Distance Measurement Benchmark

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Abstract

The need to analyze and visualize distances between objects arises in many use cases of computer-aided design. Although the problem to calculate the distance between two polygonal objects may sound simple, real-world scenarios with large models will always be challenging. It is not easy to choose the best spatial data structure to speed up distance queries.

This benchmark is designed to compare different distance measurement techniques regarding performance. In detail, this benchmark focuses on distances between triangle soups.

1 Euclidean Metric

A nonnegative function $d : X \times X \to \mathbb{R}$ describing the "distance" between objects for a given set X is called a metric, if it satisfies

$$d(x, x) = 0 \text{ and } d(x, y) = 0 \Rightarrow x = y$$

as well as the symmetry condition

$$d(x, y) = d(y, x)$$

and the triangle inequality

$$d(x, z) \leq d(x, y) + d(y, z)$$

for all $x, y, z \in X$. The most simple example, which satisfies all conditions is the discrete metric

$$d(x, y) = \begin{cases} 1, & x \neq y \\ 0, & x = y \end{cases}$$

In the field of computer-aided design (CAD) and computer graphics the Euclidean metric is of particular importance. In the following text the Euclidean distance function will be used, if not mentioned otherwise. Two points X, Y with corresponding position vectors $\mathbf{x} = \begin{pmatrix} x_1 & \ldots & x_n \end{pmatrix}^{\mathrm{T}}$ and $\mathbf{y} = \begin{pmatrix} y_1 & \ldots & y_n \end{pmatrix}^{\mathrm{T}}$ of an n-dimensional space have the Euclidean distance

$$d(X, Y) = ||\mathbf{x} - \mathbf{y}|| = \sqrt{(x_1 - y_1)^2 + \ldots + (x_n - y_n)^2}.$$

In some cases it is convenient to use the maximum metric

$$\mu(X, Y) = \max(|x_1 - y_1|, \ldots, |x_n - y_n|)$$

or the absolute value metric

$$\sigma(X, Y) = |x_1 - y_1| + \ldots + |x_n - y_n|.$$

The relationship between all these metrics in real space $\mathbb{R}^n$ is given by the inequality

$$d(X, Y) \leq \sqrt{n} \cdot \mu(X, Y) \leq \sqrt{n} \cdot \sigma(X, Y) \leq n \cdot d(X, Y),$$

for all $X, Y \in \mathbb{R}^n$. The special case $n = 1$ leads to

$$d(X, Y) = \mu(X, Y) = \sigma(X, Y) = |\mathbf{x} - \mathbf{y}|.$$

2 Distance Calculation

Although a single task to calculate the distance between two triangles is simple, the problem arises with the size and the complexity of 3D models: For two sets of triangles a distance calculation shall return all index pairs referencing a triangle in each set, for which the distance is less than or equal to all possible index pairs; i.e. for T_1, T_2 with n respectively m triangles each, the result may consist of $O(n \cdot m)$ elements. As a consequence, the worst case may be quadratic. With large meshes used in Computer-Aided Geometric Design applications, space partitioning techniques are of special interest. These data structures cannot eliminate the worst case, but the complexity of an "average" case can be reduced significantly.

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A Benchmark for Distance Measurements,
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3 Tests

This benchmark consists of several tests. Each test of a special configuration of two triangle sets with varying number of elements namely 100, 200, 500, 1 000, 2 000, 5 000, 10 000, 20 000, 50 000, 100 000, 200 000, 500 000, or 1 000 000.

3.1 Tools Test

This test is designed to represent an "average" CAD-application test. It consists of two CAD models with varying number of triangles in a good-case-scenario.

The test is composed of two tools. The first one is the "Multi-Purpose Precision Maintenance Tool" by Robert Hillan – the winning entry of Future Engineers 3-D Printing in Space Tool Challenge. The data set used in this benchmark has 8 864 vertices and 17 760 faces. It has been published by NASA 3D Resources[1].

The second tool is the "Wrench" model, which has been designed on Earth and then transmitted to space for manufacture. Its 3D representation has 7 281 vertices and 14 564 faces. The original file has been published by NASA 3D Resources[2]. Both data sets may be used for non-commercial purposes[3].

In order to create different test sizes both files have been reduced in size using quadratic edge collapse decimation for down-sized version with 100, 200, 500, 1 000, 2 000, 5 000, or 10 000 faces. The test versions with 20 000, 50 000, 100 000, 200 000, 500 000, or 1 000 000 faces have been created by uniform midpoint subdivision (to increase the number of faces by an integral factor) and quadratic edge collapse decimation (to reduce the number of faces afterwards to the predefined values) using the open source mesh processing tool MeshLab [3] (in version 1.3.4 beta 2014). The current version is available at http://www.meshlab.net. The final results are illustrated in Figure 1 and 2 showing the "Wrench" model and the "Multi-Purpose Precision Maintenance Tool" respectively.

