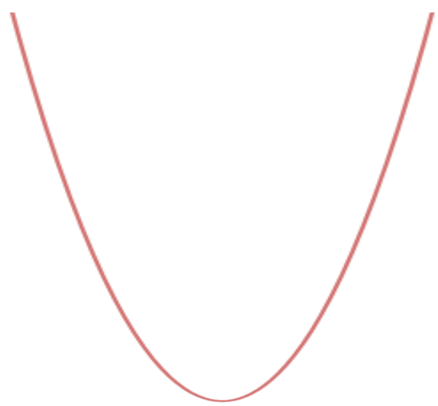
GCE/IAL applicable



Quadratic Function and equation

Content

Quadratic Function and equation

Introduction to quadratic function

$y = f(x), where\ f(x) = ax^2 + bx + c$; *where* $a, b, c\ are\ specific\ constants$.

There are some main points to explore

Complete the square

Find the symmetric axis

Find the coordinates of minimum/maximum points

Find the solutions when the function equals to 0, if any.

Main Concept 1:

Completing the square

Given a graph of the quadratic function, what can you see?

It is a curve, with specific minimum/maximum point,

and there is a symmetric axis for the graph.

Why?

When we complete the square, we will understand them all.

Given $\boldsymbol{f(x) = ax^2 + bx + c}$

$$\boldsymbol{f(x) = (ax^2 + bx) + c}$$

$$\boldsymbol{f(x) = a(x^2 + \frac{b}{a}x) + c}$$

$$\boldsymbol{f(x) = a[x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2] + c - a\left(\frac{b}{2a}\right)^2}$$

$$\boldsymbol{f(x) = a(x + \frac{b}{2a})^2 + c - a\left(\frac{b}{2a}\right)^2}$$

Exercise 1

Complete the square of the following functions

Easy parts

$\boldsymbol{a)}\ y = x^2 - 4x$

$\boldsymbol{b)}\ y = 4x^2 - 4$

$\boldsymbol{c)}\ y = x^2 - 4x + 4$

$\boldsymbol{d)}\ 2\,y = x^2 - 4x$

Challenging part

$\boldsymbol{e)}\ y = -5x^2 + 10x - 21$

$\boldsymbol{f)}\ y = 3x^2 + 8x - 1$

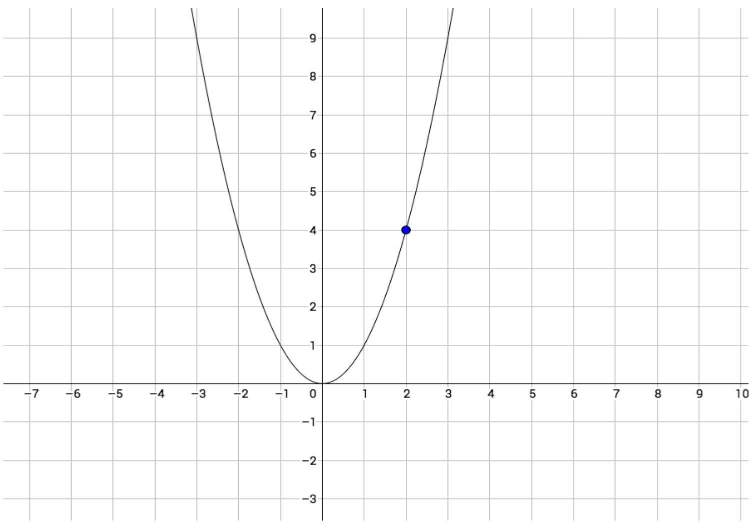
$\boldsymbol{g)}\ y = 3.5x^2 + 4.25x - 41$

$\boldsymbol{h)}\ y = -3.5x^2 - 14.25x - 71$

Main Concept 2

Finding the axis of symmetry

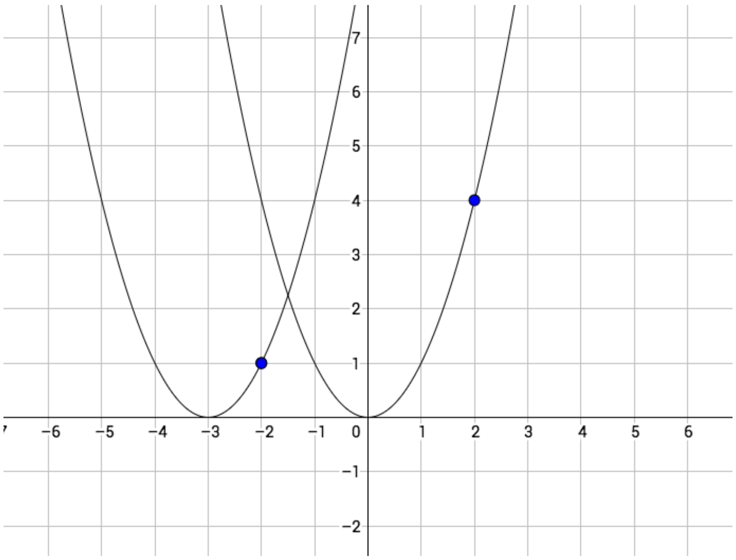
As we can see from the above deduction, $\boldsymbol{f(x) = a(x + \frac{b}{2a})^2 + c - a\left(\frac{b}{2a}\right)^2}$, so the axis of symmetry is $(\boldsymbol{x = -\frac{b}{2a}})$. We shall visualize it as follows.



This is the curve of an ordinary quadratic graph, $y = x^2$

Main Concept 2

Finding the symmetric axis

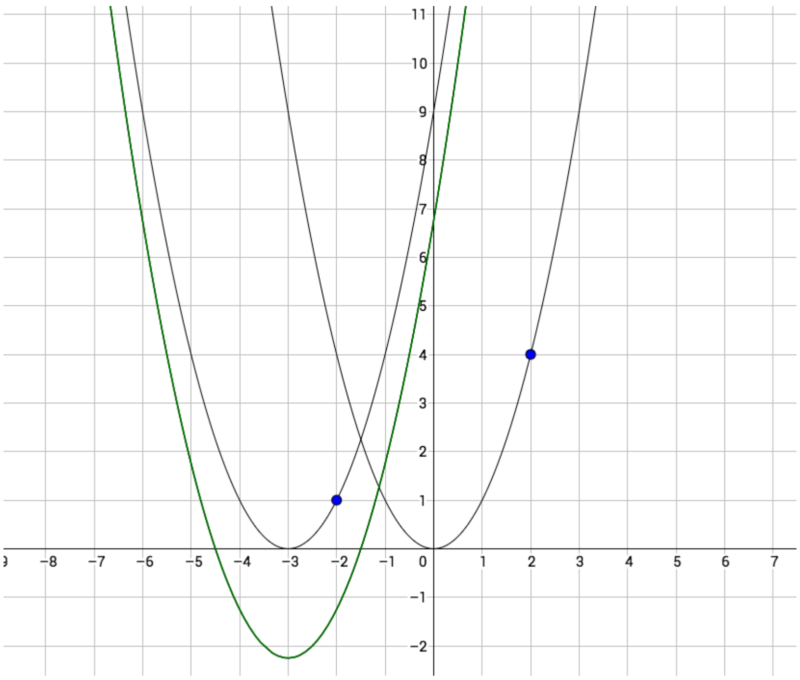


This is the curve of another graph, $\boldsymbol{y = (x+3)^2 = x^2 + 6x + 9}$

So the symmetric axis of the graph $\boldsymbol{y = (x+3)^2}$ *is* $\boldsymbol{x = -\frac{b}{2a}}$*, which is* $\boldsymbol{x = -\frac{6}{2\times 1} = -3}$

Remember, the constant c is independent in founding the positions symmetric axis.

Main Concept 2
Finding the symmetric axis



This is a graph of $\boldsymbol{y = x^2 + 6x + 6.75}$, *by completing square, it should be* $\boldsymbol{y = (x+3)^2 - 2.25}$

Exercise 2

Find the following axes of symmetry of the following function.

