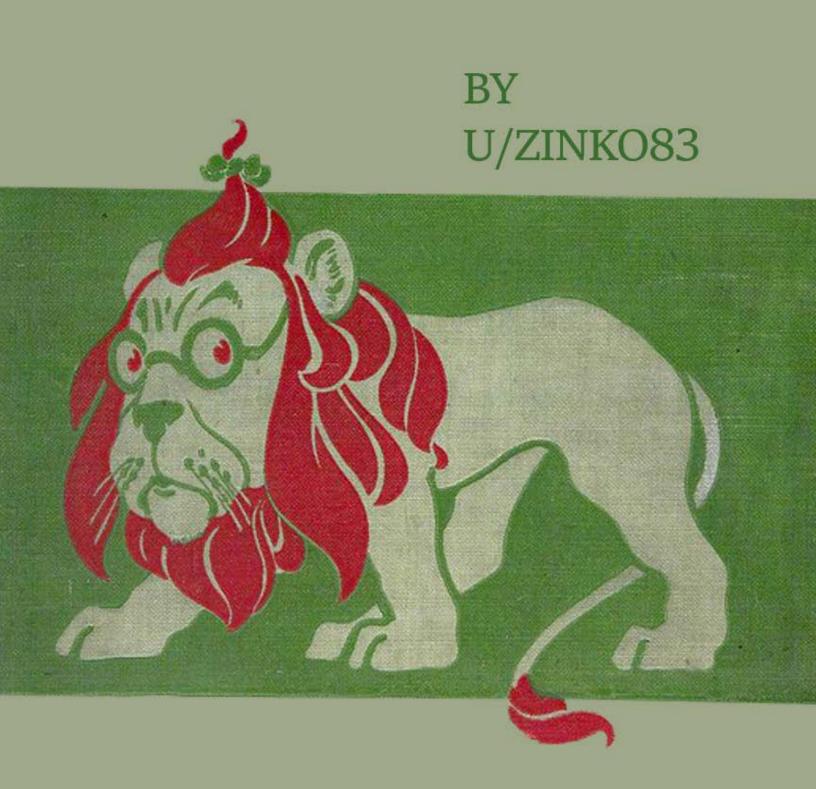
# VOLATILITY, VARIANCE, DISPERSION, Oh my!



# Volatility, Variance, Dispersion, Oh my! - Nov. 4, 2021

Possible DD

### Introduction

I have been asked to jot down some thoughts on variance by <u>u/gherkinit</u> and <u>u/Criand</u>

Variance/co-variance/volatility/simple variance/synthetic variance swaps, a year ago I had no idea what these were, over the past 3 months I and others have been diving into this black hole. <u>u/MauerAstronaut</u> has dug as deep on this as I have he is my VOLquant wanna be twin and a good friend, he has already had some post on the subject I highly recommend reading them before reading this. This subject is broad and extremely complex, the more reading the better here, without further delay let me share some thoughts.

Let me preface by first saying this post may not be for everyone, some of it may be slight speculation based off of actual events and or positions reported. Even though I will go into detail about being "long variance" it is hard to find breadcrumbs of long variance because the portfolio of options to be built as a hedge are in fact sold, or in common terms "short", meaning these positions are usually not reported limiting the evidence left. That being said those that are "short variance" do leave similar hedging breadcrumbs, that is what has led me here. Obligatory "this isn't advice" if you think it is, I feel terribly sorry for you.

#### **Brief Breakdown and Effects**

First thing is first, what is a variance swap? In simple terms it is a bet on volatility. The seller is going to receive a fixed payment called the "fair variance strike as vol" which is a fancy way of saying annualized implied volatility over a period of time (usually 30,60,90 days). The volatility strike is going to be calculated and agreed upon inception, that is going to be the fixed payment that the seller receives upon maturity. The opposite side of the trade is the buyer, the buyer is going to receive "realized variance" payment on maturity, meaning that it is going to get paid based on the volatility realized during the agreed upon period.

Here is an illustration showing the "vanilla" variance trade. Note the word "vanilla" not all trades are done this way:

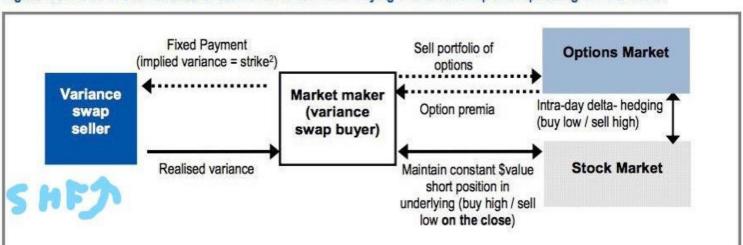


Figure 108: Flows in the market as a result of a market-maker buying a variance swap and replicating it in the market

The above came from this paper. Now to me this image illustrates a variance trade pretty well, it shows that the Market Maker buys variance as an insurance type play to their short options, then they also sell the replicating portfolio into the market to hedge against the long variance position. Easy peasy right? The buyer, especially when a Market Maker or a Broker Dealer, is pretty shielded here due to their role in the market. They basically get to roll with the tide, at the expense of their counterparty, since the effect of them delta hedging the replicating they sold as a hedge lets them stay flexible:

# 4.10: Effects of variance swap hedging

Market-makers who trade variance swaps may hedge their positions by replicating the opposite variance swap position through the replicating options portfolio. This replicating portfolio then needs to be delta hedged. The effects of deltahedging this portfolio are different to that from normal delta-hedged options for two principal reasons:

- 1. The actions of delta-hedging the options could potentially act to the disadvantage of the counterparty's position.
- 2. Since variance swap contracts typically measure close-close realised volatility, the options must be delta-hedged on the close only to capture this.

We investigate both of these properties below.

Firstly, consider a market-maker who has sold a call option to a counterparty. The market-maker will be short the option (and delta) and will therefore buy back the delta in the underlying. If the underlying rallies, the short-delta exposure from the short option will increase, and the market-maker will therefore have to buy more of the underlying in the market. Similarly, if the underlying sells off, the market-maker will have to sell the underlying. In both cases – if the position being hedged is big enough - the action of delta-hedging will have the effect of increasing volatility in the underlying magnifying both up-moves and down-moves. The argument for a put option is analogous. Similarly, if the market-maker is long options and delta-hedging, the hedging will act to suppress volatility in the underlying, potentially to the advantage of the counterparty who is short the option.

