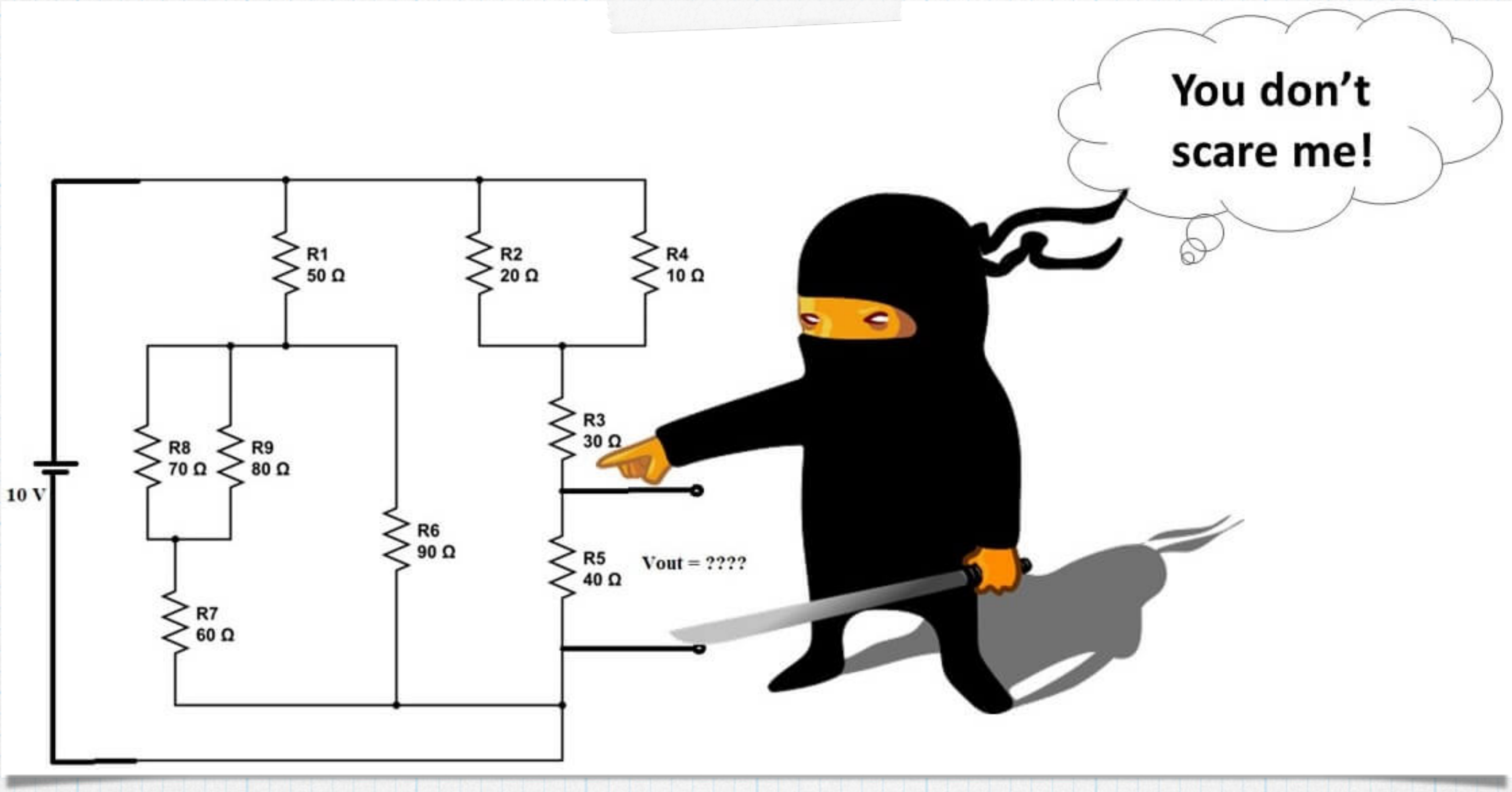
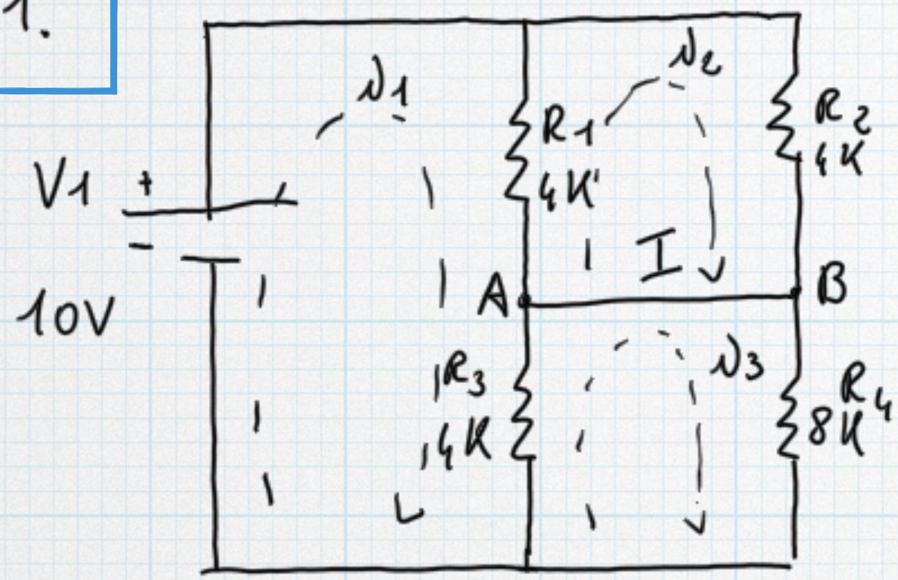


ESPERIMENTAZIONI II
Esercizi 12 Feb 2020: soluzioni

Michela Chiosso

Corso di Laurea in Fisica
Università degli Studi di Torino

$\boxed{1.}$



Mesh Analysis

$$\begin{cases} (R_1+R_3)i_1 - R_1 i_2 - R_3 i_3 = 10 \\ -R_1 i_1 + (R_1+R_2) i_2 - 0 = 0 \\ -R_3 i_1 - 0 + (R_3+R_4) i_3 = 0 \end{cases}$$

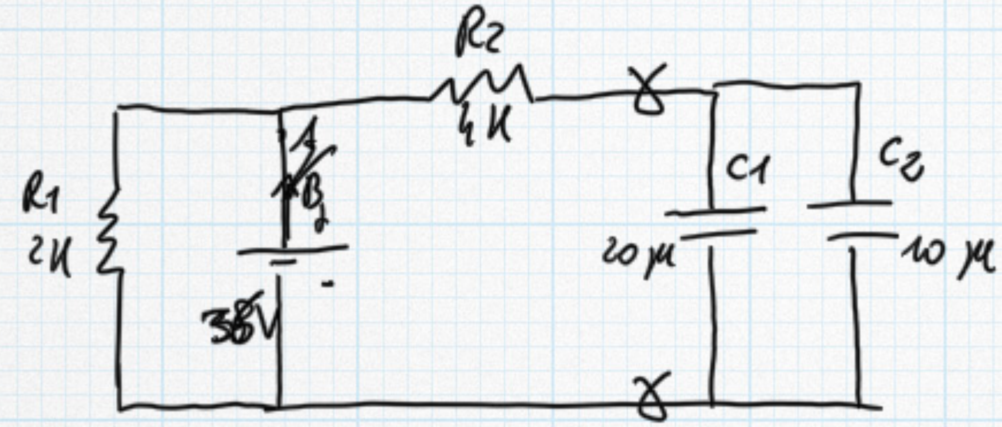
$$\begin{cases} 8K\, i_1 - 4K\, i_2 - 4K\, i_3 = 10 \\ -4K\, i_1 + 8K\, i_2 - 0 = 0 \\ -4K\, i_1 - 0 + 12K\, i_3 = 0 \end{cases}$$

$$i_2 = \frac{\begin{vmatrix} 8 & 10 & -4 \\ -4 & 0 & 0 \\ -4 & 0 & 12 \end{vmatrix}}{\begin{vmatrix} 8 & -4 & -4 \\ -4 & 8 & 0 \\ -4 & 0 & 12 \end{vmatrix}} = \frac{0 - 10\cdot(-48) - 4\cdot 0}{8\cdot 96 + 4\cdot(-48) - 4\cdot(0+32)} = \frac{480}{448} =$$

$$= 1.07\ mA \Rightarrow I = i_2 - i_3 = 0.36\ mA$$

$$i_3 = \frac{\begin{vmatrix} 8 & -4 & 10 \\ -4 & 8 & 0 \\ -4 & 0 & 0 \end{vmatrix}}{448} = \frac{0 + 4\cdot 0 + 10\cdot 32}{448} = \frac{320}{448} = 0.71\ mA$$

2



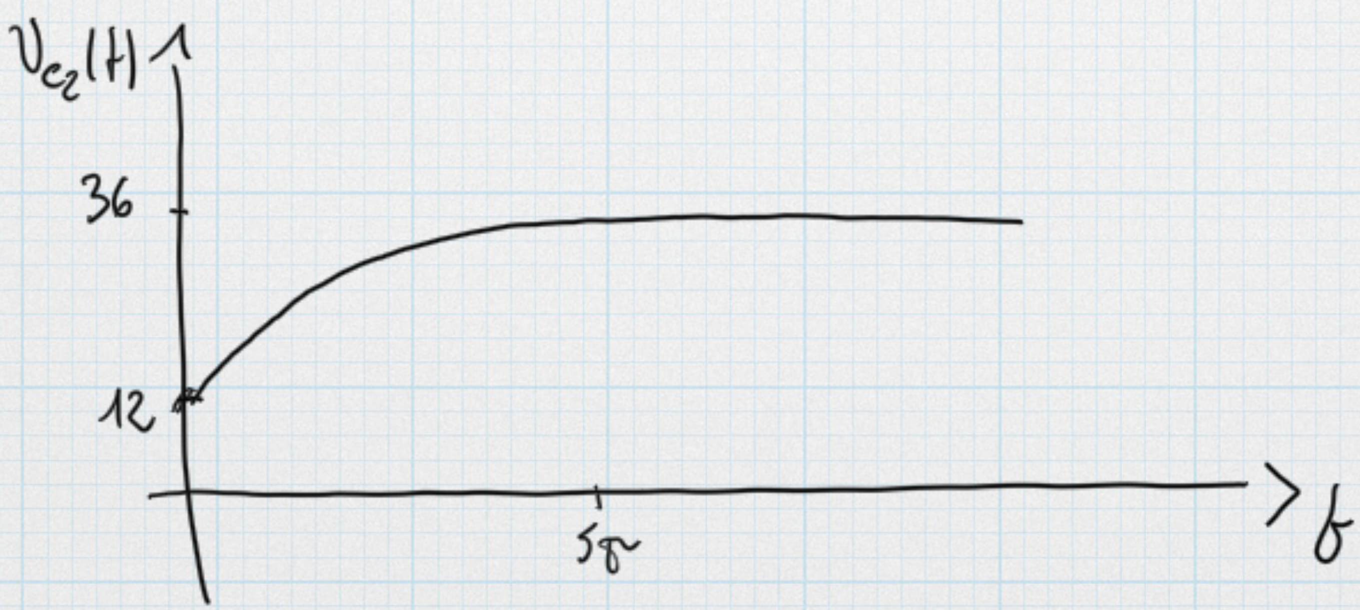
$V_{c2} = V_{c1}$ $\qquad C_{tot} = 30\,\mu F$

