Module 4:
Return & Risk for Portfolios of Multiple Assets

Return & Risk for a Multi-Asset Portfolio

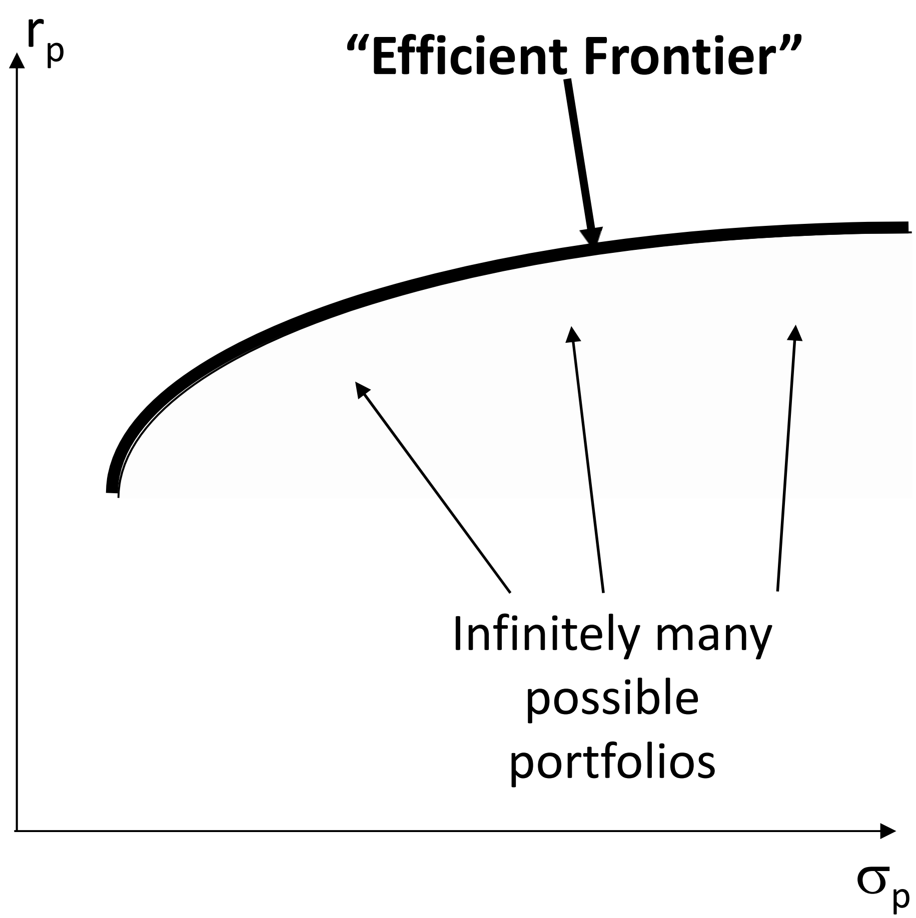
Expected Return and Standard Deviation formulas for portfolios with many assets:

$$r_p = \Sigma w_i \, r_i$$

$$\sigma_p = \left(\Sigma_i \, \Sigma_j \, w_i w_j \, \sigma_i \, \sigma_j \, \rho_{ij} \right)^{1/2}$$

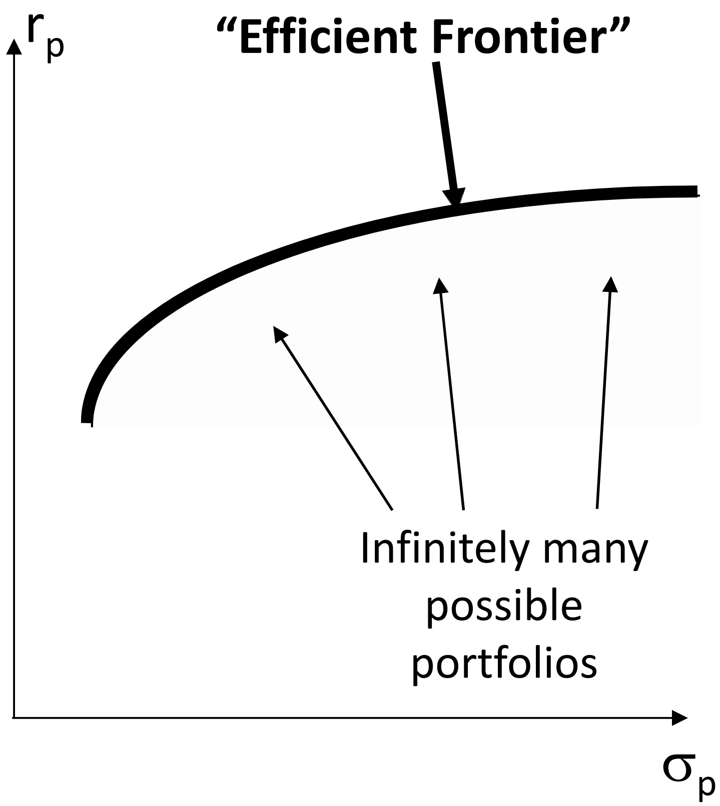
- *Portfolio return:*
  - Linear weighted average return of the assets in the portfolio
  - Does not depend on correlations between pairs of assets
- *Portfolio volatility*:
  - Non-linear: increasingly complex to calculate as more assets are added (let a computer do the work!)
  - *Does* depend on correlations between pairs of assets
  - Adding more assets incrementally increases the potential for lower portfolio volatility for any given rate of return

Return & Risk for a Multi-Asset Portfolio: The *Efficient Frontier*



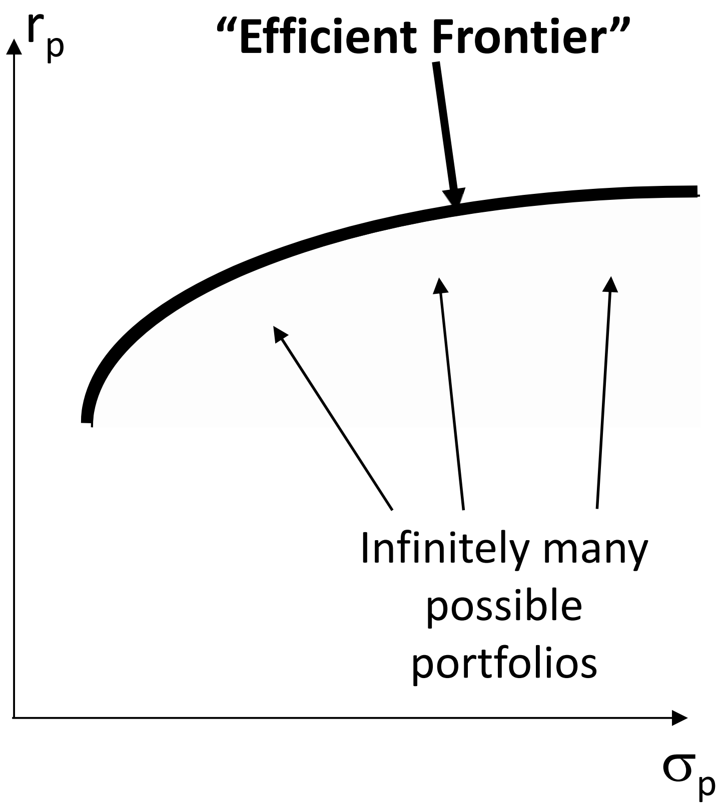
- Infinite numbers of possible portfolios in the shaded area
  - We can *estimate* risk & return characteristics of each, and hence "build" the frontier (how?)
- All possible portfolios will lie on or below a hyperbola (mathematically)
  - The "Efficient Frontier"
- Portfolios *on* the frontier have *best possible* risk-return characteristics
  - Called "optimal" portfolios
- Investors should choose portfolios that lie *on,* rather than *below*, this Frontier
  - Why?
  - How do we know which portfolios are truly optimal?!
  - Lots of caveats…

Return & Risk for a Multi-Asset Portfolio: The *Efficient Frontier: Caveats*



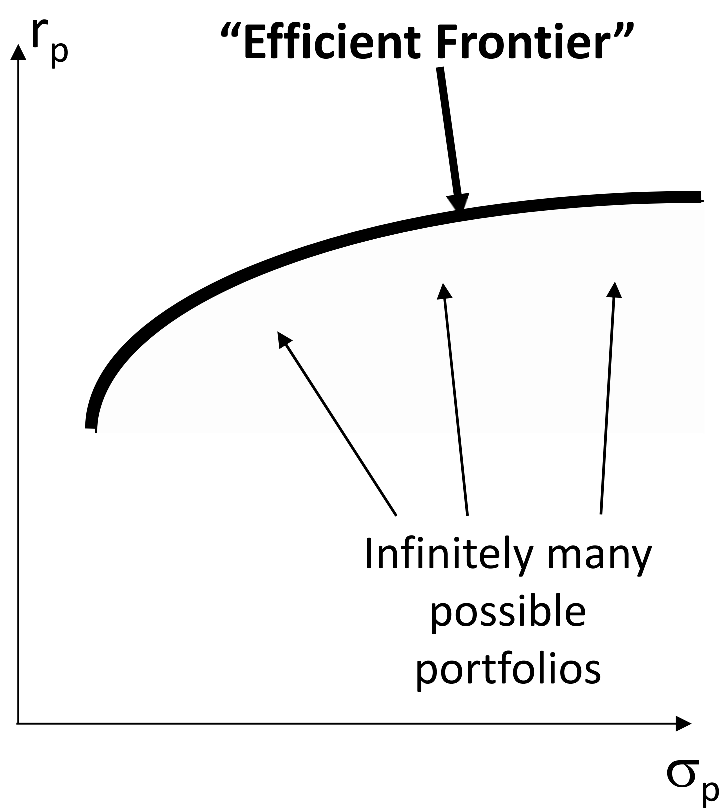
- Impossible to estimate all possible global portfolios
  - Infinitely many combinations

Return & Risk for a Multi-Asset Portfolio: The *Efficient Frontier: Caveats*



- Impossible to estimate all possible global portfolios
  - Infinitely many combinations
- Impossible to know the future return, volatility and pairwise correlations of all global assets
  - Historical data is a poor estimate at best
  - With incorrect assumptions, the efficient frontier will be in the "wrong" place

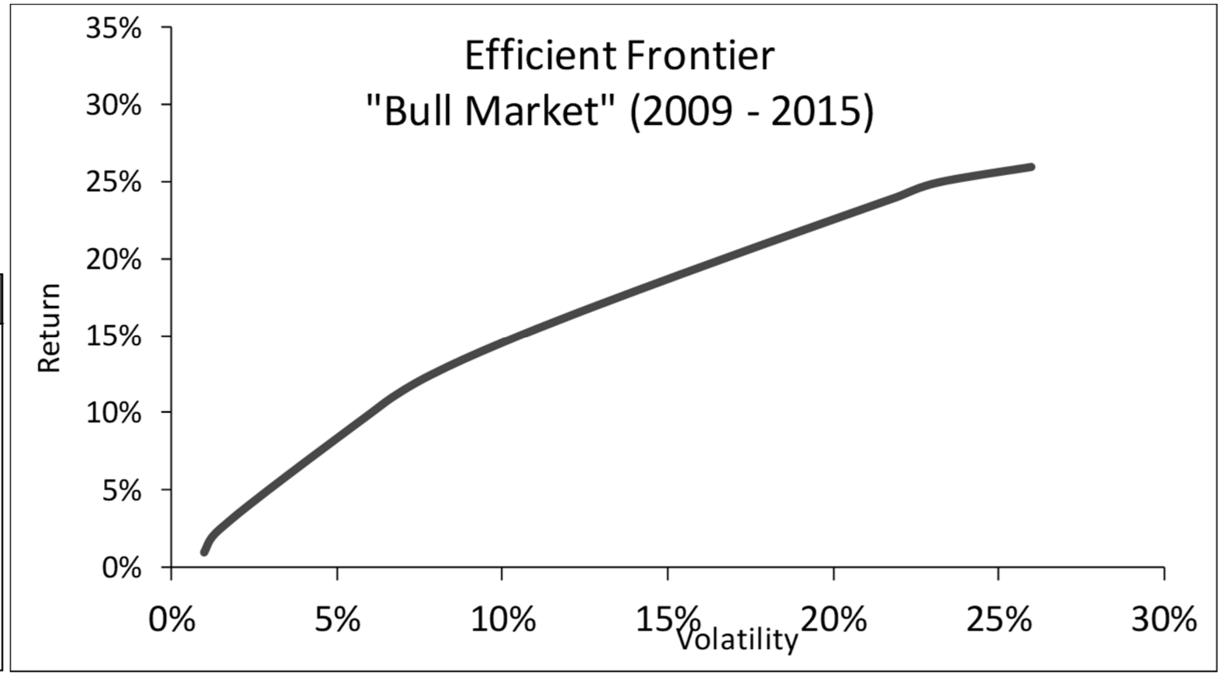
Return & Risk for a Multi-Asset Portfolio: The *Efficient Frontier: Caveats*



- Impossible to estimate all possible global portfolios
  - Infinitely many combinations
- Impossible to know the future return, volatility and pairwise correlations of all global assets
  - Historical data is a poor estimate at best
  - With incorrect assumptions, the efficient frontier will be in the "wrong" place
- Why do we care about the frontier?
  - Diversification is important!
  - Experienced financial managers *estimate* the frontier

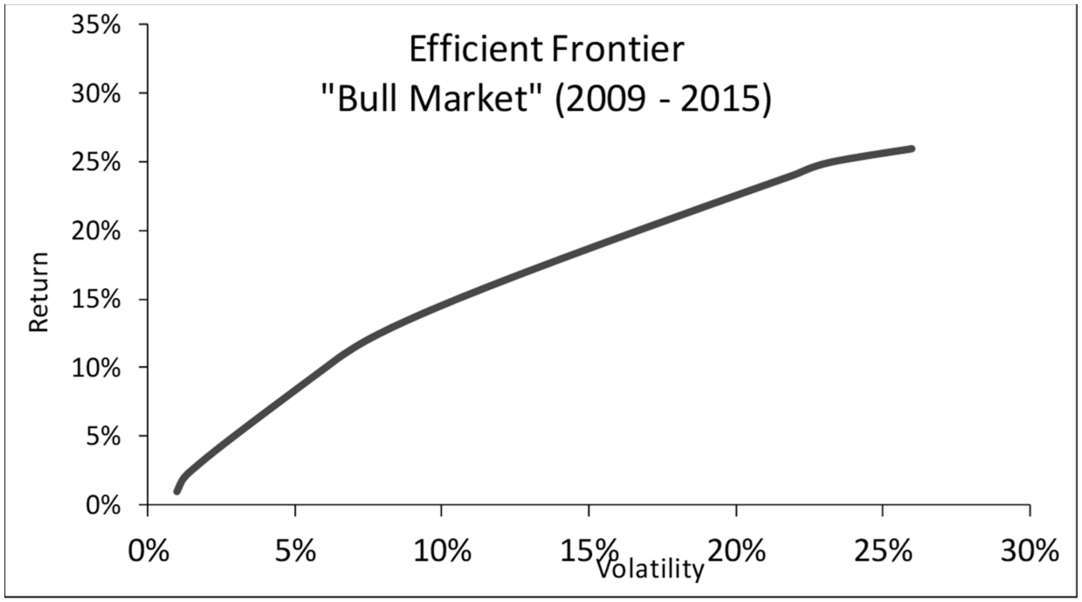
Return & Risk for a Multi-Asset Portfolio: The *Efficient Frontier: Samples*

	Avg rtn	volatility
Intl Equity	26.51%	31.27%
Commodities	-0.64%	20.89%
Corporate bonds	7.91%	5.93%
7-10yr UST	4.07%	6.92%
Inflation-sensitive	3.91%	5.75%
Real Estate	25.05%	25.30%
1-3yr UST	0.93%	0.99%

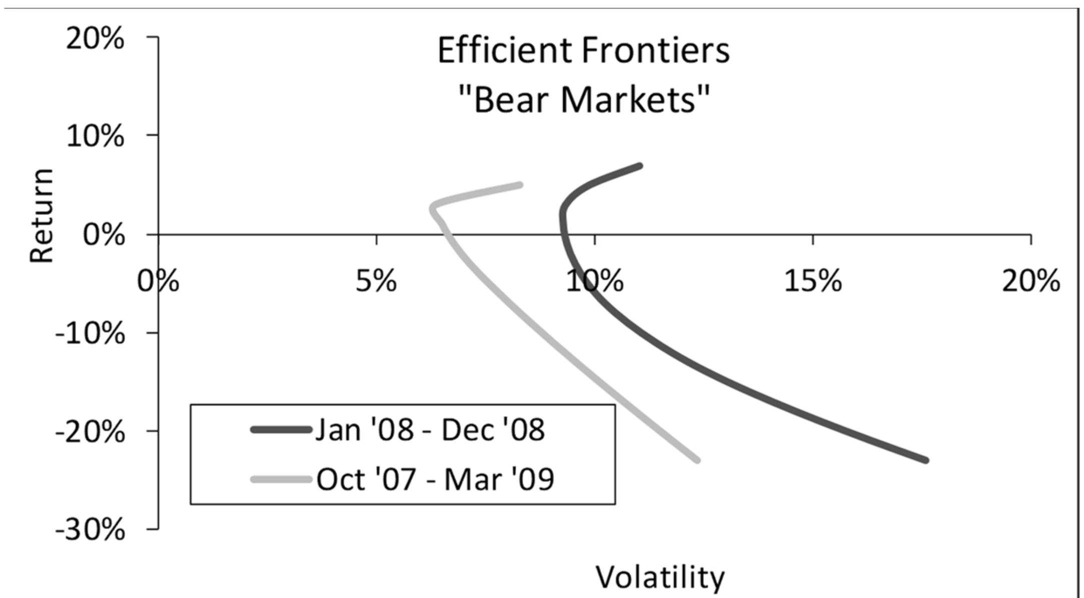


Correlation Matrix	Intl Equity	Commod	Corporate bonds	7-10yr UST	Inflation-sensitive	Real Estate	1-3yr UST
Intl Equity	100%	32%	-2%	-30%	-19%	51%	-16%
Commodities	32%	100%	-2%	-27%	-2%	43%	-15%
Corporate bonds	-2%	-2%	100%	72%	67%	11%	52%
7-10yr UST	-30%	-27%	72%	100%	79%	-27%	72%
Inflation-sensitive	-19%	-2%	67%	79%	100%	-9%	55%
Real Estate	51%	43%	11%	-27%	-9%	100%	-21%
1-3yr UST	-16%	-15%	52%	72%	55%	-21%	100%

Return & Risk for a Multi-Asset Portfolio: The *Efficient Frontier: Samples*



- From 2009 – 2015
  - Most diversified portfolios did well



- In bear markets (e.g., credit crisis)
  - Almost all asset classes and portfolios did badly!

Glossary

Bull Market: A market in which assets are going up in value.
Bear Market: A market in which assets are going down in value.

Equities (or shares): fractional ownership stake in a publicly traded company
Bonds: publicly traded debt (may be the debt of governments, corporations, municipalities etc.)

***S&P 500 Index ("S&P 500"):** A value-weighted index comprised of shares of the 500 largest US publicly traded companies, where "value" = share price x number of shares (called "Market Capitalization" or MCAP). Because it is value-weighted, it is heavily influenced by swings in the largest companies (by MCAP) in the index

***Dow Jones Index (DJIA or "DOW"):** A price-weighted index comprised of shares of the 30 largest US publicly traded companies. Because it is price-weighted (unlike the S&P 500), changes in the price of relatively larger stocks in the index do not have as much impact on the overall level of the Dow than they do on the level of the S&P

Risk Averse Investor: someone who, given the choice between two investments with equal expected returns but different volatilities, would always choose the investment with the lower volatility
Portfolio: A combination of different assets held together as an investment vehicle
Optimal Portfolio: In the context of portfolio theory, an optimal portfolio is one that offers the best possible rate of return for any given level of volatility.
Efficient Frontier: In risk-return space, a hypothetical continuous curve of "optimal" portfolios

Investopedia (www.Investopedia.com): a good online source of definitions for financial terms

** The S&P 500 can be viewed as an individual asset, because it can be purchased on a stand-alone basis (via an ETF: see Investopedia.com for a definition of ETFs). However, the S&P can also be viewed as a well diversified portfolio of US equities (as can the DOW and other US Indexes).*

Appendix

Why is the Efficient Frontier a Hyperbola?

Partial Explanation: Two-Asset Case

$r_p = w_a r_a + (1 - w_a) r_b$ (1)

$\sigma_p^2 = w_a^2 \sigma_a^2 + (1 - w_a)^2 \sigma_b^2 + 2 w_a (1 - w_a) \sigma_{ab}$ (2)

where r_a, r_b, σ_a, σ_b, and σ_{ab} are known constants which we will replace with a, b, c, d, and e to simplify

hence from (1): $w_a = \dfrac{(r_p - b)}{(a - b)}$

Substitute into (2): $\sigma_p^2 = \dfrac{(r_p - b)}{(a - b)} c^2 + \left(1 - \dfrac{(r_p - b)}{(a - b)}\right)^2 d^2 + 2e \dfrac{(r_p - b)}{(a - b)}\left(1 - \dfrac{(r_p - b)}{(a - b)}\right)$

From here, some tedious algebraic manipulation yields the formula for a hyperbola:

$$\frac{(x - h)^2}{p^2} - \frac{(y - j)^2}{q^2} = 1 \quad \textit{where } x = r_p \ \ \& \ \ y = \sigma_p$$

Even more tedious algebra can show that this generalizes to the multi-asset case