The situation with variance swaps is different. Suppose that the market is such that hedge-funds, or other market participants, have generally sold index variance swaps to market-makers (as was the case before the May 2006 correction). Note that no exchange of options has taken place here - the parties have just taken opposite sides in a contract for difference. Suppose that the volatility sellers do not hedge their variance swaps (they have sold the variance swaps specifically for the direct volatility exposure they offer). But assume that market-makers hedge their short volatility

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Part 4: Replication and Hedging

exposure. A market-maker who is long the variance swap can offset the risk by shorting the replicating portfolio of options, and delta-hedging. They will therefore be short gamma in the options market (Figure 108).

As described above, a delta-hedger who is short options will act to increase volatility in the underlying – buying as it rallies, and selling as it sells-off. However, the action of these market makers hedging their short options will not necessarily act to increase volatility in the underlying, as the counterparties they have sold options to may be counteracting this effect by themselves hedging their long volatility positions. However, the important difference between these two groups of hedgers is that the variance swap market-makers who are short options, must hedge only on the close to capture the close-close realised variance specified in the variance swap contract. In contrast, the hedgers who are long the options, will generally be free to choose when to delta-hedge, as they attempt to capture the true volatility of the underlying process.

Therefore, the overall effect of hedging these variance swaps need not have the effect of increasing *overall* market volatility, although it may if the long options positions are not being hedged (e.g. they are sold on to end-investors). However, the important point is that the hedging of long variance swap positions may act to increase close-to-close volatility, with option hedges on the close having the potential effect of magnifying daily moves. In practice, this only becomes an issue if hedgers (variance swap market-makers) are the same way round and daily moves are large enough.

In fact the expectation of these delta-hedging flows could itself act to further amplify moves into the close through possible 'feedback' effects. For instance, on big down days, the market could anticipate a sell-off into the close as a result of long variance swap hedging and take short positions into the close to profit from this anticipated effect. This then drives the underlying market down further and could increases the amount that the variance swap hedgers have to sell on the close.

These feedback effects were much talked about during the volatile market period in May 2006, when the hedging into the close of long variance swap positions, and anticipation thereof, was blamed for the amplification of some already large daily moves. In fact, in an efficient market, such effects should be propagated back through the trading day, and may be offset by intra-day delta-hedging of opposing options.

What about the seller? The sellers are usually multi-strategy hedge funds whom themselves buy and sell variance on different securities and index(s). Dispersion trading is the name of this practice, it is the practice of going short index variance and long any number of constituent stocks. Or vise versa, like anything of course. This makes a pseudo basket effect, and is thus a CORRELATION TRADE. How many stocks does GME track? Aren't they all volatile? Aren't all the index(s) and ETF(s) they are in ODD to say the least?? Picture perfect dispersion set up. Ironic isn't it...... A quick illustration to help:

. . .

This implied correlation can be traded by selling index variance and buying single-stock variance, the resulting position being short correlation. To hedge out the exposure to volatility, at least initially, the index and single-stock legs of the trade must be weighted with more index vega-notional than single-stock vega notional – the exact amounts depending on the level of implied correlation. Such a trade is known as a variance dispersion trade.

#### 7.1.1 Vanilla Dispersion Trade

The weights are chosen in the same proportion of the members in the index. In this case the exposure of the volatility is the main driver of the P&L. A long position in the strategy involves buying 50 variance swaps in the constituents of the index and being short in one variance swap in the index. A vanilla dispersion trade is long in volatility and short in correlation. Let us use the model developed in section 3 to explain this. Assume the case that there exists correlation between the two assets in the index proposed and it is different of zero. We showed that the theoretical index variance is given by:

$$\sigma_{index}^2 = (w_1\sigma_1S_1\rho + w_2\sigma_2S_2)^2t + (w_1S_1\sigma_1\sqrt{1-\rho^2})^2t$$
 (47)

A long strategy is profitable for us if the volatilities of the individual stocks increase. The correlation is bounded by one and in this extreme case the index volatility is equal to the replicated basket volatility. Therefore the volatility of the index increases at a lower rate than the one of the stocks because individual volatilities are weighted by the correlation coefficient (that is less than one). Moreover, the larger is the decrease in the correlation the bigger is the P&L. Please note that although the strategy profits directly from changes in volatility of the index versus volatility of the stocks, the driver of this movements is always generated by the correlation matrix.

The natural approach in view of the results of vanilla dispersion trading, that is, the correlation as a main driver, is to set up weights that increase the exposure to the correlation between constituents. This is called correlation-weighted dispersion trading.

I am not going to go in depth about co-variance swaps/dispersion/correlation trades in this post as it will get rambling like and there just isn't enough room. We are in the process of really breaking this down and trying to model it better. This is just to shed light on the correlation effects we've seen and a reason outside of portfolio swaps, basket shorting, etc that the strong correlation is there.

#### How is this GME related?

We ran across this link <u>SDR Services - CFTC Ticker (dtcc.com)</u> Which included these:

Equity		
Action	Product ID	Publication Timestamp
	asicPeriormance:Singleindex	
<u>NEW</u>	Equity:Swap:ParameterReturn nVolatility:SingleIndex	2021-10-20 03:44:59.0
CORRECT	Equity:Forward:PriceReturnBasicPerformance:Basket	2021-10-15 09:53:03.0
<u>NEW</u>	Equity:Swap:ParameterReturn	
CORRECT	Equity:Option:ParameterReturnVariance:SingleIndex	•
<u>NEW</u>	Equity:Swap:ParameterReturn	
CORRECT	Equity:Swap:ParameterReturn nDividend:SingleIndex	
<u>NEW</u>	Equity:Forward:PriceFournE	2021-08-27 18:12:04.0
r.	F " 0 " B' B' B	

