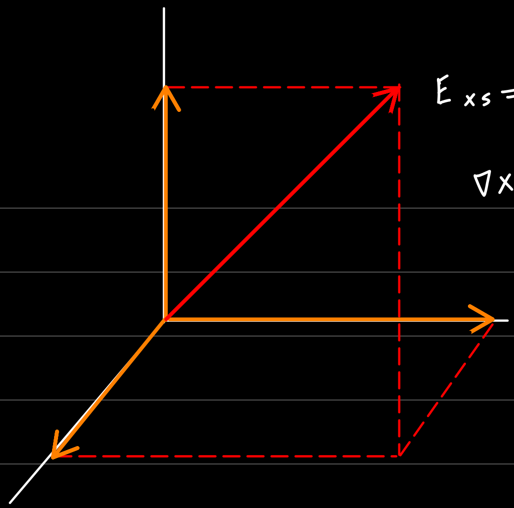
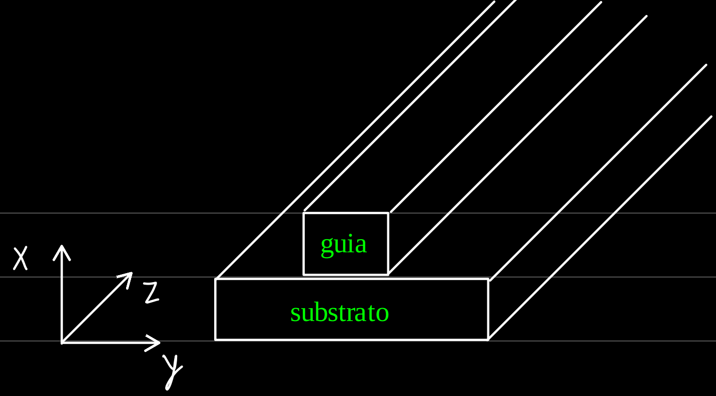


$$E_{xs} = E_{x0}\, e^{-j\beta_0 z}$$

$$\nabla \times E_x = -j\omega H_x$$



=> Dedução: Considera-se que o campo elétrico e magnético são da forma:

$$\boxed{\begin{aligned} \vec{E}(x,y,z,t) &= \vec{E}(x,y)\, e^{j\omega t - j\beta z} \\ \vec{H}(x,y,z,t) &= \vec{H}(x,y)\, e^{j\omega t - j\beta z} \end{aligned}}$$

$$\vec{E}(x,y,z,t) = \vec{E}(x,y)\cos(\omega t - \beta z)$$
$$\vec{H}(x,y,z,t) = \vec{H}(x,y)\cos(\omega t - \beta z)$$

β é o número de onda (ou constante de propagação) ao longo da direção de propagação do guia. Devido à direção de propação do guia é conveniente escrever:

$$\vec{E}(x,y) = \underbrace{\hat{x}E_x(x,y) + \hat{y}E_y(x,y)}_{\text{Transversal}} + \underbrace{\hat{z}E_z(x,y)}_{\text{Longitudinal}}$$

De modo similar, $\vec{\nabla}$ é decomposto

$$\vec{\nabla} = \underbrace{\hat{x}\partial_x + \hat{y}\partial_y}_{\text{Transversal}} + \hat{z}\partial_z = \vec{\nabla}_T + \hat{z}\partial_z = \vec{\nabla}_T - j\beta\hat{z}$$

Assim,

$$\begin{aligned} \vec{\nabla}\times\vec{E} &= -j\omega\mu\vec{H} \\ \vec{\nabla}\times\vec{H} &= j\omega\varepsilon\vec{E} \\ \vec{\nabla}\cdot\vec{E} &= 0 \\ \vec{\nabla}\cdot\vec{H} &= 0 \end{aligned} \quad \Rightarrow \quad \begin{aligned} (\vec{\nabla}_T - j\beta\hat{z})\times(\vec{E}_T + \hat{z}E_z) &= -j\omega\mu(\vec{H}_T + \hat{z}H_z) \\ (\vec{\nabla}_T - j\beta\hat{z})\times(\vec{H}_T + \hat{z}H_z) &= j\omega\varepsilon(\vec{E}_T + \hat{z}E_z) \\ (\vec{\nabla}_T - j\beta\hat{z})\times(\vec{E}_T + \hat{z}E_z) &= 0 \\ (\vec{\nabla}_T - j\beta\hat{z})\times(\vec{H}_T + \hat{z}H_z) &= 0 \end{aligned}$$

Luiz Felipe

$$\boxed{\beta = \frac{\omega}{u} = \frac{2\pi f}{\lambda f} = \frac{2\pi}{\lambda}}$$

As identidades vetoriais utilizadas nesta seção:

$$\hat{\mathbf{z}} \times (\hat{\mathbf{z}} \times \mathbf{A}_T) = -\mathbf{A}_T,$$
$$\hat{\mathbf{z}} \times (\boldsymbol{\nabla}_T A_z \times \hat{\mathbf{z}}) = \boldsymbol{\nabla}_T A_z,$$
$$\boldsymbol{\nabla}_T \times \boldsymbol{\nabla}_T A_z = 0,$$
$$\boldsymbol{\nabla}_T \times (\hat{\mathbf{z}} \times \boldsymbol{\nabla}_T A_z) = \hat{\mathbf{z}} \nabla_T^2 A_z,$$
$$\boldsymbol{\nabla}_T \cdot (\hat{\mathbf{z}} \times \boldsymbol{\nabla}_T A z) = 0.$$

1 $(\vec{\nabla}_T - j\beta\hat{z}) \times (\vec{E}_T + \hat{z} E_z) = -j\omega\mu(\vec{H}_T + \hat{z} H_z)$

2 $(\vec{\nabla}_T - j\beta\hat{z}) \times (\vec{H}_T + \hat{z} H_z) = j\omega\varepsilon(\vec{E}_T + \hat{z} E_z)$

3 $(\vec{\nabla}_T - j\beta\hat{z}) \times (\vec{E}_T + \hat{z} E_z) = 0$

4 $(\vec{\nabla}_T - j\beta\hat{z}) \times (\vec{H}_T + \hat{z} H_z) = 0$

• Eq. 1:

$$\boxed{\begin{aligned}&(\hat{x}\partial_x + \hat{y}\partial_y) \times (\hat{x} E_x(x,y) + \hat{y} E_y(x,y)) = \partial_x E_y \hat{z} - \partial_y E_x \hat{z}\\ &(\hat{x}\partial_x + \hat{y}\partial_y) \times \hat{z} E_z = -\partial_x E_z \hat{y} + \partial_y E_z \hat{x}\\ &-j\beta\hat{z} \times (\hat{x} E_x(x,y) + \hat{y} E_y(x,y)) = -j\beta E_x \hat{y} + j\beta E_y \hat{x}\end{aligned}}$$

$$\partial_x E_y \hat{z} - \partial_y E_x \hat{z} - \partial_x E_z \hat{y} + \partial_y E_z \hat{x} - j\beta E_x \hat{y} + j\beta E_y \hat{x} = -j\omega\mu\vec{H}_T - j\omega\mu H_z \hat{z}$$

$$\hat{z}(\partial_x E_y - \partial_y E_x) + \vec{\nabla} E_z \times \hat{z} - j\beta\hat{z} \times \vec{E}_T = -j\omega\mu\vec{H}_T - j\omega\mu H_z \hat{z} \quad (*)$$

$\vec{\nabla} \times \vec{E}_T$

$\hookrightarrow (\hat{x}\partial_x + \hat{y}\partial_y) \times (\hat{x} E_x + \hat{y} E_y) = \partial_x E_y \hat{z} - \partial_y E_x \hat{z}$

$$(*) \quad \vec{\nabla} \times \vec{E}_T + \underbrace{\vec{\nabla} E_z \times \hat{z} - j\beta\hat{z} \times \vec{E}_T}_{} = -j\omega\mu\vec{H}_T - j\omega\mu H_z \hat{z}$$

Comp. Longitudinais.

$\hookrightarrow$ Possuem componentes transversais (x, y)

Assim, $\vec{\nabla} \times \vec{E}_T = -j\omega\mu H_z \hat{z} \Rightarrow \boxed{\vec{\nabla} \times \vec{E} + j\omega\mu H_z \hat{z} = 0}$

$$\boxed{\vec{\nabla} E_z \times \hat{z} - j\beta\hat{z} \times \vec{E}_T = -j\omega\mu\vec{H}_T}$$

• Eq. 2: $(\vec{\nabla}_T - j\beta\hat{z}) \times (\vec{H}_T + \hat{z} H_z) = j\omega\varepsilon(\vec{E}_T + \hat{z} E_z) \quad (\#)$

$$\boxed{\begin{aligned}&(\hat{x}\partial_x + \hat{y}\partial_y) \times (\hat{x} H_x(x,y) + \hat{y} H_y(x,y)) = \partial_x H_y \hat{z} - \partial_y H_x \hat{z} = \vec{\nabla} \times \vec{H}_T \text{ (Long.)}\\ &(\hat{x}\partial_x + \hat{y}\partial_y) \times \hat{z} H_z = -\partial_x H_z \hat{y} + \partial_y H_z \hat{x} = \vec{\nabla} H_z \times \hat{z} \text{ (Transversal)}\\ &-j\beta\hat{z} \times (\hat{x} H_x(x,y) + \hat{y} H_y(x,y)) = -j\beta H_x \hat{y} + j\beta H_y \hat{x} = j\beta\vec{H}_T \times \hat{z} \text{ (Transversal)}\end{aligned}}$$

$$(\#) \quad \vec{\nabla} \times \vec{H} + \vec{\nabla} H_z \times \hat{z} + j\beta\vec{H}_T \times \hat{z} = j\omega\varepsilon\vec{E}_T + j\omega\varepsilon E_z \hat{z}$$

$$\hookrightarrow \boxed{\begin{cases}\vec{\nabla} \times \vec{H} - j\omega\varepsilon E_z \hat{z} = 0\\ \vec{\nabla} H_z \times \hat{z} - j\beta\hat{z} \times \vec{H}_T = j\omega\varepsilon\vec{E}_T\end{cases}}$$

• Eq. 3: $(\vec{\nabla}_T - j\beta\hat{z}) \times (\vec{E}_T + \hat{z}E_z) = 0$

$$\boxed{\vec{\nabla}_T \cdot \vec{E}_T - j\beta E_z = 0}$$

• Eq. 4: $(\vec{\nabla}_T - j\beta\hat{z}) \times (\vec{H}_T + \hat{z}H_z) = 0$

$$\boxed{\vec{\nabla}_T \cdot \vec{H}_T - j\beta H_z = 0}$$

Com isso, as equações finais são:

$$\nabla E_z \times \hat{\mathbf{z}} - j\beta\,\hat{\mathbf{z}} \times \mathbf{E}_T = -j\omega\mu\mathbf{H}_T \qquad (11)$$

$$\nabla H_z \times \hat{\mathbf{z}} - j\beta\,\hat{\mathbf{z}} \times \mathbf{H}_T = j\omega\varepsilon\mathbf{E}_T \qquad (12)$$

$$\nabla \times \mathbf{E}_T + j\omega\mu H_z\,\hat{\mathbf{z}} = 0 \qquad (13)$$

$$\nabla \times \mathbf{H}_T - j\omega\varepsilon E_z\,\hat{\mathbf{z}} = 0 \qquad (14)$$

$$\nabla_T \cdot \mathbf{E}_T - j\beta E_z = 0 \qquad (15)$$

$$\nabla_T \cdot \mathbf{H}_T - j\beta H_z = 0 \qquad (16)$$

• Identidades vetoriais úteis: $\hat{z} \times (\hat{z} \times \vec{A}_T) = -\vec{A}_T$

