Module 2:
Portfolio Return & Risk

Correlation and Covariance

To calculate risk (volatility) for a *portfolio* (combination) of assets we also need *Covariance.*

Covariance is a measure of the extent to which two variables move *in the same direction* over time.

Formula for covariance:

Assume two assets: *a* and *b*

Returns r_{at} and r_{bt} over multiple time periods t

average returns $r_{a,avg}$ and $r_{b,avg}$

Covariance (σ_{ab}) between *a* and *b*: $\sigma_{ab} = 1/n \; \Sigma_t \; [\, r_{at} - r_{a,avg} \,] \, [r_{bt} - r_{b,avg} \,]$

We sometimes use Correlation ρ_{ab} instead of Covariance $\boldsymbol{\rho_{ab} = \dfrac{\sigma_{ab}}{\sigma_a \, \sigma_b}}$

Correlation has a nice feature: $-1 < \rho_{ab} < 1$

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Correlation and Covariance
Example

Two assets, *a* and *b:* r_{a1} = 4.50% r_{a2} = 2.50% r_{a3} = 1.05% r_{a4} = –3.20%

r_{b1} = 3.52% r_{b2} = 4.79% r_{b3} = 3.74% r_{b4} = 1.51%

(1) A*verage* returns of *a* and *b*: $r_{a,avg}$ = 1.21% $r_{b,avg}$ = 3.39%

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$$\boldsymbol{\sigma_{ab}} = \tfrac{1}{4}\,\big[\,(4.50\% - 1.21\%)(3.52\% - 3.39\%) + (2.5\% - 1.21\%)(4.79\% - 3.39\%) + (1.05\% - 1.21\%)(3.74\% - 3.39\%) + (-3.20\% - 1.21\%)(1.51\% - 3.39\%)\,\big] = 0.026\%$$

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$$= 0.026\%$$

Correlation between *a* and *b:* $\boldsymbol{\rho_{ab}}$ = = 0.026% / (3.26% x 1.37%) = 58.51%

Correlation and Covariance
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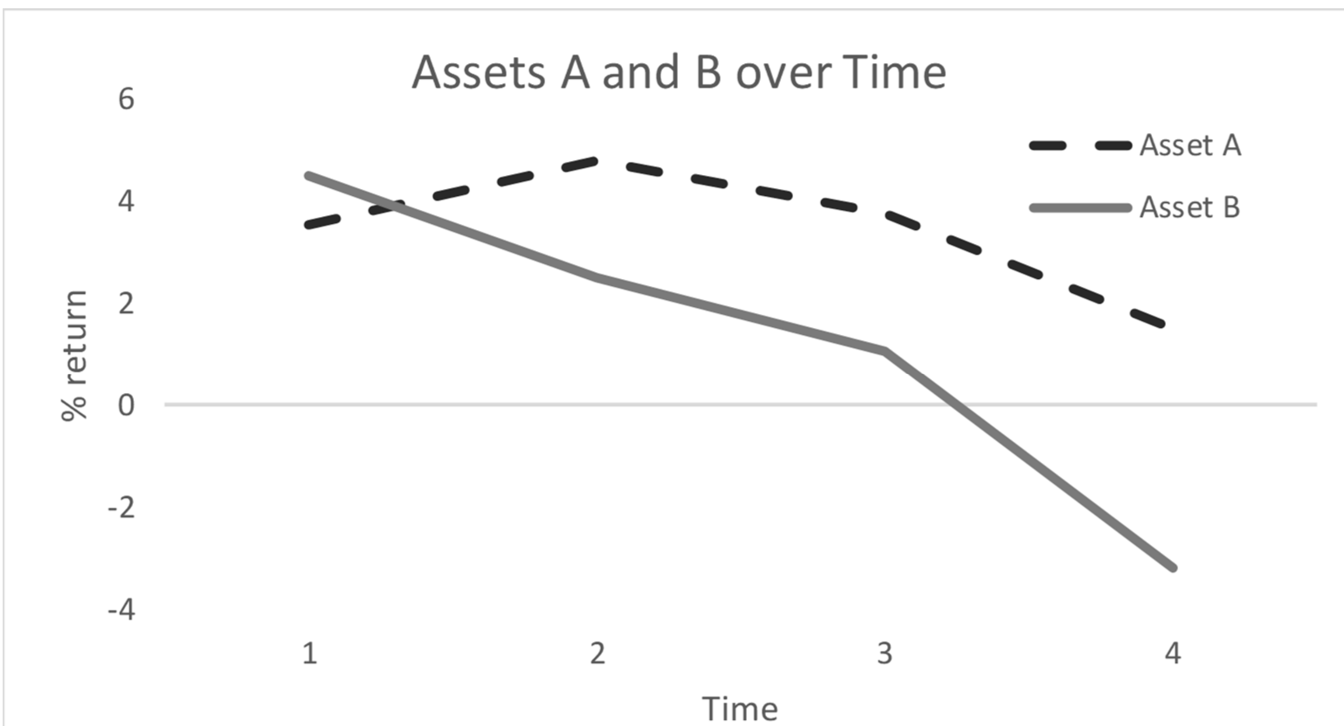
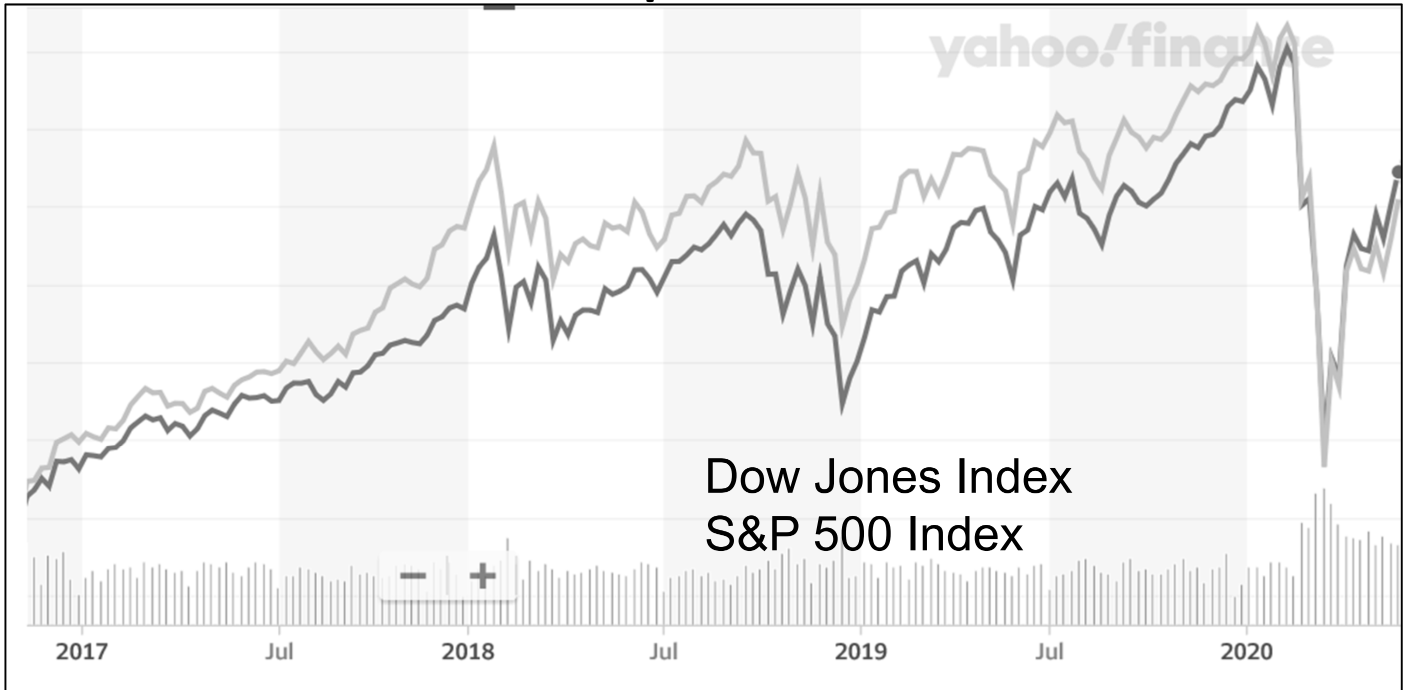


Correlation Examples



Guess the correlation between Gold & the S&P500 over this period

Statistical Relationships for Portfolios of Assets

Portfolio *return* r_P: weighted average return on the individual assets:

$r_P = \Sigma_i w_i r_i$ *with n* assets (*i = 1….n*)

Asset *i* represents w_i% of the overall portfolio value: $\Sigma_i w_i = 1$

Portfolio *volatility* is more complex.

$$\sigma_p = (\Sigma_i \Sigma_j w_i w_j \sigma_i \sigma_j \rho_{ij})^{1/2}$$

- As we add more assets, calculating portfolio volatility becomes cumbersome.
- We'll develop intuition for portfolio volatility in the two asset case
- To calculate volatility on a portfolio of multiple assets: use a computer!