Equity		
Action	Product ID	Publication Timestamp
New	Equity:Swap:ParameterReturnVariance:SingleName	2020-07-15 10:04:30.0
<u>New</u>	Equity:Swap:ParameterReturn nVolatility:SingleName	2020-05-05 10:39:48.0
<u>New</u>	Equity:Swap:ParameterReturnDividend:SingleName	2020-01-29 09:03:50.0
<u>New</u>	Equity:Swap:ParameterReturn nDividend:Basket	2016-12-14 14:51:55.0
<u>New</u>	Equity:Option:ParameterReturnVolatility:SingleIndex	2016-03-21 16:26:15.0
<u>New</u>	Equity:Option:ParameterReturnDividend:SingleName	2015-08-27 10:17:38.0
<u>New</u>	Equity:Option:ParameterReturnVariance:Basket	
<u>New</u>	Equity:Option:ParameterReturnVolatility:SingleName	<sup>1</sup> 2014-02-17 02:19:20.0

Foreign Eychange - Cash

There were more, but I think you get the point, this caught our interest so the next logical thing to do was search "GameStop Variance" that resulted in <u>GameStop</u>, <u>Variance Swaps</u>, <u>and Related Failures of Hedge Fund Risk Management (northinfo.com)</u> Well that really got me excited so then I went on to learn what this was all about.

# The Replicating Portfolio

After reading (PDF) More Than You Ever Wanted to Know About Volatility Swaps (researchgate.net) I learned the the replicating portfolio is quite important in the world of variance swaps. Two main reasons, it is the way a variance swap is priced so the two parties can come to an agreement on "fair variance" as the "variance strike", it also acts as a hedge for the forward contract the actual variance swap is. So what is it? Without getting all "mathy" it is a portfolio of OTM options (both calls and puts) that are used to best capture variance. Here is something to illustrate.

TABLE 1. The portfolio of European-style put and call options used for calculating the cost of capturing realized variance in the presence of the implied volatility skew with a discrete set of options strikes.

	Strike	Volatility	Weight	Value per Option	Contribution
	50	30	163.04	0.000002	0.0004
	55	29	134.63	0.00003	0.0035
	60	28	113.05	0.0002	0.0241
	65	27	96.27	0.0013	0.1289
	70	26	82.98	0.0067	0.5560
PUTS	75	25	72.26	0.0276	1.9939
	80	24	63.49	0.0958	6.0829
	85	23	56.23	0.2854	16.0459
	90	22	50.15	0.7384	37.0260
	95	21	45.00	1.6747	75.3616
	100	20	20.98	3.3537	70.3615
	100	20	19.63	4.5790	89.8691
	105	19	36.83	2.2581	83.1580
	110	18	33.55	0.8874	29.7752
ITS	115	17	30.69	0.2578	7.9130
CALLS	120	16	28.19	0.0501	1.4119
	125	15	25.98	0.0057	0.1476
	130	14	24.02	0.0003	0.0075
	135	13	22.27	0.000006	0.0001
		419.8671			

This illustrates it pretty well, it highlights the "weights" or number of contracts needed at each strike to build this replicating portfolio of options to price and hedge a swap on an underlying with a spot of about 100. You can always tell where the spot of the underlying is on one of these by identifying where the puts and calls "meet". Here you can tell it is 100 because it is the only strike that it tells you to buy both calls and puts on. *Cool.* Another thing to note is it says European style options, all that means is here in America you hold till maturity. It is needed for correct payoff.

Looking at this I noted, jeez they put a **lot** of emphasis on that lower strike put in this scenario don't they? That made a lightbulb turn on somewhere, I thought man GME sure does have a strange amount of open interest on lower strike puts like that, it also carries odd open interest in the higher strike ranges on the same expiry. Then I read more papers and I learn that GMEs option chains, especially on monthly expiry's are the perfect situation for someone long variance. So naturally I decided to look at options OI on January 21 2022:

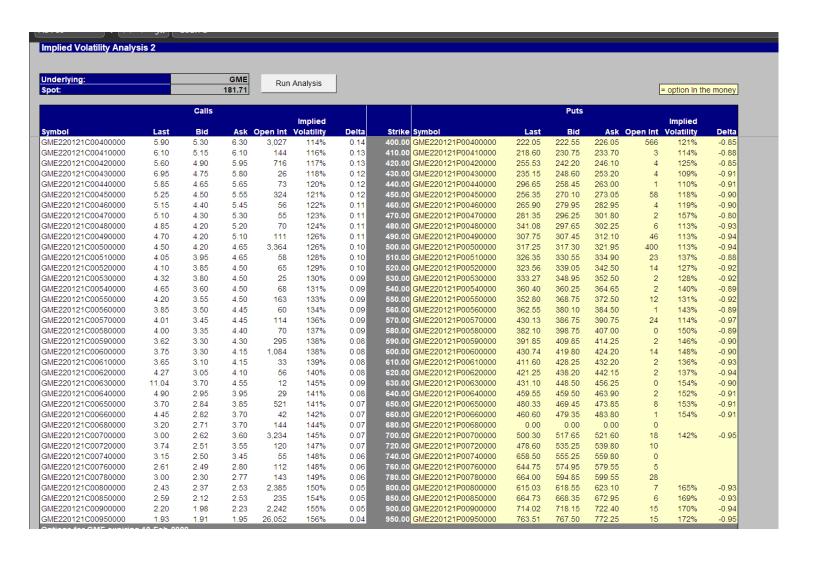
 Underlying:
 GME

 Spot:
 181.71

Run Analysis

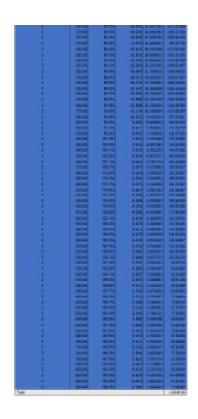
= option In the money

Calls						Puts								
20.00			200000	200	Implied	11/2/1904		2002-80			2000	-2007-1000	Implied	100
Symbol GME211217C00460000	2.70	2.60	2.71	Open Int 675	138%	Delta 0.07		Symbol GME211217P00460000	261.60	277.50	281.45	Open Int	Volatility 128%	Delta -0.95
Options for GME expirin	OF TAXABLE PARTY OF THE PARTY.	THE RESERVE TO SHARE THE PARTY OF THE PARTY	2.71	0,0	70070	0.07		OMEE 1 12 171 00 130000	201.00	277.00	201.40		12070	0.00
GME220121C00000500	185.88	178.75	183.55	1	-10000000000000000000000000000000000000			GME220121P00000500	0.01	0.00	0.01	136,222	427%	-0.00
GME220121C00001000 GME220121C00001500	191.00 306.50	178.30 181.15	183.40 186.15	3	511%	1.00		GME220121P00001000 GME220121P00001500	0.01	0.00	0.01	29,468 4,332	367% 332%	-0.00
GME220121C00001500 GME220121C00002000	198.86	180.65	185.65	1				GME220121P00001500	0.01	0.00	0.01	16,721	308%	-0.00
GME220121C00002500	129.35	180.15	185.15	0				GME220121P00002500	0.01	0.00	0.01	4,096	290%	-0.00
GME220121C00003000	161.40	179.65	184.65	0			3.00	GME220121P00003000	0.01	0.01	0.02	6,538	289%	-0.00
GME220121C00003500	160.90	179.15	184.15	3	1202%	1.00		GME220121P00003500	0.01	0.01	0.03	1,820	283%	-0.00
GME220121C00004000 GME220121C00004500	198.45 175.00	178.70 178.20	183.65 183.15	3 53	897% 783%	1.00		GME220121P00004000 GME220121P00004500	0.01	0.01	0.03	3,271 2,007	272% 272%	-0.00
GME220121C00004300	201.85	177.70	182.65	224	716%	1.00		GME220121P00004300	0.02	0.02	0.03	13,455	259%	-0.00
GME220121C00005500	38.00	177.20	182.15	4	669%	1.00		GME220121P00005500	0.03	0.03	0.04	2,336	260%	-0.00
GME220121C00007000	176.00	172.25	176.55	8		-	7.00	GME220121P00007000	0.04	0.04	0.05	3,474	247%	-0.00
GME220121C00010000	163.90	169.35	173.55	90				GME220121P00010000	0.06	0.05	0.07	9,610	226%	-0.00
GME220121C00012000 GME220121C00015000	173.63 171.90	167.35 164.30	171.55 168.60	1,747				GME220121P00012000 GME220121P00015000	0.07	0.07	0.12	1,412 6,757	220%	-0.00
GME220121C00017000	167.54	165.70	170.70	22	362%	0.99		GME220121P00017000	0.09	0.06	0.12	836	192%	-0.00
GME220121C00020000	153.47	159.40	163.60	97	00270	0.00		GME220121P00020000	0.19	0.15	0.20	4,798	191%	-0.00
GME220121C00022000	182.90	160.75	165.70	44	316%	0.98		GME220121P00022000	0.10	0.15	0.29	584	188%	-0.00
GME220121C00025000	165.30	154.35	158.65	394				GME220121P00025000	0.23	0.15	0.35	3,027	179%	-0.00
GME220121C00027000	157.80	152.35	156.65	167				GME220121P00027000	0.26	0.20	0.39	376	175%	-0.00
GME220121C00030000 GME220121C00032000	156.73 149.25	149.45 150.85	153.65 155.85	695 12	257%	0.98	30.00	GME220121P00030000 GME220121P00032000	0.24	0.20	0.40	4,911	166% 162%	-0.00
GME220121C00032000 GME220121C00035000	149.23	144.45	149.25	897	20170	0.90		GME220121P00032000	0.30	0.35	0.43	814	161%	-0.01
GME220121C00033000	165.10	144.60	148.55	114	198%	0.98		GME220121P00037000	0.42	0.35	0.55	194	157%	-0.01
GME220121C00040000	152.93	139.60	144.60	521	133%	1.00		GME220121P00040000	0.42	0.40	0.60	1,868	152%	-0.01
GME220121C00042000	139.00	141.00	146.00	144	218%	0.97	42.00	GME220121P00042000	0.69	0.40	0.66	116	148%	-0.01
GME220121C00045000	127.08	134.60	139.40	294	108%	1.00		GME220121P00045000	0.58	0.40	0.71	842	142%	-0.01
GME220121C00047000	183.05	136.10	141.10	93	203%	0.97		GME220121P00047000	0.58	0.50	0.78	217	141%	-0.01
GME220121C00050000 GME220121C00055000	131.10 134.36	130.85 125.80	134.35 129.50	600 643	135% 126%	0.99		GME220121P00050000 GME220121P00055000	0.64	0.60	0.68	4,247 4,460	135% 129%	-0.01
GME220121C00060000	125.20	120.80	124.25	1,942	113%	0.99		GME220121P00060000	0.80	0.69	1.03	763	123%	-0.02
GME220121C00065000	121.40	115.80	119.60	177	109%	0.98		GME220121P00065000	0.95	0.74	1.22	309	118%	-0.02
GME220121C00070000	131.87	110.80	114.60	202	101%	0.98	70.00	GME220121P00070000	1.05	0.91	1.26	472	112%	-0.02
GME220121C00075000	101.95	105.80	109.60	102	94%	0.98		GME220121P00075000	1.26	1.00	1.26	327	106%	-0.03
GME220121C00080000	104.44	100.80	104.75	131	89%	0.98		GME220121P00080000	1.30	1.29	1.44	576	102%	-0.03
GME220121C00085000 GME220121C00090000	99.70 99.26	95.80 90.90	99.95 95.20	263 121	85% 83%	0.98		GME220121P00085000 GME220121P00090000	1.40 1.81	1.32	1.85 2.16	300 521	98% 96%	-0.04
GME220121C00095000	99.50	86.25	90.35	72	80%	0.97		GME220121P00095000	2.20	1.85	2.10	2,222	93%	-0.05
GME220121C00100000	91.35	81.65	85.35	1,010	77%	0.96		GME220121P00100000	2.53	2.35	2.83	4,159	90%	-0.06
