quantitative Finance

for Technology-Driven Investment Decisions

ENGRMGMT & FINTECH 534

Spring 2023

Jake Vestal

Meets Tuesdays: 3:30-6:00pm EST

Lecture 1

Syllabus (on Sakai)

On Sakai!

ALL LECTURES SLIDES ARE ALWAYS POSTED ON SAKAI

- Before class, when possible
- Sometimes after class
  - If that class has a lot of “think about it” question-and-answer slides
  - Or if that class requires some up-to-date market data & analysis that makes it hard to upload beforehand

Difference between FINTECH 535 and 522

FINTECH 522

- Entry Level
- Offered in Spring
- Required by FINTECH program
- Covers a wider array of assets

FINTECH 535

- Maybe a little less Entry Level
- Uses modeling in R and Python a little more
- Not required by FINTECH
- Offered in Fall
- Focused more on stock, bonds, and equity options.

Both

- Require the use of Excel
- **HEAVY** emphasis on understanding Markowitz Portfolio Management & CAPM (first few classes)

The Plan for Today

- Course Requirements / Orientation
- Course Overview
- BASIC FINANCE
  - The Basics
  - Risk, Return, and Compounding
  - The meat and potatoes of *any* finance course

Course Requirements

- No Audits…
- Problem Set Format
  - Submit Excel Spreadsheets on Sakai
  - No late Problem Sets
- Exam Format:
  - Done in Excel
  - In person (new this semester)
  - “Open book” format – but don’t let that fool you!

Grading:

Midterm Exam	30%
Final Exam	45%
Problem Sets and Participation	25%

We will model professional behavior as befits professional people like us ☺

Problem Sets

- No Audits...
- Problem Set Format
  - Submit Excel Spreadsheets on Sakai
  - No late Problem Sets
- Exam Format:
  - Done in Excel
  - In person (new this semester)
  - “Open book” format – but don’t let that fool you!

Grading:

Midterm Exam	30%
Final Exam	45%
Problem Sets and Participation	25%

We will model professional behavior as befits professional people like us ☺

Background: Jake Vestal

B.S., Chemical Engineering, minor in Mathematics, NC State

M.E.M., Duke University

Wrote algorithm for ship condition monitoring system on Navy warships for fuel savings & performance

Data Analysis consultant

Machine learning for materials charactarization and monitoring (concrete, soil, fuel) → machine learning for funds management for high net worth individuals

Compounding

Definition: A financial instrument is said to be **compounded** when its new value (principal) is re-calculated by applying a % rate.

Expressed as a **rate** in terms of %/time

The names given -- **discount** rate, **interest** rate, rate of **return**, etc… all depend on context, but have the same conceptual meaning.

Some example interest rates:

$5\ ^{\%}/_{\text{year}}$,

$0.64\ ^{\%}/_{\text{day}}$, 0.3% continuous

All have:

- ✓ A **Value** (5, 0.64, 0.3)
- ✓ A **Basis** (yearly, daily, continuous)

Compounding

Expressed as a **rate** in terms a numeric value and a time basis.

- The **value** component tells you how much the instrument's value changes
- The **basis** component how often, in time, that change is realized (i.e., *compounded*)

For Example:

Consider the value of **$1,000** in an account that earns an interest rate of **1.2%/year**.

START: (Year 0)	**The Next Day**: (Year 0)	**11 Months Later**: (Year 0)	**18 Months Later**: (Year 1)	**40 Months Later**: (Year 3)
$\$1{,}000$	$\$1{,}000$	$\$1{,}000$	$\$1{,}012$	$\$1{,}036.40$
	no compounding	no compounding	$= \$1{,}000 \times 1.012$	$= \$1{,}000 \times 1.012 \times 1.012 \times 1.012$ $= \$1{,}000 \times (1.012)^3$

Discrete Compounding

Implicitly, we've just been using the definition of **discrete compounding**
(new principal is calculated every period)

compounding rate [%/time period]

Number of periods

Starting principal [$]

Ending principal [$]

$$P_0(1+r)^n = P_n$$

Starting period (0)

Discrete Compounding example (quarterly -> annual)

Consider an interest rate of **3%, compounded quarterly**

What is the equivalent annual rate?

Find ***r**ann* that satisfies:

$$P_1(1+r_{ann})^1 = P_2 \quad = \quad P_2 = P_1\left(1+r_{qtr}\right)^4$$

$$\cancel{P_1}(1+r_{ann})^1 = \cancel{P_1}\left(1+r_{qtr}\right)^4$$

$$r_{ann} = \left(1+r_{qtr}\right)^4 - 1$$

$$\boxed{r_{ann} = 12.6\%/\text{yr}} = (1.03)^4 - 1$$

Discrete Compounding example (annual -> quarterly)

Consider an interest rate of **12%**

What is the equivalent effective quarterly (compounded 4x per year) rate?

Find ***r_{qtr}*** that satisfies:

$$P_1(1+r_{ann})^1 = P_2 \quad = \quad P_2 = P_1\left(1+r_{qtr}\right)^4$$

$$\cancel{P}_1(1+r_{ann})^1 = \cancel{P}_1\left(1+r_{qtr}\right)^4$$

$$(1+r_{ann})^{1/4} - 1 = r_{qtr}$$

$$1.12^{1/4} - 1 = \boxed{2.87\% = r_{qtr}}$$

Discrete Compounding (formal definition)

Rate is broken out into its **value** and **basis**

n now means **frequency**

compounding rate value [%]

time (1 yr, 2 yr, etc)

Starting principal [$]

Ending principal [$]

$$P\left(1+\frac{r}{n}\right)^{nt} = P'$$

compounding frequency [1/month, 1/yr, 1/qtr…]

Continuous Compounding

$$P\left(1+\frac{r}{n}\right)^{nt} = P'$$

We want to know the principal at ANY continuous time *t*

In other words m, the compounding frequency [1/month, 1/yr, 1/qtr…] approaches infinity

We're interested in an infinitely high compounding frequency:

$$\lim_{n\to\infty}\left(P\left(1+\frac{r}{n}\right)^{nt}\right)$$

Let $n = mr.$

$$\rightarrow P\lim_{m\to\infty}\left(1+\frac{1}{m}\right)^{mrt}$$

By definition:

$$\lim_{m\to\infty}\left(1+\frac{1}{m}\right)^{m} = e$$

$$\rightarrow \boxed{Pe^{rt} = P'}$$

Compound Interest formula.

Nominal & Effective Interest Rates

Nominal: Usually calculated simply, but must read the fine print

For Example:

(See **Handout 2**)

$$\boxed{8\%/\text{yr}} = 8\%/\text{yr} \times \frac{1 \text{ year}}{4 \text{ quarters}} = 2\%/\text{quarter}$$

Well and good… but…

???

START: (qtr 0)	Qtr 1:	Qtr 2:	Qtr 3:	Qtr 4:
$\$1{,}000$	$\$1{,}020$	$\$1{,}040.40$	$\$1{,}061.21$	$\$1{,}082.43$
	$\$1{,}000 \times (1.002)^1$	$\$1{,}000 \times (1.002)^2$	$\$1{,}000 \times (1.002)^3$	$\$1{,}000 \times (1.002)^4$

(the multiple)

$$\$1{,}082.43 / \$1{,}000 = 1.08243 = \boxed{8.2\%/\text{year}}$$

Effective:
The rate that is actually realized.

Nominal & Effective Interest Rates

Bottom Line:

Nominal: Used in marketing (not everyone will bother to calculate out the effective rate), (rough) mental estimation

Effective: The real one.

Course Convention: %/yr CCR Implied Basis

Any rate (return, interest, discount, etc) mentioned on its own, without a basis, is understood to mean

% / year, effective, continuously compounded.

- Allows us to compare the returns of different investments, financial instruments & products
- Common practice in financial industry
- Any interest rate represented in this form is said to be "**annualized**" (written on a per-year basis).

Would you rather have 3% compounded quarterly (discrete), or 12% compounded annually (discrete)?

Answer:

Let’s use Handout 2

Interest Rates

Bottom Line:

Any interest rate can be converted to / understood as an equivalent effective annualized, continuously compounded rate.

IRR

Need for homework.

Let’s take a look at Handout 1 (Resources section on Sakai)

Using Solver

Need for homework.

Let's do an example in Excel

Asset Pricing and Risk Management

FINTECH 522

Jake Vestal

Class 3:

Bonds & Credit Risk

Bonds: Why do we care?

Bonds are:

- **Predictable.** They pay income periodically and at maturity.
- **Safe**(er)**.** You will *almost* certainly get your money back.
- **Not** (as) **Volatile.** Their value does not fluctuate nearly as much as other assets, and can be included in a portfolio to tame down your vol.

What is a bond?

- A bond is simply a loan.
- When corporations, municipalities, and governments issue bonds, they are seeking to borrow money from investors
- Investors pay an upfront lump sum (the loan amount) on the expectation that they will receive interest payments ("coupons") over time, as well as the "par amount" of the loan on the "maturity date" of the bond
- This is why bonds are sometimes referred to as "Fixed Income" instrument

Types of Bonds

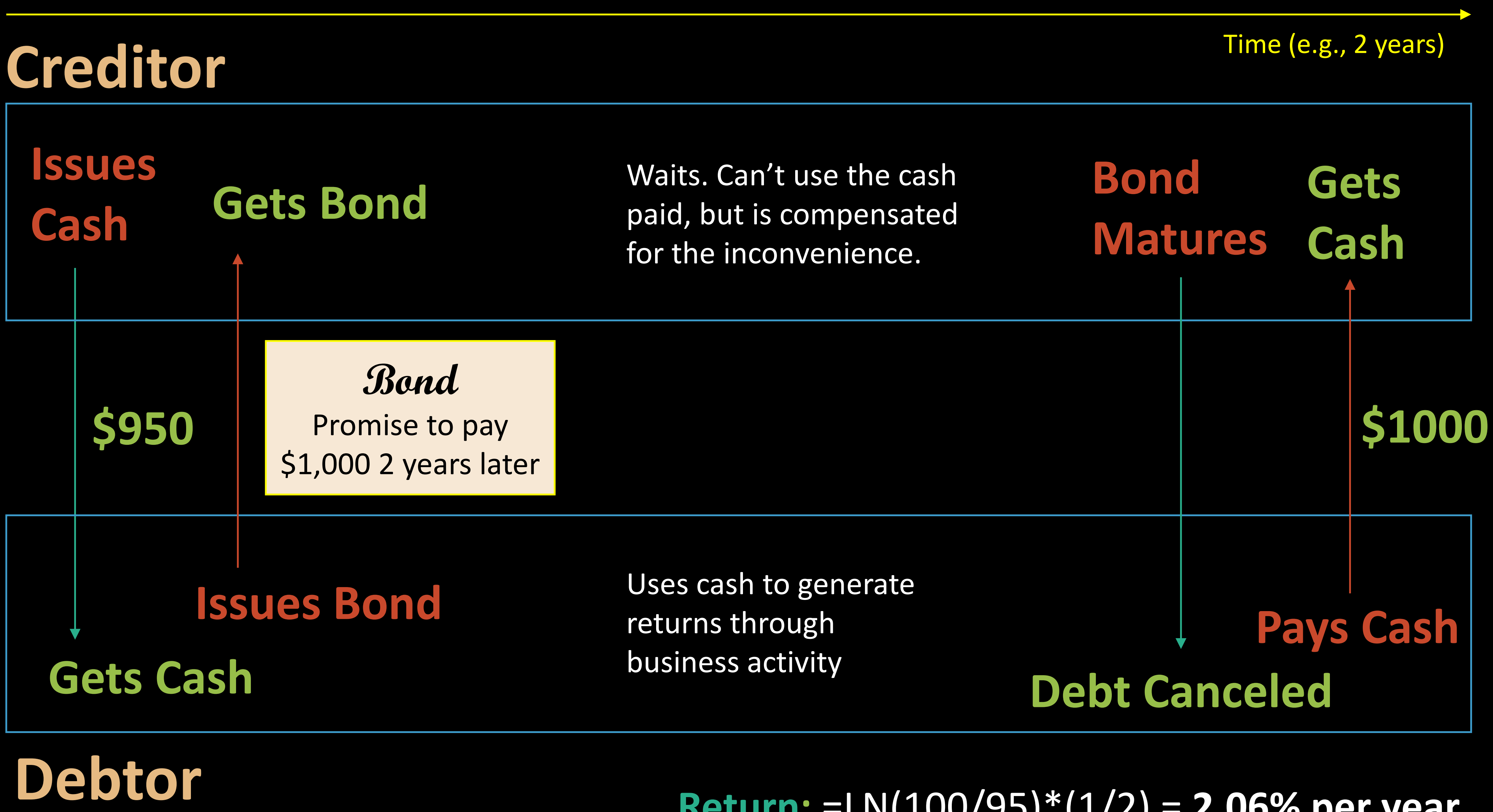
Taxable fixed income

- **Bank CDs**: safety and insurance
- **U.S. Treasuries**: safety
- **Corporate bonds**: high/medium quality
- **High yield bonds**: lower quality/high volatility

Tax-Exempt

- **Municipal Bonds (munis)**: state, federal, local

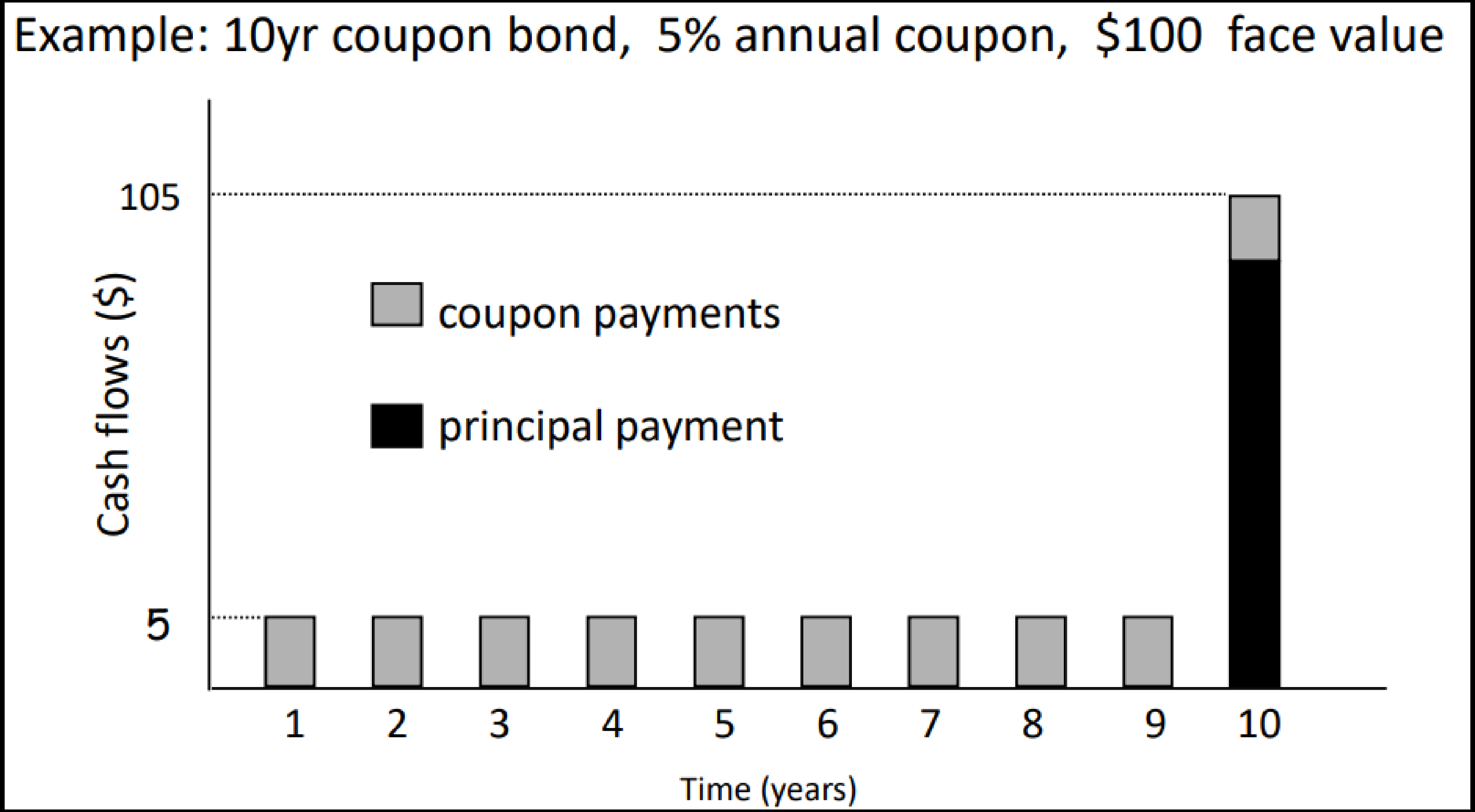
Bonds: Basic Idea



Return: =LN(100/95)*(1/2) = **2.06% per year**

Cash Flows from a Fixed Coupon Bullet Bond

- **Coupon**: A fixed interest payment in each period.
- **Face value** (also called Principal, or Par Amount) in the final period.



Bond

Involve a **promise** between a **holder** and an **issuer**

So are bonds considered **contracts**?

Securities

A word that gets tossed around a lot – often when "**instrument**" would have been more correct.

→ Two types:

Nope. Bonds are securities

1. **Debt Security**: Money that is borrowed and must be repaid, with clearly understood (and legal) terms (e.g., corporate bonds)
2. **Equity Security**: Ownership interest held by shareholders (e.g., a stock).

Courts use "**form over substance**" to determine if something is a security.

"If it behaves like a security, it's a security."

(much like **Duck Typing** in computer science)

Why do we care?

Because all securities & securities-related business in the US is regulated by the Securities Exchange Commission (SEC), and therefore involves a set of very specific (and complex) rules and regulations, all of which must be followed.

See also: The Howey Test

United States v. W.J. Howey Company (1946)

US Government Bonds

US Treasury Bills

- Up to 1 year to maturity
- Zero Coupon
- Pay face value at maturity

US Treasury Notes

- Between 2-10 yr to maturity
- Pay semi-annual coupon
- Pay face value at maturity

US Treasury Bonds

- >10 yr maturities
- Pay semi-annual coupon
- Pay face value at maturity

All US Treasury bills, notes and bonds (referred to generically as “US Treasury bonds”) are issued on a regular schedule by the US Treasury, and distributed by primary dealers (New York investment banks) to institutional investors (primarily mutual and pension funds, hedge funds, commercial banks, insurance companies and other financial institutions.

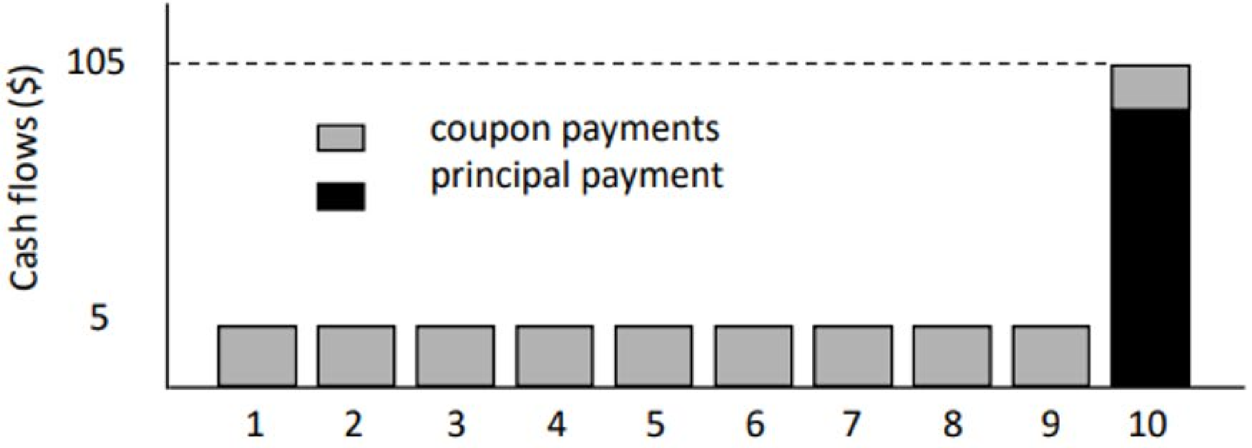
US Corporate Bonds

Bonds (debt) issued by corporations

- Pay semi-annual coupon and face amount at maturity (like USTs)
- Investment-grade vs. below investment-grade bonds
- Additional features:
  - Call provisions
  - Convertible bonds
  - Puttable bonds
- In the event of default, different classes of bonds have different claim priority on the corporation’s assets
  - Secured bonds (backed by specific assets or property)
  - Debentures (subordinate to secured debt)

Calculating the Price of a Coupon Bond

Example: Face = $100, coupon = 5% , r = 6%



Bond Price: *sum of discounted cash flows:*

$$P = \frac{5}{(1.06)} + \frac{5}{(1.06)^2} + \frac{5}{(1.06)^3} + \ldots + \frac{5}{(1.06)^{10}} + \frac{100}{(1.06)^{10}} = 92.64$$

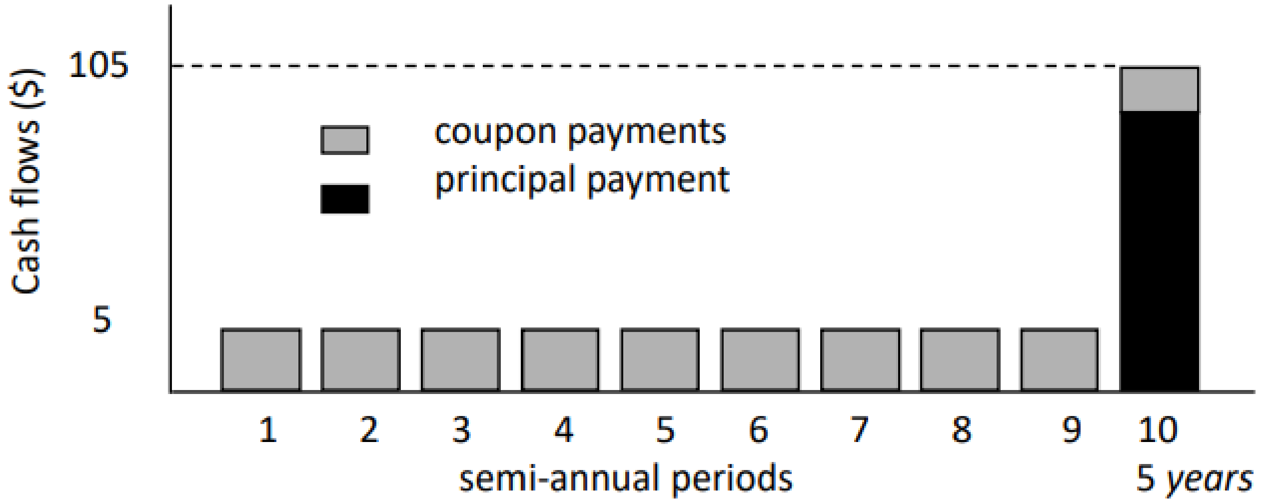
General Formula:

coupon cashflow C, Face value $100, time to maturity n years, annual discount rate y:

$$P = \frac{C}{(1+y)} + \frac{C}{(1+y)^2} + \frac{C}{(1+y)^3} + \ldots + \frac{C}{(1+y)^n} + \frac{100}{(1+y)^n}$$

US Treasury Bonds make Semi-Annual Payments

Example: 5 year *semi-annual* 10% coupon bond, discount rate 6%



$$P = \frac{5}{(1.03)} + \frac{5}{(1.03)^2} + \frac{5}{(1.03)^3} + \dots + \frac{5}{(1.03)^{10}} + \frac{100}{(1.03)^{10}}$$

$$= \frac{5}{0.03}\left[1 - (1.03)^{-10}\right] + \frac{100}{(1.03)^{10}} = 117.06$$

What assumptions do we make when calculating Present Value?

- Cash flows are of known magnitude.
- Cash flows are of known timing.
- Assume a **discount rate** for the cash flows.
  - Where does the discount rate come from?
  - What if the timing of the cash flows is not annual?

Yield to Maturity

The discount rate that equates the present value of the expected cash flows (interest and principal) to its observed price.

For a semi-annual bond:

$$\text{Price} = \frac{C/2}{\left(1 + y/2\right)} + \frac{C/2}{\left(1 + y/2\right)^2} + \cdots + \frac{100 + C/2}{\left(1 + y/2\right)^{2n}}$$

$$= \boxed{\frac{C/2}{y/2}\left[1 - \left(1 + y/2\right)^{-2n}\right] + \frac{100}{\left(1 + y/2\right)^{2n}}}$$

Wherein:

n = Years until maturity
C = Annual Coupon
y = Yield to maturity

Bond Yield – Coupon Relationship

Two different bonds

Example 1: 5yr bond, 8% coupon, 7% yield:

$$P = \frac{4}{0.035}\left[1 - (1.035)^{-10}\right] + \frac{100}{(1.035)^{10}}$$

$$= 104.16$$

Coupon > Yield, so Price > 100

Bond is trading at a *premium*

Example 2: 5yr bond, 6% coupon, 7% yield:

$$P = \frac{3}{0.035}\left[1 - (1.035)^{-10}\right] + \frac{100}{(1.035)^{10}}$$

$$= 95.84$$

Coupon < Yield, so Price < 100

Bond is trading at a *discount*

Bond Yield: Summary

- Yield is the single discount rate that discounts all of the bond's cash flows to its observed price
- Yield is a device, used by the market as a means of assessing a bond's value
- A bond's yield, <u>not its price</u>, is a measure of its value
    - Price is today's present value of coupon flows, discounted at an appropriate rate
    - <u>You cannot compare bonds on the basis of their price</u>, because the price reflects the size of the bond coupon
- Yield is a proxy for the bond's rate of return
- If (1) you hold the bond to maturity, and (2) you re-invest coupon flows at the same rate, then your realized rate of return will equal the yield
- <u>**Note that this almost never happens**</u>**!** Because it is very unlikely that you will be able to reinvest all the coupon flows at the same rate throughout the life of the bond

Bond Market Value

With most assets, Market Value = Price x Number of Units Owned.

