Visualisation of four limit cycles (adaptation)

Table of Contents

1) Reproduction of 4 limit cycle example by Kuznetsov et al.

Resolution of a proposed case with an implementation of Runge-Kutta 4.

```
% Initial conditions (t=0)
global a1 b1 c1 al1 bt1 a2 b2 c2 al2 bt2
```

```
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```
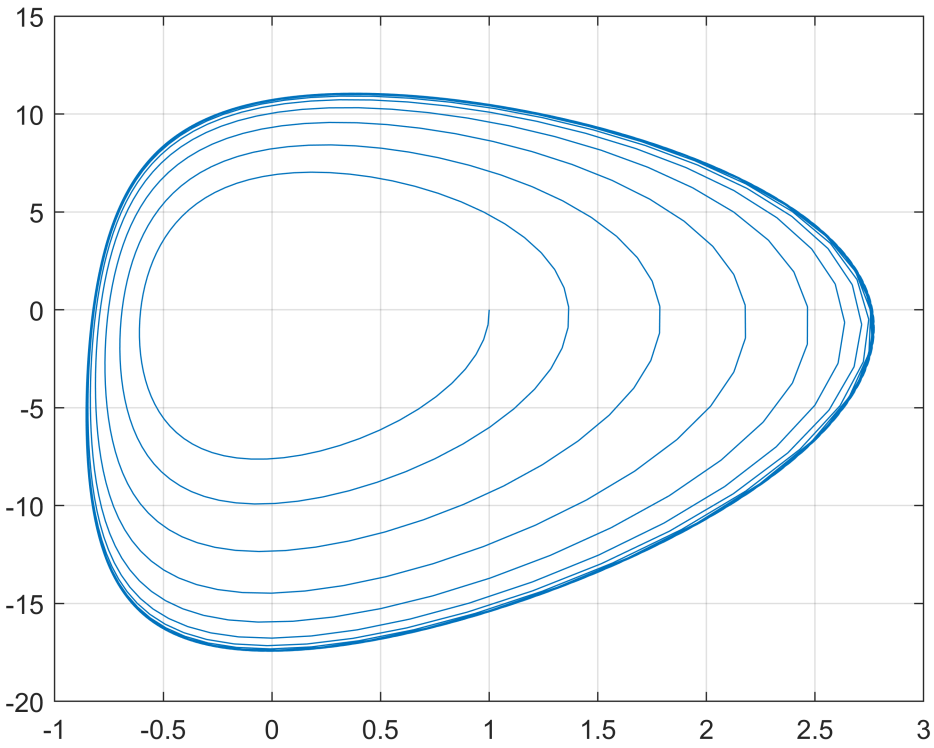

```

a1=1;b1=1;c1=0;al1=0;bt1=1;
a2=-10;b2=2.7;c2=0.4;al2=-437.5;bt2=0.003;

h=0.01;
tf=30;
N=ceil(tf/h);

t=zeros(1,N+1);
x=zeros(1,N+1);
y=zeros(1,N+1);

