IIC 2433 Minería de Datos

https://github.com/marcelomendoza/IIC2433

- KMEANS -

Clustering permite entender como se agrupan los datos

Clustering con k-means

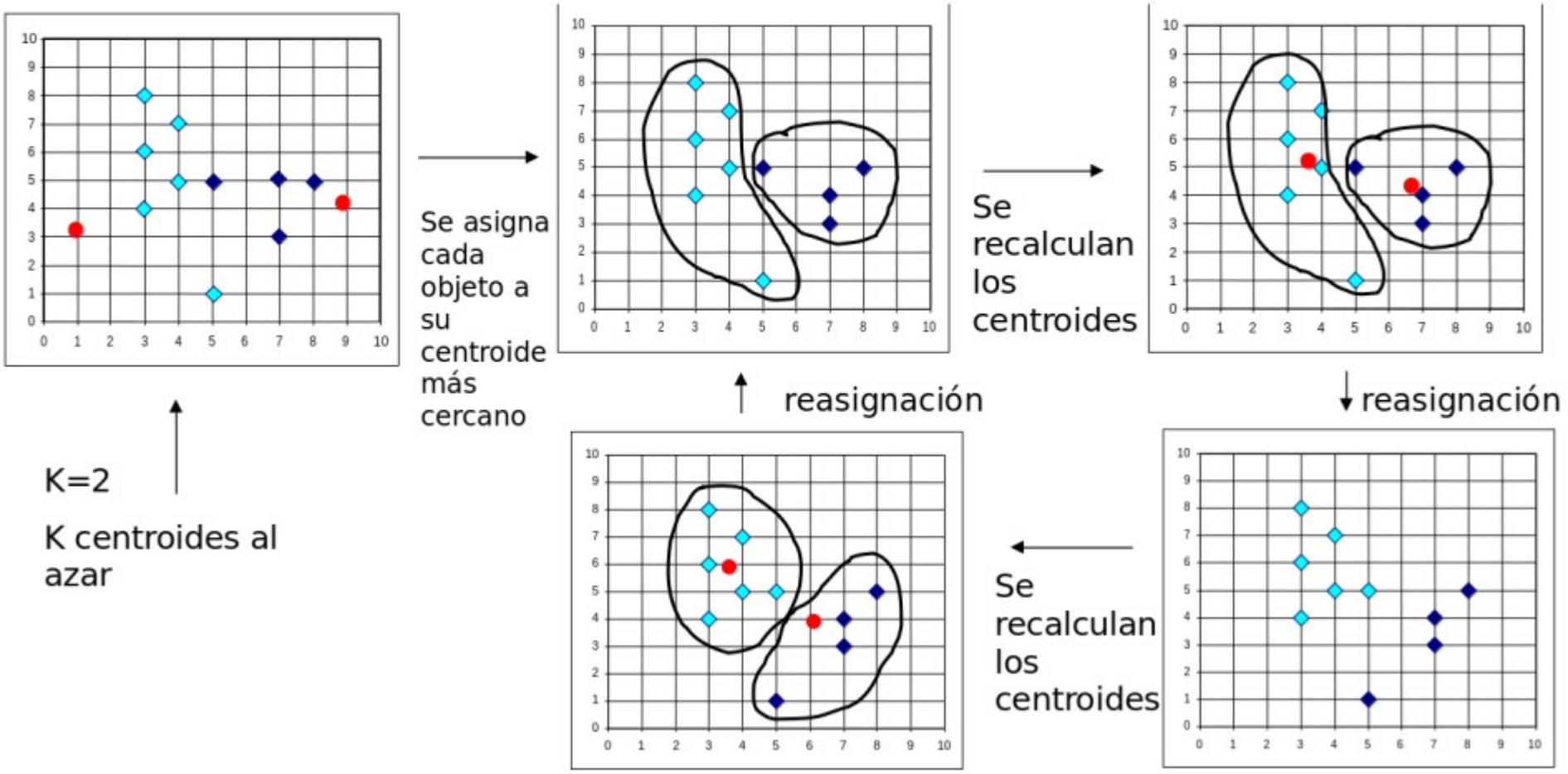
- Cada cluster en K-means es definido por un centroide.
- Objetivo: optimizar alguna noción de distancia:
  1. Intra-cluster: (Minimizar) distancia entre objetos de un cluster a su centroide.
  2. Inter-cluster: (Maximizar) distancia entre objetos de clusters distintos.
- Centroide:

$$c_i = \frac{1}{m_i} \sum_{x \in C_i} x$$

  donde C_i denota un cluster.
- Idea del algoritmo:
  - Asignación inicial: `k` centroides al azar.
  - Reasignación: asignar cada objeto a su centroide más cercano (algoritmo avaro).
  - Recomputación: recalcular los centroides. □

Clustering con k-means

Ejemplo



Clustering con k-means

Hechos importantes:

- K-means converge. (McQueen, 67)
- Criterios de parada
  1. Iteraciones: (Máximo) número de iteraciones.
  2. Error tolerado: (Optimizar) alguna noción de distancia entre objetos.
- Complejidad:
  1. K-means es $\mathtt{NP-hard}$ en cualquier espacio d-dimensional con distancia Euclideana o coseno.
  2. K-means es $\mathtt{NP-hard}$ para cualquier valor de k.

SSE: Suma de errores al ²

Clustering con k-means

k-means minimiza el SSE: implícitamente

$$\mathrm{SSE} = \sum_{i=1}^{K} \sum_{x \in C_i} (c_i - x)^2$$

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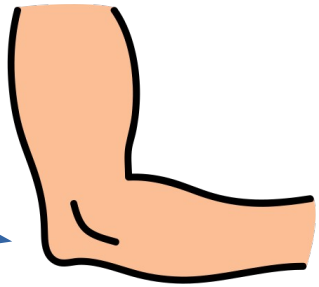
$$\sum_{x \in C_k} 2 * (c_k - x_k) = 0 \Rightarrow m_k c_k = \sum_{x \in C_k} x_k \Rightarrow c_k = \frac{1}{m_k} \sum_{x \in C_k} x_k$$