Example: Gold price = $1,000 / ounce. I purchase 5oz → **Mkt Value = $5,000 (= 1000 x 5)**

In general, it's safe to trust "units": Shares, ounces, bushels, barrels, etc

With bonds, pretend that the "units" are physical hundred dollar bills.

"Number of $100 par amounts", with "Price" quoted per $100 par amount

Example:

I purchase $5 Million par amount of a bond whose price is $101.00

Market Value = $5,000,000 * 101/100 = $5.05 Million

In general: Bond Market Value = Price x Quoted Par Amount / 100

Market Value of Bond Portfolio

* "[=]" means "has units of"

Bond Portfolio

Par Amt ($MM)	Bond	Price	Market Value ($MM)
3.00	UST 5yr	99.35	2.98
6.00	UST 10yr	99.19	5.95
-4.00	UST 30yr	92.89	-3.72
			5.22

$$= \frac{\text{Par\_Amt} \times \text{Price}}{100}$$

Total **Market Value** of bond portfolio

$\text{Par\_Amt}\ [=]\ \$$: The amount of money to be paid upon maturity.

$\text{Price}\ [=]\ \frac{\$\ \text{now}}{\$100\ \text{at maturity}}$: The current dollar amount that markets are willing to pay today for $100 worth of the specified bond. Changes constantly as bonds are traded. In this example, $99.35 will buy you one hundred dollar bill's worth of UST 5yr bonds.

$\text{Market Value}\ [=]\ \$$: If the owner fully **exited** (in other words ***unwound***, or ***liquidated***) each position, this is how much money the owner would receive/pay in todays' market.

Market Value of Bond Portfolio

Bond Portfolio

Par Amt ($MM)	Bond	Price	Market Value ($MM)
3.00	UST 5yr	99.35	2.98
6.00	UST 10yr	99.19	5.95
-4.00	UST 30yr	92.89	-3.72
			5.22

Question:
What does the "-4.00" par amount mean?

Answer: Don't forget that bonds can be **bought** or **issued**/**shorted**

- If you **buy** a bond, you're due to **receive** money
- If you **issue** or **short sell** a bond, you **owe** money

So, whoever owns this portfolio must have ***shorted*** the 30yr USTs (because they're not the US Govt, so they couldn't have issued). Maybe they're working a term structure strategy. Google "**bonds yield spread strategy**" to learn more if you're interested ☺

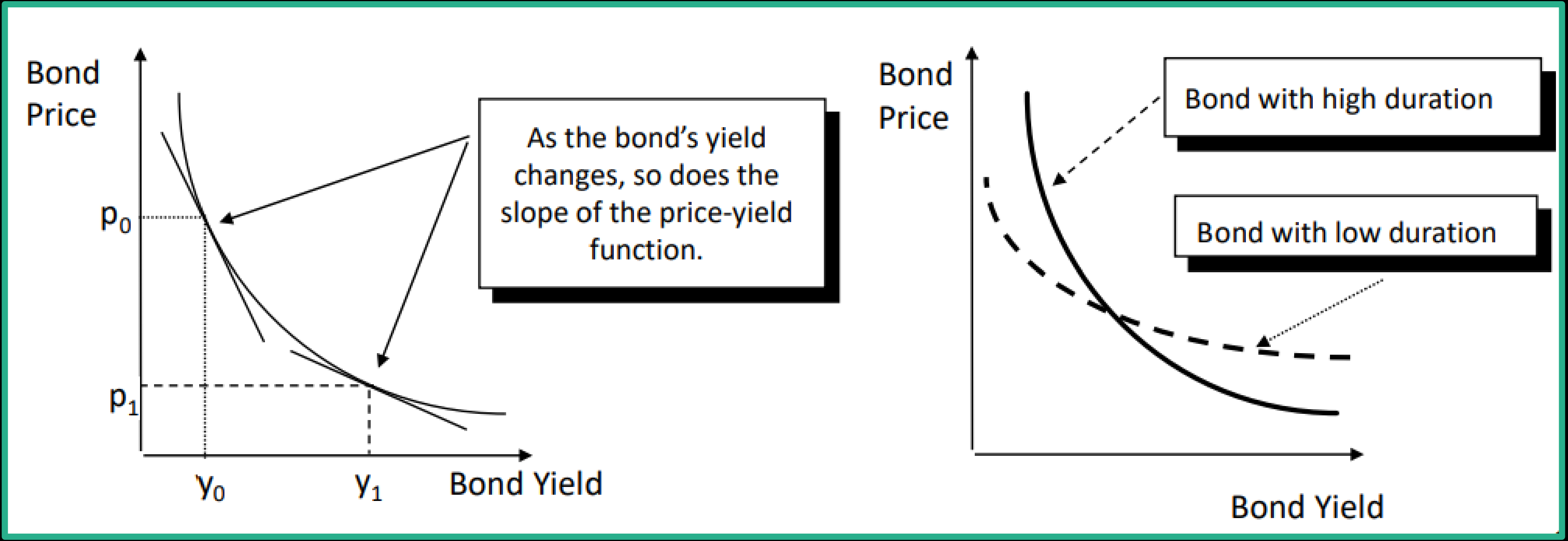
Duration

The bond's price sensitivity to changes in interest rates.

Higher duration → more price change for a given interest rate change

Duration at any given yield: estimated as the slope of the price-yield function.

$$\text{duration} = \frac{\partial P}{\partial y}$$ (note that DV01 is always negative)



Using DV01

DV01 is the approximate bond price change for a 1% change in yield.
→ also called the *dollar duration*

Example:

The 10yr UST has the following characteristics:

Price	=	102.68
Yield	=	5.81%
DV01	=	7.572

Now bond yield goes up by 20bp to 6.01%. What is the new price?

Price change = - (20/100) * 7.572
= - 1.514

It's up to you to remember that price & yield move in opposite directions: when price goes up, yield goes down & vice versa

New Price = 102.68 - 1.514
= $\boxed{101.17}$

Why use Duration when we can reprice a bond accurately?

1. Estimating market value changes

2. Weighted average portfolio duration provides insight into how an entire portfolio's value will change, based on re-pricing just benchmark bond(s)

Using Duration to Calculate Market Value Changes

Duration: a measure of the price sensitivity of a bond for a given change in yield

DV01 = change in market value of $100 par amount for a 1% change in yield

Example:

The 5yr UST has the following characteristics:

Price	102.68
DV01	3.368

Question: why are we multiplying by 10,000 here (after all, the 'par' or 'face' value that we own is 1,000,000, not 10,000)?

What is the change in market value of $1MM par amount (or face value) of the 5yr US Treasury for a 15bp increase in yield?

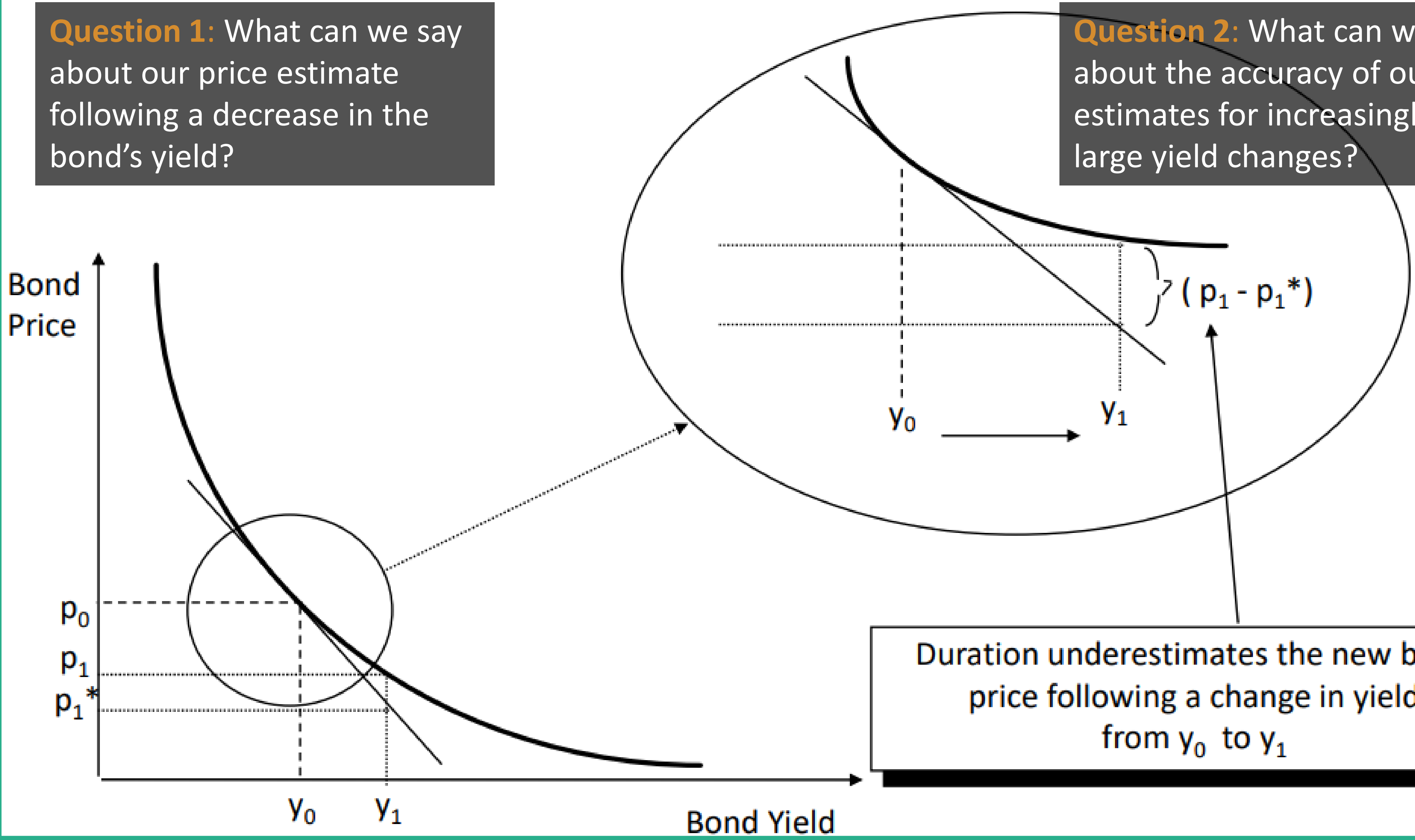
Hence, the change in market value of $1MM for a 15bp change in yield :

= **3.368 *(15/100) * 10,000 = $5,052**

Convexity

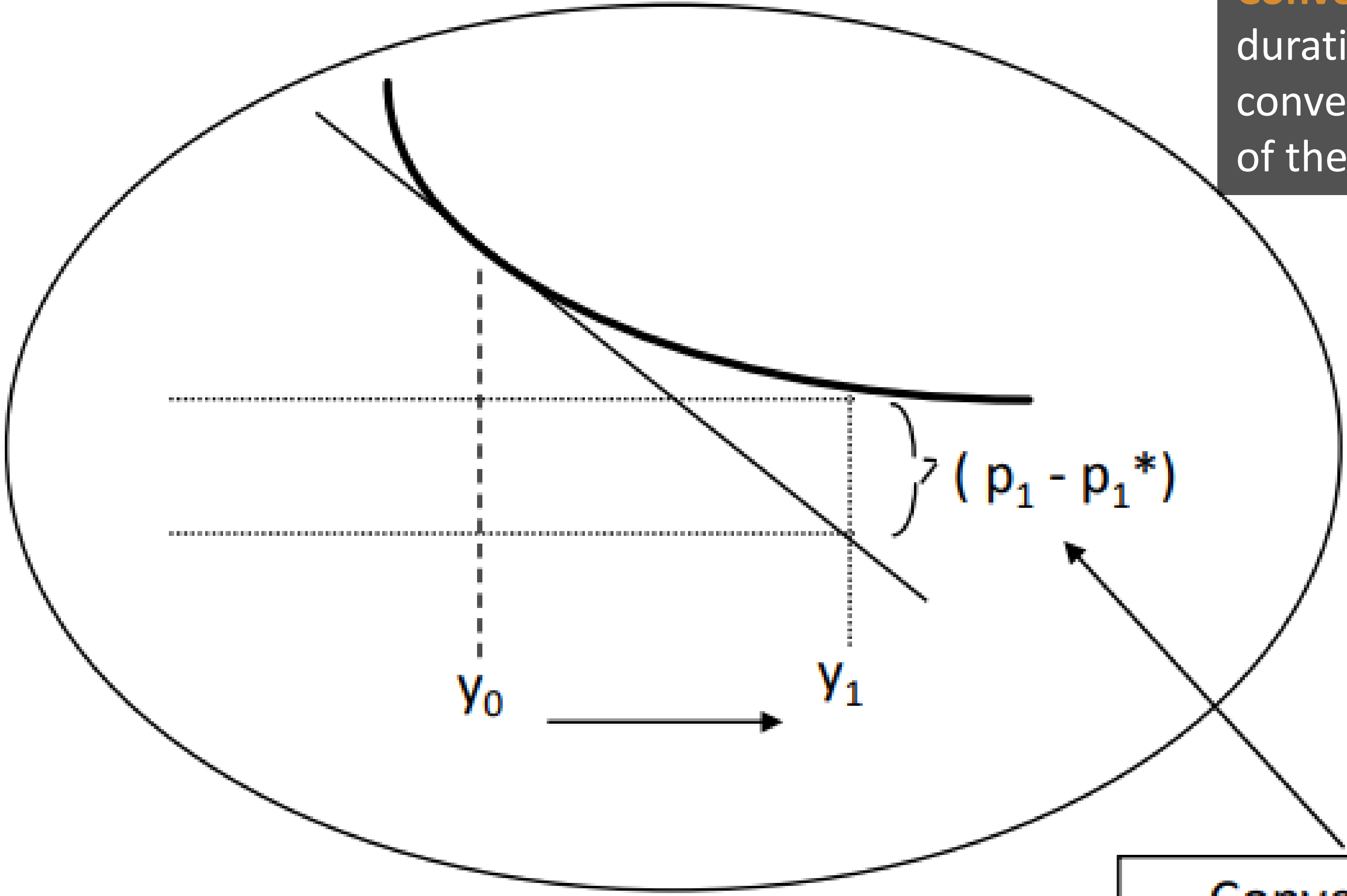
Question 1: What can we say about our price estimate following a decrease in the bond's yield?

Question 2: What can we say about the accuracy of our estimates for increasingly large yield changes?



Duration underestimates the new bond price following a change in yield from y_0 to y_1

Improving on the Duration Estimate: Convexity



Convexity: a first approximation to the duration measurement error. A bond's convexity is a measure of the "curved-ness" of the price/yield function

Question: Why is convexity **always added** to the price estimate, regardless of whether the yield change is positive or negative?

Convexity estimates the distance between p_1 and p_1^*

Using Convexity

Example:

The 10yr UST has the following characteristics:

Price	=	102.68
Yield	=	5.81%
DV01	=	7.572
Convexity	=	0.822

Now bond yield increases 20 bp to 6.01%. What is the new price?

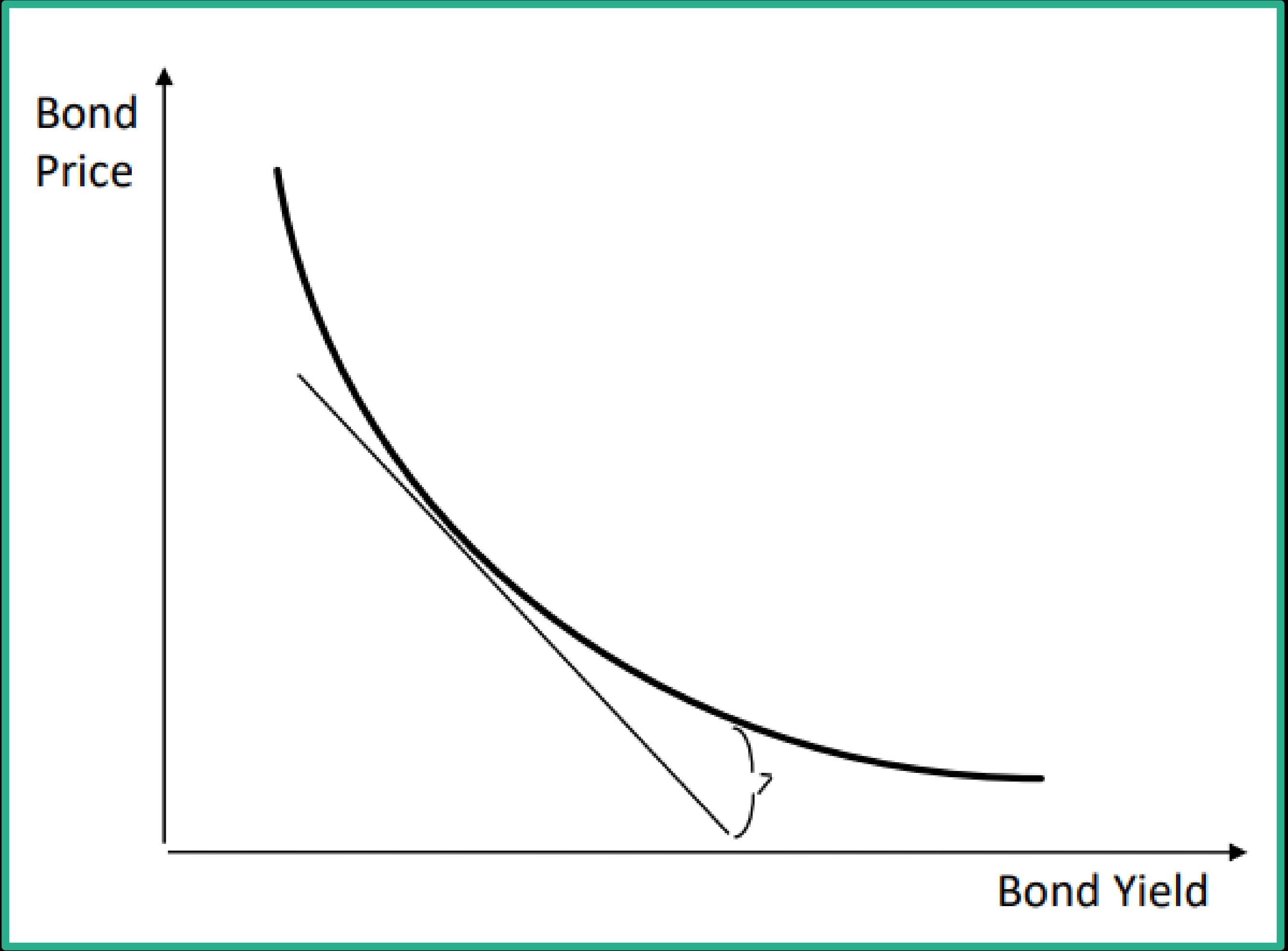
Price change from DV01 and convexity $= -(20/100) * 7.572 + \frac{1}{2}(20/100)^2 * 0.822$
$= -1.514 + 0.0164 = -1.498$

The duration estimate alone underpriced the new value of the bond by 0.01

New Price $= 102.68 - 1.498$
$= \boxed{101.18}$

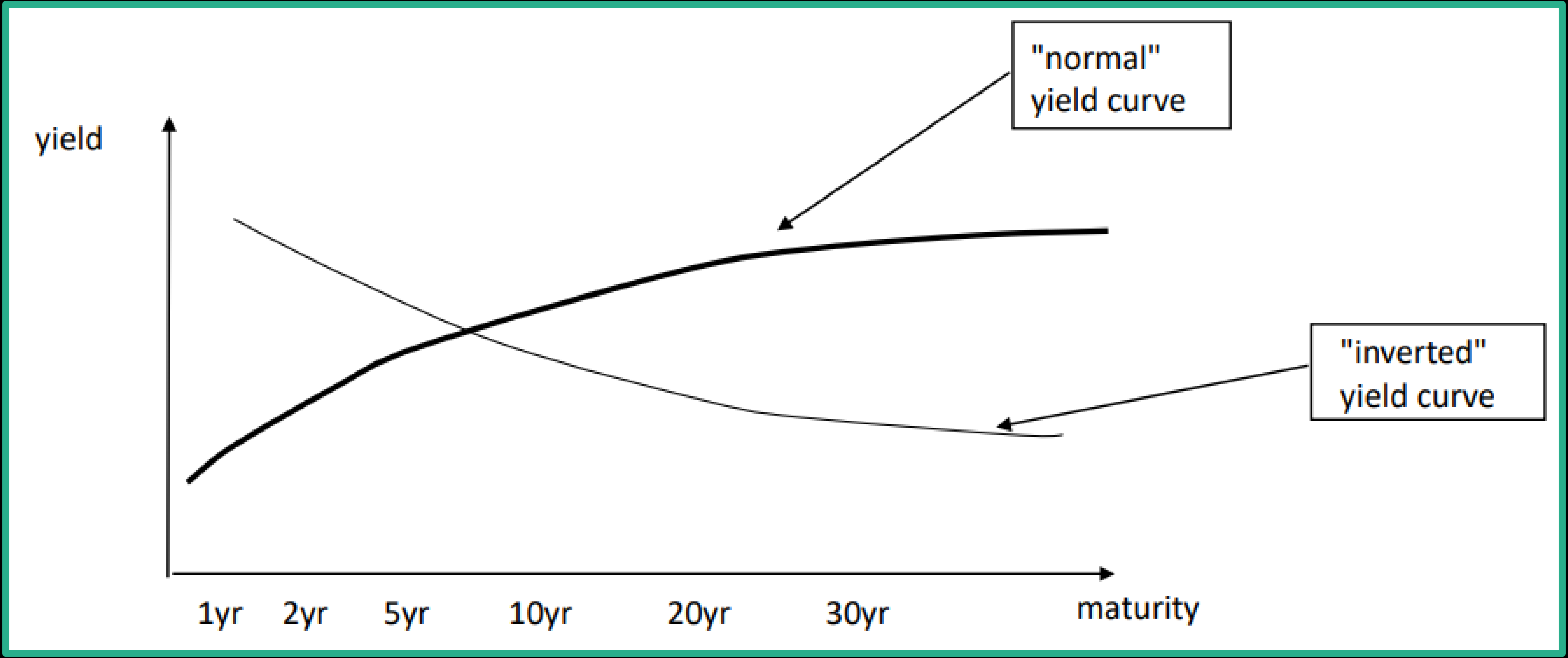
Convexity is the coefficient of the 2nd term of the Taylor series expansion of the Price-Yield function. DV01 is the first term in the same series

Convexity



- For equal maturity bonds, zeros have the most convexity.
- Doubling duration more than doubles convexity.
- For a given bond, as yield increases, convexity decreases
- Greater change in yield → greater convexity correction

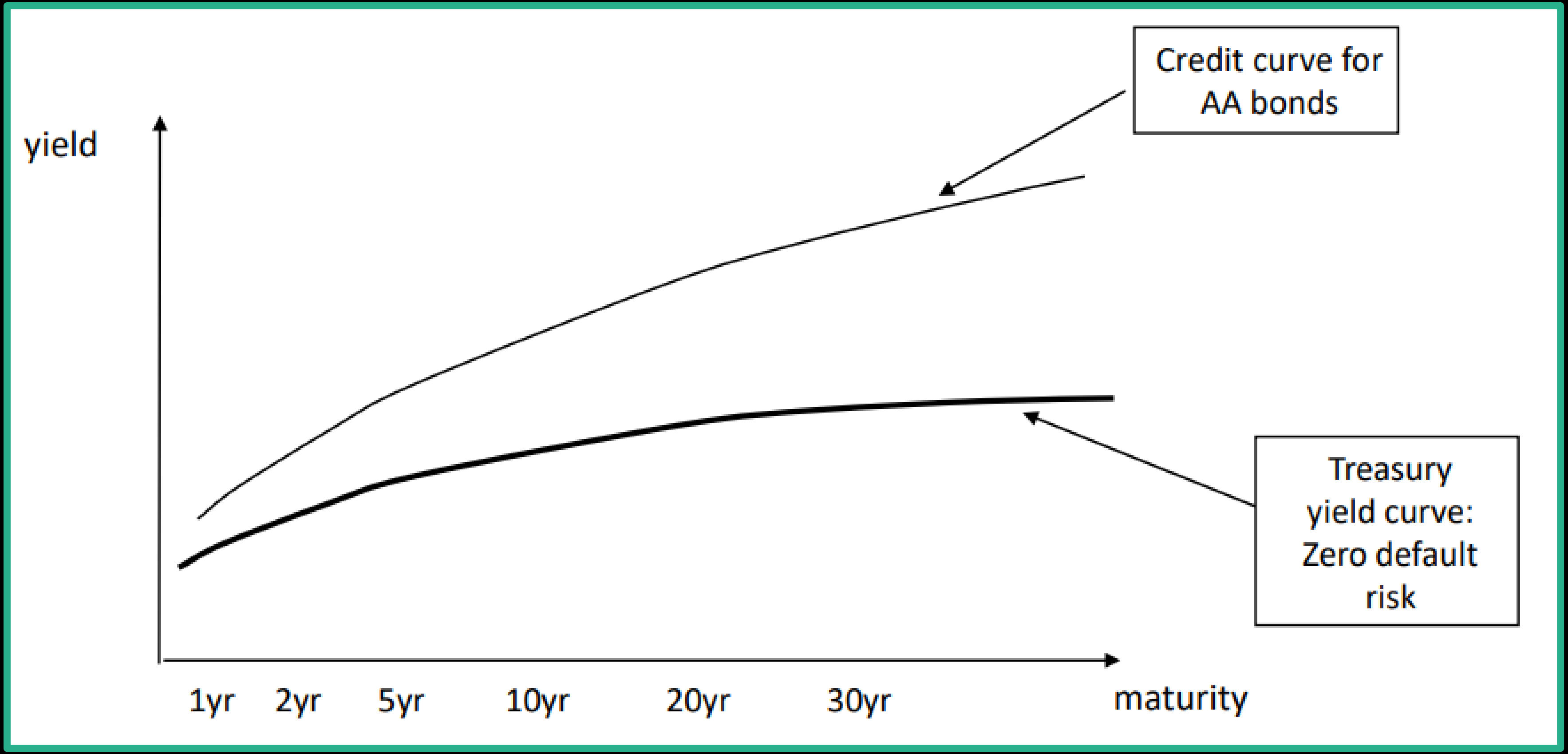
Term Structure of Interest Rates



- Relationship between the yield and the maturity of bonds in the US Treasury market
- No default risk
- Most liquid bond market

What's the yield curve look like today?

Credit Curves



- Spread between Tsy curve and credit curve is greater as maturity increases
- Curves for different credits exist
- Credit curves tend to “tighten” (get closer to the Treasury yield curve) in strong market environments, and widen in recessions (higher risk of default)

Bond Portfolio Risk Management

Bond Portfolio

Par Amt ($MM)	Bond	Price	Dollar Duration
3.0	UST 5yr	99.35	4.144
6.0	UST 10yr	99.19	7.359
-4.0	UST 30yr	92.89	12.896

- **What is the market value of this portfolio?**
- **What is the portfolio’s interest rate risk and how do I measure it?**

What's the Portfolio's Interest Rate Risk?

Par Amt ($MM)	Bond	Price	Dollar Duration	Market Value ($MM)	DV01 RISK
3	UST 5yr	99.35	4.144	2.98	124,320
6	UST 10yr	99.19	7.359	5.95	441,540
-4	UST 30yr	92.89	12.896	-3.72	-515,840
				5.22	**50,020**

Weighted Average Duration

- **Measured using Weighted Average Duration**
- **Weighted Average Duration shows change in Market Value of the portfolio for approximately 1% change in interest rates**
  - What assumption does this make about the changing yield curve?

Asset Pricing and Risk Management

FINTECH 522

Jake Vestal

Class 4:

Equities

Use the Teams Channels!

- Keep 522 separate from 533
- **Link is in Sakai Overview page, also here:**
  - https://teams.microsoft.com/l/channel/19%3au7BRwUXtx9CjLSxDAaMwgPmXgzbotB9ZyapGF6tgytc1%40thread.tacv2/General?groupId=248729ba-9b05-47c7-92d3-0117307b9e65&tenantId=cb72c54e-4a31-4d9e-b14a-1ea36dfac94c

Common Stock

Stockholders are owners of the firm and the residual claimant

- Stockholders have the right to:
  - Vote at company meetings
  - Dividends and other distributions
  - Sell their shares
- Stockholders benefit in three ways:
  - **Dividends** – periodic payments
  - **Capital gains** – appreciation in value
  - **Control** – have a say in company decisions
- Stock is issued by public corporations
  - To finance investments
  - To acquire other companies
  - To repurchase debt

Stock Transactions

Buy

"**Long**" **position**: makes money if stock increases in value

Sell

Liquidity needs: you need cash instead of stock for some reason

- Want to finance a more appealing investment
- Part of an algorithmic strategy
- Expect stock to decline in value

- "**Short**" **position**: sell a stock that you don't own, meaning that you *owe* or are *short* some number of shares of that stock.

Life Cycle of a Short Stock Position

Say you wish to short a stock XYZ.

1) **Within your trade execution system, sell more shares of XYZ than you own** (i.e., **ENTER** the short position)
2) **Your brokerage then finds that number of XYZ shares owned by someone else, sells them, and puts the cash from the sale into your account!**
3) **While the short position is open:**
   a) You may use the cash to do something else
   b) **You must pay interest on the short position** while it's open
   c) Your execution system tells you that you own a **negative** number of XYZ shares.
4) **Give the stock back to its owner**
   - i.e., **exit** the short position
   - i.e., **buy to cover**
   - You do this by issuing an order to buy the stock in your execution system.

Short Fees

You might think that the expected return $\langle E_{r,short} \rangle$ of shorting a stock equals the negative of the expected return of buying the stock long: $\langle E_{r,short} \rangle = -\langle E_{r,long} \rangle$, **but this is not the case**.

Two reasons:

1. A ***short fee*** is applied to your short position while it's open
2. Your profits from shorting are *always* considered ***short-term capital gains*** and therefore may be taxed at a higher rate than your long positions.

The thing to remember is:

For the same stock, $|\langle E_{r,short} \rangle|$ **is always less than** $|\langle E_{r,long!} \rangle|$!!

Short Fees: Example

Let's say you want to short Apple (NYSE: AAPL).

Your Execution System Says:

Fin Instr...	Shortable Action	Shrtbl Shrs Quantity	Fee rate TIF
AAPL		202,603,103	0.25%

Short Fee

Financial Instrument

Is this instrument shortable? (YES/NO)

How many shares are available for shorting?

In Apple's case: a lot. For smaller, less common stocks, this number might be lower than the amount you'd like to short.

Short Fee

- As always, **read the fine print** to be sure you understand this!
- It's common for the short fee to be *applied daily* – every calendar day, *not* every trading day – to your short position, and charged to your account *monthly*.

Short Fees: Example

Let's say you want to short Apple (NYSE: AAPL).

Your Execution System Says:

Fin Instr...	Shortable Action	Shrtbl Shrs Quantity	Fee rate TIF
AAPL		202,603,103	0.25%

Short Fee

Short Fee: Fine Print

- Sometimes hard to find! May have to have a support chat with your brokerage to verify your understanding.
- **Remember**: From your brokerage's perspective, **YOU ARE THE CUSTOMER.** You have my express permission to be *as irritating as you need to be* (in a respectful way) to customer support until you're comfortable you understand how this works!!! ☺

In Interactive Brokers' (NYSE: IBKR) case:

Short fee is represented as an annual, continuously compounded rate having a 360-day accounting basis.

→ But, for example, maybe we need the rate on a *trading year* basis:

So convert the two CCRs

$$\text{Short Fee, trading year} = \frac{360}{252}(\text{Short Fee, acct. year}) = \frac{360}{252}0.25\% = \boxed{0.36\%/\text{trd\_yr}}$$

Brokerages must overcome several challenges to offer shorting as a service.

- In order to execute your short sale, **your brokerage must be able to obtain enough shares of the stock** you want to short, either from one of the brokerage's other clients or from clients at other brokerages who collaborate to offer this service.

- Participating in a brokerage's shorting program – i.e., making shares available for shorting – is **optional** for traders. **Why would they want to do this**?

- Because **the original owner of the stock receives a small premium** in exchange for making the shares available for shorting! (in finance, *nothing* is free)

- **The original owner sees no difference in their stock positions** while the shares are being lent; e.g., if you owned 100 shares of AAPL and 50 were lent out to allow someone else to short, then you would *still* be able to sell your 100 shares at any time!

The Stock Market

- All US and global equities that are available to trade
- Value of all global equities (stocks) that are publicly tradeable is about $70 trillion
  - The US is the largest single market…
  - …but more than 50% of this value is outside the US
- Stock prices are affected by:
  - Economic environment
  - Company fundamentals (sales, earnings, news, etc.)
  - Market psychology (herd behavior, sentiment, **over** and **underreaction**)
- Stock exchanges are forums for publicly traded equities (i.e. where you can buy and sell)
  - US: NYSE, NASDAQ
  - Japan: NIKKEI
  - Hong Kong: Hang Seng
- Stock exchanges are becoming increasingly virtual (on-line trading)

Stock Indices

- Stock market indices track the "average" returns on a bundle of stocks in a particular market (geographical, e.g. US, UK, Germany) or market segments (e.g. small companies, specific industries, or other categorizations)
- How are the stocks in the index "weighted"?
  - Price weighted (DJIA): equivalent to holding one share of each company in the index
  - Market-value weighted (S&P500, NASDAQ): weighted on the basis of Market Capitalization (larger firms more heavily weighted in the index)

Financial Statement Analysis

- Three main financial statements
  - Balance Sheet
    - a snapshot at a moment in time
    - What the company owns (assets)
    - What it owes to others (liabilities)
    - Remainder: accrues to owners (equity)
  - Income Statement – a report of flows
    - How much money the company makes (revenue)
    - What are the costs of producing (expenses)
    - The difference between the two (profit)
  - Cash Flow Statement – also a report of flows
    - What cash has come in or out

Balance Sheet

Assets

($mm)	2006
Cash & Equivalents	246.6
Receivables, Net	212.0
Inventories	88.0
Other Current Assets	12.0
Current Assets	558.6
PP&E, Net	755.8
Other Assets	5.0
Total Assets	1,319.4

- Note distinction between “current assets” (some of which could be liquidated immediately) and longer term, fixed & intangible assets. Why is this distinction important?
- The Asset side of the balance sheet reflects the operations of the firm

Liabilities & Equity

($mm)	2006
Payables	148.9
Short-Term Borrowings	10.0
Other Current Liabilities	16.0
Current Liabilities	174.9
Long Term Debt	112.0
Other Liabilities	55.0
Stockholders' Equity	977.5
Total Liabilities and Equity	1,319.4

- What type of financial instrument might be included in “long-term debt”?
- Note that these are “book” values, and may not equal the firm’s “market” value
- Liabilities and equity reflect the financing aspect of the firm

Balance Sheet

Its usefulness

- Gives information about the liquidity of a company
- Tells us how "asset intensive" a company is or how many assets (machines, buildings, equipment) are necessary
- Tells us about the ability of a company to meet its long-term fixed expenses and to accomplish long-term growth

Its limitations

- Assets are recorded at historical cost rather than at market value (what you paid, not what it's "worth" or the price at which you could sell the asset)
- Resources such as employee skills and reputation are NOT recorded on balance sheet

Income & Cash Flow Statements

Revenues	1,220.30		Net Income	76.16
less COGS	756.60		D&A addback	37.80
Less S,G&A expense	303.20			
EBITDA	160.50		Change in working Capital	5.50
less D&A	37.80		*Less* Capex	(45.80)
EBIT	122.70			
Less Interest Expense	7.30		Free Cash Flow	73.66
Pre-tax Income	115.40			
Taxes	39.24			
Net Income	76.16			

Income Statement

Its usefulness

- Summarizes sales and profits and losses (P&L) over a period of time
- Lets us look at changes in key line items and ratios across time to see whether operations have been changing and in what direction

Its limitations

- Difficult to compare some ratios for companies in different industries
- Management teams have lots of options for accounting practices within the income statement
- Revenues reported don’t always equal cash collected, and expenses reported aren’t always equal to cash paid, so net income is not the same as cashflow for the period

Cash Flow Statement

- Free cash flow is what a company has left over at the end of the year - or quarter - after paying all its employees' salaries, its bills, its interest on debt, and its taxes, and after making capital expenditures to expand the business
- Investors often refer to this as the "cash" that the company is producing. The company can decide what to do with the cash (expand, pay a dividend, pay down debt, etc).

Where to Get this Info?

- Use EDGAR: https://www.sec.gov/edgar/search-and-access
- US-listed equities & financial statements in an easily scrapable & downloadable format.

Company Valuation Comparables

- Financial Analysts often use ratios to evaluate firms relative to their peers. Typical Ratios include
  - **EBITDA Multiple**: Enterprise Value / EBITDA
  - **Net Income Multiple**: Equity Value / Net Income
    - Equity value (or Market Capitalization, or MCAP) is calculated as Price per share x number of shares outstanding
    - Net Income / number of shares = Earnings per share (EPS)
    - Hence the Net Income multiple = Price/Earnings ratio (P/EPS)
- Why use ratios rather than absolute values?
- Why use ratios when we can use discounted cash flows?

* Enterprise Value = Equity Value + Net Debt. Net debt = short term debt + long term debt – cash + minority interest preferred stock Think of Enterprise Value as the value of the firm that is of interest to both owners & lenders (e.g. bond holders)

Price/Earnings Ratio

Comparison Between Firms:

1. **Firm A** has a share price of \$66. **Firm B** has a share price of \$75. Is **Firm B** more valuable? Or should we buy **Firm A** because it is "cheaper"?
2. **Firm A** has EPS of \$3.30 this year. **Firm B** has EPS of \$3.00. Is **Firm A** more valuable because its EPS is higher?
3. **Firm A** has a P/E ratio of 20. **Firm B** has a P/E ratio of 25. Should we buy **Firm B** because it has a higher P/E ratio?

EBITDA Multiple

- Financial analysts use EBITDA multiples more than any other multiple, as a way to compare firms' performance
- **What are the benefits of using EBITDA multiples rather than P/E multiples?**
  - Enterprise value encompasses the overall value of the firm to all stakeholders – both equity and bond holders
  - EBITDA captures the operations of the firm – its business-related profit – without taking into account the financing aspect. Thus it is better for comparing two firms in the same industry that might have very different financing structures

Valuation Using Comparables

Company	Ticker	Share Price (US$) [1]	Fully Diluted Shares (FY10) [2]	Market Value 3/3/11	Enterprise Value [1] 3/3/11	EBITDA [3] 2011E	EPS [3] 2011E
Caribou Coffee	CBOU	9.81	20.1	197.6	171.3	28.0	0.61
JM Smucker	SJM	69.70	118.3	8,247.6	8,780.0	1,076.0	4.66
Starbucks	SBUX	33.01	746.0	24,625.5	22,530.0	2,184.0	1.49
Peet's Coffee & Tea	PEET	47.87	13.1	625.2	552.0	50.7	1.59
Sara Lee	SLE	17.10	622.4	10,643.7	11,100.0	1,314.0	0.87
SodaStream Intl	SODA	40.65	6.3	255.3	298.9	30.0	0.90
Green Mountain Coffee	**GMCR**	**42.01**	**141.0**	**5,948.6**	**6,800.0**	**431.0**	**1.20**

Company	Ticker	P/E Ratio 2011E	EBITDA Multiple 2011E
Caribou Coffee	CBOU	16.1	6.1
JM Smucker	SJM	15.0	8.2
Starbucks	SBUX	22.2	10.3
Peet's Coffee & Tea	PEET	30.1	10.9
Sara Lee	SLE	19.7	8.4
SodaStream Internatior	SODA	45.2	10.0
Mean		**24.7**	**9.0**
Median		**20.9**	**9.2**
High		**45.2**	**10.9**
Low		**15.0**	**6.1**
Green Mountain Coffee	**GMCR**	**35.0**	**15.8**

- How do Green Mountain's multiples look, relative to their competitors?
- Does it look like a good equity investment? What about as a potential acquisition target financing structures?

[1]Yahoo finance
[2]Company 10K
[3]Thomson Reuters estimates

Valuation Using P/E Multiples

- Compare comparables median P/E with GMCR P/E (why use median rather than mean?)
  - Median comparables P/E is 20.9
  - GMCR P/E (current share price / 2011 est. earnings) = 35
  - Why is GMCR P/E so much higher?
- Equivalently: GMCR market value relative to MCAP implied by peers
  - GMCR 2011 estimated EPS is 1.20, with 141.0 Mill shares outstanding: estimated total earnings = $169.2 Mill
  - GMCR MCAP = 20.9 x 169.2 = $3,536 (using comparables P/E)
  - GMCR actual MCAP (current share price x # shares) = $5,948.6
  - Thus market share price implies higher value for GMCR
- Is GMCR overvalued?

Valuation Using EBITDA Multiples

EBITDA Multiples comparison

- Median comparables EBITDA multiple is 9.2
- GMCR 2011 estimated EBITDA multiple is 15.8 (EV / 2011 estimated EBITDA)

Equivalently: Enterprise Value implied by peer multiple

- Implied GMCR Enterprise Value using peer multiple: 9.2 x 431 = $3,968
- GMCR actual Enterprise Value = $6,800

- Is GMCR overvalued?
- Note that both EBITDA multiple and P/E ratio for GMCR are higher than peers' average. Would it be surprising for one of these multiples to be greater than peer average, while the other was lower?

Asset Pricing and Risk Management

FINTECH 522

Jake Vestal

Class 5:

Equities + Portfolio Theory

Quick Review: Financial Metrics

Equity Value (or Market Capitalization, or MCAP):

- MCAP = price per share * number of shares outstanding
- **Not all share are freely traded**; e.g., shares insiders **:** see "*float*"

EBITDA:

- Earnings before taxes, interest, depreciation, and amortization
- Comes from the income statement

Net Income:

- Revenue - {what it took for the firm to earn that revenue}
  - COGS, SG&A Expense, Interest Expense, Taxes

Net Debt:

- = short term debt + long term debt – cash + minority interest preferred stock

Enterprise Value:

- EV = Equity Value + Net Debt
- "How much would it cost to buy the firm, given its capital structure?"

Quick Review: Ratios

EBITDA Multiple:
- EV / EBITDA

Net Income Multiple:
- Equity Value / Net Income

Earnings Per Share (EPS):
- Net Income / # of shares

$$\text{Net Income Multiple} \equiv \frac{\text{Equity Value}}{\text{NI}} = \frac{\text{Price per Share} \times \#\ \text{Shares}}{\text{NI}} = \frac{\text{Price per Share}}{\text{EPS}}$$

Sometimes you want to **calculate** a ratio to gain information about a firm
Sometimes you want to **assume** a ratio to evaluate "what-ifs"

Analyzing Series of Returns: Equities

Continuously Compounded Rate of Return

"CCR"

$$\ln\left(\frac{P_2}{P_1}\right)$$

- Is the "rt" in $P_2 = P_1e^{rt}$
- If you don't believe me:

$$P_2 = P_1e^{rt} = P_1e^{\ln\left(\frac{P_2}{P_1}\right)} = \cancel{P_1}\left(\frac{P_2}{\cancel{P_1}}\right)$$

$$P_2 = P_2 \checkmark$$

- Also called the "Log Return"
- Has "units" of percent (%)
- Easy to convert
- Symmetric
- Plays well with lognormal distributions

A pretty decent blog post about why we use log returns can be found here: https://lucaslouca.com/Why-Use-Logarithmic-Returns-In-Time-Series-Modelling/#:~:text=Logarithmic%20returns%20are%20useful%20for,will%20cancel%20each%20other%20out.

OHLC(V) Data

For a specified time period (also called a **bar size**)
(day, month, hour, 5 minute, etc)

Open: First price at which an asset traded
High: Highest price that someone paid
Low: Lowest price that someone sold
Close: Last price at which an asset traded
Volume: The total number of units (stock, contracts, etc) that were traded

Adjusted Close (1 of 2)

DIVIDENDS

A company may choose to return excess profits to its shareholders in the form of additional stock, or cash. When/if this happens, some sources (e.g., Yahoo) deduct the value of the dividend from the closing share price.

Adjusted Close (2 of 2)

SPLITS

A company may want to change the number of shares issued (usually increase). For example, if a company issued a *two-for-one* split, the number of shares issued on the market would double… therefore, they'd be worth half as much as before the split.

Some sources incorporate this information into the **Adj Close** of the trading day before the split, by multiplying by the *split ratio* (2 in this example).

If you are trying to re-create what trades would have executed in the past…

…i.e., you're NOT looking for an overall metric of your return including splits and dividends…

NEVER use Adjusted Close, ALWAYS use actual price.

And don’t ever use Yahoo Finance data to make decisions with real money

For demonstration and debugging purposes though, it’s great.

Excel Breakout: Log Returns

Using Adjusted Close Prices

Analyzing a SERIES of returns

We've now got a spreadsheet open, displaying a series of returns in time.

It'd be nice to be able to summarize them so as to compare to other investments.

We will look at two (2) ways to do so

The Arithmetic Mean Rate of Return

(1 of 2)

A series of returns might look like this:

r = [1.3%/week, 3.0%/week, 2.3 %/week, 2.8 %/week, 1.9 %/week]

(**bold** means it's a time series of returns)

In this example, the Arithmetic Mean Rate of Return would be the simple average of the returns:

$$r = (1.3 + 3.0 + 2.3 + 2.8 + 1.9)/5$$

$$= \mathbf{2.3\%\ per\ week}$$

But Arithmetic Mean Rate of Return doesn't always tell the most accurate story…

r = [+25%/qtr, -20%/qtr, +25%/qtr, -20%/qtr]

The Arithmetic Mean Rate of Return is 2.5% per quarter!

Does this make sense??

qtr = "quarter"