Trovo tensione e R equivalente di Thevenin

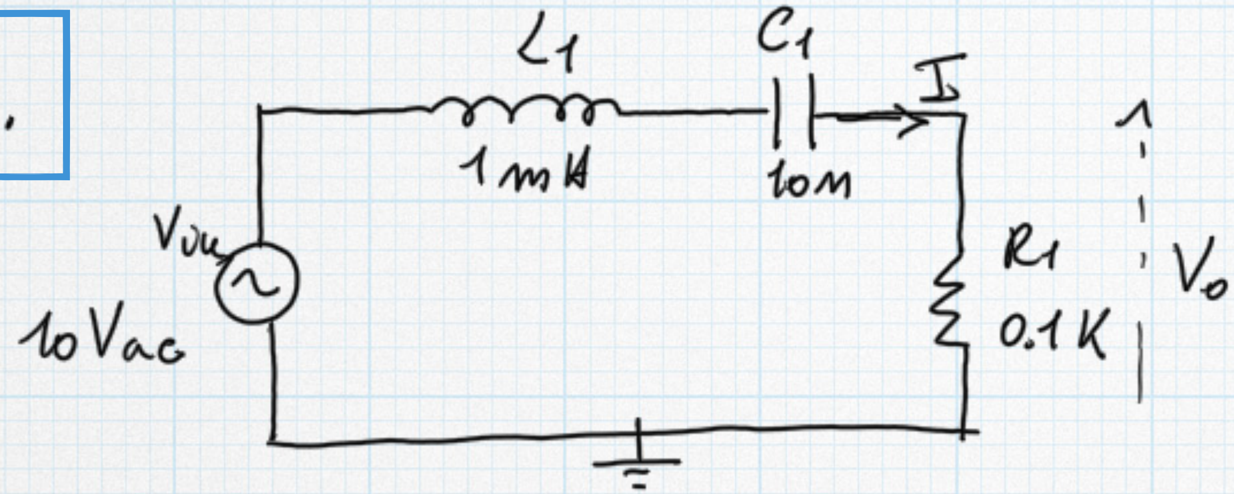
$V_{eq} = 36\ V$ $\qquad R_{eq} = R_2 = 4K$

$\tau = 4 \cdot 10^3 \cdot 3 \cdot 10^{-5} = 12 \cdot 10^{-2} = 120\ msec$

$V_{c2}(t) = v_{ctot}(t) = 36 - (36 - 12) \cdot e^{-\frac{t}{\tau}}$



3.



$$I = \frac{V_{in}}{R + j\left(\omega L - \frac{1}{\omega C}\right)} = \frac{V_{in} \cdot \left[R - j\left(\omega L - \frac{1}{\omega C}\right)\right]}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$tg(45°) = 1 \Rightarrow tg(\varphi) = \frac{-\left(\omega L - \frac{1}{\omega C}\right)}{R} = 1 \Rightarrow$$

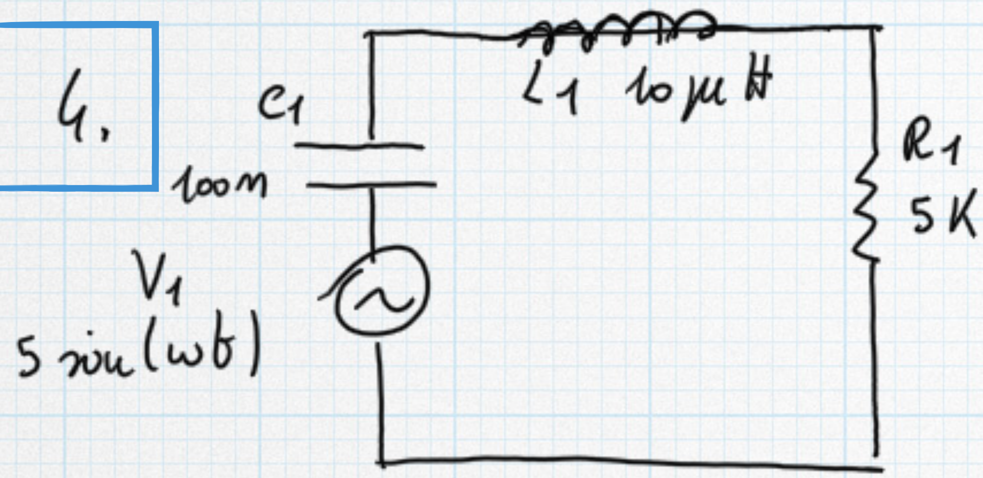
$$\Rightarrow \frac{1}{\omega C} - \omega L - R = 0$$

$$\omega^2 L + \omega R - \frac{1}{C} = 0 \Rightarrow \omega = \frac{-R \pm \sqrt{R^2 + 4\frac{L}{C}}}{2L} \Rightarrow$$

$$\Rightarrow \omega = \frac{-R + R\sqrt{1 + 4\frac{L}{R^2C}}}{2L} = \frac{R}{2L} \cdot \left[\sqrt{1 + \frac{4L}{R^2C}} - 1\right] =$$

$$= \frac{10^2\,\Omega}{2 \cdot 10^{-3}\,H} \cdot \left[\sqrt{1 + \frac{4 \cdot 10^{-3}\,H}{10^4\,\Omega \cdot 10^{-8}\,F}} - 1\right] = 2{,}7 \cdot 10^5 \Rightarrow f = 43\ KHz$$

4.



$$P_{R_1} = I_{R_1}^2 \cdot R$$

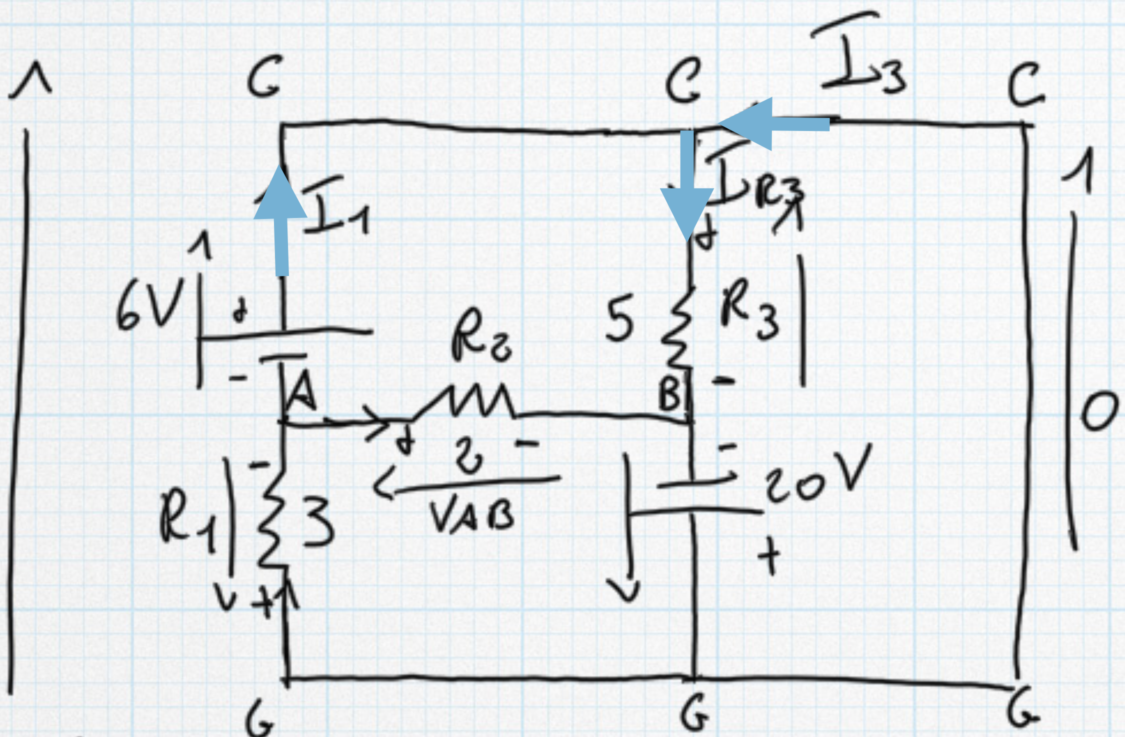
$$I_{R_1} = \frac{V_1}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

I_{R_1} è massima (e dunque P_{R_1} è massima) per $\omega L = \frac{1}{\omega C}$ $\Rightarrow$

$$\Rightarrow \quad f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi \cdot \sqrt{10^{-5} H \cdot 10^{-7} F}} = \frac{1}{2\pi} \cdot 10^{6} = 1.6 \cdot 10^{5}\ Hz$$

$$P_{MAX} = I_{R_1}^2 \cdot R = \frac{V_1^2}{R_1^2} \cdot R = \frac{V_1^2}{R_1} = 5\ mW$$

5.