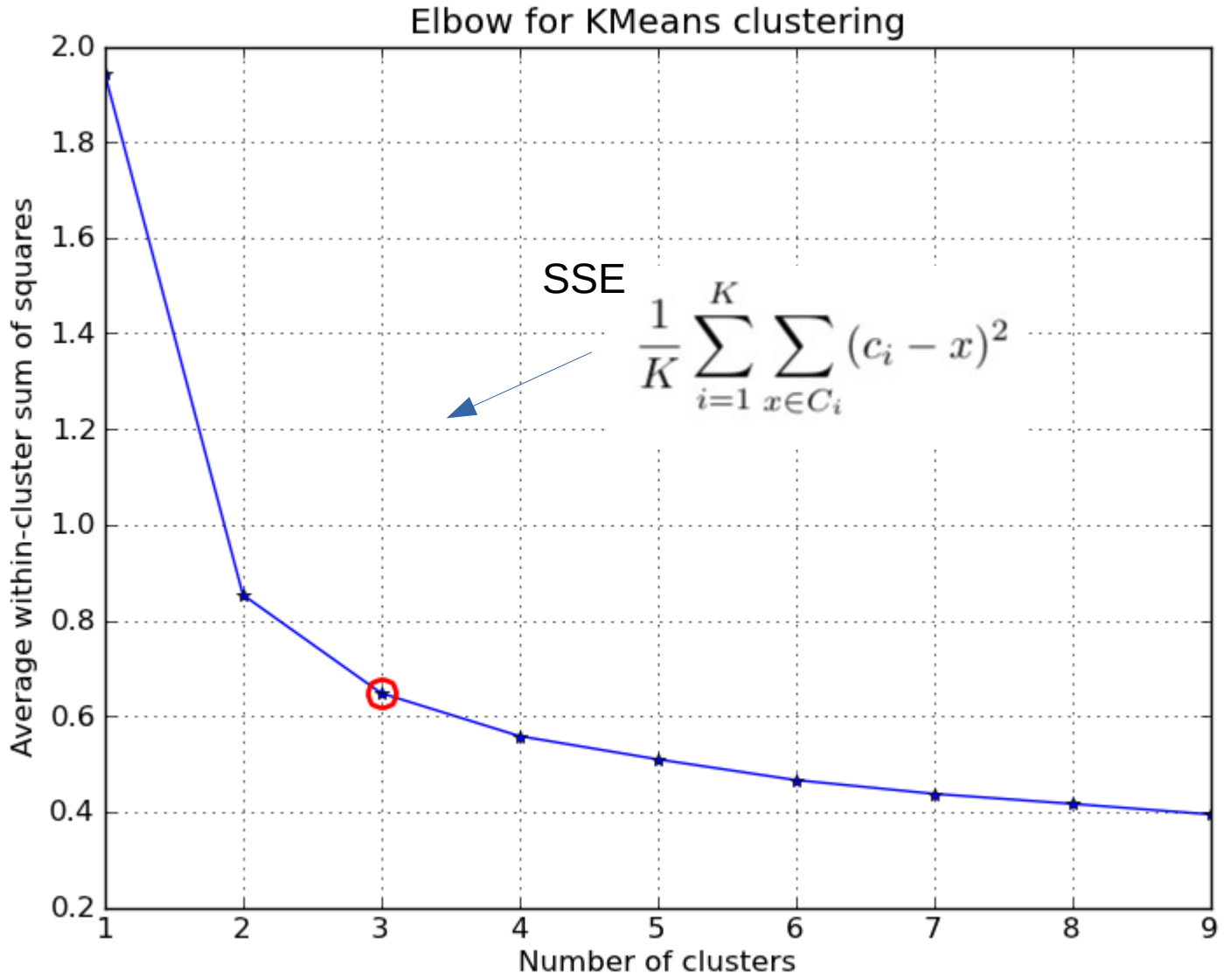
m_k: # elementos en el clúster

¿Cuántos prototipos usamos?

Variar k buscando el codo



ELBOW (codo):



SSE

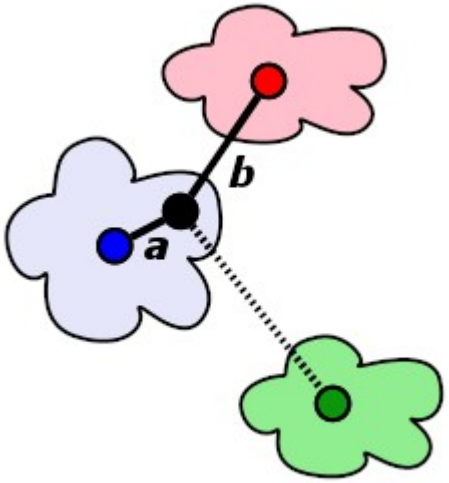
$$\frac{1}{K}\sum_{i=1}^{K}\sum_{x \in C_i}(c_i - x)^2$$

¿Cuántos prototipos usamos?

