Say you have 4 vectors: $u_1, u_2, u_3, u_4 \in \mathbb{R}^4$

4 dimensional = 4 coordinates = 4 column vectors

Vectors are independent if they correspond to different directions

$\lambda_1 u_1 + \lambda_2 u_2 + \lambda_3 u_3 = 0$

Suppose I did this and not all λ are zero, $(\lambda_1 \neq 0)$

$\lambda_1 u_1 = -\lambda_2 u_2 - \lambda_3 u_3$

$\lambda_1 = (-\lambda_2/\lambda_2) u_2 - (\lambda_3/\lambda_1) u_3$

$\therefore$ u_1 depends on u_2, u_3

So for independence all scalars have to be zero

- if you have independence than those vectors span the entire space

- Spanning family: collection of vectors s.t. every other vector in space can be written as a linear combinations. (not neceissarily unique)
- Any 2 spanning sets that are linearly independent, then you have a basis.

maximal independent set = spanning

$\mathbb{R}^2$ $\underset{e_1}{\begin{pmatrix}1\\0\end{pmatrix}} \underset{e_2}{\begin{pmatrix}0\\1\end{pmatrix}}$ = basis

* When you multiply a matrix by a column vector, your combining the columns in terms of the column vector coefficients

$H^T H = D \begin{pmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$ HW1 Q2 hint explaination

$Hx = \emptyset \Rightarrow \underbrace{H^T H}_{D} x = H^T \emptyset$

$= 0$

$Dx = 0$

$\begin{pmatrix} 4 & & & \\ & 4 & & \\ & & 2 & \\ & & & 2 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 4x_1 \\ 4x_2 \\ 2x_3 \\ 2x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \left.\begin{matrix} x_1 = 0 \\ x_2 = 0 \\ x_3 = 0 \\ x_4 = 0 \end{matrix}\right\}$ linear independence

linear independence $\Leftrightarrow$ invertibility

$m \times n$ Matrix C

$\underset{n}{A(x+y)} = \underset{m}{Ax + Ay}$ $\qquad$ $\underset{n}{A(\lambda x)} = \underset{m}{\lambda(Ax)}$

linear transformation example: taking direvatives * derivative of constant = zero

- represent by a matrix

Image = range of map: Apply linear map to all vectors

Kernel = set of vectors that map to zero vector

less trivial example of kernel:

$\begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \mathbb{R}^4 - P$ $\qquad$ $P \cdot \begin{pmatrix} x_1 \\ x_2 \\ \vdots \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ 0 \end{pmatrix}$

$$\begin{pmatrix} 0 & \\ & 0 \end{pmatrix} = \quad \begin{pmatrix} x_3 \\ x_4 \end{pmatrix} \quad \begin{pmatrix} 0 \end{pmatrix}$$

$$\text{Kernel}(P) = \begin{pmatrix} 8 \\ x_3 \\ x_4 \end{pmatrix} \cdot x_1 \cdot x_4 \in \mathbb{R}$$

★ When Kernel & Image live in same space, they must be linearly independent
- gives us the entire space (rank-nullity theorem)

HW I Q 5: