

HOW VARIANCE SWAPS CAN EXPLAIN OI IN FAR OTM PUTS

A DUE DILIGENCE



BY U/MAUERAASTRONAUT

How Variance Swaps can explain OI in far OTM Puts and many other of the Weirdnesses that were observable this year. - Nov. 7 ,2021

Due Diligence

I have no education in Finance whatsoever, I am just someone on the internet who likes maths and hard problems. Thus, I will also focus more on the maths side, because I am rather smooth regarding accounting, legal or trading topics.

This post is a little rushed. I wanted to be more thorough, but especially since [u/gherkinit](#) and [u/Criand](#) referred to [u/Zinko83](#) (my brother in arms since the beginning of my quant journey; I do the math shit, he does the modeling using a commercial toolbox) and myself multiple times by now, the interest in my previous posts and our blowing-up inboxes prompted us to pump this out now. And while I am writing this, Zinko rushed on me as well. If you've read my Cheatsheet, please be aware that some of the opinions I expressed there I no longer consider to be perfectly accurate. I apologize that I wasn't really active on Reddit much, but I promise that I'll at least try to get back to everyone who contacted me or had questions since my last post.

This post will clean up with many beliefs I've seen apes express even until recently. However, it is not a bear thesis, so I guess no \$1100 bounty for me. Instead, we believe that it can serve as a unifying theory for many idiosyncracies and weirdnesses we were able to observe.

You can find some of our sources at the end of this post.

If you are ape that cannot read, look for text in italics, at least until the GME section.

Introduction

Ever since the January Sneeze, many people wondered where short interest on the stock went, why there was this incredible open interest, especially in far OTM puts (but also calls), how HFs like Melvin Capital, who claimed to have closed their short positions, managed to sustain losses afterwards, and how a gigawhale like Citadel could be threatened by a short position in essentially one idiosyncratic stock.

It was speculated that this was due to naked short selling, and since there was a missing explanation for the far OTM puts it was speculated that these were used to hide naked shorts.

However, not-so-recent theories argued that SHFs were, in fact, not naked short, but unloaded their bags onto Market Makers and Prime Brokers instead. I believe that to be true and will also be providing a mechanism that allows them to stay naked based on deemed-to-own regulations.

Many of the things that happened this year can be explained by players like Citadel Advisors being short variance swaps. Variance swaps can explain pretty much the entire options chain in GME, why the stock did, and sometimes still does, experience major intraday swings to only close slightly up or down for the day, why GME and many other stocks (especially from the small and mid cap sector) experienced major volatility and constant selloff patterns that sometimes spanned months, losses for SHFs after allegedly closing their positions, and many other things.

However, I have not found an explanation for certain correlations, like Shitty Floors stock, that would satisfy me. Such an unsatisfying answer would be "manipulation", mostly because markets are efficient. As such, I am still looking for a good hypothesis that I can base on arbitrage or the delta-hedging of certain derivatives.

Background

What are variance swaps?

Variance swaps are an instrument that whales use to bet on or against volatility of an underlying. Also, even retail has direct access to the volatility of some products, for instance through VIX.

In stochastics, the variance is the expected squared deviance, usually called Standard Deviation, of some random variable from the mean. In finance, the deviance is called volatility, while its square, like in stochastics, is called variance. The expected volatility/variance is commonly called Implied Volatility/Variance (IV/IVar), while the vol/var that actually happened over a period is called Realized Volatility/Variance (RV/RVar). *Wut mean? Ape believe banana value will go up and down a lot, so IV high. However, banana did not go up or down a lot, so RV low.*

A variance swap is a forward contract that pays the RV (minus some strike) at maturity. The RV is proportional to the sum of daily, squared log returns. *Ape take percentage relative to previous day (1.04 for a 4% gain), apply logarithm, then power of 2, then sum up.*

Who is interested in longing/shorting volatility? (Note that I might be using volatility and variance interchangeable because they basically refer to the same thing, except when talking about specific products.)

Many market participants are interested in buying volatility. Many reasons involve hedging (against a crash, for instance), some others gambling or the incorporation into specific strategies. Buying options can also be considered long volatility. Generally, long volatility products are unprofitable, because most of the time markets are relatively calm and not volatile. *Ape bet on banana variance, but banana value do not go up and down a lot, so ape lose.*

Which brings us to the sellers of volatility, because this is highly profitable. In fact, it is twice as profitable as stonks (risk-adjusted---meaning very favourable Sharpe-Ratio) and can, for instance, replace a major portion of bonds in the standard equity-bond portfolio to further enhance its risk-adjusted returns. Why? Because most of the time markets are calm without much volatility. Many sellers of volatility are HFs. Think of it like insurance: Most of the time nothing happens and the premiums are free money. But if something happens, you have to pay a lot. *Whale bet against banana variance and banana value do not go up and down a lot, so whale win.*

How would you hedge being short variance?

There are several possibilities:

1. You don't. Markets are calm most of the time, and since you have tons of experience, quants and sometimes HFT, you feel confident that, overall, you will survive major volatility.
2. You hedge by buying ATM options (mostly puts) and rebalance when the stock moves/is expected to move. Most short volatility funds do this. (I call it lazy hedging.)
3. You buy a replicating portfolio (RP) of options, extremely heavy to the put side. This is very uncommon, because maintaining this portfolio is eating most, if not all, of the premiums that you were paid.

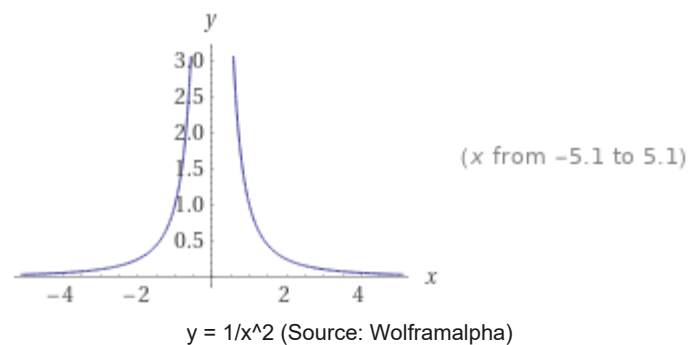
The Replicating Portfolio

This portfolio was introduced by Demeterfi in 1999. It is my impression that this work changed modern finance forever (it is also cited by every single paper that talks about volatility products), so if you've got math skills, I highly recommend you take a look. For the RP you have to choose a boundary strike S^* (the expected stock price at maturity). It is not

super important, but influences the size of your cash position and the portfolio value at maturity. The RP consists of several parts (options and futures on the underlying you want to hedge):

1. Static long position in European options with the same maturity as the swap over the entire range of strikes (from zero to infinity), weighted $1/K^2$, meaning the inverse square of the strike price. Below the boundary strike buy puts, above buy calls.
2. Static short position in futures/forwards, weighted $1/S^*$, meaning the inverse of the boundary strike. *Short less bananas with a higher strike.*
3. A dynamic long position in shares that is kept at a constant value. *Sell some bananas when bananas become expensive, buy bananas when they are cheap. Buy low, sell high.*
4. Cash (equivalents).

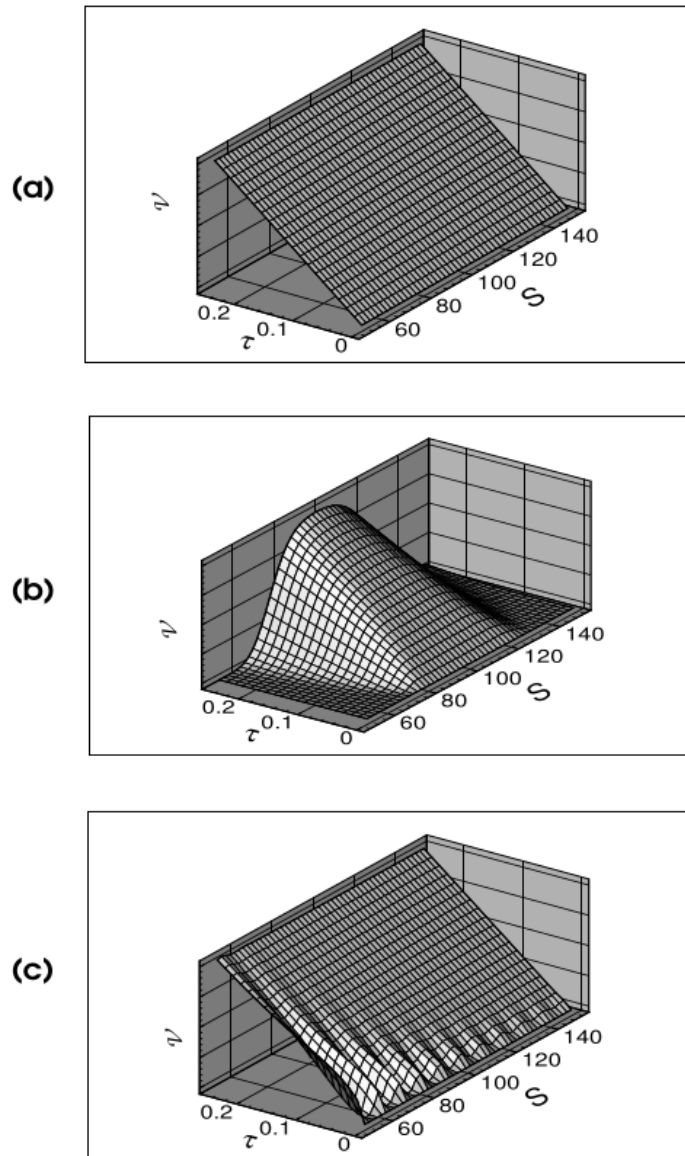
To illustrate the distribution, here is the plot of the function $1/x^2$ (ignore the part left of the y axis).