$\boldsymbol{a)\ y = x^2 + 4x}$

$\boldsymbol{e)\ y = -5x^2 + 10x + 21}$

$\boldsymbol{b)\ y = 4x^2 + 4}$

$\boldsymbol{f)\ y = 3x^2 + 8x + 1}$

$\boldsymbol{c)}\ \boldsymbol{y = x^2 + 4x + 4}$

$\boldsymbol{g)}\ \boldsymbol{y = 3.5x^2 + 4.25x + 41}$

$\boldsymbol{d)}\ \boldsymbol{2\,y = x^2 + 4x}$

$\boldsymbol{h)}\ \boldsymbol{y = -3.5x^2 - 14.25x + 71}$

Main Concept 3

Finding the vertex of the curve

When $(x = -\frac{b}{2a})$, then put it into the equation, we get

$$f\left(-\frac{b}{2a}\right) = a(-\frac{b}{2a})^2 + b(-\frac{b}{2a}) + c$$

$$f\left(-\frac{b}{2a}\right) = \frac{ab^2}{4a^2} - \frac{b^2}{2a} + c$$

$$f\left(-\frac{b}{2a}\right) = \frac{ab^2 - 2a(b^2) + c(4a^2)}{4a^2}$$

$$f\left(-\frac{b}{2a}\right) = \frac{4ca^2 - ab^2}{4a^2}$$

$$f\left(-\frac{b}{2a}\right) = \frac{4ac - b^2}{4a}$$

Therefore the minimum/maximum point is given by $(\frac{-b}{2a}, \frac{4ac-b^2}{4a})$or $(\frac{-b}{2a}, \frac{-\triangle}{4a})$

Main Concept 3
Finding the vertex of the curve

Exercise 3

$a)\ y = x^2 - 4x$

$e)\ y = -5x^2 + 10x - 21$

$b)\ y = 4x^2 - 4$

$f)\ y = 3x^2 + 8x - 1$

$\boldsymbol{c)\ y = x^2 - 4x + 4}$

$\boldsymbol{g)\ y = 3.5x^2 + 4.25x - 41}$

$\boldsymbol{d)\ 2\,y = x^2 - 4x}$

$\boldsymbol{h)\ y = -3.5x^2 - 14.25x - 7}$

Main Concept 4

Solving the quadratic equations:

The ability to solve equations concerning the quadratic functions are important topics of this chapter.

Usually, we will have two sides, $y_1 = f(x)$ and $y_2 = g(x)$, $f(x)$ could be a quadratic function, and $g(x)$ could be a constant including 0, a linear function or a quadratic function.

Let us explore more about quadratic equation, but first please remind the below contents

$$f(x) = ax^2 + bx + c$$

$$f(x) = (ax^2 + bx) + c$$

$$f(x) = a(x^2 + \frac{b}{a}x) + c$$

$$f(x) = a[x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\Big] + c - a\left(\frac{b}{2a}\right)^2$$

$$f(x) = a(x + \frac{b}{2a})^2 + c - a\left(\frac{b}{2a}\right)^2,$$

Main Concept 4

Finding the quadratic equations:

if f(x)=0, then $a\left(x+\frac{b}{2a}\right)^2 = a\left(\frac{b}{2a}\right)^2 - c$,

$$\left(x+\frac{b}{2a}\right) = \sqrt{\frac{b^2-4ac}{4a^2}}$$

$$x = -\frac{b}{2a} \pm \sqrt{\frac{b^2-4ac}{4a^2}}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} \; or \; x = \frac{-b\pm\sqrt{\Delta}}{2a}$$

where the Δ is called the determinant.

Note that there are some possible cases happened,

1. Distinct real roots
2. Repeated roots
3. No solutions (no real roots)

1 happens when the determinant $\Delta > 0$

2 happens when the determinant $\Delta = 0$

3 happens when the determinant $\Delta < 0$

Exercise 4

Main Concept 4

Finding the quadratic equations:

Find the solutions of the following equations

$a)\ x^2 - 1 = 0$

$b)\ x^2 - 1 = 3$

$c)\ 3x^2 + 9x = 0$

$f)\ 2x^2 + 7x - 1 = 5x + 10$

$g)\ 2x^2 + 7x - 1 = x^2 + 5x + 10$

$h)\ 4x^2 + 9x - 1 = 4x^2 + 5x + 10$

Main Concept 4

Finding the quadratic equations:

$\boldsymbol{d}$) $\mathbf{3x^2 + 8x - 9 = 0}$

$\boldsymbol{e}$) $\mathbf{3x^2 + 8x - 1 = 2x + 5}$

$\boldsymbol{i}$) $\mathbf{3x^2 + 4x - 9 = x^2 + 5x + 9}$

$\boldsymbol{j}$) $\mathbf{2x^2 + 7x - 1 = x^2 + 5x - 1}$

Main Concept 4

Finding the quadratic equations:

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Davis Tang

Main Concept 4

Finding the quadratic equations: