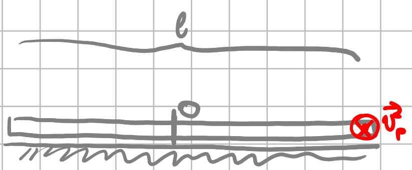
43.3

Hp

$m = 1kg$
$\ell = 1m$
$v_p = 10 m/s$
$m_p = 0.1kg$
M_{att}



Th

1) ω @ urto
2) M_{att} @ $\omega = 0$ dopo 2 giri

$$m_p v_p = (m_p + m) v \rightarrow \boxed{v = \frac{m_p v_p}{m_p + m}}$$

cons. mom ang. $I_{TOT}\,\omega = \frac{\ell}{2}(m + m_p) v$

con $I_{TOT} = \frac{1}{12} m \frac{1}{4}\ell^2 + \frac{1}{4}\ell^2 m_p = \frac{1}{4}\ell^2\left(\frac{1}{12}m + m_p\right)$

$$\Rightarrow \omega = \frac{\cancel{\frac{1}{2}}\cancel{\ell}\cancel{(m+m_p)}\cdot}{\frac{1}{2}\cancel{\frac{1}{4}}\ell^{\cancel{2}}\left(\frac{1}{12}m + m_p\right)} \frac{m_p v_p}{\cancel{(m_p+m)}} = \frac{2 m_p v_p}{\left(\frac{1}{12}m + m_p\right)\ell}$$

2) $\frac{1}{2} I_{TOT}\,\omega^2 = W_{att} \rightarrow \frac{1}{2} I_{TOT}\,\omega^2 = M_{att}\,\vartheta = M_{att}\,4\pi$

$$\Rightarrow M_{att} = \frac{\frac{1}{2}\cdot\frac{1}{4}\cancel{\ell^2}\cancel{\left(\frac{1}{12}m + m_p\right)}}{\cancel{4}\pi} \cdot \frac{\cancel{4} m_p^2 v_p^2}{\left(\frac{1}{12}m + m_p\right)^{\cancel{2}}\cancel{\ell^2}} = \frac{m_p^2 v_p^2}{8\pi\left(\frac{1}{12}m + m_p\right)}$$

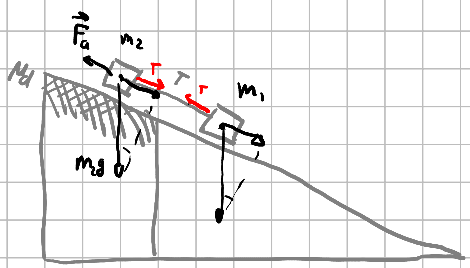
43.2

Hp

$m_1 = 3kg$
$m_2 = 1kg$

$\vartheta = \frac{\pi}{6}$

$\mu_d = 0.3$



Th

1) a_{m_2}, T

2) F @ v_0 cost

$$1) \begin{cases} m_2 a = T + m_2 g \sin\vartheta - m_2 g \cos\vartheta\,\mu \\ m_1 a = m_1 g \sin\vartheta - T \end{cases} \rightarrow a = \frac{m_1 g \sin\vartheta - T}{m_1}$$

$$m_2 \frac{m_1 g \sin\vartheta - T}{m_1} = T + m_2 g \sin\vartheta - m_2 g \cos\vartheta\,\mu$$

$$\cancel{-m_2 g \sin\vartheta} + \frac{m_2}{m_1}T + T = \cancel{-m_2 g \sin\vartheta} + m_2 g \cos\vartheta\,\mu$$

$$\Rightarrow \frac{m_2 T + m_1 T}{m_1} = m_2 g \cos\vartheta\,\mu \Rightarrow T\left(\frac{m_2 + m_1}{m_1}\right) = m_2 g \cos\vartheta\,\mu$$

$$\Rightarrow T = \frac{m_2 m_1 g \cos\vartheta\,\mu}{m_2 + m_1}$$

$$\Rightarrow a = \frac{m_1 g \sin\vartheta}{m_1} - \frac{m_2 g \cos\vartheta\,\mu}{m_2+m_1} = g\sin\vartheta - \frac{m_2}{m_2+m_1} g\cos\vartheta\,\mu = a$$

2) verso l'alto, attrito nel senso opposto $\Rightarrow a' = g\sin\vartheta + \frac{m_2}{m_2+m_1} g\cos\vartheta\,\mu$

$$F-(m_1+m_2)a' = 0 \Rightarrow F = (m_1+m_2)\,g\sin\vartheta + m_2 g\cos\vartheta\,\mu$$

43.1

Hp

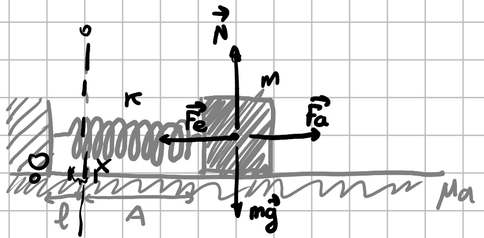
$\kappa = 19.6\ \frac{N}{m}$

$\ell = 40\ cm$

$m = 100\ g$

$\mu_d = 0.5$

$A = 30\ cm$



Th

1) x_{min}

2) $v(\ell)$

metto lo zero in ℓ

$$\frac{1}{2}\kappa A^2 = \mu m g(A-x) + \frac{1}{2}\kappa x^2$$

$$\Rightarrow \frac{1}{2}\kappa A^2 = \mu m g A - \mu m g x + \frac{1}{2}\kappa x^2 \Rightarrow \frac{1}{2}\kappa x^2 - \mu m g\, x + A\left(\mu m g - \frac{1}{2}\kappa A\right) = 0$$

$$x = \frac{\mu m g \pm \sqrt{(\mu m g)^2 - 2\kappa A\left(\mu m g - \frac{1}{2}\kappa A\right)}}{\kappa} = \begin{cases} 0.3 \\ -0.25 \end{cases}$$ minore

con lo zero in O -0.25 diventa $\boxed{15\ cm}$ ✓ $= b$

2) $\frac{1}{2}\kappa(\ell-b)^2 = \frac{1}{2}m v^2 + \mu m g(\ell-b) \Rightarrow v = \sqrt{\frac{\kappa(\ell-b)^2 - \mu m g(\ell-b)}{m}}$

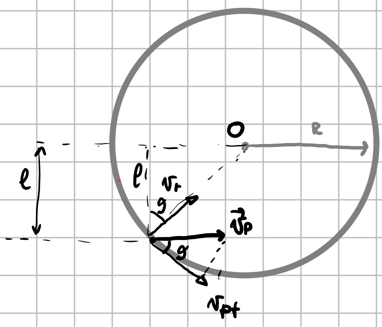
42.3

$m = 11.0\ kg$

$R = 49.0\ cm$

$m_p = 1.6\ kg$

$v = 2.70\ m/s$



$R\cos\vartheta = \ell$

$\cos\vartheta = \ell/R$

$v_{pt} = v_p\cos\vartheta$
$= v_p\,\ell/R$

Th

1) ω dopo urto

2) $\Delta\vec{P}$

1) $m_p v_{pt} = (m+m_p)\,v \rightarrow v = \frac{m_p v_{pt}}{m+m_p} = \frac{m_p v_p}{m+m_p}\frac{\ell}{R}$

