1 Supplementary resources

1.1 Software and Post-hoc Comparison Procedures Description

Throughout this study we used the software R Studio under R version 4.1.2 (R Core Team, 2021). The datasets simulation will be done using the parameters of control presented bellow. We will take into account two different kinds of PCPs: pairwise (step-wise and simultaneous) PCPs and the control group PCP. We present here all those Post-hoc Comparison Procedures. Additionally, a practical session in R will be presented and will focus on how to perform those tests after fitting a LMEM. We then used fake data as presented in next lines.

Illustrative data: For this practical we consider the following data on maize production (yield). We deal with 05 repeated measures treated with 03 different fertilizers, say $(G1, G2, G3)$, repeated five (05) times within six (06) different blocks (B1, B2, B3, B4, B5 and B6). The first 10 lines of database are presented bellow:

```
load('data.rda')
head(d2, 10)

##    block group yield
## 1     B1    G1  6.01
## 2     B1    G1  5.10
## 3     B1    G1  7.77
## 4     B1    G2  5.37
## 5     B1    G2  6.84
## 6     B1    G2  5.52
## 7     B1    G3  5.80
## 8     B1    G3  5.75
## 9     B1    G3  4.72
## 10    B2    G1  7.72
```

We first performed the LMEM using the function **lme()** from the package **nlme** (Pinheiro *et al.*, 2021). The fitted model is stored in the object **model** what we will use next for the PCPs running.

```
require(nlme)
model <- lme(yield ~ group, random = ~1|block , data = d2)
summary(model)
```

```
## Linear mixed-effects model fit by REML
##   Data: d2
```

```
##         AIC      BIC    logLik
##   149.7223 159.3814 -69.86113
##
## Random effects:
##  Formula: ~1 | block
##         (Intercept) Residual
## StdDev:   0.2764528 0.846088
##
## Fixed effects:  yield ~ group
##                  Value Std.Error DF   t-value p-value
## (Intercept)  6.250000 0.2291462 46 27.275169  0.0000
## groupG2     -0.125000 0.2820293 46 -0.443216  0.6597
## groupG3     -0.845556 0.2820293 46 -2.998112  0.0044
##  Correlation:
##         (Intr) gropG2
## groupG2 -0.615
## groupG3 -0.615  0.500
##
## Standardized Within-Group Residuals:
##         Min          Q1         Med          Q3         Max
## -2.08323965 -0.67585052  0.09802814  0.51102697  2.31346622
##
## Number of Observations: 54
## Number of Groups: 6
```

```
anova(model)
```

```
##             numDF denDF   F-value p-value
## (Intercept)     1    46 1351.1802  <.0001
## group           2    46    5.2375  0.0089
```

Let's consider the following notations:

- X is the random variable which represents the skin pigmentation;
- k the number of groups (families);
- x_{ij} the j^{th} skin pigmentation measured in the i^{th} group;
- n_i the number of observations in the i^{th} group;
- for a group i, the arithmetic mean x_i and sample variance s_i^2 are defined by the following formulas:

$$\bar{x_i} = \frac{1}{n_i}\sum_{j=1}^{n_i} x_{ij} \text{ and } s_i^2 = \frac{1}{n_i - 1}\sum_{j=1}^{n_i}(x_{ij} - \bar{x_i})^2$$

.

To compare two group means $\bar{x_a}$ and $\bar{x_b}$, we define $\alpha = 0.05$, the significance level for the following test in practical session. Let's then define the post-hoc comparison procedures involved in this study. In Appendix, we provide R code lines for running the procedures based on the illustrative example.

1.1.1 Simultaneous Pairwise PCPs

Tukey's Honestly Significant Difference (HSD)

Also sometimes called *Tukey A*, very similar to the SNK test and uses also the Studentized range statistic, this test is developed by Tukey (1953) and designed for equal variances and group sizes. The modified of the initial procedure including unequal group sizes (Tukey-Kramer) is commonly used in practice. Sauder (2017) wrote that the HSD test leads to a significance difference when the next condition is satisfied.

$$|\bar{x}_a - \bar{x}_b| \geqslant Q_{a,b} S_{\epsilon,k,f} \tag{1.1}$$

where:

1. $Q_{a,b} = S_{pp}\sqrt{\frac{1}{2}\left(\frac{1}{n_a} + \frac{1}{n_b}\right)}$
2. $S_{\epsilon,k,f}$ is the Student range statistic for probability $\epsilon = 1 - \alpha$, range difference k and degree of freedom of error f from ANOVA table

The function **HSD.test()** could be also used to perform *Tukey's HDS* procedure using anova model derived from the LMEM. It is also provided by the package **agricolae** and the command line in R is as following:

Scheffé Procedure

The American statistician Scheffé (1953) developed his own test when he was performing analysis of variance. The Scheffé (1999) showed that the procedure detects a significant difference when:

$$|\bar{x}_a - \bar{x}_b| > Q_{a,b}\sqrt{2(k-1)F^{1-\alpha}_{k-1,f}} \tag{1.2}$$

where:

1. $Q_{a,b} = S_{pp}\sqrt{\frac{1}{2}\left(\frac{1}{n_a} + \frac{1}{n_b}\right)}$

2. $F^{1-\alpha}_{k-1,f}$ is the $1-\alpha$ quantile of Fisher distribution with degree of freedom $df1 = k-1$ and $df2 = f$

Sidak

Sidak (1967) developed a modification of the Bonferroni procedure to provide a little more power than Dunn's test (Bonferroni). The critical level per comparison $\alpha^{'}$ is slightly increased to do the job. Then the procedure adjusts the nominal level for each comparison using α_c (Sidak, 1967). The *Sidak* procedure will then detect mean difference if the following is satisfied:

$$|\bar{x}_a - \bar{x}_b| > Q_{a,b}\sqrt{2} \times t^{\alpha_c}_f \tag{1.3}$$

where:

1. $t^{\alpha_c}_f$ is the critical value at $\alpha_c = 1-(1-\alpha)^{1/k^*}$ level of t-distribution with degree of freedom f, the degree of freedom of the mean square error term from the ANOVA table

2. $Q_{a,b} = S_{pp}\sqrt{\frac{1}{2}\left(\frac{1}{n_a} + \frac{1}{n_b}\right)}$

Bonferroni

Dunn (1959, 1961) widely popularized the procedure which used the Bonferroni inequalities (hence the name). The decision of rejecting equality of the two group means $\bar{x}_a$ and $\bar{x}_b$ is taken when:

$$|\bar{x}_a - \bar{x}_b| > Q_{a,b}\sqrt{2} \times t^{\alpha^{'}}_f \tag{1.4}$$

where:

1. $t^{\alpha^{'}}_f$ is the critical value at $\alpha^{'} = \alpha/m$ (Dunn, 1961) adjusted level ($m = C^k_n$ being the total number of pairwise comparisons) of t-distribution with degree of freedom f, the degree of freedom of the mean square error term from the ANOVA table

2. $Q_{a,b} = S_{pp}\sqrt{\frac{1}{2}\left(\frac{1}{n_a} + \frac{1}{n_b}\right)}$

Fisher's Least Significant Difference (LSD)

The procedure is one of the older procedure, it is developed by Fisher (1935) in his study of analysis of variance. Sauder (2017) used the following condition to know that there is a significant means difference:

$$|\bar{x}_a - \bar{x}_b| > Q_{a,b}\sqrt{2} \times t_f^{\alpha} \tag{1.5}$$

where:

1. t_f^{α} is the critical value at α level of t-distribution with degree of freedom f, the degree of freedom of the mean square error term from the ANOVA table

2. $Q_{a,b}$ the pooled standard error as defined above

Duncan Multiple range test

Duncan (1955)'s Multiple Range test (DMRT) is similar to the SNK test and it uses an adjustment of the Studentized range value's alpha level $\alpha^{'}$ equal to $\alpha^{'} = 1 - (1-\alpha)^{k-1}$. Where k is the number of groups in comparison. The test detects significant difference when the difference means satisfies (Sauder, 2017):

$$|\bar{x}_a - \bar{x}_b| \geqslant S_{\alpha^{'},r,f} Q_{a,b} \tag{1.6}$$

where:

1. $S_{\alpha^{'},r,f}$ is the Studentized Range Statistic at level $\alpha^{'}$, number of groups r and degree of freedom of error f from ANOVA table.

2. $Q_{a,b} = S_{pp}\sqrt{\frac{1}{2}\left(\frac{1}{n_a} + \frac{1}{n_b}\right)}$

1.1.2 Stepwise PCPs

Ryan-Einot-Gabriel-Welsch *Range* test (REGWQ)

The test is a stepwise comparison test introduced after the series of three statistical papers: those of the American Thomas A. Ryan (1960), Israeli Israel Einot and the German-born American K. Ruben Gabriel (Einot & Gabriel, 1975) and the american

Roy E. Welsch (1977). To maintain the error rate at each step, the test adjusts the significance levels for each step between means such as:

$$\gamma_r = \begin{cases} \alpha & \text{if } r \geqslant k-1 \\ 1-(1-\alpha)^{r/k} & \text{if } r < k-1 \end{cases} \tag{1.7}$$

where r is the number of means in a subset.