$\hat{z} \times (\vec{\nabla}_T A_z \times \hat{z}) = \vec{\nabla}_T A_z$

• Expressando as componentes transversais em função das longitudinais:

Eq. (11):

$$\vec{\nabla} E_z \times \hat{z} - j\beta\hat{z} \times \vec{E}_T = -j\omega\mu\vec{H}_T$$

$$\hat{z} \times (\vec{\nabla} E_z \times \hat{z}) - \hat{z} \times (j\beta\hat{z} \times \vec{E}_T) = \hat{z} \times (-j\omega\mu\vec{H}_T)$$

$$\vec{\nabla}_T E_z + j\beta\vec{E}_T = -j\omega\mu(H_x\hat{y} - H_y\hat{x})$$

$$\vec{\nabla}_T E_z + j\beta\vec{E}_T = j\omega\mu H_y\hat{x} - j\omega\mu H_x\hat{y}$$

$$\vec{\nabla}_T E_z + j\beta\vec{E}_T = -j\omega\mu\,\hat{z} \times \vec{H}_T \quad (1)$$

Eq. (12):

$$\vec{\nabla} H_z \times \hat{z} - j\beta\,\hat{z} \times \vec{H}_T = j\omega\varepsilon\vec{E}_T$$

$$\hat{z} \times (\vec{\nabla} H_z \times \hat{z} - j\beta\,\hat{z} \times \vec{H}_T) = \hat{z} \times (j\omega\varepsilon\vec{E}_T)$$

$$\vec{\nabla}_T H_z + j\beta\vec{H}_T = j\omega\varepsilon E_x\hat{y} - j\omega\varepsilon E_y\hat{x}$$

$$\vec{\nabla}_T H_z + j\beta\vec{H}_T = j\omega\varepsilon\,\hat{z} \times \vec{E}_T \quad (2)$$

De (2):

$$\vec{H}_T = \frac{\omega\varepsilon}{\beta}\hat{z}\times\vec{E}_T + \frac{j}{\beta}\vec{\nabla}_T H_z \quad (3)$$

$$(3)\to(1):\ \vec{\nabla}_T E_z + j\beta\vec{E}_T = -j\omega\mu\,\hat{z}\times\vec{H}_T = -j\omega\mu\,\hat{z}\times\left[\frac{\omega\varepsilon}{\beta}\hat{z}\times\vec{E}_T + \frac{j}{\beta}\vec{\nabla}_T H_z\right]$$

$$\vec{\nabla}_T E_z + j\beta\vec{E}_T = +j\frac{\omega^2\mu\varepsilon}{\beta}\vec{E}_T + \frac{\omega\mu}{\beta}\hat{z}\times\vec{\nabla}H_z$$

$$j\beta\vec{E}_T - j\frac{\omega^2\mu\varepsilon}{\beta}\vec{E}_T = \frac{\omega\mu}{\beta}\hat{z}\times\vec{\nabla}H_z - \vec{\nabla}_T E_z$$

$$\vec{E}_T\cdot\left[\frac{j\beta^2 - j\omega^2\mu\varepsilon}{\beta}\right] = \frac{\omega\mu}{\beta}\hat{z}\times\vec{\nabla}H_z - \vec{\nabla}_T E_z$$

$$\vec{E}_T = \frac{\omega\mu\,\hat{z}\times\vec{\nabla}_T H_z}{j(\beta^2-\omega^2\mu\varepsilon)} - \frac{\beta\vec{\nabla}_T E_z}{j(\beta^2-\omega^2\mu\varepsilon)}$$

The quantity k is the wavenumber a uniform plane wave would have in the propagation mediu

$$\hat{z}\times\vec{E}_T = -\frac{\omega\mu\vec{\nabla}_T H_z}{j(\beta^2-\omega^2\mu\varepsilon)} - \frac{\beta\,\hat{z}\times\vec{\nabla}_T E_z}{j\underbrace{(\beta^2-\omega^2\mu\varepsilon)}_{\to -k_c^2}}\ ; \quad k=\omega\sqrt{\mu\varepsilon}, \quad k_c^2 = k^2-\beta^2$$

em que kc é o número de onda de corte e pode ser interpretado fisicamente como a componente transversal do vetor de onda de magnitude k = ωsqrt(με):

$$\boxed{\hat{z}\times\vec{E}_T = -j\frac{\beta}{k_c^2}\hat{z}\times\vec{\nabla}_T E_z - j\frac{\omega\mu}{k_c^2}\vec{\nabla}_T H_z}$$

Para determinar a outra equação, deve-se partir de (1):

$$\vec{\nabla}_T E_z + j\beta\vec{E}_T = -j\omega\mu\,\hat{z}\times\vec{H}_T$$

$$\vec{E}_T = \left[-j\omega\mu\,\hat{z}\times\vec{H}_T - \vec{\nabla}_T E_z\right]/j\beta$$

$$\vec{E}_T = \frac{j}{\beta}\vec{\nabla}_T E_z - \frac{\omega\mu}{\beta}\hat{z}\times\vec{H}_T \quad (4)$$

Tomando (4) -> (3):

$$\vec{H_T} = \frac{\omega\varepsilon}{\beta}\hat{z}\times\vec{E_T} + \frac{j}{\beta}\vec{\nabla}_T H_z = \frac{j}{\beta}\vec{\nabla}_T H_z + \frac{\omega\varepsilon}{\beta}\hat{z}\times\left[\frac{j}{\beta}\vec{\nabla}_T E_z - \frac{\omega\mu}{\beta}\hat{z}\times\vec{H_T}\right]$$

$$\vec{H_T} = \frac{j}{\beta}\vec{\nabla} H_z + \frac{j\omega\varepsilon}{\beta^2}\hat{z}\times\vec{\nabla} E_z + \frac{\omega^2\mu\varepsilon}{\beta^2}\vec{H_T}$$

$$\vec{H_T}\left[1 - \frac{\omega^2\mu\varepsilon}{\beta^2}\right] = \frac{j}{\beta}\vec{\nabla} H_z + \frac{j\omega\varepsilon}{\beta^2}\hat{z}\times\vec{\nabla} E_z$$

$$\vec{H_T} = \frac{j\beta}{(\beta^2-\omega^2\mu\varepsilon)}\vec{\nabla} H_z + \frac{j\omega\varepsilon}{\underbrace{(\beta^2-\omega^2\mu\varepsilon)}_{\to\, -k_c^2}}\hat{z}\times\vec{\nabla} E_z$$

$$\boxed{\vec{H_T} = -\frac{j\omega\varepsilon}{k_c^2}\hat{z}\times\vec{\nabla}_T E_z - \frac{j\beta}{k_c^2}\vec{\nabla} H_z}$$

Assim,

$$\boxed{\hat{z}\times\vec{E_T} = -j\frac{\beta}{k_c^2}\hat{z}\times\vec{\nabla}_T E_z - j\frac{\omega\mu}{k_c^2}\vec{\nabla}_T H_z}$$

$$\boxed{\vec{H_T} = -\frac{j\omega\varepsilon}{k_c^2}\hat{z}\times\vec{\nabla}_T E_z - \frac{j\beta}{k_c^2}\vec{\nabla} H_z}$$

Podem ser escritas como:

$$\boxed{\begin{aligned}\hat{z}\times\vec{E_T} &= -j\frac{\beta}{k_c^2}\left[\hat{z}\times\vec{\nabla}_T E_z + \frac{\omega\mu}{\beta}\vec{\nabla} H_z\right]\\ \vec{H_T} &= -j\frac{\beta}{k_c^2}\left[\frac{\omega\varepsilon}{\beta}\hat{z}\times\vec{\nabla}_T E_z + \vec{\nabla} H_z\right]\end{aligned}} \quad (5)$$

- Relação de dispersão: $k^2 = k_c^2 + \beta^2 \Rightarrow \omega^2\mu\varepsilon = \omega_c^2\mu\varepsilon + \beta^2$

$$\omega_c^2 = \frac{\omega^2\mu\varepsilon}{\mu\varepsilon} - \frac{\beta^2}{\mu\varepsilon} \Rightarrow \omega_c^2 = \omega^2 - \frac{1}{\mu\varepsilon}\beta^2$$

$$c = \frac{1}{\sqrt{\mu \varepsilon}}$$

Logo, $\boxed{\omega_c = \sqrt{\omega^2 - c^2\beta^2}}$; $\omega_c \rightarrow$ freq. angular de corte.

Velocidade de fase: $V_p = \frac{\omega}{\beta} = \omega \cdot \frac{c^2}{\omega^2 - \omega_c^2} = \frac{c^2}{1 - (\omega_c/\omega)^2}$

$$V_p = \frac{c}{\sqrt{1 - (\omega_c/\omega)^2}}$$

Velocidade de grupo:

$$V_g = \frac{\partial \omega}{\partial \beta} = \frac{\partial}{\partial \beta}\left([\omega_c^2 + c^2\beta^2]^{1/2}\right) = \frac{1}{2} \cdot [\omega_c^2 + c^2\beta^2]^{-1/2} \cdot 2c^2\beta$$

$$V_g = \frac{c^2\beta}{[\omega_c^2 + c^2\beta^2]^{1/2}} = c^2 \cdot \frac{1}{\frac{1}{\beta}[\omega_c^2 + c^2\beta^2]^{1/2}} = c^2 \cdot \frac{1}{\left[\frac{\omega_c^2}{\beta^2} + c^2\right]^{1/2}}$$

$$V_g = c\sqrt{1 - (\omega_c/\omega)}$$

- Definindo as impedâncias transversais:

$\eta_{TE} = \frac{\omega \mu}{\beta} = \eta \frac{\omega}{\beta c}$; $\eta_{TM} = \frac{\beta}{\omega \varepsilon} = \eta \frac{\beta c}{\omega}$; onde $\eta = \sqrt{\mu/\varepsilon}$
↳ imp. do meio

Só que, $c = \frac{1}{\sqrt{\mu\varepsilon}} \Rightarrow \eta = \left[\frac{\mu^2}{\mu\varepsilon}\right]^{1/2} = [c^2\mu^2]^{1/2} = c\mu$

$\boxed{\eta/c = \mu}$

e também, $\frac{\eta}{c} = \mu = \frac{1}{c^2\varepsilon} \Rightarrow \boxed{\eta c = 1/\varepsilon}$

- Propriedades: $\eta_{TE}\,\eta_{TM} = \eta^2$; $\frac{\eta_{TE}}{\eta_{TM}} = \frac{\omega^2}{\beta^2 c^2}$

Sabe-se:

$$\omega_c = \sqrt{\omega^2 - c^2\beta^2} \Rightarrow \omega_c = \omega\sqrt{1 - \frac{c^2\beta^2}{\omega^2}} \Rightarrow \omega_c^2 = \omega^2\left[1 - \frac{c^2\beta^2}{\omega^2}\right]$$

$$\frac{c^2\beta^2}{\omega^2} = 1 - \frac{\omega_c^2}{\omega^2} \Rightarrow \boxed{\frac{\beta c}{\omega} = \sqrt{1 - \frac{\omega_c^2}{\omega^2}}}$$