$I\omega = L = r \cdot mv \rightarrow \omega = \frac{rmv}{I} = \frac{\cancel{R}(\cancel{m+m_p})}{R^2(\frac{1}{2}m+m_p)} \cdot \frac{m_p v_p}{\cancel{m+m_p}} \cdot \frac{\ell}{\cancel{R}}$

$r = R$
$I = \frac{1}{2}mR^2 + m_pR^2$
$m = m + m_p$

$$\Rightarrow \omega = \frac{m_p v_p}{\frac{1}{2}m + m_p} \cdot \frac{\ell}{R^2}$$

2) $\Delta P_x : P_{dx} - P_{px} = (m+m_p)v - m_p v_p = \cancel{(m+m_p)} \frac{m_p v_p}{\cancel{m+m_p}} \frac{\ell}{R} - m_p v_p$

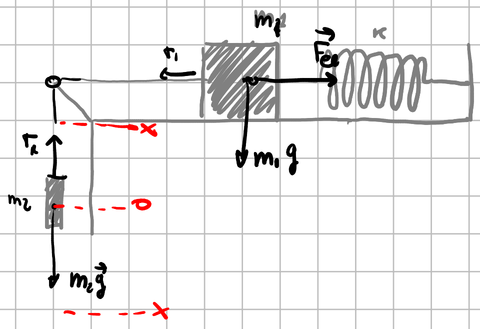
$= m_p v_p (\frac{\ell}{R} - 1)$?

3) $m_p v_p = (m+m_p) v_{CM} \rightarrow v_{CM} = \frac{m_p v_p}{m+m_p}$

42.2

Hp

$m_1 = 2kg$
$k = 98 \; N/m$
$m_2 = 1kg$



Th

1) x, T

2) periodo τ

$T = m_2 g \rightarrow T - kx = 0 \rightarrow kx = m_2 g \rightarrow x = \frac{m_2 g}{k}$

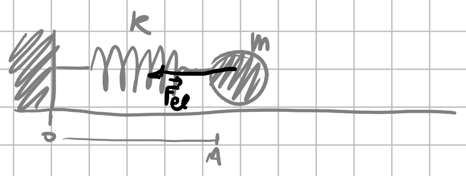
2) la molla oscilla attorno alla posizione di equilibrio
$\Rightarrow S_{max} = 2x$

$\omega = \sqrt{\frac{k}{m}} \Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m_1+m_2}{k}}$

42.1

Hp

m
k
A



Th

1) P(t)

2) T

$x(t) = A\sin(\sqrt{\frac{k}{m}}t) \Rightarrow F_{el} = -kA\sin(\sqrt{\frac{k}{m}}t)$

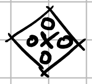
$v(t) = \sqrt{\frac{k}{m}}A\cos(\sqrt{\frac{k}{m}}t) \Rightarrow P = Fv = +kA\sin(\sqrt{\frac{k}{m}}t) \cdot \sqrt{\frac{k}{m}}A\cos(\sqrt{\frac{k}{m}}t)$

$= \cdot kA^2\sqrt{\frac{k}{m}}\sin \cdot \cos \cdot -$

$= \frac{1}{2}kA^2\sqrt{\frac{k}{m}}\sin\left(2\sqrt{\frac{k}{m}}t\right)$

2) $T = 2\pi/\omega = 2\pi\sqrt{\frac{m}{k}} \Rightarrow P(T/8) = \cdot kA^2\sqrt{\frac{k}{m}}\sin\frac{\pi}{4}\cos\frac{\pi}{4}$

$= kA^2\sqrt{\frac{k}{m}}\,\frac{1}{2}$



$$P_m = \frac{1}{T}\int_0^{T}\frac{1}{2}kA^2\sqrt{\frac{k}{m}}\sin\left(2\sqrt{\frac{k}{m}}t\right)dt = -\frac{1}{T}\frac{1}{2}kA^2\cancel{\sqrt{\frac{k}{m}}}\underbrace{\cos\left(2\sqrt{\frac{k}{m}}t\right)}\cdot\frac{1}{2}\cancel{\sqrt{\frac{m}{k}}}$$

$$\cos\left(2\cancel{\sqrt{\frac{k}{m}}}\cdot 2\pi\cancel{\sqrt{\frac{m}{k}}}\right) = 0$$

$= 0$

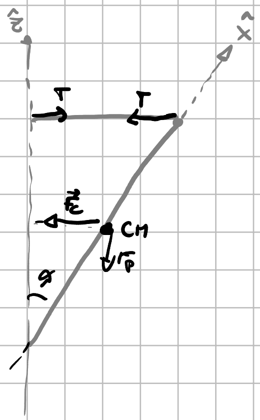
41.3

Hp

l, m, ϑ, ω

$\lambda = cx$

$dm = \lambda dx = cx\,dx$



Th

1) C.M.
2) M_p
3) M_c

$$x_{CM} = \frac{\int_0^m x\,dm}{\int_0^m dm} = \frac{\int_0^l x^2 c\,dx}{\int_0^l cx\,dx} = \frac{\frac{1}{3}l^3 c}{\frac{1}{2}l^2 c} = \frac{2}{3}l$$

2) $M_p = mg\,x_{CM}\cdot\sin\vartheta = \frac{2}{3}mgl\sin\vartheta$

3) $a_c = \omega^2 r = \omega^2 x\sin\vartheta$

$F_c = ma_c \Rightarrow dF_c = \omega^2 x\sin\vartheta\,dm \Rightarrow dF_c = \omega^2 x\sin\vartheta\,cx\,dx$
$= \omega^2 x^2 c\sin\vartheta\,dx$

$M_c = F_c\cdot b\cos\vartheta \Rightarrow dM_c = x\cos\vartheta\cdot\omega^2 x^2 c\sin\vartheta\,dx$
$= \cos\vartheta\sin\vartheta\,\omega^2 c\,x^3\,dx$

$\Rightarrow M_c = \frac{1}{4}l^4\cos\vartheta\sin\vartheta\,\omega^2 c$?