You may have noticed several things:

- I said something about infinity. This is why, in practice, you buy options over the entire available strike range, and then buy some more calls at the highest strike to hedge the distance to infinity.
- Stock options commonly are American. This, however, doesn't matter too much, because they converge to the same value at maturity.
- Yes, short futures/forwards. *Let me be clear: The RP is not a vehicle to hide SI. It is to hedge old short exposure to variance, where, coincidentally, short futures/forwards play a role. However, MMs could use those forwards as deemed-to-own to circumvent close-outs.*
- The RP value goes down as it nears maturity. This is true, its end value is proportional to the log return between the boundary strike and the stock price. The actual returns (for the payout) are generated by rebalancing the shares position EoD (or intraday when you feel like it) and storing the proceeds in cash. It is needed for mark-to-market (margin). *Whale keep RP so Prime Whale don't take away their fake bananas. Whale make money by playing banana value.*
- If you have basic knowledge of options, you might wonder: "Why not a straddle?" I'm glad you asked, because that brings us to Figure 3 of the Demeterfi paper.

FIGURE 3. The variance vega, \mathcal{V} , of a portfolio of puts and calls, weighted inversely proportional to the square of the strike level, and chosen to replicate a three-month variance swap. (a) An infinite number of strikes. (b) Strikes from \$75 to \$125, uniformly spaced \$1 apart. (c) Strikes from \$20 to \$200, uniformly spaced \$10 apart.



Distributions of vega: (a) Ideal, (b) Lazy hedging, (c) RP. (Source: Demeterfi, 1999.)

Since we are trying to hedge volatility with a portfolio of options, the greek we have to look at is **vega**. Vega represents the sensitivity of an option to changes in IV, and it is highest ATM. The axes in the above graphic are time to maturity, vega exposure and the stock price.

(a) This is the ideal vega distribution. Notice how it is not influenced by the stock price and goes down linearly with time. *Banana value do not change RP value, but banana value going up and down a lot do, but less at the end.* **(b)** The distribution you achieve by lazy hedging (which is roughly similar to straddles). Notice how exposure to vega goes down hard if the stock price comes near your strike limit (in the example, the range is 80 to 120). *This is super important. If banana value move outside of hedge, whale fuk.* **(c)** The distribution achieved by the RP. Notice that it has fluctuations that become more violent with declining stock price and closer to maturity. This happens because there are no fractional

