1. Mean of rand(1:8) = (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8)/8 = 4.5

Mean of exponential = 1/λ

λ = 1/4.5

scale = 1/ λ = 4.5

Therefore: rand(1:8) -> rand(Exponential(4.5))

1. Mean of rand(3:12) = (3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12)/10 = 7.5

Variance = (12-3)2/12 = 27/4

Mean of Erlang = k/λ = 7.5

Variance of Erlang = k/λ2 = 27/4

Rearrange Mean: λ = k/7.5

Rearrange Variance: k = (27/4)∙λ2

k = (27/4)(k/7.5)2

k = (27/4)(k2/7.52)

k = 27k2/225

225 = 27k

**k = 8.333**

**k = 8 (rounded)**

λ = k/7.5

λ = 8.33333/7.5

**λ = 1.11111**

scale = 1/ λ

scale = 0.9

Therefore: rand(3:12) -> rand(Erlang(8, 0.9))

*Sidenote*: because k is rounded, the mean is actually around 7.2 and not the 7.5 as expected when using rand(Erlang(8, 0.9))

Given that k = 8, there are 8 applications of an exponential distribution with λ = 1.11111.

1. Sample output from program (in blue):

the mean of the total wait time in the simple sim was: 475.59

the mean of the total wait time in the realistic sim was: 514.94

the standard deviation of the total wait time in the simple sim was: 144.57957824134238

the standard deviation of the total wait time in the realistic sim was: 231.34332438471992

the 99% confidence interval of the total wait time in the simple sim was: (437.6175406695578, 513.5624593304422)

the 99% confidence interval of the total wait time in the realistic sim was: (454.1798630330483, 575.7001369669517)

the F test of the simple and real systems yielded: 4.568319794007181e-6

the Welch T test of the simple and real systems yielded: 0.15106746751741082

The results of the F test show that the difference in the variances are statistically significant.

Because the difference in the variances are statistically significant, I used the Welch T test to determine that the difference in the means is not statistically significant. The value from the ttest varies from different executions of the program, but it tended to not be statistically significant (values close to 0.90 and values close to 0.003 were observed with values on the low end being less common).

1. After 50 simulations, the F test becomes less consistent on repeated executions of the program, but still tends to be less than 0.01 (statistically significant difference). The T test is very inconsistent and tends to be well over 0.01 (not statistically significant).

After 100 simulations see observations above.

After 311 simulations the F test becomes 0.0. The T test is still inconsistent across repeated executions of the program.

After 600 simulations the T test is more consistently less than 0.01, but still has some values over 0.01 on repeated executions of the program. At this point I’m comfortable stating the T test’s statistically significant difference.

1. See “[TwoCentersSystem] derived\_stats.jl”
2. For most of the math see Q1 a) and b)

Mean of rand(4:8) = (4 + 5 + 6 + 7 + 8)/5 = 76

Variance = (8-4)2/12 = 4/3

Mean of Erlang = k/λ = 6

Variance of Erlang = k/λ2 = 4/3

Rearrange Mean: λ = k/6

Rearrange Variance: k = (4/3)∙λ2

k = (4/3)(k/6)2

k = (4/3)(k2/62) 4/3

k = 4k2/108

k = k2/27

**k = 27**

λ = k/6

λ = 27/6

**λ = 4.5**

scale = 1/ λ

scale = 2/9

Therefore: rand(4:8) -> rand(Erlang(27, 2/9))

1. Example output:

The mean of wait time from the first server is: 153.46 with a 95% confint of: (148.08772027360388, 158.83227972639614)

The mean of wait time from the second server is: 88.94 with a 95% confint of: (87.64581256463082, 90.23418743536918)

The mean of wait time from both servers is: 121.2 with a 95% confint of: (115.92379276905658, 126.47620723094343)

The mean of service time from the first server is: 153.46 with a 95% confint of: (148.08772027360388, 158.83227972639614)

The mean of service time from the second server is: 88.94 with a 95% confint of: (87.64581256463082, 90.23418743536918)

The mean of service time from both servers is: 121.2 with a 95% confint of: (115.92379276905658, 126.47620723094343)

The probability of waiting from the first server is: 0.99 with a 95% confint of: (0.9701578304841358, 1.0098421695158641)

The probability of waiting from the second server is: 1.0 with a 95% confint of: (1.0, 1.0)

The probability of waiting from both servers is: 0.995 with a 95% confint of: (0.9851402172787412, 1.0048597827212589)

The probability of an idle server from the first server is: 0.91 with a 95% confint of: (0.8529292592958191, 0.967070740704181)

The probability of an idle server from the second server is: 1.0 with a 95% confint of: (1.0, 1.0)

The probability of an idle server from both servers is: 0.955 with a 95% confint of: (0.9260213102887644, 0.9839786897112355)

The probability of an busy server from the first server is: 0.99 with a 95% confint of: (0.9701578304841358, 1.0098421695158641)

The probability of an busy server from the second server is: 1.0 with a 95% confint of: (1.0, 1.0)

The probability of an busy server from both servers is: 0.995 with a 95% confint of: (0.9851402172787412, 1.0048597827212589)