Silhouette:

Congruencia de x_i a C_i: $$a(i) = \frac{1}{|C_i| - 1} \sum_{j \in C_i, i \neq j} d(i,j)$$

Congruencia de x_i a otros clusters: $$b(i) = \min_{k \neq i} \frac{1}{|C_k|} \sum_{j \in C_k} d(i,j)$$



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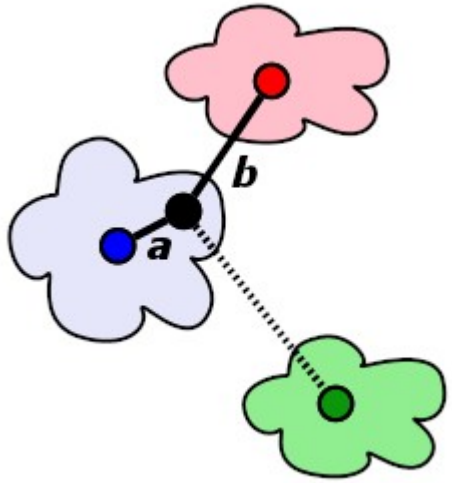

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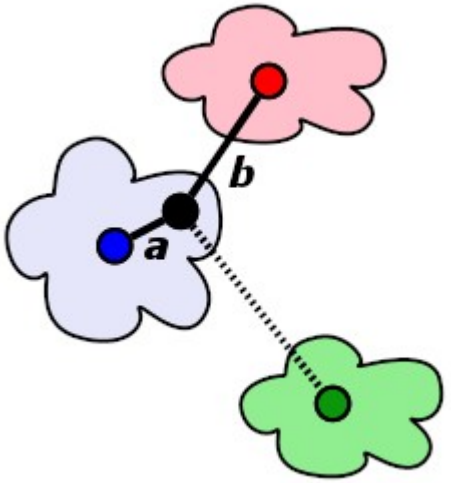

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¿Intervalo?

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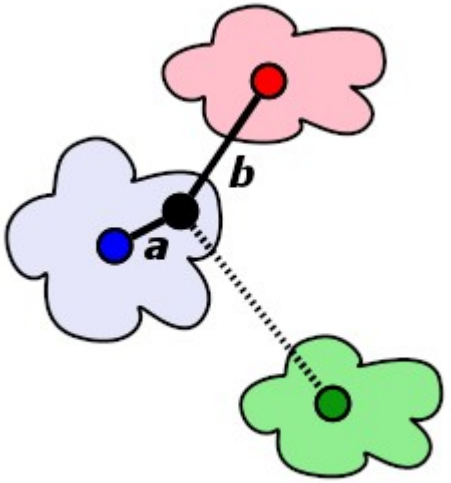

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[-1, 1]

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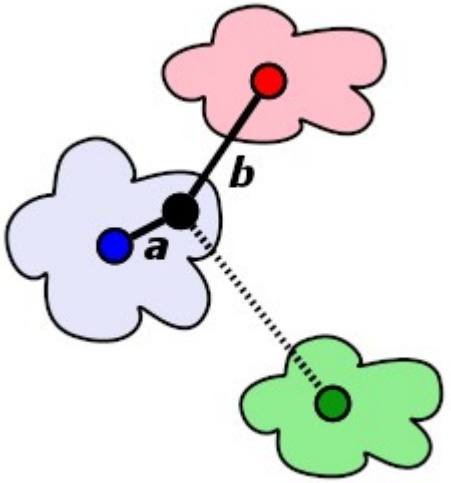

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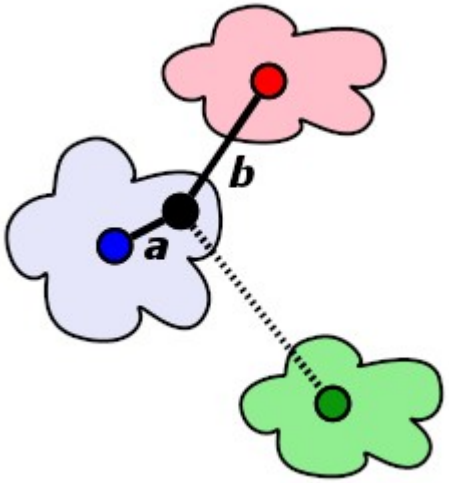

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[-1, 1]