t(1)=0;
x(1)=1;
y(1)=0;
```

```
[x,y] = RK4(t,x,y);
```

2) Grid in (x,y) plane

```
plot(x,y)
grid on
hold on

% t(1)=0;
% x(1)=3;
% y(1)=0;
%
% [x,y] = RK4(t,x,y);
%
% plot(x,y)
% grid on
hold off
```



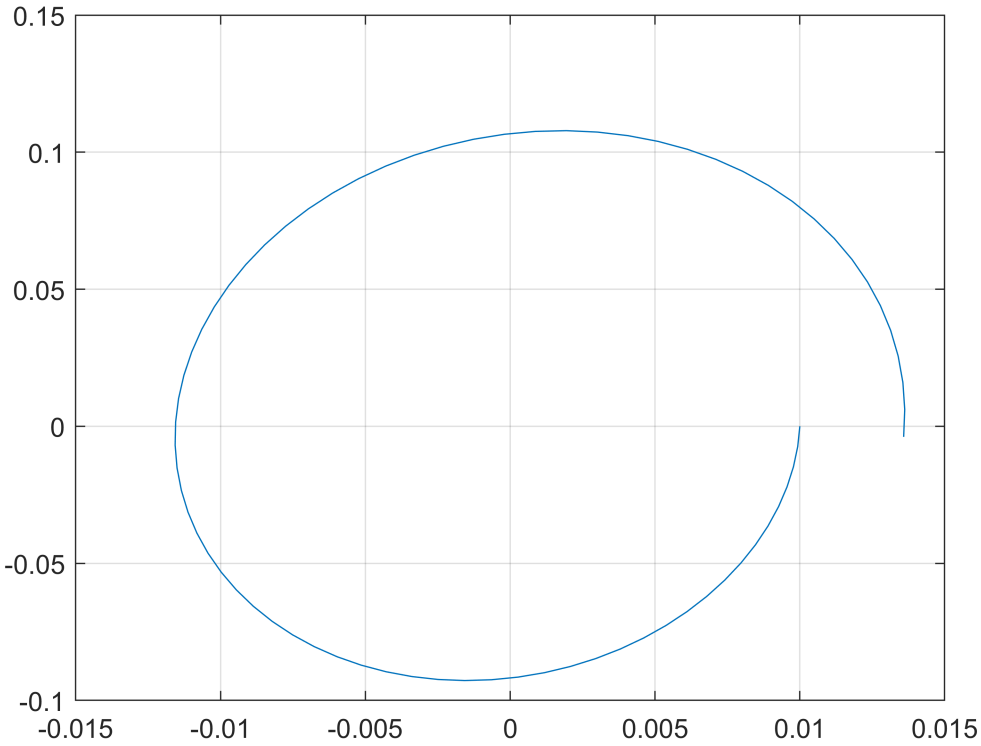
3) Time integration

```
a1=1;b1=1;c1=0;al1=0;bt1=1;
a2=-10;b2=2.7;c2=0.4;al2=-437.5;bt2=0.003;
```

```
h=0.01;
i=1;
x=[]; x(1) = 0.01;
y=[]; y(1) = 0;


[x,y] = RK4cycle(x,y,h);
```

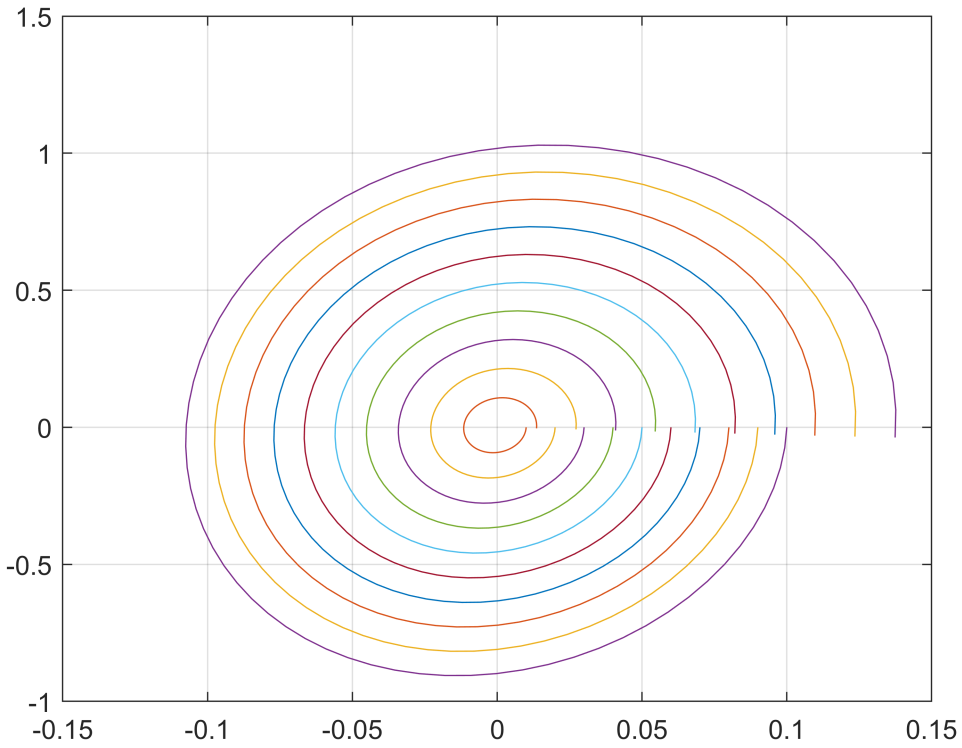
Plot x(t) vs x(t+2)-x(t)