Another way to look at it…

Start Here

No return yet

Earned zero dollars, $0

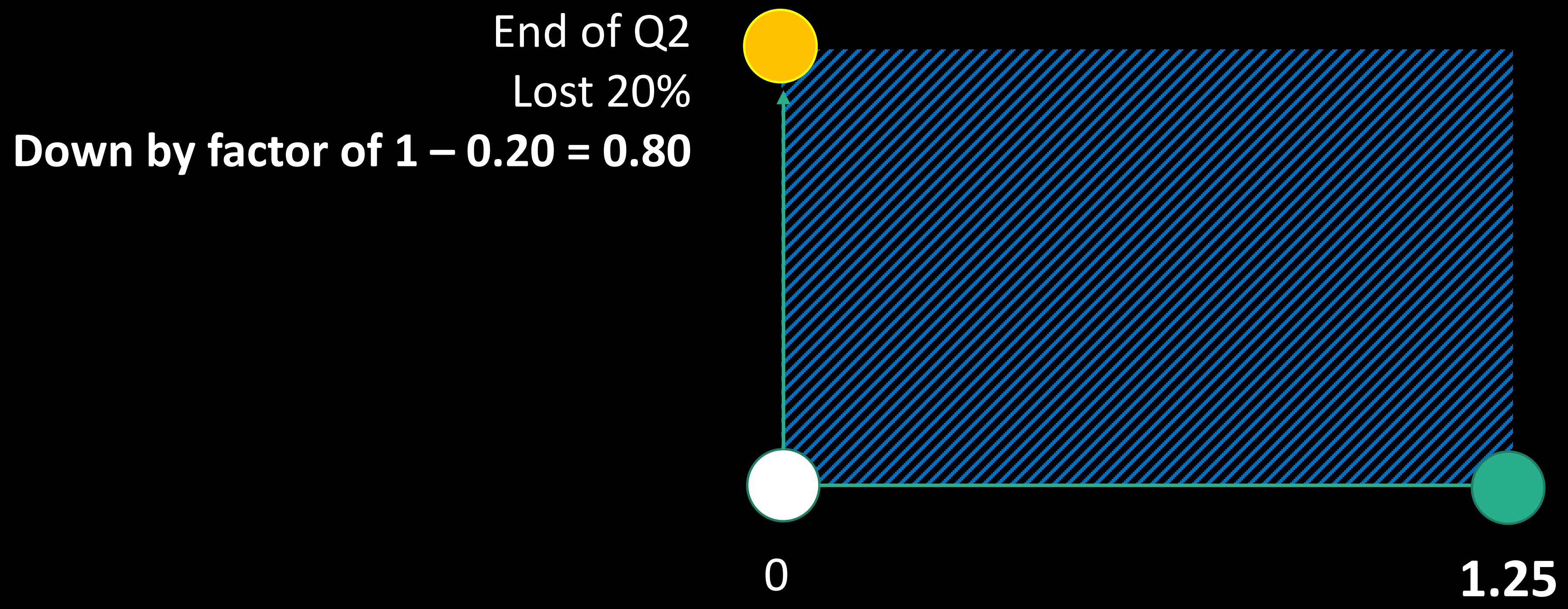
Another way to look at it…

$r_{n=1}$ = 25%/qtr

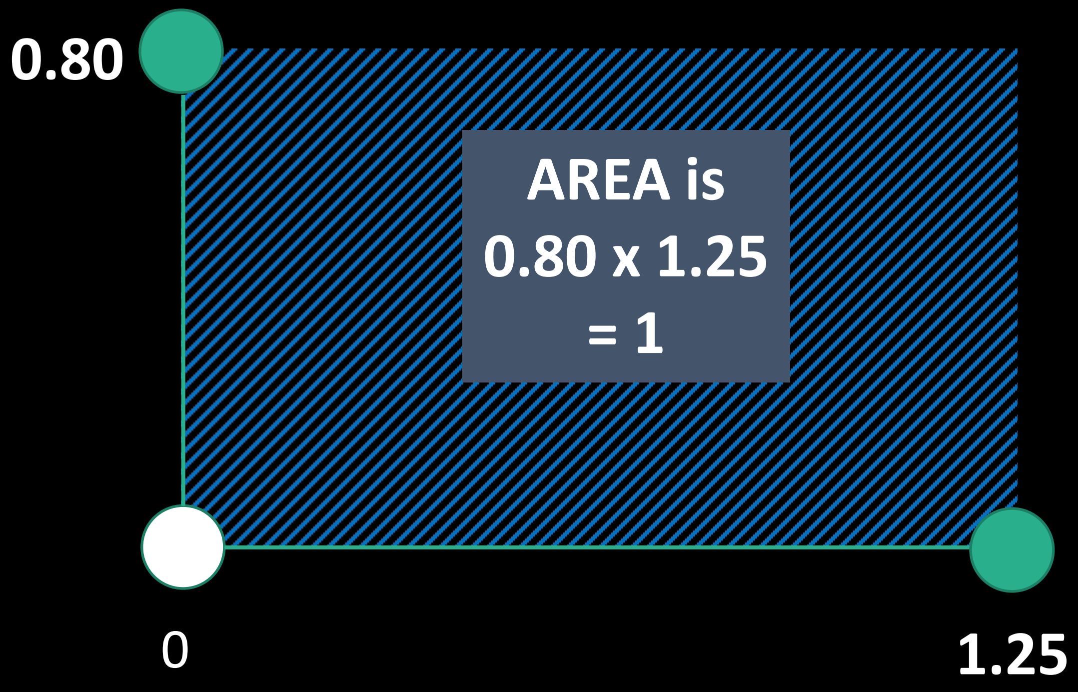
0

End of Q1
Earned +25
Up by factor of 1 + 0.25 = 1.25

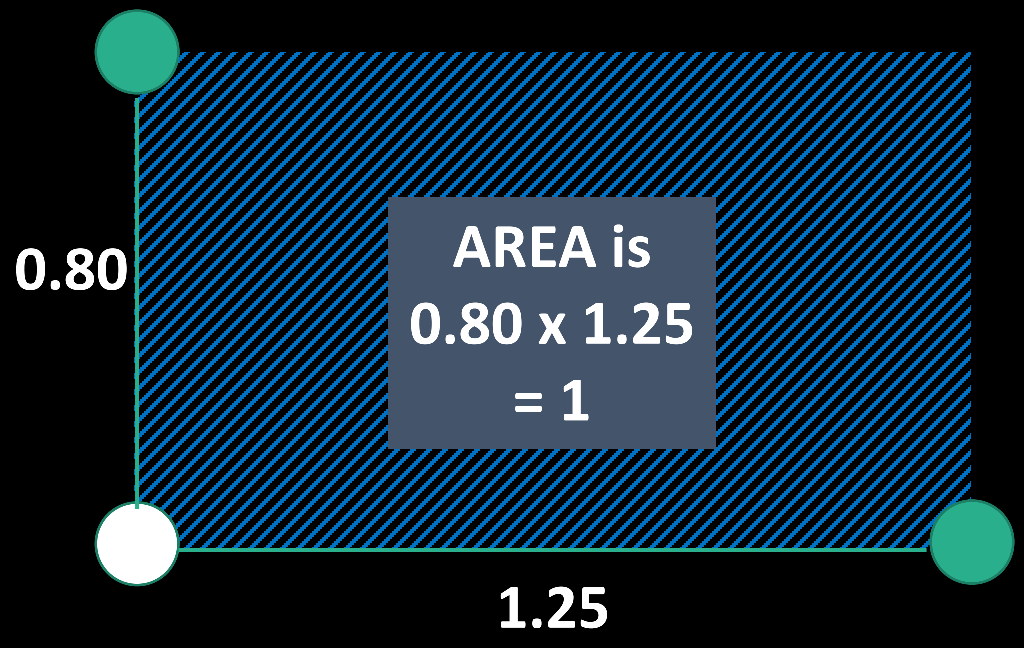
Another way to look at it…



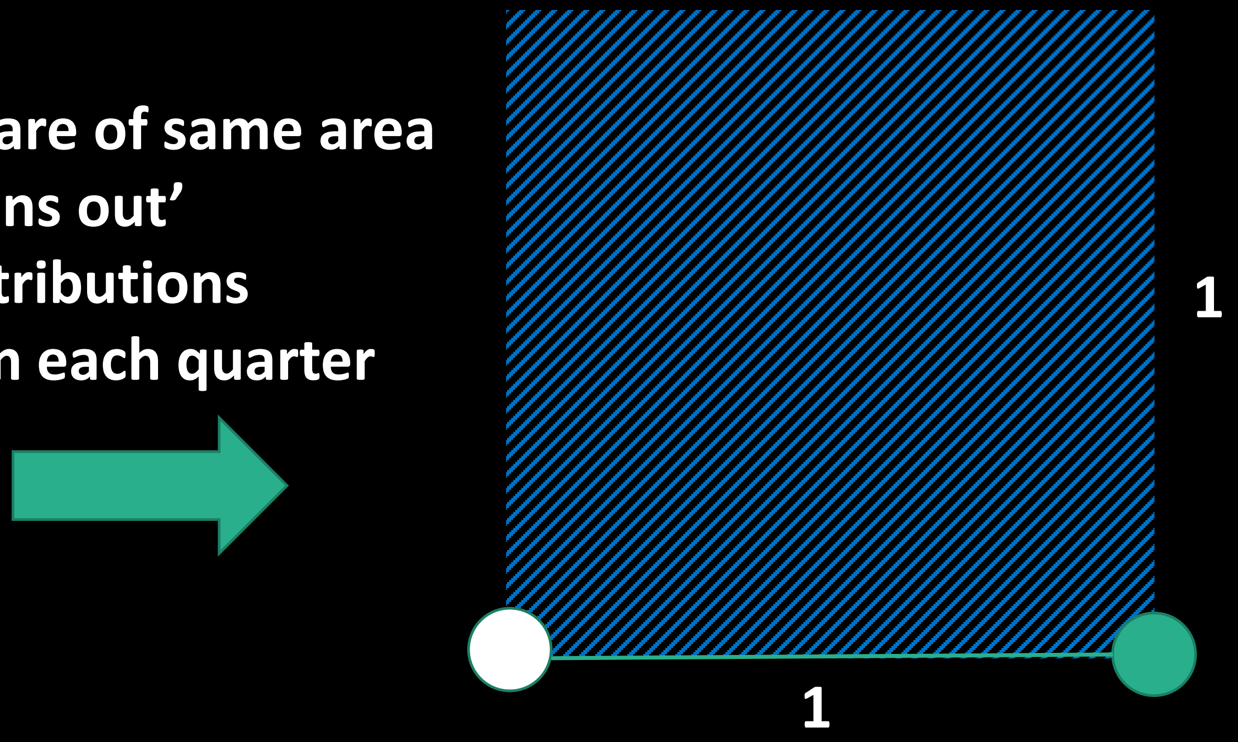
Another way to look at it…



Another way to look at it…



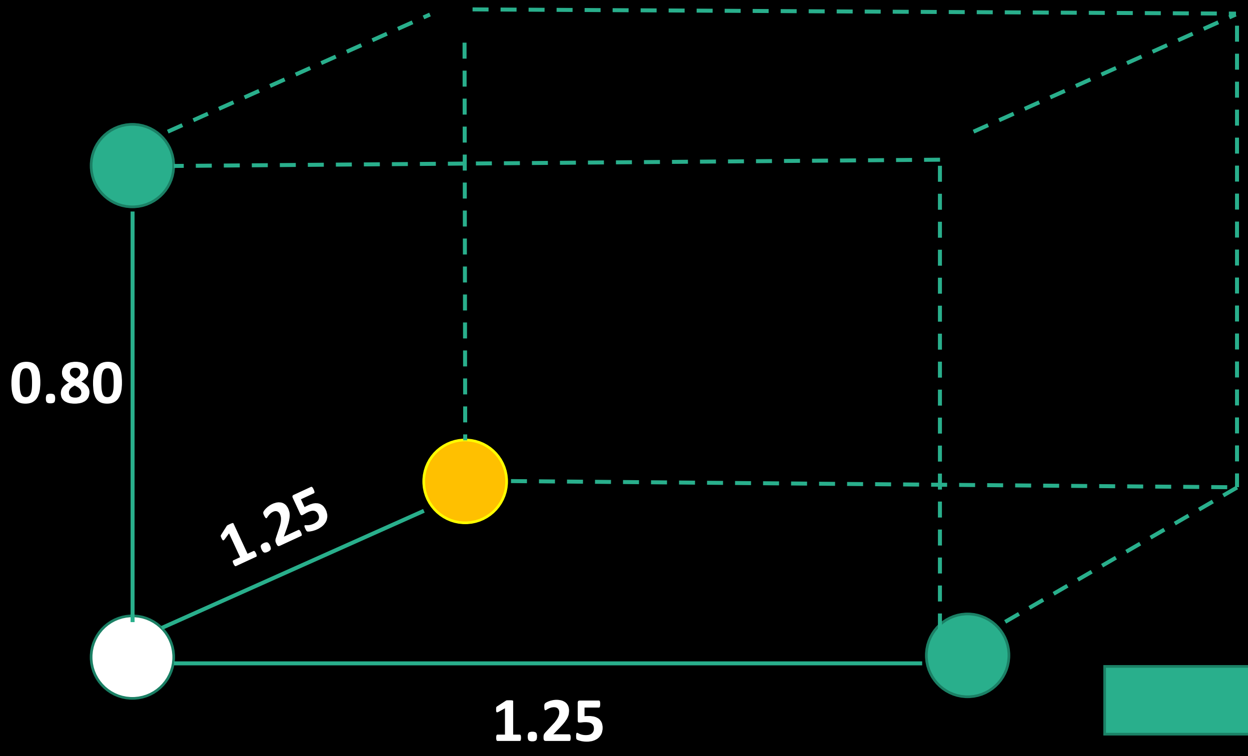
Square of same area 'evens out' contributions from each quarter



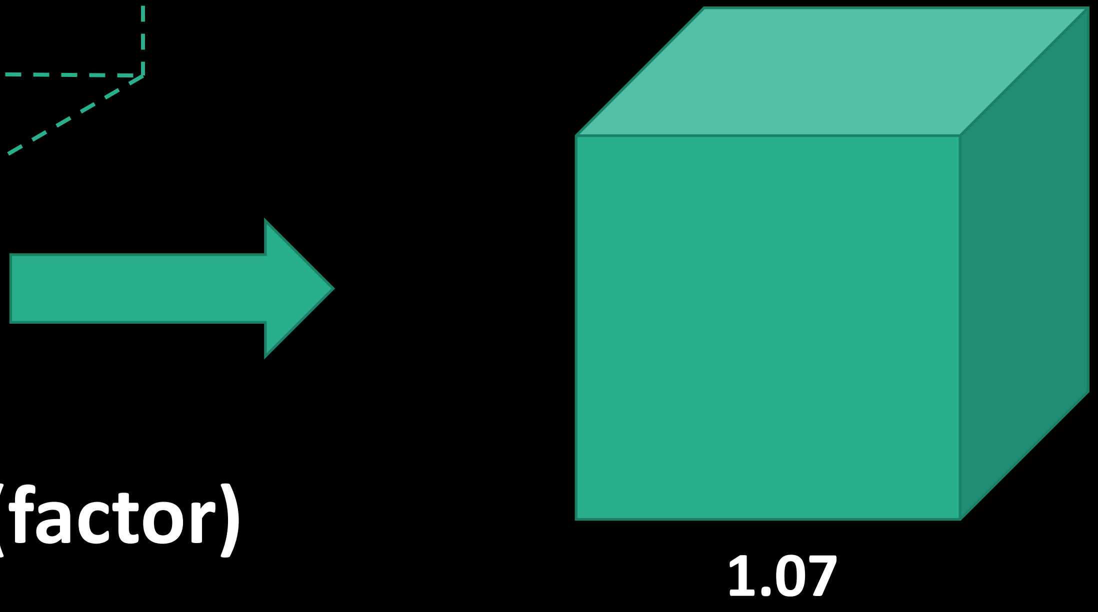
Length of a side of square = 1 (factor)

Subtract 1 to get %: 1 – 1 = 0

Add another dimension…



Even out sides,
Equivalent volume cube…



Length of a side of cube = 1.07 (factor)
Subtract 1 to get %: 1 – 1 = 1.07

…And so on.

By “averaging geometrically” in this way, we arrive at…

$$GMRR = \sqrt[n]{(1 + r_1)(1 + r_2)(1 + r_3)\ldots(1 + r_n)} - 1$$

The Geometric Mean Rate of Return!

Excel Breakout: analyze a series

For the stock, calculate:

- Average rate of return
- Geometric mean rate of return

A Word on Data

You will, of course, need to obtain data from somewhere. Here are a few sources you might consider

Data Subscription Services:

Two kinds:

Market Data (also called **Live** or **Real-Time**): streaming data updated constantly

Historical Data: Data from the past, only need to download once for a given date range

- **Refinitiv:**
  - Go to the Ford Library (Fuqua) site and find it in "databases"
  - Register for it
  - Has a great Python API and a standalone Workstation app
- **Siblis Research:** https://siblisresearch.com/
  - Provides several historical datasets in Excel format that you may find useful
- **Xignite:** https://www.xignite.com/product/historical-stock-prices/#/ProductOverview
  - Easy-to-use REST API for querying data from Excel, Python, R, etc.

Paper Traders:

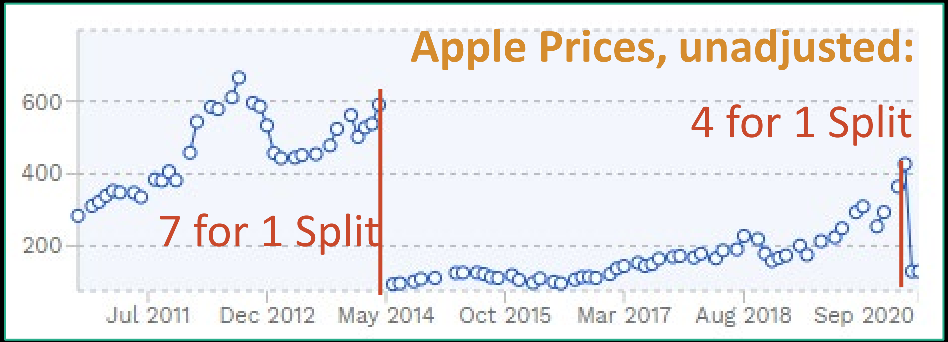
A **Paper Trading** account is a "sandbox account" that uses real market data (often 15 min delayed) and allows you to make trades using simulated money so that you can test out an algorithm before you run it live on real money. Most (if not all) of these services offer some kind of historical data.

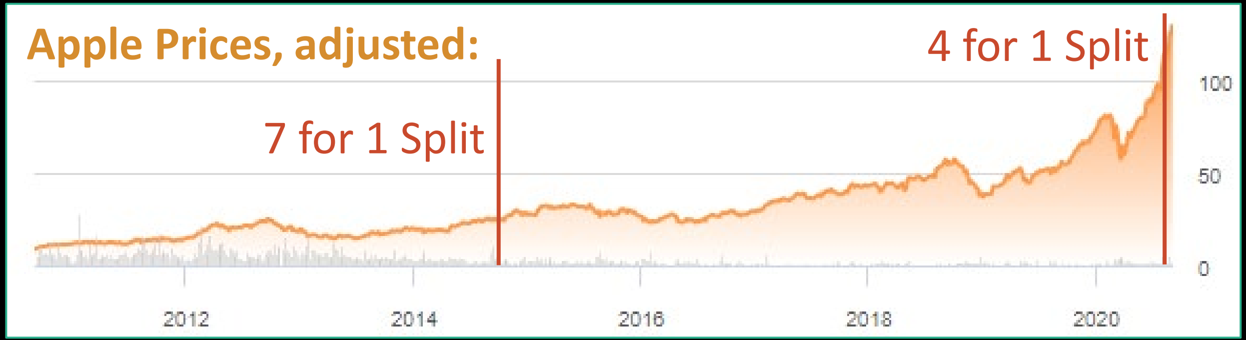
- **Interactive Brokers** (https://www.interactivebrokers.com/en/software/omnibrokers/topics/papertrader.htm)
- **TradeStation** (https://www.tradestation.com/platforms-and-tools/simulated-trading/)
- **NinjaTrader** (https://ninjatrader.com/Simulate)
- **WeBull** (https://www.webull.com/)
- **thinkorswim** (https://www.tdameritrade.com/tools-and-platforms/thinkorswim/desktop.page

A Word on Data 2

Whatever service you choose, be sure you have all the data you need and understand how it's presented

- **Prices:** You might find that you can implement a strategy via limit orders; therefore, you might not really need to pay for a real-time market data subscription (i.e., *not* 15-minute delayed prices)
- **Historical Prices:** Unadjusted is best for backtesting because:
  - You often care about whether or not a trade would have filled in the past – adjusted prices won't tell you this
  - The way data providers adjust prices for splits, dividends, and M&A might not be what you think or what you'd like
- **Dividends:** Make sure you can get data for past dividends. Future dividends, as they're announced, are nice to have
- **Splits:** You'll need data for splits:





- **STOCK TICKERS CAN CHANGE OVER TIME:** You might want historical prices for United Technologies Corp, a multinational industrial conglomerate traded under symbol UTX on the New York Stock Exchange (NYSE).
  - On 03 Apr 2020, UTX merged with Raytheon (RTN) to create a new company called Raytheon Technologies, traded as "RTX"
  - Merger immediately spun off Carrier Global Corp. (CARR) and Otis Worldwide Corp. (OTIS)
  - … but if you query your data provider for "UTX", you *might* get data up to the present day! Why???
    → It's because your data provider is somehow adjusting for the merger
  - Be sure you understand how your data provider handles this, and take it into account.

DISCLAIMER: Choosing making investment-related decisions is a very personal endeavor that must be decided by you, the investor, based on your own needs and personal financial situation. This slide (and any other information in this course) is provided for **informational purposes only** and is in no way to be taken as investment advice or endorsement of any company, institution, or service.

A Word on Data 3

I have personally found problems with data from:

- **Yahoo Finance:**
  - Very hard to tell how they adjust for dividends & corporate actions so that you can un-adjust
  - Not all the companies you might want are available (especially delisted)

- **Compustat:**
  - Sometimes incomplete, sometimes simply inaccurate.
  - Pretty decent, though, for compiled financial information (quarterly & annual reports, lists of companies and statistics by industry, market share, etc)

Dividends

Anatomy of a Dividend Announcement

You see a dividend announcement on a website, your data feed, or your brokerage's news feed:

Company: Exxon Mobil Corp. (XOM)

Declare Date: When XOM announced the dividend. Usually very regular and predictable (same time every year)

Dividend Amount ($/share)

Declare Date	Ex-Div Date	Record Date	Pay Date	Frequency	Amount
7/29/2020	8/12/2020	8/13/2020	9/10/2020	Quarterly	0.8700

Payable Date: Date on which the dividend is paid out

Record Date: Must be an XOM shareholder on this date to get dividend

Ex-Div Date: Must *purchase* the stock *before* this date to get the dividend.

Instant Dividends: Many brokerages will credit your account by the dividend amount if you own the stock on the ex-div date: you don't have to wait until the pay date.

Might also see: (No new info, not really that useful)

Forward Yield: $\dfrac{\text{Dividends for the Year}}{\text{current share price}}$

How Dividends Work

From previous slide

Declare Date	Ex-Div Date	Record Date	Pay Date	Frequency	Amount
7/29/2020	8/12/2020	8/13/2020	9/10/2020	Quarterly	0.8700

Exxon Mobil (NYSE: XOM)

What Actually Happens

Also called Ex-Date, Ex-Div date, etc; it's the date *before which* you have to buy into the stock to get the dividend.

29 Jul, 2020	. . .	12 Jul, 2020	overnight	13 Jul, 2020	. . .	10 Sep, 2020
Announcement Date		**Ex-Dividend Date**	settling...	**Date of Record**		**Pay Date**

FACT: The **settlement period** is the time it takes between your purchase of an asset and the day on which you appear as registered owner of that asset.

Settlement Period has gotten shorter over history (used to be 5 business days); for many stocks it's now 1 business day.

→ **Bottom line:** if you want to get the dividend, you have to buy it *before* the ex-date.

On the ex-date itself, the seller gets the dividend.

Many firms now offer 'instant dividends'

Understanding Adjusted Close: DIVIDENDS

Exxon Mobil (NYSE: XOM)

$0.75 dividend paid on 11 Aug, 2015

Date	Open	High	Low	Close
Sep 01, 2015	73.30	75.47	71.51	74.35
Aug 11, 2015				0.73 Dividend
Aug 01, 2015	78.70	79.29	66.55	75.24

Using the bare close:

Clearly your return from 01 Aug – 01 Sep was ln($74.35/$75.24) = -1.19% right?

No. if you bought on 01 Aug you earned $0.73/share in dividends.

Industry standard uses "adjusted close":

→ Adjusted close method: $\ln(\$74.35 / \underbrace{(\$75.24 - \$0.73)}_{\text{This price (\$74.51) is the "adjusted close"}}) = -0.215\%$

Use for backtesting (very slightly different, more accurate):

→ Actual return: ln(($74.35 + $0.73) / $75.24) = -0.213%

Splits

Anatomy of a Split Announcement

Apple (NASDAQ: AAPL) recently underwent a **4-for-1 split**. Below is the split announcement from their website.

② Stock Split

Record Date: Date on which all AAPL shareholders are owed extra shares.

Split Date: After CoB on this day (a Friday), AAPL shareholders will receive 3 additional shares for every 1 share of AAPL stock owned.

Ex Date: The split is now "official"; i.e., AAPL shares trade at their new prices.

Split Ratio: There will be 4 times as many AAPL shares after the split as there were before.

Why have you decided to split Apple's stock?
We want Apple stock to be more accessible to a broader base of investors.

Has Apple stock ever split before?
This will be Apple's fifth stock split since going public. Apple's common stock split on a 2-for-1 basis on May 15, 1987, June 21, 2000 and February 18, 2005; and on a 7-for-1 basis on June 6, 2014.

What is the effective date of the split?
There are several key dates.
The Record Date – August 24, 2020 – determines which shareholders are entitled to receive additional shares due to the split.
The Split Date – August 28, 2020 – shareholders are due split shares after the close of business on this date.
The Ex Date – August 31, 2020 – the date determined by Nasdaq when Apple common shares will trade at the new split-adjusted price.

How does a 4-for-1 stock split actually work?
A 4-for-1 split means that three additional shares of stock are issued for each share in existence on the Record Date of August 24, 2020.

Understanding Adjusted Close: SPLITS

What actually happens:

Apple Inc. (NASDAQ: AAPL)

There are now more shares of AAPL in play, but the market isn't open.

30 Jul, 2020	…	24 Aug, 2020	…	28 Aug, 2020 (Friday)	31 Aug, 2020 (Monday)
Announcement Date		**Record Date**		**Split Date** (after hours)	**Ex Date** Shares trade at new price.

Record date doesn't really matter much for traders. Used by AAPL accountants to actually issue shares to current shareholders….

…but not all traders understand this ☺. That means there might be a scramble to buy AAPL shares before this date because traders *think* they need to in order to get the split.

Why is this? Because when a shareholder sells AAPL stock after the record date, they also transfer their right to receive the split to the buyer.

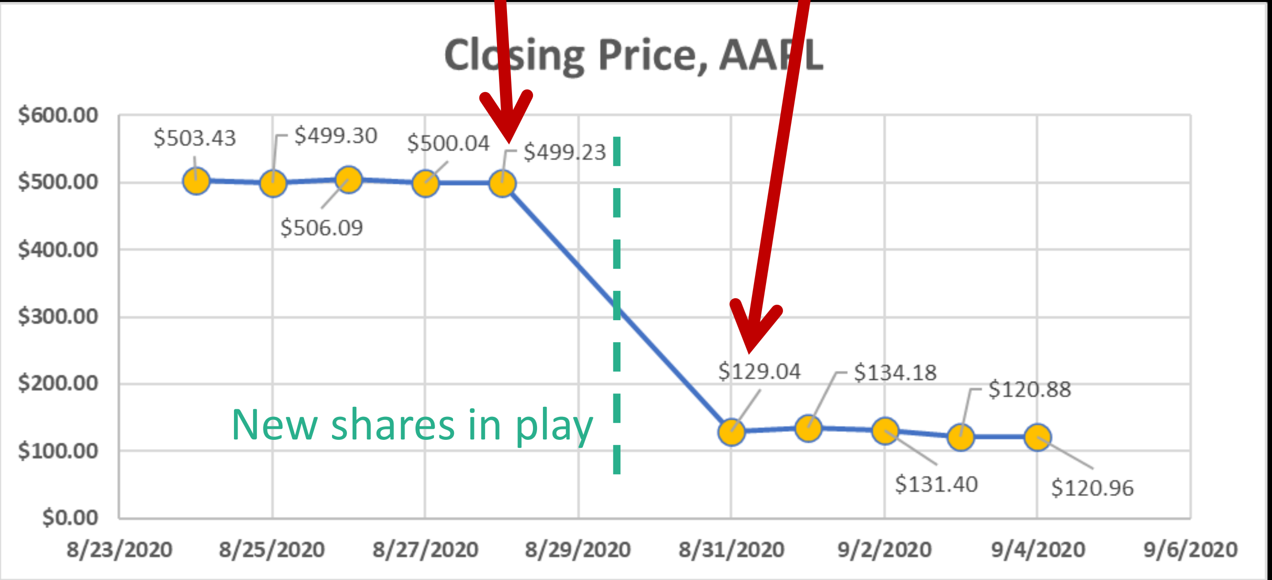
So take note: **Splits don't behave exactly like dividends in this way.**

Understanding Adjusted Close: SPLITS

Apple Inc. (NASDAQ: AAPL)

Split Date 29 Aug 2020

Ex Date 31 Aug 2020



Adjusted Close for splits is done on a basis of the **new prices.**

For example, closing price on 27 Aug 2020 was $500.04.

The **adjusted close** would be: $\frac{\$500.04}{4} = \boxed{\$125.01}$

Split Ratio: 4-to-1

There will be 4 times as many AAPL shares after the split as there were before.

Before Split: approx. $501.62/sh

After Split: approx. $127.29/sh

$\$501.62/\$127.29 \cong 4$ Because 4x as many shares (supply & demand)

If you owned 100 shares before the split, how many do you own after the split?

Answer: 400 (you are issued 300 more)

Therefore, you must adjust ALL past prices according to splits that took place in between that price's timestamp and today.

Excel Breakout: analyze a series

Unadjusted data!

