Part III — Mechanics

Based on lectures by Brian
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These notes are not endorsed by the lecturers, and I have modified them (often significantly) after lectures. They are nowhere near accurate representations of what was actually lectured, and in particular, all errors are almost surely mine.

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11.1 Forces which vary with speed

Proposition.

$$\mathbf{a} = \mathbf{v}\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}\mathbf{x}}$$

Proof.

$$\mathbf{a} = \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}\mathbf{t}} \times \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}\mathbf{x}} = \mathbf{v}\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}\mathbf{x}}$$

□

12 Elastic strings and springs

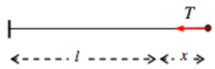
12.1 Hooke's Law

Law (Hooke's Law)**.** There are two cases for using Hooke's Law

(i) Elastic strings: The tension T in an elastic string is

$$T = \frac{\lambda x}{l}$$

where
l is the natural (unstretched) length of the string,
x is the extension and
λ is the modulus of elasticity

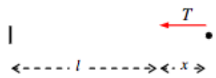


When the string is slack there is no tension.

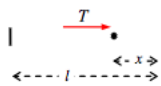
(ii) Elastic springs: The tension, or thrust, T is an elastic spring is

$$T = \frac{\lambda x}{l}$$

where
l is the natural (unstretched) length of the string,
x is the extension or compression and
λ is the modulus of elasticity



Tension (stretched)



Thrust (compressed)

12.2 Energy stored in an elastic string or spring

Like kinematics,If there is force F and displacement traveled δs, the Work done is $\delta W = F\delta s$. Similarly, If the tension force is T and string/spring extended/stretched, then

$$\delta W \approx T\delta x$$

Total work done in exrending from $x = 0$ to $x = X$ is approximately

$$\sum_{0}^{X} T\delta x$$

and , as $\delta x \to 0$, the total work done:

$$W = \int_0^X T\mathrm{d}x = \int_0^X \frac{\lambda x}{l}\mathrm{d}x = \frac{\lambda x^2}{2l}$$

The expression of Total work done is also called the Elastic Potential Energy

13 Further dynamics

13.1 Impulse of a variable force

$$\delta I \approx F(t)\delta t$$

The total impulse from time t_1 to t_2 is

$$I \approx \sum_{t_1}^{t_2} F(t)\delta t$$

and as $\delta t \to 0$, the total impulse is

$$I = \int_{t_1}^{t_2} F(t)\mathrm{d}t$$

Also, as $F(t) = ma = m\frac{\mathrm{d}v}{\mathrm{d}t}$

$$\int_{t_1}^{t_2} F(t)\mathrm{d}t = \int_{U}^{V} m\mathrm{d}v = mV - mU$$

13.2 Work done by a variable force

$$\delta W \approx G(x)\delta x$$

and the total work done in moving from a displacement x_1 to x_2 is

$$W \approx \sum_{x_1}^{x_2} G(x)\delta x$$

and as $\delta x \to 0$, the total work done is

$$W = \int_{x_1}^{x_2} G(x)\mathrm{d}x$$

Also $G(x) = ma = m\frac{\mathrm{d}v}{\mathrm{d}x} = m\frac{\mathrm{d}x}{\mathrm{d}t} \times \frac{\mathrm{d}v}{\mathrm{d}x} = mv\frac{\mathrm{d}v}{\mathrm{d}x}$

$$\int_{x_1}^{x_2} G(x)\mathrm{d}x = \int_{U}^{V} mv\mathrm{d}v = \frac{1}{2}mV^2 - \frac{1}{2}mU^2$$

13.3 Newton's Law of Gravitation

Law. The force of attraction between two bodies of masses M_1 and M_2 is directly proportional to the product of their masses and inversely proportional to the square of the distance, d , between them:

$$F = \frac{GM_1M_2}{d^2}$$

where G is a constant known as the constant of Gravitation

13.4 Finding k in $F = \frac{k}{x^2}$

$$F = ma = \frac{k}{d^2}$$

13.5 Simple harmonic motion S.H.M.

14 Motion in a circle

14.1 Angular velocity

14.2 Acceleration

Types of problems

14.3 Motion in a vertical circle

Types of problems

15 Statics of rigid bodies 2

15.1 Centre of mass

16 Relative motion

17 Elastic collisions in two dimensions

18 Resisted motion of a particle moving in a straight line

19 Damped and forced harmonic motion

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21 Applications of vectors in mechanics

22 Variable mass

23 Moments of inertia of a rigid body

24 Rotation of a rigid body about a fixed smooth axis