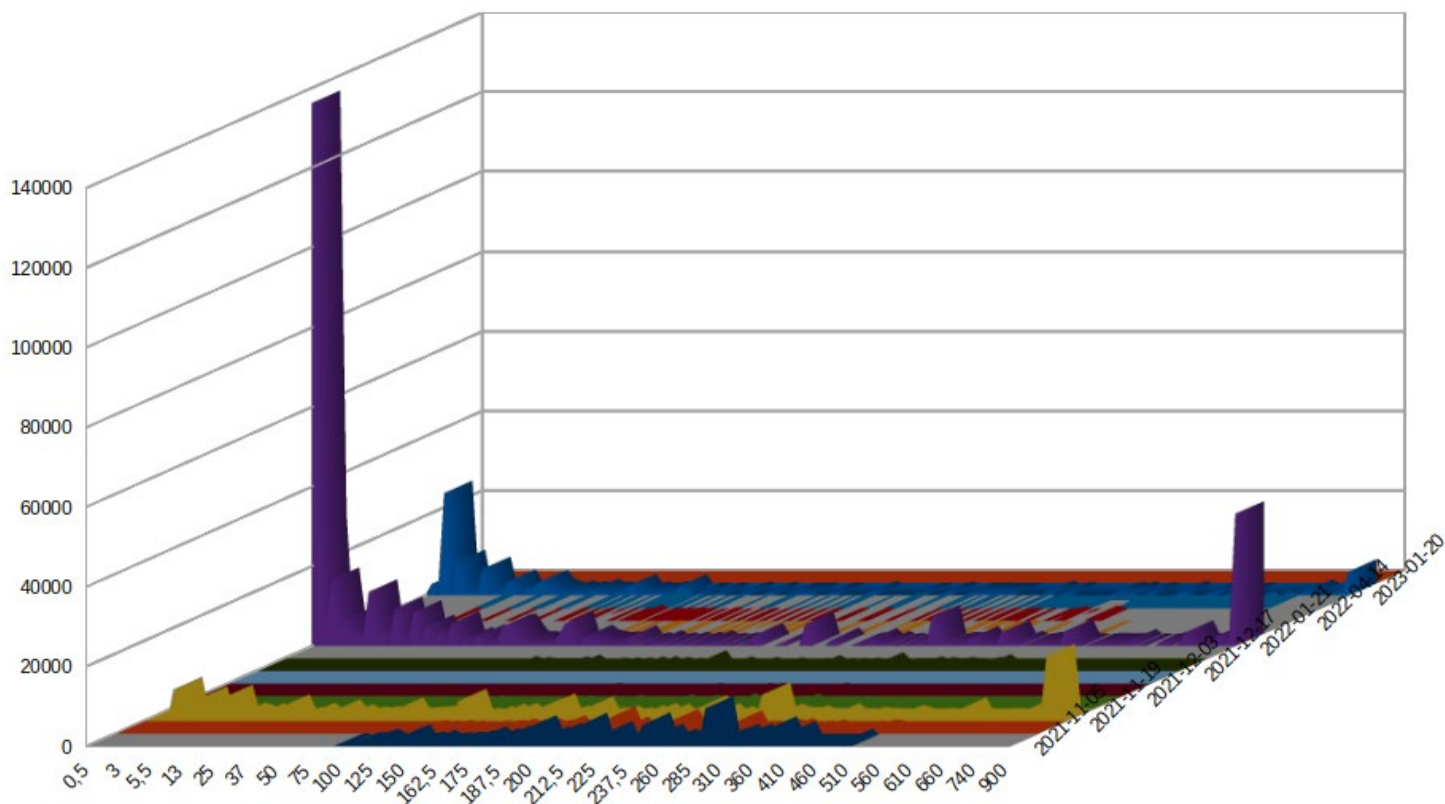
options contracts.

Properties of variance (swaps)

- Daily returns are squared. This implies that many small movements have less of an impact than one large movement and could help explain the long-ass selloffs we could observe on GME and many smaller companies. *Whale more fuk if banana value did go up or down by a lot, from close to close.*
- If trading on a stock slows (for instance after getting delisted), volatility and variance go down. For this reason we think it profitable to also sell variance swaps on companies you are trying to kill. *Selling banana variance make more money if banana become rotten.*
- IV tends to overestimate RV, so the RP is providing a cushion against RV.
- If the underlying moves outside of your strike range, you have a problem. You can try to hedge by buying more options, fast. *Again, if banana value move outside of hedge, whale fuk.*
- It is not easy to get out of variance swaps. Assuming you can find someone to sell one to you, going long a new variance swap does not hedge old exposure, only the future. *Whale remain fuk.*

