In[1]:=
```
(* :Title: QEDComptonScatteringTree                                    *)

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   Copyright (C) 2014-2016 Vladyslav Shtabovenko
*)

(* :Summary:  Computation of the matrix element squared for Compton
                             scattering in QED at tree level

(* -------------------------------------------------------------------- *)
```

Load FeynCalc and FeynArts

In[2]:=
```
If[ $FrontEnd === Null,
        $FeynCalcStartupMessages = False;
        Print["Computation of the matrix element squared for Compton scatterin
];
$LoadFeynArts= True;
<<FeynCalc`
$FAVerbose = 0;
```

FeynCalc 9.2.0. For help, use the
documentation center, check out the wiki or write to the mailing list.

See also the supplied examples. If you use FeynCalc in your research, please cite

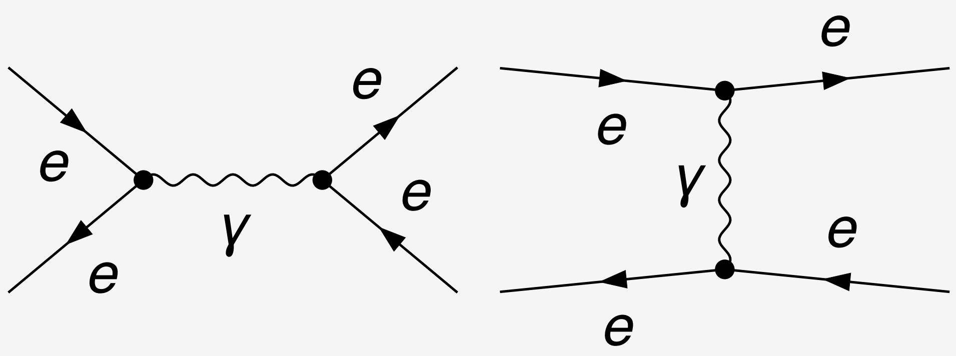
- V. Shtabovenko, R. Mertig and F. Orellana,
  Comput. Phys. Commun., 207C, 432–444, 2016, arXiv:1601.01167
- R. Mertig, M. Böhm, and A. Denner, Comput. Phys. Commun., 64, 345–359, 1991.

FeynArts 3.9 patched for use with FeynCalc, for documentation use the
manual or visit www.feynarts.de.

Generate Feynman diagrams

In[6]:=

```
topCompton = CreateTopologies[0, 2 -> 2];
diagsCompton = InsertFields[topCompton, {F[2, {1}], -F[2, {1}]} ->
        {F[2, {1}], -F[2, {1}]}, InsertionLevel -> {Classes},
        Model -> "SM", ExcludeParticles -> {S[1], S[2], V[2]}];
Paint[diagsCompton, ColumnsXRows -> {2, 1}, Numbering -> None,SheetHeader->Non
ImageSize->{512,256}];
```



Obtain corresponding amplitudes

In[9]:=

```
ampCompton=FCFAConvert[CreateFeynAmp[diagsCompton, Truncated -> False],
IncomingMomenta->{p1,k1},OutgoingMomenta->{p2,k2},TransversePolarizationVector
UndoChiralSplittings->True,ChangeDimension->4,List->False, SMP->True]
```

Out[9]=

$$\frac{1}{(\overline{k2}+\overline{p2})^2}\bar{g}^{Lor1\,Lor2}\left(\varphi(-\overline{k1},m_e)\right).\left(i\,e\,\bar{\gamma}^{Lor1}\right).\left(\varphi(\overline{p1},m_e)\right)\left(\varphi(\overline{p2},m_e)\right).\left(i\,e\,\bar{\gamma}^{Lor2}\right).\left(\varphi(-\overline{k2},m_e)\right)-\frac{1}{(\overline{k2}-\overline{k1})^2}\bar{g}^{Lor1\,Lor2}\left(\varphi(-\overline{k1},m_e)\right).\left(i\,e\,\bar{\gamma}^{Lor2}\right).\left(\varphi(-\overline{k2},m_e)\right)\left(\varphi(\overline{p2},m_e)\right).\left(i\,e\,\bar{\gamma}^{Lor1}\right).\left(\varphi(\overline{p1},m_e)\right)$$