The test is based on the Studentized Range Statistic. The test detects a significant difference means if:

$$|\bar{x_a} - \bar{x_b}| \geqslant S_{pp}\sqrt{\frac{1}{n_a} + \frac{1}{n_b}} \times S_{\gamma_r, r, f} \tag{1.8}$$

where:

1. $S_{1-\gamma_r,k,f}$ is the Studentized Range Statistic at level $1-\gamma_r$, number of steps between the two means in comparison k and degree of freedom of error f from ANOVA table;

2. S_{pp} is the square root of the Mean Square Error in ANOVA table

Benjamini-Hochberg (BH) procedure

Addressing the control of increasing type I error rate when testing simultaneously a family of hypothesis, Benjamini & Hochberg (1995) derived the *BH* procedure (from the authors names). This procedures is a modified of *Bonferroni* procedure using a modified adjusted significance level. Let be $\hat{p_1} \leqslant \hat{p_2} \leqslant \cdots \leqslant \hat{p_m}$ the ordered observed p-values for respectively the tested null hypotheses $H_1^0, H_2^0, \ldots, H_m^0$. For BH, the null hypothesis $H_i^0, i \in \{1, 2, \ldots, m\}$, is rejected when :

$$\hat{p_i}^{BH} = m * \hat{p_i}/i \leqslant \alpha \tag{1.9}$$

Holm-Bonferroni procedure

The emeritus professor in Biostatistics, Sture Holm, developed a *sequentially rejective Bonferroni* procedure (Holm, 1979). Let $\hat{p_1}, \hat{p_2}, \ldots, \hat{p_m}$ be the ordered p-values of respectively to tested hypotheses $H_1, H_2, \ldots, H_m$ for all the m pairwise comparisons. An hypothesis i comparing two group means $\bar{x_a}$ and $\bar{x_b}$ is rejected if the following condition is satisfied (Sauder, 2017):

$$\hat{p_i}^{Holm} = (m + 1 - i)\,\hat{p_i} \leqslant \alpha, \forall i \in \{1, 2, \ldots, m\} \tag{1.10}$$

In original paper Holm (1979) the p-values are used in such way:

$$|\bar{x}_a - \bar{x}_b| \geqslant Q_{a,b}\sqrt{2} \times t_f^{\alpha^{Holm}} \tag{1.11}$$

where:

1. $t_f^{\alpha^{Holm}}$ is the critical value at $\alpha^{Holm} = \alpha / (m + 1 - i)$ adjusted level of t-distribution with degree of freedom f defined in Equation (1.4)
2. $Q_{a,b} = S_{pp}\sqrt{\frac{1}{2}\left(\frac{1}{n_a} + \frac{1}{n_b}\right)}$

Benjamini-Yekutieli (BY) procedure

The problem of dependency of test statistics has been explored by Yekutieli & Benjamini (1999) who found that a simple conservative modification of the *BH* procedure controls the false discovery rate for all other forms of dependency. The modification takes into account the increasing by $\sum_{j=1}^{m} \frac{1}{j}$ of the FDR. Then, for a given null hypothesis H_i^0 comparing $\bar{x_a}$ against $\bar{x_b}$, the procedure is modified such that it detects a significant difference when the p-value $\hat{p_i}$ from the t-statistic multiple comparison test satisfies:

$$\hat{p_i}^{BY} = \hat{p_i}^{BH} * \sum_{j=1}^{m} \frac{1}{j} = \frac{m * \hat{p_i}}{i} \sum_{j=1}^{m} \frac{1}{j} \leqslant \alpha \tag{1.12}$$

where $\hat{p_i}^{BH}$ is the BH modified p-value in Equation (1.9) and α is the initial significance level.

Hochberg procedure

Hochberg (1988) derived a *sharper* procedure for multiple comparisons for family-wise error rate control as compared to the one made by Hommel (1988). The test is based on the *Bonferroni* procedure with p-values adjustment. We still consider $H_1^0, H_2^0, \ldots, H_m^0$ as the $m = C_n^k$ tested null hypothesis for pairwise comparison of k group means and $\hat{p_i}, \forall i = 1, 2, \ldots, m$ the ordered associated p-values computed after multiple comparison$t$-statistic. The test leads to a significant means difference when the following condition is satisfied:

$$\hat{p_i}^{Hoch} = (m - i + 1) * \hat{p_i} \leqslant \alpha \tag{1.13}$$

Another way is to perform *Bonferroni* procedure (Equation 1.4) using the modified significance level $\alpha_{Hoch} = \alpha / (m - i + 1)$.

Hommel procedure

Another Bonferroni procedure modified to control the family-wise error rate is developed by Hommel (1988). Described as more powerful than *Hochberg* procedure, the *Hommel* procedure is performed by first ordering the p-values $\hat{p_1}, \ldots, \hat{p_m}$ obtained from usual pairwise multiple comparison test. Next, we compute

$$j = \max\{i \in \{1, \ldots, n\} : p_{m \hat{-} i+l} \geqslant \frac{l\alpha}{i}, \forall l = 1, \ldots, i\} \tag{1.14}$$

All the m null hypotheses H_i^0 are rejected (significant difference between two means in comparison) if j does not exist else, the null hypotheses H_i^0 s.t. $\hat{p_i}^{Hom} = j * \hat{p_i} \leqslant \alpha$ are rejected.

Student-Newman-Keuls (SNK)

The SNK takes its name from three authors: Student (1927), Newman (1939) and Keuls (1952). The test uses a step-down logic to compare group means producing an increasing of the Type I error rate when the number of groups increases when the assumptions are met (Einot & Gabriel, 1975). When comparing two groups, there is significant difference when :

$$|\bar{x}_a - \bar{x}_b| \geqslant S_{pp} \sqrt{\frac{1}{2}\left(\frac{1}{n_a} + \frac{1}{n_b}\right)} S_{\epsilon,r,f} \tag{1.15}$$

where :

1. $S_{\epsilon,r,f}$ is the Studentized range statistic with parameters: probability $\epsilon = 1 - \alpha$, the number of steps between the ordered means being compared r and the degree of freedom of error f;

2. S_{pp} is the square root of the MSE from ANOVA table

1.1.3 Control group PCP

Dunnett t test

To compare k treatment groups to a control group, the Canadian statistician Charles Dunnett (1955) developed his test for multiple comparison. The Dunnett's two-sides test detects an significant difference between the control group 0 and treatment group i when:

$$|\bar{x}_i - \bar{x}_0| > d_k^\alpha \times s \times \sqrt{\left(\frac{1}{n_i} + \frac{1}{n_0}\right)} \tag{1.16}$$

where:

1. $s^2 = \dfrac{\sum_{i=0}^{k} \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i)^2}{\sum_{i=0}^{k} (n_i - 1)}$
2. d_k^α is the upper $\alpha \times 100\%$ point of the distribution of $T = \max_{1 \leqslant i \leqslant k} \{|T_i|\}$ and $T_i = \dfrac{\bar{x}_i - \bar{x}_0}{s\sqrt{\frac{1}{n_i} + \frac{1}{n_0}}}$

1.2 Additional Tables and Figures

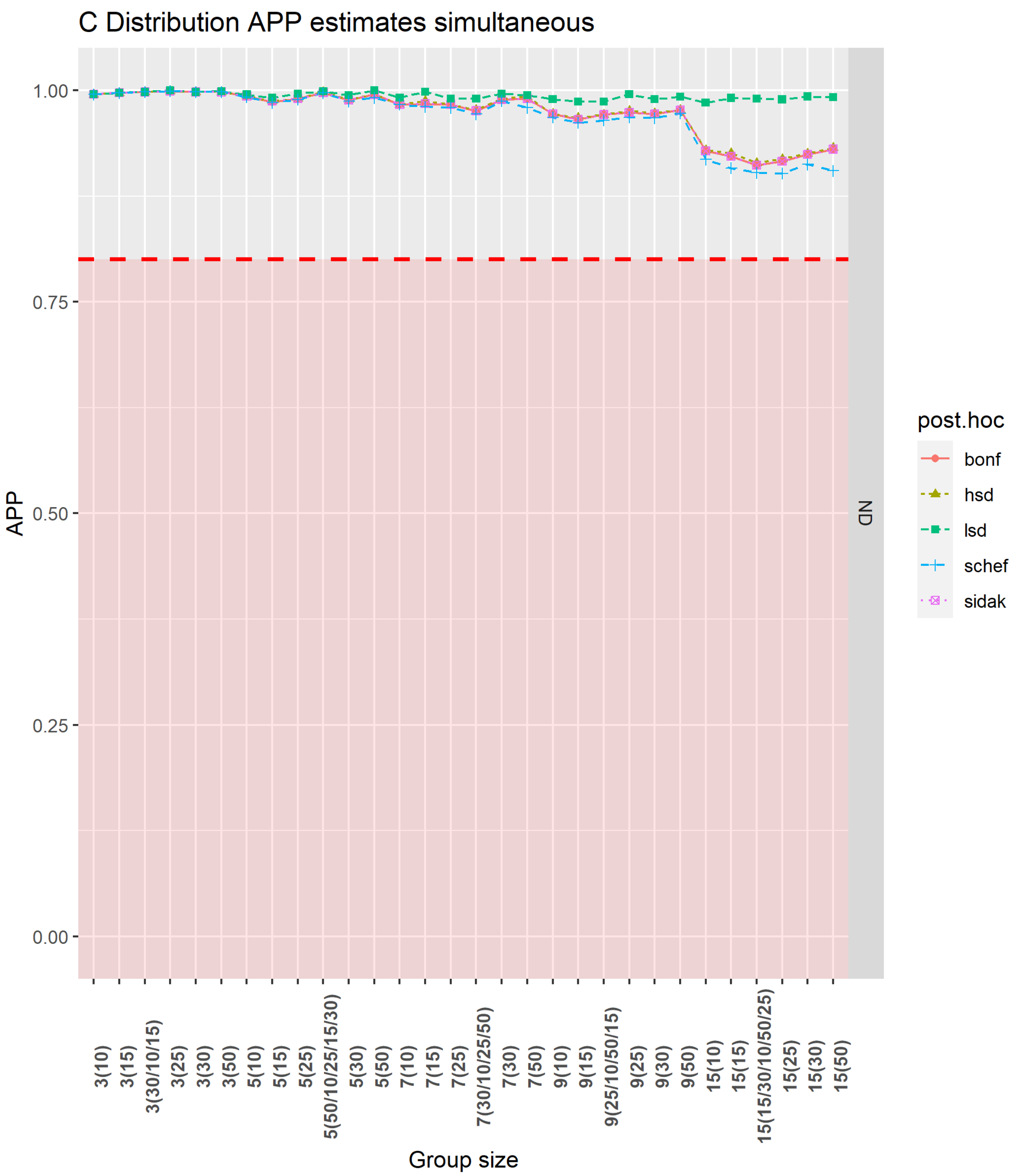

Figure 1: All Pairwise Power Estimates for Simultaneous Procedures
Note: *Red area* represents the non desirable region for the estimates. **UV** and **EV** stand respectively for *Unequal Variances* and *Equal Variances*.

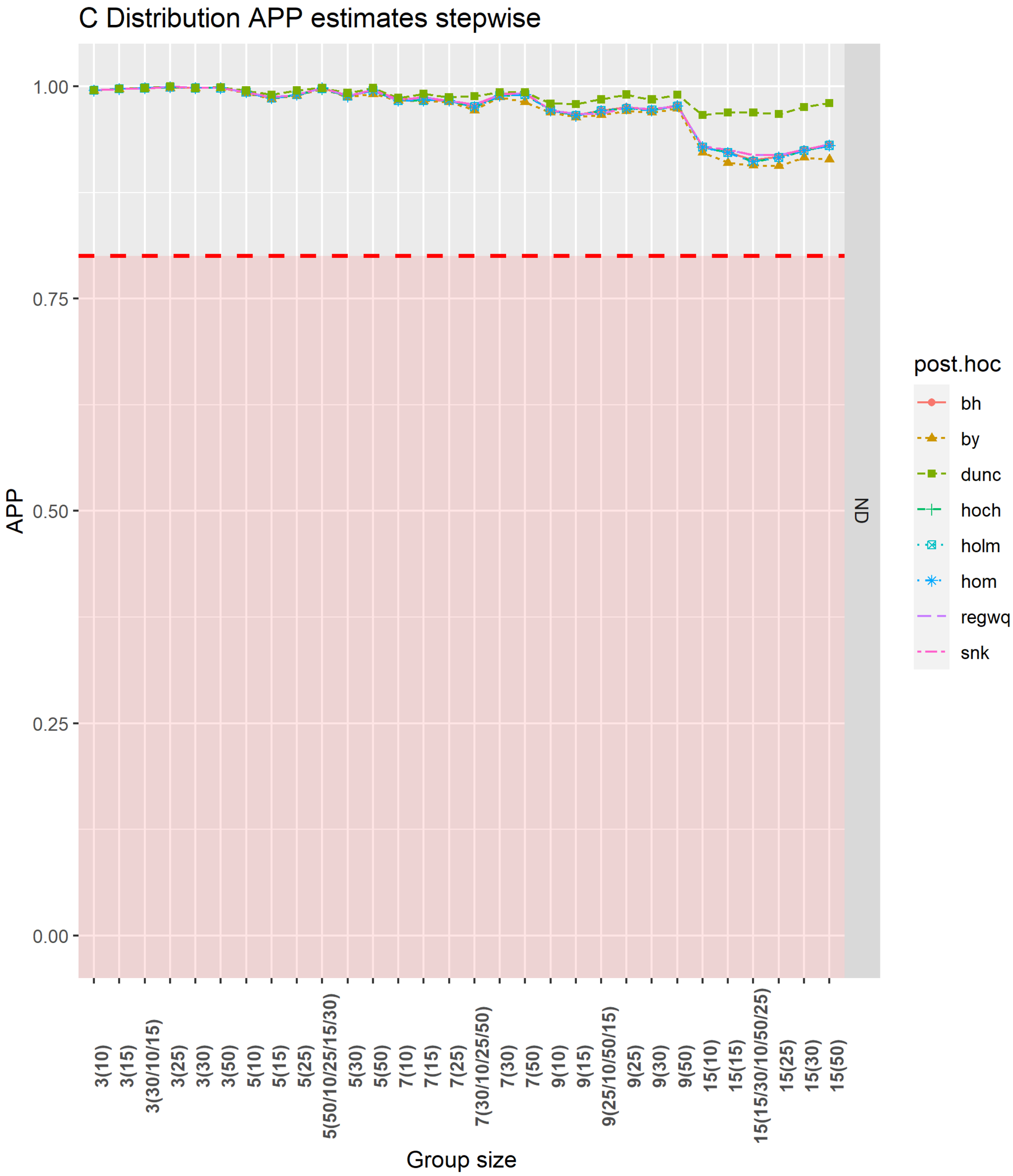

Figure 2: All Pairwise Power Estimates for Stepwise Procedures
Note: *Red area* represents the non desirable region for the estimates. **UV** and **EV** stand respectively for *Unequal Variances* and *Equal Variances*.

Table 1: Type I error rate (ALPHA) estimate for Contaminated Normal (C) distribution

GN	Bal	GS	DT	nb	**bh**	**bonf**	**by**	**dunc**	**dunnet**	**hoch**	**holm**	**hom**	**hsd**	**lsd**	**regwq**	**schef**	**sidak**	**snk**
3	Bg	3(50)	C	150	0.122	0.116	0.080	0.212	0.124	0.118	0.116	0.122	0.136	0.243	0.146	0.104	0.119	0.146
3	Bg	3(30)	C	90	0.093	0.087	**0.062**	0.159	0.083	0.089	0.087	0.093	0.094	0.195	0.108	0.081	0.089	0.108
3	Bg	3(25)	C	75	0.065	**0.062**	**0.038**	0.129	0.080	**0.063**	**0.062**	**0.064**	0.070	0.154	0.081	**0.058**	**0.063**	0.081
3	Bg	3(15)	C	45	0.104	0.101	0.068	0.164	0.108	0.103	0.101	0.104	0.110	0.189	0.119	0.088	0.102	0.119
3	Bg	3(10)	C	30	0.204	0.201	0.176	0.257	0.202	0.202	0.201	0.204	0.208	0.276	0.214	0.189	0.201	0.214
3	Ubg	3(30/10/15)	C	55	0.071	0.065	**0.043**	0.134	0.075	0.066	0.065	0.069	0.072	0.154	0.088	**0.061**	0.067	0.088
7	Bg	7(50)	C	350	0.084	0.074	0.028	0.341	0.085	0.075	0.074	0.077	0.097	**0.499**	0.098	0.022	0.076	0.100
7	Bg	7(30)	C	210	0.072	**0.063**	0.020	0.327	0.066	**0.063**	**0.063**	**0.063**	0.079	**0.501**	0.081	0.009	**0.064**	0.084
7	Bg	7(25)	C	175	**0.049**	**0.045**	0.013	0.293	**0.053**	**0.045**	**0.045**	**0.045**	**0.057**	**0.507**	**0.058**	0.010	**0.045**	**0.061**
7	Bg	7(15)	C	105	**0.045**	**0.039**	0.011	0.284	**0.043**	**0.039**	**0.039**	**0.040**	**0.048**	**0.463**	**0.050**	0.007	0.040	**0.052**
7	Bg	7(10)	C	70	**0.042**	**0.039**	0.012	0.282	**0.051**	**0.039**	**0.039**	**0.039**	**0.052**	0.464	**0.053**	0.007	**0.040**	**0.053**
7	Ubg	7(30/10/25/50)	C	200	**0.052**	**0.049**	0.018	0.343	**0.058**	**0.049**	**0.049**	**0.052**	**0.059**	0.481	0.085	0.007	**0.052**	0.085
5	Bg	5(50)	C	250	0.101	0.088	**0.038**	0.303	0.107	0.088	0.088	0.093	0.106	0.384	0.109	**0.046**	0.088	0.117
5	Bg	5(30)	C	150	0.081	0.074	0.033	0.257	0.088	0.074	0.074	0.077	0.092	0.366	0.095	0.033	0.074	0.098
5	Bg	5(25)	C	125	0.070	**0.061**	0.022	0.229	0.071	**0.062**	**0.061**	**0.062**	0.073	0.319	0.077	0.025	**0.061**	0.080
5	Bg	5(15)	C	75	**0.055**	**0.052**	0.022	0.205	**0.058**	**0.052**	**0.052**	**0.052**	**0.064**	0.307	**0.064**	0.023	**0.053**	0.067
5	Bg	5(10)	C	50	0.066	**0.063**	0.028	0.205	0.071	**0.063**	**0.063**	0.065	0.072	0.312	0.073	0.030	**0.064**	0.075
5	Ubg	5(50/10/25/15/30)	C	130	**0.055**	**0.050**	0.018	0.215	0.079	**0.050**	**0.050**	**0.050**	0.066	0.340	0.083	0.019	**0.050**	0.088
9	Bg	9(50)	C	450	0.067	**0.059**	0.013	0.456	0.067	**0.059**	**0.059**	**0.059**	0.074	0.672	0.076	0.002	**0.059**	0.076
9	Bg	9(30)	C	270	**0.058**	**0.050**	0.010	0.407	**0.063**	**0.050**	**0.050**	**0.050**	0.071	0.659	0.071	0.001	**0.051**	0.071
9	Bg	9(25)	C	225	**0.057**	**0.050**	0.021	0.406	0.067	**0.050**	**0.050**	**0.051**	0.068	0.641	0.068	0.003	**0.051**	0.069
9	Bg	9(15)	C	135	**0.048**	**0.044**	0.009	0.368	**0.055**	**0.044**	**0.044**	**0.044**	**0.056**	0.591	**0.058**	0.001	**0.044**	**0.059**
9	Bg	9(10)	C	90	**0.050**	**0.041**	0.013	0.349	**0.052**	**0.041**	**0.041**	**0.041**	**0.057**	0.591	**0.057**	0.004	**0.042**	**0.057**
9	Ubg	9(25/10/50/15)	C	210	**0.050**	**0.039**	0.010	0.383	**0.056**	**0.039**	**0.039**	**0.039**	**0.053**	0.576	0.088	0.001	**0.040**	0.088
15	Bg	15(50)	C	750	**0.046**	**0.044**	0.012	0.555	0.075	**0.044**	**0.044**	**0.044**	**0.061**	0.847	**0.061**	0.000	**0.044**	**0.061**
15	Bg	15(30)	C	450	**0.046**	**0.044**	0.012	0.560	**0.053**	**0.044**	**0.044**	**0.044**	**0.058**	0.842	**0.058**	0.000	**0.044**	**0.058**
15	Bg	15(25)	C	375	**0.054**	**0.049**	0.013	0.515	**0.054**	**0.049**	**0.049**	**0.049**	**0.062**	0.832	**0.062**	0.000	**0.049**	**0.062**
15	Bg	15(15)	C	225	0.029	0.023	0.006	0.541	**0.055**	0.023	0.023	0.023	0.037	0.844	0.037	0.000	0.023	0.038
15	Bg	15(10)	C	150	0.033	0.027	0.005	0.492	**0.049**	0.027	0.027	0.027	**0.050**	0.810	**0.050**	0.000	0.029	**0.050**
15	Ubg	15(15/30/10/50/25)	C	355	**0.044**	**0.038**	0.007	0.528	**0.057**	**0.038**	**0.038**	**0.038**	**0.050**	0.822	0.086	0.000	**0.039**	0.086

Note: Values in **bold** indicates control of the estimates at $0.05 \pm 1.96 \times \sqrt{0.05 \times 0.95/1000}$

Table 2: Type I error rate (ALPHA) estimates for Truncated Normal (T) distribution

GN	Bal	GS	DT	nb	**bh**	**bonf**	**by**	**dunc**	**dunnet**	**hoch**	**holm**	**hom**	**hsd**	**lsd**	**regwq**	**schef**	**sidak**	**snk**
3	Bg	3(50)	T	150	0.304	0.295	0.239	0.411	0.288	0.298	0.295	0.303	0.316	0.441	0.342	0.275	0.296	0.342
3	Bg	3(30)	T	90	0.205	0.202	0.158	0.301	0.201	0.204	0.202	0.205	0.216	0.340	0.228	0.190	0.204	0.228
3	Bg	3(25)	T	75	0.191	0.184	0.133	0.277	0.196	0.189	0.184	0.190	0.200	0.310	0.214	0.170	0.185	0.214
3	Bg	3(15)	T	45	0.123	0.112	0.080	0.224	0.127	0.116	0.112	0.118	0.130	0.248	0.148	0.103	0.113	0.148
3	Bg	3(10)	T	30	0.107	0.101	**0.064**	0.179	0.119	0.103	0.101	0.106	0.112	0.209	0.125	0.086	0.103	0.125
3	Ubg	3(30/10/15)	T	55	0.134	0.130	0.086	0.209	0.139	0.130	0.130	0.133	0.138	0.249	0.143	0.115	0.130	0.143
7	Bg	7(50)	T	350	0.453	0.424	0.252	0.783	0.361	0.424	0.424	0.429	0.474	0.876	0.479	0.188	0.429	0.482
7	Bg	7(30)	T	210	0.256	0.225	0.113	0.630	0.217	0.225	0.225	0.230	0.259	0.779	0.265	0.082	0.231	0.267
7	Bg	7(25)	T	175	0.268	0.247	0.139	0.610	0.194	0.247	0.247	0.247	0.280	0.764	0.282	0.106	0.254	0.284
7	Bg	7(15)	T	105	0.159	0.144	**0.062**	0.505	0.156	0.144	0.144	0.146	0.173	0.675	0.174	**0.036**	0.148	0.178
7	Bg	7(10)	T	70	0.122	0.108	**0.043**	0.452	0.115	0.108	0.108	0.110	0.134	0.604	0.136	0.021	0.110	0.140
7	Ubg	7(30/10/25/50)	T	200	0.256	0.237	0.138	0.556	0.218	0.237	0.237	0.239	0.274	0.765	0.232	0.092	0.240	0.234
5	Bg	5(50)	T	250	0.389	0.372	0.248	0.635	0.320	0.373	0.372	0.379	0.410	0.702	0.415	0.265	0.374	0.424
5	Bg	5(30)	T	150	0.278	0.259	0.157	0.553	0.238	0.260	0.259	0.263	0.294	0.652	0.301	0.160	0.261	0.307
5	Bg	5(25)	T	125	0.225	0.210	0.133	0.467	0.213	0.210	0.210	0.213	0.228	0.591	0.235	0.139	0.210	0.242
5	Bg	5(15)	T	75	0.161	0.149	0.081	0.378	0.147	0.149	0.149	0.152	0.176	0.498	0.181	0.088	0.150	0.185
5	Bg	5(10)	T	50	0.127	0.120	**0.062**	0.329	0.117	0.120	0.120	0.123	0.140	0.463	0.142	0.065	0.121	0.144
5	Ubg	5(50/10/25/15/30)	T	130	0.196	0.183	0.111	0.399	0.230	0.183	0.183	0.184	0.206	0.537	0.182	0.122	0.183	0.189
9	Bg	9(50)	T	450	0.508	0.468	0.277	0.866	0.376	0.468	0.468	0.472	0.515	0.940	0.519	0.129	0.469	0.523
9	Bg	9(30)	T	270	0.325	0.291	0.139	0.772	0.235	0.291	0.291	0.291	0.340	0.888	0.342	**0.047**	0.292	0.345
9	Bg	9(25)	T	225	0.279	0.251	0.117	0.722	0.202	0.251	0.251	0.252	0.302	0.866	0.307	**0.037**	0.256	0.309
9	Bg	9(15)	T	135	0.170	0.154	**0.064**	0.586	0.148	0.154	0.154	0.154	0.188	0.775	0.189	0.021	0.156	0.189
9	Bg	9(10)	T	90	0.134	0.122	**0.039**	0.548	0.124	0.122	0.122	0.124	0.145	0.734	0.145	0.013	0.123	0.148
9	Ubg	9(25/10/50/15)	T	210	0.225	0.207	0.096	0.601	0.204	0.207	0.207	0.208	0.244	0.831	0.219	0.033	0.209	0.221
15	Bg	15(50)	T	750	0.523	0.481	0.246	0.980	0.324	0.481	0.481	0.481	0.557	0.996	0.559	0.004	0.481	0.559
15	Bg	15(30)	T	450	0.372	0.329	0.127	0.906	0.268	0.329	0.329	0.329	0.387	0.982	0.387	0.003	0.331	0.387
15	Bg	15(25)	T	375	0.261	0.218	0.071	0.895	0.219	0.218	0.218	0.224	0.276	0.983	0.276	0.000	0.228	0.276
15	Bg	15(15)	T	225	0.185	0.147	**0.047**	0.798	0.142	0.147	0.147	0.149	0.205	0.954	0.205	0.002	0.152	0.205
15	Bg	15(10)	T	150	0.143	0.123	0.032	0.735	0.125	0.123	0.123	0.123	0.155	0.912	0.154	0.000	0.126	0.154
15	Ubg	15(15/30/10/50/25)	T	355	0.261	0.224	0.076	0.815	0.150	0.224	0.224	0.226	0.293	0.971	0.229	0.000	0.232	0.229

Note: Values in **bold** indicates control of the estimates at $0.05 \pm 1.96 \times \sqrt{0.05 \times 0.95/1000}$

Table 3: Any-pairwise-Power (ANP) estimates for Contaminated Normal (C) distribution

GN	Bal	GS	DT	nb	**bh**	**bonf**	**by**	**dunc**	**dunnet**	**hoch**	**holm**	**hom**	**hsd**	**lsd**	**regwq**	**schef**	**sidak**	**snk**
3	Bg	3(50)	C	150	0.350	0.350	0.350	0.350	0.350	0.350	0.350	0.350	0.350	0.350	0.350	0.350	0.350	0.350
3	Bg	3(30)	C	90	0.420	0.420	0.420	0.420	0.420	0.420	0.420	0.420	0.420	0.420	0.420	0.420	0.420	0.420
3	Bg	3(25)	C	75	0.482	0.482	0.482	0.483	0.482	0.482	0.482	0.482	0.482	0.483	0.483	0.482	0.482	0.483
3	Bg	3(15)	C	45	0.663	0.663	0.663	0.663	0.663	0.663	0.663	0.663	0.663	0.663	0.663	0.663	0.663	0.663
3	Bg	3(10)	C	30	0.798	0.798	0.798	0.799	0.793	0.798	0.798	0.798	0.798	0.799	0.799	0.798	0.798	0.799
3	Ubg	3(30/10/15)	C	55	0.743	0.743	0.743	0.743	0.743	0.743	0.743	0.743	0.743	0.743	0.742	0.743	0.743	0.742
7	Bg	7(50)	C	350	0.056	0.056	0.056	0.057	0.056	0.056	0.056	0.056	0.056	0.057	0.056	0.056	0.056	0.056
7	Bg	7(30)	C	210	0.063	0.062	0.062	0.063	0.062	0.062	0.062	0.062	0.063	0.063	0.063	0.062	0.062	0.063
7	Bg	7(25)	C	175	0.065	0.065	0.065	0.065	0.065	0.065	0.065	0.065	0.065	0.065	0.065	0.064	0.065	0.065
7	Bg	7(15)	C	105	0.076	0.076	0.076	0.077	0.076	0.076	0.076	0.076	0.076	0.077	0.076	0.076	0.076	0.076
7	Bg	7(10)	C	70	0.091	0.091	0.091	0.091	0.091	0.091	0.091	0.091	0.091	0.092	0.091	0.091	0.091	0.091
7	Ubg	7(30/10/25/50)	C	200	0.068	0.068	0.068	0.069	0.068	0.068	0.068	0.068	0.068	0.069	0.068	0.068	0.068	0.068
5	Bg	5(50)	C	250	0.113	0.113	0.113	0.113	0.113	0.113	0.113	0.113	0.113	0.113	0.113	0.113	0.113	0.113
5	Bg	5(30)	C	150	0.134	0.134	0.134	0.134	0.134	0.134	0.134	0.134	0.134	0.134	0.134	0.134	0.134	0.134
5	Bg	5(25)	C	125	0.142	0.142	0.142	0.142	0.141	0.142	0.142	0.142	0.142	0.142	0.142	0.142	0.142	0.142
5	Bg	5(15)	C	75	0.172	0.172	0.172	0.172	0.172	0.172	0.172	0.172	0.172	0.172	0.172	0.172	0.172	0.172
5	Bg	5(10)	C	50	0.219	0.219	0.219	0.219	0.219	0.219	0.219	0.219	0.219	0.219	0.219	0.219	0.219	0.219
5	Ubg	5(50/10/25/15/30)	C	130	0.164	0.164	0.164	0.164	0.164	0.164	0.164	0.164	0.164	0.164	0.164	0.164	0.164	0.164
9	Bg	9(50)	C	450	0.033	0.033	0.033	0.033	0.033	0.033	0.033	0.033	0.033	0.033	0.033	0.033	0.033	0.033
9	Bg	9(30)	C	270	0.035	0.035	0.035	0.036	0.035	0.035	0.035	0.035	0.035	0.036	0.035	0.035	0.035	0.035
9	Bg	9(25)	C	225	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037
9	Bg	9(15)	C	135	0.042	0.042	0.042	0.043	0.042	0.042	0.042	0.042	0.042	0.043	0.042	0.042	0.042	0.042
9	Bg	9(10)	C	90	0.049	0.049	0.049	0.049	0.049	0.049	0.049	0.049	0.049	0.050	0.049	0.049	0.049	0.049
9	Ubg	9(25/10/50/15)	C	210	0.041	0.041	0.041	0.041	0.041	0.041	0.041	0.041	0.041	0.041	0.041	0.041	0.041	0.041
15	Bg	15(50)	C	750	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011
15	Bg	15(30)	C	450	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012
15	Bg	15(25)	C	375	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012	0.012
15	Bg	15(15)	C	225	0.013	0.013	0.013	0.014	0.013	0.013	0.013	0.013	0.013	0.014	0.013	0.013	0.013	0.013
15	Bg	15(10)	C	150	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015	0.015
15	Ubg	15(15/30/10/50/25)	C	355	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013

Table 4: Any-pairwise-Power (ANP) estimates for Truncated Normal (T) distribution

GN	Bal	GS	DT	nb	**bh**	**bonf**	**by**	**dunc**	**dunnet**	**hoch**	**holm**	**hom**	**hsd**	**lsd**	**regwq**	**schef**	**sidak**	**snk**
3	Bg	3(50)	T	150	0.343	0.343	0.343	0.343	0.337	0.343	0.343	0.343	0.343	0.343	0.343	0.343	0.343	0.343
3	Bg	3(30)	T	90	0.335	0.335	0.335	0.335	0.333	0.335	0.335	0.335	0.335	0.335	0.335	0.335	0.335	0.335
3	Bg	3(25)	T	75	0.334	0.334	0.334	0.334	0.333	0.334	0.334	0.334	0.334	0.334	0.334	0.334	0.334	0.334
3	Bg	3(15)	T	45	0.341	0.341	0.341	0.342	0.341	0.341	0.341	0.341	0.341	0.342	0.341	0.341	0.341	0.341
3	Bg	3(10)	T	30	0.384	0.384	0.384	0.384	0.384	0.384	0.384	0.384	0.384	0.385	0.384	0.384	0.384	0.384
3	Ubg	3(30/10/15)	T	55	0.353	0.353	0.353	0.353	0.353	0.353	0.353	0.353	0.353	0.353	0.353	0.353	0.353	0.353
7	Bg	7(50)	T	350	0.055	0.054	0.054	0.055	0.049	0.055	0.055	0.055	0.055	0.055	0.055	0.054	0.054	0.055
7	Bg	7(30)	T	210	0.051	0.051	0.051	0.051	0.049	0.051	0.051	0.051	0.051	0.051	0.051	0.050	0.051	0.051
7	Bg	7(25)	T	175	0.050	0.050	0.049	0.050	0.048	0.050	0.050	0.050	0.050	0.050	0.050	0.049	0.050	0.050
7	Bg	7(15)	T	105	0.049	0.049	0.049	0.050	0.048	0.049	0.049	0.049	0.049	0.050	0.049	0.049	0.049	0.049
7	Bg	7(10)	T	70	0.051	0.051	0.051	0.051	0.051	0.051	0.051	0.051	0.051	0.051	0.051	0.051	0.051	0.051
7	Ubg	7(30/10/25/50)	T	200	0.051	0.050	0.050	0.051	0.049	0.050	0.050	0.050	0.050	0.051	0.050	0.050	0.050	0.050
5	Bg	5(50)	T	250	0.108	0.108	0.107	0.108	0.103	0.108	0.108	0.108	0.108	0.108	0.108	0.107	0.108	0.108
5	Bg	5(30)	T	150	0.103	0.103	0.102	0.103	0.101	0.103	0.103	0.103	0.103	0.103	0.103	0.102	0.103	0.103
5	Bg	5(25)	T	125	0.102	0.102	0.102	0.102	0.100	0.102	0.102	0.102	0.102	0.102	0.102	0.101	0.102	0.102
5	Bg	5(15)	T	75	0.102	0.102	0.102	0.103	0.102	0.102	0.102	0.102	0.102	0.103	0.102	0.102	0.102	0.102
5	Bg	5(10)	T	50	0.110	0.110	0.110	0.110	0.109	0.110	0.110	0.110	0.110	0.110	0.110	0.110	0.110	0.110
5	Ubg	5(50/10/25/15/30)	T	130	0.102	0.102	0.102	0.102	0.102	0.102	0.102	0.102	0.102	0.102	0.102	0.102	0.102	0.102
9	Bg	9(50)	T	450	0.035	0.034	0.034	0.035	0.029	0.034	0.034	0.034	0.034	0.035	0.034	0.032	0.034	0.034
9	Bg	9(30)	T	270	0.030	0.030	0.030	0.031	0.028	0.030	0.030	0.030	0.030	0.031	0.030	0.029	0.030	0.030
9	Bg	9(25)	T	225	0.030	0.030	0.029	0.030	0.028	0.030	0.030	0.030	0.030	0.030	0.030	0.029	0.030	0.030
9	Bg	9(15)	T	135	0.029	0.029	0.029	0.029	0.028	0.029	0.029	0.029	0.029	0.029	0.029	0.029	0.029	0.029
9	Bg	9(10)	T	90	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.030	0.029	0.030	0.030
9	Ubg	9(25/10/50/15)	T	210	0.030	0.030	0.029	0.030	0.028	0.030	0.030	0.030	0.030	0.030	0.029	0.029	0.030	0.029
15	Bg	15(50)	T	750	0.014	0.014	0.013	0.014	0.010	0.014	0.014	0.014	0.014	0.014	0.014	0.011	0.014	0.014
15	Bg	15(30)	T	450	0.012	0.011	0.011	0.012	0.010	0.011	0.011	0.011	0.011	0.012	0.011	0.010	0.011	0.011
15	Bg	15(25)	T	375	0.011	0.011	0.011	0.012	0.010	0.011	0.011	0.011	0.011	0.012	0.011	0.010	0.011	0.011
15	Bg	15(15)	T	225	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.011	0.010	0.010	0.010	0.010
15	Bg	15(10)	T	150	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.011	0.010	0.010	0.010	0.010
15	Ubg	15(15/30/10/50/25)	T	355	0.011	0.011	0.011	0.011	0.010	0.011	0.011	0.011	0.011	0.011	0.011	0.010	0.011	0.011

Table 5: Any-pairwise-Power (ANP) estimates for Exponential (E) distribution

GN	Bal	GS	DT	nb	**bh**	**bonf**	**by**	**dunc**	**dunnet**	**hoch**	**holm**	**hom**	**hsd**	**lsd**	**regwq**	**schef**	**sidak**	**snk**
3	Bg	3(50)	E	150	0.335	0.335	0.335	0.335	0.335	0.335	0.335	0.335	0.335	0.335	0.335	0.335	0.335	0.335
3	Bg	3(30)	E	90	0.336	0.336	0.336	0.336	0.336	0.336	0.336	0.336	0.336	0.336	0.336	0.336	0.336	0.336
3	Bg	3(25)	E	75	0.336	0.336	0.336	0.336	0.336	0.336	0.336	0.336	0.336	0.336	0.336	0.336	0.336	0.336
3	Bg	3(15)	E	45	0.337	0.337	0.337	0.337	0.337	0.337	0.337	0.337	0.337	0.337	0.337	0.337	0.337	0.337
3	Bg	3(10)	E	30	0.339	0.339	0.339	0.339	0.339	0.339	0.339	0.339	0.339	0.339	0.339	0.339	0.339	0.339
3	Ubg	3(30/10/15)	E	55	0.337	0.337	0.337	0.337	0.337	0.337	0.337	0.337	0.337	0.337	0.337	0.337	0.337	0.337
7	Bg	7(50)	E	350	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048
7	Bg	7(30)	E	210	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048
7	Bg	7(25)	E	175	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048
7	Bg	7(15)	E	105	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048
7	Bg	7(10)	E	70	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048
7	Ubg	7(30/10/25/50)	E	200	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048
5	Bg	5(50)	E	250	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101
5	Bg	5(30)	E	150	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101
5	Bg	5(25)	E	125	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101
5	Bg	5(15)	E	75	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101
5	Bg	5(10)	E	50	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101
5	Ubg	5(50/10/25/15/30)	E	130	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101	0.101
9	Bg	9(50)	E	450	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028
9	Bg	9(30)	E	270	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028
9	Bg	9(25)	E	225	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028
9	Bg	9(15)	E	135	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028
9	Bg	9(10)	E	90	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028
9	Ubg	9(25/10/50/15)	E	210	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028	0.028
15	Bg	15(50)	E	750	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010
15	Bg	15(30)	E	450	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010
15	Bg	15(25)	E	375	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010
15	Bg	15(15)	E	225	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010
15	Bg	15(10)	E	150	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010
15	Ubg	15(15/30/10/50/25)	E	355	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.010

Table 6: All-Pairwise-Power (APP) estimates for Contaminated Normal (C) distribution

GN	Bal	GS	DT	nb	**bh**	**bonf**	**by**	**dunc**	**dunnet**	**hoch**	**holm**	**hom**	**hsd**	**lsd**	**regwq**	**schef**	**sidak**	**snk**
3	Bg	3(50)	C	150	0.998	0.998	0.998	0.999	0.998	0.998	0.998	0.998	0.998	0.999	0.998	0.998	0.998	0.998
3	Bg	3(30)	C	90	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998
3	Bg	3(25)	C	75	0.999	0.999	0.998	1.000	0.999	0.999	0.999	0.999	0.999	1.000	1.000	0.999	0.999	1.000
3	Bg	3(15)	C	45	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997
3	Bg	3(10)	C	30	0.995	0.995	0.995	0.995	0.995	0.995	0.995	0.995	0.995	0.995	0.995	0.995	0.995	0.995
3	Ubg	3(30/10/15)	C	55	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.997	0.998	0.998	0.997
7	Bg	7(50)	C	350	0.991	0.990	0.981	0.993	0.981	0.990	0.990	0.990	0.992	0.994	0.992	0.980	0.990	0.992
7	Bg	7(30)	C	210	0.991	0.989	0.987	0.993	0.986	0.989	0.989	0.989	0.990	0.996	0.990	0.987	0.989	0.990
7	Bg	7(25)	C	175	0.983	0.983	0.981	0.987	0.982	0.983	0.983	0.983	0.983	0.990	0.983	0.979	0.983	0.983
7	Bg	7(15)	C	105	0.984	0.983	0.982	0.991	0.981	0.983	0.983	0.983	0.987	0.998	0.987	0.981	0.984	0.987
7	Bg	7(10)	C	70	0.985	0.983	0.983	0.986	0.985	0.983	0.983	0.983	0.983	0.991	0.983	0.983	0.983	0.984
7	Ubg	7(30/10/25/50)	C	200	0.976	0.976	0.972	0.988	0.978	0.976	0.976	0.976	0.977	0.990	0.979	0.972	0.976	0.979
5	Bg	5(50)	C	250	0.995	0.995	0.991	0.998	0.994	0.995	0.995	0.995	0.995	1.000	0.995	0.991	0.995	0.996
5	Bg	5(30)	C	150	0.988	0.988	0.987	0.992	0.988	0.988	0.988	0.988	0.989	0.994	0.989	0.987	0.988	0.989
5	Bg	5(25)	C	125	0.990	0.990	0.989	0.995	0.988	0.990	0.990	0.990	0.990	0.996	0.990	0.989	0.990	0.990
5	Bg	5(15)	C	75	0.986	0.986	0.985	0.990	0.985	0.986	0.986	0.986	0.987	0.991	0.987	0.985	0.987	0.987
5	Bg	5(10)	C	50	0.993	0.993	0.992	0.995	0.994	0.993	0.993	0.993	0.993	0.995	0.993	0.992	0.993	0.993
5	Ubg	5(50/10/25/15/30)	C	130	0.998	0.997	0.997	0.998	0.997	0.997	0.997	0.998	0.998	0.999	0.998	0.997	0.997	0.998
9	Bg	9(50)	C	450	0.978	0.977	0.974	0.990	0.974	0.977	0.977	0.977	0.977	0.992	0.977	0.973	0.977	0.977
9	Bg	9(30)	C	270	0.972	0.972	0.969	0.985	0.969	0.972	0.972	0.972	0.973	0.989	0.973	0.967	0.972	0.973
9	Bg	9(25)	C	225	0.976	0.974	0.971	0.990	0.976	0.974	0.974	0.974	0.976	0.995	0.976	0.968	0.974	0.976
9	Bg	9(15)	C	135	0.966	0.966	0.964	0.979	0.966	0.966	0.966	0.966	0.967	0.987	0.967	0.962	0.966	0.967
9	Bg	9(10)	C	90	0.972	0.972	0.969	0.980	0.972	0.972	0.972	0.972	0.972	0.989	0.972	0.968	0.972	0.972
9	Ubg	9(25/10/50/15)	C	210	0.971	0.971	0.966	0.985	0.966	0.971	0.971	0.971	0.971	0.987	0.967	0.964	0.971	0.967
15	Bg	15(50)	C	750	0.931	0.930	0.914	0.980	0.919	0.930	0.930	0.930	0.932	0.992	0.932	0.905	0.930	0.932
15	Bg	15(30)	C	450	0.925	0.924	0.916	0.975	0.923	0.924	0.924	0.924	0.925	0.993	0.925	0.912	0.924	0.925
15	Bg	15(25)	C	375	0.917	0.916	0.906	0.968	0.916	0.916	0.916	0.916	0.919	0.989	0.919	0.902	0.916	0.919
15	Bg	15(15)	C	225	0.923	0.922	0.910	0.969	0.924	0.922	0.922	0.922	0.926	0.991	0.926	0.908	0.922	0.926
15	Bg	15(10)	C	150	0.928	0.928	0.922	0.966	0.929	0.928	0.928	0.928	0.929	0.985	0.929	0.918	0.928	0.929
15	Ubg	15(15/30/10/50/25)	C	355	0.913	0.912	0.907	0.969	0.913	0.912	0.912	0.912	0.914	0.990	0.919	0.903	0.912	0.919

Table 7: All-Pairwise-Power (APP) estimates for Truncated Normal (T) distribution

GN	Bal	GS	DT	nb	**bh**	**bonf**	**by**	**dunc**	**dunnet**	**hoch**	**holm**	**hom**	**hsd**	**lsd**	**regwq**	**schef**	**sidak**	**snk**
3	Bg	3(50)	T	150	0.998	0.998	0.998	0.998	0.996	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998
3	Bg	3(30)	T	90	0.997	0.997	0.996	0.997	0.996	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997
3	Bg	3(25)	T	75	0.999	0.999	0.999	1.000	0.999	0.999	0.999	0.999	0.999	1.000	0.999	0.999	0.999	0.999
3	Bg	3(15)	T	45	0.994	0.994	0.994	0.996	0.993	0.994	0.994	0.994	0.994	0.996	0.994	0.994	0.994	0.994
3	Bg	3(10)	T	30	0.994	0.994	0.994	0.995	0.994	0.994	0.994	0.994	0.994	0.996	0.995	0.994	0.994	0.995
3	Ubg	3(30/10/15)	T	55	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
7	Bg	7(50)	T	350	0.992	0.992	0.987	0.997	0.963	0.992	0.992	0.992	0.992	0.999	0.992	0.979	0.992	0.992
7	Bg	7(30)	T	210	0.977	0.977	0.968	0.990	0.961	0.977	0.977	0.977	0.977	0.997	0.977	0.964	0.977	0.977
7	Bg	7(25)	T	175	0.973	0.972	0.967	0.994	0.963	0.972	0.972	0.972	0.973	0.998	0.973	0.962	0.972	0.973
7	Bg	7(15)	T	105	0.983	0.982	0.970	0.993	0.970	0.982	0.982	0.982	0.983	0.996	0.984	0.969	0.982	0.984
7	Bg	7(10)	T	70	0.978	0.978	0.976	0.989	0.976	0.978	0.978	0.978	0.978	0.990	0.978	0.976	0.978	0.978
7	Ubg	7(30/10/25/50)	T	200	0.984	0.983	0.977	0.989	0.975	0.983	0.983	0.983	0.983	0.993	0.972	0.976	0.983	0.972
5	Bg	5(50)	T	250	0.993	0.992	0.989	0.995	0.986	0.992	0.992	0.992	0.992	0.996	0.993	0.988	0.992	0.993
5	Bg	5(30)	T	150	0.994	0.993	0.992	0.996	0.987	0.993	0.993	0.993	0.994	0.998	0.994	0.991	0.993	0.994
5	Bg	5(25)	T	125	0.997	0.997	0.995	0.999	0.993	0.997	0.997	0.997	0.998	0.999	0.998	0.995	0.997	0.998
5	Bg	5(15)	T	75	0.990	0.990	0.990	0.994	0.988	0.990	0.990	0.990	0.990	0.995	0.990	0.990	0.990	0.991
5	Bg	5(10)	T	50	0.989	0.989	0.986	0.995	0.986	0.989	0.989	0.989	0.990	0.995	0.990	0.987	0.989	0.990
5	Ubg	5(50/10/25/15/30)	T	130	0.994	0.994	0.991	0.995	0.992	0.994	0.994	0.994	0.994	0.996	0.993	0.990	0.994	0.993
9	Bg	9(50)	T	450	0.988	0.985	0.966	0.997	0.939	0.985	0.985	0.985	0.990	0.998	0.991	0.934	0.986	0.991
9	Bg	9(30)	T	270	0.971	0.966	0.950	0.992	0.948	0.966	0.966	0.966	0.973	0.997	0.973	0.931	0.966	0.973
9	Bg	9(25)	T	225	0.976	0.973	0.959	0.990	0.956	0.973	0.973	0.974	0.976	0.997	0.976	0.938	0.974	0.976
9	Bg	9(15)	T	135	0.978	0.978	0.971	0.992	0.964	0.978	0.978	0.978	0.978	0.993	0.978	0.960	0.978	0.978
9	Bg	9(10)	T	90	0.966	0.963	0.957	0.985	0.963	0.963	0.963	0.963	0.963	0.990	0.964	0.952	0.963	0.966
9	Ubg	9(25/10/50/15)	T	210	0.978	0.975	0.962	0.992	0.962	0.975	0.975	0.975	0.978	0.995	0.964	0.944	0.975	0.964
15	Bg	15(50)	T	750	0.977	0.971	0.936	0.999	0.885	0.971	0.971	0.971	0.977	1.000	0.977	0.747	0.972	0.977
15	Bg	15(30)	T	450	0.952	0.944	0.913	0.995	0.887	0.944	0.944	0.944	0.951	0.998	0.951	0.790	0.944	0.951
15	Bg	15(25)	T	375	0.940	0.934	0.883	0.993	0.881	0.934	0.934	0.936	0.944	0.998	0.944	0.783	0.937	0.944
15	Bg	15(15)	T	225	0.934	0.930	0.905	0.984	0.915	0.930	0.930	0.930	0.934	0.997	0.935	0.873	0.930	0.935
15	Bg	15(10)	T	150	0.928	0.924	0.907	0.980	0.908	0.924	0.924	0.924	0.932	0.994	0.932	0.880	0.925	0.932
15	Ubg	15(15/30/10/50/25)	T	355	0.946	0.942	0.908	0.989	0.873	0.942	0.942	0.942	0.947	0.998	0.919	0.823	0.942	0.919

Table 8: False Discovery Rate (FDR) estimates for Contaminated Normal (C) distribution

GN	Bal	GS	DT	nb	**bh**	**bonf**	**by**	**dunc**	**dunnet**	**hoch**	**holm**	**hom**	**hsd**	**lsd**	**regwq**	**schef**	**sidak**	**snk**
3	Bg	3(50)	C	150	0.062	0.059	0.041	0.106	0.062	0.060	0.059	0.062	0.068	0.122	0.074	0.052	0.060	0.074
3	Bg	3(30)	C	90	0.048	0.044	0.032	0.080	0.043	0.046	0.044	0.048	0.048	0.098	0.055	0.042	0.046	0.055
3	Bg	3(25)	C	75	0.033	0.032	0.020	0.064	0.040	0.032	0.032	0.032	0.035	0.076	0.040	0.029	0.032	0.040
3	Bg	3(15)	C	45	0.054	0.053	0.036	0.084	0.056	0.054	0.053	0.054	0.058	0.098	0.062	0.046	0.054	0.062
3	Bg	3(10)	C	30	0.122	0.120	0.105	0.152	0.120	0.120	0.120	0.122	0.124	0.164	0.128	0.113	0.120	0.128
3	Ubg	3(30/10/15)	C	55	0.036	0.034	0.022	0.068	0.038	0.034	0.034	0.035	0.037	0.078	0.046	0.032	0.034	0.046
7	Bg	7(50)	C	350	0.046	0.041	0.023	0.170	0.051	0.042	0.041	0.043	0.051	0.246	0.052	0.021	0.042	0.052
7	Bg	7(30)	C	210	0.040	0.036	0.016	0.164	0.040	0.036	0.036	0.036	0.044	0.248	0.045	0.011	0.037	0.046
7	Bg	7(25)	C	175	0.032	0.030	0.016	0.150	0.035	0.030	0.030	0.030	0.036	0.253	0.037	0.016	0.030	0.038
7	Bg	7(15)	C	105	0.030	0.028	0.015	0.143	0.030	0.028	0.028	0.028	0.030	0.226	0.031	0.014	0.028	0.032
7	Bg	7(10)	C	70	0.028	0.028	0.015	0.146	0.032	0.028	0.028	0.028	0.034	0.232	0.034	0.012	0.028	0.034
7	Ubg	7(30/10/25/50)	C	200	0.038	0.036	0.023	0.172	0.040	0.036	0.036	0.038	0.040	0.238	0.052	0.018	0.038	0.052
5	Bg	5(50)	C	250	0.052	0.046	0.023	0.150	0.056	0.046	0.046	0.048	0.054	0.189	0.056	0.027	0.046	0.060
5	Bg	5(30)	C	150	0.046	0.043	0.023	0.130	0.050	0.043	0.043	0.044	0.051	0.184	0.052	0.023	0.043	0.054
5	Bg	5(25)	C	125	0.040	0.035	0.016	0.115	0.041	0.035	0.035	0.035	0.041	0.159	0.043	0.018	0.035	0.044
5	Bg	5(15)	C	75	0.034	0.032	0.018	0.106	0.036	0.032	0.032	0.032	0.038	0.155	0.038	0.019	0.032	0.040
5	Bg	5(10)	C	50	0.036	0.034	0.018	0.104	0.038	0.034	0.034	0.035	0.039	0.157	0.040	0.019	0.035	0.040
5	Ubg	5(50/10/25/15/30)	C	130	0.028	0.026	0.011	0.108	0.040	0.026	0.026	0.026	0.034	0.169	0.042	0.011	0.026	0.044
9	Bg	9(50)	C	450	0.044	0.040	0.020	0.226	0.046	0.040	0.040	0.040	0.048	0.330	0.049	0.015	0.040	0.049
9	Bg	9(30)	C	270	0.043	0.038	0.021	0.204	0.046	0.038	0.038	0.038	0.048	0.324	0.048	0.018	0.039	0.048
9	Bg	9(25)	C	225	0.040	0.038	0.025	0.201	0.045	0.038	0.038	0.038	0.046	0.312	0.046	0.018	0.038	0.046
9	Bg	9(15)	C	135	0.040	0.039	0.024	0.187	0.044	0.039	0.039	0.039	0.044	0.290	0.044	0.020	0.039	0.045
9	Bg	9(10)	C	90	0.038	0.034	0.022	0.179	0.040	0.034	0.034	0.034	0.042	0.291	0.042	0.018	0.035	0.042
9	Ubg	9(25/10/50/15)	C	210	0.039	0.034	0.022	0.192	0.044	0.034	0.034	0.034	0.040	0.283	0.059	0.019	0.034	0.059
15	Bg	15(50)	C	750	0.059	0.058	0.053	0.260	0.078	0.058	0.058	0.058	0.065	0.384	0.065	0.052	0.058	0.065
15	Bg	15(30)	C	450	0.062	0.062	0.052	0.266	0.066	0.062	0.062	0.062	0.067	0.384	0.067	0.048	0.062	0.067
15	Bg	15(25)	C	375	0.070	0.068	0.058	0.248	0.070	0.068	0.068	0.068	0.072	0.376	0.072	0.054	0.068	0.072
15	Bg	15(15)	C	225	0.056	0.054	0.052	0.260	0.066	0.054	0.054	0.054	0.058	0.384	0.058	0.051	0.054	0.058
15	Bg	15(10)	C	150	0.054	0.052	0.045	0.242	0.061	0.052	0.052	0.052	0.062	0.377	0.062	0.044	0.052	0.062
15	Ubg	15(15/30/10/50/25)	C	355	0.068	0.066	0.054	0.252	0.074	0.066	0.066	0.066	0.070	0.372	0.084	0.054	0.066	0.084