Variance Swaps and GME

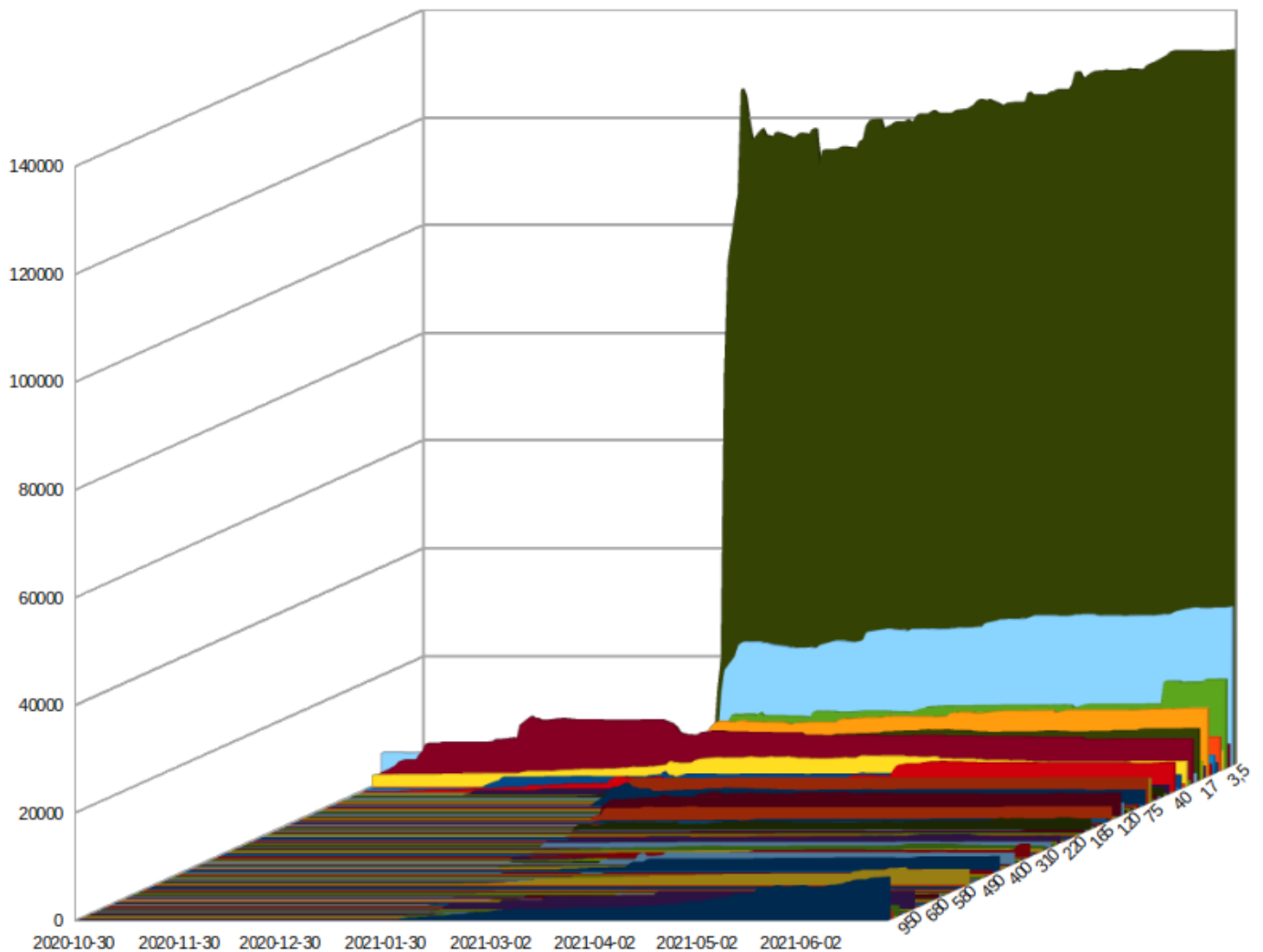
If you now wonder "What does this have to do with my favourite stonk?!", please remind yourself of that function graph I showed above, and the notion about buying calls at the highest strike to hedge against infinity. *Buckle up and also read non-italics text.*



OI on GME, 2021-11-04. (Source: own work)

Look at this shit. And make sure to do a double take to count the digits on the vertical axis (for the OI; the others are