3.2 Rosetta Test

This test is designed to represent a big-and-large model CAD-application test. It consists of two geometric models in a configuration with very different scales.

The first model consists of the ESA Nav-Cam shape model of comet 67P/Churyumov-Gerasimenko. The Rosetta mission has reached the nucleus of comet 67P/Churyumov-Gerasimenko in early August 2014, allowing a detailed mapping of its surface with the onboard imaging system, up to resolutions of 50cm or even better in some areas.

Shape reconstruction techniques have been used by Jorda et al. to build a very detailed 3D model [4]. The original 3D reconstruction has been published by ESA[4] under a Creative Commons Attribution-ShareAlike 3.0 IGO License[5]. It comprehends 833 538 vertices and 1 667 073 faces. In order to have a varying model size the shape model of the comet has been reduced using quadratic edge collapse decimation (see Figure 3) to 100, 200, 500, 1 000, . . . , and 1 000 000 faces implemented in the open source mesh processing tool MeshLab [3] (in version 1.3.4 beta 2014).

The second test model is a fixed size model; i.e. it is used in a fixed resolution in all tests. It has the shape of the space probe Rosetta built by the European Space Agency (see Figure 4). The file has been published by NASA 3D Resources[6] for non-commercial purposes[7].

Besides the model size with respect to the number of vertices and faces, the problem in this test configuration is its approximately correct scaling and positional correlation. The comet consists of two lobes connected by a narrower neck, with the larger lobe measuring $\approx$ 4.1km and the smaller one about $\approx$ 2.6km in diameter, whereas the space probe has a diameter of $\approx$ 32m, respectively $\approx$ 0.032km. Furthermore, the local circumstances are reflected by a measurement distance of $\approx$ 15km; in short, the test scenario consists of a small, far-away part measured towards a large, big dataset.

[1]see https://nasa3d.arc.nasa.gov/detail/mpmt
[2]see https://nasa3d.arc.nasa.gov/detail/wrench-mis
[3]see https://www.nasa.gov/multimedia/guidelines/index.html
[4]see http://blogs.esa.int/rosetta/2015/08/13/a-shape-model-whats-that
[5]see https://creativecommons.org/licenses/by-sa/3.0/igo/
[6]see https://nasa3d.arc.nasa.gov/detail/eoss-rosetta
[7]see https://www.nasa.gov/multimedia/guidelines/index.html