- Logo, $\eta_{TE} = \eta\frac{\omega}{\beta c} \Rightarrow \eta_{TE} = \eta \cdot \frac{1}{\sqrt{1 - \omega_c^2/\omega^2}}$

$$\eta_{TM} = \eta\frac{\beta c}{\omega} \Rightarrow \eta_{TM} = \eta \cdot \sqrt{1 - \omega_c^2/\omega^2}$$

- Com essa definições as equações do sistema (5) são reescritas:

$$\boxed{\begin{aligned} \hat{z}\times\vec{E_T} &= -j\frac{\beta}{k_c^2}\left[\hat{z}\times\vec{\nabla}_T E_z + \frac{\omega\mu}{\beta}\vec{\nabla} H_z\right] \\ \vec{H_T} &= -j\frac{\beta}{k_c^2}\left[\frac{\omega\varepsilon}{\beta}\hat{z}\times\vec{\nabla}_T E_z + \vec{\nabla} H_z\right]\end{aligned}} \quad (5)$$

$$\boxed{\begin{aligned} \hat{z}\times\vec{E_T} &= -j\frac{\beta}{k_c^2}\left[\hat{z}\times\vec{\nabla}_T E_z + \eta_{TE}\vec{\nabla} H_z\right] \\ \vec{H_T} &= -j\frac{\beta}{k_c^2}\left[\frac{1}{\eta_{TM}}\hat{z}\times\vec{\nabla}_T E_z + \vec{\nabla} H_z\right]\end{aligned}} \quad (6)$$

- Usando a indentidade vetorial $\hat{z}\times(\hat{z}\times\vec{E_T}) = -\vec{E_T}$, obtém-se:

$$-\vec{E_T} = -j\frac{\beta}{k_c^2}\left[-\vec{\nabla}_T E_z + \eta_{TE}\,\hat{z}\times\vec{\nabla} H_z\right]$$

$$\vec{E_T} = -j\frac{\beta}{k_c^2}\left[\vec{\nabla}_T E_z - \eta_{TE}\,\hat{z}\times\vec{\nabla} H_z\right]$$

logo, tem-se o sistema resultante:

$$\boxed{\begin{aligned}\vec{E_T} &= -j\frac{\beta}{k_c^2}\left[\vec{\nabla}_T E_z - \eta_{TE}\,\hat{z}\times\vec{\nabla} H_z\right]\\ \vec{H_T} &= -j\frac{\beta}{k_c^2}\left[\vec{\nabla}_T H_z + \frac{1}{\eta_{TM}}\,\hat{z}\times\vec{\nabla}_T E_z\right]\end{aligned}} \quad (7)$$

Substituindo essas equação nas equações 13-16:

$\nabla E_z \times \hat{\mathbf{z}} - j\beta\,\hat{\mathbf{z}}\times\mathbf{E}_T = -j\omega\mu\mathbf{H}_T$	(11)	$\nabla\times\mathbf{E}_T + j\omega\mu H_z\,\hat{\mathbf{z}} = 0$	(13)
$\nabla H_z \times \hat{\mathbf{z}} - j\beta\,\hat{\mathbf{z}}\times\mathbf{H}_T = j\omega\varepsilon\mathbf{E}_T$	(12)	$\nabla\times\mathbf{H}_T - j\omega\varepsilon E_z\,\hat{\mathbf{z}} = 0$	(14)
		$\nabla_T\cdot\mathbf{E}_T - j\beta E_z = 0$	(15)
		$\nabla_T\cdot\mathbf{H}_T - j\beta H_z = 0$	(16)

$$\nabla_T\times\nabla_T A_z = 0,$$
$$\nabla_T\times(\hat{\mathbf{z}}\times\nabla_T A_z) = \hat{\mathbf{z}}\nabla_T^2 A_z,$$
$$\nabla_T\cdot(\hat{\mathbf{z}}\times\nabla_T Az) = 0.$$

Assim, p/ 13:

$$\vec{\nabla}\times\vec{E_T} = -j\frac{\beta}{k_c^2}\left[\cancelto{0}{\vec{\nabla}\times\vec{\nabla}_T E_z} - \eta_{TE}\,\vec{\nabla}\times\hat{z}\times\vec{\nabla} H_z\right]$$

$$\vec{\nabla}\times\vec{E_T} = -j\frac{\beta}{k_c^2}\left[-\eta_{TE}\,\hat{z}\,\nabla_T^2 H_z\right]$$

$$\vec{\nabla}\times\vec{E_T} + j\omega\mu H_z\hat{z} = j\frac{\beta}{k_c^2}\eta_{TE}\,\hat{z}\,\nabla_T^2 H_z + j\omega\mu H_z\hat{z}$$

$$\vec{\nabla}\times\vec{E_T} + j\omega\mu H_z\hat{z} = j\frac{\cancel{\beta}}{k_c^2}\frac{\omega\mu}{\cancel{\beta}}\,\hat{z}\,\nabla_T^2 H_z + j\omega\mu H_z\hat{z}$$

$$\boxed{\vec{\nabla}\times\vec{E_T} + j\omega\mu H_z\hat{z} = j\frac{\omega\mu}{k_c^2}\,\hat{z}\left[\nabla_T^2 H_z + k_c^2 H_z\right]}$$

p/ 14:

$$\vec{\nabla}_T\times\vec{H_T} = -j\frac{\beta}{k_c^2}\left[\cancelto{0}{\vec{\nabla}_T\times\vec{\nabla}_T H_z} + \frac{1}{\eta_{TM}}\vec{\nabla}_T\times\hat{z}\times\vec{\nabla}_T E_z\right]$$

$$\vec{\nabla}_T\times\vec{H_T} = -j\frac{\beta}{k_c^2}\left[+\frac{\omega\varepsilon}{\beta}\,\hat{z}\,\nabla_T^2 E_z\right]$$

$$\vec{\nabla}_T\times\vec{H_T} - j\omega\varepsilon\hat{z}E_z = -j\frac{\omega\varepsilon}{k_c^2}\hat{z}\,\nabla_T^2 E_z - j\omega\varepsilon\hat{z}E_z$$

$$\boxed{\vec{\nabla}_T\times\vec{H_T} - j\omega\varepsilon\hat{z}E_z = -j\frac{\omega\varepsilon}{k_c^2}\hat{z}\left[\nabla_T^2 E_z + k_c^2 E_z\right]}$$

$\nabla_T \times \nabla_T A_z = 0,$
$\nabla_T \times (\hat{\mathbf{z}} \times \nabla_T A_z) = \hat{\mathbf{z}}\nabla_T^2 A_z,$
$\nabla_T \cdot (\hat{\mathbf{z}} \times \nabla_T Az) = 0.$

p/15:
$$\vec{\nabla}_T \cdot \vec{E}_T = -j\frac{\beta}{k_c^2}\left[\vec{\nabla}_T \cdot \vec{\nabla}_T E_z - \eta_{TE}\cancel{\vec{\nabla}_T \cdot \hat{z} \times \vec{\nabla} H_z}^{\,0}\right]$$

$$\vec{\nabla}_T \cdot \vec{E}_T = j\frac{\beta}{k_c^2}\nabla_T^2 E_z$$

$$\vec{\nabla}_T \cdot \vec{E}_T - j\beta E_z = j\frac{\beta}{k_c^2}\nabla_T^2 E_z - j\beta E_z$$

$$\boxed{\vec{\nabla}_T \cdot \vec{E}_T - j\beta E_z = j\frac{\beta}{k_c^2}\left[\nabla_T^2 E_z - k_c^2 E_z\right]}$$

p/16:
$$\vec{\nabla}_T \cdot \vec{H}_T = -j\frac{\beta}{k_c^2}\left[\vec{\nabla}_T \cdot \vec{\nabla}_T H_z + \frac{1}{\eta_{TM}}\cancel{\vec{\nabla}_T \cdot \hat{z} \times \vec{\nabla}_T E_z}^{\,0}\right]$$

$$\vec{\nabla}_T \cdot \vec{H}_T = -j\frac{\beta}{k_c^2}\nabla_T^2 H_z$$

$$\boxed{\vec{\nabla}_T \cdot \vec{H}_T - j\beta H_z = -j\frac{\beta}{k_c^2}\left[\nabla_T^2 H_z + k_c^2 H_z\right]}$$

Resumo da equações:

$$\left.\begin{aligned}
\nabla_T \times \mathbf{E}_T + j\omega\mu\,\hat{\mathbf{z}}H_z &= \frac{j\omega\mu}{k_c^2}\,\hat{\mathbf{z}}\left(\nabla_T^2 H_z + k_c^2 H_z\right) \\
\nabla_T \times \mathbf{H}_T - j\omega\epsilon\,\hat{\mathbf{z}}E_z &= -\frac{j\omega\epsilon}{k_c^2}\,\hat{\mathbf{z}}\left(\nabla_T^2 E_z + k_c^2 E_z\right) \\
\nabla_T \cdot \mathbf{E}_T - j\beta E_z &= -\frac{j\beta}{k_c^2}\left(\nabla_T^2 E_z + k_c^2 E_z\right) \\
\nabla_T \cdot \mathbf{H}_T - j\beta H_z &= -\frac{j\beta}{k_c^2}\left(\nabla_T^2 H_z + k_c^2 H_z\right)
\end{aligned}\right\} \quad (8)$$

- Assim, as componente de campo H_z e E_z devem satisfazer as equações de Helmholtz

$$\boxed{\begin{aligned}\nabla_T^2 E_z + k_c^2 E_z = 0\\ \nabla_T^2 H_z + k_c^2 H_z = 0\end{aligned}}$$

após obter a solução pelas condições de fronteiras de E_z e H_z as componentes de campo H_T e E_T são obtidas pela equações apresentadas em (8). Para obter o campo completo, (x, y, z, t), basta multiplicar pelo fato $e^{j\omega t - j\beta z}$.