Table 9: False Discovery Rate (FDR) estimates for Truncated Normal (T) distribution

GN	Bal	GS	DT	nb	**bh**	**bonf**	**by**	**dunc**	**dunnet**	**hoch**	**holm**	**hom**	**hsd**	**lsd**	**regwq**	**schef**	**sidak**	**snk**
3	Bg	3(50)	T	150	0.146	0.142	0.115	0.197	0.139	0.143	0.142	0.146	0.152	0.211	0.164	0.132	0.142	0.164
3	Bg	3(30)	T	90	0.103	0.102	0.080	0.150	0.102	0.102	0.102	0.103	0.108	0.170	0.114	0.096	0.102	0.114
3	Bg	3(25)	T	75	0.096	0.092	0.066	0.138	0.098	0.094	0.092	0.095	0.100	0.154	0.107	0.085	0.092	0.107
3	Bg	3(15)	T	45	0.064	0.058	0.043	0.112	0.066	0.060	0.058	0.061	0.067	0.124	0.076	0.054	0.059	0.076
3	Bg	3(10)	T	30	0.056	0.053	0.035	0.092	0.062	0.054	0.053	0.056	0.059	0.106	0.064	0.046	0.054	0.064
3	Ubg	3(30/10/15)	T	55	0.067	0.065	0.044	0.104	0.070	0.065	0.065	0.066	0.069	0.124	0.072	0.058	0.065	0.072
7	Bg	7(50)	T	350	0.191	0.179	0.111	0.324	0.170	0.179	0.179	0.181	0.200	0.360	0.202	0.090	0.181	0.203
7	Bg	7(30)	T	210	0.127	0.114	0.068	0.286	0.118	0.114	0.114	0.116	0.128	0.349	0.131	0.056	0.116	0.132
7	Bg	7(25)	T	175	0.138	0.129	0.082	0.284	0.110	0.129	0.129	0.129	0.144	0.354	0.144	0.070	0.132	0.146
7	Bg	7(15)	T	105	0.084	0.078	0.045	0.244	0.090	0.078	0.078	0.079	0.091	0.322	0.091	0.034	0.080	0.093
7	Bg	7(10)	T	70	0.070	0.064	0.034	0.224	0.068	0.064	0.064	0.064	0.076	0.298	0.078	0.022	0.064	0.080
7	Ubg	7(30/10/25/50)	T	200	0.127	0.118	0.076	0.264	0.114	0.118	0.118	0.120	0.136	0.358	0.122	0.056	0.120	0.124
5	Bg	5(50)	T	250	0.178	0.172	0.117	0.288	0.151	0.172	0.172	0.174	0.188	0.317	0.190	0.126	0.172	0.194
5	Bg	5(30)	T	150	0.136	0.128	0.080	0.268	0.122	0.128	0.128	0.130	0.144	0.314	0.148	0.082	0.129	0.150
5	Bg	5(25)	T	125	0.112	0.104	0.068	0.229	0.108	0.104	0.104	0.106	0.112	0.290	0.116	0.070	0.104	0.120
5	Bg	5(15)	T	75	0.084	0.078	0.044	0.188	0.078	0.078	0.078	0.080	0.091	0.246	0.094	0.048	0.078	0.095
5	Bg	5(10)	T	50	0.068	0.064	0.038	0.164	0.064	0.064	0.064	0.066	0.074	0.229	0.074	0.038	0.064	0.076
5	Ubg	5(50/10/25/15/30)	T	130	0.098	0.092	0.059	0.197	0.116	0.092	0.092	0.093	0.104	0.264	0.092	0.064	0.092	0.096
9	Bg	9(50)	T	450	0.186	0.174	0.120	0.306	0.172	0.174	0.174	0.176	0.188	0.332	0.188	0.088	0.174	0.190
9	Bg	9(30)	T	270	0.156	0.144	0.088	0.334	0.130	0.144	0.144	0.144	0.161	0.382	0.162	0.060	0.144	0.163
9	Bg	9(25)	T	225	0.138	0.126	0.075	0.327	0.114	0.126	0.126	0.126	0.148	0.387	0.150	0.051	0.128	0.151
9	Bg	9(15)	T	135	0.091	0.084	0.046	0.278	0.088	0.084	0.084	0.084	0.100	0.366	0.100	0.032	0.084	0.100
9	Bg	9(10)	T	90	0.081	0.077	0.042	0.266	0.078	0.077	0.077	0.078	0.088	0.351	0.087	0.032	0.078	0.088
9	Ubg	9(25/10/50/15)	T	210	0.114	0.108	0.064	0.277	0.113	0.108	0.108	0.108	0.122	0.380	0.119	0.046	0.108	0.120
15	Bg	15(50)	T	750	0.135	0.130	0.104	0.220	0.162	0.130	0.130	0.130	0.142	0.222	0.143	0.198	0.130	0.143
15	Bg	15(30)	T	450	0.158	0.150	0.100	0.312	0.166	0.150	0.150	0.150	0.164	0.337	0.164	0.139	0.150	0.164
15	Bg	15(25)	T	375	0.130	0.118	0.101	0.314	0.154	0.118	0.118	0.120	0.132	0.342	0.132	0.142	0.120	0.132
15	Bg	15(15)	T	225	0.116	0.103	0.075	0.348	0.109	0.103	0.103	0.104	0.125	0.406	0.124	0.074	0.105	0.124
15	Bg	15(10)	T	150	0.102	0.096	0.067	0.328	0.106	0.096	0.096	0.096	0.106	0.397	0.105	0.068	0.097	0.105
15	Ubg	15(15/30/10/50/25)	T	355	0.132	0.120	0.086	0.313	0.136	0.120	0.120	0.122	0.143	0.366	0.136	0.110	0.124	0.136

Table 10: Analysis of variance of ALPHA estimate in N distribution for *Dunnett*

Source	DF	SSE	MSE	F ratio	Pr(>F)	
Bal	1	0.0151	0.01505	2.849	0.10215	
Variance	1	0.0556	0.05561	10.526	0.00296	**
GN	4	0.3069	0.07672	14.523	1.27e-06	***
GS	24	0.3275	0.01364	2.583	0.00792	**
Residuals	29	0.1532	0.00528			

```
Signif. codes: '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Application Session in R

```
load('data.rda')
head(d2, 20)

##    block group yield
## 1     B1    G1  6.01
## 2     B1    G1  5.10
## 3     B1    G1  7.77
## 4     B1    G2  5.37
## 5     B1    G2  6.84
## 6     B1    G2  5.52
## 7     B1    G3  5.80
## 8     B1    G3  5.75
## 9     B1    G3  4.72
## 10    B2    G1  7.72
## 11    B2    G1  6.03
## 12    B2    G1  6.45
## 13    B2    G2  7.55
## 14    B2    G2  5.73
## 15    B2    G2  6.72
## 16    B2    G3  5.40
## 17    B2    G3  6.15
## 18    B2    G3  5.96
## 19    B3    G1  5.83
## 20    B3    G1  7.43

require(nlme)
```

```r
model <- lme(yield ~ group, random = ~1|block , data = d2)

summary(model)

anova(model)

library(emmeans)

emmeans(model, pairwise ~ group, adjust = "tukey")

emmeans(model, pairwise ~ group, adjust = "bonf")

emmeans(model, pairwise ~ group, adjust = "BH")

emmeans(model, pairwise ~ group, adjust = "Holm")

emmeans(model, pairwise ~ group, adjust = "BY")

emmeans(model, pairwise ~ group, adjust = "none")

emmeans(model, pairwise ~ group, adjust = "hoch")

emmeans(model, pairwise ~ group, adjust = "hommel")

emmeans(model, pairwise ~ group, adjust = "sidak")

require(agricolae)

lme.aov <- aov(yield ~ group + block , data = d2)

SNK.test(lme.aov, "group", group = F, console = T)

LSD.test(lme.aov, "group", group = F, console = T)

duncan.test(lme.aov, "group", group = F, console = T)

scheffe.test(lme.aov, "group", group = F, console = T)

require(multcomp)

post <- multcomp::glht(model, linfct = mcp(group = "Dunnett"),
test = adjusted('none'))
summary(post)
```

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