Trovo V_{R3}:

$0 + 20 - V_{R3} = 0 \Rightarrow V_{R3} = 20$

Soluzione 1

La tensione fra C e G è nulla, perché ai capi di un cortocircuito

$0 + V_{R1} - 6V = 0 \Rightarrow V_{R1} = 6V$

Trovo V_{AB} risolvendo l'equazione di Kirchoff alla maglia

$V_{AB} + V_{R1} - 20V = 0 \Rightarrow V_{R1} = 20 - V_{AB} \Rightarrow \underline{V_{AB} = 20 - V_{R1} = (20 - 6) = 14\ V}$

Equazione Kirchoff al nodo A:

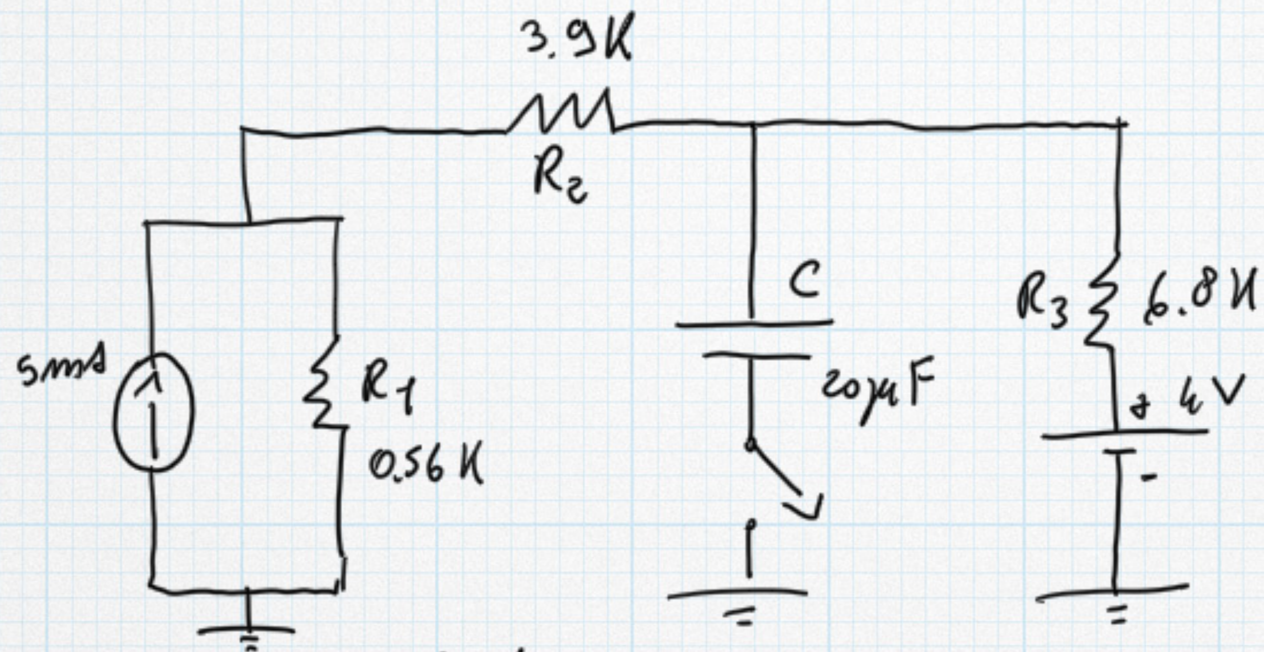
$I_{R1} - I_{R2} - I_1 = 0 \Rightarrow I_1 = \frac{6V}{3} - \frac{14}{2} = -5A$

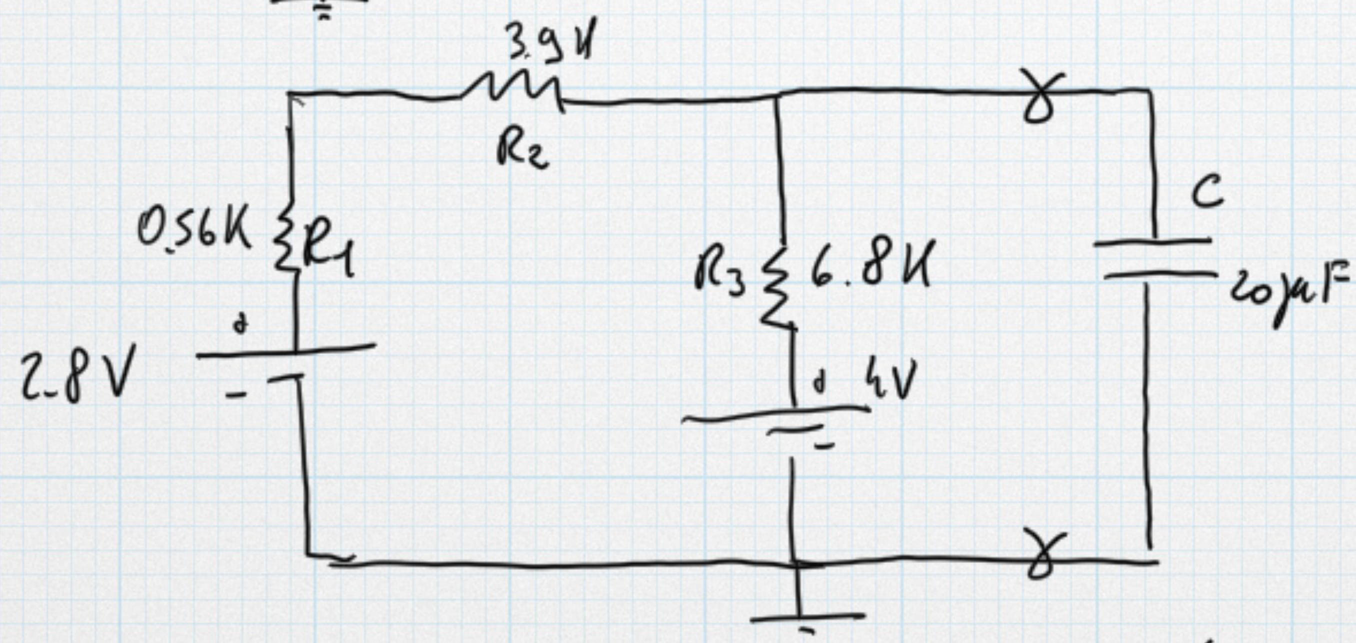
Eq. Kirchoff nodo C:

$I_3 - I_{R3} + I_1 = 0 \Rightarrow \underline{I_3} = -I_1 + I_{R3} = 5A + \frac{20V}{5\Omega} = \underline{9A}$

Soluzione 2: Risolvo con mesh analisi

$\boxed{6.}$





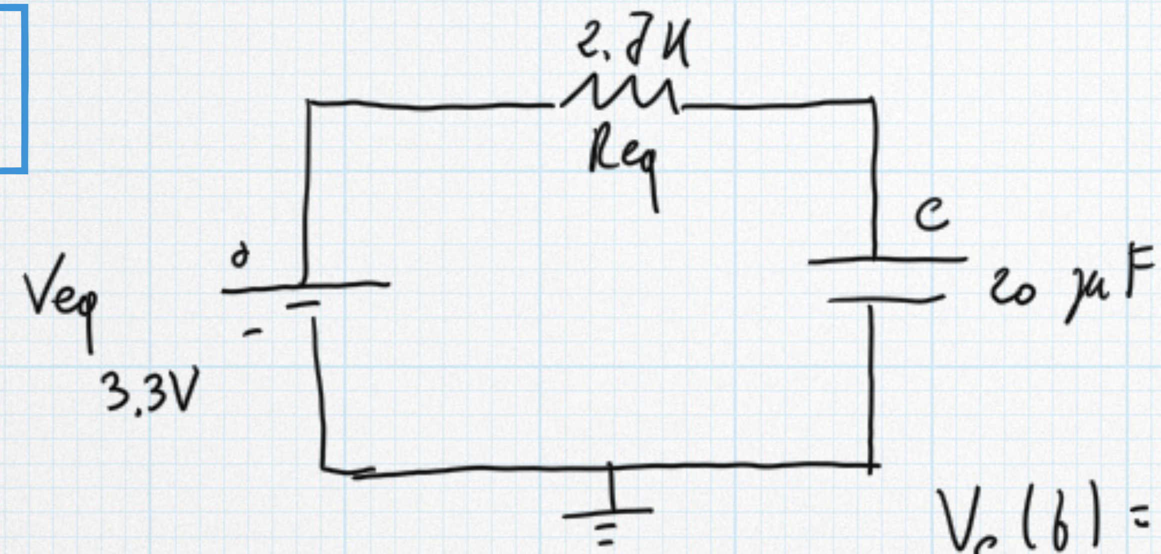
Applico Millman per trovare Veq

$$V_{eq} = \frac{\frac{2.8}{4.5} + \frac{4}{6.8}}{\frac{1}{4.5} + \frac{1}{6.8}} = \frac{0.62 + 0.59}{0.22 + 0.15} = 3.3\ V$$

$$R_{eq} = (R_1 + R_2) // R_3 = $$
$$= 2.7\ K \implies$$

CONTINUA

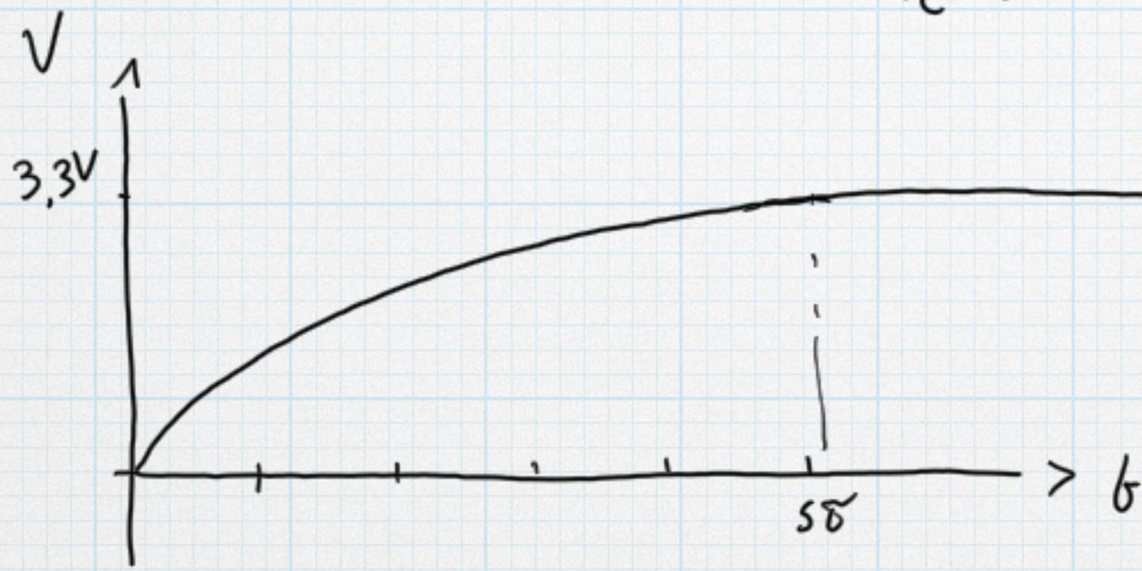
6.