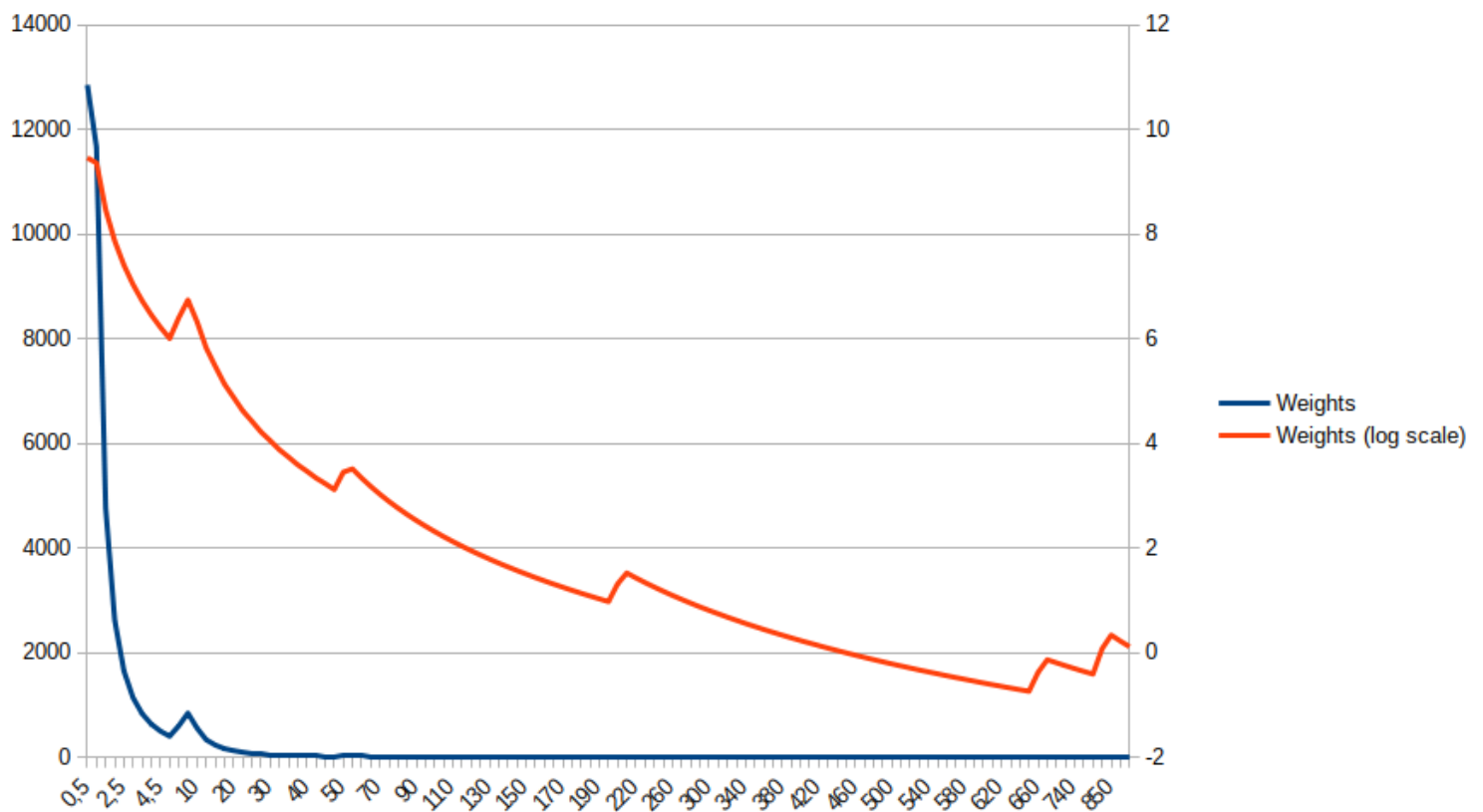
strikes and expiries). The purple one is the January expiry. It dominates the others so strongly that the over 8000 open contracts on the smallest strike for November 19 look really tiny. The blue row in the back is the 2023 LEAP. That's how far out this currently goes.



OI for 2022-01-21 over time. (Source: own work)

For reference, the above graphic illustrates OI for the January 2022 LEAP between October 2020 and July 2021. Also note how OI seemed to move with the stock price until around mid January. One can assume lazy hedging. But here's the thing: Even if they were perfectly hedged, the stock breaking the highest strike multiple times would still have fucked whoever had to hedge. This was not only a short squeeze, it was also a volatility squeeze.

To conclude this subsection, I asked Zinko to build me an RP. I chose the 2nd of February for the January 2022 expiry, because at that time OI seems to have stabilized.



Hypothetical RP built on 2021-02-02, expiring 2022-01-21. Log scale so you can see better what's happening. (Source: own work)

I would have loved to show you the portfolio performance, but both of my suitable datasets have errors. More on the portfolio in a bit.

Who might be short variance on GME?

Let me refer you to the presentation by Northfield and the article by Volquant, linked in the references. Northfield alleges, without specifying, that SHFs were not only short the stock, but also selling variance swaps on it. From Volquant we can learn that Citadel kind of always has been the insurance company for Wall Street, which almost killed them in 2008 and supposedly threatened them in January.