- Para o sistema de coordenadas cartesiano:

$$\boxed{\begin{aligned}(\nabla_x^2 + \nabla_y^2) E_z + k_c^2 E_z = 0\\ (\nabla_x^2 + \nabla_y^2) H_z + k_c H_z = 0\end{aligned}}$$

p/ E_x e E_y: $\vec{E_T} = -j\dfrac{\beta}{k_c^2}\left[\vec{\nabla}_T E_z - \eta_{TE}\,\hat{z}\times\vec{\nabla} H_z\right]$

$$E_x\hat{x} + E_y\hat{y} = -j\frac{\beta}{k_c^2}\left[\partial_x E_z\hat{x} + \partial_y E_z\hat{y} - \eta_{TE}\,\partial_x H_z\hat{y} + \eta_{TE}\,\partial_y H_z\hat{x}\right]$$

logo,

$$\boxed{\begin{aligned}E_x &= -j\frac{\beta}{k_c^2}\left[\partial_x E_z + \eta_{TE}\,\partial_y H_z\right]\\ E_y &= -j\frac{\beta}{k_c^2}\left[\partial_y E_z - \eta_{TE}\,\partial_x H_z\right]\end{aligned}}$$

p/ H_x e H_y: $\vec{H_T} = -j\dfrac{\beta}{k_c^2}\left[\vec{\nabla}_T H_z + \dfrac{1}{\eta_{TM}}\hat{z}\times\vec{\nabla}_T E_z\right]$

$$H_x\hat{x} + H_y\hat{y} = -j\frac{\beta}{k_c^2}\left[\nabla_x H_z\hat{x} + \nabla_y H_z\hat{z} + \frac{1}{\eta_{TM}}\partial_x E_z y + \frac{1}{\eta_{TM}}\partial_y E_y(-\hat{x})\right]$$

$$H_x = -j\frac{\beta}{k_c^2}\left[\nabla_x H_z - \frac{1}{\eta_{TM}}\partial_y E_y\right];\quad H_y = -j\frac{\beta}{k_c^2}\left[\nabla_y H_z + \frac{1}{\eta_{TM}}\partial_x E_z\right]$$

• Resumo:

$$(\partial_x^2 + \partial_y^2)E_z + k_c^2 E_z = 0$$
$$(\partial_x^2 + \partial_y^2)H_z + k_c^2 H_z = 0$$

$$E_x = -\frac{j\beta}{k_c^2}(\partial_x E_z + \eta_{TE}\,\partial_y H_z)$$
$$E_y = -\frac{j\beta}{k_c^2}(\partial_y E_z - \eta_{TE}\,\partial_x H_z)$$

$$H_x = -\frac{j\beta}{k_c^2}(\partial_x H_z - \frac{1}{\eta_{TM}}\,\partial_y E_z)$$
$$H_y = -\frac{j\beta}{k_c^2}(\partial_y H_z + \frac{1}{\eta_{TM}}\,\partial_x E_z)$$

• Soluções para diferentes modos:

1) Modo TEM: $k_c^2 = 0$; $k_c^2 = k^2 - \beta^2 \Rightarrow k = \beta \rightarrow \omega\sqrt{\mu\varepsilon} = \beta$

$$\frac{\omega}{c} = \beta$$

$$\boxed{\omega = \beta c}$$

ainda, $\eta_{TE} = \frac{\omega\mu}{\beta} = \eta\,\frac{\omega}{\beta c} = \eta$; $\eta_{TM} = \frac{\beta}{\omega\varepsilon} = \eta\,.\,\frac{\beta c}{\omega} = \eta$

• Resumo:

$$(\partial_x^2 + \partial_y^2)\cancel{E_z}^{\,0} + k_c^2 \cancel{E_z}^{\,0} = 0$$
$$(\partial_x^2 + \partial_y^2)\cancel{H_z}^{\,0} + k_c^2 \cancel{H_z}^{\,0} = 0$$

$$\nabla E_z \times \hat{\mathbf{z}} - j\beta\,\hat{\mathbf{z}} \times \mathbf{E}_T = -j\omega\mu\mathbf{H}_T \quad (11)$$
$$\nabla H_z \times \hat{\mathbf{z}} - j\beta\,\hat{\mathbf{z}} \times \mathbf{H}_T = j\omega\varepsilon\mathbf{E}_T \quad (12)$$
$$\nabla \times \mathbf{E}_T + j\omega\mu H_z\,\hat{\mathbf{z}} = 0 \quad (13)$$
$$\nabla \times \mathbf{H}_T - j\omega\varepsilon E_z\,\hat{\mathbf{z}} = 0 \quad (14)$$
$$\nabla_T \cdot \mathbf{E}_T - j\beta E_z = 0 \quad (15)$$
$$\nabla_T \cdot \mathbf{H}_T - j\beta H_z = 0 \quad (16)$$

$$\vec{H_T} = \frac{\beta}{\omega\mu}\,\hat{z} \times \vec{E_T} = \frac{1}{\eta_{TE}}\,\hat{z} \times \vec{E_T} = \frac{1}{\eta}\,.\,\hat{z} \times \vec{E_T}$$

$$\boxed{\vec{H_T} = \frac{1}{\eta}\,.\,\hat{z} \times \vec{E_T}}$$

O campo elétrico pode ser determinado por

$$\boxed{\vec{\nabla_T} \times \vec{E_T} = 0 \quad e \quad \vec{\nabla_T} \cdot \vec{E_T} = 0}$$

2) Modo TM: ($H_z = 0$ e $E_z \neq 0$)

$$(\partial_x^2 + \partial_y^2)E_z + k_c^2 E_z = 0$$
$$(\partial_x^2 + \partial_y^2)\cancel{H_z}^{\to 0} + k_c^2 \cancel{H_z}^{\to 0} = 0$$

$$\vec{E_T} = -j\frac{\beta}{k_c^2}\left[\vec{\nabla}_T E_z - \eta_{TE}\, \hat{z} \times \cancel{\vec{\nabla} H_z}^{\to 0}\right] \Rightarrow \vec{E_T} = -j\frac{\beta}{k_c^2}\vec{\nabla}_T E_z$$

• Determinando $\vec{H_T}$:

$$\nabla E_z \times \hat{\mathbf{z}} - j\beta\,\hat{\mathbf{z}} \times \mathbf{E}_T = -j\omega\mu\mathbf{H}_T \qquad (11)$$
$$\nabla H_z \times \hat{\mathbf{z}} - j\beta\,\hat{\mathbf{z}} \times \mathbf{H}_T = j\omega\varepsilon\mathbf{E}_T \qquad (12)$$
$$\nabla \times \mathbf{E}_T + j\omega\mu H_z\,\hat{\mathbf{z}} = 0 \qquad (13)$$
$$\nabla \times \mathbf{H}_T - j\omega\varepsilon E_z\,\hat{\mathbf{z}} = 0 \qquad (14)$$
$$\nabla_T \cdot \mathbf{E}_T - j\beta E_z = 0 \qquad (15)$$
$$\nabla_T \cdot \mathbf{H}_T - j\beta H_z = 0 \qquad (16)$$

Pode-se escrever (11) e (12) como:

$$j\beta\hat{z} \times \vec{E_T} - j\omega\mu\vec{H_T} = \vec{\nabla} E_z \times \hat{z} \Rightarrow \beta\hat{z} \times \vec{E_T} - \omega\mu\vec{H_T} = -j\vec{\nabla} E_z \times \hat{z}$$

$$\Rightarrow \beta\hat{z} \times \vec{E_T} - \omega\mu\vec{H_T} = j\hat{z} \times \vec{\nabla} E_z$$

$$\beta\,\hat{\mathbf{z}} \times \boldsymbol{E}_T - \omega\mu\boldsymbol{H}_T = j\hat{\mathbf{z}} \times \boldsymbol{\nabla}_T E_z$$
$$\omega\epsilon\,\hat{\mathbf{z}} \times \boldsymbol{E}_T - \beta\boldsymbol{H}_T = -j\boldsymbol{\nabla}_T H_z$$

ainda

$$\vec{H_T} = -\frac{j}{\omega\mu}\hat{z} \times \vec{\nabla}_T E_z + \frac{\beta}{\omega\mu}\hat{z} \times \vec{E_T} = -\frac{j}{\beta}\frac{1}{\eta_{TE}} \cdot \hat{z} \times \vec{\nabla}_T E_z + \frac{1}{\eta_{TE}}\vec{z} \times \vec{E_T}$$

Caminho errado!

$$\boldsymbol{H}_T - \frac{1}{\eta_{TM}}\hat{\mathbf{z}} \times \boldsymbol{E}_T = \frac{j}{\beta}\boldsymbol{\nabla}_T H_z$$
$$\boldsymbol{E}_T - \eta_{TE}\boldsymbol{H}_T \times \hat{\mathbf{z}} = \frac{j}{\beta}\boldsymbol{\nabla}_T E_z$$

$$\beta\,\hat{\mathbf{z}}\times\mathbf{E}_T-\omega\mu\mathbf{H}_T=j\hat{\mathbf{z}}\times\nabla_T E_z$$
$$\omega\epsilon\,\hat{\mathbf{z}}\times\mathbf{E}_T-\beta\mathbf{H}_T=-j\nabla_T \cancel{H_z}^{\to 0}$$

$$\Rightarrow \beta H_T = \omega\varepsilon\,\hat{z}\times\vec{E_T}$$
$$H_T = \frac{1}{\eta_{TM}}\,\hat{z}\times\vec{E_T}$$

$\Rightarrow$ Em resumo:

$$\nabla_T^2 E_z + k_c^2 E_z = 0$$
$$\mathbf{E}_T = -j\frac{\beta}{k_c^2}\nabla_T E_z$$
$$\mathbf{H}_T = \frac{1}{\eta_{\text{TM}}}\,\hat{\mathbf{z}}\times\mathbf{E}_T.$$

Modo TM

3) Modo TE: ($E_z = 0$ e $H_z \neq 0$)

$$(\partial_x^2+\partial_y^2)\cancel{E_z}^{\to 0}+k_c^2\cancel{E_z}^{\to 0}=0$$
$$(\partial_x^2+\partial_y^2)H_z+k_c^2H_z=0$$

- $\vec{\nabla}_T H_z + k_c^2 H_z = 0$

$$\vec{H_T} = -j\frac{\beta}{k_c^2}\left[\vec{\nabla}_T H_z + \cancel{\frac{1}{\eta_{TM}}\hat{z}\times\vec{\nabla}_T E_z}^{\to 0}\right]$$

- $\vec{H_T} = -j\dfrac{\beta}{k_c^2}\vec{\nabla}_T H_z$

$$\beta\,\hat{\mathbf{z}}\times\mathbf{E}_T-\omega\mu\mathbf{H}_T=j\hat{\mathbf{z}}\times\cancel{\nabla_T E_z}^{\to 0}$$
$$\omega\epsilon\,\hat{\mathbf{z}}\times\mathbf{E}_T-\beta\mathbf{H}_T=-j\nabla_T H_z$$

$$\beta\hat{z}\times\vec{E_T} = \omega\mu\vec{H_T} \Rightarrow \hat{z}\times\vec{E_T} = \eta_{TE}\vec{H_T} \Rightarrow -\vec{E_T} = \eta_{TE}\,\hat{z}\times\vec{H_T}$$

- $\vec{E_T} = \eta_{TE}\,\vec{H_T}\times\hat{z}$

Logo, em resumo:

$$\nabla_T^2 H_z + k_c^2 H_z = 0$$

$$\mathbf{H}_T = -j\frac{\beta}{k_c^2}\boldsymbol{\nabla}_T H_z$$

$$\mathbf{E}_T = \eta_{\mathsf{TE}}\,\mathbf{H}_T \times \hat{\mathbf{z}}.$$

Modo TE

4) Modos híbridos:

Análise - *slab* simétrico ($n_c = n_s$)

Etapas de análise (de acordo com a geometria do problema):

1. aplicação das equações de Maxwell;
2. determinação das componentes dos campos;
3. aplicação das condições de contorno;
4. determinação da equação característica;
5. a partir da solução da equação característica, determinar os parâmetros dos modos, como índice de refração e constante de propagação.

De acordo com as características do guia, como homogeneidade e simetria, é possível classificar a solução, como TEM ou TE, e determinar diretamente as componentes dos campos.