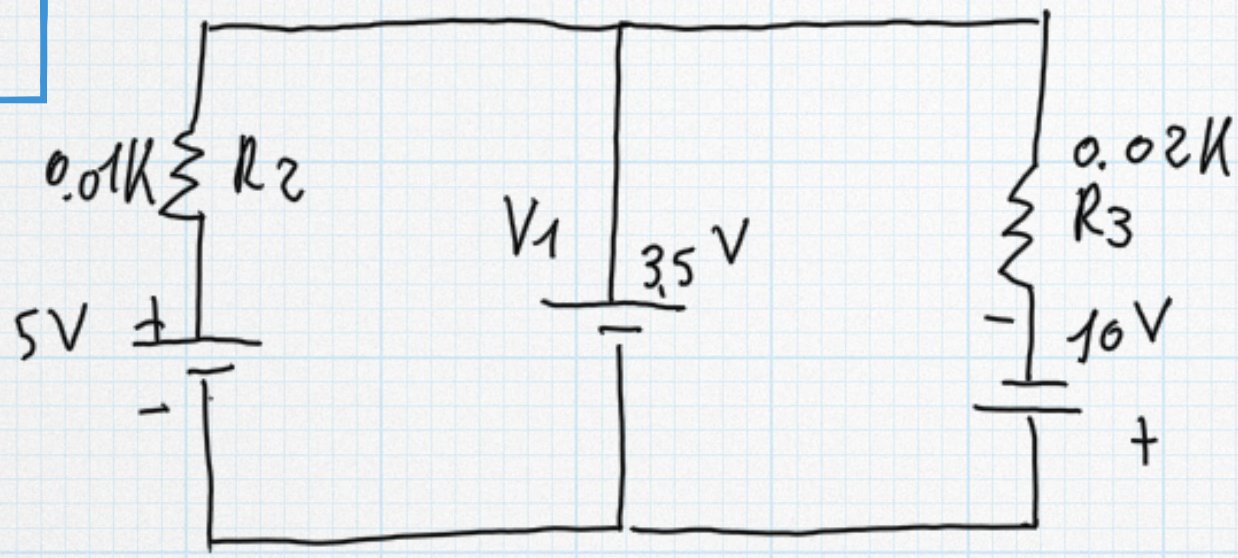
$$\tau = R_{eq} \cdot C = 2{,}7 \cdot 10^{3} \cdot 2 \cdot 10^{-5} =$$

$$= 5{,}4 \cdot 10^{-2} = 54 \text{ msec}$$

$$V_C(t) = V_{eq} \cdot \left(1 - e^{-\frac{t}{\tau}}\right)$$



7.

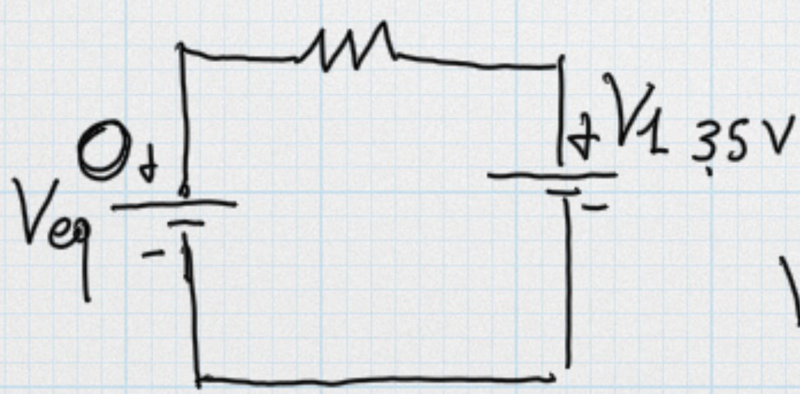


Applico Thevenin e trovo la tensione equivalente con MILLMAN

$$V_{eq} = \frac{\frac{5}{10} - \frac{10}{20}}{\frac{1}{10} + \frac{1}{20}} = \frac{\frac{1}{2} - \frac{1}{2}}{\frac{1}{10} + \frac{1}{20}} = 0$$