We don't know for certain if Melvin or Citadel in particular were short variance on GME (they could also have lost again by going short through whatever means again), but Northfield's description of the matter kind of reminded me of Melvin. Also, when the Covid crash wiped out some existing volatility funds, Kenny apparently decided to go balls deep into selling more of it (see References).

Discussion

If you paid attention, there are some more things to note.

OI looks different than sample RP

The graph of our sample portfolio looks way smoother than actual OI. This is true, and reality is actually in line with Demeterfi who suggests going into strikes evenly spaced, while our tool implicitly optimizes against the fluctuations shown in Figure 3c. The irregularities that can be observed in our model portfolio seem to happen everytime the distance

between strikes changes. However, not all strikes are (almost) exclusively high put OI. Some have high call OI. One can speculate that these are the boundary strikes for instances of the RP, and that the calls are part of synthetic forwards.

Discrepancies in maturity

I said something about the RP having the same maturity as the actual variance swap, and yet we are seeing OI similar to the RP almost every week. Does that mean they are selling incredibly short dated var swaps, even in January? I don't think so (I feel obligated to point out that this has been a matter of dispute with my colleagues). In fact, JPM indicates that you can combine a portfolio of variance swaps to fit the yield curves of arbitrary variance swaps on the same underlying. This means, of course, that you can do the same thing with your hedge. Why would you do this?

- Piling all your resources into one expiry makes you vulnerable when you have to roll. In fact, I believe that this is the strongest driver of price action already. In my first post I said that I didn't believe that this was the case, but as it turns out we have been looking in the absolutely wrong place.
- Diversifying expiries reduces fluctuations.
- I believe it can probably help with position limits.
- Apes are proving that they can stay retarded longer than SHFs can remain solvent.
- Other reasons that I don't know of because, as I said, I have no background in finance.

Open Interest for January 2022

There is insane OI for January. This could be because of several things:

- A lot of variance swaps are expiring in January. This may be the case. Remember that Jim Cramer was actually promoting the GME squeeze before it went nuts. One can speculate that SHFs were interested in getting the IV up for selling in mid January to reap more premiums, but didn't anticipate it going so horribly wrong.
- There are not a lot of expiries that offer the entire strike range. In fact, the next one after January 2022 is January 2023; June 2022 starts at \$10. *This could imply that we are going to see major price movements again, just to make more strikes available.* (In italics so those of you that can't read can read it.)
- It is because of "kicking the can" on the next cycle.

Cycles

By now likely all of you know about the "cycles", where GME is expected to print fat green dildoes every three months. But did you notice that the same thing is happening on a much smaller scale almost every month around the same time? The entire market does it, more or less. Previously I recommended another, now deleted post, which might have played a role as well (and likely did for February), but I have developed a new thesis since then.

We move when a RP gets built. That's the thesis. Why do we move harder every three months? Because according to my thesis the size of the move depends on the new hedges required, and hence it is stronger for expiries that have been around for longer and have more strikes available (both incentivizing larger positions).

Coincidentally that seems to be true for February, May, August and November. And let's not forget January, because that's the month LEAPs expire on any stock.

Please understand that this is anecdotal and that I have not been able to prove this yet, because we are severely understaffed (and our historical data is bad). That said and considering things like the recent increases in January OI, don't expect November cycle to happen (on time).

The correlations

I can not yet explain why Shitty Floors would track GME as closely as it does. There are ways to set up variance swaps to profit from covariance/correlation, which is generally called dispersion trading. However, the hedging of variance swaps doesn't cause correlations. This seems to be also true for other instruments like variance options or volatility swaps. The latter involve active trading of variance swaps/replicating options so might have more influence on the stock price than vanilla variance swaps, but it doesn't change a thing regarding correlating stocks.

As I said in the introduction, "manipulation" is not a satisfying explanation, so that leaves us with arbitrage or other instruments that yet have to be discovered.

WhAt'S tHe Si?!?

Honestly, I don't care much about that, but I know you retards do. These roughly 2300 shares in Citadel Advisor's 13F may or may not be indicative of a position in short forwards amounting to a net position of millions of shares short. Yes, the shares position in the RP is that tiny compared to the amount you go short in forwards.

WE SHOULD NOT BE WITNESSING THIS

In caps because it's the most important thing in this section.

First and foremost, as stated earlier, it is really uncommon to build a complete RP because of the expenses. It takes millions of Dollars to maintain these positions. Instead, players usually build the lazy hedge where the variance vega distribution looks like in Figure 3b, or use even other strategies.