```
 plot(x,y,"-")
 grid on
```

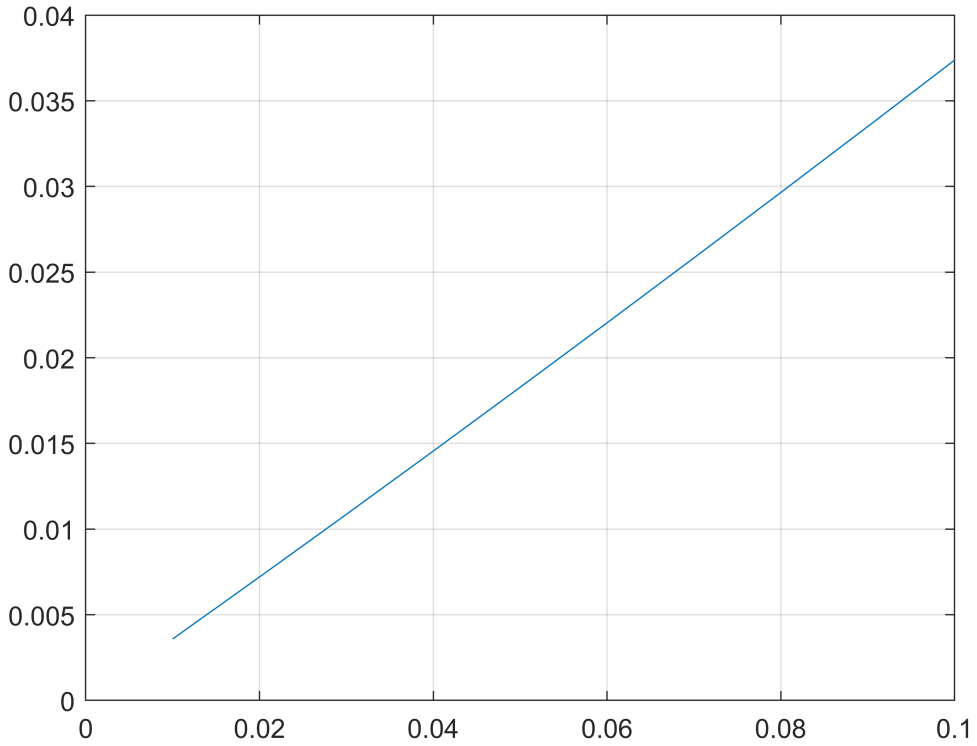



```
a1=1;b1=1;c1=0;al1=0;bt1=1;
a2=-10;b2=2.7;c2=0.4;al2=-437.5;bt2=0.003;