Unpolarized process e^- gamma -> e^- gamma

In[10]:=
```
SetMandelstam[s, t, u, p1, k1, -p2, -k2, SMP["m_e"], 0];
sqAmpCompton = (ampCompton (ComplexConjugate[ampCompton]//FCRenameDummyIndices
        PropagatorDenominatorExplicit//Expand//DoPolarizationSums[#,k1,0,Extra
        1/2]&//DoPolarizationSums[#,k2,0]&//Contract//FermionSpinSum[#, ExtraF
        ReplaceAll[#, DiracTrace :> Tr] & // Contract//Simplify//TrickMandelst
```

Out[11]=

$$
\begin{aligned}
&\Big(32 e^4 \Big(6 \overline{k2}^4 m_e^4 + 8 (\overline{k2}\cdot\overline{p2})^2 m_e^4 + 2 \overline{p2}^4 m_e^4 + 12 \overline{k2}^2 (\overline{k2}\cdot\overline{p2}) m_e^4 + 6 \overline{k2}^2 \overline{p2}^2 m_e^4 + \\
&8 (\overline{k2}\cdot\overline{p2}) \overline{p2}^2 m_e^4 + 2 (\overline{k1}\cdot\overline{p1}) \overline{k2}^4 m_e^2 + (\overline{k1}\cdot\overline{p2}) \overline{k2}^4 m_e^2 + 2 \overline{k2}^2 (\overline{k2}\cdot\overline{p2})^2 m_e^2 - \\
&(\overline{p1}\cdot\overline{p2}) \overline{p2}^4 m_e^2 + \overline{k2}^4 (\overline{k2}\cdot\overline{p1}) m_e^2 + 2 \overline{k2}^4 (\overline{k2}\cdot\overline{p2}) m_e^2 + 2 (\overline{k1}\cdot\overline{p1}) \overline{k2}^2 (\overline{k2}\cdot\overline{p2}) m_e^2 + \\
&2 (\overline{k1}\cdot\overline{p2}) \overline{k2}^2 (\overline{k2}\cdot\overline{p2}) m_e^2 + 2 \overline{k2}^2 (\overline{k2}\cdot\overline{p1}) (\overline{k2}\cdot\overline{p2}) m_e^2 - 2 \overline{k2}^4 (\overline{p1}\cdot\overline{p2}) m_e^2 - \\
&4 (\overline{k2}\cdot\overline{p2})^2 (\overline{p1}\cdot\overline{p2}) m_e^2 - 6 \overline{k2}^2 (\overline{k2}\cdot\overline{p2}) (\overline{p1}\cdot\overline{p2}) m_e^2 + (\overline{k1}\cdot\overline{p1}) \overline{k2}^2 \overline{p2}^2 m_e^2 + (\overline{k1}\cdot\overline{p2}) \overline{k2}^2 \overline{p2}^2 m_e^2 + \\
&\overline{k2}^2 (\overline{k2}\cdot\overline{p1}) \overline{p2}^2 m_e^2 + \overline{k2}^2 (\overline{k2}\cdot\overline{p2}) \overline{p2}^2 m_e^2 - 3 \overline{k2}^2 (\overline{p1}\cdot\overline{p2}) \overline{p2}^2 m_e^2 - 4 (\overline{k2}\cdot\overline{p2}) (\overline{p1}\cdot\overline{p2}) \overline{p2}^2 m_e^2 + \\