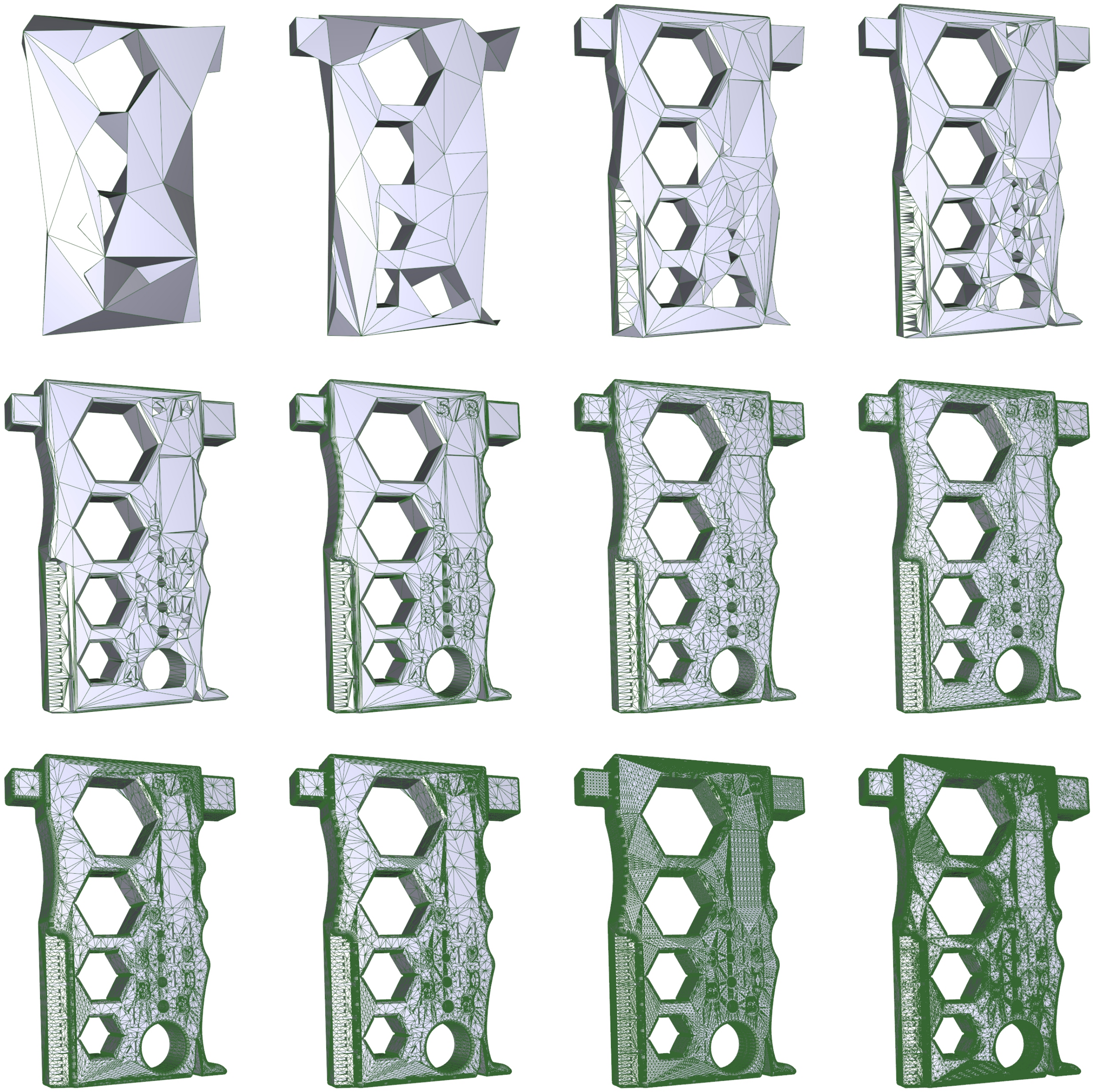

Figure 1: The "Tools Test" representing a "typical" measurement in the context of computer-aided design consists of two tool models with varying number of faces. Figure 2 shows the corresponding second part of the distance measurment test.

Figure 2: The "Tools Test" measures the distance between two tools. The one shown in this Figure and the other one shown in Figure 1. Both models have 100, 200, 500, 1000, 2000, ..., 1 000 000 faces and are rendered in wire-frame mode to outline the mesh topology. For the sake of clarity, the 1 000 000 faces versions are not shown as their renderings are almost completely green due to the not shaded, green wires.