h=0.01;
x2=[];
xdif=[];
hold on
grid on
for i=1:1:10
    h=0.01;
    x=[]; x(1) = i*h;
    y=[]; y(1) = 0;
    [x,y] = RK4cycle(x,y,h);
```

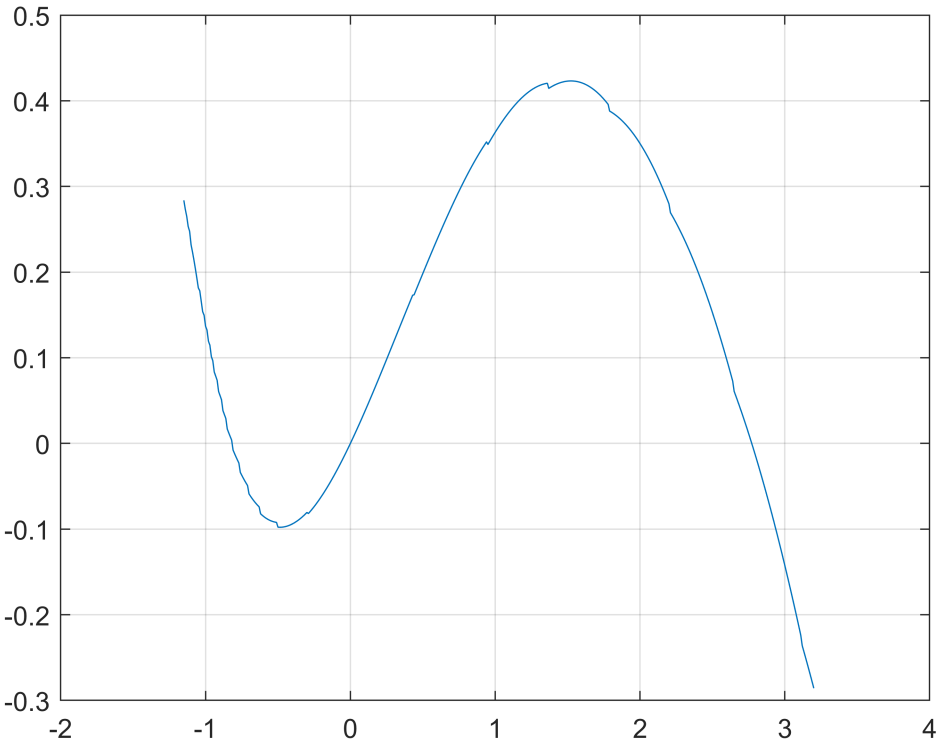
```
    x2(i)=i*h;
    xdif(i)=x(end)-x(1);
    plot(x,y,"-")
end
hold off
```



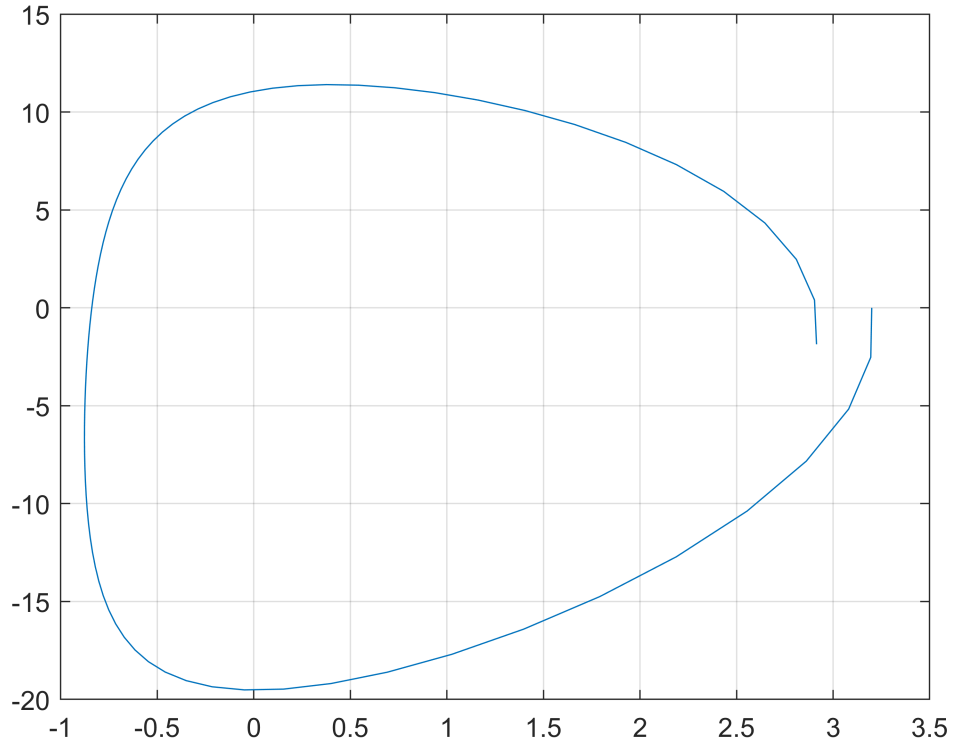