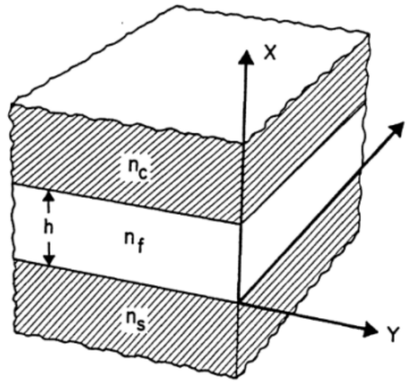


Figura 6: Guia dielétrico de placas planas assimétrico.

A Fig. 6 ilustra um guia dielétrico retangular formado por três camadas dielétricas, cujo índice de refração é n_i ($i = c, f, s$). O guia é simétrico ($n_c = n_s$), em que $n_f > n_s$ e, portanto, satisfaz a condição de reflexão interna total. Assim, o campo propaga-se no dielétrico, cujo índice é igual a n_f. Tomando a Eq. 25 e Eq. 26 (geometria retangular) e considerando $\partial/\partial y = 0$, estas equações são reescritas como:

$$\vec{E_T} = -j\frac{\beta}{k_c^2}\left[\vec{\nabla}_T E_z - \eta_{TE}\,\hat{z}\times\vec{\nabla} H_z\right]$$

$$\vec{H_T} = -j\frac{\beta}{k_c^2}\left[\vec{\nabla}_T H_z + \frac{1}{\eta_{TM}}\hat{z}\times\vec{\nabla}_T E_z\right]$$

$$\vec{E_T} = -j\frac{\beta}{k_c^2}\left[\frac{d}{dx}E_z\hat{x} - \eta_{TE}\frac{d}{dx}H_z\hat{y}\right]$$

$$\vec{H_T} = -j\frac{\beta}{k_c^2}\left[\frac{d}{dx}H_z\hat{x} + \frac{1}{\eta_{TM}}\frac{d}{dx}E_z\hat{y}\right]$$

Assim, $E_x = -j\dfrac{\beta}{k_c^2}\dfrac{d}{dx}E_z\hat{x}$ e $E_y = j\dfrac{\beta}{k_c}\eta_{TE}\dfrac{d}{dx}H_z$

$H_x = -j\dfrac{\beta}{k_c^2}\dfrac{d}{dx}H_z$ e $H_y = -j\dfrac{\beta}{k_c^2}\dfrac{1}{\eta_{TM}}\dfrac{d}{dx}E_z$

ou

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = -j\frac{\beta}{k_c}\begin{bmatrix} 1 & 0 \\ 0 & -\eta_{TE} \end{bmatrix}\begin{bmatrix} dE_z/dx \\ dH_z/dx \end{bmatrix} \quad e \quad \begin{bmatrix} H_x \\ H_y \end{bmatrix} = -j\frac{\beta}{k_c^2}\begin{bmatrix} 1 & 0 \\ 0 & 1/\eta_{TM} \end{bmatrix}\begin{bmatrix} dH_z/dx \\ dE_z/dx \end{bmatrix}$$

As componentes longitudinais: $\frac{d^2E_z}{dx^2} + k_c^2 E_z = 0$ e $\frac{d^2H_z}{dx^2} + k_c^2 H_z = 0$

Observa-se que há dois conjuntos de equações, portanto, pode-se dividir em:

Então, podemos dividir as equações em dois grupos:

Modo TM:

$$E_x = -j\frac{\beta}{k_c^2}\frac{\mathrm{d}E_z}{\mathrm{d}x}$$
$$H_y = -j\frac{\omega\varepsilon}{k_c^2}\frac{\mathrm{d}E_z}{\mathrm{d}x}$$
$$\frac{\mathrm{d}^2E_z}{\mathrm{d}x^2} + k_c^2E_z = 0.$$

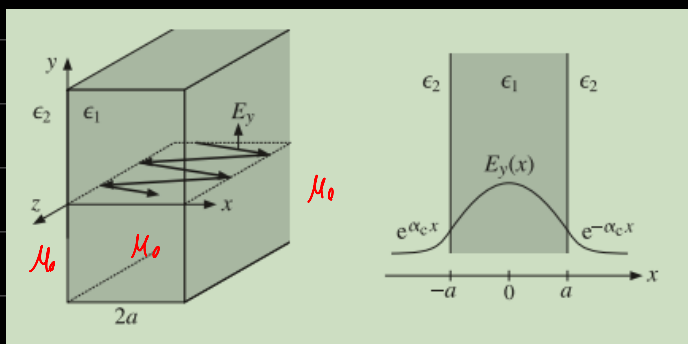
Modo TE:

$$E_y = j\frac{\omega\mu}{k_c^2}\frac{\mathrm{d}H_z}{\mathrm{d}x}$$
$$H_x = -j\frac{\beta}{k_c^2}\frac{\mathrm{d}H_z}{\mathrm{d}x}$$
$$\frac{\mathrm{d}^2H_z}{\mathrm{d}x^2} + k_c^2H_z = 0.$$

• No interior e exterior do guia as equações de Helmholtz são:

$\frac{d^2}{dx^2}H_z + k_{c1}^2 H_z = 0$ e $\frac{d}{dx^2}H_z + k_{c2}^2 H_z = 0$
↳ exterior

• Solucionando a ED:
$r^2 + k_{c1}^2 = 0 \Rightarrow r = \pm j k_{c1}$
logo, $H_z(x) = A\cos(k_{c1}x) + B\,\mathrm{sen}(k_{c1}x)$



• Para o exterior, assume-se que $k_{c2} = -\alpha_c^2$
logo, $r^2 - \alpha_c^2 = 0 \Rightarrow r = \pm \alpha_c$
$|x| \geq a$ $\quad H_z(x) = Ce^{\alpha_c x} + De^{-\alpha_c x}$

Adotando: $k_{c1} = k_c$ e $k_{c2} = -\alpha_c \Rightarrow k_c^2 = k_0^2 n_1^2 - \beta^2$; $-\alpha_c^2 = k_0^2 n_2^2 - \beta^2$
$\alpha_c^2 = \beta^2 - k_0^2 n_2^2$

O campo deve decair fora do core exponencialmente. Assim:

$\frac{d}{dx}H_z(x) + k_c^2 H_z(x) = 0 \ (|x| \leq a)$ e $\frac{d}{dx}H_z(x) - \alpha_c H(x) = 0 \ (|x| \geq a)$

$$\rightarrow H_z(x) = E\cos(k_c x - \theta)$$

Com isso:

$$\begin{cases} H_z(x) = A\cos(k_c x) + B\,\text{sen}(k_c x) & ; |x| \leq a \\ H_z(x) = C e^{-\alpha_c x} & ; x \geq a \\ H_z(x) = D e^{\alpha_c x} & ; x \leq a \end{cases}$$

$(E = H_1;\ C = H_2;\ D = H_3)$

• Para

$$H_x(x) = -\frac{j\beta}{k_c^2}\left(-k_c E\,\text{sen}(k_c x - \theta)\right) = \frac{j\beta}{k_c^2} k_c E\,\text{sen}(k_c x - \theta) \quad ; |x| \leq a$$

$$H_x(x) = -\frac{j\beta}{k_c^2} - \alpha_c C e^{-\alpha_c x} = \frac{j\beta}{k_c^2}\alpha_c C e^{-\alpha_c x} \quad ; x \geq a$$

$$H_x(x) = -\frac{j\beta}{k_c^2}.\left[D\alpha_c e^{\alpha_c x}\right] = -\frac{j\beta}{\underbrace{k_c^2}_{\rightarrow -\alpha_c^2}} D\alpha_c e^{\alpha_c x} \quad ; x \leq a$$

$$\begin{cases} H_x(x) = \dfrac{j\beta}{k_c^2} k_c E\,\text{sen}(k_c x - \theta) & ; |x| \leq a \\ H_x(x) = -\dfrac{j\beta}{\alpha_c} C e^{-\alpha_c x} & ; x \geq a \\ H_x(x) = \dfrac{j\beta}{\alpha_c} D e^{\alpha_c x} & ; x \leq -a \end{cases}$$

$(E = H_1;\ C = H_2;\ D = H_3)$

$$\begin{cases} H_x(x) = \dfrac{j\beta}{k_c} H_1\,\text{sen}(k_c x - \theta) & ; |x| \leq a \\ H_x(x) = -\dfrac{j\beta}{\alpha_c} H_2 e^{-\alpha_c x} & ; x \geq a \\ H_x(x) = \dfrac{j\beta}{\alpha_c} H_3 e^{\alpha_c x} & ; x \leq -a \end{cases}$$

• Para $E_y(x)$ o modo TE se tem: $E_y(x) = -\eta_{TE} H_x(x)$

$$\begin{cases} H_x(x) = \dfrac{j\beta}{k_c} H_1\,\text{sen}(k_c x - \theta) & ; |x| \leq a \\ H_x(x) = -\dfrac{j\beta}{\alpha_c} H_2 e^{-\alpha_c x} & ; x \geq a \\ H_x(x) = \dfrac{j\beta}{\alpha_c} H_3 e^{\alpha_c x} & ; x \leq a \end{cases} \Rightarrow \begin{aligned} E_y(x) &= -\frac{j\beta}{k_c} H_1 \eta_{TE}\,\text{sen}(k_c x - \theta) \\ E_y(x) &= \frac{j\beta}{\alpha_c} H_2 \eta_{TE} e^{-\alpha_c x} \\ E_y(x) &= -\frac{j\beta}{\alpha_c} H_3 \eta_{TE} e^{\alpha_c x} \end{aligned}$$

Logo, $\begin{cases} E_y(x) = E_1 \operatorname{sen}(k_c x - \theta) & ; \ |x| \leq a \\ E_y(x) = E_2 e^{-\alpha_c x} & ; \ x \geq a \\ E_y(x) = E_3 e^{\alpha_c x} & ; \ x \leq -a \end{cases}$

The boundary conditions state that the tangential components of the magnetic and electric fields, that is, H_z, E_y, are continuous across the dielectric interfaces at $x = -a$ and $x = a$. Similarly, the normal components of the magnetic field $B_x = \mu_0 H_x$ and therefore also H_x must be continuous. Because $E_y = -\eta_{TE} H_x$ and η_{TE} is the same in both media, the continuity of E_y follows from the continuity of H_x. The continuity of H_z at $x = a$ and $x = -a$ implies that: η_1

$$H_x(x) = j\frac{\beta}{k_c} H_1 \operatorname{sen}(k_c x - \theta)$$
$$H_x(x) = -j\frac{\beta}{\alpha_c} H_2 e^{-\alpha_c x}$$
$$H_x(x) = j\frac{\beta}{\alpha_c} H_3 e^{\alpha_c x}$$

$$\Rightarrow \begin{cases} \dfrac{H_1}{k_c} \operatorname{sen}(k_c x - \theta) = -\dfrac{1}{\alpha_c} H_2 e^{-\alpha_c x} & (x = a) \\ \dfrac{H_1}{k_c} \operatorname{sen}(k_c x - \theta) = \dfrac{1}{\alpha_c} H_3 e^{\alpha_c x} & (x = -a) \end{cases}$$

$$(8) \quad \begin{cases} \dfrac{H_1}{k_c} \operatorname{sen}(k_c a - \theta) = -\dfrac{1}{\alpha_c} H_2 e^{-\alpha_c a} \\ \dfrac{H_1}{k_c} \operatorname{sen}(-k_c a - \theta) = \dfrac{1}{\alpha_c} H_3 e^{-\alpha_c a} \end{cases}$$

