Optimal Coordination of Vehicles at Traffic Intersections

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Introduction

Coordination of automated vehicle at intersection is described as using vehicle automation and communication, the number of accidents in such scenarios can be reduced and both energy efficiency and infrastructure utilization can be improved. The first thing we focus on the case that control the automated vehicle across the intersection without collision. Just like the Illustration, Several cars running on the road are going to across the intersection without any traffic lights. What we will do is cooperatively deciding a policy can employed on vehicle to explicitly coordinate the use of the zone where collisions can occur (e.g., the inside of the intersection). In the fully automated case, such control systems would remove the need for traffic lights, signs and rules and thereby enable continuous flows of traffic.

In particular, we specify this problem with several assumptions:

1. The precedence order is given
2. No more than one vehicle at intersection all the time and assure the vehicles go through one by one.
3. Minimal changes from initial states(The strong limit of acceleration and as small change from reference velocity as possible).

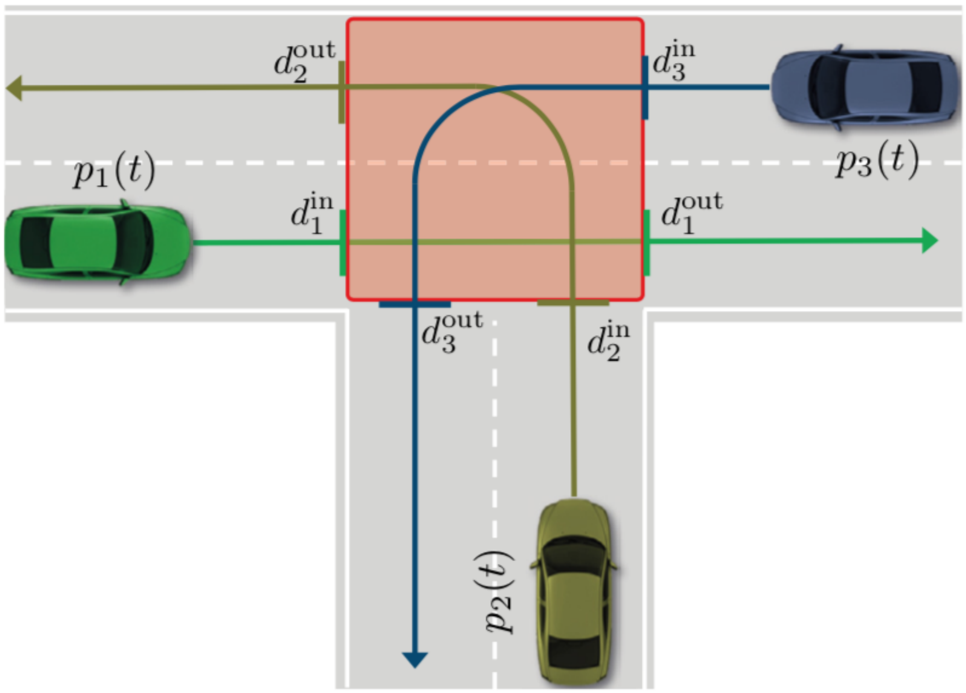

Figure 1: Schematic illustration of the intersection scenarios

Problem Formulation

A) Collision Avoidance

The intersection can be model as a interval (p_{in}, p_{out}),then for each vehicle i on the intersection at time t can be expressed like that: $p_i(t) \in [p_{in}, p_{out}]$. From the previous assumption to avoid collision, we have the first condition

$$(p(t_{i,in}), p(t_{i,out}))^T = (p_{in}, p_{out})^T$$

$$t_{i,in} \geq t_{i-1,out}$$

B) Newton's Law of motion

Apply the Newton's law of motion for each vehicle i, we have the second part of constrain:

$$\dot{v} = a$$

$$\dot{p} = v + at$$

C) Objective Function

We add a additional term of $t_{out} - t_{in}$ to minimize the cost time for each vehicle to across the intersection.In order to Minimize the changes from initial states, we have the objective function in quadratic form :

$$min \int_0^t ||v - v_{ref}||_Q^2 + ||a||_R^2 + \underline{||t_{out} - t_{in}||_V^2}$$