41.2

Hp

$m_A = 4.5 kg$

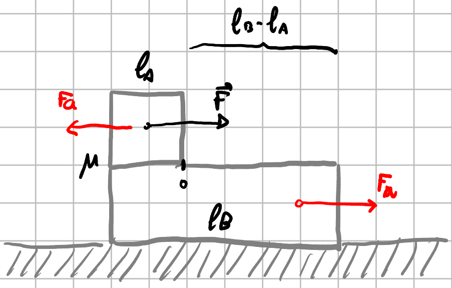
$m_B = 9.0 kg$

$\mu = 0.2$

$F = 22 N$

$l_A = 15 cm$

$l_B = 30 cm$



Th

1) a_A , a_B

2) $l_A = l_B$

3) μ_{min}

1) $\begin{cases} m_A a_a = F - F_a \\ m_B a_b = F_a \end{cases} \rightarrow \begin{cases} m_A a_A = F - m_A g \mu & \rightarrow a_A = \left(\frac{F}{m_A} - \mu g\right) \\ m_B a_B = m_A g \mu & \rightarrow a_B = \frac{m_A}{m_B} \mu g \end{cases}$

2) $v_A(t) = \left(\frac{F}{m_A} - \mu g\right) t \quad \rightarrow \quad x_A(t) = \frac{1}{2}\left(\frac{F}{m_A} - \mu g\right) t^2 \rightarrow$

$v_B(t) = \left(\frac{m_A}{m_B} \mu g\right) t \quad \rightarrow \quad x_B(t) = \frac{1}{2}\left(\frac{m_A}{m_B} \mu g\right) t^2$

$$s(t) = (x_A - x_B) = \frac{1}{2}\left(\overbrace{\frac{F}{m_A} - \mu g}^{a_A} - \overbrace{\frac{m_A}{m_B} \mu g}^{a_B}\right) t^2$$

$$s(\tau) = l_B - l_A \Rightarrow \frac{1}{2}\left(\overbrace{\frac{F}{m_A} - \mu g}^{a_A} - \overbrace{\frac{m_A}{m_B} \mu g}^{a_B}\right) \tau^2$$

$$\tau = \sqrt{\frac{2(l_B - l_A)}{a_A - a_B}}$$

3) $a_A = a_B \rightarrow \frac{F}{m_A} - \mu_{min} g = \frac{m_A}{m_B} \mu_{min} g \Rightarrow \frac{F}{m_A} = \mu_{min}\left(g + \frac{m_A}{m_B} g\right) \Rightarrow \mu_{min} = \frac{F}{m_A g \left(1 + \frac{m_A}{m_B}\right)}$

$$\mu_{min} = \frac{F}{m_A g \left(\frac{m_B + m_A}{m_B}\right)} = \frac{F m_B}{m_A g (m_B + m_A)}$$

$\boxed{41.1}$?

$\boxed{40.3}$

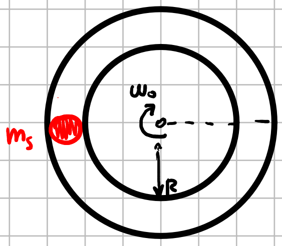
Hp

$m_0, R, \omega_0 \rightsquigarrow v_0 = R\omega_0$

$m = kt$

Th

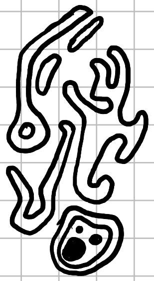
τ @ $\omega(\tau) = \frac{1}{2}\omega_0$



$$\left[I_{rot} = m_0 R^2 + kt R^2 = R^2(m_0 + kt) \right]$$

cons. mom angolare

$$I_{rot}\,\omega = R m_{rot} v_0 \rightarrow \cancel{R^2}(m_0 + kt)\,\omega = \cancel{R^2} m_0 \omega_0 \Rightarrow \omega = \frac{m_0 \omega_0}{m_0 + kt}$$



$$\omega(\tau) = \frac{1}{2}\omega_0 \Rightarrow \omega(\tau) = \frac{m_0 \cancel{\omega_0}}{m_0 + k\tau} = \frac{1}{2}\cancel{\omega_0} \rightarrow 2m_0 = \cancel{\frac{1}{2}} m_0 + \cancel{\frac{1}{2}} k\tau \Rightarrow \tau = \frac{m_0}{k}$$

$$\alpha(t) = \frac{d\omega}{dt} = -\frac{m_0 \omega_0}{(m_0 + kt)^2} k$$

40.2

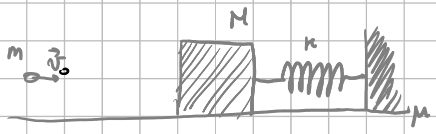
Hp

M

K

Th

δ_{max}



cons. qdm

$$m v_0 = (M+m) v \rightarrow \boxed{v = \frac{m}{M+m} v_0}$$

$$\frac{1}{2} k \delta_{max}^2 + \mu m g\, \delta_{max} - \frac{1}{2} m v = 0$$

$\boxed{\text{con } m = M+m}$

$$\delta = \frac{-\mu m g \pm \sqrt{(\mu m g)^2 + k m v}}{k}$$

✓

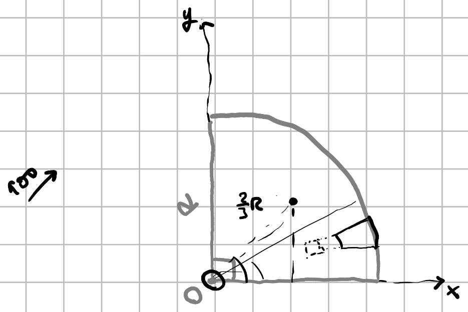
$\boxed{40.1}$?

39.3

Hp

m, R, ρ

$\theta = \frac{\pi}{2}$



Th

1) $\vec{r}_{CM}$

2) I_o

3) T

$dm = \rho dS = \rho r d\theta dr$

1)
$$x_{CM} = \frac{\int r\,dm}{\int dm} = \frac{\int r\cos\theta \cdot \rho r\,d\theta\,dr}{\int \rho r\,d\theta\,dr} = \frac{\cancel{\rho} \cdot \int_0^R r^2 dr \int_0^{\pi/2} \cos\theta\,d\theta}{\cancel{\rho} \int_0^R r\,dr \int_0^{\pi/2} d\theta}$$

$$= \frac{\frac{1}{3}R^3}{\frac{1}{2}R^2 \cdot \frac{\pi}{2}} = \frac{4}{3} \cdot \frac{R}{\pi}$$

$$y_{CM} = \frac{4}{3}\frac{R}{\pi}$$

$$\Rightarrow c.m. \left(\frac{4}{3}\frac{R}{\pi} , \text{"} \right)$$

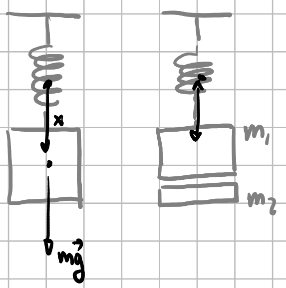
2) $I_o = \int r^2 dm = \int r^2 \rho r\,dr\,d\theta = \rho \int_0^R r^3 dr \int_0^{\pi/2} d\theta = \frac{\cancel{4}m}{\cancel{\pi R^2}} \cdot \frac{1}{\cancel{4}} R^{\cancel{4}\,2} \cdot \frac{\cancel{\pi}}{2} = \frac{1}{2} m R^2$

3) $T = 2\pi \sqrt{\frac{I_o}{m g r_{CM}}} = \cancel{2}\pi \sqrt{\frac{\frac{1}{2}\cancel{m}R^{\cancel{2}}}{\cancel{m} g \frac{\cancel{4}}{3}\frac{\cancel{R}}{\pi}\sqrt{2}}} = \pi \sqrt{\pi \frac{3}{2} \cdot \frac{R}{g\sqrt{2}}}$

39.1

$m_1 = \frac{2}{3} m$

$m_2 = \frac{1}{3} m$



Th

1) a_{max}

$m_2 g = m_1 a \rightarrow \frac{1}{\cancel{3}} m g = \frac{2}{\cancel{3}} \cancel{m} a \Rightarrow a = \frac{1}{2} g$

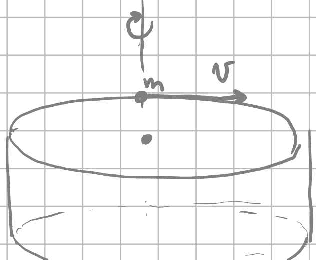
38.3

Hp

$m_c = 0.2$ kg

$R = 0.15$ m

$\omega_0 = 20$ rad/s



Th

1) ω @ $m_c' = m_c - m$

2) W

$m = 5\,g$

1) **con mom angol**

$$I_0 \omega_0 = I_1 \omega_1 \rightarrow \omega_1 = \frac{I_0}{I_1}\omega_0 = \frac{m_c}{m_c - m}\omega_0$$

con
$$\begin{array}{|l|} I_0 = \frac{1}{2} m_c R^2 \\ I_1 = \frac{1}{2}(m_c - m) R^2 \end{array}$$

2) $$W = \Delta E_K = \frac{1}{2} I_1 \omega_1^2 - \frac{1}{2} I_0 \omega_0^2 = \frac{1}{2}\left(I_1 \omega_1^2 - I_0 \omega_0^2\right) =$$

$$= \frac{1}{2}\left[\frac{1}{2}\cancel{(m_c - m)} R^2 \frac{m_c^2}{(m_c - m)^{\cancel{2}}} \omega_0^2 - \frac{1}{2} m_c R^2 \omega_0^2\right]$$

$$= \frac{1}{4} R^2 m_c \omega_0^2 \left(\frac{m_c}{m_c - m} - 1^{\,-m+m}\right)$$

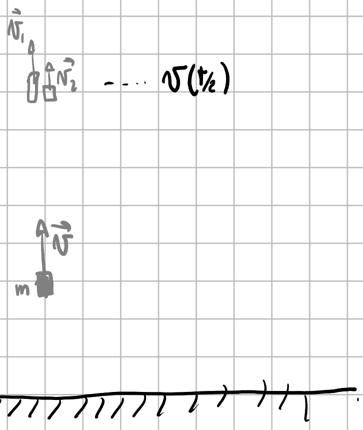
$$= \frac{1}{4} R^2 m_c \omega_0^2 \cdot \frac{m}{m_c - m}$$

38.1

Hp

$m, v = 30\,m/s$

$m_1 = \frac{2}{3} m$
$m_2 = \frac{1}{3} m$



Th

1) v_2

2) τ @ $y(\tau) = 0$

CIMA $$\frac{1}{2} m v^2 = \cancel{m} g h \Rightarrow h = \frac{1}{2}\frac{v^2}{g}$$

METÁ $$\frac{1}{\cancel{2}} \cancel{m} v^2 = \frac{1}{\cancel{2}} \cancel{m} v_{metá}^2 + \cancel{m} g h / \cancel{2} \rightarrow v_{metá}^2 = \cancel{2}(v^2 - gh) \rightarrow v_{metá} = \sqrt{v^2 - gh}$$

$$\Rightarrow v_{metá} = \sqrt{v^2 - \frac{1}{2} v^2} = \frac{1}{\sqrt{2}} v = v'$$

Cons qdm

$$\begin{cases} (m_1 + m_2) v' = m_1 v_1 + m_2 v_2 \\ \frac{1}{\cancel{2}}(m_1 + m_2) v'^2 = \frac{1}{\cancel{2}} m_1 v_1^2 + \frac{1}{\cancel{2}} m_2 v_2^2 \end{cases}$$

$$\begin{cases} m_1 v_1 + m_2 v_2 - a \\ m_1 v_1^2 + m_2 v_2^2 = a v' \end{cases} \rightarrow v_1 = \frac{a - m_2 v_2}{m_1}$$

$$\rightarrow \quad \cancel{m_1}\left(\frac{a^2 - 2am_2v_2 + m_2^2v_2^2}{m_1^{\cancel{2}}}\right) + m_2v_2^2 = av'$$

$$\frac{a^2}{m_1} - \frac{2am_2}{m_1}v_2 + \frac{m_2^2}{m_1}v_2^2 + m_2v_2^2 - av' = 0$$

$$v_2^2\left(\frac{m_2^2}{m_1} + m_2\right) - \left(\frac{2am_2}{m_1}\right)v_2 + \frac{a^2}{m_1} - av' = 0$$

$$v_2 = \frac{\frac{2am_2}{m_1} \pm \sqrt{\left(\frac{2am_2}{m_1}\right)^2 - 4\left(\frac{m_2^2}{m_1} + m_2\right)\left(\frac{a^2}{m_1} - av'\right)}}{2\left(\frac{m_2^2}{m_1} + m_2\right)}$$

2) $v_2(t) = v_2 - gt$

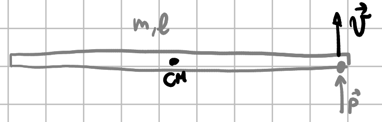
$y_1(t) = v_2t - \frac{1}{2}gt^2 \quad \rightarrow \quad y_2(\tau) = 0 \quad \rightarrow \quad \frac{1}{2}gt^{\cancel{2}} - v_2\cancel{t} = 0 \quad \rightarrow \quad t = \frac{2v_2}{g}$

37.2

Hp

$l = 1.0\,m$
$m = 3.3\,kg$

$P = 5.6\,Ns$



Th

1) v_{CM}
2) ω
3) $E_{k\,rot}$

1) Cons qdm : $P = mv_{CM} \rightarrow v_{CM} = \frac{P}{m}$

2) Cons mom ang : $I\omega = \frac{1}{2}lP \rightarrow \omega = \frac{\frac{1}{2}\cancel{l}P}{\frac{1}{12}ml^{\cancel{2}}} = \frac{6P}{ml}$