```
plot(x2,xdif);
grid on
```




```
h=0.01;
x2=[];
xdif=[];
j=1;
for i=-115:1:320
    h=0.01;
    x=[]; x(1) = i*h;
    y=[]; y(1) = 0;
    [x,y] = RK4cycle(x,y,h);
    x2(j)=i*h;
    xdif(j)=x(end)-x(1);
    j=j+1;
end
plot(x2,xdif);
grid on
```

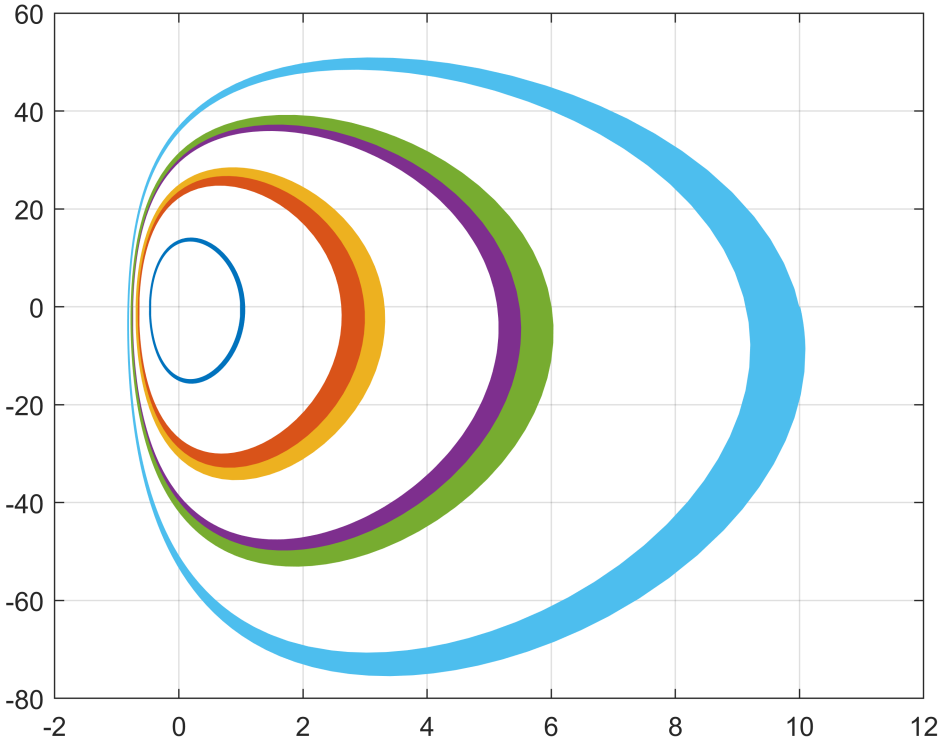



```
plot(x,y)
grid on
```



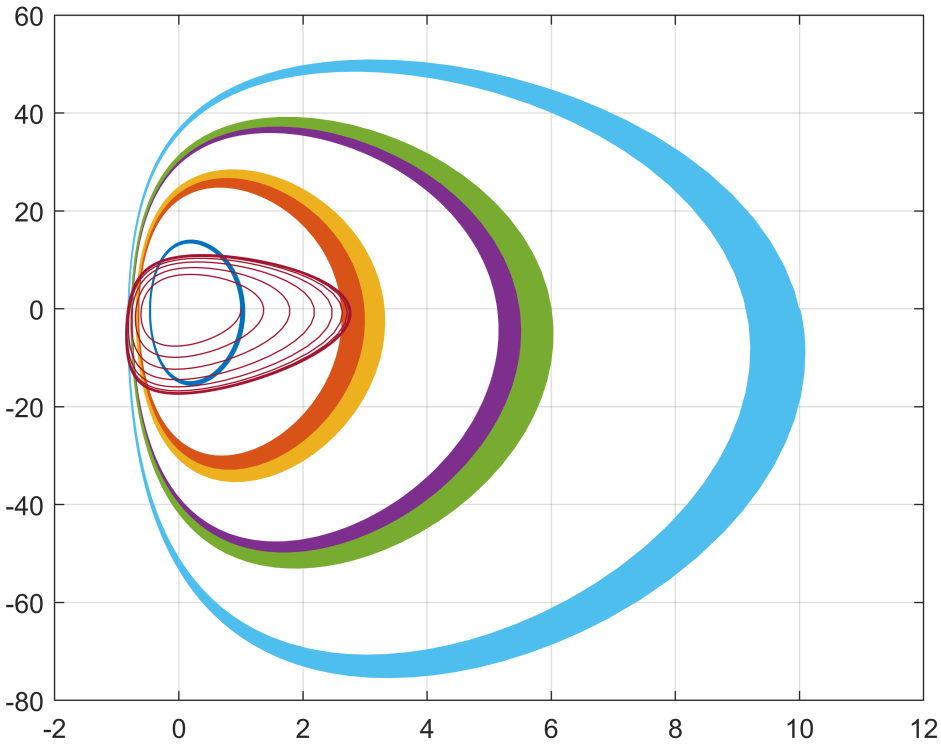
Adaptation of Kuznetsov et al.

