1) a, b, c, d = scalars

$$a\begin{bmatrix}1\\2\\-1\\-2\end{bmatrix} + B\begin{bmatrix}2\\3\\0\\-1\end{bmatrix} + c\begin{bmatrix}1\\2\\1\\4\end{bmatrix} + d\begin{bmatrix}1\\3\\-1\\0\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

linear independence = linear scalar combination adds up to $\emptyset$

write matrix in ecaelon form. not in lecture

$$\left[\begin{array}{cccc|c}1&2&1&1&0\\2&3&2&3&0\\-1&0&1&-1&0\\-2&-1&4&0&0\end{array}\right] \rightarrow \begin{array}{cccc}a_1&a_2&a_3&a_4\\ \hline 1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{array} = \text{identity matrix}$$

$-2R_1 + R_2 \rightarrow R_2$

so on....

all scalars evaluate to zero, so matrix is linearly independent

or

$$a\begin{bmatrix}1\\2\\-1\\-2\end{bmatrix} = B\begin{bmatrix}2\\3\\0\\-1\end{bmatrix} + c\begin{bmatrix}1\\2\\1\\4\end{bmatrix} + d\begin{bmatrix}1\\3\\-1\\0\end{bmatrix} \begin{matrix}\to 0\\=0\\=0\\=0\end{matrix}$$

Solve this system for scalars vectors get zero. = linear independent

$$\left.\begin{aligned} 1 &= 2B + c + d\\ 2 &= 3B + 2c + 3d\\ -1 &= c - d\\ -2 &= -B + 4c\end{aligned}\right\}$$ no solution, $\therefore$ linearly independent

find coordinates of vector $x = (7, 14, -1, 2)$ w/ basis (a, b, c, d)

$$x\begin{bmatrix}7\\14\\-1\\2\end{bmatrix} = a\begin{bmatrix}1\\2\\-1\\-2\end{bmatrix} + B\begin{bmatrix}2\\3\\0\\-1\end{bmatrix} + c\begin{bmatrix}1\\2\\1\\4\end{bmatrix} + d\begin{bmatrix}1\\3\\-1\\0\end{bmatrix}$$

$$\begin{aligned} 7 &= a + 2B + c + d\\ 14 &= 2a + 3B + 2c + 3d\\ -1 &= -a + c - d\\ 2 &= -2a - B + 4c\end{aligned}$$

$$\left.\begin{aligned} a&=0\\ c&=1\\ b&=2\\ d&=2\end{aligned}\right\}$$ coordinates of vector X

Determinate $\neq 0$, linear independent

---

2)

$$\begin{array}{cccc} a & b & c & d\\ 1 & 1 & 1 & 0\\ 1 & 1 & -1 & 0\\ 1 & -1 & 0 & 1\\ 1 & -1 & 0 & -1\end{array}$$

columns$(A) = 4$
rows$(A') = 4$
$\therefore$ you can multiply them

$$A\begin{bmatrix}1&1&1&0\\1&1&-1&0\\1&-1&0&1\\1&-1&0&-1\end{bmatrix} A'\begin{bmatrix}1&1&1&0\\1&1&-1&0\\1&-1&0&1\\1&-1&0&-1\end{bmatrix} = \begin{bmatrix}3&1&0&1\\1&3&0&-1\\1&-1&2&-1\\-1&1&2&1\end{bmatrix}$$

$a = b + c + d$

$(1\cdot1)+(1\cdot1)+(1\cdot1)+(0\cdot1) = 3$
$(1\cdot1)+(1\cdot1)+(1\cdot-1)+(0\cdot-1) = 1$
$(1\cdot1)+(1\cdot1)+(1\cdot0)+(0\cdot0) = 0$
$(1\cdot0)+(1\cdot0)+(1\cdot1)+(0\cdot-1) = 1$

and so on.....

$$\left.\begin{aligned} 1 &= b + c\\ 1 &= b - c\\ 1 &= -b + d\\ 1 &= -b - d\end{aligned}\right\}$$

unsolvable $\therefore$ linearly independent

---

3) Vector space

Vectors V = commutative in addition
Scalars F = field under multiplication

- $1v = v$
- $a(bv) = (ab)v$
- $a(u+v) = au + av$
- $(a+b)v = av + bv$

abelian group axioms

(associative) $u + (v+w) = (u+v) + w$

identity 2) $\exists$ vector $0 \in E$ s.t. $v + 0 = 0 + v = w$
inverse 3) $\forall v \in V \ \exists -v \in E$ s.t. $v + (-v) = -v + v = 0$
Commutative 4) $v + u = u + v$

Which axiom is violated?

addition property: $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$
mult. property: $\lambda \cdot (x, y) = (\lambda x, y)$

the other two axioms not included?
in that case, multiplication axiom is violated?

4a) given: A^{-1} exists.

associative

$$A^{-} \cdot A x = 0 A^{-}$$
$$(A^{-} \cdot A) = I x$$
$$I x = 0$$
$$x = 0$$

$\longrightarrow$ identity matrix property $(A \cdot A^{-}) = I$
- anything times zero matrix = zero matrix
- Matrix mult. = associative

4b) Inverse: for any square matrix A of dimension n, if matrix B s.t. $AB = BA = I_n$ exists, then its unique and inverse of A. matrix B denoted as A^{-1}

Help!

5) Since f is a bijection $\exists$ $u, v \in U$ s.t. $f(u) = x$, $f(v) = y$

$\therefore$

$$f^{-1}(x+y) = f^{-1}(f(u) + f(v))$$
$$= f^{-1}(f(u+v))$$
$$= u + v$$
$$= f^{-1}(x) + f^{-1}(y)$$

$\longrightarrow$ linearity of f'

now we show this holds w/ scalars λ:

$$f^{-1}(\lambda x) = f^{-1}(\lambda f(u))$$
$$= f^{-1}(f(\lambda u))$$
$$= \lambda u$$
$$= \lambda f^{-}(x)$$

<u>Definition one</u>
$f(x+y) = f(x) + f(y)$

<u>Definition two</u>
$f(\lambda x) = \lambda f(x)$

$$A^T A x = 0$$
$$x^T A^T A x = x^T 0$$
$$x = 0$$