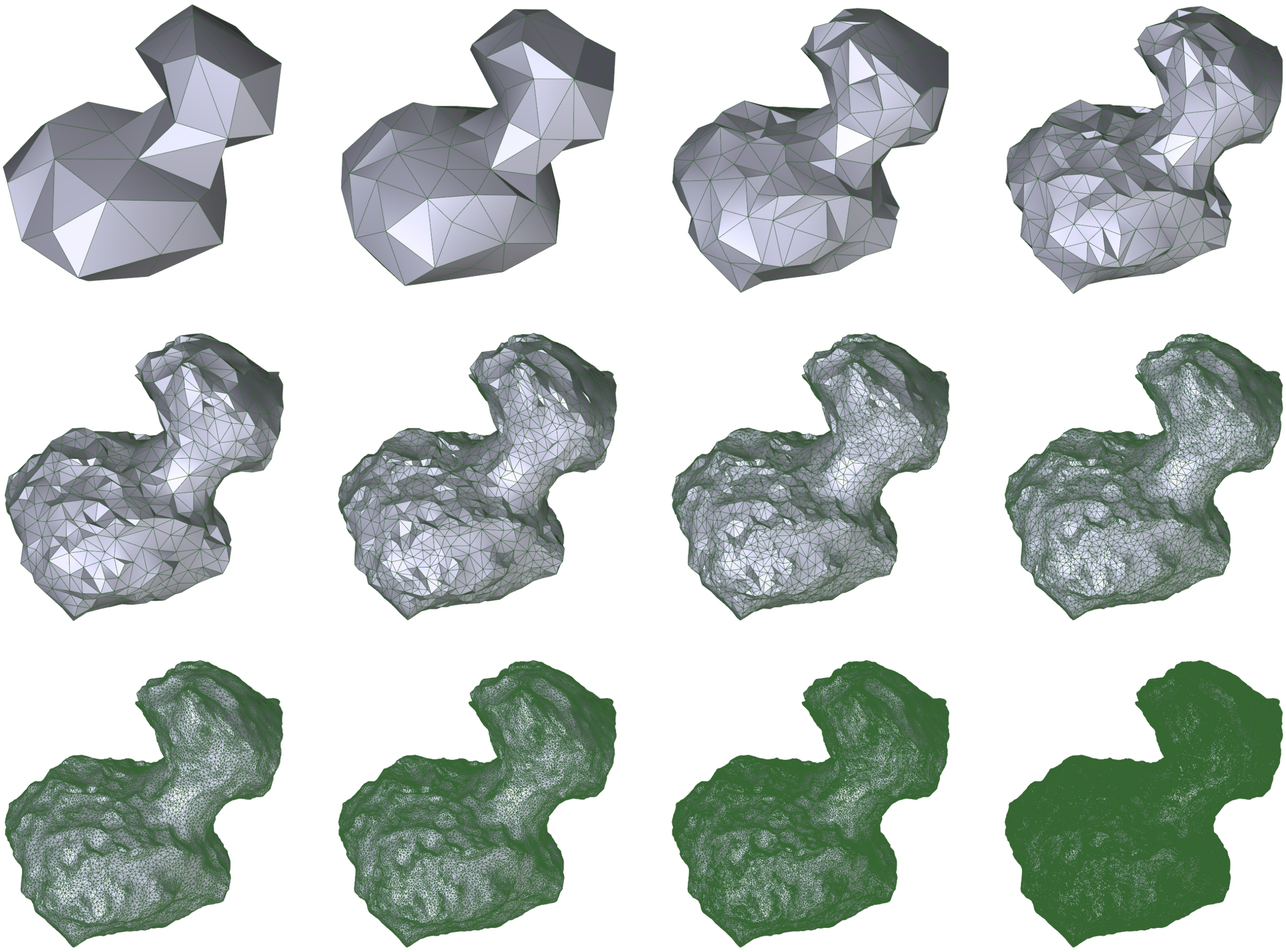

Figure 3: The “Rosetta Test” representing a big-and-large scale measurement uses the shape model of comet 67P/Churyumov-Gerasimenko in different resolutions.

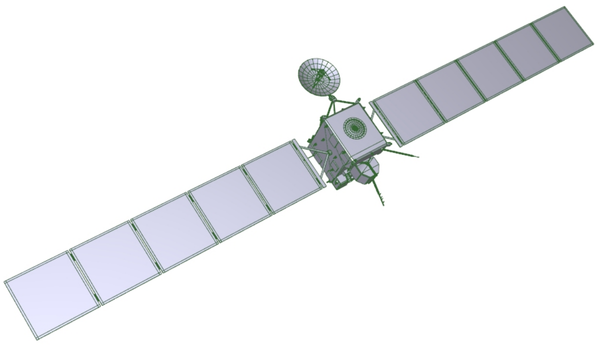

Figure 4: The “Rosetta test” shows the Rosetta space probe and the comet 67P/Churyumov-Gerasimenko (shown in Figure 3).

3.3 Spirit Test

This test is designed to represent a heterogeneously tessellated model test. It consists of two geometric models in a configuration with elements (triangles) in very different scales.

The first model shows the Gusev Cater on Mars – the landing site of the Spirit Rover (see Figure 5). The original data set shows 178km × 237km with exaggerated z-axis for better visualization effects [5]. The file with 244 447 vertices and 488 890 faces has been published by NASA 3D Resources[8] for non-commercial purposes[9].

The second test model is the corresponding Mars Exploration Rovers. The file has also been published by NASA 3D Resources[10] for non-commercial purposes[11].

In order to have a varying model size with heterogeneous tessellation the 3D models have been reduced using quadratic edge collapse decimation implemented in the open source mesh processing tool MeshLab [3] (in version 1.3.4 beta 2014). Furthermore, randomly selected faces of the models have been subdivided using the mesh processing tool Blender [2].

Besides the model size with respect to the number of vertices and faces, the problem in this test configuration is its heterogeneous tesselation; i.e. any spatial data structure to speed-up the distance calculation has to cope with non-uniform elements.

3.4 Sphere Test

This test comprehends a challenging configuration. Each test consists of a triangulation of random points uniformly distributed on a sphere scaled by two different factors (radius $r_1 = 1$ and $r_2 = 2$). Consequently, a test configuration with n elements in each data set should report $O(n)$ pairs with a distance ≈ 1.

The test case has been generated algorithmically with two uniformly distributed random numbers $u \in [-1, 1]$ and $v \in [0, 1]$. These values are combined to three dimensional, random point

$$P\,(\cos v\sqrt{1-u^2} \;\mid\; \sin v\sqrt{1-u^2} \;\mid\; u).$$

These random point sets have been processed by "The Quickhull algorithm for convex hulls" [1] using version 2015.2 available at http://www.qhull.org. The scaling with radius $r_1 = 1$ and $r_2 = 2$ is performed on the quickhull triangulation. Therefore, the topology of two meshes to compare is always the same. Consequently, Figure 7 shows only one mesh per test size.

