Module 3:
Portfolio Return & Risk
The Two-Asset Case

Statistical Relationships for a Two-Asset Portfolio

	Asset *a*	Asset *b*	
Expected return on each asset	r_a	r_b	
Risk (volatility)	σ_a	σ_b	
% holding in each asset	w_a	w_b	$w_a + w_b = 1$

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Covariance between A & B: σ_{ab}

Correlation between A & B: $\rho_{ab} = \sigma_{ab} / (\sigma_a \sigma_b)$

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Portfolio return r_P and volatility σ_P are then calculated as:

$$r_p = w_a r_a + w_b r_b \qquad w_a + w_b = 1$$

$$\sigma_p = [\, w_a^2 \sigma_a^2 + w_b^2 \sigma_b^2 + 2 w_a w_b \sigma_a \sigma_b \rho_{ab} \,]^{1/2}$$

Statistical Relationships for a Two-Asset Portfolio: *Calculation Check*

		Asset A	Asset B
Expected return	r	5%	8%
Risk (volatility)	σ	9%	15%
Correlation between A & B	ρ	50%	

r_P = _________

σ_P = _________

Please calculate the return and volatility for the portfolio of A & B when $w_a = 40\%$

$r_p = w_a r_a + w_b r_b$ $\quad\quad w_a + w_b = 1$

$\sigma_p = [\, w_a^2\sigma_a^2 + w_b^2\sigma_b^2 + 2 w_a w_b \sigma_a \sigma_b \rho_{ab} \,]^{1/2}$

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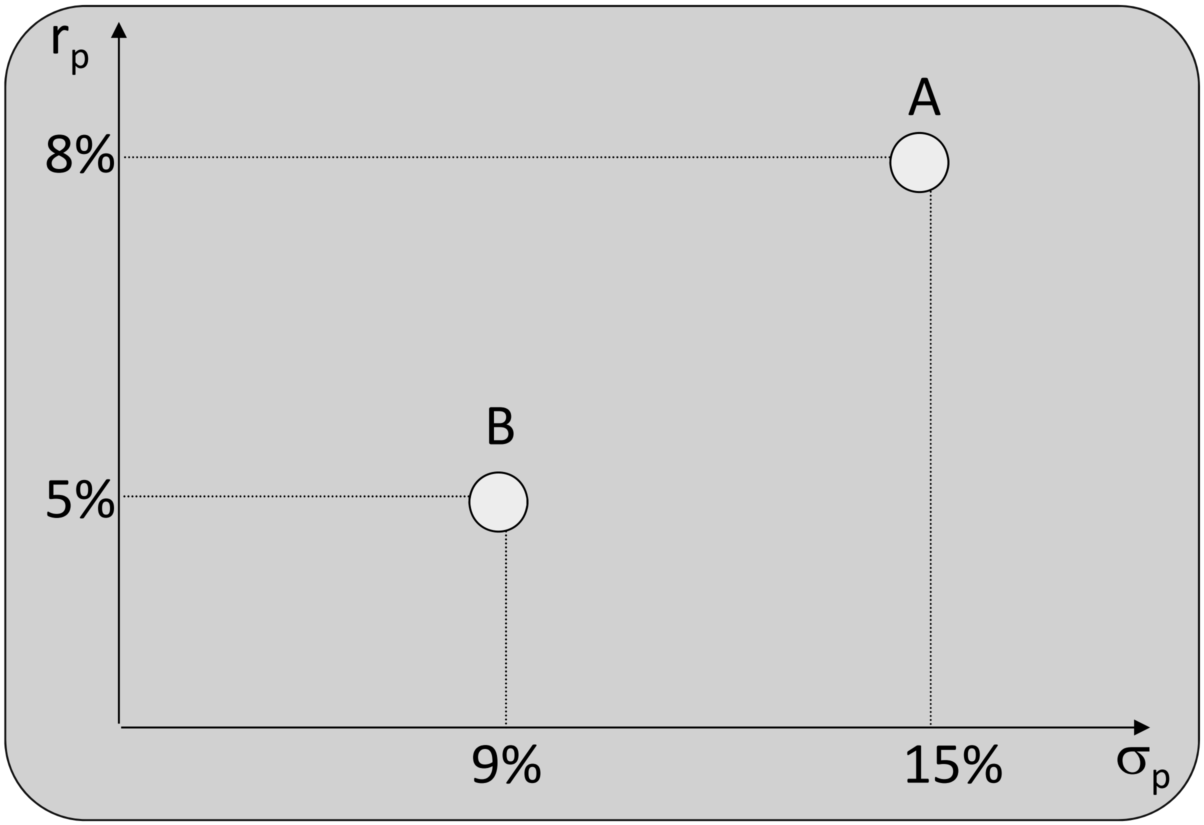
Return & Risk for a Two-Asset Portfolio