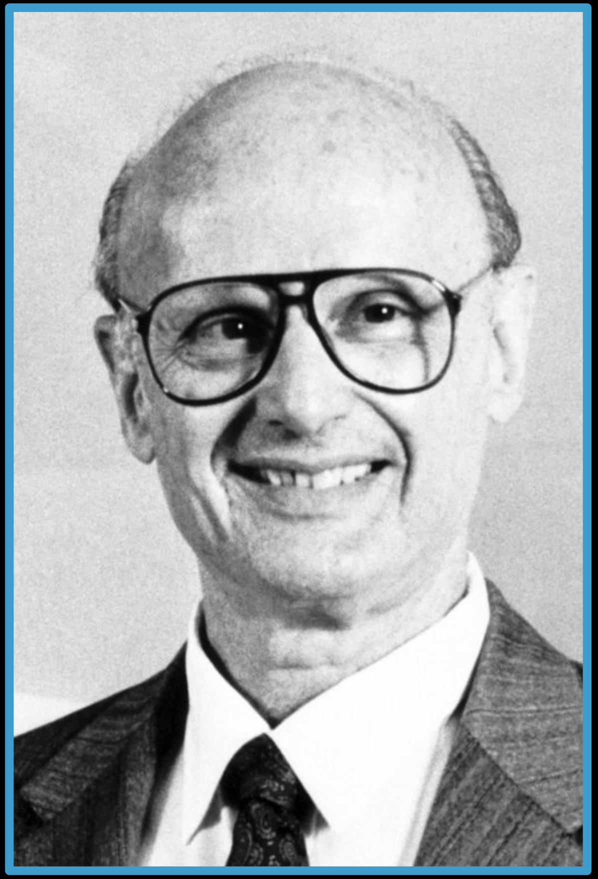
For the stock, calculate:

- Average rate of return
- Geometric mean rate of return

Portfolio Risk, Return, & Diversification

"Diversification is the only free lunch in investing"

- **Harry Markowitz**



Harry Markowitz's Contribution:

1. Consider a set of investment opportunities
   (stocks, algo strategies, bonds, doesn't matter)
2. Think about a time period (e.g., *n* days into the future)
3. Write down, for each asset:
   your expected return over the next period
   your expected risk over the next period
4. Write down, for each *pair* of assets:
   your expected correlation of returns over the next period

You do 3. and 4. using *whatever* machine learning, 'best practice', gut estimate, or any other method you choose. Bottom line is, whatever it is, **you must believe in it**.

4. Markowitz showed that given the above, **there exists an optimal allocation of your money that should be placed on each of the investment opportunities you identified**.

And the things that make this great:

1. It’s philosophically sound so long as you’re sure you believe what you believe
2. Easy(ish) to calculate
3. You can use it to trade with today
   - And many in industry do exactly that.
   - Unlike other strategies (e.g., Kelley Betting) that are fascinating and promising but difficult to implement

Markowitz Portfolio Theory at-a-glance

Investment	Expected Return	Expected Risk
E.g., buy a stock, run a strategy, enter an investment	Return that YOU EXPECT for that investment over the next period	Risk that YOU EXPECT for that investment over the next period
A	12.5%	17%
B	9%	15%
C	5%	11%
D	19%	28%
E	3%	2%

The covariance that YOU EXPECT between the returns of each investment during the next period

	A	B	C	D	E
A	$\mathrm{Cov}(A,A)$	$\mathrm{Cov}(B,A)$	$\mathrm{Cov}(C,A)$	$\mathrm{Cov}(D,A)$	$\mathrm{Cov}(E,A)$
B	$\mathrm{Cov}(A,B)$	$\mathrm{Cov}(B,B)$	$\mathrm{Cov}(C,B)$	$\mathrm{Cov}(D,B)$	$\mathrm{Cov}(E,B)$
C	$\mathrm{Cov}(A,C)$	$\mathrm{Cov}(B,C)$	$\mathrm{Cov}(C,C)$	$\mathrm{Cov}(D,C)$	$\mathrm{Cov}(E,C)$
D	$\mathrm{Cov}(A,D)$	$\mathrm{Cov}(B,D)$	$\mathrm{Cov}(C,D)$	$\mathrm{Cov}(D,D)$	$\mathrm{Cov}(E,D)$
E	$\mathrm{Cov}(A,E)$	$\mathrm{Cov}(B,E)$	$\mathrm{Cov}(C,E)$	$\mathrm{Cov}(D,E)$	$\mathrm{Cov}(E,E)$

Vary Allocation
Set Risk or Return

A	B	C	D	E
50%	0%	20%	25%	5%

Your Portfolio
The allocation of resources that you should take so as to maximize the return you get for your level of risk.

Note
The **true values** of the three pieces of information you must specify (exp risk, return, and cov) are **unknown and unknowable**, but once you determine what you believe, the optimal portfolio follows as a logical consequence.

Passive professional equity investing

- Index-Fund Investing return **equals benchmark**
- **John Bogle** senior thesis (1951) "*Mutual Funds can make no claims to superiority over the Market Averages.*"
- **Malkiel**, *Returns from Investing in Equity Mutual Funds 1971 to 1991* (1995)
- **Malkiel**, *Random Walk Down Wall Street* (1973, 1995, 2016)
- Equity Index Funds and ETFs **get 9 out of 10 dollars invested in 2017 (NYTimes)**

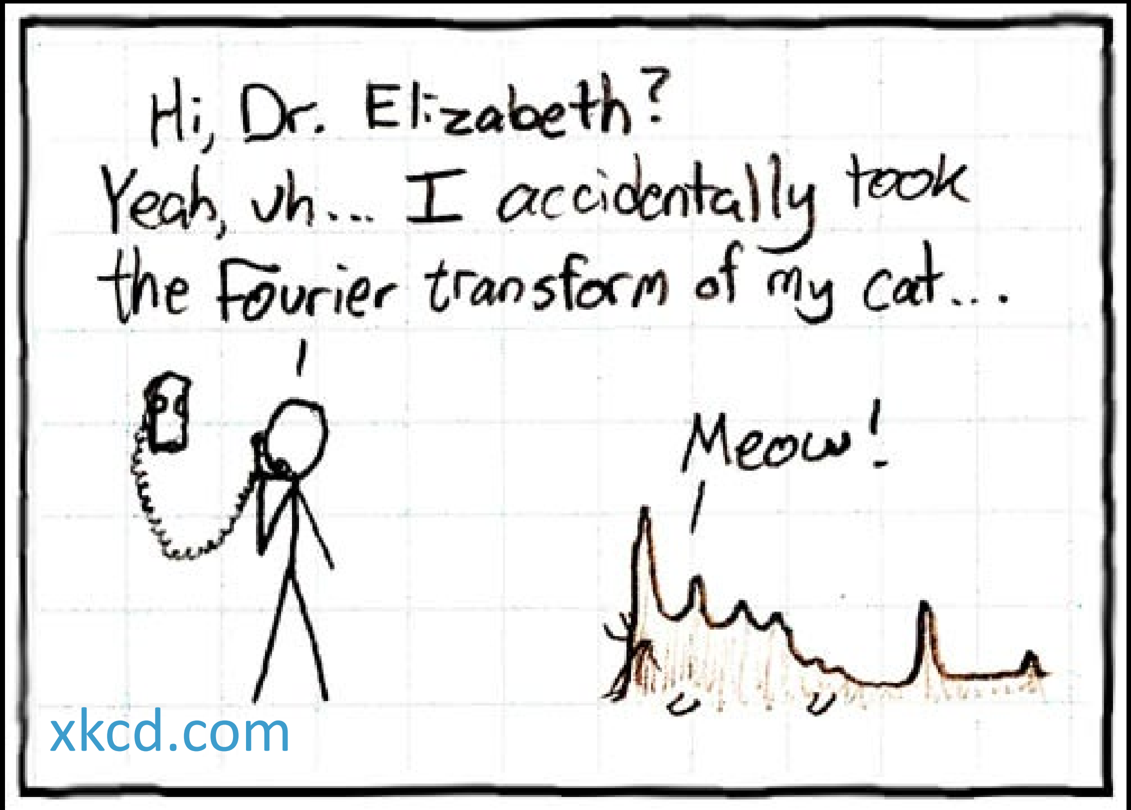
REPORTS SPIVA U.S. Scorecard

Report 1: Percentage of U.S. Equity Funds Outperformed by Benchmarks

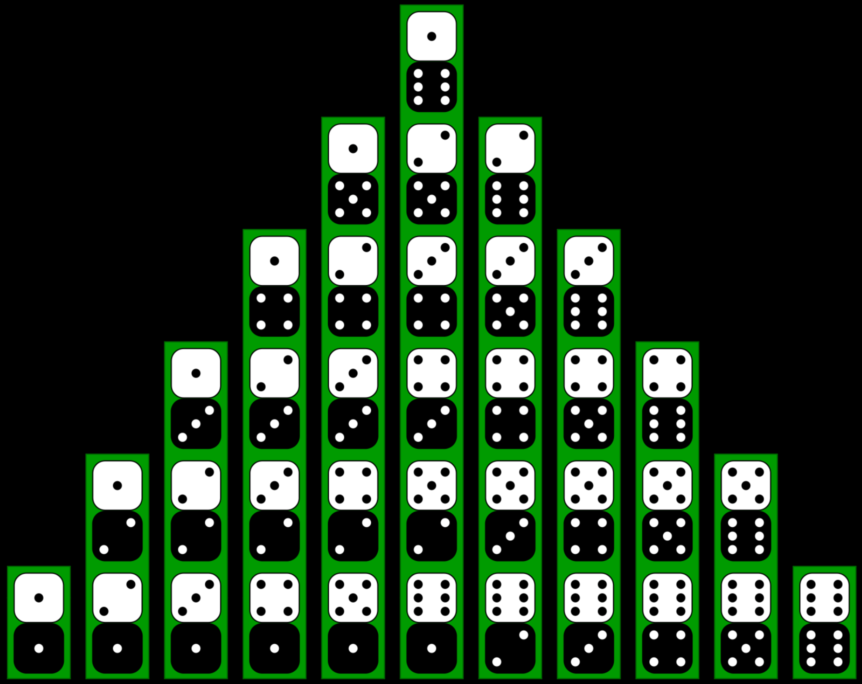
FUND CATEGORY	COMPARISON INDEX	1-YEAR (%)	3-YEAR (%)	5-YEAR (%)	10-YEAR (%)	15-YEAR (%)
All Domestic Funds	S&P Composite 1500	63.43	83.40	86.72	86.65	83.74
All Large-Cap Funds	S&P 500	63.08	80.56	84.23	89.51	92.33
All Mid-Cap Funds	S&P MidCap 400	44.41	86.34	85.06	96.48	94.81
All Small-Cap Funds	S&P SmallCap 600	47.70	88.83	91.17	95.71	95.73

The Notion of "Spaces"

1) Representing a Cat in Fourier Space

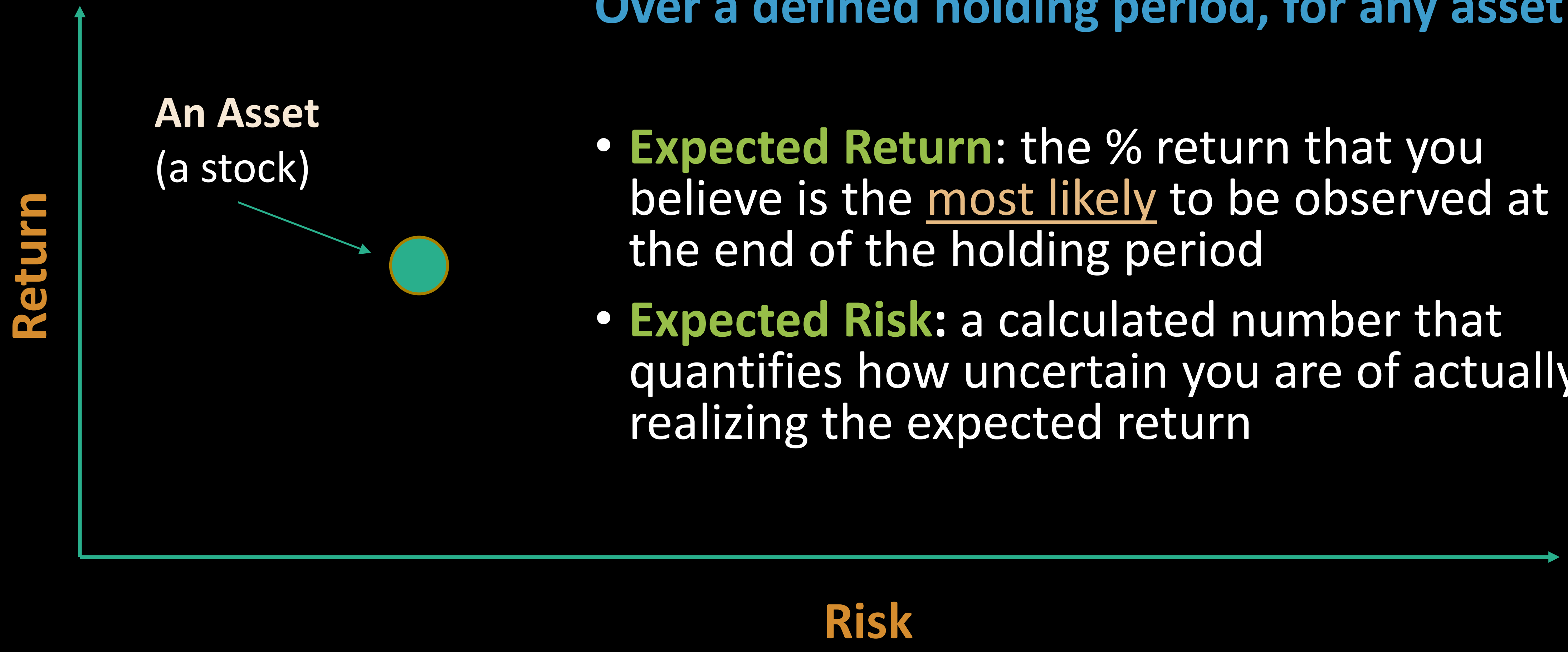


2) Representing outcomes of rolling a pair of dice in Probability Space



Portfolio Space

The Foundation of Financial Risk Analysis



Over a defined holding period, for any asset:

- **Expected Return**: the % return that you believe is the most likely to be observed at the end of the holding period
- **Expected Risk**: a calculated number that quantifies how uncertain you are of actually realizing the expected return

What return can I get if I am willing to accept 0% volatility of return risk?

Answer: the RISK-FREE rate of return.

The interest paid by US Govt. on short-term borrowing

Assumption:
Investors are Risk Averse and Profit Maximizing

- Between two assets with the **same** forecast **volatility**, investors will choose the one with **higher** expected **return** (**Profit Maximizing**).
- Between two assets with the **same** expected **return**, they will choose the one with **lower** forecast **volatility** (**Risk Averse**).
- Map volatility on the x-axis (E-W), expected return on the y-axis (N-S). Investors want to be in the "upper left"

"Risk-Averse" = "Northwest" Corner

Assumption:
Market Prices are Martingales (all else being equal)

Definition:

The best estimate for the next value is the current value.


30%
25%
20%
15%
10%
5%
0%
Return
0.00%
5.00%
10.00%
15.00%
20.00%
25.00%
30.00%
35.00%
40.00%
SE
Standard Deviation of Returns

To Maximize Risk-Adjusted Return

First - Set projected volatility on x-axis, expected return on y-axis

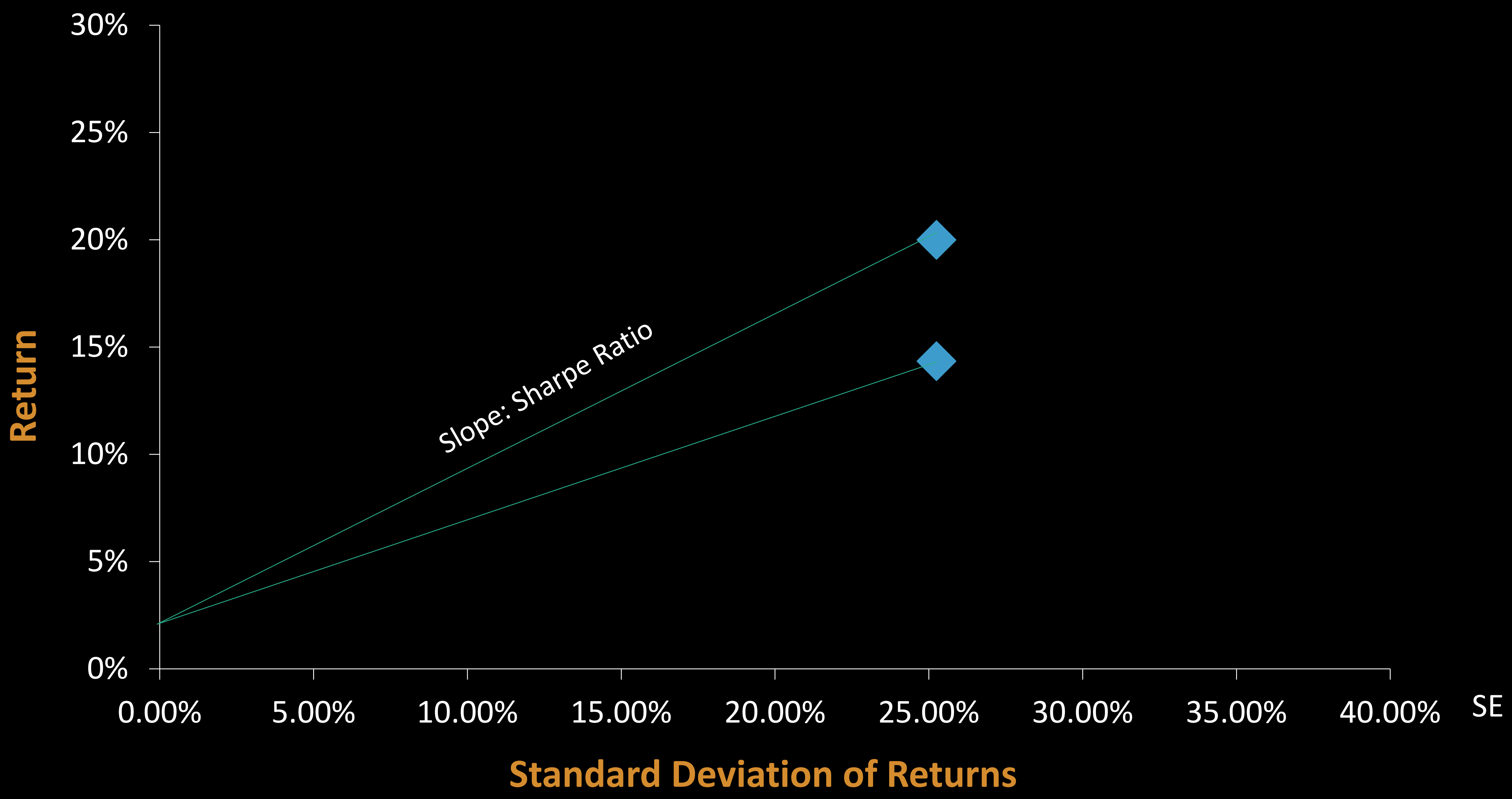
Second - Locate the "risk-free return point" on graph: **x = 0, y = risk-free rate**

(currently about 2% in US)

Goal:

Find portfolio point (x,y) with **maximum slope** from risk-free return point

The difference between the expected return and the risk-free return, divided by the expected volatility of return, is the **Sharpe Ratio** (more on this later)!



30%
25%
20%
15%
10%
5%
0%
Return
Slope: Sharpe Ratio
0.00%
5.00%
10.00%
15.00%
20.00%
25.00%
30.00%
35.00%
40.00%
SE
Standard Deviation of Returns

What about Return of a portfolio?

Portfolio Return

Weighted average of the individual expected returns

$$\langle r \rangle = w_1 r_1 + w_2 r_2 + w_3 r_3 \ldots$$

What about Variance of a portfolio?

Portfolio Variance of Returns: 2 Assets

- Variance of **combination** of two assets is dependent upon Volatility (SD) of each, weight of each, and Covariance.
- Note that weights w(a) + w(b) sum to 1
- **Variance** of the combined assets:

$$\boldsymbol{\sigma}^{\mathbf{2}}_{\text{Portfolio}} = w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_A w_B \text{Cov}(A, B)$$

Covariance of B & A

Weight Matrix, Squared

$$\begin{array}{c} \\ A \\ B \end{array}\begin{array}{c} \begin{array}{cc} \quad A \quad & \quad B \quad \end{array} \\ \begin{bmatrix} w_A^2 & w_A w_B \\ w_A w_B & w_B^2 \end{bmatrix} \end{array}$$

Covariance Matrix

$$\begin{array}{c} \\ A \\ B \end{array}\begin{array}{c} \begin{array}{cc} \quad A \quad & \quad B \quad \end{array} \\ \begin{bmatrix} \sigma_A^2 & \rho_{B,A}\sigma_B\sigma_A \\ \rho_{A,B}\sigma_A\sigma_B & \sigma_B^2 \end{bmatrix} \end{array}$$

Covariance of A & B

Covariance of A &A,
Covariance of B & B
...which is just the variance

1) Multiply element-wise

$$\begin{bmatrix} \sigma_A^2 & \rho_{B,A}\sigma_B\sigma_A \\ \rho_{A,B}\sigma_A\sigma_B & \sigma_B^2 \end{bmatrix}$$

$$\begin{bmatrix} w_A^2 & w_B w_A \\ w_A w_B & w_B^2 \end{bmatrix}$$

Element-wise multiplication, then sum, has a name: ”**Frobenius Product**”

2) Sum them up

Equal

$$w_A^2\sigma_A^2 + w_A w_B \rho_{A,B}\sigma_A\sigma_B + w_B w_A \rho_{B,A}\sigma_B\sigma_A + w_B^2\sigma_B^2$$

3) Portfolio Variance:

$$\boxed{w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_A w_B \mathrm{Cov}(A,B)}$$

Three Stocks

A B & C

Element-wise Multiply...

$$
\begin{array}{c}
\begin{array}{cccc}
 & \mathbf{A} & \mathbf{B} & \mathbf{C}
\end{array}
\\
\begin{array}{c}
\mathbf{A} \\ \mathbf{B} \\ \mathbf{C}
\end{array}
\begin{bmatrix}
\sigma_A^2 & \sigma_{B,A} & \sigma_{C,A} \\
\sigma_{A,B} & \sigma_B^2 & \sigma_{C,B} \\
\sigma_{A,C} & \sigma_{A,B} & \sigma_C^2
\end{bmatrix}
\end{array}
$$

$$
\begin{array}{c}
\begin{array}{cccc}
 & \mathbf{A} & \mathbf{B} & \mathbf{C}
\end{array}
\\
\begin{array}{c}
\mathbf{A} \\ \mathbf{B} \\ \mathbf{C}
\end{array}
\begin{bmatrix}
w_A^2 & w_B w_A & w_C w_A \\
w_A w_B & w_B^2 & w_C w_B \\
w_A w_C & w_B w_C & w_C^2
\end{bmatrix}
\end{array}
$$

Then sum...

$$
\sigma^2_{\textbf{Portfolio}} = w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + w_C^2\sigma_C^2 + 2w_A w_B \mathrm{Cov}(A,B) + 2w_A w_C \mathrm{Cov}(A,C) + 2w_B w_C \mathrm{Cov}(B,C)
$$

…and so on for *n* stocks.

3 Factors in Stock Price Volatility

- Market Risk: Stock price moves linked to **stock's overall universe**
- Sector Risk: Stock price moves linked to companies in same **industry sector**
- Idiosyncratic: Stock price moves **unrelated** to either; specific to stock

But returns of assets are correlated.

Correlation Coefficient

Definition: The slope of the line-of-best-fit that minimizes the sum-of-squares differences between two random variables.