3.5 Intersection Test

This test comprehends a worst-case configuration; i.e. each data set consists of triangles which pass through the $X\,(\sqrt{2}\,|\,-\pi\,|\,e)$. Consequently, all triangle pairs referencing a triangle in each set have distance zero. As all these pairs have to be reported, the quadratic reporting effort has the same complexity as a naive brute-force solution.

The test case has been generated algorithmically with a random quaternion based on a three-dimensional random vector $\mathbf{s}$. Each coordinate $\mathbf{s}_i$ $(i = 1, 2, 3)$ is a scalar value uniformly distributed between zero and one. With $\sigma_1 = \sqrt{1 - \mathbf{s}_1}$, $\sigma_2 = \sqrt{\mathbf{s}_1}$, $\theta_1 = 2\pi\mathbf{s}_2$, and $\theta_2 = 2\pi\mathbf{s}_3$ the random quaternion is

$$\begin{aligned} w &= \sigma_2 \cos\theta_2, \\ x &= \sigma_1 \sin\theta_1, \\ y &= \sigma_1 \cos\theta_1, \\ z &= \sigma_2 \sin\theta_2 \end{aligned}$$

with corresponding rotation matrix $\mathbf{R}$:

$$\begin{pmatrix} w^2+x^2-y^2-z^2 & 2xy-2wz & 2xz+2wy \\ 2xy+2wz & w^2-x^2+y^2-z^2 & 2yz-2wx \\ 2xz-2wy & 2yz+2wx & w^2-x^2-y^2+z^2 \end{pmatrix}$$

This matrix is applied to three vectors

$$\begin{aligned} \mathbf{a} &= \mathbf{R} \cdot \left(\ \cos(1/6\,\pi) \quad \sin(1/6\,\pi) \quad 0\ \right)^{\mathrm{T}}, \\ \mathbf{b} &= \mathbf{R} \cdot \left(\ \cos(5/6\,\pi) \quad \sin(5/6\,\pi) \quad 0\ \right)^{\mathrm{T}}, \\ \mathbf{c} &= \mathbf{R} \cdot \left(\ \cos(9/6\,\pi) \quad \sin(9/6\,\pi) \quad 0\ \right)^{\mathrm{T}}. \end{aligned}$$

These vectors are scaled independently by a random variable uniformly distributed between 0 and 100 (for $\mathbf{a}$), respectively between 1 and 100 (for $\mathbf{b}$ and $\mathbf{c}$). Finally, the scaled vectors $\mathbf{a}'$, $\mathbf{b}'$, and $\mathbf{c}'$ are added to the position vector of the common point X to form the position vectors of the vertices of one triangle. Figure 8 shows the final results.