- Graphical demonstration of range of possible portfolios of two assets
  - Asset A has higher return and higher risk than Asset B
  - What range of portfolios can we create from these two assets?



$r_p = w_a r_a + w_b r_b$ $\qquad w_a + w_b = 1$

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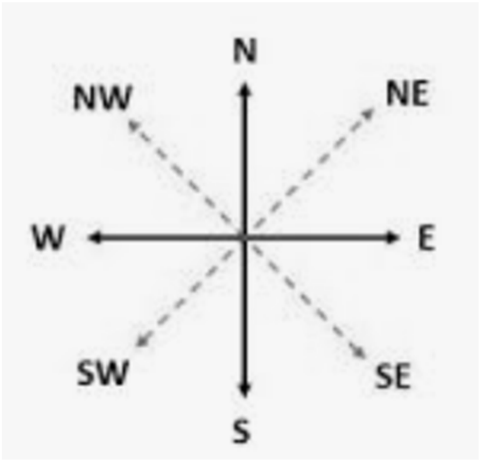

$$r_p = w_a r_a + w_b r_b \qquad w_a + w_b = 1$$

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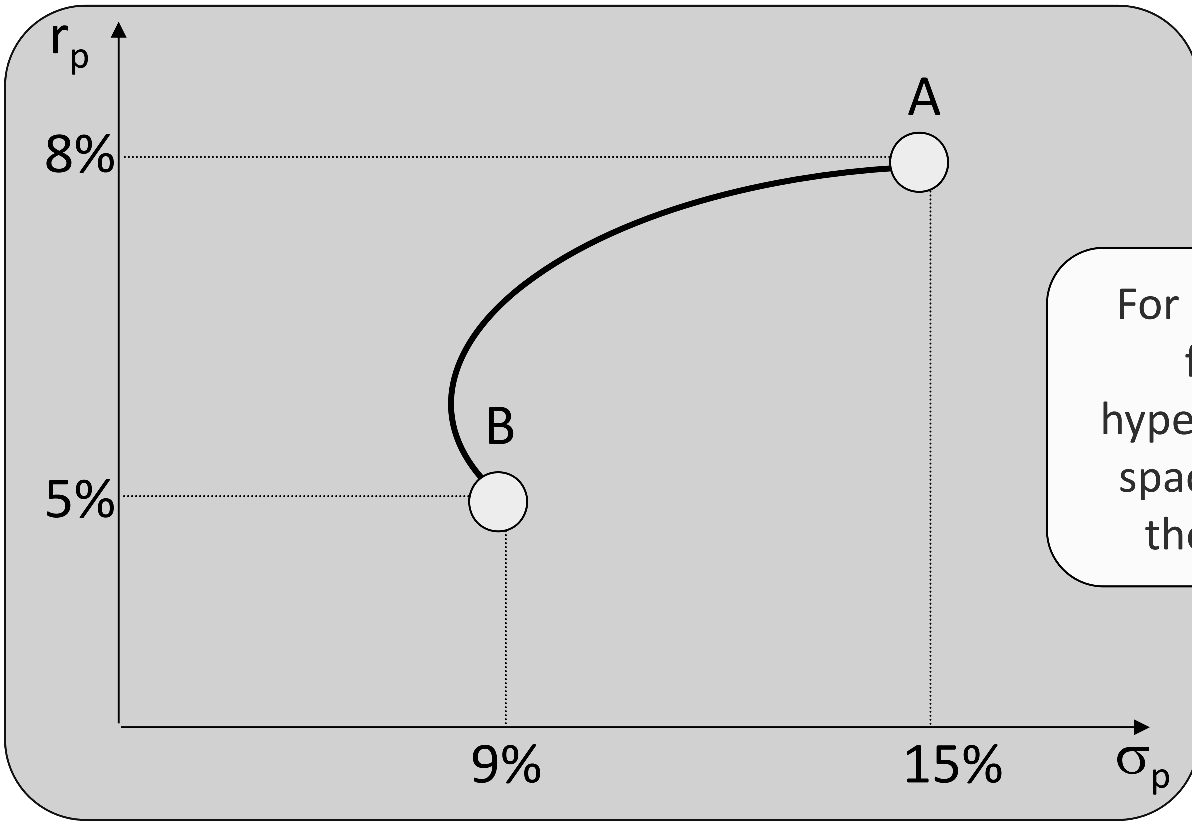
Return & Risk for a Two-Asset Portfolio

- Graphical demonstration of range of possible portfolios of two assets
  - The white curve demonstrates the range of all possible portfolios
  - Portfolio risk-return outcome driven by selection of w_a (and hence w_b)



For partial proof that these formulas generate a hyperbola in return-volatility space, see the Appendix at the back of the handout

$$r_p = w_a r_a + w_b r_b \qquad w_a + w_b = 1$$

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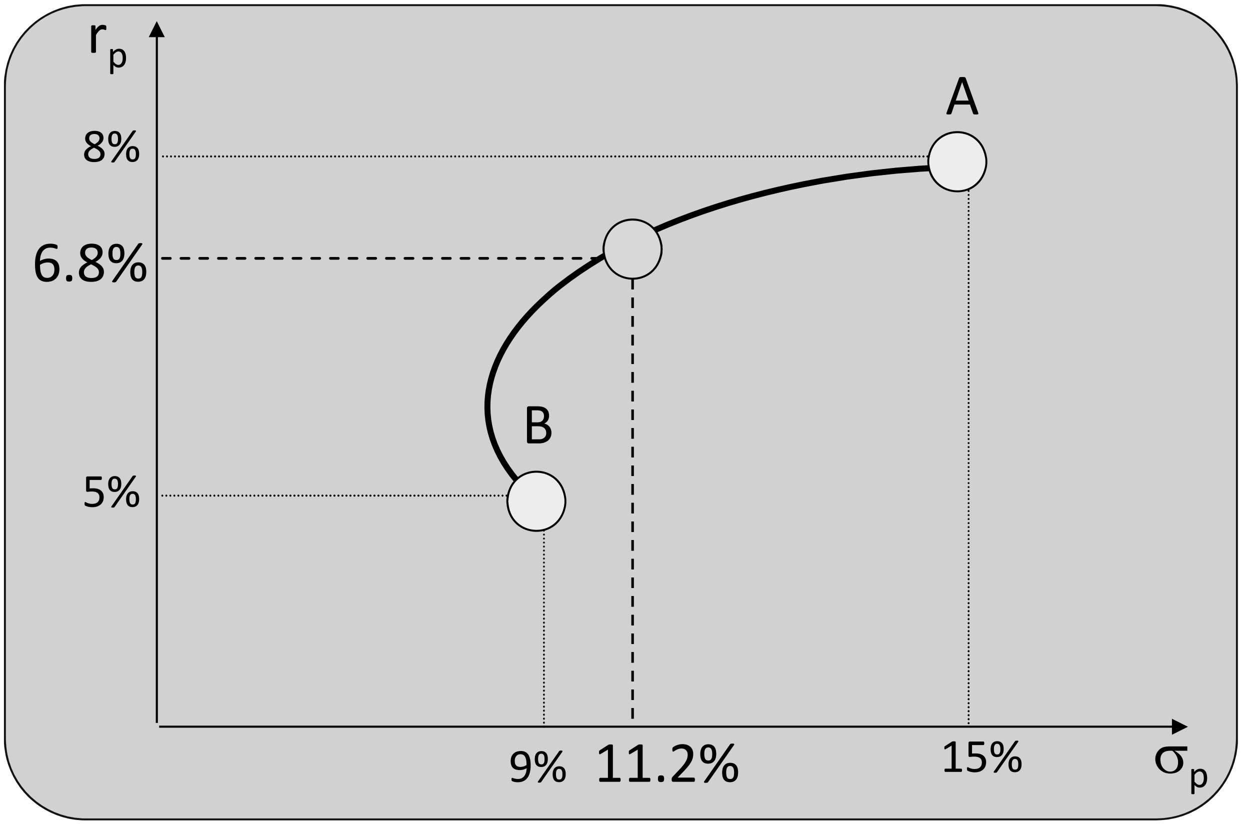
Return & Risk for a Two-Asset Portfolio
Example

If $\rho = 0.5$ & $w_a = 0.4$ as before: $r_p = 6.8\%$, $\sigma_p = 11.2\%$



$r_p \quad = w_a r_a + w_b r_b \qquad\qquad w_a + w_b = 1$

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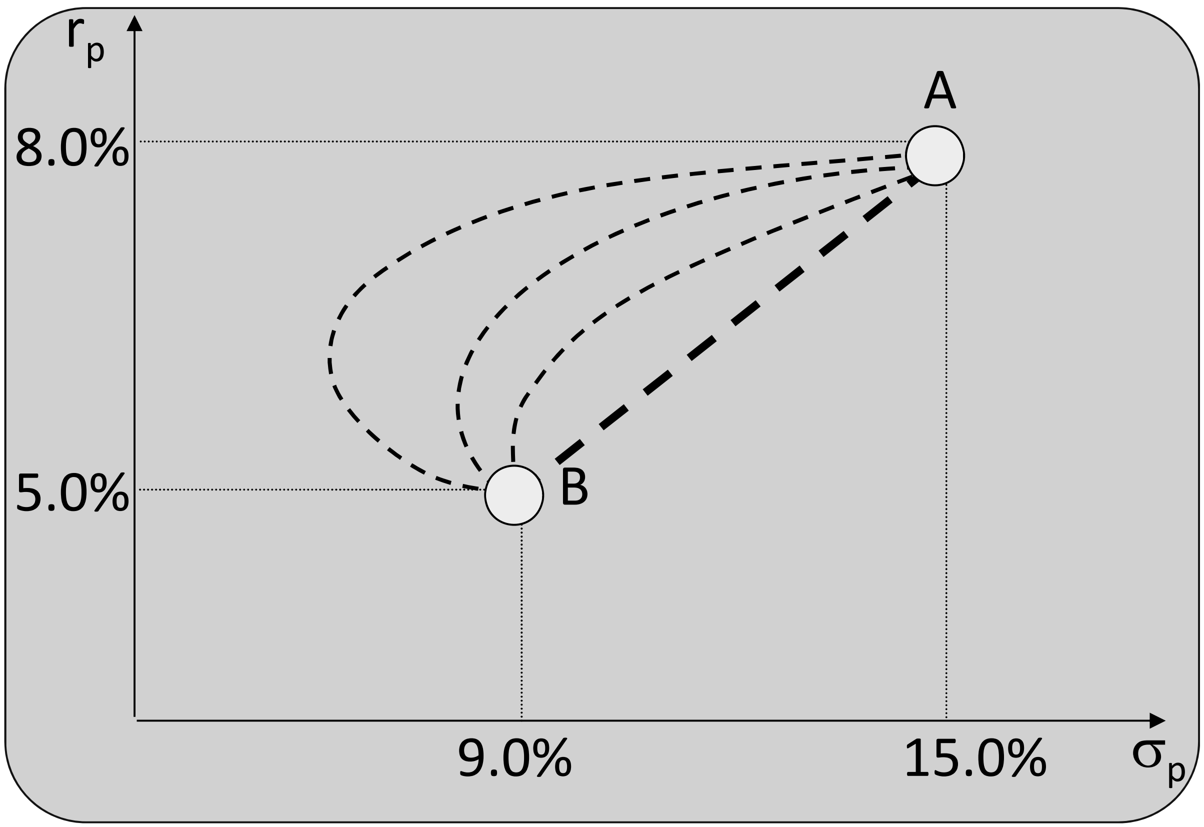
Return & Risk for a Two-Asset Portfolio
Concept Check

(1) Which statistical variable drives curvature?

(i) r_a	(iv) σ_a
(ii) r_b	(v) σ_b
(iii) w_a	(vi) ρ_{ab}



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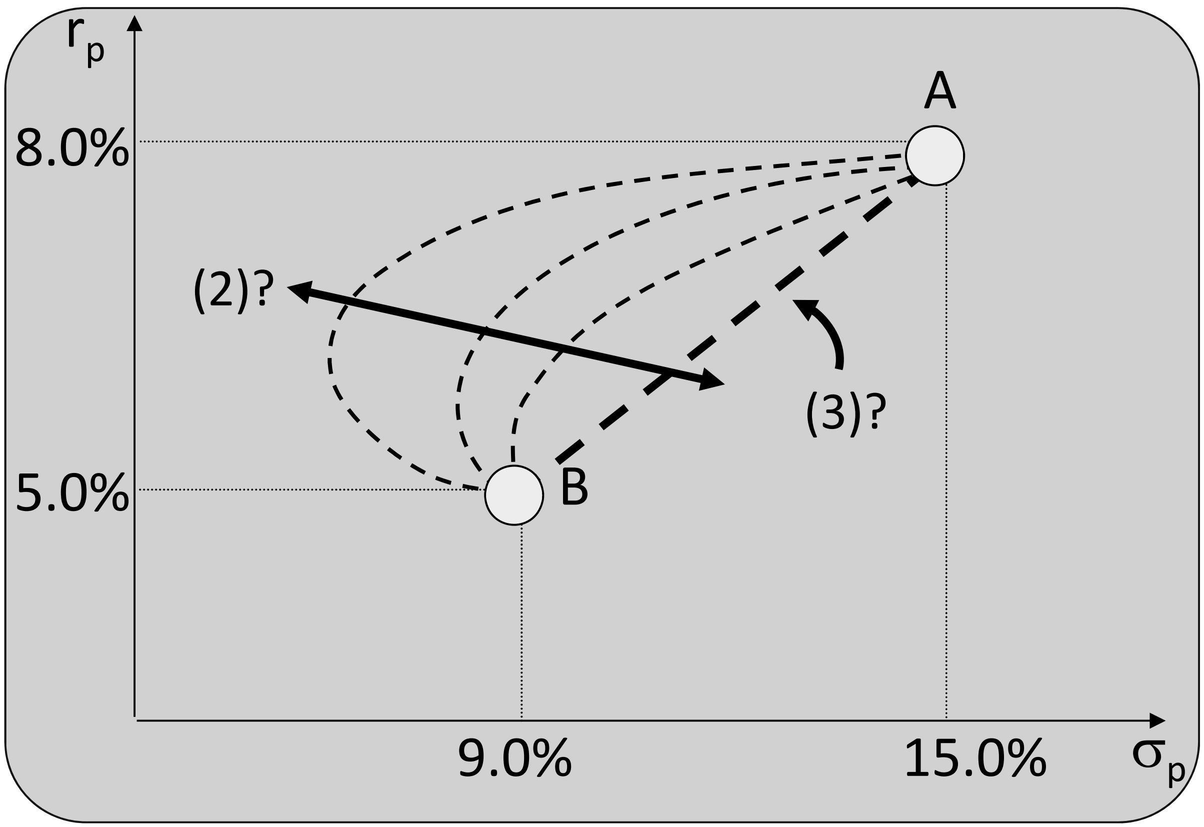

(2) As the variable *decreases*, which way does the curve stretch?

(3) Under what circumstances would the curve from A to B be linear (so that investors' portfolio options are limited to the straight line between A & B)?

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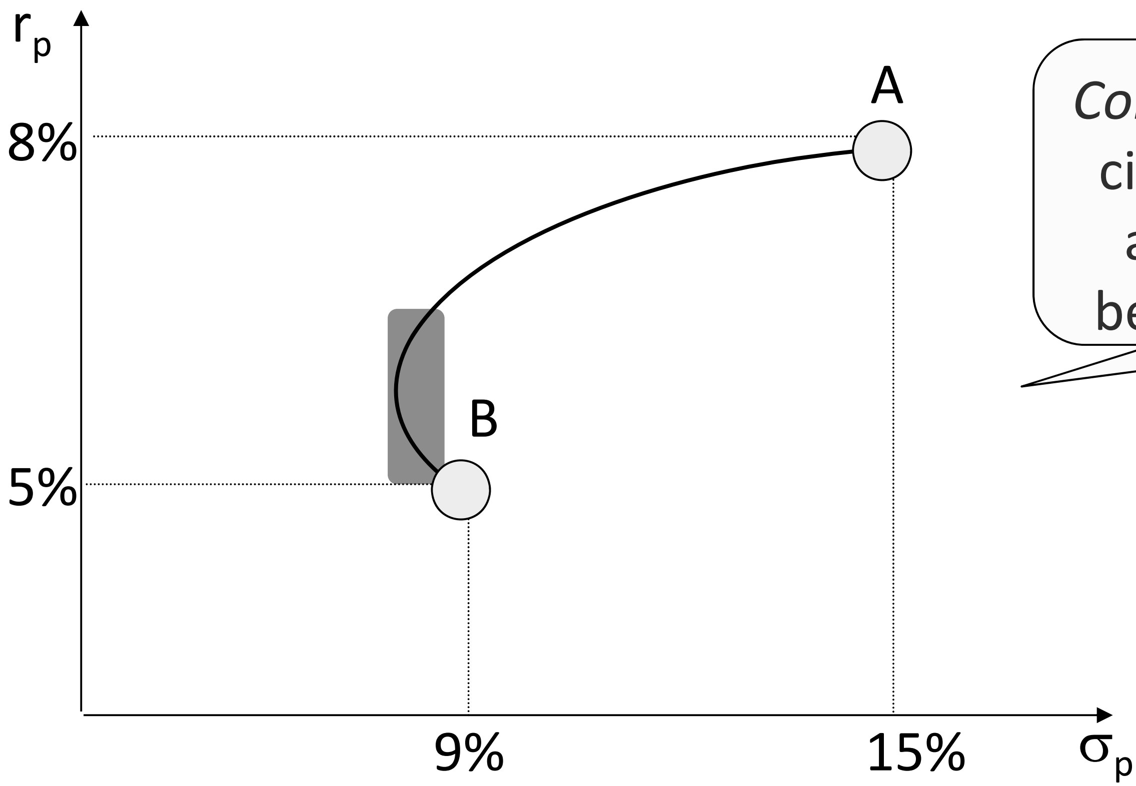
Return & Risk for a Two-Asset Portfolio

- When $\rho_{ab} < 1$, portfolio possibilities exist that have ***lower* volatility *and higher* return**, than Asset B alone
  - Shaded region of the curve

- This is the **benefit of diversification**: combinations of assets *almost invariably* offer better risk-return profiles than individual assets



Concept Check: under what circumstances will a two-asset portfolio have **no** benefit of diversification?

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