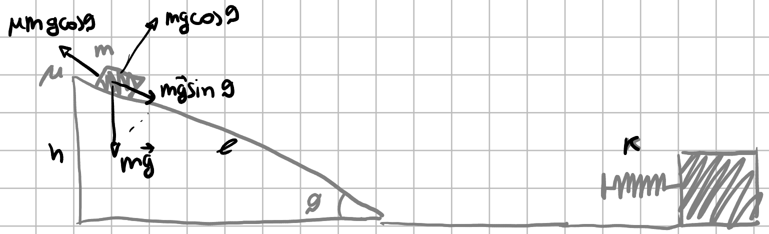
3) $E_{k_{rot}} = E_{TRAS} + E_{ROT} = \frac{1}{2}mv_{CM}^2 + \frac{1}{2}I\omega^2$

$$= \frac{1}{2}\cancel{m}\frac{P^2}{m^{\cancel{2}}} + \frac{1}{2}\cancel{\frac{1}{12}}\cancel{m}\cancel{l^2}\cdot\frac{\overset{3}{\cancel{36}}P^2}{m^{\cancel{2}}\cancel{l^2}} = \frac{1}{3}\frac{P^2}{m} + \frac{3}{2}\frac{P^2}{m} = \frac{11}{6}\frac{P^2}{m}$$

37.1

Hp

$m = 0.5\,kg$
$h = 2m$
$\theta = 30°$
$\mu = 0.2$
$k = 4\cdot10^3\,N/m$



Th

1) W_{att}
2) δ_{max}
3) δ' @ $N=0$

$\ell \sin\theta = h \quad \rightarrow \quad \ell = \frac{h}{\sin\theta} = 2h$

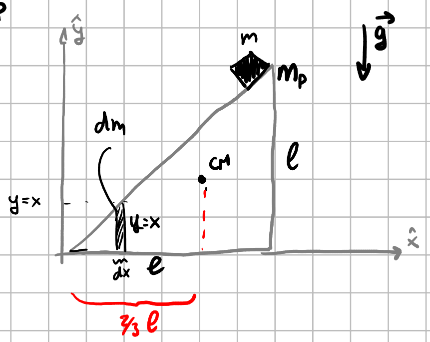
1) $\Rightarrow \quad W_{att} = \mu mg\cos\theta \, 2h$

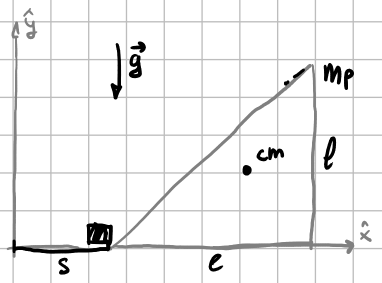
2) $mgh = \mu mg\cos\theta \, 2h + \frac{1}{2} k \delta^2 \quad \rightarrow \quad \delta = \sqrt{\frac{2(mgh - \mu mg\cos\theta \, 2h)}{k}}$

3) $\delta' = \sqrt{\frac{2mgh}{k}}$

36.2

Hp





Th

1) x_{cm}
2) s
3) v_x @ blocco a $y=0$

1) $x_{CM} = \frac{\int x\,dm}{\int dm}$ con $dm = \rho\, ds = \rho x\, dx \quad \rightarrow \quad x_{CM} = \frac{\cancel{\rho}\int_0^{\ell} x^2 dx}{\cancel{\rho}\int_0^{\ell} x\, dx} = \frac{\frac{1}{3}\ell^3}{\frac{1}{2}\ell^2} = \frac{2}{3}\ell$

2) conservaz. qdm sui 2 assi

x: $\quad m v = (m + m_p) v_{CM} \quad \Rightarrow \quad v_{CM} = \frac{m v}{m + m_p}$

$\boxed{v_x = \frac{\ell}{t}} \quad \boxed{v_{CM} = \frac{s}{t}} \quad \Rightarrow \quad \frac{s}{\cancel{t}} = \frac{m}{m+m_p}\frac{\ell}{\cancel{t}} \quad \Rightarrow \quad s = \frac{m}{m+m_p}\ell$

y: $\quad m v_y = 0$

3) $mg\ell = \frac{1}{2} m v^2 + \frac{1}{2}(m+m_p) v_{CM}^2 \quad \rightarrow \quad \boxed{v_{CM} = \frac{m v}{m + m_p}} \; , \; \boxed{v = v_x\sqrt{2}}$

$\cancel{m} g \ell = \cancel{m} v_x^2 + \cancel{(m+m_p)} \frac{m^{\cancel{2}} v_x^2}{(m+m_p)^{\cancel{2}}}$

$\rightarrow v_x^2 \left(1 + \frac{m}{m+m_p}\right) = g\ell \quad \Rightarrow \quad v_x = \sqrt{\frac{g\ell(m+m_p)}{2m + m_p}}$?

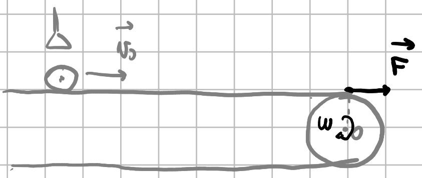
$\left(\frac{2m + m_p}{m + m_p}\right)$

36.1

Hp

$\frac{dm}{dt} = 310 \ \text{kg/s} = r$

$v_0 = 3.40 \ \text{m/s}$



Th

1) F @ v_{cost}

2) P

v costante se $a=0$ → $\boxed{F = \frac{dP}{dt} = \frac{dm}{dt} v + \frac{dv}{dt} m}$

impongo $\frac{dv}{dt} = 0 \Rightarrow F = \frac{dm}{dt} v = rv$

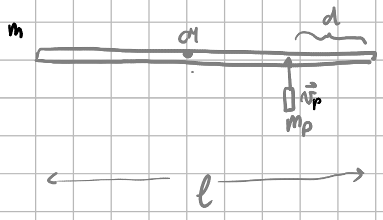
$P = \frac{95}{100} Fv = 0.95 \ rv^2$

35.2

Hp

m, l, d

m_p, v_p



Th

1) m_p @ $v_{p'} = 0$

(m, l, d)

cons qdm e energia

$$\begin{cases} m_p v_p = m v_{CM} \\ \frac{1}{2} m_p v_p^2 = \frac{1}{2} m v_{CM}^2 + \frac{1}{2} I \omega^2 \end{cases}$$

cons. mom ang.

$$\left(\frac{\ell}{2} - d\right) m_p v_p = I\omega$$

2

$$\begin{cases} m_p v_p = m v_{CM} \\ \frac{1}{2} m_p v_p^2 = \frac{1}{2} m v_{CM}^2 + \frac{1}{2} I \omega^2 \end{cases} \rightarrow \boxed{v_{CM} = \frac{m_p v_p}{m}}$$

$$\hookrightarrow \frac{1}{2} m_p v_p^2 = \frac{1}{2} \cancel{m} \frac{m_p^2 v_p^2}{m^2} + \frac{1}{24} I \omega^2$$

Trovo ω da $\left(\frac{\ell}{2} - d\right) m_p v_p = I\omega \rightarrow \boxed{\omega = \frac{\left(\frac{\ell}{2} - d\right) m_p v_p}{I}}$

↘

$$\cancel{\tfrac{1}{2}}m_p v_p^2 = \cancel{\tfrac{1}{2}}\frac{m_p^2 v_p^2}{m} + \cancel{\tfrac{1}{2}}\cancel{I}\frac{\left(\frac{\ell}{2}-d\right)^2 m_p^2 v_p^2}{I^{\cancel{2}}}$$

$$\cancel{m_p}\cancel{v_p^2} = \frac{m_p^{\cancel{2}}\cancel{v_p^2}}{m} + \frac{12\left(\frac{\ell}{2}-d\right)^2 m_p^{\cancel{2}}\cancel{v_p^2}}{\ell^2 m}$$

$$\boxed{?}$$

Perchè ho $\left(\frac{\ell}{2}-d\right)^2$ al posto di d !

$$1 = \left(\frac{1}{m} + \frac{12\left(\frac{\ell}{2}-d\right)^2}{\ell^2 m}\right)m_p \implies m_p = \left(\frac{\overbrace{\ell^2+12\left(\frac{\ell}{2}-d\right)^2}}{\ell^2 m}\right) =$$

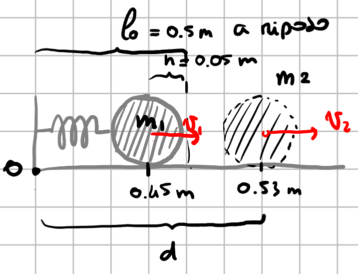
35.1

Hp

$m_1 = 1\,kg$

$\ell_0 = 50\,cm$

$m_2 = 3m_1$



Th

1) δ_{max} post urto

$$\rightarrow \quad \tfrac{1}{2}kh^2 = \tfrac{1}{2}m_1 v_1^2 + \tfrac{1}{2}k(d-\ell_0)^2$$ ⤳ da qui ricavo v_1

$$\cancel{\tfrac{1}{2}}m_1 v_1^2 = \cancel{\tfrac{1}{2}}k\left(h^2-(d-\ell_0)^2\right) \implies v_1^2 = \frac{k}{m_1}\left[h^2-(d-\ell_0)^2\right]$$

$$\begin{cases} m_1 v_1 = m_2 v_2 + m_1 v_1' \\ \tfrac{1}{2}m_1 v_1^2 = \tfrac{1}{2}m_2 v_2^2 + \tfrac{1}{2}m_1 v_1'^2 \end{cases} \rightarrow \begin{cases} \cancel{m_1}v_1 = 3\cancel{m_1}v_2 + \cancel{m_1}v_1' \\ \cancel{m_1}v_1^2 = 3\cancel{m_1}v_2^2 + \cancel{m_1}v_1'^2 \end{cases} \begin{cases} v_1 = 3v_2 + v_1' \\ v_1^2 = 3v_2^2 + v_1'^2 \end{cases} \rightarrow v_2 = \frac{v_1 - v_1'}{3}$$

$$v_1^2 = \cancel{3}\,\frac{v_1^2 + v_1'^2 - 2v_1 v_1'}{\cancel{9}\;3} + v_1'^2 \rightarrow \overbrace{v_1^2} = \overbrace{\tfrac{1}{3}v_1^2} + \underbrace{\tfrac{1}{3}v_1'^2} - \underbrace{\tfrac{2}{3}v_1 v_1'} + \underbrace{v_1'^2}$$

$$\cancel{\tfrac{2}{3}}v_1^2 = \cancel{\tfrac{4}{3}}^{2}v_1'^2 - \cancel{\tfrac{2}{3}}v_1 v_1' \rightarrow v_1^2 = 2v_1'^2 - v_1 v_1'$$

$$2v_1'^2 - v_1 v_1' - v_1^2 = 0 \rightarrow v_1' = \frac{v_1 \pm \sqrt{v_1^2 + 8v_1^2}}{4} = \frac{v_1 \pm 3v_1}{4}$$

$$v_1' = v_1$$

$$v_1' = -\tfrac{1}{2}v_1 = v$$

$$\implies \cancel{\tfrac{1}{2}}k(d-\ell_0)^2 + \cancel{\tfrac{1}{2}}\frac{m_1}{k}v^2 = \cancel{\tfrac{1}{2}}k\delta^2$$

$$\delta^2 = (d-l_0) + \frac{\cancel{m}}{\cancel{k}} \frac{1}{4} \cdot \frac{\cancel{k}}{\cancel{m_1}} \left[h^2 - (d-l_0)^2\right]$$

$$\delta^2 = (d-l_0)^2 + \frac{1}{4}h^2 - \frac{1}{4}(d-l_0)^2 = \frac{1}{4}h^2 + \frac{3}{4}(d-l_0)^2$$

$$\delta = \sqrt{\frac{1}{4}h^2 + \frac{3}{4}(d-l_0)^2}$$

34.2

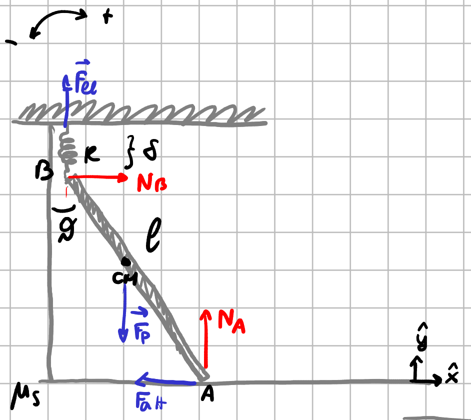
Hp

$m = 1kg$

$K = 4\ N/m$

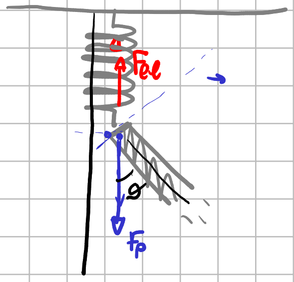
$\delta = 2m$

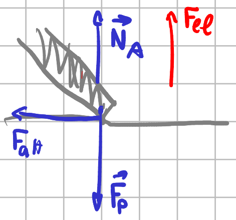
$\vartheta = 63.5°$



Th

1) N_A , N_B





F
x: $N_B - F_{att} = 0 \rightarrow N_B = F_a$
y: $N_a - F_p + F_e = 0 \rightarrow N_A = F_p - F_e$

M: $\sum M = N_B \cancel{l} \cos\vartheta + F_e \cancel{l} \sin\vartheta - F_p \frac{\cancel{l}}{2} \sin\vartheta + F_{att} \cancel{l} \cos\vartheta - N_A \cancel{l} \sin\vartheta = 0$

$$N_B \cos\vartheta + k\delta\sin\vartheta - \frac{mg}{2}\sin\vartheta + N_B\cos\vartheta - mg\sin\vartheta + k\delta\sin\vartheta = 0$$

$$\cancel{2} N_B \cos\vartheta = \cancel{2}\frac{mg}{2}\sin\vartheta - \cancel{2}k\delta\sin\vartheta \Rightarrow N_B = \left(\frac{mg}{2} - k\delta\right)\tan\vartheta$$