Another aspect of the incomplete hedges is that it is not signaling to even retards like us what is apparently going on, leaving more room for ambiguity (and making it harder for competing firms to figure out was going on).

That leaves the question if it is incredibly important to someone to make other firms believe that a RP is maintained in the options chain, or if they for some reason are forced to do so.

Conclusion

GME options chain is indicative of the "Replicating Portfolio" used to hedge short variance exposure. This portfolio involves long puts and calls, short forwards and a rather small position in shares. Assuming open naked shorts on GME, willing counterparties for short forwards would be market makers or prime brokers trying to get around close-outs through deemed-to-own clauses.

Short variance exposure can explain many events that happened this year around the stock, but not correlations to other tickers. We believe it to be a major driver of GME's stock price.

It is suspicious that the options chain looks like this in such an obvious way, since doing a complete hedge usually burns the premiums collected, and also is sending very clear signals which investment firms usually try to circumvent.

Edit: Whales bet that banana wouldn't go up and down a lot, but it did. Open banana options suggest this, and maybe help explain where fake bananas came from. Banana value moves because whales are trying to not get fukd. Whales normally are not that obvious, which tells us something. The question is what.

References

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Morgan: [http://quantlabs.net/academy/download/free_quant_institutional_books_/\[JP%20Morgan\]%20Variance%20Swaps.pdf](http://quantlabs.net/academy/download/free_quant_institutional_books_/[JP%20Morgan]%20Variance%20Swaps.pdf)
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Appendix A: The "2500 Straddle"

On Monday, there was a large floor trade of both 2500 put and call contracts, at the strike price of \$220. That evening, this sub saw a post claiming that this was a straddle and some whale was expecting major price movements.

Even if you ignore that straddles are ATM, it was not a straddle. At roughly the same time, a block trade worth 250k shares went through a Dark Pool. These block trades have become very rare on GME. If this trade would have been a straddle, the net delta would have been 0.08, equaling 20k shares that would have hit the market. If we assume that one of these trades was short options, the net delta would have been 1, resulting in 250k shares traded. *It was a synthetic forward, and the shares are part of a married trade.*

Now we have to contemplate which direction.

1. A short forward could indicate that these forwards are part of an RP being built (with the forward price of \$220). [u/Criand](#) be my witness, I made a bet how this would play out for the stonk. (Not saying because that makes them adapt.)
2. A long forward would mean that some whale sold shorted shares to a MM. On the one hand that means that those of you that have subscriptions to services like Ortex should actually be able to see changes in SI on Monday. On the other, it is my understanding that it was speculated that upwards momentum this week came from APs being forced to buy shares for ETFs.

Appendix B: How Hedge Funds operate

I'm sure most of you have heard of the "Long/Short Portfolio" before, but don't really know what it means. It means that HFs go long a stock that they believe in, and at the same time short a very similar stock that they have less faith in. This helps eliminate market risk. Examples:

- Hedgie is bullish on CLF, so they short MT for "free money".
- Hedgie is bullish on AMZN, so they short GME, EXPR and others to hedge against problems in gaming, clothing and other markets.

I am telling you this, because, of course, the same thing can be done in volatility trading. Dispersion and spread trading like talked above are examples of this. I think it possible that many idiosyncracies seen this year can be attributed to such strategies, although some of the issues I identified for correlations also hold here.

Appendix C: Stock splits

It is our opinion that stock splits are good for the ones shorting variance, because unlike shares, there's not going to be a split in the variance swap. That means that a narrower range of strikes and therefore less capital is required to maintain the hedges.

So apart from other reasons that were discussed back then, this might provide an answer to those that were hoping for stock splits earlier this year.

Appendix ZZZ: Thank you!

Thank you to [u/Leenixus](#) for paying with his own money for the tool Zinko is using. It is Hoadley, btw, and it has a lot of functionality that can come in handy if you are serious about becoming a super serious investor. (I have no affiliations with them, and they don't even have stonks.)

Thank you to [u/Zinko83](#), because he's the guy I've been working on this with for the last two months or so, and to [u/sweatysuits](#) for actively participating in our discussions, and to the rest of the FWFB Discord.

But most importantly, thank you to Kenny, ballSAC, the Toy from Bulgaria, Melvin, and whoever was in bed with them. Without you guys I would have never learned this shit and would be as clueless as the average consumer of CNBC. You guys are the best, seriously.

And thank you, the reader, for making it this far!