```
clear all; syms x
global a1 b1 c1 al1 bt1 a2 b2 c2 al2 bt2

a1=1;b1=1;c1=0;al1=0;bt1=1;
a2=-10;b2=2.7;c2=0.4;al2=-437.5;bt2=0.003;
x0_1=1; x0_2=3; x0_3=5.5; x0_4=10;
acc=0.00001; y0_1=-2000; y0_2=-4000;
len1=40*pi; len2=70*pi; len3=-30*pi; len4=-30*pi; len5=25*pi; len6=15*pi; len7=-1*pi; len8=-1*p

RelTol=acc; AbsTol=acc; InitialStep=acc;
options=odeset('RelTol', RelTol ,'AbsTol', AbsTol, 'InitialStep', InitialStep, 'NormControl','o


x0=x0_1 ; y0=0;[T, XY]= ode45(@fQsys, [0 len1], [x0 y0], options);
plot(XY(: ,1), XY(: ,2));
hold on; grid on;
x0=x0_2 ; y0=0;[T, XY]= ode45(@fQsys, [0 len2], [x0 y0], options);
plot(XY(: ,1), XY(: ,2));
hold on; grid on;

x0=x0_2 ; y0=0;[ T, XY ]= ode45(@fQsys ,[0 len3 ] ,[ x0 y0 ], options);
plot(XY(: ,1), XY(: ,2));
hold on; grid on;
x0=x0_3 ; y0=0;[ T, XY ]= ode45(@fQsys ,[0 len4 ] ,[ x0 y0 ], options);
plot(XY(: ,1), XY(: ,2));
hold on; grid on;

x0=x0_3 ; y0=0;[T, XY]= ode45(@fQsys ,[0 len5] ,[x0 y0], options);
plot(XY(: ,1), XY(: ,2));
hold on; grid on;
x0=x0_4 ; y0=0;[T, XY]= ode45(@fQsys ,[0 len6] ,[x0 y0], options);
plot(XY(: ,1), XY(: ,2));
hold on; grid on;
```



Result comparison

```
h=0.01;
tf=30;
N=ceil(tf/h);

t=zeros(1,N+1);
x=zeros(1,N+1);
y=zeros(1,N+1);

t(1)=0;
x(1)=1;
y(1)=0;

[x,y] = RK4(t,x,y);
plot(x,y); grid on; hold off
```



Functions

```
function dz=fQsys(t, z)
    global a1 b1 c1 al1 bt1 a2 b2 c2 al2 bt2
    dz=zeros(2,1); % z=(z(1),z(2))=(x,y)

    dz(1)=(a1 * z(1)^2+ b1 * z(1)* z(2)+ c1 * z(2)^2+ al1 * z(1)+ bt1 * z(2));
    dz(2)=(a2 * z(1)^2+ b2 * z(1)* z(2)+ c2 * z(2)^2+ al2 * z(1)+ bt2 * z(2));
end

function [x,y] = RK4(t,x,y)

    global a1 b1 c1 al1 bt1 a2 b2 c2 al2 bt2

    % ODE
    dxdt=@(t,x,y) a1*x^2 + b1*x*y + c1*y^2 + al1*x + bt1*y;
    dydt=@(t,x,y) a2*x^2 + b2*x*y + c2*y^2 + al2*x + bt2*y;

    % Step size
    h=0.01;
    tf=30;
    N=ceil(tf/h);

    % Runge-Kutta 4
```

```
        for i=1:N
            t(i+1)=t(i)+h;

            k1x=dxdt(t(i), x(i), y(i));
            k1y=dydt(t(i), x(i), y(i));

            k2x=dxdt(t(i)+h/2, x(i)+h/2*k1x, y(i)+h/2*k1y);
            k2y=dxdt(t(i)+h/2, x(i)+h/2*k1x, y(i)+h/2*k1y);

            k3x=dxdt(t(i)+h/2, x(i)+h/2*k2x, y(i)+h/2*k2y);
            k3y=dxdt(t(i)+h/2, x(i)+h/2*k2x, y(i)+h/2*k2y);

            k4x=dxdt(t(i)+h, x(i)+h*k3x, y(i)+h*k3y);
            k4y=dxdt(t(i)+h, x(i)+h*k3x, y(i)+h*k3y);

            x(i+1)=x(i) + h/6*(k1x+2*k2x+2*k3x+k4x);
            y(i+1)=y(i) + h/6*(k1y+2*k2y+2*k3y+k4y);
        end
end

function [x,y] = RK4cycle(x,y,h)

    global a1 b1 c1 al1 bt1 a2 b2 c2 al2 bt2

    % ODE
    dxdt=@(t,x,y) a1*x^2 + b1*x*y + c1*y^2 + al1*x + bt1*y;
    dydt=@(t,x,y) a2*x^2 + b2*x*y + c2*y^2 + al2*x + bt2*y;