- The long way: Graph in Excel, plot line, get R^2, take square root
- The easy way: **CORREL()**

Some Pairs of Stock Price Changes are More Correlated Than Others

Perfectly Correlated: R = 1

Negatively Correlated R < 0

No Correlation: R = 0

- Stock prices changes of companies of **same size** in same **market sectors** tend to have high correlations

- All stocks generally have > 0 correlation with their market

Covariance

Measures the extent to which two random variables change together.

$$\mathrm{Cov}(A,B) = \frac{1}{n}\sum_{i=1}^{n}(A_i - \bar{A})\,(B_i - \bar{B})$$

→ Use the **COVARIANCE.P()** function in Excel

Covariance and R

$$-1 \leq R \leq 1$$

A "Standardized covariance"

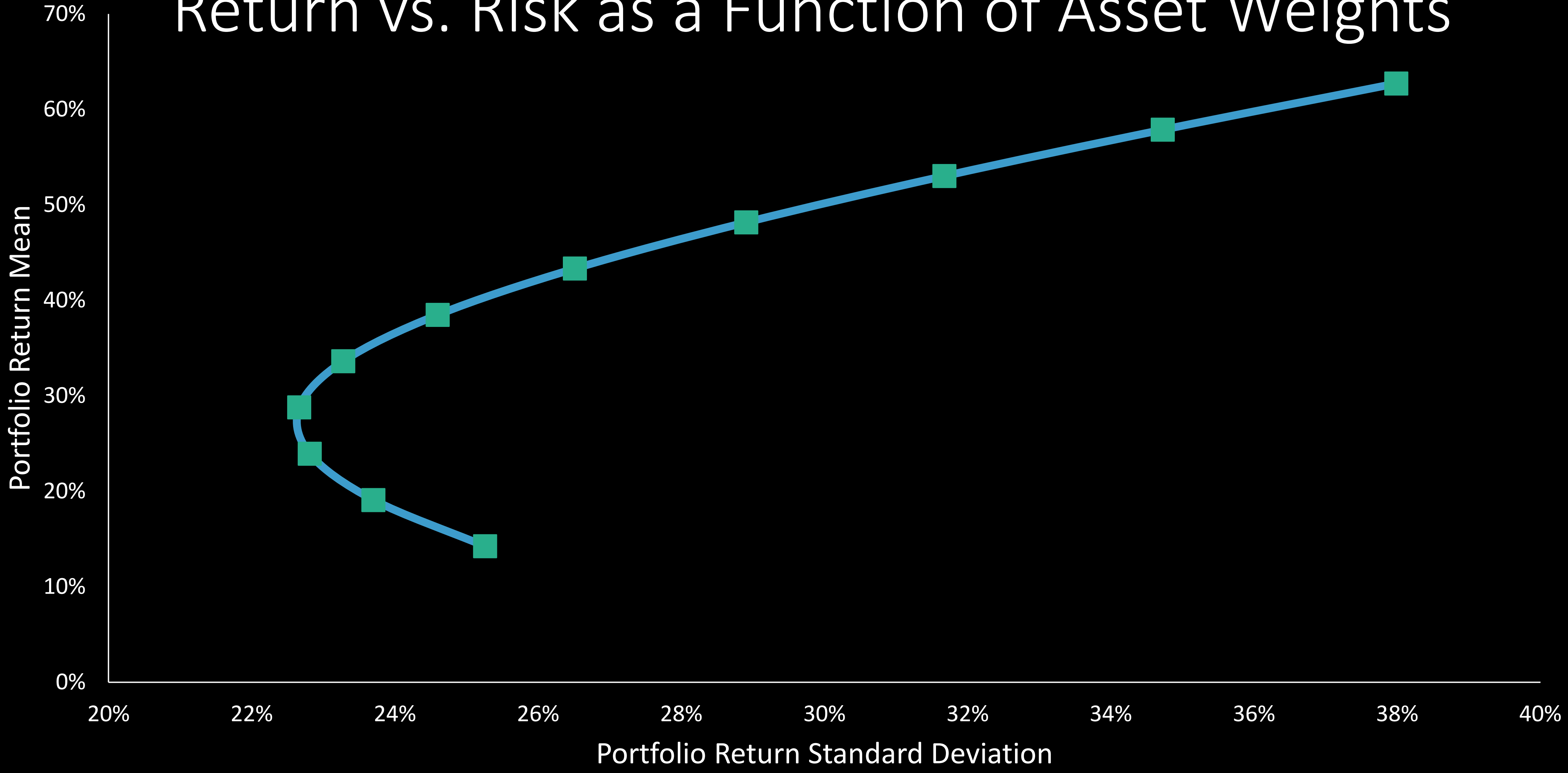
$$\mathrm{Cov}(A, B) = \frac{1}{n}\sum_{i=1}^{n}(A_i - \bar{A})\,(B_i - \bar{B}) = \sigma_{A,B} = R\sigma_A\sigma_B$$

In other words…

$$\mathrm{Cov}(A, B) = R\sigma_A\sigma_B$$

The Efficient Frontier

Return vs. Risk as a Function of Asset Weights



Interpreting the Graph

One end point is 100% GM stock, 0% Microsoft stock

Other end point is 0% GM stock, 100% MS stock

The 9 other marked points on the curve are:

- w(GM) = .9, w(MS) = .1,
- w(GM) = .8, w(MS) = .2,
- w(GM) = .7, w(MS) = .3,
- And so on

The graph contains some **bad choices**

The upward sloping portion of the graph is called the Efficient Frontier

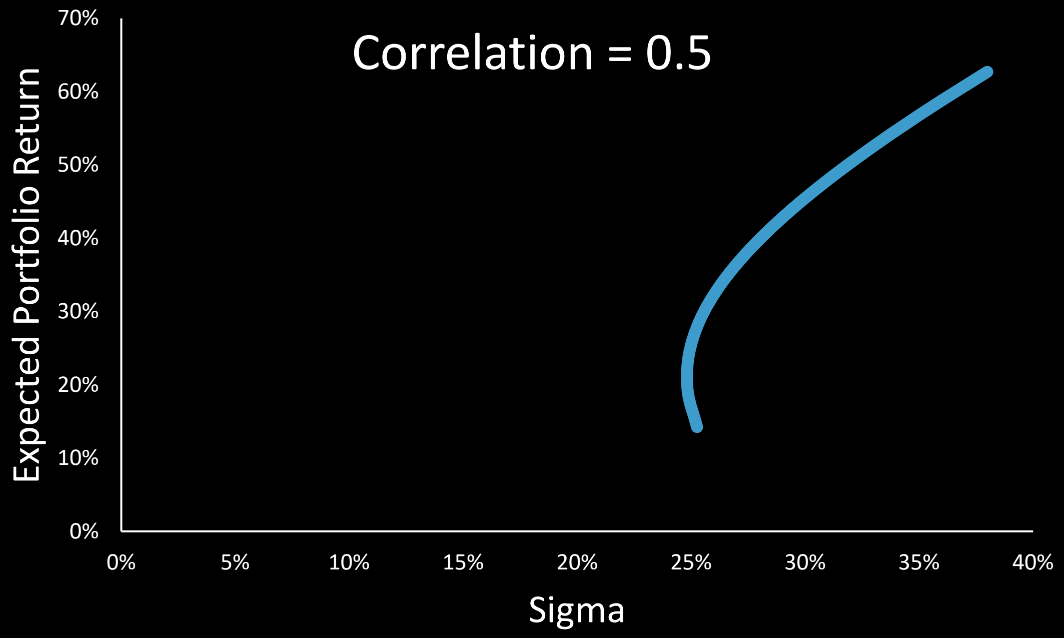
"Northwest is Best!"

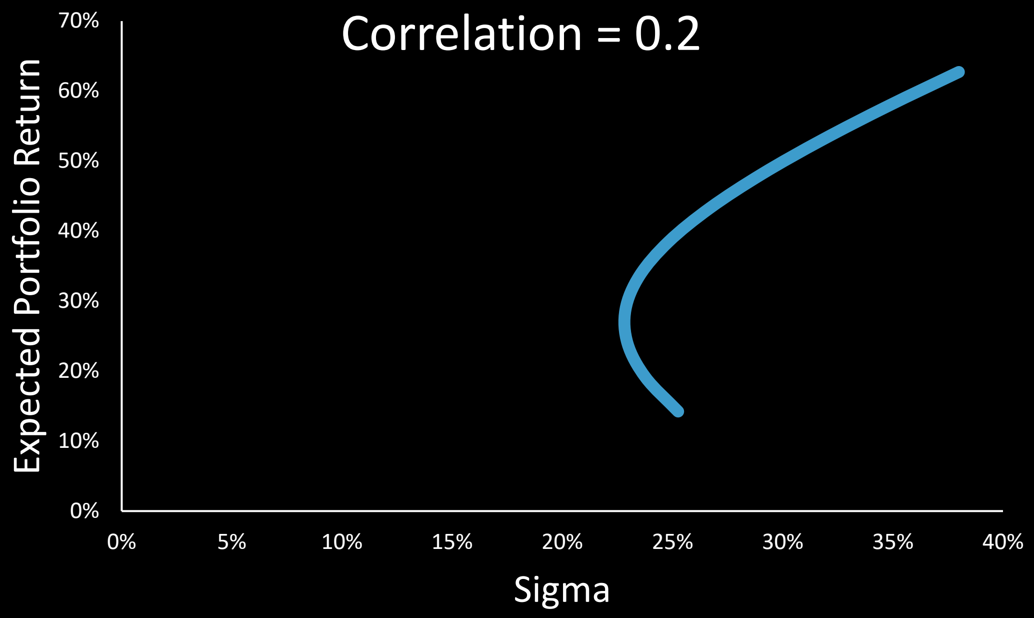
The “Efficient Frontier”

We assume:

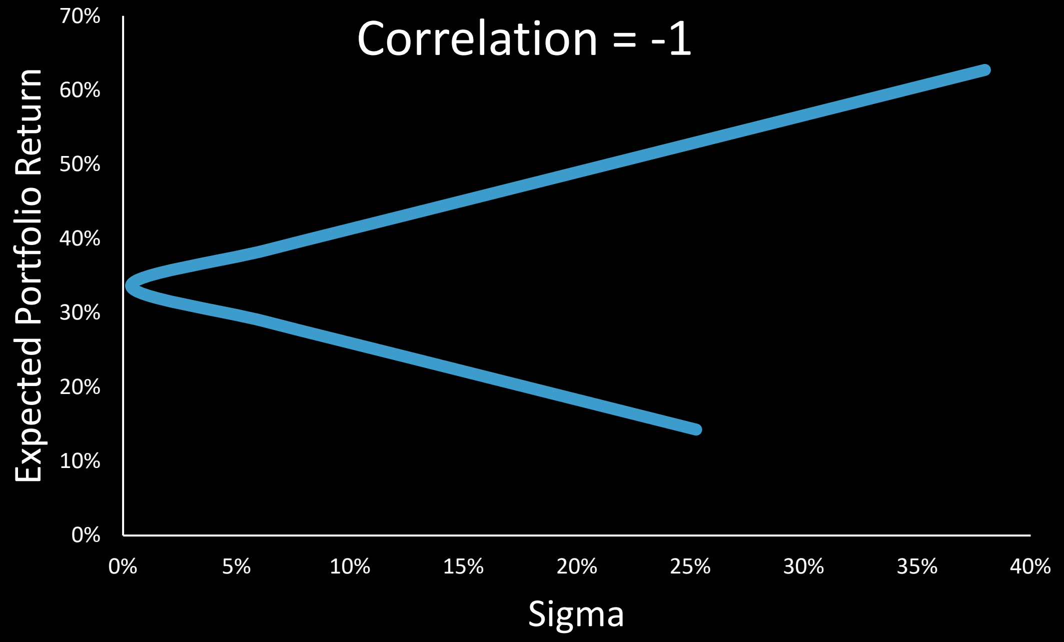
- Investors are “profit-maximizing” and “risk-averse”
- Given 2 portfolios with same SD of returns, investors pick the one with higher expected return
- Given 2 portfolios with same expected return, investors pick the one with lower SD of returns

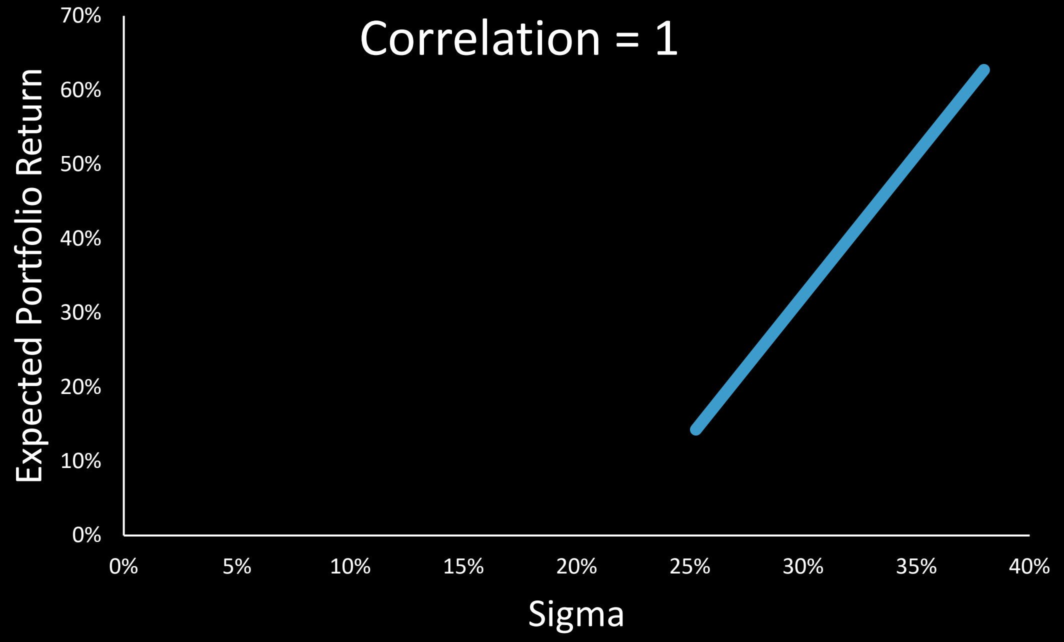
Correlation’s Impact on the Efficient Frontier



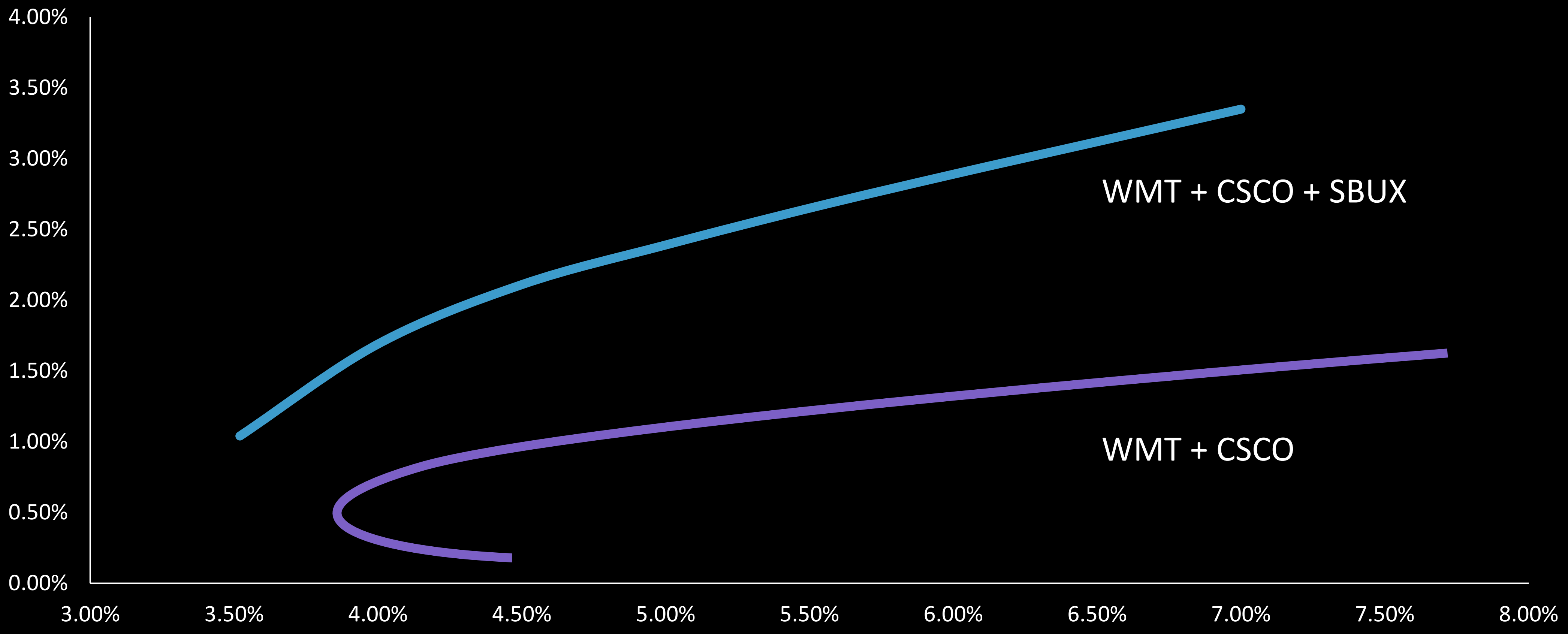


Correlation's Impact on the Efficient Frontier





Adding a 3rd Asset Moves Efficient Frontier



Consequences:

The better you are at forecasting what the expected return, vol, and correlation of returns will be in the next period = the more you can beat the market.*

That means its possible to make good risk-adjusted returns in the long run even if you're limited to a set of poor investment assets/strategies.

*However, beware! It's difficult to improve – even with a great model – on the basic assumption that "whatever these parameters were duing the last period is what they'll be during the next" when you're dealing with new, out-of-sample data.

Portfolio Theory: Key Assumptions

(1) Definition of “risk” used is “volatility of period-over-period returns”

- Historical return/ volatility time series data are assumed to be reasonable basis for forecasting the future
- Distribution of price changes is assumed to follow a Gaussian, or normal probability distribution

(2) All investors are assumed to be “risk-averse”

Standard for graphing:

- Projected volatility – x axis
- Projected return – y axis

Volatility of Returns == Standard Deviation of Returns

Standard deviation of a series of numbers; e.g., monthly or annual returns

Standard deviation is a measure of how **dispersed** a group of numbers is around their mean

Number of observations

Standard Deviation; volatility [%/time]

population mean (average)

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2}$$

The i^{th} observation

Quick side note: not all dangers are included in Portfolio Theory definition of “Risk”

Risk is used to mean “random outcome with known properties”

Mean, standard deviation, shape of probability distribution all assumed known

Not included in this definition:

- Liquidity Risk: no buyer exists, at any price, when you want to sell – a “liquidity hole”
- Uncertainty: probability distribution of future outcomes unknown
- Counterparty Risk: Default, partial default (bonds & derivatives)
- Political Risk: Expropriation, Hyperinflation
- Fraud Risk: Ponzi Schemes (Madoff Fund)

Portfolio Weightings

- By definition, sum of all weightings = 1
- No single component's weight > 1 (absolute value)
- Weighting <0
  - Short Selling
  - **Assume no short selling** (longs only), unless explicitly stated otherwise in a problem statement, for this course

QUESTION

Let's say we designed a trading competition in which everyone spun up a trading account and traded stocks all semester. At the end of the semester, who wins?

Student 1

Puts all cash into a risky startup, the return on which fluctuates wildly during the period – sometimes up, sometimes down by large amounts

On any given day, student might be up by as much as 60%, or down by as much as -60%.

On the official end of the competition, student is up by a whopping 75%.

Got Lucky. The competition just happened to end on a good day for Student 1. By the end of next week, Student 1 was down to -15% return.

Student 2

Diversifies risk by allocating cash across a variety of assets in different industries.

Value of the portfolio goes steadily up – a healthy, reliable return with low variance.

At the end of the competition, Student 2 is up by a very respectable 12%.

Should win. Student 2 was consistently earning a return with low volatility. By the end of next week, Student 2 was up by a little more than 12%.

How can we quantify this?

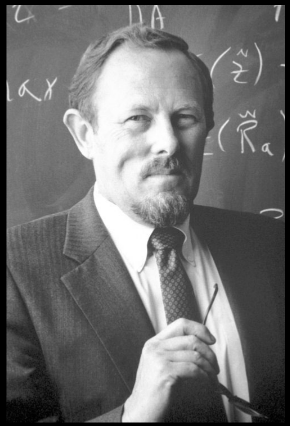
The Sharpe Ratio: Definition

We'd like to develop a metric that has units of **RETURN / RISK** so that we can quantify how well an investment is doing for the risk being taken.

$$\text{Sharpe Ratio} = \frac{E(r_p) - r_f}{\sigma_p}$$

1) The Return
Difference between the portfolio's **return** and the **risk-free rate** (i.e.; how much better are we doing than risk-free?)

2) The Risk
Portfolio's **volatility** (expected standard deviation of returns)

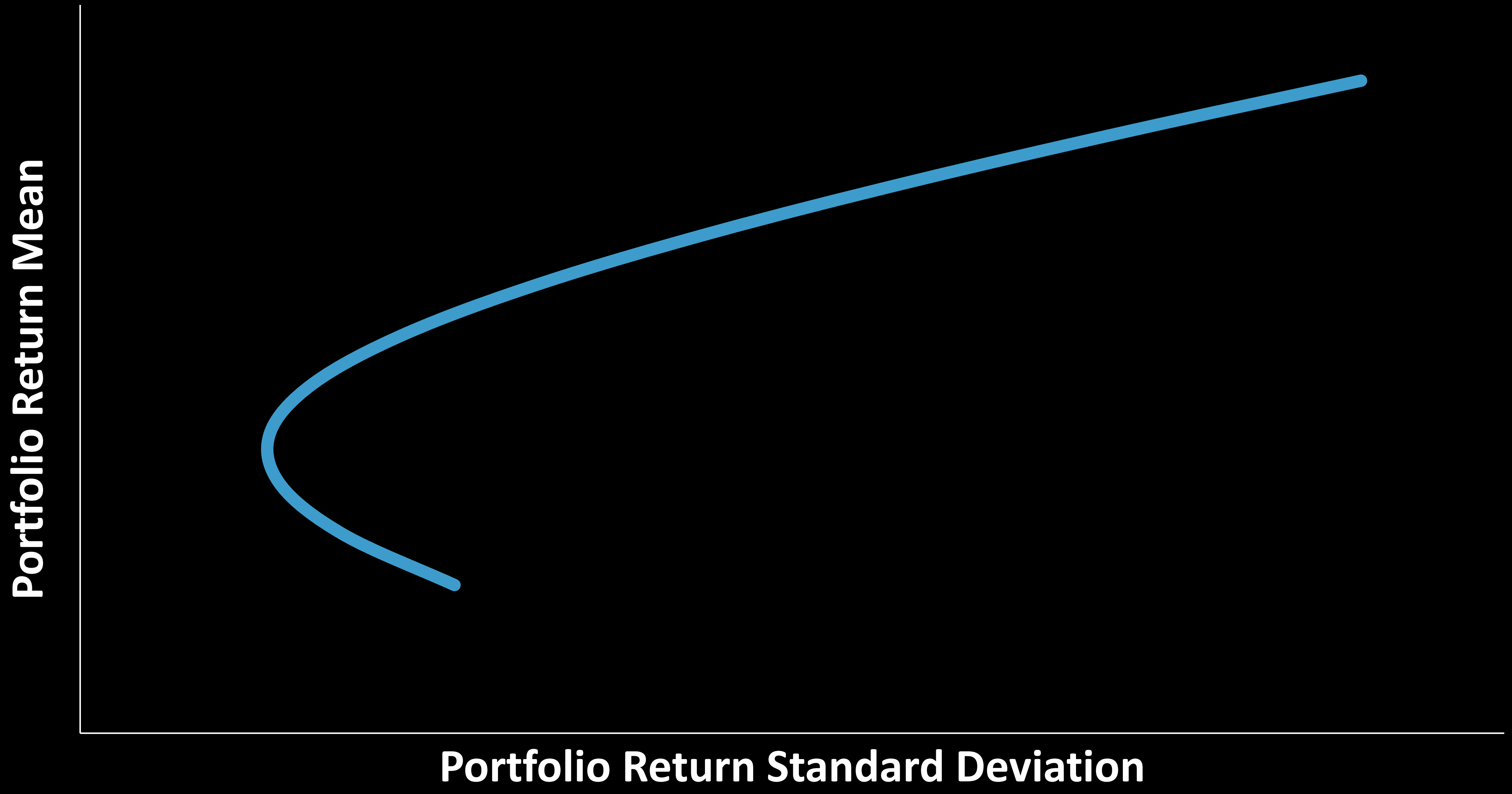


3) Give it a name
William F. Sharpe

Original Paper (1966):
http://web.stanford.edu/~wfsharpe/art/sr/sr.htm

Expected OR Historical
Can measure Sharpe ratio in the past to compare performance, or use expected values to balance a portfolio today

Tradeoffs Between Return and Volatility



Sharpe Ratio

The term "market" as used here is taken to mean "all of the assets you're taking into consideration for your portfolio." Also called your "universe".

By weighting portfolios containing stocks & risk-free bonds, can achieve best available risk/return on the CML (Capital Market Line).

Portfolios that fall on that line have the highest ratio of Return vs. Risk available in a given market.

- Slope of the CML is defined by the best-return portfolios*
- Slope of the CML = best Sharpe Ratio available in your universe*
- Most common metric used to express **risk-adjusted returns**

* Except for the case in which you're leveraged at a rate higher than the risk-free rate r_f.

Let's explore these concepts in an Excel breakout.

Excel Breakout

Let’s explore the Efficient Frontier.

Asset Pricing and Risk Management

FINTECH 522

Jake Vestal

Class 6:

Equities + Portfolio Theory pt. 2

Asking Questions about HW and Lectures

X not email

X not direct message

X coming up after class

- [x] Ask during class
- [x] Ask in a Teams channel

Reason:

It’s a big class and you’re probably not the only student who has that question. Ask it publicly where other students can see it and learn!

EVENING with the BOND KING

Next week is our review session, but we’ve also got a special guest:

Richard King

One of the top minds in bonds & fixed income.

He’ll be here to talk about the bond world some more – very cool experience for us! After he speaks for an hour or so we’ll do our review session.

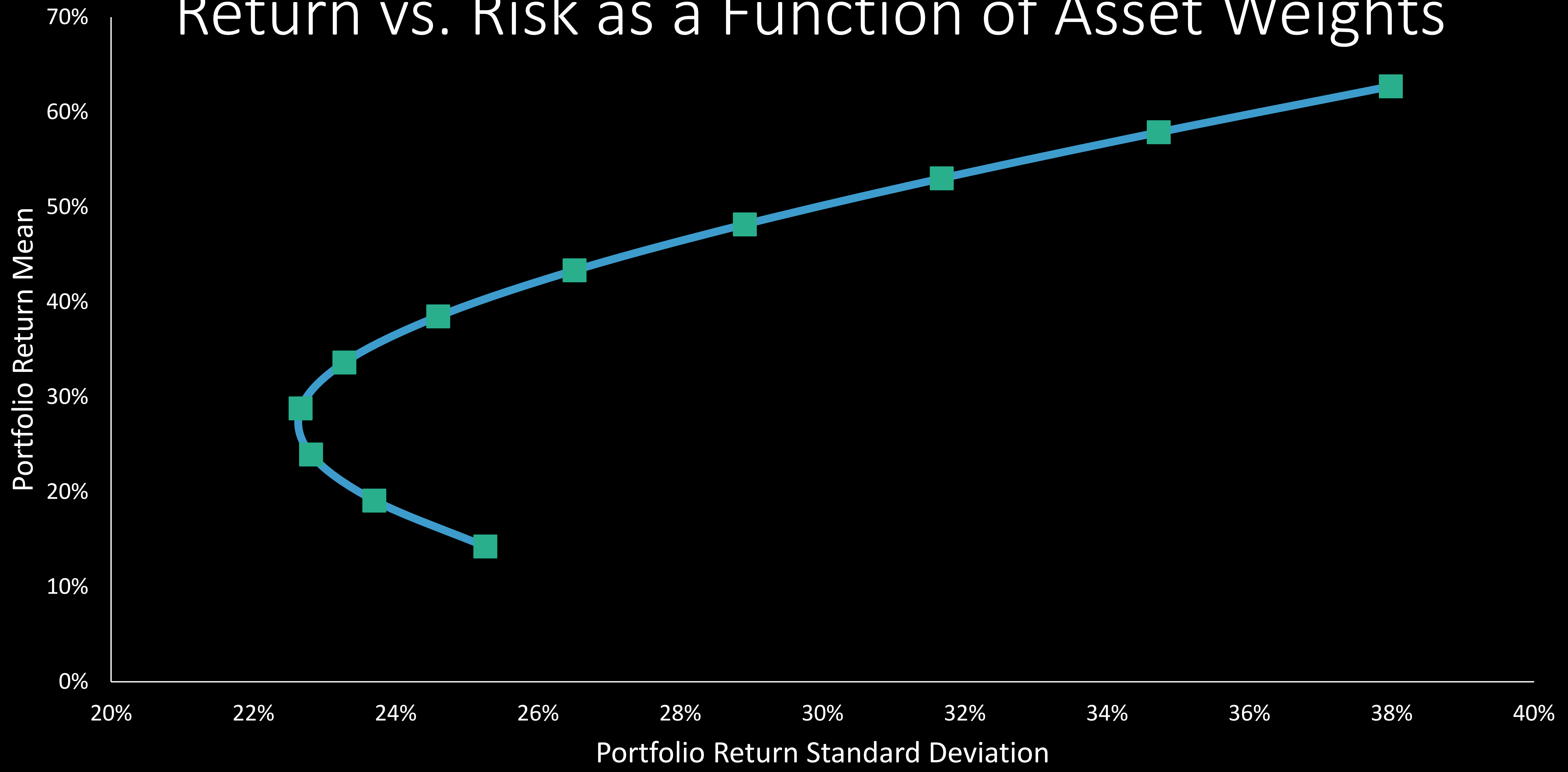
Moving on to Portfolios

Alpha & Beta
Security Market Line
Efficient Fronteir
Capital Market Line

Portfolio Weightings

- By definition, sum of all weightings = 1
- No single component’s weight > 1 (absolute value)
- Weighting <0
  - Short Selling
  - **Assume no short selling** (longs only), unless explicitly stated otherwise in a problem statement, for this course

Return vs. Risk as a Function of Asset Weights



Let’s take a look at HW4

Interpreting the Graph

One end point is 100% GM stock, 0% Microsoft stock

Other end point is 0% GM stock, 100% MS stock

The 9 other marked points on the curve are:

- w(GM) = .9, w(MS) = .1,
- w(GM) = .8, w(MS) = .2,
- w(GM) = .7, w(MS) = .3,
- And so on

The graph contains some **bad choices**

The upward sloping portion of the graph is called the Efficient Frontier

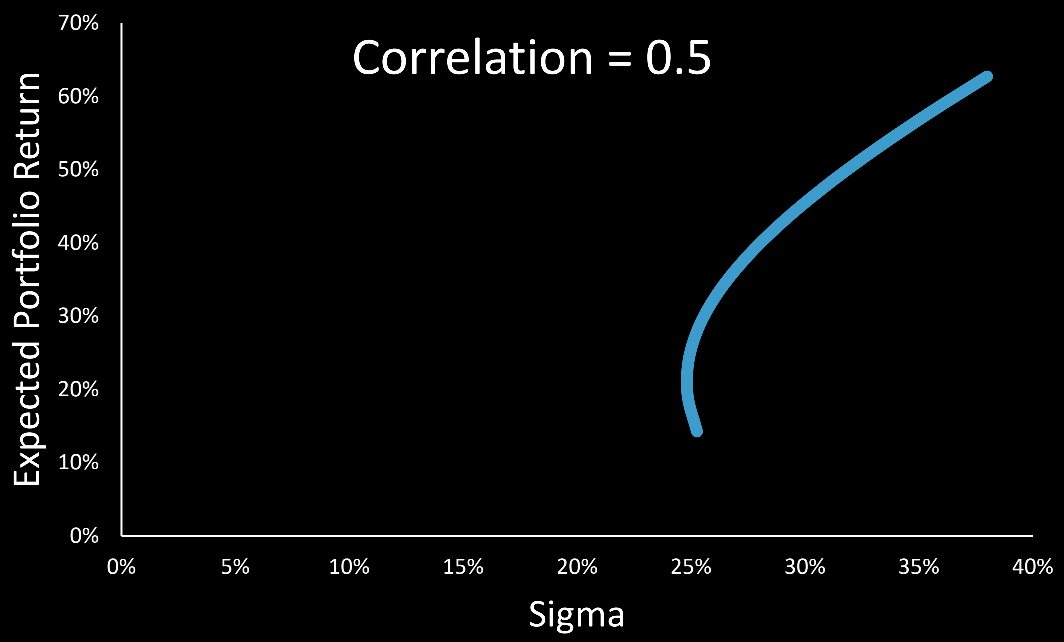
"Northwest is Best!"

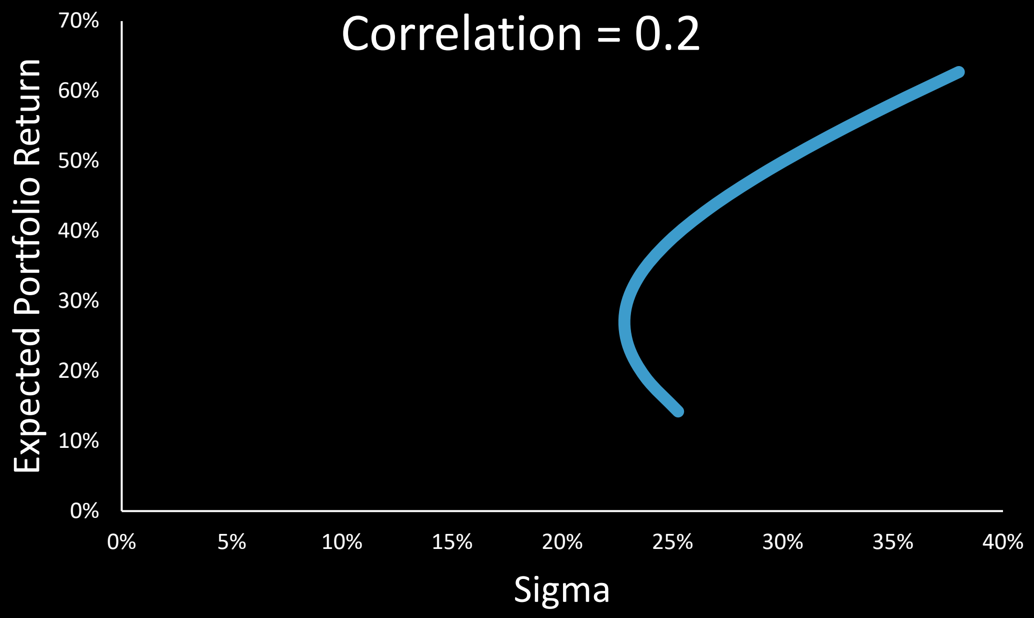
The “Efficient Frontier”

We assume:

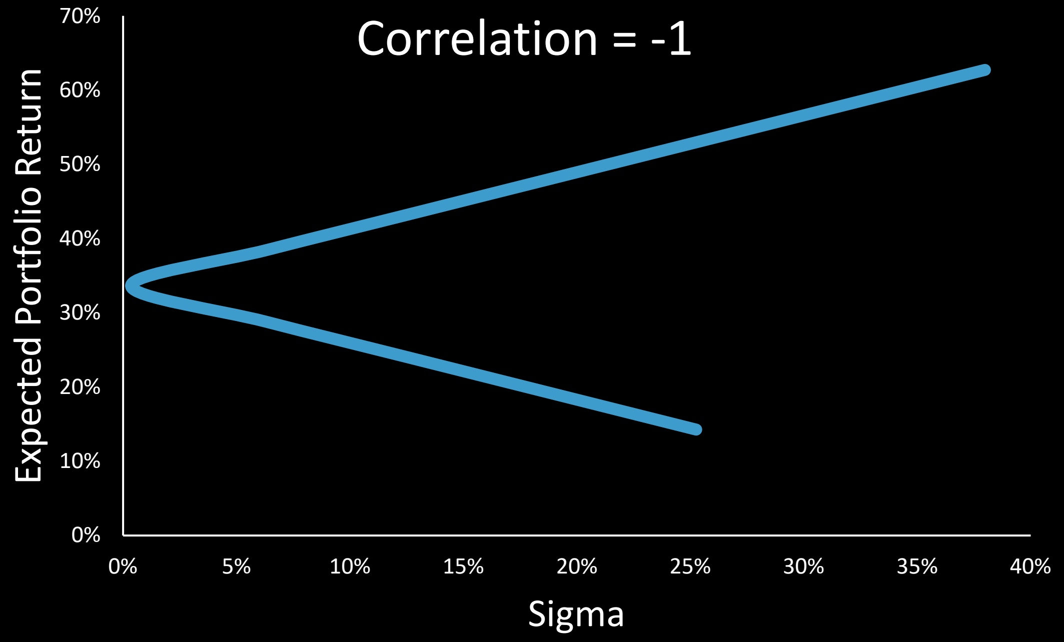
- Investors are “profit-maximizing” and “risk-averse”
- Given 2 portfolios with same SD of returns, investors pick the one with higher expected return
- Given 2 portfolios with same expected return, investors pick the one with lower SD of returns

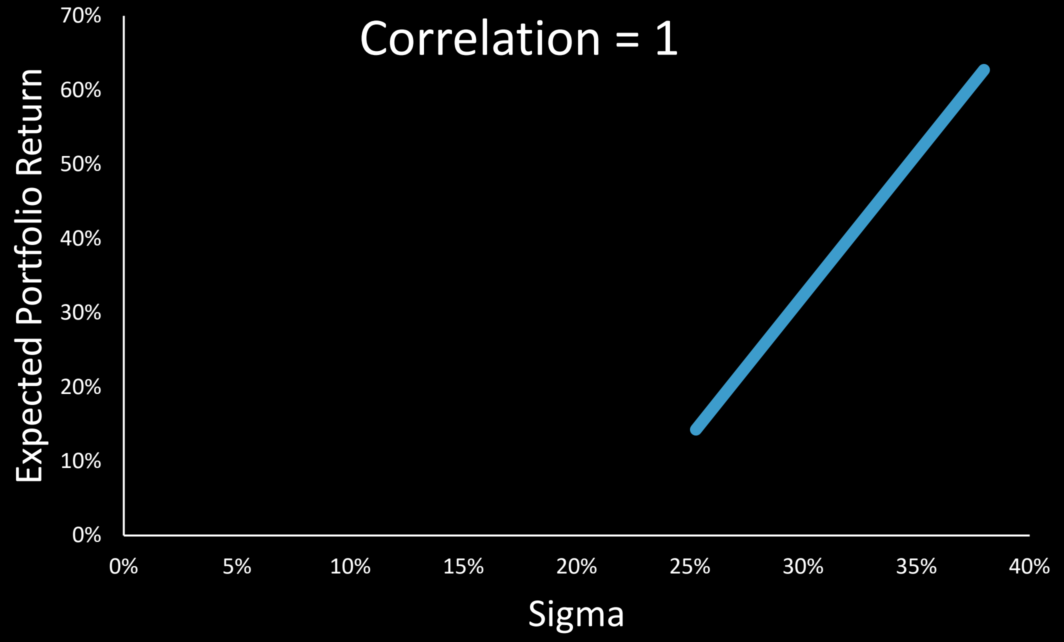
Correlation's Impact on the Efficient Frontier



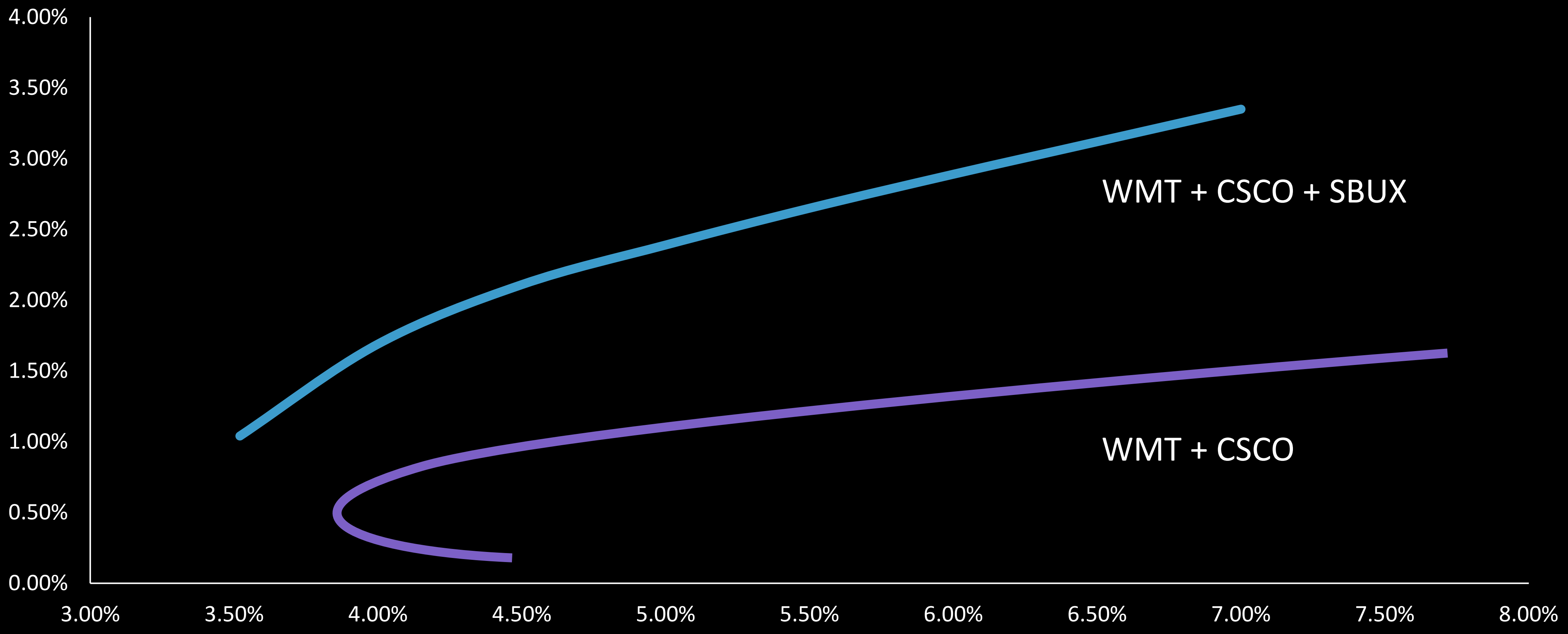


Correlation’s Impact on the Efficient Frontier





Adding a 3rd Asset Moves Efficient Frontier



Consequences:

The better you are at forecasting what the expected return, vol, and correlation of returns will be in the next period = the more you can beat the market.*

That means its possible to make good risk-adjusted returns in the long run even if you're limited to a set of poor investment assets/strategies.

*However, beware! It's difficult to improve – even with a great model – on the basic assumption that "whatever these parameters were duing the last period is what they'll be during the next" when you're dealing with new, out-of-sample data.

QUESTION

Let's say we designed a trading competition in which everyone spun up a trading account and traded stocks all semester. At the end of the semester, who wins?

Student 1

Puts all cash into a risky startup, the return on which fluctuates wildly during the period – sometimes up, sometimes down by large amounts

On any given day, student might be up by as much as 60%, or down by as much as -60%.

On the official end of the competition, student is up by a whopping 75%.

Got Lucky. The competition just happened to end on a good day for Student 1. By the end of next week, Student 1 was down to -15% return.

Student 2

Diversifies risk by allocating cash across a variety of assets in different industries.

Value of the portfolio goes steadily up – a healthy, reliable return with low variance.

At the end of the competition, Student 2 is up by a very respectable 12%.

Should win. Student 2 was consistently earning a return with low volatility. By the end of next week, Student 2 was up by a little more than 12%.

How can we quantify this?

The Sharpe Ratio: Definition

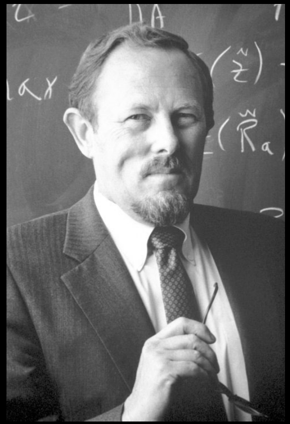
We'd like to develop a metric that has units of **RETURN** / **RISK** so that we can quantify how well an investment is doing for the risk being taken.

$$\text{Sharpe Ratio} = \frac{E(r_p) - r_f}{\sigma_p}$$

1) The Return
Difference between the portfolio's **return** and the **risk-free rate** (i.e.; how much better are we doing than risk-free?)

2) The Risk
Portfolio's **volatility** (expected standard deviation of returns)

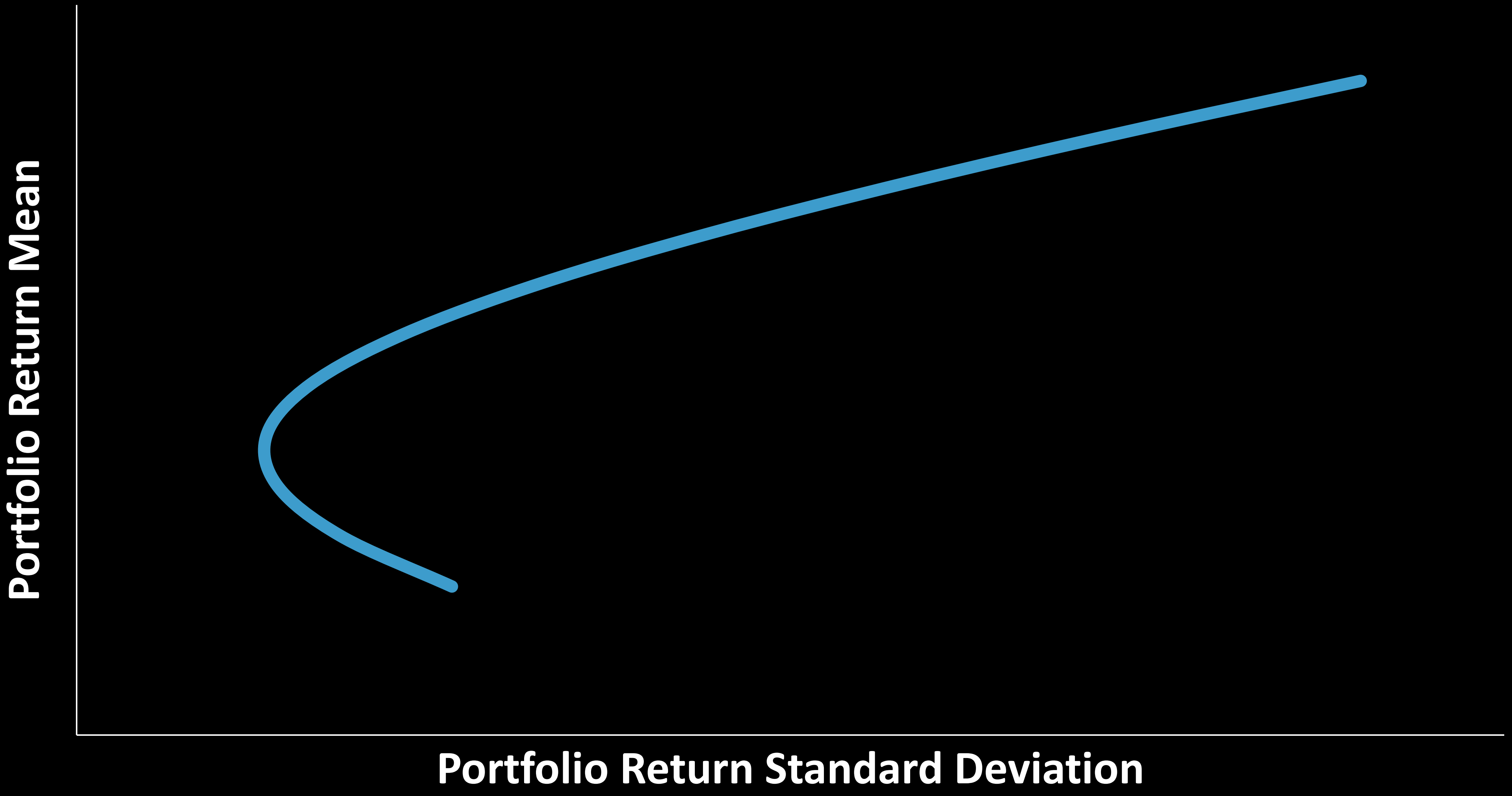
Expected OR Historical
Can measure Sharpe ratio in the past to compare performance, or use expected values to balance a portfolio today



3) Give it a name
William F. Sharpe

Original Paper (1966):
http://web.stanford.edu/~wfsharpe/art/sr/sr.htm

Tradeoffs Between Return and Volatility



Sharpe Ratio

The term "market" as used here is taken to mean "all of the assets you're taking into consideration for your portfolio." Also called your "universe".

By weighting portfolios containing stocks & risk-free bonds, can achieve best available risk/return on the CML (Capital Market Line).

Portfolios that fall on that line have the highest ratio of Return vs. Risk available in a given market.

- Slope of the CML is defined by the best-return portfolios*
- Slope of the CML = best Sharpe Ratio available in your universe*
- Most common metric used to express **risk-adjusted returns**

* Except for the case in which you're leveraged at a rate higher than the risk-free rate r_f.

Let's explore these concepts in an Excel breakout.

Excel Breakout

Let's tie it all together.

Stock A: _____

Exp rtn:

Exp vol:

Stock B: _____

Exp rtn:

Exp vol:

Correlation: $(R_{CB,TXN})$ =

Risk-free rate: $r_f =$

Alpha & Beta

After Markowitz published in 1952

- People realized: more stocks, more betas, better risk adjustment
- Adding stocks into the universe allows for more selection opportunity to find the best efficient frontier
- Theoretically, the “best portfolio” whose universe included all available stocks would have weightings assigned to all stocks
- Such a “best portfolio” would not necessarily assign non-zero weights to every stock- some are left out (weight = 0).

Conclusion: No subset of any given universe can produce a more optimal portfolio than the one calculated on the universe as a whole.

Alpha & Beta: Motivation

Sometimes we want to compare an investment's returns against those of a different asset (i.e., a **benchmark**).

We can do this by graphing the returns of the benchmark on the x-axis, and those of the asset on the y-axis.

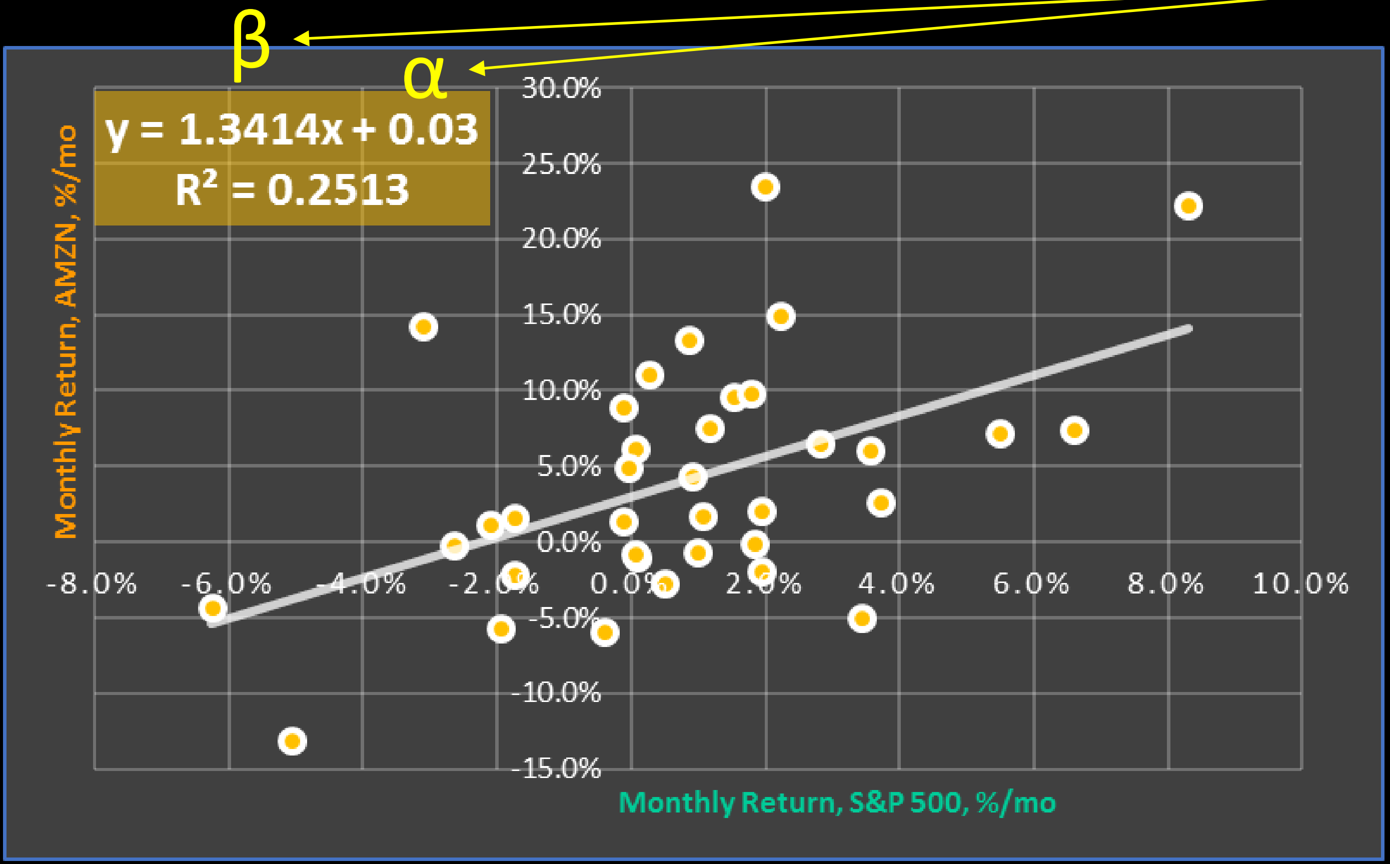
Alpha & Beta: Definition

x-axis: Historical returns of a **Benchmark** (S&P 500)

y-axis: Historical returns of an **Asset** (AMZN)

Returns

	A	B	C
1	datadate	SP500	AMZN
2	20150131	-3.1%	14.2%
3	20150228	5.5%	7.2%
4	20150331	-1.7%	-2.1%
5	20150430	0.9%	13.4%
6	20150531	1.0%	1.8%
7	20150630	-2.1%	1.1%
8	20150731	2.0%	23.5%
9	20150831	-6.3%	-4.3%
10	20150930	-2.6%	-0.2%
11	20151031	8.3%	22.3%
12	20151130	0.1%	6.2%
13	20151231	-1.8%	1.7%
14	20160131	-5.1%	-13.2%
15	20160229	-0.4%	-5.9%
16	20160331	6.6%	7.4%
17	20160430	0.3%	11.1%
18	20160531	1.5%	9.6%
19	20160630	0.1%	-1.0%
20	20160731	3.6%	6.0%
21	20160831	-0.1%	1.4%
22	20160930	-0.1%	8.9%
23	20161031	-1.9%	-5.7%
24	20161130	3.4%	-5.0%
25	20161231	1.8%	-0.1%
26	20170131	1.8%	9.8%
27	20170228	3.7%	2.6%
28	20170331	0.0%	4.9%
29	20170430	0.9%	4.3%
30	20170531	1.2%	7.5%
31	20170630	0.5%	-2.7%
32	20170731	1.9%	2.0%
33	20170831	0.1%	-0.7%
34	20170930	1.9%	-2.0%
35	20171031	2.2%	15.0%
36	20171130	2.8%	6.5%
37	20171231	1.0%	-0.6%

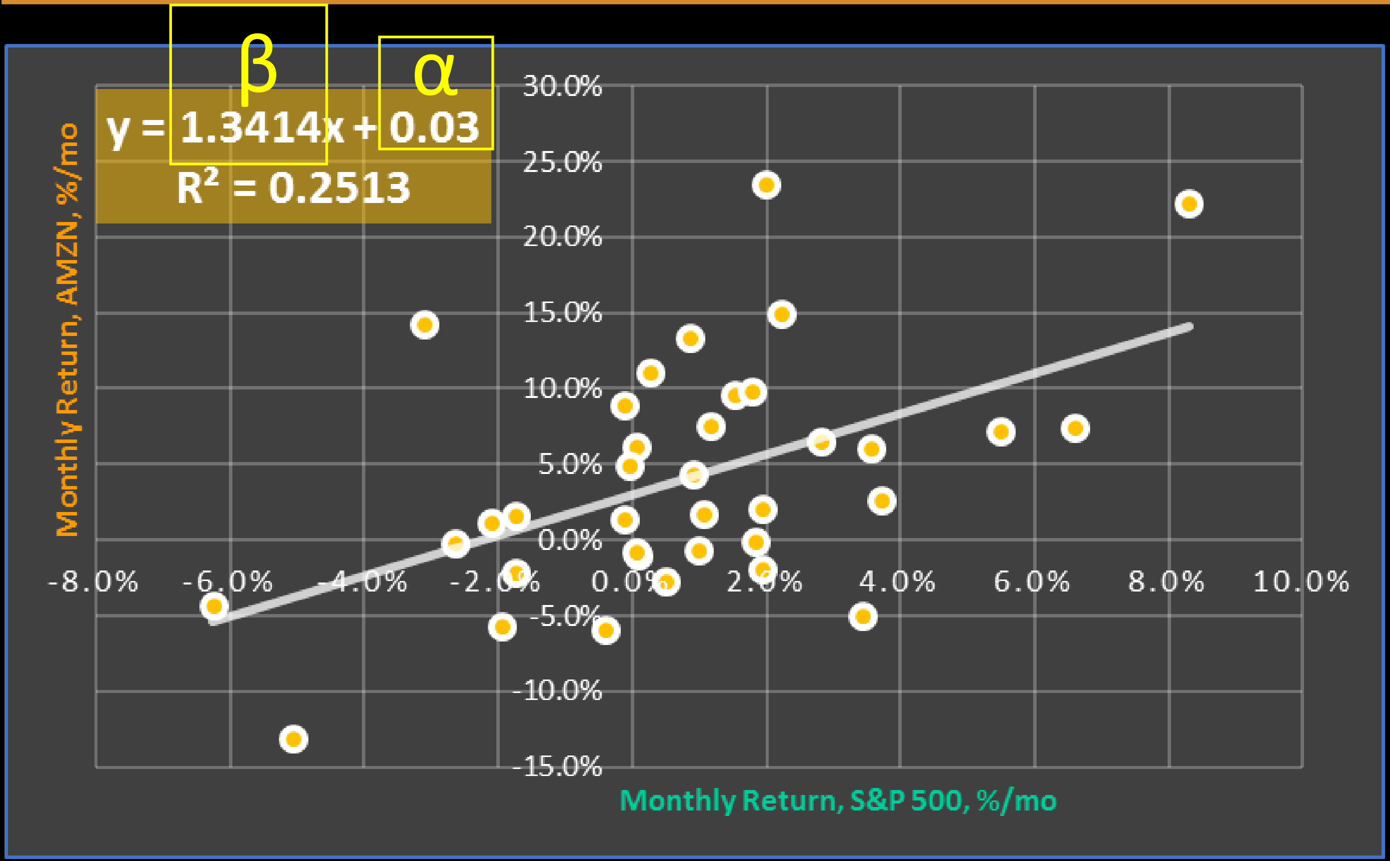


Alpha (**α**): The y-intercept
Beta (**β**): The slope
…of the line-of-best-fit between an **asset** and an appropriate **benchmark**.

Benchmark can be:

- An index
- Another stock
- A strategy
- …any financial instrument for which we have historical returns

Alpha & Beta: Interpretation



What are we saying by drawing a regression line through these returns?

There is **mutual information** between **AMZN** and **SP500** prices – knowing whether one increases or decreases improves our ability to predict the other

What happens if SP500 Return set to 0?

If the S&P returns nothing, AMZN still returns 0.036% (α). (and that's good!)

- **α** is therefore sometimes called the "excess return" over the benchmark
- If these returns are all log returns, what is Amazon's excess return, **annualized**?
  = 0.03% * 12 = **0.36%**

β measures the sensitivity of AMZN's returns to those of the S&P 500; i.e., Market Risk

- **β** of **1.34** shows that **AMZN** was quite sensitive to the market during this time period
- **β > 1** termed **aggressive**
- **β < 1** termed **defensive**

Alpha and Beta can refer either to Historical Measures or Forecasts

In Capital Market (CAPM) theory, **over the long run**:

- Every asset will have a return equal to the product of its Beta and the difference between the market and the risk-free rate of return
- Every asset will therefore have forecast Alpha of 0
  - Alpha is defined as an asset's return minus the product of its beta and the difference between the market and risk-free return
  - historical data show Alphas > 0 and Alphas <0

Because Alpha gives a fund's return when the overall market's return is zero, it provides a decent measure of a money manager's performance over the market.

In other words: why put your money in a mutual fund vs. simply buying shares of an S&P500 index yourself?

→ Because you believe that the mutual fund will earn you a better alpha.

→ But mutual funds charge fees, and when you take *that* into account…

The Efficient Market Hypothesis

Malkiel (1995) results

Analysis from Malkiel, "Returns from Investing in Equity Mutual Funds 1971-1991" Journal of Finance (1995) (posted on Sakai)

- Considered *all* US mutual funds in existence from 1971 to 1995, except those investing in foreign securities or in one particular industry sector (gold stocks, pharma, etc)
- Accounted for management and gross expenses charged by funds to arrive at the actual return an investor would have received
- The mean Alpha of all US equity mutual funds surviving 1972-1991 is **0%**!
- Capital Market (CAPM) Theory says that over long time intervals, measured alpha will approch 0%.

Survivorship Bias

In 1992, Malkiel did not just look at the mutual funds that existed, acquire their past data, and start calculating returns (doing so would have injected "survivorship bias" into the analysis).

Eliminate survivorship bias by including the funds that went out of business during the time period being studied.

Annualized Returns From 1982-1991:

- S&P 500 returns: **17.52%**
- Mean return on mutual funds in existence in 1982 ***that were still in business*** in 1992: **17.09%** (slightly worse than market)
- When mutual funds that went out of business during the time period are included (adjust for survivorship bias), mean return was: **15.69%**

Survivorship Bias

Of 239 funds surviving 1982-1991, compared against S&P returns:

- Funds with positive, statistically significant alpha: **0**
- Funds with negative, statistically significant alpha: **19**

Efficient Market Hypothesis

Essentially says: For markets made up of assets that:

- are highly liquid (i.e. frequently traded), and
- for which the information relevant to determining price is public and widely available, and
- there is a low barrier to entry (easy for many traders to access the market)

… no observed risk-adjusted excess returns are earned in the long run by money managers that actively buy & sell (i.e., mutual funds).

- Guessing "heads or tails" eight times in a row – a "skill" that one in 256 people possess
- Good luck naturally appears, both to the lucky themselves, and to others, to be skill
- This is what finance writer Nassim Taleb calls, "**Fooled by Randomness**."

Efficient Market Hypothesis

In an efficient market, no excess risk-adjusted returns possible:

- by analysis of past price movements – WEAK FORM
- By analysis of publicly-available information – SEMI-STRONG FORM
- By use of any information, public or “insider” – STRONG FORM

Capital Market Line (breakout)

In theory, the market is only willing to give you one best “price” (in terms of risk) that you’ll pay to earn a return.

That “price” is given by the slope of the Capital Market Line, which is the Sharpe Ratio of the best portfolio available in a market.

Let’s explore with Excel.

Capital Market Line

When to hire active money managers:

When they manage money in markets that are *less than* efficient (e.g., **alternative asset classes)**

- Markets with inefficient transactions (each one unique)
- Markets with few highly-informed participants
- Markets where relevant information is expensive
- Algorithmically identify and exploit mispricing
    - Warning: Even when the market is “wrong”, and you take a position to exploit a price inefficiency, the market can remain “wrong” until *after* you go broke
- Illiquid Investments (benefit from lack of “Liquidity premium”)
    - Buyouts, Venture Capital, etc
    - Needs specialized knowledge and access in order to participate

Manager should be able to explain, concisely and clearly, what the inefficiency is that they’re exploiting, why they have an “edge” over everyone else, and, perhaps most importantly, *how they’ll know when that “edge” stops working.*

Technical Analysis

Try to identify for “patterns” (or “fractals”, as some say) in stock prices, observe a “pattern” that is partially complete, then take a position assuming that the pattern will repeat

- Called “chart traders”, “pattern traders”, or “chartists”
- Looked down upon by most professionals
- Relies upon past price patterns to predict future prices.
- Does not work (except possibly in high-frequency trading cases)
- Do you really believe that **past price patterns contain information to reliably predict future price patterns**?

Don’t trust any trading strategy unless you can write down clearly and concisely why it works (or at least have a theory), and how you’ll know when it stops working.

Fundamentals Analysis

- *Does* sometimes work – due to the **weak** & **semi-strong** forms of the Efficient Market Hypothesis (EMH)
- EMH **strong** form says that it can't work
- *The Big Short* by finance writer Michael Lewis is an example in which fundamental analysis payed of big…

  …but the money managers almost went broke before it did!

Fundamentals Analysis

- *Does* sometimes work – due to the **weak** & **semi-strong** forms of the Efficient Market Hypothesis (EMH)
- EMH **strong** form says that it can’t work
- *The Big Short* by finance writer Michael Lewis is an example in which fundamental analysis payed of big…

  …but the money managers almost went broke before it did!

Efficient Market Hypothesis & Security Market Line

We already know:

α_i: the (excess) return of a security *i* when that of the benchmark is 0

β_i: 'conversion factor' between (excess) returns of benchmark and *i*

In Symbols:

$$E(r_i) - r_f = \beta_i\left[E(r_M) - r_f\right] + \alpha_i$$

Efficient Market Hypothesis & Security Market Line

The **Efficient Market Hypothesis** states that all relevant, publicly available information is already reflected in the market prices

When true:

long-term alpha for active managers = 0

$$E(r_i) - r_f = \beta_i\left[E(r_M) - r_f\right] + \cancel{\alpha_i}$$

Long-term return of a security should be its beta * the market's excess return + the risk

Efficient Market Hypothesis & Security Market Line

All relevant, publicly available information is already reflected in the market prices

When true:

long-term alpha for active managers = 0

$$\boxed{\text{SML: } E(r_i) = \beta_i\left[E(r_M) - r_f\right] + r_f}$$

Long-term return of a security should be its beta * the market's excess return + the risk

Active Manager's long-term Sharpe Ratio is expected to be same as the market portfolio

Excel Breakout: SML

Discounted Cash Flows & Rates of Return

Future Value

Why is a dollar today worth more than a dollar tomorrow?

Example:
Receive \$100 today. Invest at 2% per annum. Calculate Future Value:

$$100 + 2\% \times 100 = 100\,(1 + 0.02) = \$102$$

And in two years' time?

$$102 + (2\% \times 102) = 104.04$$

Generalize:
Present Value *PV* dollars, *t* years, *r* is interest rate

Future Value *FV* is:

$$\boxed{FV = PV\,(1 + r)^t}$$

Future Value Example

If you have \$100 today, and invest it at 2% per year, what will be its future value in three years?

$$FV = PV\,(1 + r)^t$$

End of: Year 1: $FV = \$100\,(1 + 0.02)^1 = \$102$

Year 2: $FV = \$100\,(1 + 0.02)^2 = \$102\,(1.02) = \$104.04$

Year 3: $FV = \$100\,(1 + 0.02)^3 = \$104.04\,(1.02) = \$106.12$

Worked Examples: Future Value of a Single Cashflow

Example 1*:* Suppose that a wealthy relative gives you \$20,000 to help provide for your newborn child's university fees. You are able to invest this money at 5% p.a. (per annum) until your child is ready to begin college. How much will be in the account 18 years from now?

$$FV = PV\,(1 + r)^t$$

$$FV = 20{,}000\,(1 + 5\%)^{18}$$

$$= \$48{,}132.38$$

Example 2*:* A woman invests \$1,000 at 10% p.a. and plans to hold this investment for 5 years. How much will she have at the end of the holding period?

$$FV = PV\,(1 + r)^t$$

$$FV = 1{,}000\,(1 + 10\%)^5$$

$$= \$1{,}610.51$$

Future Value with Compounding

If you have \$100 today, and invest it at 2% per year ***with semi-annual compounding***, what will be its future value in two years?

"Compounding" means that you receive partial interest *more often* than just once per year.

2% per year with semi-annual compounding means 1% every 6 months

End of:	1st 6 months:	FV = \$100 $(1 + 0.01)^1$ = \$101
	2nd 6 months:	FV = \$101 $(1 + 0.01)^1$ = \$102.01
	3rd 6 months:	FV = \$102.01 $(1 + 0.01)^1$ = \$103.03
	4th 6 months:	FV = \$103.03 $(1 + 0.01)^1$ = \$104.06

Equivalently: FV = 100 $(1.01)^4$ = 104.06

Future Value with Compounding

General formula for Future Value with Compounding:

$$FV = PV\,(1 + r/m)^{t*m} \quad = \; PV\,(1 + q)^{n}$$

Where: *r* is the annual (or "quoted") rate,

t is the number of years

m is the number of compounding periods per year

q is the interest rate per compounding period (= r/m)

n is the number of compounding periods (= t*m)

y is the "Effective Annual Rate" = $(1 + q)^{m} - 1$

Examples: Future value of $100 after 1 year with 4% *quoted* rate, with different *compounding* periods:

no compounding	FV = 100 (1.04) = $104	y = 4.00%
semi-annual compounding	FV = 100 $(1.02)^{2}$ = $104.04	y = 4.04%
Monthly compounding	FV = 100 $(1.0033)^{12}$ = $104.07	y = 4.07%

Future Value with Compounding

In some settings, we use *continuous* compounding:

$FV = PV\,(1 + r/m)^{t*m}$ where $m \rightarrow \infty$ (∞ = *infinity,* i.e., very large!)

$= PV\, e^{r}$

For continuous compounding over *t* years:

$$FV = PV\, e^{rt}$$

Example: $500 invested for 3 years at 5% with continuous compounding:

$FV = 500\, e^{0.05*3}$

$= 580.92$

Compare with $500 invested for 3 years at 5% with *no* compounding:

$FV = 500\,(1.05)^{3} = \$578.81$

Future Value of a Series of Identical Cashflows

Some financial investments pay a stream of cash flows at equally spaced intervals, and of an equal amount.

Example: You put $100 per month into a vacation fund at 6% per year, with monthly compounding. How much will you have available after one year?

$$FV = 100[(1 + 0.5\%)^{12} – 1] / 0.5\% = \$1{,}233.56$$

More generally: the future value of a series of identical cash flows of $*C* for *t* years at *r*% per year with *m* compounding periods per year is:

$$\begin{aligned} FV &= C\,[(1 + r/m)^{t*m} – 1] / (r/m) \\ &= C\,[(1 + q)^{n} – 1]/q \end{aligned}$$

where: C = size of each cash flow
q = periodic interest rate (= r / m)
n = total number of periods (t * m)

Future Value of a Series of Identical Cashflows

Example: Your financial advisor tells you that you'll need to have $2 million to fund your retirement. You plan to work for another 30 years before retiring. You will make 30 annual contributions to a pension plan. The first contribution will be made one year from now, and the last will be made 30 years from now, on the day you retire. How much will each contribution have to be to ensure that you have $2 million in your pension plan account on your retirement day if the pension plan guarantees a return of 5% p.a.?

FV = 2,000,000
r = 5%
C = ??

so $2{,}000{,}000 = C\ [(1.05)^{30} - 1]/0.05$

Hence C = $30,102.87

Present Value

Example:
Receive \$100 in two years time. What is that worth today?

$FV = 100$, $t = 2$, $r = 2\%$

$$PV\,(1 + 0.02)^2 = 100$$

hence $$PV = \frac{100}{(1.02)^2}$$

$$= 96.12$$

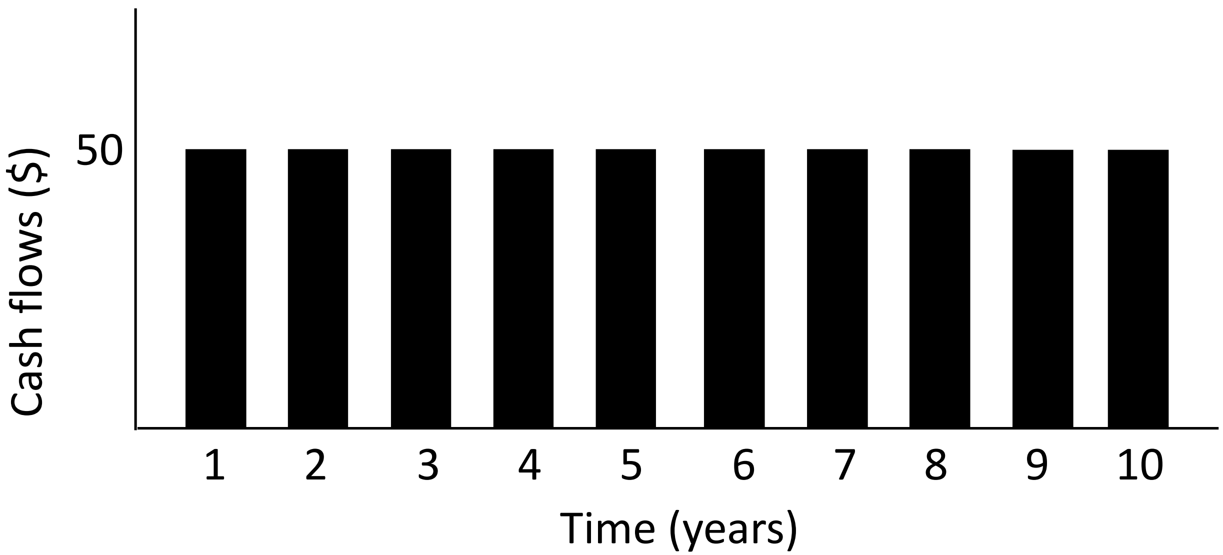
Generalize:

$$\boxed{PV = \frac{FV}{(1 + r)^t}}$$

We call r the "discount rate" since we are *discounting* the future cash flow to today

Present Value with Multiple Cash Flows

Example: identical annual payment of \$50 each year for 10 years
Annual discount rate = 6%



$$P = \frac{50}{(1.06)} + \frac{50}{(1.06)^2} + \frac{50}{(1.06)^3} + ... + \frac{50}{(1.06)^{10}} = 368.00$$

Multiple Cash Flows

$$P \;=\; \frac{50}{(1.06)} + \frac{50}{(1.06)^2} + \frac{50}{(1.06)^3} + \ldots. + \frac{50}{(1.06)^{10}} \;=\; 368$$

$$= \frac{50}{0.06}\,[1 - (1.06)^{-10}]$$

$$= \;368.00$$

Formula for identical cash flows *C* every year for *t* years with discount rate *r*:

$$\boxed{PV = \frac{C}{r}\,[1 - (1 + r)^{-t}]}$$

Multiple Cash Flows and Compounding

Example: identical payment of $20 every six months for 5 years
Annual discount rate = 6% *with semi-annual compounding*

$$P = \frac{20}{(1.03)} + \frac{20}{(1.03)^2} + \frac{20}{(1.03)^3} + \dots + \frac{20}{(1.03)^{10}}$$

$$= \frac{20}{0.03}[1 - (1.03)^{-10}]$$

$$= 170.60$$

$$PV = \frac{C}{r/m}\left[1 - (1 + r/m)^{-t*m}\right] = \frac{C}{q}\left[1 - (1+q)^{-n}\right]$$

where $q = r/m$, $n = t * m$

Example: Mortgage Payments

Suppose you take out a 30 year mortgage loan of $400,000. The loan has a annual rate of 7.45%, fixed over the life of the loan, and compounded *monthly*. What is the fixed monthly payment required to pay off the loan?

For this example, we wish to solve for *C* given

$q = \frac{7.45\%}{12} = 0.62\%$ $\quad n = 30 * 12 = 360$ $\quad \text{PV} = 400{,}000.$

$$400{,}000 = \frac{C}{0.0062}\left[1 - (1.0062)^{-360}\right]$$

Hence C = $2,780.44

Present Value of a Perpetuity

A perpetuity is a stream of cashflows that continue forever. The formula for the Present Value of a perpetuity is:

$$PV = C / r$$

Example 1: In the early 1900's the Canadian Government issued $100 par value 2% Consol bonds. The holder of these bonds is entitled to receive a coupon (or interest) payment of $2 per year forever. If the current appropriate discount rate is 5% p.a. and the next coupon is due one year from now, how much is one of the Consols worth?

$$\begin{aligned} PV &= \$2 / 5\% \\ &= \$40 \end{aligned}$$

Example 2: Your company anticipates the introduction of environmental protection laws in three years time. Under these laws you will have to pay a yearly environmental tax of $5,000, an obligation that will continue indefinitely. If the prevailing interest rate is 6% p.a. what is the present value of your company's obligations under this law (the first payment will be four years from now)?

$$PV_3 \text{ (\textit{as of 3 years from now})} = 5{,}000/0.06 = \$83{,}333.33$$

Hence PV_0 *(as of today)*

$$\begin{aligned} &= 83{,}333.33 / (1.06)^3 \\ &= 69{,}968.27 \end{aligned}$$

Future Value and Present Value Formulas

Future value of a single cash flow: $FV = PV\,(1 + r)^t$

Future value of a cash flow when interest rates are compounded: $FV = PV\,(1 + r/m)^{t*m} = PV\,(1 + q)^n$

Future value of a cash flow with continuous compounding: $FV = PV\,e^{rt}$

Future value of a stream of identical cash flows *C* each year until time *t*: $FV = C\,[(1 + r)^n – 1]/r$

Present Value of a single future cash flow: $PV = FV / (1 + r)^t$

Present value of a future cash flow when interest rates are compounded: $PV = FV/ (1 + r/m)^{t*m}$
$= FV\,(1 + q)^n$

Present value of a cash flow with continuous compounding: $PV = FV\,e^{-rt}$

Present value of a perpetuity: $PV = C/r$

r is the annual (or "quoted") rate,
t is the number of years
m is the number of compounding periods per year
q is the interest rate per compounding period (= r/m)
n is the number of compounding periods (= t*m)
y is the "Effective Annual Rate" = $(1 + q)^m - 1$