33.2

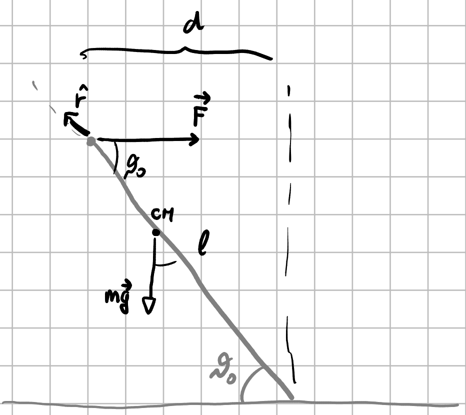
Hp

$m = 1kg$

$l = 80cm$

$\lambda = k\sqrt{r}$

$\vartheta_0 = 40°$



Th

1) F

2) W@ F = 2F

con $\vartheta = \frac{\pi}{2} - \vartheta_0$

1) $r_{CM} = \frac{\int r\,dm}{\int dm}$ con $dm = k\sqrt{r}\,dr \rightarrow = \frac{\int r k r^{1/2} dr}{\int k r^{1/2} dr} = \frac{\int_0^l r^{3/2} dr}{\int_0^l r^{1/2} dr} = \frac{\frac{2}{5} l^{5/2}}{\frac{2}{3} l^{3/2}}$

$= \frac{3}{5}\ell$

Per equilibrio: $mg\frac{3}{5}\ell\cos\theta_0 - F\ell\sin\theta \rightarrow F = \frac{3}{5}mg\frac{\cos\theta}{\sin\theta}$

2) $W = 2Fd = \frac{6}{5}mg\frac{\cos\theta}{\sin\theta}\ell\cos\theta$

33.1

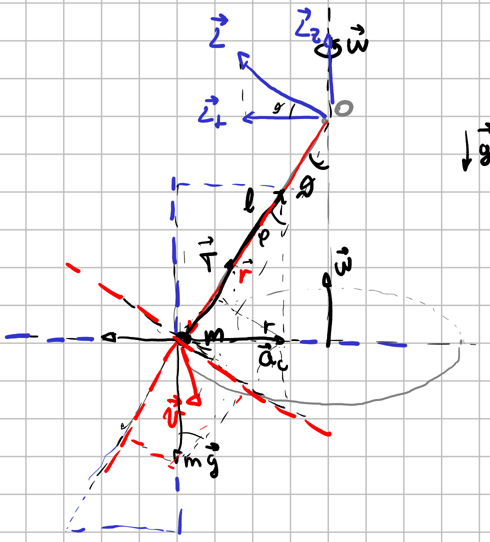
Hp

m

ℓ

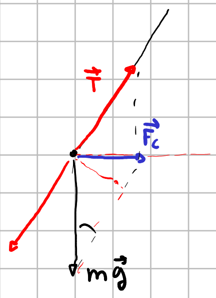
g

θ



Th

L_z



$L_z = \ell m v$

$$\begin{cases} T\cos\theta = mg \\ T\sin\theta = m\frac{v^2}{r} \end{cases} \Rightarrow v = \sqrt{\frac{\ell g\sin^2\theta}{\cos\theta}}$$

$$\begin{cases} mg\cos\theta = T \\ m\frac{v^2}{r}\cos\theta = mg\sin\theta \end{cases} \rightarrow v^2\cos\theta = \ell g\sin^2\theta \Rightarrow v = \sqrt{\ell g\frac{\sin^2\theta}{\cos\theta}}$$

$$L_z = \ell m\sqrt{\ell g\frac{\sin^2\theta}{\cos\theta}} = \sqrt{\ell^3 m^2 g\frac{\sin^2\theta}{\cos\theta}}$$

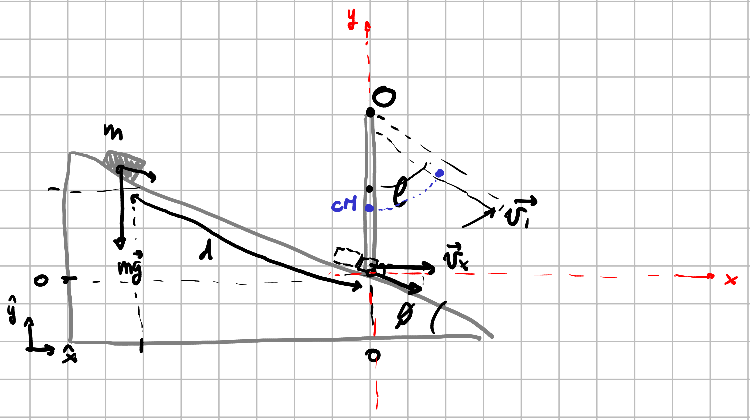
33.2

Hp

$m_a = 3m$

ℓ

m



Th

1) $\vec F_{cm}$ prima e dopo l'urto

1) $x_{cm} = 0$

$$y_{cm} = \frac{\frac{\ell}{2}3m}{4m} = \frac{3}{8}\ell$$

2) m arriva a velocità v: $\cancel{m}gd\sin\theta = \frac{1}{2}\cancel{m}v^2 \Rightarrow v = \sqrt{2gd\sin\theta}$

la componente x di v: $v_x = \sqrt{2gd\sin\theta}\cos\theta$

<u>cons. qdm</u>: $mv_x = (m+3m)v_1 \Rightarrow v_1 = \frac{1}{4}\sqrt{2gd\sin\theta}\cos\theta$

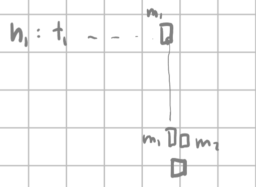
<u>cons mom ang</u>: $I_{TOT}\omega = (m+3m)v_1\ell \rightarrow \omega = \frac{\cancel{4m}\cdot\cancel{\frac{1}{4}}\sqrt{2gd\sin\theta}\cos\theta\cdot\cancel{\ell}}{\cancel{m}\ell^{\cancel{2}}+\cancel{m}\ell^{2}}$

$$\omega = \frac{\sqrt{2dg\sin\theta}\cos\theta}{2\ell}$$

<u>cons en.</u> $\frac{1}{2}I\omega^2 = mgh \rightarrow h = \frac{\frac{1}{2}(2m\ell^2)\omega^2}{4mg}$

<u>32.1</u>

<u>Hp</u>



<u>Th</u>

h_2 @ m_2 in t_2

v_0, $m_1 = m_2 = m$

$t_1 - m_1 : h_1$

$(m_1+m_2)v_0 = m_1v_1 + m_2v_2 \rightarrow 2\cancel{m}v_0 = \cancel{m}v_1 + \cancel{m}v_2 \rightarrow 2v_0 = v_1 + v_2$
$\rightarrow v_2 = 2v_0 - v_1$