    % Runge-Kutta 4

        for i=1:1:2

            k1x=dxdt(i*h, x(i), y(i));
            k1y=dydt(i*h, x(i), y(i));

            k2x=dxdt(i*h+h/2, x(i)+h/2*k1x, y(i)+h/2*k1y);
            k2y=dxdt(i*h+h/2, x(i)+h/2*k1x, y(i)+h/2*k1y);

            k3x=dxdt(i*h+h/2, x(i)+h/2*k2x, y(i)+h/2*k2y);
            k3y=dxdt(i*h+h/2, x(i)+h/2*k2x, y(i)+h/2*k2y);

            k4x=dxdt(i*h+h, x(i)+h*k3x, y(i)+h*k3y);
            k4y=dxdt(i*h+h, x(i)+h*k3x, y(i)+h*k3y);

            x(i+1)=x(i) + h/6*(k1x+2*k2x+2*k3x+k4x);
            y(i+1)=y(i) + h/6*(k1y+2*k2y+2*k3y+k4y);

        end

        i=3;
```

```
while y(i)*y(i-1)>0

    k1x=dxdt(h*i, x(i), y(i));
    k1y=dydt(h*i, x(i), y(i));

    k2x=dxdt(h*i+h/2, x(i)+h/2*k1x, y(i)+h/2*k1y);
    k2y=dxdt(h*i+h/2, x(i)+h/2*k1x, y(i)+h/2*k1y);

    k3x=dxdt(h*i+h/2, x(i)+h/2*k2x, y(i)+h/2*k2y);
    k3y=dxdt(h*i+h/2, x(i)+h/2*k2x, y(i)+h/2*k2y);

    k4x=dxdt(h*i+h, x(i)+h*k3x, y(i)+h*k3y);
    k4y=dxdt(h*i+h, x(i)+h*k3x, y(i)+h*k3y);

    x(i+1)=x(i) + h/6*(k1x+2*k2x+2*k3x+k4x);
    y(i+1)=y(i) + h/6*(k1y+2*k2y+2*k3y+k4y);
    i=i+1;
end

j=i;

for i=i:1:i+2

    k1x=dxdt(i*h, x(i), y(i));
    k1y=dydt(i*h, x(i), y(i));

    k2x=dxdt(i*h+h/2, x(i)+h/2*k1x, y(i)+h/2*k1y);
    k2y=dxdt(i*h+h/2, x(i)+h/2*k1x, y(i)+h/2*k1y);

    k3x=dxdt(i*h+h/2, x(i)+h/2*k2x, y(i)+h/2*k2y);
    k3y=dxdt(i*h+h/2, x(i)+h/2*k2x, y(i)+h/2*k2y);

    k4x=dxdt(i*h+h, x(i)+h*k3x, y(i)+h*k3y);
    k4y=dxdt(i*h+h, x(i)+h*k3x, y(i)+h*k3y);

    x(i+1)=x(i) + h/6*(k1x+2*k2x+2*k3x+k4x);
    y(i+1)=y(i) + h/6*(k1y+2*k2y+2*k3y+k4y);

end

i=j+2;
while y(i)*y(i-1)>0

    k1x=dxdt(h*i, x(i), y(i));
    k1y=dydt(h*i, x(i), y(i));

    k2x=dxdt(h*i+h/2, x(i)+h/2*k1x, y(i)+h/2*k1y);
    k2y=dxdt(h*i+h/2, x(i)+h/2*k1x, y(i)+h/2*k1y);
```

```
            k3x=dxdt(h*i+h/2, x(i)+h/2*k2x, y(i)+h/2*k2y);
            k3y=dxdt(h*i+h/2, x(i)+h/2*k2x, y(i)+h/2*k2y);

            k4x=dxdt(h*i+h, x(i)+h*k3x, y(i)+h*k3y);
            k4y=dxdt(h*i+h, x(i)+h*k3x, y(i)+h*k3y);

            x(i+1)=x(i) + h/6*(k1x+2*k2x+2*k3x+k4x);
            y(i+1)=y(i) + h/6*(k1y+2*k2y+2*k3y+k4y);
            i=i+1;
        end
end
```