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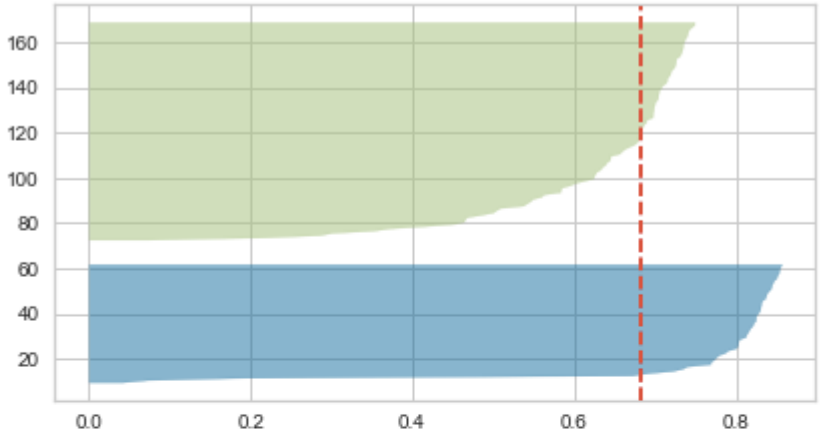
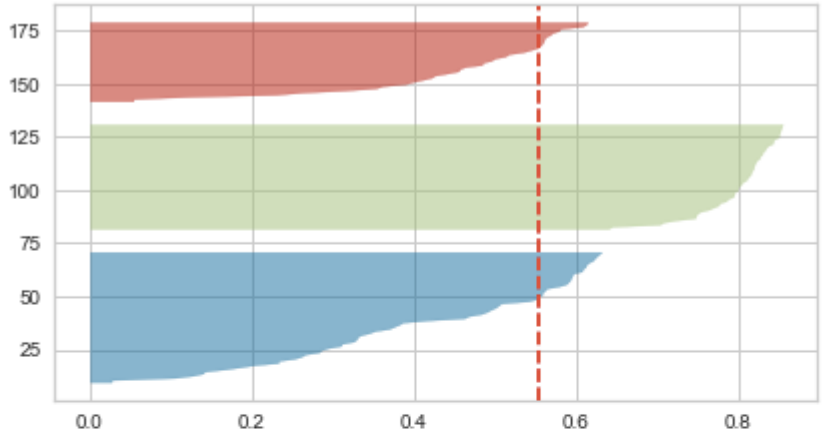
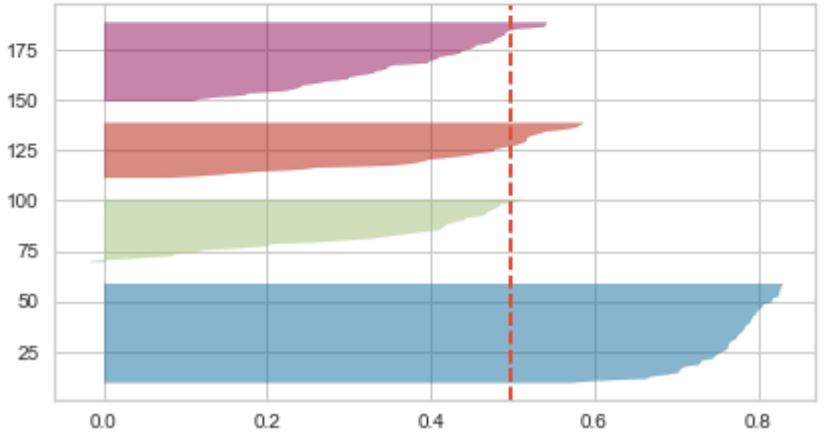
Silhouette promedio:

k=2

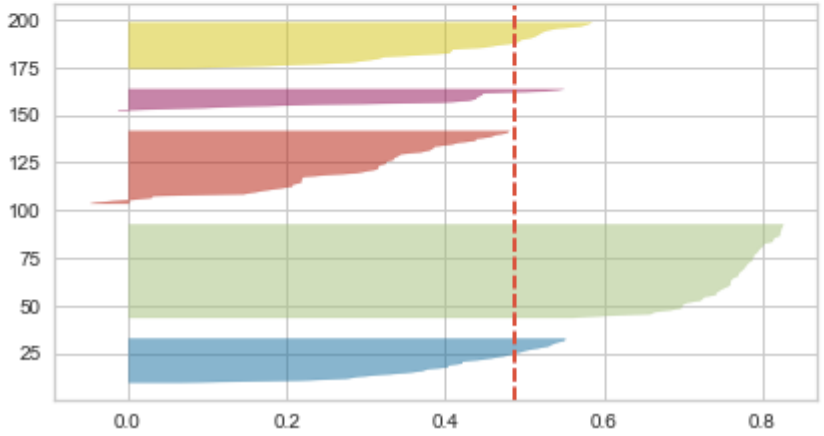


k=3





k=4



k=5

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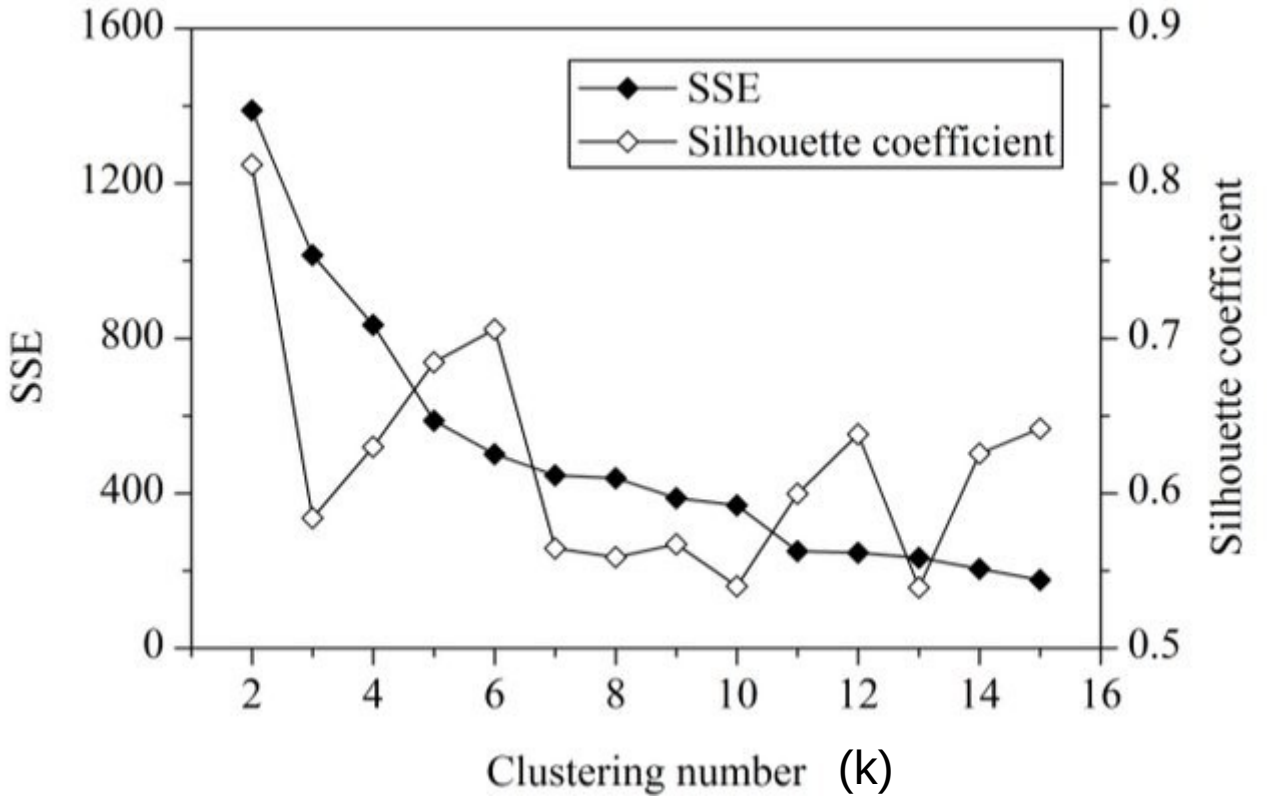
Silhouette v/s ELBOW:



¿Con cuál **k** se quedan?