&4 (\overline{k1}\cdot\overline{p1}) (\overline{k2}\cdot\overline{p2})^3 + 4 (\overline{k1}\cdot\overline{p1}) \overline{k2}^2 (\overline{k2}\cdot\overline{p2})^2 + 4 (\overline{k1}\cdot\overline{p2}) (\overline{k2}\cdot\overline{p1}) (\overline{k2}\cdot\overline{p2})^2 + \\
&(\overline{k1}\cdot\overline{p2}) (\overline{k2}\cdot\overline{p1}) \overline{p2}^4 + (\overline{k1}\cdot\overline{p1}) (\overline{k2}\cdot\overline{p2}) \overline{p2}^4 + 4 (\overline{k1}\cdot\overline{p2}) \overline{k2}^4 (\overline{k2}\cdot\overline{p1}) + (\overline{k1}\cdot\overline{p1}) \overline{k2}^4 (\overline{k2}\cdot\overline{p2}) + \\
&8 (\overline{k1}\cdot\overline{p2}) \overline{k2}^2 (\overline{k2}\cdot\overline{p1}) (\overline{k2}\cdot\overline{p2}) + 4 (\overline{k1}\cdot\overline{k2})^3 (\overline{p1}\cdot\overline{p2}) + 4 (\overline{k1}\cdot\overline{p1}) (\overline{k2}\cdot\overline{p2})^2 \overline{p2}^2 + \\
&4 (\overline{k1}\cdot\overline{p2}) \overline{k2}^2 (\overline{k2}\cdot\overline{p1}) \overline{p2}^2 + 2 (\overline{k1}\cdot\overline{p1}) \overline{k2}^2 (\overline{k2}\cdot\overline{p2}) \overline{p2}^2 + 4 (\overline{k1}\cdot\overline{p2}) (\overline{k2}\cdot\overline{p1}) (\overline{k2}\cdot\overline{p2}) \overline{p2}^2 + \\
&\overline{k1}^4 \big((2 m_e^2 + \overline{k1}\cdot\overline{p1} + \overline{k2}\cdot\overline{p2}) m_e^2 + (\overline{k1}\cdot\overline{p2}) (\overline{k2}\cdot\overline{p1}) + (\overline{k1}\cdot\overline{k2}) (\overline{p1}\cdot\overline{p2})\big) + 2 (\overline{k1}\cdot\overline{k2})^2 \\
&\quad \Big(\big(4 m_e^2 + 2 (\overline{k1}\cdot\overline{p1}) + 4 (\overline{k2}\cdot\overline{p2}) + \overline{p2}^2\big) m_e^2 + 2 (\overline{k1}\cdot\overline{p2}) (\overline{k2}\cdot\overline{p1}) + \overline{k2}^2 \big(m_e^2 - 2 (\overline{p1}\cdot\overline{p2})\big)\Big) - (\overline{k1}\cdot\overline{k2}) \\
&\quad \Big(-(\overline{p1}\cdot\overline{p2} - 2 m_e^2) \overline{k2}^4 + m_e^2 \big(12 m_e^2 + 6 (\overline{k1}\cdot\overline{p1}) + 2 (\overline{k2}\cdot\overline{p1}) + 12 (\overline{k2}\cdot\overline{p2}) - 2 (\overline{p1}\cdot\overline{p2}) + 3 \overline{p2}^2\big) \overline{k2}^2 + \\
&\quad\quad \big(2 (\overline{k2}\cdot\overline{p2}) + \overline{p2}^2\big) m_e^2 \big(4 m_e^2 + 2 (\overline{k1}\cdot\overline{p1}) + 2 (\overline{k2}\cdot\overline{p1}) + 4 (\overline{k2}\cdot\overline{p2}) - 2 (\overline{p1}\cdot\overline{p2}) + \overline{p2}^2\big) + 2 (\overline{k1}\cdot\overline{p2}) \\
&\quad\quad\quad \Big(\big(2 (\overline{k2}\cdot\overline{p2}) + \overline{p2}^2\big) \big(m_e^2 + 2 (\overline{k2}\cdot\overline{p1})\big) + \overline{k2}^2 \big(m_e^2 + 4 (\overline{k2}\cdot\overline{p1})\big)\Big)\Big) + \overline{k1}^2 \Big(-4 (\overline{p1}\cdot\overline{p2}) (\overline{k1}\cdot\overline{k2})^2 - \\
&\quad\quad \Big(\big(8 m_e^2 + 4 (\overline{k1}\cdot\overline{p1}) + 6 (\overline{k2}\cdot\overline{p2}) + \overline{p2}^2\big) m_e^2 + 4 (\overline{k1}\cdot\overline{p2}) (\overline{k2}\cdot\overline{p1}) + \overline{k2}^2 \big(m_e^2 - 2 (\overline{p1}\cdot\overline{p2})\big)\Big) (\overline{k1}\cdot\overline{k2}) + \\
&\quad\quad (\overline{k1}\cdot\overline{p2}) \Big(\big(2 (\overline{k2}\cdot\overline{p2}) + \overline{p2}^2\big) \big(m_e^2 + 2 (\overline{k2}\cdot\overline{p1})\big) + \overline{k2}^2 \big(m_e^2 + 4 (\overline{k2}\cdot\overline{p1})\big)\Big) + \\
&\quad\quad m_e^2 \Big((\overline{k1}\cdot\overline{p1}) \big(3 \overline{k2}^2 + 2 (\overline{k2}\cdot\overline{p2}) + \overline{p2}^2\big) + \big(2 (\overline{k2}\cdot\overline{p2}) + \overline{p2}^2\big) \big(2 m_e^2 + \overline{k2}\cdot\overline{p1} + \overline{k2}\cdot\overline{p2} - \overline{p1}\cdot\overline{p2}\big) + \\
&\quad\quad\quad \overline{k2}^2 \big(6 m_e^2 + \overline{k2}\cdot\overline{p1} + 3 (\overline{k2}\cdot\overline{p2}) - \overline{p1}\cdot\overline{p2}\big)\Big)\Big)\Big)\Big) \Big/ \\
&\Big(\big(\overline{k1}^2 - 2 (\overline{k1}\cdot\overline{k2}) + \overline{k2}^2\big)^2 \big(\overline{k2}^2 + 2 (\overline{k2}\cdot\overline{p2}) + \overline{p2}^2\big)^2\Big)
\end{aligned}
$$