Do sistema (8) tem-se: $H_2 = -H_3 = -\dfrac{\alpha_c}{k_c} H_1 e^{\alpha_c a} \operatorname{sen}(-k_c a - \theta)$

$$H_2 = -H_3 = \frac{\alpha_c}{k_c} H_1 e^{\alpha a} \operatorname{sen}(k_c a + \theta) \quad (9)$$

$$H_z(x) = H_1 \cos(k_c x - \theta)$$
$$H_z(x) = H_2 e^{-\alpha_c x}$$
$$H_z(x) = H_3 e^{\alpha_c x}$$

$$\Rightarrow H_1 \cos(k_c a - \theta) = H_2 e^{-\alpha_c a} \quad (x = a)$$
$$H_2 = e^{\alpha_c a} \cos(k_c a - \theta) H_1 \quad (10)$$

- $(10) = (9) \Rightarrow H_1 e^{\alpha_c a} \cos(k_c a - \theta) = \dfrac{\alpha_c}{k_c} H_1 e^{\alpha_c a} \operatorname{sen}(k_c a + \theta)$

$$\alpha_c = k_c \frac{\cos(k_c a - \theta)}{\operatorname{sen}(k_c a + \theta)}$$

Definição do guia

O guia dielétrico planar é formado por N camadas de material dielétrico de espessura h_i e índice de refração n_i $(i = f, n, s)$. Nosso primeiro estudo consiste em um guia dielétrico planar de 3 (três) camadas, conforme ilustra a Fig. 7. A relação entre os índices é

$$0 < n_c < n_s < n_f. \tag{79}$$

Por esta relação, devido à reflexão interna total, o campo é confinado majoritariamente no núcleo (índice n_f). Entretanto, há campo evanescente na casca (índice n_c) e no substrato (índice n_s). É importante destacar que μ_r (permeabilidade magnética) em todos os guias dielétricos é unitária.

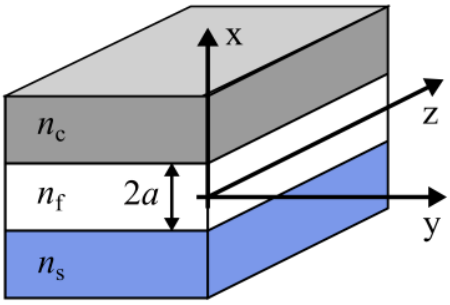


Figura 7: Ilustração do guia dielétrico planar e a escolha do sistema de coordenadas. Observe que a origem da coordenada x é o centro da camada cujo índice é n_f.

Para o modo TE:

$$\nabla_T^2 H_z + k_c^2 H_z = 0$$
$$\mathbf{H}_T = -j\frac{\beta}{k_c^2}\boldsymbol{\nabla}_T H_z$$
$$\mathbf{E}_T = \eta_{\mathrm{TE}}\,\mathbf{H}_T \times \hat{\mathbf{z}}.$$

$$\frac{\partial}{\partial y} = 0$$

assim, $\frac{d^2 H_z}{dx^2} + k_c^2 H_z = 0$; $H_x = -j\frac{\beta}{k_c^2}\frac{d H_z}{dx}$; $E_x = 0$

$E_x = \eta_{TE} \cdot H_x \hat{x} \times \hat{z} \Rightarrow E_y = +\eta_{TE}\, j\frac{\beta}{k_c^2}\frac{d}{dx}H_z$

$$\begin{cases} \dfrac{d^2 H_z}{dx^2} + k_i^2 H_z = 0 \\ H_x = -j\dfrac{\beta}{k_i^2}\dfrac{dH_z}{dx} \\ E_y = j\eta_{\mathrm{TE}}\dfrac{\beta}{k_i^2}\dfrac{dH_z}{dx} \end{cases}$$

- Definindo: $k_i^2 = k_0^2 n_i^2 - \beta^2$ $(i = f, n, s)$ e $\eta_{TE} = \omega\mu/\beta$
  $\beta = k_0 n_{eff}$; n_{eff} é o índice de refração efetivo do modo no guia.

- As componentes de H_z devem ser iguais, funções contínuas, para as três regiões do guia. Assim, as condições de contorno serão satisfeitas.

1) $\frac{d^2}{dx^2}H_z(x) + k_c^2 H_z(x) = 0$ $\quad x \geq a \Rightarrow h^2 + k_c^2 = 0 \rightsquigarrow h = \pm jk$

↳ o campo deve decair na casca.

Logo, $k_c^2 = -\alpha_c^2$

assim, $\frac{d H_z(x)}{dx} - \alpha_c^2 H_z(x) = 0 \Rightarrow r^2 - \alpha_c^2 = 0 \Rightarrow r = \pm \alpha_c$

portanto, $\boxed{H_z(x) = A e^{\alpha_c x} + A' e^{-\alpha_c x} \quad (x \geqslant a)}$

2) Para o core: $\frac{d^2 H_z(x)}{dx^2} + k_F^2 H_z(x) = 0 \Rightarrow r = \pm j k_F$

logo, $\boxed{H_z(x) = B\cos(k_F x) + C\sin(k_F x) \quad (|x| \leq a)}$

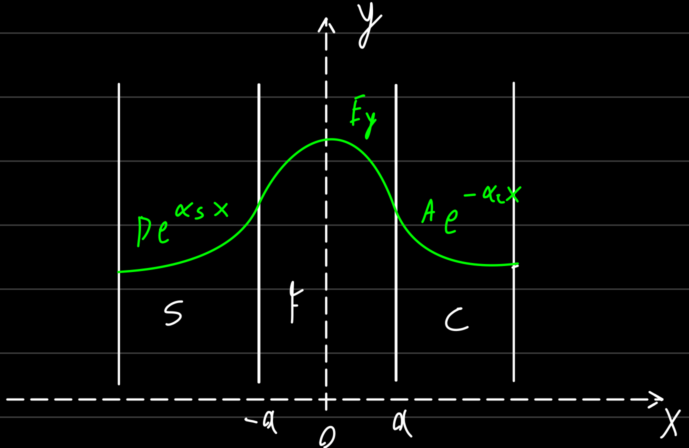
3) Para o substrato: $\frac{d^2 H_z(x)}{dx^2} - \alpha_s^2 H_z(x) = 0 \Rightarrow r = \pm \alpha_s$

$\boxed{H_z(x) = D e^{\alpha_s x} + D' e^{-\alpha_s x} \quad (x \leq -a)}$

Resumo:

$$H_z(x) = \begin{cases} Ae^{-\alpha_c x} + A' e^{\alpha_c x}, & x \geqslant a \\ B\cos(k_f x) + C\sin(k_f x), & |x| \leqslant a \\ De^{\alpha_s x} + D' e^{-\alpha_s x}, & x \leqslant -a \end{cases}$$

Condições:



Para essas condições serem satisfeitas, tem-se:

$$H_z(x) = \begin{cases} Ae^{-\alpha_c x} + \underbrace{\cancel{A' e^{\alpha_c x}}}_{\to 0}, & x \geqslant a \\ B\cos(k_f x) + C\sin(k_f x), & |x| \leqslant a \\ De^{\alpha_s x} + \underbrace{\cancel{D' e^{-\alpha_s x}}}_{\to 0}, & x \leqslant -a \end{cases}$$

$A \operatorname{sen} x + B\cos x = \sqrt{A^2+B^2}\,\sin(x + tg^{-1}(B/A))$

Logo,

$$H_z(x) = \begin{cases} Ae^{-\alpha_c x}, & x \geqslant a \\ H_0 \sin(k_f x + \phi), & |x| \leqslant a \\ De^{\alpha_s x}, & x \leqslant -a \end{cases}$$

Aplicando as condições de fronteira: $H_0 \sin(k_f a + \varnothing) = A e^{-\alpha_c a}$ $\quad (x = a)$

$$A = H_0 \sin(k_f a + \varnothing)\, e^{\alpha_c a}$$

e

$$H_0 \sin(-k_f a + \varnothing) = D e^{-\alpha_s a} \quad (x = -a)$$

$$D = H_0 \sin(-k_f a + \varnothing)\, e^{\alpha_s a}$$

Desse modo,

$$\boxed{H_z(x) = \begin{cases} H_0 \sin(k_f a + \varnothing)\, e^{\alpha_c a}\, e^{-\alpha_c x} & , \; x \geq a \\ H_0 \sin(k_f x + \varnothing) & , \; |x| \leq a \\ -H_0 \sin(k_f a - \varnothing)\, e^{\alpha_s a}\, e^{\alpha_s x} & , \; x \leq a \end{cases}}$$

$$\begin{cases} \dfrac{\mathrm{d}^2 H_z}{\mathrm{d}x^2} + k_i^2 H_z = 0 \\ H_x = -j \dfrac{\beta}{k_i^2} \dfrac{\mathrm{d}H_z}{\mathrm{d}x} \\ E_y = j\eta_{\mathrm{TE}} \dfrac{\beta}{k_i^2} \dfrac{\mathrm{d}H_z}{\mathrm{d}x} \end{cases}$$

• Para determinar $H_x(x)$:

Casca: $H_x(x) = j \dfrac{\beta \alpha_c}{-\alpha_c^2} H_0 \sin(k_f a + \varnothing)\, e^{\alpha_c a}\, e^{-\alpha_c x}$

$$H_x(x) = -j H_0 \beta \alpha_c^{-1} \sin(k_f a + \varnothing)\, e^{\alpha_c a}\, e^{-\alpha_c x} \quad (x \geq a)$$

Core: $H_x(x) = -j \dfrac{\beta}{k_f^2} k_f H_0 \cos(k_f x + \varnothing)$

$$H_x(x) = -j \beta H_0 k_f^{-1} \cos(k_f x + \varnothing) \quad (|x| \leq a)$$

Substrato: $H(x) = -j \dfrac{\beta}{-\alpha_s^2} \left(-\alpha_s H_0 \sin(k_f a - \varnothing)\, e^{\alpha_s a}\, e^{\alpha_s x}\right)$

$$H(x) = -j H_0 \beta \alpha_s^{-1} \sin(k_f a - \varnothing)\, e^{\alpha_s a}\, e^{\alpha_s x} \quad (x \leq -a)$$

• Para $E_y(x)$: $E_y = +\eta_{TE}\, j \dfrac{\beta}{k_c^2} \dfrac{d H_z}{dx}$

$$\boxed{H_z(x) = \begin{cases} H_0 \sin(k_f a + \varnothing)\, e^{\alpha_c a}\, e^{-\alpha_c x} & , \; x \geq a \\ H_0 \sin(k_f x + \varnothing) & , \; |x| \leq a \\ -H_0 \sin(k_f a - \varnothing)\, e^{\alpha_s a}\, e^{\alpha_s x} & , \; x \leq -a \end{cases}}$$

$$\eta_{TE} = \omega \mu / \beta$$

Casca: $E_y(x) = j H_0 \omega \mu \alpha_c^{-1} \sin(k_f a + \varnothing)\, e^{-\alpha_c (x - a)}$ $\quad (x \geq a)$

