\gamma\cdot \mathrm{p2}) + (\gamma\cdot \mathrm{p2}).\gamma^\mu.0)\right)\right)$$

In[29]:=
```
Γ_ren = PaVeReduce[Γ[p1 - p3, M, μ] - Γ0] // Simplify
```

Out[29]=
$$-\frac{ie^3\left(-2\,(D-2)\,\gamma^\mu\,\mathrm{B}_0\left(0,M^2,M^2\right)+\cdots 12 \cdots\right)}{32\pi^2}$$

large output | show less | show more | show all | set size limit...

In[30]:=
```
VertexM = e Spinor[Momentum[p3], m].GAD[ν].Spinor[Momentum[p1], m]
    MT[μ, ν]
  ---------- Spinor[Momentum[p4], M].Γren.Spinor[Momentum[p2], M]
  SP[p1 - p3]
```

Out[30]=

$$\frac{e\left(\varphi\left(\overline{\text{p3}},m\right)\right).\gamma^{\nu}.\left(\varphi\left(\overline{\text{p1}},m\right)\right)\left(\varphi\left(\overline{\text{p4}},M\right)\right).\left(-\frac{\text{<< 1 >>}}{32\,\pi^2}\right).\left(\varphi\left(\overline{\text{p2}},M\right)\right)\bar{g}^{\mu\nu}}{\left(\overline{\text{p1}}-\overline{\text{p3}}\right)^2}$$

large output | show less | show more | show all | set size limit...

In[31]:=
```
1/4 ChangeDimension[Contract[FermionSpinSum[ComplexConjugate[M0] * VertexM]], D] //
  DiracSimplify;
% /. e^6 → alpha^3 * 64 * π^3 // FullSimplify;
Xv = PaVeReduce[%, PaVeAutoReduce → True]
```

Out[33]=

$$\frac{32\,(2-D)\,M^2\,\pi\left(m^4+2\,M^2\,m^2-2\,s\,m^2-t\,m^2+M^4+s^2-2\,M^2\,s+s\,t\right)\text{alpha}^3}{\left(4\,M^2-t\right)t^2}-\frac{1}{\left(4\,M^2-t\right)t^2}$$

$$4\,(2-D)\,\pi\left(8\,M^6+16\,m^2\,M^4-16\,s\,M^4-4\,t\,M^4+8\,m^4\,M^2+8\,s^2\,M^2+4\,D\,t^2\,M^2-8\,t^2\,M^2-16\,m^2\,s\,M^2+16\,s\,t\,M^2-D\,t^3+2\,t^3-4\,s\,t^2-4\,m^4\,t-4\,s^2\,t+8\,m^2\,s\,t\right)\text{B}_0\left(0,\,M^2,\,M^2\right)\text{alpha}^3-\frac{1}{\left(4\,M^2-t\right)t^2}$$

$$4\,\pi\left(48\,M^6+96\,m^2\,M^4-96\,s\,M^4-20\,t\,M^4+48\,m^4\,M^2+48\,s^2\,M^2+20\,D\,t^2\,M^2-40\,t^2\,M^2-96\,m^2\,s\,M^2-8\,m^2\,t\,M^2+88\,s\,t\,M^2-5\,D\,t^3+10\,t^3-20\,s\,t^2-20\,m^4\,t-20\,s^2\,t+40\,m^2\,s\,t\right)\text{B}_0\left(M^2,\,0,\,M^2\right)\text{alpha}^3+\frac{1}{\left(4\,M^2-t\right)t^2}4\,\pi\left(32\,M^6+64\,m^2\,M^4-64\,s\,M^4+4\,D\,t\,M^4-28\,t\,M^4+32\,m^4\,M^2+32\,s^2\,M^2-4\,D^2\,t^2\,M^2+36\,D\,t^2\,M^2-56\,t^2\,M^2-64\,m^2\,s\,M^2-8\,D\,m^2\,t\,M^2+24\,m^2\,t\,M^2-8\,D\,s\,t\,M^2+88\,s\,t\,M^2+D^2\,t^3-9\,D\,t^3+14\,t^3+4\,D\,s\,t^2-28\,s\,t^2+4\,D\,m^4\,t-28\,m^4\,t+4\,D\,s^2\,t-28\,s^2\,t-8\,D\,m^2\,s\,t+56\,m^2\,s\,t\right)\text{B}_0\left(t,\,M^2,\,M^2\right)\text{alpha}^3+\frac{1}{t^2}8\,(1-D)\,M^2\,\pi\left(4\,m^4+8\,M^2\,m^2-8\,s\,m^2+4\,M^4+4\,s^2+D\,t^2-2\,t^2-8\,M^2\,s+4\,s\,t\right)\text{C}_0\left(0,\,M^2,\,M^2,\,M^2,\,M^2,\,0\right)\text{alpha}^3-\frac{1}{t^2}$$

$$8\,\pi\left(2\,M^2-t\right)\left(4\,m^4+8\,M^2\,m^2-8\,s\,m^2+4\,M^4+4\,s^2+D\,t^2-2\,t^2-8\,M^2\,s+4\,s\,t\right)\text{C}_0\left(M^2,\,M^2,\,t,\,M^2,\,0,\,M^2\right)\text{alpha}^3+\frac{1}{t^2}$$

$$8\,M^2\,\pi\left(-8\,D\,m^4+4\,m^4-16\,D\,M^2\,m^2+8\,M^2\,m^2+16\,D\,s\,m^2-8\,s\,m^2+4\,D\,t\,m^2-8\,D\,M^4+4\,M^4-8\,D\,s^2+4\,s^2-D^2\,t^2+3\,D\,t^2-2\,t^2+16\,D\,M^2\,s-8\,M^2\,s-8\,D\,s\,t+4\,s\,t\right)\text{C}_1\left(M^2,\,M^2,\,0,\,M^2,\,0,\,M^2\right)\text{alpha}^3+\frac{1}{t^2}$$

$$32\,(2-D)\,M^2\,\pi\left(m^4+2\,M^2\,m^2-2\,s\,m^2-t\,m^2+M^4+s^2-2\,M^2\,s+s\,t\right)\text{C}_{11}\left(M^2,\,M^2,\,0,\,M^2,\,0,\,M^2\right)\text{alpha}^3$$