GME220121C00105000	82.00	77.15	81.15	113	79%	0.95	105.00	GME220121P00105000	2.85	2.75	3.45	75	89%	-0.07
GME220121C00110000	79.00	72.80	76.65	113	77%	0.93	110.00	GME220121P00110000	3.30	3.30	4.05	378	87%	-0.08
GME220121C00115000	75.90	68.70	72.20	866	78%	0.92		GME220121P00115000	4.40	4.05	4.80	1,370	86%	-0.10
GME220121C00120000	67.66 67.00	65.20 60.45	68.10 64.15	105 171	79% 77%	0.90		GME220121P00120000	5.30 6.27	4.90 5.75	5.75 6.80	904 552	85% 84%	-0.12 -0.13
GME220121C00125000 GME220121C00130000	64.60	57.30	60.30	110	79%	0.86		GME220121P00125000 GME220121P00130000	7.30	6.90	8.00	272	83%	-0.15
GME220121C00135000	56.80	53.70	56.70	90	78%	0.83		GME220121P00135000	8.74	8.35	9.45	278	83%	-0.18
GME220121C00140000	56.20	49.70	53.25	364	77%	0.81		GME220121P00140000	10.38	9.90	11.00	239	83%	-0.20
GME220121C00145000	55.74	47.20	49.95	80	80%	0.78		GME220121P00145000	12.35	11.75	12.75	220	84%	-0.22
GME220121C00150000	46.10	43.90	46.80	883	79%	0.76		GME220121P00150000	14.80	13.70	14.75	1,300	83%	-0.25
GME220121C00155000	44.85	40.75	43.95	122	79%	0.73	155.00	GME220121P00155000	15.25	15.95	17.10	149	84%	-0.28
GME220121C00160000 GME220121C00165000	41.55 38.10	38.35 35.90	41.20 38.65	154 135	80% 80%	0.70		GME220121P00160000 GME220121P00165000	18.30 20.27	18.10 20.60	19.05 21.65	811 226	84% 84%	-0.30
GME220121C00170000	35.65	33.90	36.25	193	81%	0.65		GME220121P00170000	22.73	23.20	24.35	132	84%	-0.35
GME220121C00175000	36.10	31.85	34.05	199	82%	0.62		GME220121P00175000	26.00	25.85	27.60	89	85%	-0.38
GME220121C00180000	30.50	30.50	31.50	389	83%	0.60		GME220121P00180000	28.77	28.90	30.20	166	86%	-0.40
GME220121C00185000	28.47	28.20	29.85	96	83%	0.57		GME220121P00185000	29.15	31.85	33.05	111	86%	-0.43
GME220121C00190000 GME220121C00195000	28.20 26.00	26.35 24.95	28.15 26.55	274 101	83% 84%	0.55		GME220121P00190000 GME220121P00195000	35.90 39.73	35.05 38.55	36.30 39.90	166 114	86% 87%	-0.45
GME220121C00193000 GME220121C00200000	23.90	23.90	24.35	1,277	85%	0.52		GME220121P00193000 GME220121P00200000	41.09	41.80	43.00	470	87%	-0.47
GME220121C00210000	21.20	20.75	22.20	343	86%	0.46		GME220121P00210000	47.70	49.20	50.60	398	89%	-0.54
GME220121C00220000	19.10	18.60	19.70	321	87%	0.42	220.00	GME220121P00220000	54.05	56.70	58.50	139	90%	-0.58
GME220121C00230000	17.10	16.45	17.60	636	88%	0.38		GME220121P00230000	63.00	64.10	66.20	662	90%	-0.61
GME220121C00240000	15.73	14.70	15.95	178	89%	0.35		GME220121P00240000	70.10	72.30	74.75	80	92%	-0.64
GME220121C00250000 GME220121C00260000	13.80 12.70	13.10 12.10	14.40 13.15	1,363 482	90% 92%	0.32		GME220121P00250000 GME220121P00260000	78.50 97.60	80.80 89.55	83.80 92.55	132 68	94% 95%	-0.67 -0.69
GME220121C00260000 GME220121C00270000	13.20	10.85	12.05	117	94%	0.30		GME220121P00260000	95.95	98.45	101.55	63	95%	-0.69
GME220121C00270000	10.95	10.00	11.15	407	95%	0.25		GME220121P00280000	118.03	107.55	110.45	57	99%	-0.74
GME220121C00290000	9.75	9.40	10.20	405	96%	0.24		GME220121P00290000	127.27	116.75	119.70	43	100%	-0.75
GME220121C00300000	9.00	8.70	9.45	3,775	97%	0.22		GME220121P00300000	124.94	126.05	129.55	1,272	103%	-0.76
GME220121C00310000	8.50	8.10	8.95	279	99%	0.21		GME220121P00310000	142.62	135.45	138.30	49	103%	-0.78
GME220121C00320000	8.30	7.60	8.55	200	101%	0.20		GME220121P00320000	143.00	144.95	148.20	80	106%	-0.79
GME220121C00330000 GME220121C00340000	8.40 8.00	7.15 6.75	8.15 7.75	259 128	103% 104%	0.19		GME220121P00330000 GME220121P00340000	163.34 171.05	154.50 164.10	157.35 167.35	56 42	108%	-0.80 -0.81
GME220121C00340000 GME220121C00350000	7.45	6.50	7.75	1,383	104%	0.18		GME220121P00340000 GME220121P00350000	182.39	173.75	176.20	573	110%	-0.82
GME220121C00360000	6.75	6.10	7.20	85	107%	0.16		GME220121P00360000	192.20	192.95	195.35	23	146%	-0.72
GME220121C00370000	7.00	5.90	6.90	76	109%	0.15		GME220121P00370000	188.00	191.70	194.60	5	107%	-0.86
GME220121C00380000	6.20	5.65	6.70	202	110%	0.15		GME220121P00380000	212.45	200.30	204.70	7	107%	-0.87
GME220121C00390000	6.35	5.45	6.50	206	112%	0.14		GME220121P00390000	219.09	212.75	216.20	18	119%	-0.84
GME220121C00400000	5.90	5.30	6.30	3,027	114%	0.14		GME220121P00400000	222.05	222.55	226.05	566	121%	-0.85
GME220121C00410000 GME220121C00420000	6.10 5.60	5.15 4.90	6.10 5.95	144 716	116% 117%	0.13		GME220121P00410000 GME220121P00420000	218.60 255.53	230.75 242.20	233.70 246.10	3	114% 125%	-0.88
10WEEE0 121000420000	0.00	4.30	5.85	110	111/0	0.10	420.00	SINEEZO 12 IF 00420000	200.00	242.20	240.10	4	12070	-0.00



That just seems oddly familiar to the example above doesn't it? Almost textbook how wide this option chain is, and the OI spread across strikes just as the papers recommend. If only I knew what this looked like for GME:





This example replicating portfolio of GME was made by me approximately 2 weeks ago, as well as the pictures of the OI was around the same time, so the OI may have moved some, nothing to deter the point its just worth noting. This is for a 90 swap ending Jan 21 2022 it recommends a replicating portfolio, and gives a fair variance as volatility which is used for the "variance strike".

So just to recap, hedge funds sell variance making them short, which in turn requires them to hold a portfolio of long OTM options to hedge the short swap. This should be making lightbulbs turn on, if it doesn't go check Citadel Advisors, Susquehanna, Simplex holdings and see they hold not only puts but calls come back and stare at the replicating above, it will click eventually. If not, never fear <u>u/MauerAstronaut</u> is making a post about the options OI and how it relates to the replicating in more depth soon.

# **Dynamic and Imperfect Hedging**

Variance swaps require a log contract and would thus need an infinite amount of strikes to be perfectly hedged, without going into all the mathy details this means that narrow strike options chain = **bad** for them. Illustrations:

Imperfect Replication Due to Limited Strike Range Variance replication requires a log contract. Since log contracts are not traded in practice, we replicate the payoff with traded standard options in a limited strike range. Because these strikes fail to duplicate the log contract exactly, they will capture less than the true realized variance. Therefore, they have lower value than that of a true log contract, and so produce an inaccurate, lower estimate of the fair variance.

In Table 4 below we show how the estimated value of fair variance is affected by the range of strikes that make up the replicating portfolio. The fair variances are estimated from (1) a replicating portfolio with a narrow range of strikes, ranging from 75% to 125% of the initial spot level, and (2) a portfolio with a wide range of strikes, from 50% to 200% of the initial spot level. In both cases the strikes are uniformly spaced, one point apart. (The fair variance is calculated according to Equation 26, except that the integrals are replaced by sums over the available option strikes whose weights are chosen according to the procedure of Appendix A). We assume here that implied volatility is 25% per year for all strikes, with no volatility skew, so that all options are valued at the same implied volatility. We also assume a continuously compounded annual interest rate of 5%.

For both expirations, the wide strike range accurately approximates the actual square of the implied volatility. However, the narrow strike range underestimates the fair variance, more dramatically so for longer expirations.

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#### QUANTITATIVE STRATEGIES RESEARCH NOTES

TABLE 4. The effect of strike range on estimated fair variance.

Expiration	Wide strike range (50% - 200%)	Narrow strike range (75% - 125%)
Three-month	$(25.0)^2$	$(24.9)^2$
One-year	$(25.0)^2$	$(23.0)^2$

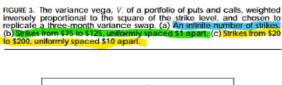
In the section entitled Replicating Variance Swaps. First Steps on page 6, we have already discussed one approach to understanding why the narrow strike range fails to capture variance. As shown in Figure 3, the vega and gamma of a limited strike range both fall to zero when the index moves outside the strike range, and the strategy then fails to accrue realized variance as the stock price moves. Consequently, the estimated variance is lower than the true fair value for both expirations above, and the reduction in value is greater for the one-year case. Over a longer time period it is more likely that the stock price will evolve outside the strike range.

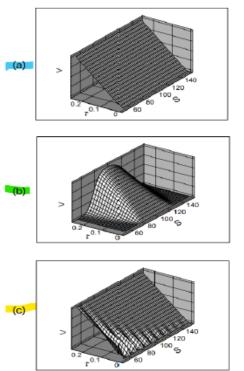
Imperfect Hedges

It takes an infinite number of strikes, appropriately weighted, to replicate a variance swap. In practice, this isn't possible, even when the stock and options market satisfy all the Black-Scholes assumptions: there are only a finite number of options available at any maturity. Figure 1 illustrates that a finite number of strikes fails to produce a uniform // as the stock price moves outside the range of the available strikes. As long as the stock price remains within the strike range, trading the imperfectly replicated log contract will allow variance to accrue at the correct rate. Whenever the stock price moves outside, the reduced vega of the imperfectly replicated log contract will make it less responsive than a true variance swap.

Figure 3 shows how the variance vega of a three-month variance swap is affected by imperfect replication. Figure 3a shows the ideal variance vega that results from a portfolio of puts and calls of all strikes from zero to infinity, weighted in inverse proportion to the strike squared. Here the variance vega is independent of stock price level, and decreases linearly with time to expiration, as expected for a swap

whose value is proportional to the remaining variance  $\sigma^2 \tau$  at any time. Figure 3b shows strikes from \$75 to \$125, uniformly spaced \$1 apart. Here, deviation from constant variance vega develops at the tail of the strike range, and the deviation is greater at earlier times. Finally, Figure 3c shows the vega for strikes from \$20 to \$200, spaced \$10 apart. Now, although the range of strikes is greater, the coarser spacing causes the vega surface to develop corrugations between strike values that grow more pronounced closer to expiration.





I will now attempt to explain the above illustration (figure 3) in ape speak as well as one can.

- A) Perfect hedge if an infinite number of strikes existed, doesn't exist so its for reference
- B) This is what happens when you have a narrow strike range, you are not as hedged because of it.
- C) This is when you have more strikes available in a wider range. It mimics the reference one in a) much better.

#### More Illustrations:

## 4.9: Replication and hedging in practice

As demonstrated above, a variance swap can be statically hedged with a portfolio of out-of-the-money (European-style) options, weighted according to the inverse squares of their strikes. This makes it easy, in theory, to calculate the fair value of a variance swap, assuming option prices are available across the entire range of strikes. In practice, traded strikes are not continuous, although for major liquid indices they are closely spaced (0.4% notional apart for the S&P, 1% for the FTSE, 1.4% for the Euro Stoxx). A more serious limitation is the lack of liquidity in OTM strikes, especially for puts, as these provide a relatively large component of the variance swap price in the presence of steep put skews. S&P options are listed down to a strike of 600 (c45% of current spot), FTSE to 3525 (c60% of spot) and Euro Stoxx down to 600 (c15% of spot), although in reality, liquidity does not even reach this far.