Core: $E_y(x) = j H_0 \omega \mu k_f^{-1} \cos(k_f x + \phi) \quad (|x| \leq a)$

Substrato: $E_y(x) = + j H_0 \omega \mu \alpha_s^{-1} \sin(k_f a - \phi) e^{\alpha_s (x+a)} \quad (x \leq -a)$

- Em resumo:

$$H_z(x) = H_0 \begin{cases} \sin(k_f a + \phi) e^{-\alpha_c (x-a)}, & x \geqslant a \\ \sin(k_f x + \phi), & |x| \leqslant a \\ -\sin(k_f a - \phi) e^{\alpha_s (x+a)}, & x \leqslant -a \end{cases}$$

$$H_x(x) = -j H_0 \beta \begin{cases} \alpha_c^{-1} \sin(k_f a + \phi) e^{-\alpha_c (x-a)}, & x \geqslant a \\ k_f^{-1} \cos(k_f x + \phi), & |x| \leqslant a \\ \alpha_s^{-1} \sin(k_f a - \phi) e^{\alpha_s (x+a)}, & x \leqslant -a \end{cases}$$

$$E_y(x) = j H_0 \omega \mu \begin{cases} \alpha_c^{-1} \sin(k_f a + \phi) e^{-\alpha_c (x-a)}, & x \geqslant a \\ k_f^{-1} \cos(k_f x + \phi), & |x| \leqslant a \\ \alpha_s^{-1} \sin(k_f a - \phi) e^{\alpha_s (x+a)}, & x \leqslant -a \end{cases}$$

- As condições de contorno de $E_y(x)$ resulta:

(1): $\dfrac{1}{k_f} \cos(k_f a + \phi) = \dfrac{1}{\alpha_c} \sin(k_f a + \phi) \rightarrow \tan(k_f a + \phi) = \dfrac{\alpha_c}{k_f} \quad (x = a)$

(2): $\dfrac{1}{k_f} \cos(-a k_f + \phi) = \dfrac{1}{\alpha_s} \sin(k_f a - \phi) \rightarrow \tan(k_f a - \phi) = \dfrac{\alpha_s}{k_f} \quad (x = -a)$

Obtém-se:

$$\tan(k_f a + \phi) = \frac{\alpha_c}{k_f} \Rightarrow \quad k_f a + \phi = \arctan(\alpha_c / k_f)$$

$$+$$

$$\tan(k_f a - \phi) = \frac{\alpha_s}{k_f} \Rightarrow \quad k_f a - \phi = \arctan(\alpha_s / k_f)$$

$$\|$$

$$2 k_f a = \arctan(\alpha_c / k_f) + \arctan(\alpha_s / k_f)$$

Assim, $2 k_f a = \arctan(\alpha_c / k_f) + \arctan(\alpha_s / k_f) \quad (11)$

sabendo que: $\arctan(a) \pm \arctan(b) = \arctan\left(\dfrac{a \pm b}{1 \mp ab}\right)$

logo, de (11): $2 k_f a = tg^{-1}\left(\dfrac{\alpha_c / k_f + \alpha_s / k_f}{1 - \alpha_c \alpha_s / k_f^2}\right) \Rightarrow \tan(2 k_f a) = \dfrac{\alpha_c + \alpha_s}{k_f - \alpha_c \alpha_s / k_f}$

$$\boxed{\tan(2 k_f a) = \frac{k_f(\alpha_c + \alpha_s)}{k_f^2 - \alpha_c \alpha_s} \quad (12)}$$

Para praticidade nas manipulações algébricas, são definidos os parâmetros modais $u = k_f h$, $v = \alpha_s h$ e $w = \alpha_c h$. Esses parâmetros são positivos (ou nulos) e adimensionais. A partir desses parâmetros modais, a Eq. (12) é reescrita como

(Adota-se $h = a$)

$u = k_f a$, $v = \alpha_s a$ e $w = \alpha_c a$

$$\tan(2u) = \frac{u/a\,[v/a + w/a]}{(u/a)^2 - vw/a^2} = \frac{u\,[v + w]}{u^2 - vw}$$

Logo,

$$\boxed{\tan(2u) = \frac{u\,[v+w]}{u^2 - vw} \quad (13)}$$

Solução gráfica para a eq. (13):

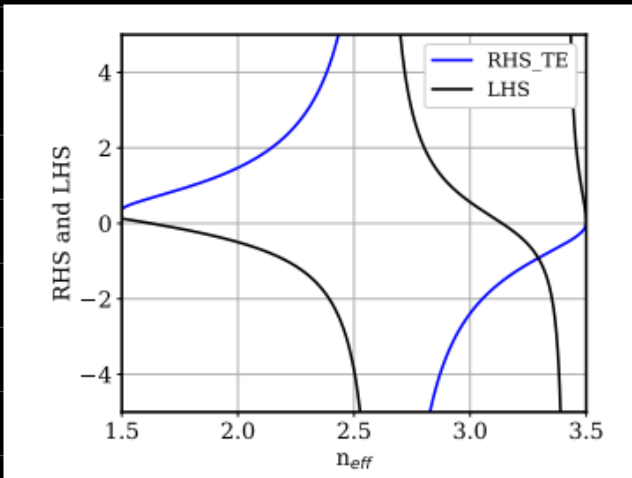


Figura 8: Curvas da equação característica, considerando $n_f = 3.5$, $n_s = 1.45$, $n_c = 1.0$, $a = 0,25\ \mu$m e $\lambda = 1550$ nm.

lembrando que: $k_i^2 = k_0^2 n_i^2 - \beta^2$

e $\beta = k_0 n_{eff}$

assim:

$$\begin{cases} u = a k_f = a k_0 \sqrt{n_f^2 - n_{eff}^2} \\ w = a \alpha_c = a k_0 \sqrt{n_{eff}^2 - n_c^2} \\ v = a \alpha_s = a k_0 \sqrt{n_{eff}^2 - n_s^2} \end{cases}$$

=> Determinando outros parâmetros importantes:

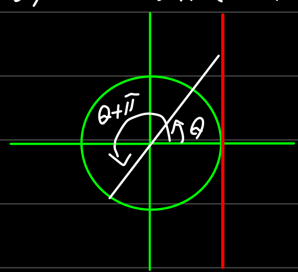
$$\tan(k_f a + \phi) = \frac{\alpha_c}{k_f}$$

$$\tan(k_f a - \phi) = \frac{\alpha_s}{k_f}$$

$$\tan(u + \phi) = \frac{w}{u}$$

$$\tan(u - \phi) = \frac{v}{u}$$

$$\tan(\theta) = \tan(\theta + m\pi)$$



$$\tan(u + \phi) = \tan(u + \phi + m\pi) = \frac{w}{u} \Rightarrow u + \phi + m\pi = \tan^{-1}\left(\frac{w}{u}\right) \quad (14)$$

$$\tan(u - \phi) = \tan(u - \phi + m\pi)\ \frac{v}{u} \Rightarrow u - \phi + m\pi = \tan^{-1}\left(\frac{v}{u}\right) \quad (15)$$

$$(14)+(15):\quad 2u = m\pi + \tan^{-1}\left(\frac{w}{u}\right) + \tan^{-1}\left(\frac{v}{u}\right)$$

$-(14)-(15)$: $-2\phi = m\pi - \tan^{-1}\left(\frac{w}{u}\right) + \tan^{-1}\left(\frac{v}{u}\right)$

$$2\phi = m\pi + \tan^{-1}\left(\frac{w}{u}\right) - \tan^{-1}\left(\frac{v}{u}\right)$$

Em resumo:

$$2u = m\pi + \arctan\left(\frac{w}{u}\right) + \arctan\left(\frac{v}{u}\right)$$

$$2\phi = m\pi + \arctan\left(\frac{w}{u}\right) - \arctan\left(\frac{v}{u}\right)$$

p/ $m=0$

$$2u = \tan^{-1}\Big(\underbrace{\frac{w}{u}}_{a}\Big) + \tan^{-1}\Big(\underbrace{\frac{v}{u}}_{b}\Big) \Rightarrow 2u = \tan^{-1}a + \tan^{-1}b$$

$$2u = \tan^{-1}\left(\frac{a+b}{1-ab}\right) \Rightarrow \tan(2u) = \frac{a+b}{1-ab} = \frac{w/u + v/u}{1 - wv/u^2}$$

$$\tan(2u) = \frac{u(w+v)}{u^2 - wv}$$

Partindo do parâmetro modal u, temos (mostre!)

$$u = \sqrt{1 - \underbrace{\left(\frac{\beta^2 - k_0^2 n_s^2}{k_0^2 n_f^2 - k_0^2 n_s^2}\right)}_{b}} \underbrace{\left[k_0 a \sqrt{\left(n_f^2 - n_s^2\right)}\right]}_{V},$$

↳ ? Demonstrar

- b: Constante de propagação normalizada
- V: Frequencia normalizada

$$u = \sqrt{1-b}\,V \quad ; \quad v = \sqrt{b}\,V \quad ; \quad u^2 + v^2 = V^2$$

- Relação entre o parâmetro modal w e os demais:

$$\left(\frac{w}{u}\right)^2 = \left(\frac{\alpha_c a}{k_f a}\right)^2 = \frac{\beta^2 - k_0^2 n_c^2}{k_0^2 n_f^2 - \beta^2} = \frac{\beta^2 - k_0^2 n_c^2 + k_0^2 n_s^2 - k_0^2 n_s^2}{k_0^2 n_f^2 - \beta^2 + k_0^2 n_s^2 - k_0^2 n_s^2} \Rightarrow$$

$$\frac{\beta^2 - k_0^2 n_s^2 + k_0^2 n_s^2 - k_0^2 n_c^2}{k_0^2 n_f^2 - k_0^2 n_s^2 + k_0^2 n_s^2 - \beta^2}$$

$$\frac{\beta^2 - k_0^2 n_s^2 + k_0^2 n_s^2 - k_0^2 n_c^2}{k_0^2 n_f^2 - k_0^2 n_s^2 + k_0^2 n_s^2 - \beta^2} = \frac{\dfrac{\beta^2 - k_0^2 n_s^2 + k_0^2 n_s^2 - k_0^2 n_c^2}{k_0^2 n_f^2 - k_0^2 n_s^2}}{\dfrac{k_0^2 n_f^2 - k_0^2 n_s^2 + k_0^2 n_s^2 - \beta^2}{k_0^2 n_f^2 - k_0^2 n_s^2}} =$$

$$\frac{\overbrace{\dfrac{\beta^2 - k_0^2 n_s^2}{k_0^2 n_f^2 - k_0^2 n_s^2}}^{b} + \overbrace{\dfrac{k_0^2 n_s^2 - k_0^2 n_c^2}{k_0^2 n_f^2 - k_0^2 n_s^2}}^{\delta}}{\dfrac{k_0^2 n_f^2 - k_0^2 n_s^2}{k_0^2 n_f^2 - k_0^2 n_s^2} + \dfrac{k_0^2 n_s^2 - \beta^2}{k_0^2 n_f^2 - k_0^2 n_s^2}} = \frac{b + \delta}{1 - b}$$

Portanto, $\delta = \dfrac{k_0^2 n_s^2 - k_0^2 n_c^2}{k_0^2 n_f^2 - k_0^2 n_s^2} = \dfrac{n_s^2 - n_c^2}{n_f^2 - n_s^2}$

$\delta = \dfrac{n_s^2 - n_c^2}{n_f^2 - n_s^2}$ denominado de parâmetro de assimetria. Caso $n_s = n_c$ (guia simétrico), então $\delta = 0$.