- m_1 e m_2 partono con velocita v_1 e v_2 t.c. $v_2 = 2v_0 - v_1$
  $\Rightarrow$ con velocità $\boxed{v_1}$ e $\boxed{2v_0 - v_1}$

$y_1(t) = v_1t - \frac{1}{2}gt^2$ con $v_1(t) = v_1 - gt$

$y_2(t) = (2v_0 - v_1)t - \frac{1}{2}gt^2$

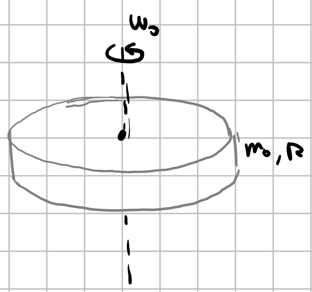
$y_1(t_1) = v_1t_1 - \frac{1}{2}gt_1^2 = h_1 \Rightarrow \boxed{v_1 = \frac{h_1}{t_1} + \frac{1}{2}gt_1}$

$y_2(t_1) = \left(2v_0 - \frac{h_1}{t_1} + \frac{1}{2}gt_1\right)t_1 - \frac{1}{2}gt_1^2$

$= 2v_0t_1 - h_1$

?

31.2



Hp

m_0, R, ω_0

$m = k e^{t/\tau}$

Th

1) $\omega(t)$

2) P @ $\omega = \text{cost} = \omega_0$

1) $m_c(t) = m_0 + k e^{t/\tau}$

cons mom angol.

$$I_0\omega_0 = I_1\omega_1 \rightarrow \cancel{\tfrac{1}{2}} m_0 \cancel{R^2}\omega_0 = \cancel{\tfrac{1}{2}}(m_0 + ke^{t/\tau})\cancel{R^2}\omega(t)$$

$$\Rightarrow \omega(t) = \frac{m_0\omega_0}{m_0 + ke^{t/\tau}} \rightarrow v(t) = \frac{m_0\omega_0 R}{m_0 + k e^{t/\tau}}$$

2) ω cost. se $a = 0 = \frac{dv}{dt}$

Trovo la forza: $F = \frac{dP}{dt} = \frac{d}{dt}\left(m(t)v(t)\right) = \frac{dm}{dt}v(t) + \cancel{\frac{dv}{dt}m(t)}$

impongo $\frac{dv}{dt} = 0$

$$F = \frac{dm}{dt}v(t) = \frac{k}{\tau}e^{t/\tau}v(t)$$

$$\Rightarrow P = Fv = \frac{k}{\tau}e^{t/\tau}v^2(t)$$

NON CÈ SOLUZIONE

31.1

Hp

A, ω, m

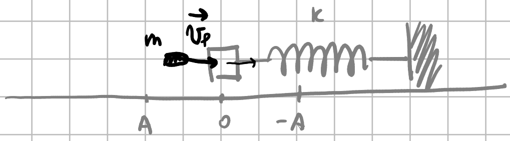
moj: m_p, v_p

Th

1) v_{cm}

2) E dissipata

3) A'



$x(t) = A\sin(\omega t) \rightarrow v(t) = \omega A\cos(\omega t)$

Nella posiz. la velocità é massima $\rightarrow \boxed{v = \omega A}$

⇒ Velocità relativa al proiettile: $v = v_p - \omega A$

Cons. qdm: $m_p(v_p - \omega A) = (m+m_p)\,v_{CM}$

$$\Rightarrow \quad v_{CM} = \frac{m_p(v_p - \omega A)}{m + m_p}$$

2) Energia dissipata: $\frac{1}{2}m_p v_p^2 + \frac{1}{2}m(\omega A)^2 = \frac{1}{2}(m+m_p)v_{CM}^2 + E_{diss}$

$$E_{diss} = \frac{1}{2}\left(m_p v_p^2 + m\omega^2 A^2 - \cancel{(m+m_p)}\frac{m_p^2(v_p-\omega A)^2}{(m+m_p)^{\cancel{2}}}\right)$$

$$= \frac{1}{2}\left(m_p v_p^2 + m\omega^2 A^2 - \frac{m_p^2(v_p-\omega A)^2}{m+m_p}\right)$$

3) Cons. energia: $\cancel{\frac{1}{2}}(m+m_p)v_{CM}^2 = \cancel{\frac{1}{2}}kA'^2$

$$\Rightarrow A'^2 = \frac{(m+m_p)v_{CM}^2}{k} \qquad \text{con} \quad \omega = \sqrt{\frac{k}{m}} \rightarrow k = \omega^2 m$$

$$\Rightarrow A' = \sqrt{\frac{\cancel{(m+m_p)}v_{CM}^2}{\omega^2\cancel{(m+m_p)}}} = \frac{v_{CM}}{\omega}$$

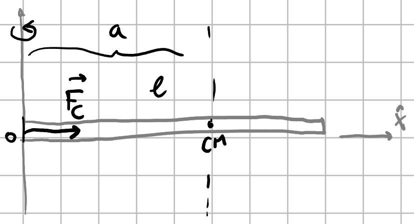
30.1

Hp

ℓ, $\lambda = cx$

$dm = \lambda dx = cx\,dx$

$$\int_0^m dm = \lambda dx = \int_0^\ell cx\,dx = \overset{\overset{m}{\shortparallel}}{\frac{1}{2}c\ell^2} \rightarrow c = 2m/\ell^2$$



Ib

1) m
2) I_o, I_{CM}

2) N @ ω cost

$$I_o = \int x^2 dm = \int_0^\ell x^3 c\,dx = \frac{1}{4}c\ell^4 = \frac{1}{2}m\ell^2$$

2) CM: $$\frac{\int x\,dm}{\int dm} = \frac{\int_0^\ell c x^2 dx}{\int_0^\ell cx\,dx} = \frac{\frac{1}{3}\ell^{\cancel{3}}\cancel{c}}{\frac{1}{2}\ell^{\cancel{2}}\cancel{c}} = \frac{2}{3}\ell$$

$$I_o = I_{CM} + m_{tot}a^2 \quad \Rightarrow \quad I_{CM} = I_o - m_{tot}a^2$$

$$= \frac{1}{2}m\ell^2 - \frac{1}{2}c\ell^2\frac{4}{9}\ell^2 = \frac{1}{2}m\ell^2 - \frac{\cancel{1}}{\cancel{2}}\frac{\cancel{2}m}{\cancel{\ell^2}}\ell^{\overset{2}{\cancel{4}}}\frac{4}{9}$$

$$= \frac{1}{2}m\ell^2 \ominus \frac{4}{9}m\ell^2 = \frac{9-8}{18}m\ell^2 = \frac{1}{18}m\ell^2$$

?

nella soluzione è $\oplus$

3) $N = F_c = m\omega^2\frac{2}{3}\ell$