Homework for week 2: Basic Convolution

1 Convolution (2 points)

Recall the definition of convolution,

$$g = I \otimes f \tag{1}$$

where $\boldsymbol{I}$ and $\boldsymbol{f}$ represents the image and kernel respectively.

Typically, when kernel f is a 1-D vector, we get

$$g(i) = \sum_m I(i-m)f(m) \tag{2}$$

where i is the index in the 1-D dimension.

If the kernel f is a 2-D kernel, we have

$$g(i,j) = \sum_{m,n} I(i-m, j-n)f(m,n) \tag{3}$$

where i and j are the row and column indices respectively.

In this section, you need to perform the convolution **by hand**, get familiar with convolution in both 1-D and 2-D as well as its corresponding properties.

Note: All convolution operations in this section follow except additional notifications: 1. Zero-Padding, 2. Same Output Size, 3. An addition or multiplication with 0 will count as one operation.

For this problem, we will use the following 3×3 image:

$$I = \begin{bmatrix} 0.0 & 1.0 & -1.0 \\ 2.0 & 1.0 & 0.0 \\ 0.0 & 3.0 & -1.0 \end{bmatrix} \tag{4}$$

You are given two 1-D vectors for convolution:

$$f_x = \begin{bmatrix} -1.0 & 0.0 & 1.0 \end{bmatrix} \tag{5}$$
$$f_y = \begin{bmatrix} 1.0 & 1.0 & 1.0 \end{bmatrix}^T \tag{6}$$

Let $g_1 = I \otimes f_x \otimes f_y$, $f_{xy} = f_x \otimes f_y$ and $g_2 = I \otimes f_{xy}$.

Note: f_{xy} should be of full output size.

- **Question 1.1:** Compute g_1 and g_2 (At least show two steps for each convolution operation and intermediate results), and verify the associative property of convolution.
- **Question 1.2:** How many operations are required for computing g_1 and g_2 respectively? addition and multiplication times in your result.
- **Question 1.3:** What does convolution do to this image?

Question 1.1

$I \otimes f_x$:

$$\begin{bmatrix} 0.0 & 1.0 & -1.0 \\ 2.0 & 1.0 & 0.0 \\ 0.0 & 3.0 & -1.0 \end{bmatrix} \otimes \begin{bmatrix} -1.0 & 0.0 & 1.0 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 1 \\ -1 & 2 & 1 \\ -3 & 1 & 3 \end{bmatrix}$$

$$(0*0)+(1*-1)+(0*1) = -1$$
$$(0*1)+(1*0)+(-1*-1) = 1$$
$$(-1*0)+(1*1)+(-1*0) = 1$$

$$(0*2)+(-1*1)+(0*1) = -1$$
$$(0*1)+(1*2)+(0*-1) = 2$$
$$(0*0)+(1*1)+(0*-1) = 1$$

$$(0*0)+(3*-1)+(0*1) = -3$$
$$(3*0)+(1*0)+(-1*-1) = 1$$
$$(-1*0)+(1*3)+(0*-1) = 3$$

$I \otimes f_x \otimes f_y$:

$$\begin{bmatrix} -1 & 1 & 1 \\ -1 & 2 & 1 \\ -3 & 1 & 3 \end{bmatrix} \otimes \begin{bmatrix} 1.0 & 1.0 & 1.0 \end{bmatrix}^T = \begin{bmatrix} -2 & 3 & 2 \\ -5 & 4 & 5 \\ -4 & 3 & 4 \end{bmatrix} = G_1$$

$$(-1*1)+(1*-1)+(0*1) = -2$$
$$(-1*1)+(1*-1)+(-1*-3) = -5$$
$$(-3*1)+(-1*1)+(-1*0) = -4$$

$$(1*1)+(2*1)+(0*1) = 3$$
$$(2*1)+(1*1)+(1*1) = 4$$
$$(1*1)+(1*2)+(0*1) = 3$$

$$(1*1)+(1*1)+(0*1) = 2$$
$$(1*1)+(1*1)+(1*3) = 5$$
$$(1*3)+(1*1)+(0*1) = 4$$

$f_x \otimes f_y$:

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \otimes \begin{bmatrix} -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned}
(-1*1)+(0*0)+(0*0) &= -1 \\
(-1*0)+(1*0)+(0*1) &= 0 \\
(1*1)+(0*0)+(0*0) &= 1 \\
\\
(-1*1)+(0*0)+(0*0) &= -1 \\
(-1*0)+(1*0)+(0*1) &= 0 \\
(1*1)+(0*0)+(0*0) &= 1 \\
\\
(-1*1)+(0*0)+(0*0) &= -1 \\
(-1*0)+(1*0)+(0*1) &= 0 \\
(1*1)+(0*0)+(0*0) &= 1
\end{aligned}$$

$f_{yx} \otimes I$:

$$\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 0 \\ 0 & 3 & -1 \end{bmatrix} = \begin{bmatrix} -2 & 3 & 2 \\ -5 & 4 & 5 \\ -4 & 3 & 4 \end{bmatrix} = G_2$$

$$\begin{aligned}
(0*0)+(-1*1)+(2*0)+(-1*1)+(1*0)+(0*0)+(1*0)+(1*0)+(0*1) &= -2 \\
(1*2)+(1*0)+(0*-1)+(1*0)+(3*0)+(-1*-1)+(1*0)+(0*0)+(0*-1) &= 3 \\
(-1*0)+(0*0)+(1*1)+(1*1)+(-1*0)+(0*0)+(-1*0)+(1*0)+(0*-1) &= 2 \\
(2*0)+(0*0)+(0*0)+(-1*1)+(1*-1)+(3*1-)+(1*0)+(1*0)+(0*1) &= -5 \\
(1*0)+(0*1)+(-1*-1)+(1*2)+(1*0)+(0*-1)+(1*0)+(3*0)+(-1*-1) &= 4 \\
(1*1)+(-1*0)+(1*1)+(0*0)+(3*1)+(0*-1)+(-1*0)+(-1*0)+(0*-1) &= 5 \\
(0*0)+(0*2)+(1*-1)+(3*-1)+(1*0)+(0*1)+(1*0)+(0*0)+(0*-1) &= -4 \\
(2*1)+(1*0)+(0*-1)+(1*0)+(3*0)+(-1*-1)+(1*0)+(0*0)+(0*-1) &= 3 \\
(-1*0)+(3*1)+(1*1)+(0*0)+(-1*0)+(0*-1)+(-1*0)+(0*0)+(0*1) &= 4
\end{aligned}$$

Verification of the commutative property of convolution:
$I \otimes f = f \otimes I$
since $G_1 = G_2$ we can say:
$I \otimes f_x \otimes f_y = I \otimes f_{xy} \longrightarrow f_x \otimes f_y \otimes I$

Question 1.2

G_1: $45 + 45 = 90$ operations
G_2: $153 + 45 = 198$ operations

Question 1.3

Convolution brightens this image.

2 Kernel Estimation (2 points)

Recall the special case of convolution discussed in class: The Impulse function. Using an impulse function, it is possible to 'shift' (and sometimes also 'scale') an image in a particular direction.

For example, when the following image

$$I = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \tag{7}$$

is convolved with the kernel,

$$f = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{8}$$

it results in the output:

$$g = \begin{bmatrix} e & f & 0 \\ h & i & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{9}$$

Another useful trick to keep in mind is the decomposition of a convolution kernel into scaled impulse kernels. For example, a kernel

$$f = \begin{bmatrix} 0 & 0 & 7 \\ 0 & 0 & 0 \\ 0 & 4 & 0 \end{bmatrix} \tag{10}$$

can be decomposed into

$$f_1 = 7 * \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } f_2 = 4 * \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

- **Question:** Using the two tricks listed above, estimate the kernel f **by hand** which when convolved with an image

$$I = \begin{bmatrix} 1 & 5 & 2 \\ 7 & 8 & 6 \\ 3 & 9 & 4 \end{bmatrix} \tag{11}$$

results in the output image

$$g = \begin{bmatrix} 29 & 43 & 10 \\ 62 & 52 & 30 \\ 15 & 45 & 20 \end{bmatrix} \tag{12}$$

Hint: Look at the relationship between corresponding elements in g and I.

$I \otimes f = G$:

$$\begin{bmatrix} 1 & 5 & 2 \\ 7 & 8 & 6 \\ 3 & 9 & 4 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 29 & 43 & 10 \\ 62 & 52 & 30 \\ 15 & 45 & 20 \end{bmatrix} = G$$

First, the decomposition trick allowed me to noticed that the last row and last column of G are multiplies of 5, therefore 5 must be in the kernel. I then experimented by putting 5 in every position in my null f until

I ruled out every position except for center. After performing the multiples of 5 operations on I, the top 2x2 appeared to me as multiples of 3.

$$
\begin{aligned}
29-(5*1)&=24\\
43-(5*5)&=18\\
62-(7*5)&=27\\
52-(8*5)&=12
\end{aligned}
$$

Now giving me:

$$
\begin{bmatrix} 24 & 18 & 0 \\ 27 & 12 & 0 \\ 0 & 0 & 0 \end{bmatrix}
$$

Lastly, due to the impulse function trick, I know that the 3 must be in the bottom right corner, so when flipped and multiplied out it gives me multiples of three in that 2x2 section. This gives me:

$$
f=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix}
$$

3 Edge Moving (2 points)

Object Recognition is one of the most popular applications in Computer Vision. The goal is to identify the object based on a template or a specific pattern of the object that has been learnt from a training dataset. Suppose we have a standard template for a "barrel" which is a 3×3 rectangle block in a 4×4 image. We also have an input 4×4 query image. Now, your task is to verify if the image in question contains a barrel. After preprocessing and feature extraction, the query image is simplified as I_Q and the barrel template is I_T.

$$I_Q = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, I_T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

Instinctively, the human eye can automatically detect a potential barrel in the top left corner of the query image but a computer can't do that right away. Basically, if the computer finds that the difference between query image's features and the template's features are minute, it will prompt with high confidence: 'Aha! I have found a barrel in the image'. However, in our circumstance, if we directly compute the pixel wise distance D between I_Q and I_T where

$$D(I_Q, I_T) = \sum_{i,j} (I_Q(i,j) - I_T(i,j))^2 \tag{13}$$

we get $D = 10$ which implies that there's a huge difference between the query image and our template. To fix this problem, we can utilize the power of the convolution. Let's define the 'mean shape' image I_M which is the blurred version of I_Q and I_T.

$$I_M = \begin{bmatrix} 0.25 & 0.5 & 0.5 & 0.25 \\ 0.5 & 1 & 1 & 0.5 \\ 0.5 & 1 & 1 & 0.5 \\ 0.25 & 0.5 & 0.5 & 0.25 \end{bmatrix}$$

- **Question 3.1:** Compute two 3×3 convolution kernels f_1, f_2 **by hand** such that $I_Q \otimes f_1 = I_M$ and $I_T \otimes f_2 = I_M$ where $\otimes$ denotes the convolution operation. (Assume zero-padding)
- **Question 3.2:** For a convolution kernel $f = (f_1 + f_2)/2$, we define $I'_Q = I_Q \otimes f$ and $I'_T = I_T \otimes f$. Compute I'_Q, I'_T and $D(I'_Q, I'_T)$ **by hand**. Compare it with $D(I_Q, I_T)$ and briefly explain what you find.

3.1

$$f_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & .25 & .25 \\ 0 & .25 & .25 \end{bmatrix}$$

$$f_2 = \begin{bmatrix} .25 & .25 & 0 \\ .25 & .25 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

I computed f_1 through brute force process of elimination. I noticed that the kernel had to be in multiples of $\frac{1}{4}$ and placed $\frac{1}{4}$ in the bottom right index of f_1, then flipped the kernel and multiplied appropriately. After all outside edges of I_Q being checked I took advantage of the center index of 3x3 1's. This confirmed that the center index had to be $\frac{1}{4}$ because it was the only way to properly perform the convolution operations and get a 1.

After computing f_1, f_2 was simple to compute because I_Q and I_T are inverses. I hypothesized that all I needed to do is invert f_1 to get f_2 and that was correct. Above are the kernels of f_1, f_2 before they are flipped for operations.

$I_Q \otimes f_1$:

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 \\ 0 & .25 & .25 \\ 0 & .25 & .25 \end{bmatrix} = \begin{bmatrix} .25 & .5 & .5 & .25 \\ .5 & 1 & 1 & .5 \\ .5 & 1 & 1 & .5 \\ .25 & .5 & .5 & .25 \end{bmatrix} = I_M$$

$I_T \otimes f_2$:

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} .25 & .25 & 0 \\ .25 & .25 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} .25 & .5 & .5 & .25 \\ .5 & 1 & 1 & .5 \\ .5 & 1 & 1 & .5 \\ .25 & .5 & .5 & .25 \end{bmatrix} = I_M$$

3.2

$f = \frac{(f_1+f_2)}{2}$:

$$f = \frac{1}{2} * \begin{bmatrix} (\frac{1}{4}+0) & (\frac{1}{4}+0) & (0+0) \\ (\frac{1}{4}+0) & (\frac{1}{4}+\frac{1}{4}) & (\frac{1}{4}+0) \\ (0+0) & (\frac{1}{4}+0) & (\frac{1}{4}+0) \end{bmatrix} = \begin{bmatrix} \frac{1}{8} & \frac{1}{8} & 0 \\ \frac{1}{8} & \frac{1}{4} & \frac{1}{8} \\ 0 & \frac{1}{8} & \frac{1}{8} \end{bmatrix}$$

$I_Q \otimes f$:

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} \cdot\frac{1}{8} & \frac{1}{8} & 0 \\ \frac{1}{8} & \frac{1}{4} & \frac{1}{8} \\ 0 & \frac{1}{8} & \frac{1}{8} \end{bmatrix} = \begin{bmatrix} \frac{5}{8} & \frac{3}{4} & \frac{1}{2} & \frac{1}{8} \\ \frac{3}{4} & 1 & \frac{3}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{3}{4} & \frac{5}{8} & \frac{1}{4} \\ \frac{1}{8} & \frac{1}{4} & \frac{1}{4} & \frac{1}{8} \end{bmatrix} = I'_Q$$

Work shown for 3 steps of convolution operation and immediate results:

$$(\frac{1}{4}*1)+(\frac{1}{8}*1)+(\frac{1}{8}*1)+(\frac{1}{8}*1)+(0*0)+(\frac{1}{8}*0)+(\frac{1}{8}*0)+(\frac{1}{8}*0)+(0*0) = \frac{5}{8}$$
$$(\frac{1}{8}*1)+(\frac{1}{4}*1)+(\frac{1}{8}*1)+(0*1)+(\frac{1}{8}*1)+(\frac{1}{8}*1)+(\frac{1}{8}*0)+(\frac{1}{8}*0)+(0*0) = \frac{3}{4}$$
$$(\frac{1}{8}*1)+(\frac{1}{4}*1)+(\frac{1}{8}*0)+(0*1)+(\frac{1}{8}*1)+(\frac{1}{8}*0)+(\frac{1}{8}*0)+(\frac{1}{8}*0)+(0*0) = \frac{1}{2}$$

$$\cdots$$

$I_T \otimes f$:

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} \frac{1}{8} & \frac{1}{8} & 0 \\ \frac{1}{8} & \frac{1}{4} & \frac{1}{8} \\ 0 & \frac{1}{8} & \frac{1}{8} \end{bmatrix} = \begin{bmatrix} \frac{1}{8} & \frac{1}{4} & \frac{1}{4} & \frac{1}{8} \\ \frac{1}{4} & \frac{5}{8} & \frac{3}{4} & \frac{1}{2} \\ \frac{1}{4} & \frac{3}{4} & 1 & \frac{3}{4} \\ \frac{1}{8} & \frac{1}{4} & \frac{3}{4} & \frac{5}{8} \end{bmatrix} = I'_T$$

Work shown for 3 steps of convolution operation and immediate results:

$$(\frac{1}{4}*0)+(\frac{1}{8}*0)+(\frac{1}{8}*0)+(\frac{1}{8}*1)+(0*0)+(\frac{1}{8}*0)+(\frac{1}{8}*0)+(\frac{1}{8}*0)+(0*0)=\frac{1}{8}$$
$$(\frac{1}{8}*0)+(\frac{1}{4}*0)+(\frac{1}{8}*0)+(0*0)+(\frac{1}{8}*1)+(\frac{1}{8}*1)++(\frac{1}{8}*0)+(\frac{1}{8}*0)+(0*0)=\frac{1}{4}$$
$$(\frac{1}{8}*0)+(\frac{1}{4}*0)+(\frac{1}{8}*0)+(0*1)+(\frac{1}{8}*1)+(\frac{1}{8}*1)+(\frac{1}{8}*0)+(\frac{1}{8}*0)+(0*0)=\frac{1}{4}$$

$$\cdots$$

$$D(I_Q, I_T)$$

$$\begin{aligned}
(1-0)^2 &= 1\\
(1-0)^2 &= 1\\
(1-0)^2 &= 1\\
(0-0)^2 &= 0\\
(1-0)^2 &= 1\\
(1-1)^2 &= 0\\
(1-1)^2 &= 0\\
(0-1)^2 &= 1\\
(1-0)^2 &= 1\\
(1-1)^2 &= 0\\
(1-1)^2 &= 0\\
(0-1)^2 &= 1\\
(0-0)^2 &= 0\\
(0-1)^2 &= 1\\
(0-1)^2 &= 1\\
(0-1)^2 &= 1\\
&= 10
\end{aligned}$$

$$D(I'_Q, I'_T)$$

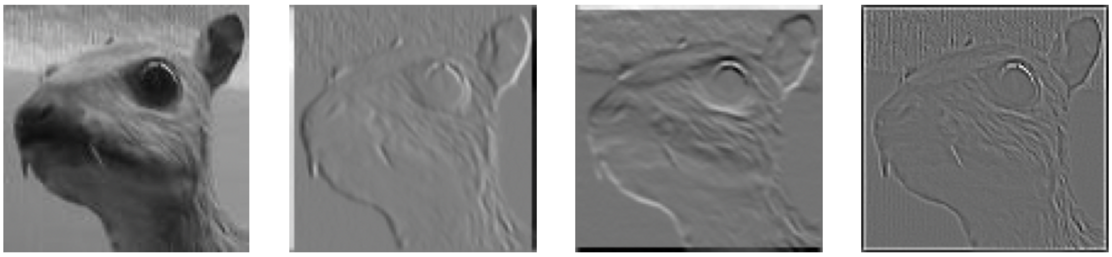
$$\begin{aligned}
(\frac{5}{8} - \frac{1}{8})^2 &= \frac{1}{4} \\
(\frac{3}{4} - \frac{1}{4})^2 &= \frac{1}{4} \\
(\frac{1}{2} - \frac{1}{4})^2 &= \frac{1}{16} \\
(\frac{1}{8} - \frac{1}{8})^2 &= 0 \\
(\frac{3}{4} - \frac{1}{4})^2 &= \frac{1}{4} \\
(1 - \frac{5}{8})^2 &= \frac{9}{64} \\
(\frac{3}{4} - \frac{3}{4})^2 &= 0 \\
(\frac{1}{4} - \frac{1}{2})^2 &= \frac{1}{16} \\
(\frac{1}{2} - \frac{1}{4})^2 &= \frac{1}{16} \\
(\frac{3}{4} - \frac{3}{4})^2 &= 0 \\
(\frac{5}{8} - 1) &= \frac{9}{64} \\
(\frac{1}{4} - \frac{3}{4})^2 &= \frac{1}{4} \\
(\frac{1}{8} - \frac{1}{8})^2 &= 0 \\
(\frac{1}{4} - \frac{1}{2})^2 &= \frac{1}{16} \\
(\frac{1}{4} - \frac{3}{4})^2 &= \frac{1}{4} \\
(\frac{1}{8} - \frac{5}{8})^2 &= \frac{1}{4} \\
&= 2.453125
\end{aligned}$$

Brief Description:

The non-zero terms and the zero terms respectively correlate from $D(I_Q, I_T)$ to $D(I'_Q, I'_T)$. This allows the computer to prompt 'I have found a barrel in the image' because after convoluting as prescribed, the difference in query image and template becomes minute. This is because of the algebraic manipulation of kernels f_1 and f_2 into f.

4 Match the Kernels (2 points)

- **Question 4.1 Match the corresponding kernels for the output images.**



Input Image (a) (b) (c)

1	2	1
0	0	0
-1	-2	-1

(6)

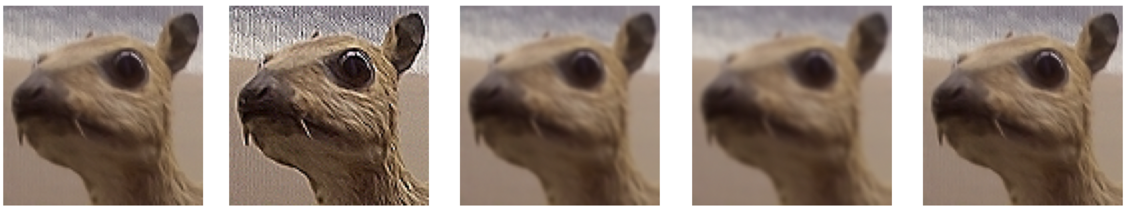
1	0	-1
2	0	-2
1	0	-1

(2)

-1	-1	-1
-1	8	-1
-1	-1	-1

(3)

- **Question 4.2 Match the corresponding kernels for the output images.**



Input Image (d) (e) (f) (g)

(1/256 *

1	4	6	4	1
4	16	24	16	4
6	24	36	24	6
4	16	24	16	4
1	4	6	4	1

(5)

0	0	0
0	1	0
0	0	0

(7)

0	-1	0
-1	5	-1
0	-1	0

(4)

(1/16) *

1	2	1
2	4	2
1	2	1

(1)

$$a = (6)$$
$$b = (2)$$
$$c = (3)$$

$$f = (5)$$
$$g = (7)$$
$$4 = (d)$$
$$e = (1)$$

5 Boundary Conditions (2 points)

For this problem, we will use the following 3×3 image:

$$I = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 1 & 1 \end{bmatrix} \tag{14}$$

You are given 2-D convolution filter:

$$f = \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \tag{15}$$

Let $g = I \underbrace{\otimes f \cdots \otimes f}_{\text{infinity times}}$. The output image g has the same size as I.

- **Question 5.1:** When zero-padding is used, what's the output image g. (Give the verification process)

$I \otimes f$:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 1 & 1 \end{bmatrix} \otimes \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{6}{9} & 1 & \frac{7}{9} \\ \frac{8}{9} & \frac{12}{9} & 1 \\ \frac{6}{9} & 1 & \frac{7}{9} \end{bmatrix} = \text{*take the limit*} \longrightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$(\frac{1}{9}) + (\frac{1}{9}) + (\frac{1}{9}) + (\frac{3}{9}) + (0) + (0) + (0) + (0) + (0) = \frac{6}{9}$$
$$(\frac{1}{9}) + (\frac{1}{9}) + (\frac{1}{9}) + (\frac{1}{9}) + (\frac{3}{9}) + (\frac{2}{9}) + (0) + (0) + (0) = 1$$
$$(\frac{1}{9}) + (\frac{1}{9}) + (\frac{3}{9}) + (\frac{2}{9}) + (0) + (0) + (0) + (0) + (0) = \frac{7}{9}$$
$$(\frac{1}{9}) + (\frac{1}{9}) + (\frac{1}{9}) + (\frac{3}{9}) + (\frac{1}{9}) + (\frac{1}{9}) + (0) + (0) + (0) = \frac{8}{9}$$
$$(\frac{1}{9}) + (\frac{1}{9}) + (\frac{1}{9}) + (\frac{1}{9}) + (\frac{3}{9}) + (\frac{2}{9}) + (\frac{1}{9}) + (\frac{1}{9}) + (\frac{1}{9}) = \frac{12}{9}$$
$$(\frac{1}{9}) + (\frac{1}{9}) + (\frac{3}{9}) + (\frac{2}{9}) + (\frac{1}{9}) + (\frac{1}{9}) + (0) + (0) + (0) = 1$$
$$(\frac{1}{9}) + (\frac{3}{9}) + (\frac{1}{9}) + (\frac{1}{9}) + (0) + (0) + (0) + (0) + (0) = \frac{6}{9}$$
$$(\frac{1}{9}) + (\frac{3}{9}) + (\frac{2}{9}) + (\frac{1}{9}) + (\frac{1}{9}) + (\frac{1}{9}) + (0) + (0) + (0) = 1$$
$$(\frac{3}{9}) + (\frac{2}{9}) + (\frac{1}{9}) + (\frac{1}{9}) + (0) + (0) + (0) + (0) + (0) = \frac{7}{9}$$

As each convolution occurs, all indices of g get closer and closer to 0. After even one iteration, this pattern can be spotted and we take the limit.