[8] see https://nasa3d.arc.nasa.gov/detail/SpiritLanding

[9] see https://www.nasa.gov/multimedia/guidelines/index.html

[10] see https://nasa3d.arc.nasa.gov/detail/spirit

[11] see https://www.nasa.gov/multimedia/guidelines/index.html

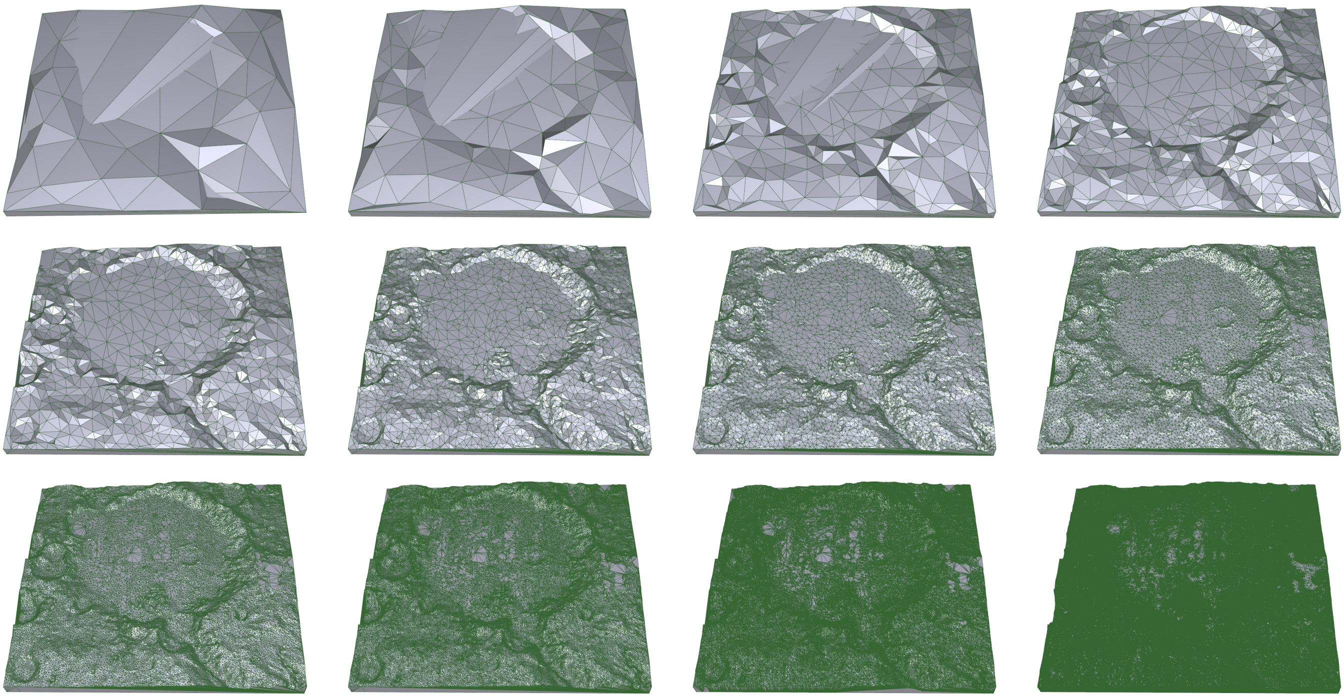

Figure 5: The landing site 3D data used in the “Spirit Test” have been processed to create a heterogeneous tessellation as visible in the renderings.

