

# TL

## Amplitude

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In[*]:= Ma = I e^2 Spinor[Momentum[p3], m].GA[v].Spinor[Momentum[p1], m]
          MT[μ, ν]
          SP[p1 - p3] Spinor[Momentum[p4], M].GA[μ].Spinor[Momentum[p2], M]
Out[*]:= 
$$i e^2 \bar{g}^{\mu\nu} (\varphi(\bar{p}_3, m)) \cdot \bar{\gamma}^\nu \cdot (\varphi(\bar{p}_1, m)) (\varphi(\bar{p}_4, M)) \cdot \bar{\gamma}^\mu \cdot (\varphi(\bar{p}_2, M))$$

          
$$\frac{1}{(\bar{p}_1 - \bar{p}_3)^2}$$

In[*]:= X0 = 
$$\frac{1}{4} \text{Contract}[\text{FermionSpinSum}[\text{Ma} * \text{ComplexConjugate}[\text{Ma}]]] /. e^4 \rightarrow \text{alpha}^2 * 16 \pi^2 //$$

          DiracSimplify // FullSimplify
Out[*]:= 
$$\frac{32 \pi^2 \text{alpha}^2 (2(m^2 + M^2 - s)^2 + 2 s t + t^2)}{t^2}$$

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## Electron vertex

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In[12]:= Γ[q_, p_, m_, μ_] :=
          -I e^2 PaVeReduce[TID[GAD[ρ].(GSD[p + q + k] + m).GAD[μ].(GSD[p + k] + m).GAD[ρ] ×
          FAD[{k, SmallVariable[λ]}, {p + q + k, m}, {p + k, m}], k, ToPaVe → True]]
In[13]:= δ1[m_] := ((-e^2) / (8 π^2)) (1 / (2 Epsilon) + (1 / 2) Log[4 π] +
          (1 / 2) Log[ScaleMu^2 / m^2] + (5 - EulerGamma) / 2 + Log[SmallVariable[λ]^2 / m^2])
In[14]:= ΓRenE := 
$$\frac{1}{16 \pi^4} \Gamma[p2 - p4, p1, m, \nu] + \text{GAD}[\nu] \times \delta 1[m]$$

In[15]:= VertexERen = I e^2 Spinor[Momentum[p3], m].ΓRenE.Spinor[Momentum[p1], m]
          MT[μ, ν]
          SP[p1 - p3] Spinor[Momentum[p4], M].GAD[μ].Spinor[Momentum[p2], M];
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In[18]:=  $\frac{1}{4}$  ChangeDimension [Contract [FermionSpinSum [ComplexConjugate [Ma] * VertexEREN]], D] //
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DiracSimplify;
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X2 = % /. e^6 -> alpha^3 * 64 * pi^3 // FullSimplify
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Out[19]=  $-\frac{1}{\epsilon t^2 (t - 4 m^2)^2} 4 \text{alpha}^3 \pi$ 
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$$\left( -8 (D - 2) \epsilon (4 m^2 - t) \left( (m^2 + M^2 - s)^2 + (s - M^2) t \right) B_0(0, m^2, m^2) m^2 - (4 m^2 - t) \left( 8 (\epsilon (D + 2 (-6 + \gamma)) - 2) m^6 + \right. \right. \\ \left. \left. 4 (4 (\epsilon (D + 2 (-6 + \gamma)) - 2) (M^2 - s) - \epsilon (-5 + \gamma) t + t) m^4 + 4 (2 (\epsilon (D + 2 (-6 + \gamma)) - 2) (M^2 - s)^2 + \right. \right. \\ \left. \left. (D - 2) (\epsilon (-5 + \gamma) - 1) t^2 + 2 (-\epsilon (D + \gamma - 7) M^2 + M^2 + \epsilon (D + 3 \gamma - 17) s - 3 s) t \right) m^2 - \right. \\ \left. (\epsilon (-5 + \gamma) - 1) t (4 (M^2 - s)^2 + (D - 2) t^2 + 4 s t) - \epsilon (4 m^2 - t) \right. \\ \left. \left( 4 (m^2 + M^2 - s)^2 + (D - 2) t^2 + 4 s t \right) \left( \log \left( \frac{4 \pi \mu^2}{m^2} \right) + 2 \log \left( \frac{\lambda^2}{m^2} \right) \right) \right) - \\ 2 \epsilon B_0(m^2, m^2, \lambda^2) \left( (D - 2) t^3 - 4 ((D - 2) (m - M) (m + M) + (D - 3) s) t^2 - 4 (D - 3) m^4 + \right. \\ \left. 2 ((5 D - 9) M^2 - 5 D s + 11 s) m^2 + (D - 3) (M^2 - s)^2 t + 32 (D - 2) m^2 (m^2 + M^2 - s)^2 \right) \lambda^2 + \\ 2 (4 m^2 - t) \left( -(D - 2) t^3 + 4 ((D - 2) m^2 - s) t^2 - 4 (m^4 - 4 s m^2 + (M^2 - s)^2) t + 8 m^2 (m^2 + M^2 - s)^2 \right) + \\ \epsilon (B_0(t, m^2, m^2) \left( 2 ((D - 2) t^3 + 4 (s - (D - 2) m^2) t^2 + 4 (m^4 + ((6 - 4 D) M^2 + 2 (2 D - 5) s) m^2 + (M^2 - s)^2) t + \right. \\ \left. 16 (D - 2) m^2 (m^2 + M^2 - s)^2 \right) \lambda^2 + \\ (4 m^2 - t) \left( (D - 7) (D - 2) t^3 - 4 (D - 7) ((D - 2) m^2 - s) t^2 + 4 (D - 7) m^4 - \right. \\ \left. 2 ((D - 3) M^2 + (D - 11) s) m^2 + (D - 7) (M^2 - s)^2 t + 32 m^2 (m^2 + M^2 - s)^2 \right) + \\ C_0(m^2, m^2, t, m^2, \lambda^2, m^2) \left( 2 ((D - 2) t^3 + 4 (s - (D - 2) m^2) t^2 + 4 (m^4 + \right. \\ \left. ((6 - 4 D) M^2 + 2 (2 D - 5) s) m^2 + (M^2 - s)^2) t + 16 (D - 2) m^2 (m^2 + M^2 - s)^2 \right) \lambda^4 + \\ (t - 4 m^2) \left( -(D - 8) (D - 2) t^3 + 4 (D - 8) ((D - 2) m^2 - s) t^2 - 4 (D - 8) m^4 + \right. \\ \left. 2 (D M^2 - 3 D s + 16 s) m^2 + (D - 8) (M^2 - s)^2 t + 16 (D - 4) m^2 (m^2 + M^2 - s)^2 \right) \lambda^2 - \\ \left. \left. 2 (2 m^2 - t) (t - 4 m^2)^2 (4 (m^2 + M^2 - s)^2 + (D - 2) t^2 + 4 s t) \right) \right)$$

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In[20]:= PaXEvaluateUV[X2] // FullSimplify
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Out[20]=  $\frac{64 \pi \text{alpha}^3 \lambda^2 \left( (m^2 + M^2 - s)^2 + t (s - M^2) \right)}{t^2 (4 m^2 - t) \epsilon_{UV}}$ 
```