In[12]:=
```
sqAmpComptonPeskin = (2SMP["e"]^4(SP[p1,k2]/SP[p1,k1]+SP[p1,k1]/SP[p1,k2]+
        2SMP["m_e"]^2 (1/SP[p1,k1]-1/SP[p1,k2])+SMP["m_e"]^4 (1/SP[p1,k1]-1/SP
Print["Check with Peskin and Schroeder, Eq 5.87: ", If[(sqAmpComptonPeskin-sqAr
        "CORRECT.", "!!! WRONG !!!"]];
```

Check with Peskin and Schroeder, Eq 5.87: !!! WRONG !!!

In[14]:=

```
masslessSqAmpCompton = (sqAmpCompton /. {SMP["m_e"] -> 0})//Simplify
```

Out[14]=

$$\begin{aligned}
&\Big(32\, e^4 \Big(4 \left(\overline{p1}\cdot\overline{p2}\right)\left(\overline{k1}\cdot\overline{k2}\right)^3 + 4 \left(\overline{k1}\cdot\overline{k2}\right)^2 \Big(\left(\overline{k1}\cdot\overline{p2}\right)\left(\overline{k2}\cdot\overline{p1}\right) - \overline{k2}^2 \left(\overline{p1}\cdot\overline{p2}\right)\Big) + \\
&\quad 4 \left(\overline{k1}\cdot\overline{p1}\right)\left(\overline{k2}\cdot\overline{p2}\right)^3 + 4\, \overline{k2}^2 \left(\overline{k1}\cdot\overline{p1}\right)\left(\overline{k2}\cdot\overline{p2}\right)^2 + 4 \left(\overline{k1}\cdot\overline{p2}\right)\left(\overline{k2}\cdot\overline{p1}\right)\left(\overline{k2}\cdot\overline{p2}\right)^2 + \\
&\quad 4\, \overline{p2}^2 \left(\overline{k1}\cdot\overline{p1}\right)\left(\overline{k2}\cdot\overline{p2}\right)^2 + \overline{k1}^2 \Big(-4 \left(\overline{p1}\cdot\overline{p2}\right)\left(\overline{k1}\cdot\overline{k2}\right)^2 + \\
&\qquad \left(\overline{k1}\cdot\overline{k2}\right)\Big(2\, \overline{k2}^2 \left(\overline{p1}\cdot\overline{p2}\right) - 4 \left(\overline{k1}\cdot\overline{p2}\right)\left(\overline{k2}\cdot\overline{p1}\right)\Big) + 2 \left(\overline{k1}\cdot\overline{p2}\right)\left(\overline{k2}\cdot\overline{p1}\right)\Big(2 \left(\overline{k2}\cdot\overline{p2}\right) + 2\, \overline{k2}^2 + \overline{p2}^2\Big)\Big) + \\
&\quad \left(\overline{k1}\cdot\overline{k2}\right)\Big(\overline{k2}^4 \left(\overline{p1}\cdot\overline{p2}\right) - 4 \left(\overline{k1}\cdot\overline{p2}\right)\left(\overline{k2}\cdot\overline{p1}\right)\Big(2 \left(\overline{k2}\cdot\overline{p2}\right) + 2\, \overline{k2}^2 + \overline{p2}^2\Big)\Big) + \overline{p2}^4 \left(\overline{k1}\cdot\overline{p2}\right)\left(\overline{k2}\cdot\overline{p1}\right) + \\
&\quad \overline{p2}^4 \left(\overline{k1}\cdot\overline{p1}\right)\left(\overline{k2}\cdot\overline{p2}\right) + 4\, \overline{k2}^4 \left(\overline{k1}\cdot\overline{p2}\right)\left(\overline{k2}\cdot\overline{p1}\right) + \overline{k2}^4 \left(\overline{k1}\cdot\overline{p1}\right)\left(\overline{k2}\cdot\overline{p2}\right) + \\
&\quad 8\, \overline{k2}^2 \left(\overline{k1}\cdot\overline{p2}\right)\left(\overline{k2}\cdot\overline{p1}\right)\left(\overline{k2}\cdot\overline{p2}\right) + \overline{k1}^4 \Big(\left(\overline{k1}\cdot\overline{p2}\right)\left(\overline{k2}\cdot\overline{p1}\right) + \left(\overline{k1}\cdot\overline{k2}\right)\left(\overline{p1}\cdot\overline{p2}\right)\Big) + \\
&\quad 4\, \overline{k2}^2\, \overline{p2}^2 \left(\overline{k1}\cdot\overline{p2}\right)\left(\overline{k2}\cdot\overline{p1}\right) + 2\, \overline{k2}^2\, \overline{p2}^2 \left(\overline{k1}\cdot\overline{p1}\right)\left(\overline{k2}\cdot\overline{p2}\right) + 4\, \overline{p2}^2 \left(\overline{k1}\cdot\overline{p2}\right)\left(\overline{k2}\cdot\overline{p1}\right)\left(\overline{k2}\cdot\overline{p2}\right)\Big)\Big) \Big/ \\
&\Big(\Big(-2 \left(\overline{k1}\cdot\overline{k2}\right) + \overline{k1}^2 + \overline{k2}^2\Big)^2 \Big(2 \left(\overline{k2}\cdot\overline{p2}\right) + \overline{k2}^2 + \overline{p2}^2\Big)^2\Big)
\end{aligned}$$