$$R_{eq} = R_2 \parallel R_3 = \frac{10 \cdot 20}{30} \simeq 6{,}7\,\Omega$$

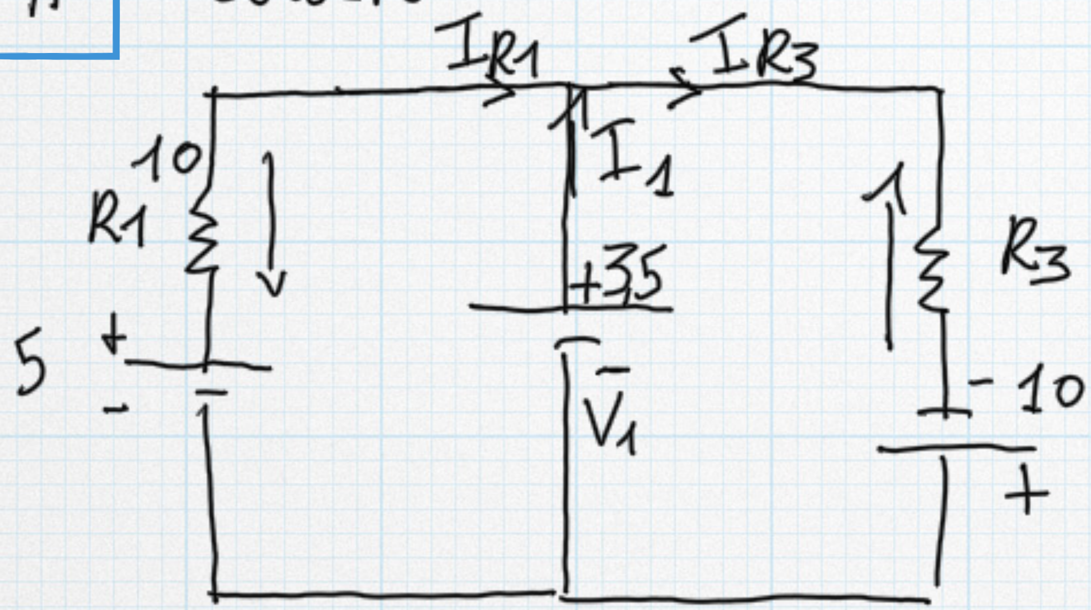
Il circuito diventa:



$$\Rightarrow I_1 = \frac{3.5\,V}{6.7\,\Omega} = 0.52\,A$$

V_1 eroga potenza $P = 1.8\,W$

$\boxed{7.}$ Soluzione 2



$$I_{R_1} = \frac{(5-3,5)V}{10\,\Omega} = 0,15\,A$$

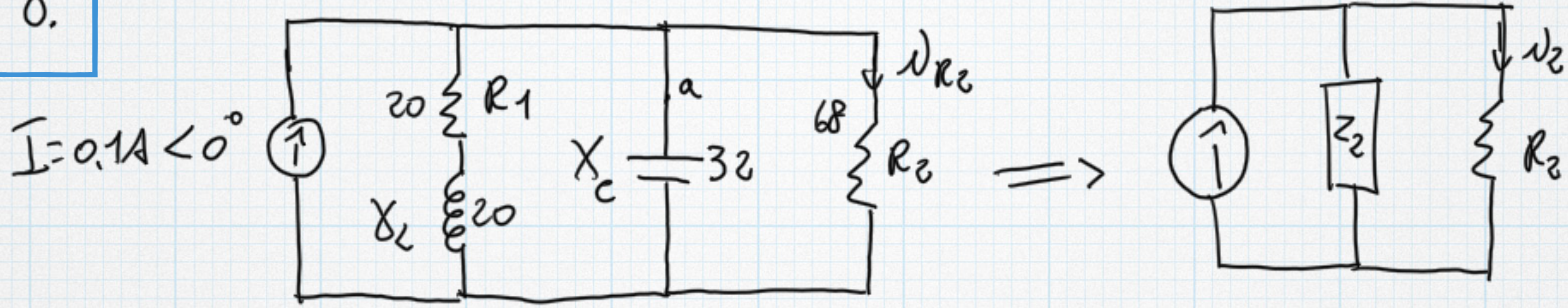
$$I_{R_3} = \frac{(3,5+10)V}{20\,\Omega} = 0,67\,A$$

$$I_{R_1} + I_1 - I_{R_3} = 0 \Rightarrow I_1 = I_{R_3} - I_{R_1} = 0,52\,A$$

$$P = 3.5\,V \cdot 0.52\,A = 1.8\,W$$

V_1 eroga potenza

8.



$$Z_1 = R_1 + Z_L = R_1 + jX_L = 20(1+j)$$

$$Z_2 = Z_1 \parallel Z_c = Z_1 \parallel (-jX_c) = \frac{20\cdot(1+j)\cdot -j32}{20(1+j) - j32} =$$

$$= \frac{20\cdot(32-32j)}{20 - j\cdot 12} = \frac{5\cdot(32-32j)}{5-j\cdot 3}$$

$$v_{R2} = \frac{I\cdot Z_2}{Z_2 + R_2} = \frac{0.1\cdot \dfrac{5\cdot(32-32j)}{5-3j}}{68 + \dfrac{5\cdot(32-32j)}{5-3j}} =$$

$$= \frac{0.1 \cdot 5\cdot(32-32j)}{340 - 204j + 160 - 160j} = \frac{16\cdot(1-j)}{500 - 364j} =$$

$\Rightarrow$ CONTINUA

$$= \frac{4 \cdot (1-j)}{125 - 91j} = \frac{4 \cdot (1-j) \cdot (125 + 91j)}{125^2 + 91^2} =$$

$$= \frac{4 \cdot (125 + 91j - 125j + 91)}{125^2 + 91^2} =$$

$$= \frac{4 \cdot (216 - 34j)}{125^2 + 91^2}$$

$$|I| = \frac{4 \cdot \sqrt{216^2 + 34^2}}{23.906} = \frac{874.6}{23.906} = 0.037 A$$

$$|V_{ab}| = |V_{R_2}| = 0.037 A \cdot 68 \Omega = 2.5 V$$

$$\varphi = \arctg\left(\frac{-34}{216}\right) \approx -9^\circ$$

$$i_{R_2}(t) = 0.037 A \cdot \sin(\omega t - 9^\circ) = \Im m\left(e^{\omega t} \cdot e^{j\varphi}\right)$$