Desse modo, $\left(\dfrac{w}{u}\right) = \dfrac{b+\delta}{1-b} \Rightarrow \dfrac{w}{u} = \sqrt{\dfrac{b+\delta}{1-b}} \Rightarrow \dfrac{w}{\sqrt{1-b}\,V} = \dfrac{\sqrt{b+\delta}}{\sqrt{1-b}}$

$w = \sqrt{b+\delta}\,V \Rightarrow w^2 = (b+\delta)V^2 \; ; \; b = (v/V)^2$

$w^2 = \left(\dfrac{v^2}{V^2} + \delta\right)V^2 = v^2 + \delta V^2$

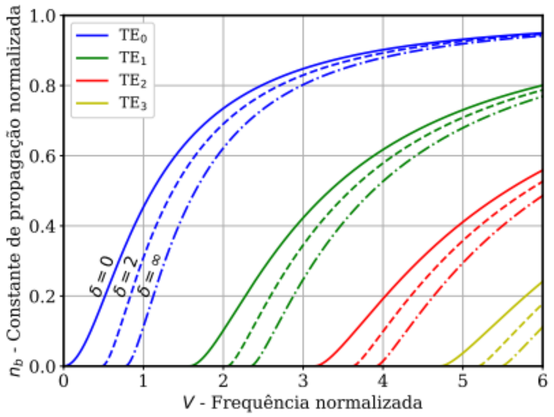
$w^2 - v^2 = \delta V^2$, ou seja,

$$\frac{w}{u} = \sqrt{\frac{b+\delta}{1-b}} \quad \rightarrow \quad w^2 - v^2 = \delta V^2$$

- Como $u = \sqrt{1-b}\,V$ a equação $2u = m\pi + \tan^{-1}\left(\frac{w}{u}\right) + \tan^{-1}\left(\frac{v}{u}\right)$ pode ser escrita como:

$$2\sqrt{1-b}\,V = m\pi + \tan^{-1}\left(\sqrt{\frac{b+\delta}{1-b}}\right) + \tan^{-1}\left(\frac{\sqrt{b}\,V}{\sqrt{1-b}\,V}\right)$$

$$2V\sqrt{1-b} = m\pi + \arctan\left(\sqrt{\frac{b+\delta}{1-b}}\right) + \arctan\left(\sqrt{\frac{b}{1-b}}\right)$$



Curvas universais para diferentes valores do parâmetro de assimetria - modos TE.

Frequência de corte

A frequência normalizada de corte V_c^m é determinada quando $b = 0$ e, portanto, a partir da Eq. 101:

$$2V_c^m = m\pi + \arctan(\sqrt{\delta}) \rightarrow f_c^m = \frac{m\pi + \arctan(\sqrt{\delta})}{4\pi \frac{a}{c_0}\sqrt{n_f^2 - n_s^2}} \tag{102}$$

Observe que para $\delta = 0$ (guia simétrico), as equações anteriores são reescritas como

$$V_c^m = m\frac{\pi}{2} \quad \text{e} \quad f_c^m = \frac{m\pi}{4\pi \frac{a}{c_0}\sqrt{n_f^2 - n_s^2}}$$

A quantidade de modos TE guiados pode ser obtida considerando que $V_c^m \leqslant V$ ou[6]

$$m \leqslant \frac{2V - \arctan(\sqrt{\delta})}{\pi} \rightarrow M = \text{floor}\left(\frac{2V - \arctan(\sqrt{\delta})}{\pi}\right) \tag{103}$$

em que M é o índice do modo de mais alta ordem e, portanto, haverá $M + 1$ modos.

[6] floor(x) é o maior inteiro menor que x.

Para o modo TM:

$$E_z(x) = E_0 \begin{cases} \sin(k_f a + \phi) e^{-\alpha_c (x-a)}, & x \geqslant a \\ \sin(k_f x + \phi), & |x| \leqslant a \\ -\sin(k_f a - \phi) e^{\alpha_s (x+a)}, & x \leqslant -a \end{cases}$$

$$E_x(x) = -jE_0\beta \begin{cases} \alpha_c^{-1} \sin(k_f a + \phi) e^{-\alpha_c (x-a)}, & x \geqslant a \\ k_f^{-1} \cos(k_f x + \phi), & |x| \leqslant a \\ \alpha_s^{-1} \sin(k_f a - \phi) e^{\alpha_s (x+a)}, & x \leqslant -a \end{cases}$$

$$H_y(x) = -jE_0\omega \begin{cases} \dfrac{n_c^2}{\alpha_c} \sin(k_f a + \phi) e^{-\alpha_c (x-a)}, & x \geqslant a \\ \dfrac{n_f^2}{k_f} \cos(k_f x + \phi), & |x| \leqslant a \\ \dfrac{n_s^2}{\alpha_s} \sin(k_f a - \phi) e^{\alpha_s (x+a)}, & x \leqslant -a \end{cases}$$

• Aplicando as condições de fronteira para $H_y(x)$:

$$\frac{n_f^2}{k_f}\cos(k_f a + \phi) = \frac{n_c^2}{\alpha_c}\sin(k_f a + \phi) \quad (x=a) \Rightarrow \tan(k_f a + \phi) = \frac{\alpha_c}{k_f}\overbrace{\frac{n_f^2}{n_c^2}}^{\to p_c} = \frac{\alpha_c p_c}{k_f}$$

$$\frac{n_f^2}{k_f}\cos(k_f(-a) + \phi) = \frac{n_s^2}{\alpha_s}\sin(k_f a - \phi) \quad (x=-a) \Rightarrow \tan(k_f a - \phi) = \frac{\alpha_s}{k_f}\frac{n_f^2}{n_s^2} = \frac{\alpha_s p_s}{k_f}$$

Dessa forma,

$$\begin{cases} \tan(k_f a + \phi) = \dfrac{\alpha_c p_c}{k_f} \\ \tan(k_f a - \phi) = \dfrac{\alpha_s p_s}{k_f} \end{cases} \Rightarrow \begin{array}{l} k_f a + \phi = \tan^{-1}\left(\dfrac{\alpha_c p_c}{k_f}\right) \ (*) \\ k_f a - \phi = \tan^{-1}\left(\dfrac{\alpha_s p_s}{k_f}\right) \ (\#) \end{array}$$

$$(*) + (\#): \quad 2k_f a = \tan^{-1}\left(\frac{\alpha_c p_c}{k_f}\right) + \tan^{-1}\left(\frac{\alpha_c p_s}{k_f}\right)$$

$$2k_f a = \tan^{-1}\left(\frac{\dfrac{\alpha_c p_c}{k_f} + \dfrac{\alpha_s p_s}{k_f}}{1 - \dfrac{\alpha_c \alpha_s p_c p_s}{k_f^2}}\right)$$

$$\tan(2k_f a) = \frac{k_f(\alpha_c p_c + \alpha_s p_s)}{k_f^2 - \alpha_c \alpha_s p_c p_s}$$

Definindo: $\begin{cases} u = k_f a \\ w = \alpha_c a \\ v = \alpha_s a \end{cases} \Rightarrow \boxed{\tan(2u) = \dfrac{u(w p_c + v p_s)}{u^2 - w v p_c p_s} \quad (A)}$

A solução de (A) são as constantes de propagação β dos modos TM.

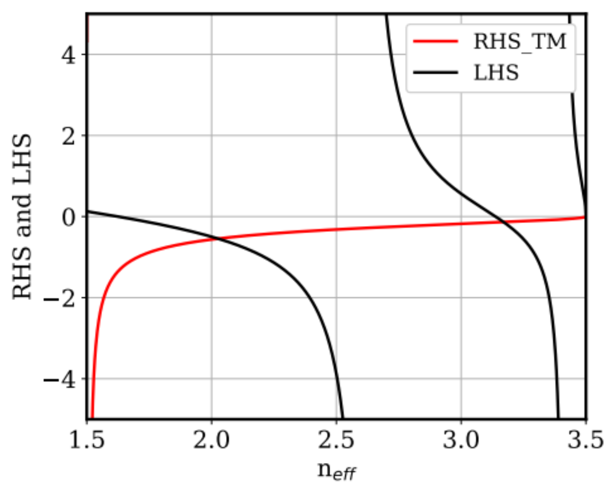
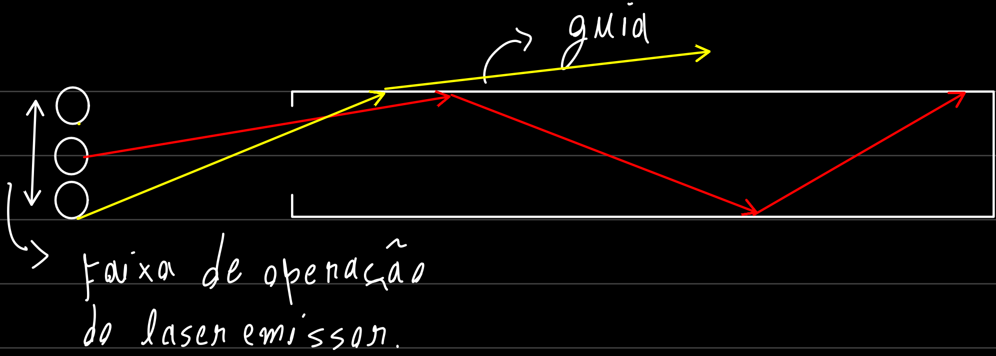
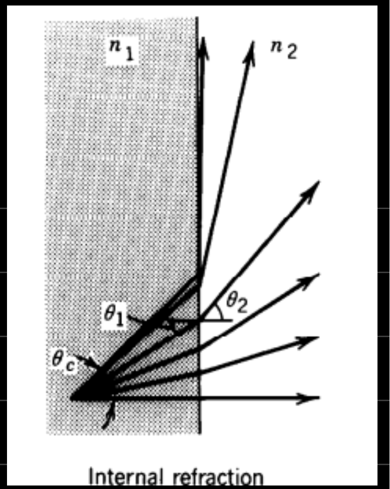


Figura 10: Curvas da equação característica, considerando $n_f = 3.5$, $n_s = 1.45$, $n_c = 1.0$, $a = 0,5$ μm e $\lambda = 1550$ nm.

Luiz Felipe

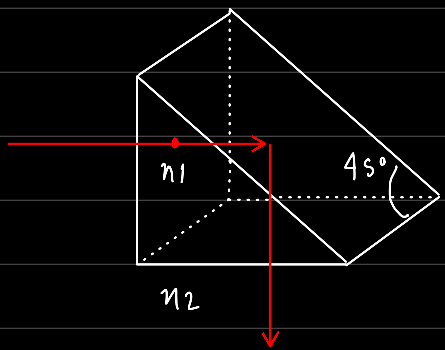




Internal refraction

critical angle: $\theta_2 = 90°$, $n_1 \sin\theta_1 = n_2 \sin\theta_2$ (Lei de Snell)

$$\theta_1 = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$



$n_2 = 1$ $\qquad$ $\theta_1 = \arcsin(1/\sqrt{2}) = 45°$

$n_1 = \sqrt{2}$