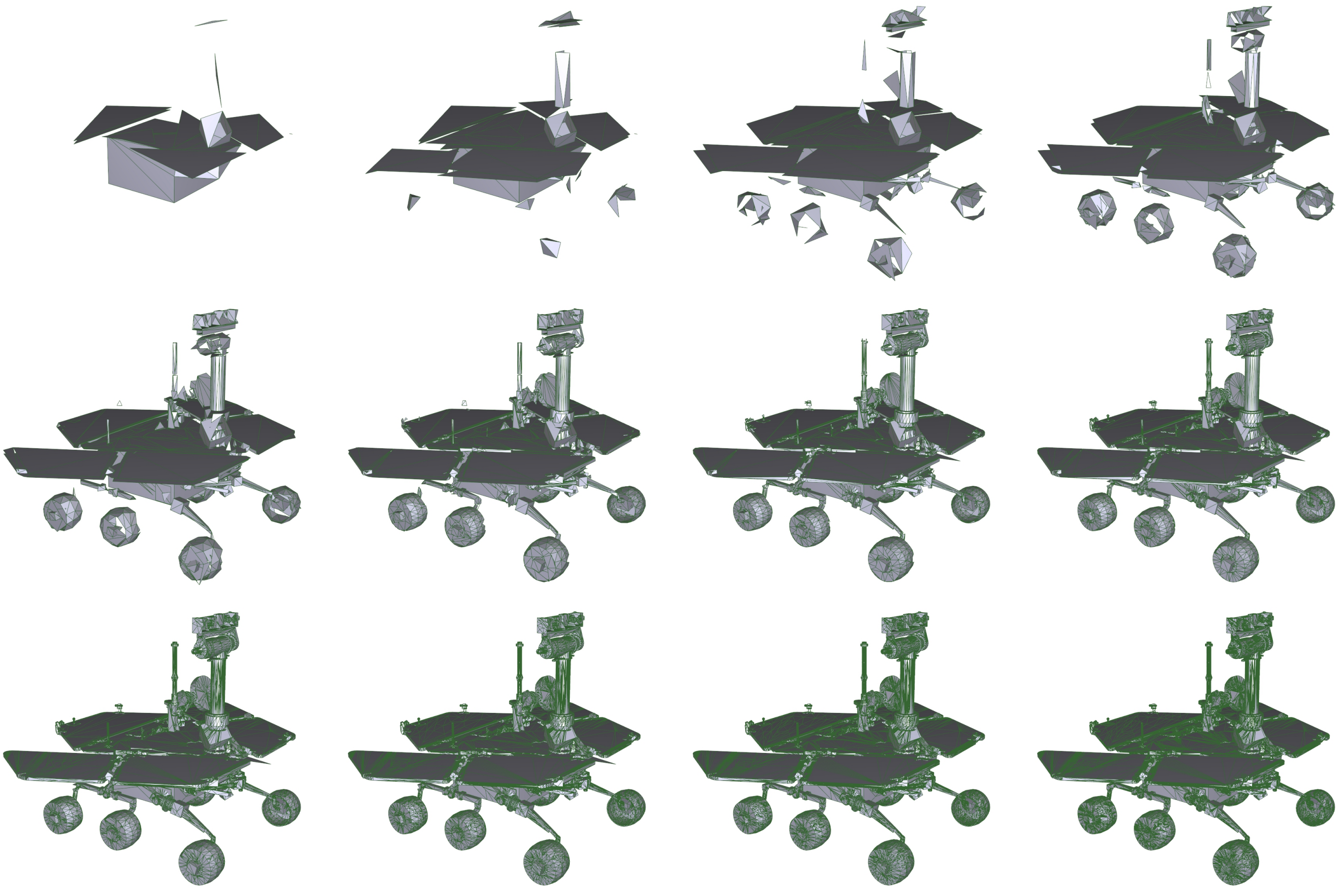

Figure 6: Not only the landing site dataset (see Figure 5) but also the rover dataset has been reduced and resampled heterogeneously. Due to the drastic reduction in size, the small size datasets $< 5\,000$ faces have severe topological errors.

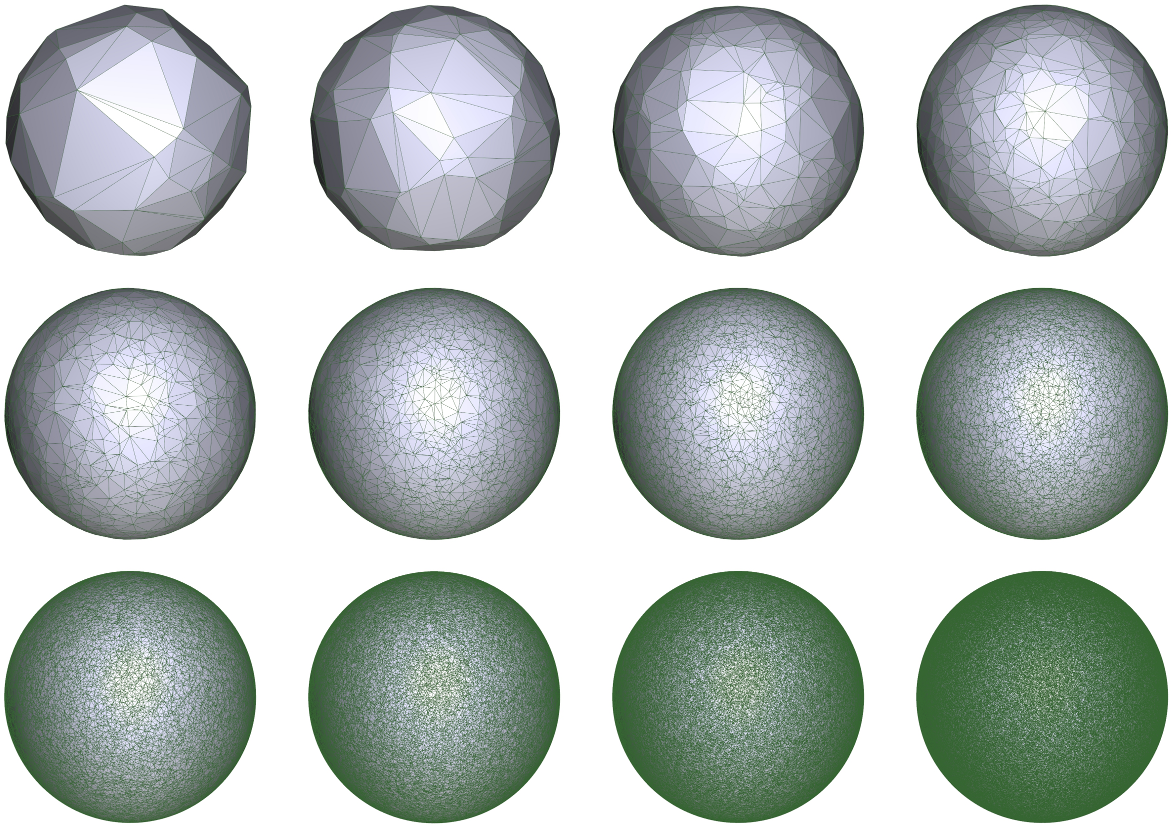

Figure 7: The “Sphere Test” consists of triangulated points on a sphere. Each data set has been generated with 100, 200, 500, . . . , 500 000 input points (top left to bottom right). The resulting triangulations vary in size with approximately twice as much faces as vertices.