Then , the full finite horizon optimal control problem is:

$$\begin{aligned} &\min \int_0^t ||v - v_{ref}||_Q^2 + ||a||_R^2 + \underline{||t_{out} - t_{in}||_V^2} \\ &s.t. \ (p(t_{i,in}), p(t_{i,out}))^T = (p_{in}, p_{out})^T \\ &\qquad t_{i,in} \geq t_{i-1,out} \\ &\qquad \dot{v} = a \\ &\qquad \dot{p} = v + at \end{aligned}$$

Solution Method

I will use Linear-Quadratic Regulator as a main tool to solve the optimal control problem.I first discretize the continuous-time optimal problem into the discrete-time optimal problem and use the solver in $CasADi$ to solve the discretized nonlinear programing problem. The discretized problem is that:

$$\begin{aligned} &\min \sum_{k=0}^{N-1} Q(v_i^{ref} - v_{i,k})^2 + Ra_{i,k}^2 + \underline{V(t_{out} - t_{in})^2} \\ &s.t. \ \ (p(t_{i,in}), p(t_{i,out}))^T = (p_{in}, p_{out})^T; t_{i,in} \geq t_{i-1,out} \\ &\qquad v_{i,k+1} = v_{i,k} + Ta_{i,k}; \ \ p_{i,k+1} = p_{i,k} + Tv_{i,k} + \frac{1}{2}Ta_{i,k}^2 \end{aligned}$$

Just like the formula shows, the input parameters for this solver is the $T_{1,in}, T_{i,out}$, $a_{i,k}$,$v_{i,k}$,$p_{i,k}$ for each vehicle in the discretized time interval $[0, T_{out}]$.

Numerical Results

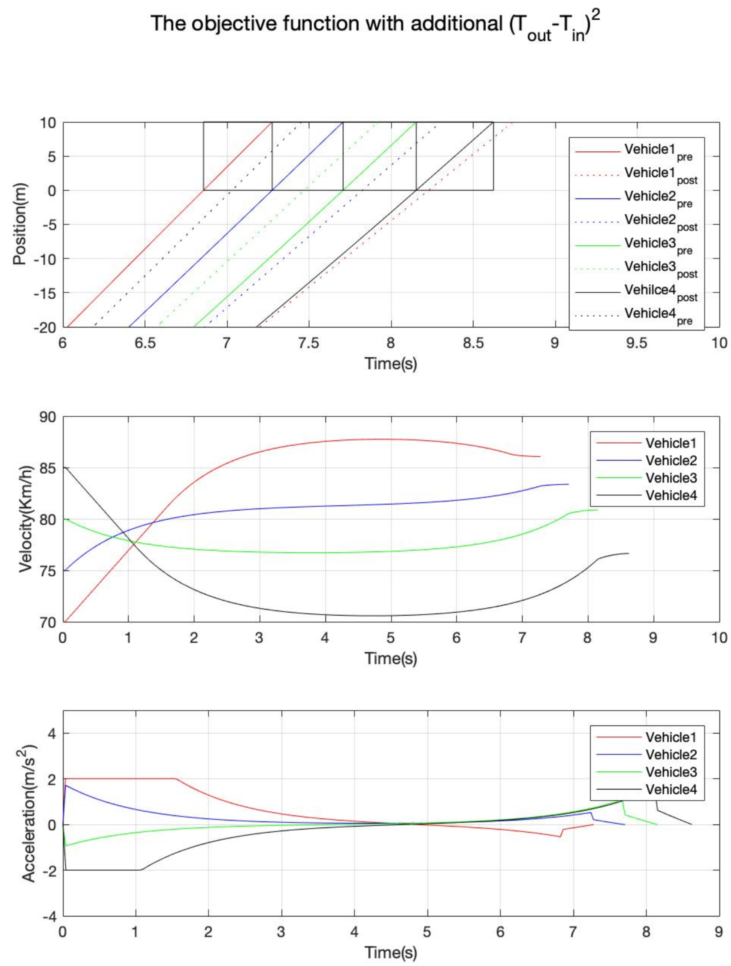

Figure 2: Numerical result

Conclusion

From the above numerical result, we have seen the performance of the each vehicle's state under our control. Those lead us to several conclusion:

- The only thing we control of the vehicles is acceleration, and it can have a great impact on this problem. But there is the need to strong constrain to avoid collision which means more time to coordinate them.
- Setting a central coordination controller is not practical for numerous vehicles, then we need a distributed algorithm for this problem.