The problem with the lack of OTM puts can be seen from following through the practical example of setting up the replicating portfolio above. The long futures position is used to create a pay-out which is equivalent to a long log contract plus realised variance. In opposition the long options/short forward position is used to create a short log contract and pay the fixed strike. Supposing the market falls significantly, the delta-hedge will be long the log contract (a will hence lose), while the options should counteract this by being short the log contract. However, if not enough downside puts were used, the options portfolio will not fully reflect the short log-contract exposure needed and hence the overall hedge will lose money. This lack of liquidity at the wings has led to the development of conditional variance swaps which can remove exposure to volatility once the underlying moves into areas where vanilla options are illiquid.

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European Equity Derivatives Strategy 17 November 2006

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Part 4: Replication and Hedging

In practice, market-makers will not attempt to hedge with the entire strip of options but typically will use only two or three - including one close to the money and one or more OTM, but liquid, puts. Alternatively they could approach the replicating by hedging the vega with an OTM put whose implied volatility coincides with the variance swap strike - close to the money for 1-month maturity, 95% for 3-6 months, 90% for 1-year, 85% for 2-3 years etc. In this case they would also look to buy back the wings/convexity separately. Other proxies, such as CDS or EDS could also be used instead of the more deeply out-of-the-money puts.

One problem with this kind of approach is that the partial hedge is no longer static, and must be dynamically managed. For example if the market sells off towards the strike, the market maker will have to trade further OTM puts to ensure that their exposure to volatility, in the form of dollar-gamma, remains constant. This makes the actual variance swap replication more akin to a combination of alternatives 2 and 3 listed on p79. Here the constant dollar gamma would be maintained by a combination of holding a portfolio which has roughly constant dollar gamma if the underlying does not move too much, and re-hedging by trading more options if the underlying does move significantly.

In practice, pricing models will often only price contributing options for the variance swap hedge between 1 and 99 delta. For example, on the Euro Stoxx (6-month maturity) this would only price puts down to 2700 (ref 3822) or around 70% of spot. The difference from this truncated and the full replication pricing is typically in the region of 5-25bps.

Another limitation comes from the discrete nature of adjusting the delta hedge on the close, which introduces possible errors due to large daily moves. However these moves are actually of order the cube of the move (as in jumps for the continuous case, see section 4.7) and hence are negligible for all but very large moves. Also as previously noted if interest rate changes are related to changes in the spot underlying, this can also have an impact on the ability to accurate replicate the log-contract and realised volatility

This and other differences between the theoretical and mid-market price of a variance swap contribute to the so-called variance swap basis, see Section 4.7. This basis tends to vary across maturities, being larger for longer maturities. At shorter dates a value of around 0.5 vegas would be typical.

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Ok all that just to say that dynamic hedging tail risk is hard without those *really* deep OTM puts. It forces whomever is short the swap to buy more calls to get the exposure needed, which could force the stock upward which is usually not wanted. I will attempt to illustrate this with GME now:

The above is an illustration I made to try to show what I described above in practice. I chose these historical "as of" dates (gathered from Market Chameleon) because they were the beginning of a snowball tumbling down a mountain that turns into a bigger snowball later. Basically they are the start of any major run up that we have had this year excluding February, it had a very wide options chain yet totally took off. February is an anomaly in many ways so I ignore it. Fuck February.

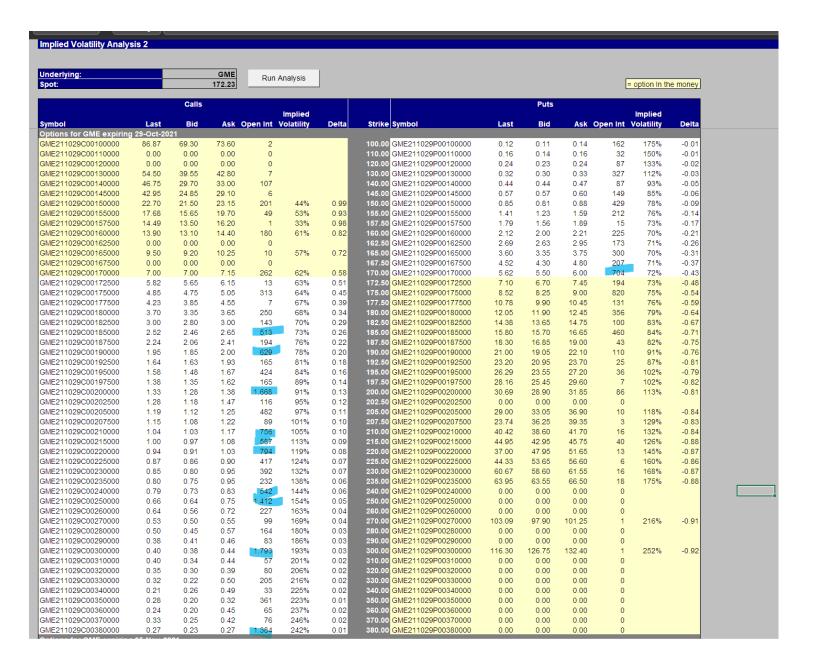
If you look you will see that every one of these run ups had a limited chain, comparatively to what GMEs monthly chains look like. After each run up gets started (just like this most recent) the option chain(s) get expanded into more strikes, for several reasons, I just note this to why I picked these dates to capture what they were before expansion. This is key, especially as of the latest run ups, because it forces them to buy more calls for their dynamic hedge vs puts because that's where the vega and IV exposure lies to get their portfolio greeks where they need to be at the close every day since everything is measured close to close.

To illustrate this I put a theoretical 7 day swap in my model using the 10/29/2021 chain:

Inputs Calculation_type					
Asset_price		172.23			
Days		7			
Risk_free_rate		2.0%			
Div_yield		0.0%			
Results	Calc_type	Value			
Fair variance (strike)	VAR	0.762195			
Fair variance as volatility	VOL	87.304%			
Cost of options portfolio	C	7619.11			
					_
Results - Options portfolio Put/Call	Strike	Volatility	Weighte	Option Val	Contribut
1	100.00	174.9%	1048.106	0.135547	142.06
1	110.00	149.7%	865.447	0.153472	132.82
1	120.00	133.1%	726.733	0.247876	180.13
1	130.00	112.0%	618.908	0.316937	196.15
1	140.00	93.2%	409.356	0.455082	186.29
1	145.00	85.2% 78.5%	248.152 231.875	0.594358	147.49 194.20
i	155.00	75.6%	164.592	1.395886	229.75
1	157.50	73.5%	105.113	1.736722	182.55
1	160.00	70.4%	101.854	2.088826	212.75
1	162.50	71.1%	98.744	2.802782	276.75
1	165.00	70.1%	95,774	3.521088 4.578727	337.22 425.52
1	167.50	72.0%	124,981	5.713995	714.14
2	170.00	72.0%	34.759	8.009188	-278.38
2	172.50	63.2%	87.626	5.915851	518.38
2	175.00	64.0%	85.140	4.874399	415.00
2	177.50	67.0%	82.758	4.201546	347.71
2 2	180.00	69.8%	80.475 78.285	3.477246 2.909111	279.83
2	185.00	72.6%	76.184	2.531760	192.87
2	187.50	75.8%	74.165	2.240941	166.20
2	190.00	77.7%	72.226	1.923756	138.94
2	192.50	81.4%	70.362	1.763097	124.05
2 2	195.00	84.1%	68.570 66.844	1.572314	107.81
2	200.00	90.6%	65.184	1.335071	87.02
2	202.50	95.4%	63.584	1.320316	83.95
2	205.00	97.3%	62.043	1.175496	72.93
2	207.50	101.4%	60.556	1.145634	69.37
2	210.00	105.1%	87.993	1.107065	97.41
2 2	215.00	112.8%	112:833 107:761	1.059025 0.977577	119.49 105.34
2	225.00	124.1%	103.024	0.876323	90.28
2	230.00	131.7%	98.592	0.872557	86.02
2	235.00	138.4%	94.440	0.854139	80.66
7	240.00	143.7%	133.989 166.991	0.801835	107.43
2 2	250.00 260.00	163.1%	166.991	0.712597	118.99 99.33
2	270.00	169.5%	143.151	0.536193	76.75
2	280.00	180.1%	133.102	0.532343	70.85
2	290.00	185.9%	124.076	0.456556	56.64
2	300.00	193.2%	115.937	0.421263	48.84
2	310.00	200.7%	108.574	0.397461	43.15
2	320.00 330.00	206.1%	101.891 95.807	0.354030	36.07 35.46
	340.00	225.2%	90.252	0.386436	34.87
2 2		The second secon			
	350.00	223.5%	85.166	0.283428	24.13
2 2 2	350.00 360.00	237.4%	80.498	0.351788	28.31
2 2	350.00	1000			24.13 28.31 27.77 19.00

1=put 2=call

**OOF** that's a lot of 2's as well as some weights behind them. As you see from the graphic I posted above it was the same the week after (this week) and you see what has happened. When I made this I needed to confirm my bias even stronger and pull up the OI for the week I just made this and it was:



Note, this was OI before the run up matching the tail risk replicating I modeled. It was pretty clear upward movement was coming.

# Citadel and Volatility

Anyone could do a quick duck go containing "Citadel Volatility" scroll through and see, he has been hiring volatility talent for a while. No one more actually. More images:

By Hema Parmar

19 March 2020, 19:20 CET

Citadel is raising money for a new fund designed to capitalize on volatility in fixed-income markets.

The firm headed by billionaire Ken Griffin registered the Citadel Relative Value Fixed Income Fund with the U.S. Securities and Exchange Commission on March 13, according to a regulatory filing. It didn't report any gathered money or a target.

The new fund aims to take advantage of opportunities created by the recent market tumult sparked by the coronavirus pandemic, according to a person briefed on the matter. The fund will focus on relative value trading, which involves spotting securities trading outside of their normal price range and wagering that the discrepancy will diminish over time.

Citadel fell about 3% this month through March 13th in its main multi-strategy hedge fund, but it's still positive for the year. People familiar with the matter asked not to be identified, because the information is not public. A spokeswoman on Chicago-based Citadel declined to comment on the fundraising plans, which were reported earlier by Institutional Investor.

Original Article: https://www.institutionalinvestor.com/article/b1kt7jz0815czc/citadel-launching-new-fund-to-take-advantage-of-market-conditions

The timing and the context of this article someone transcribed for me is quite telling. A good dispersion trading strategy is going short credit (fixed income) vol and go long equity vol. So just to be clear Citadel has entities whom systematically short volatility (Citadel Advisors) one whom is buying volatility (Citadel Securities) and also one for credit volatility (Citadel RVFIF) its almost like Citadel is *built* around volatility and dispersion trading, hint: it is. They have a history in it, shown <a href="here">here</a>, and considering how they have expanded into a broad spectrum as pointed out above have learned how to get a grasp on it.

#### Conclusion

Variance swaps, or volatility based swaps in general, seem to play a key role in this trade. Considering Citadel's entrenchment with it, it's easy to see to me anyway, how he was cocky enough to take on Melvin's position. Thinking he could hedge it away, internalizing all the risk and profiting off of dispersion trading and systematic variance shorting until everyone got bored and they could get out cheap.

The problem is growing for them, people have held and bought more, making the risk that got/gets internalized much heavier to carry, meaning they have to release that risk back into the market sometime causing unwanted and unmanageable tail risk to hedge away which can in turn make the problem worse (see this week). DRS is having an

affect simply because it basically marks registered shares as insider shares thus removing them circulation making delta hedging (which is daily on replicating portfolios) much harder and more costly.

This is my short, and probably not only, thoughts on variance. I have had a lot of help from <u>u/turdfurg23</u>, <u>u/sweatysuits</u>, and <u>u/atlasmxz</u> I cannot thank them enough. There have been many others I am sure that I'm forgetting, know ahead of time I'm sorry. Thank you for your time.