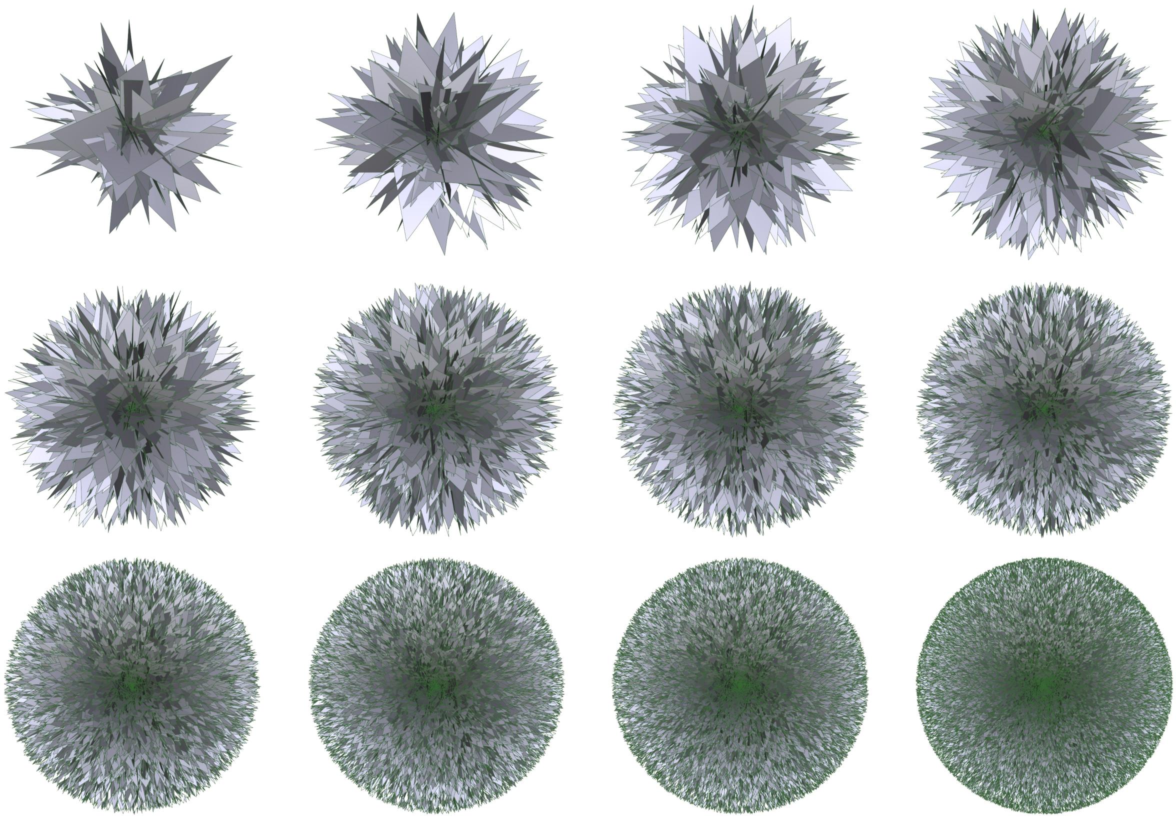

Figure 8: The “Intersection Test” consists of triangle sets which have one intersection point in common. Each data set is rendered in solid, wireframe mode and comprehends 100, 200, 500, . . . , 500 000 triangles (top left to bottom right).

References

[1] C. Bradford Barber, David P. Dobkin, and Hannu Huhdanpaa. The quickhull algorithm for convex hulls. *ACM Transactions on Mathematical Software*, 22(4):469–483, 1996.

[2] Blender Documentation Team. *Blender 2.78 Manual*. Blender Documentation Team, 2017.

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[4] L. Jorda, R. W. Gaskell, S. F. Hviid, C. Capanna, O. Groussin, P. Gutiérrez, L. Philippe, F. Scholten, F. Preusker, H. U. Keller, K. Joerg, E. Kuhrt, S. Mottola, H. Sierks, C. Snodgrass, N. Thomas, J. B. Vincent, S. Marchi, and M. Massironi. The Shape of Comet 67P/Churyumov-Gerasimenko from Rosetta/Osiris Images. *AGU Fall Meeting Abstracts*, 47:P41C:3943, 2014.

[5] Mirjam van Kan Parker, Tanja Zegers, Thomas Kneissl, Boris Ivanov, Bernard Foing, and Gerhard Neukum. 3d structure of the gusev crater region. *Earth and Planetary Science Letters*, 294:411–423, 2010.