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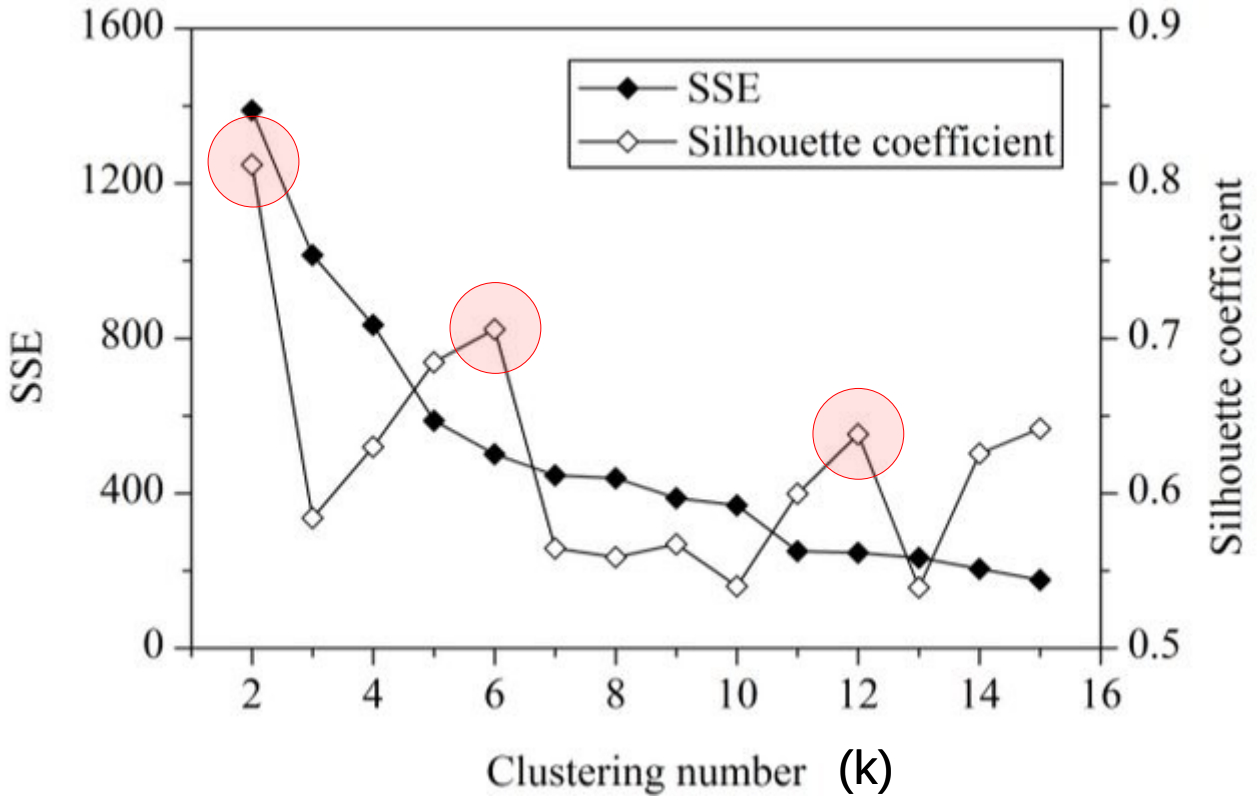

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