# Euler's Method

Aiden Rosenberg

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## Problem 1

1. (a) Given the differential equation $\frac{dy}{dx} = x + 2$ and $y(0) = 3$. Find an approximation for $y(1)$ by using Euler's Method with two equal steps.

Table 1: $\Delta x = 0.5$

| $x$ | $y$ | $\frac{dy}{dx}$ | $\Delta y$ |
|---|---|---|---|
| 0 | 3 | 3 | 1.5 |
| 0.5 | 4.5 | 5 | 2.25 |
| 1 | 6.25 | 7.25 | 3.625 |

$$y(1) \approx 6.25$$

(b)

$$\int \frac{dy}{dx}\,dx = \int x + 2\,dx$$

$$y = \frac{x^2}{2} + 2x + C\Big|_{(0,3)} \implies C = 3$$

$$y = \frac{x^2}{2} + 2x + 3\Big|_{x=1} = 5.5$$

(c) The difference between the estimate and the exact value is $\frac{1}{4}$; to increase accuracy smaller steps can be used.

| $x$ | 2.0 | 2.5 | 3.0 |
|---|---|---|---|
| $f'(x)$ | 0.4 | 0.6 | 0.8 |
| $f(x)$ | 5 | | |

2. Suppose a continuous function $f$ and its derivative $f'$ have the values given in the following table. Given that $f(2) = 5$, use Euler's Method with two steps of size $\Delta x = 0.5$ to approximate the value of $f(3)$.

Table 2: $\Delta x = 0.5$

| $x$ | $y$ | $\frac{dy}{dx}$ | $\Delta y$ |
|---|---|---|---|
| 2 | 5 | 0.4 | 0.2 |
| 2.5 | 5.2 | 0.6 | 0.3 |
| 3 | 5.5 | | |

(a)

(b)

$$f(3) \approx 5.5$$

3. Given the differential equation $\frac{dy}{dx} = \frac{1}{x+2}$ and $y(0) = 1$. Find an approximation for $y(1)$ by using Euler's Method with two steps $\Delta x = 0.5$.

Table 3: $\Delta x = 0.5$

| $x$ | $y$ | $\frac{dy}{dx}$ | $\Delta y$ |
|---|---|---|---|
| 0 | 1 | 0.5 | 0.25 |
| 0.5 | 1.25 | 0.4 | 0.2 |
| 1 | 1.45 | | |

(a)

(b)

$$y(1) \approx 1.45$$

4. Given the differential equation $\frac{dy}{dx} = x+y$ and $y(1) = 3$. Find an approximation for $y(2)$ by using Euler's Method with two steps $\Delta x = 0.5$.

(a)

(b)

$$y(2) \approx 8.25$$

Table 4: $\Delta x = 0.5$

| $x$ | $y$ | $\frac{dy}{dx}$ | $\Delta y$ |
|---|---|---|---|
| 1 | 3 | 4 | 2 |
| 1.5 | 5 | 6.5 | 3.25 |
| 2 | 8.25 | | |

5. The curve passing through (2,0) satisfies the differential equation $\frac{dy}{dx} = 4x + y$. Find an approximation to $y(3)$ using Euler's Method with two equal steps.

Table 5: $\Delta x = 0.5$

| $x$ | $y$ | $\frac{dy}{dx}$ | $\Delta y$ |
|---|---|---|---|
| 2 | 0 | 8 | 4 |
| 2.5 | 4 | 14 | 7 |
| 3 | 11 | | |

(a)

(b)

$$y(3) \approx 11$$

6. Suppose a continuous function $f$ and its derivative $f'$ have the values given in the following table. Use Euler's Method with two equal steps to approximate the value of $f(4.4)$.

| $x$ | 4 | 4.2 | 4.4 |
|---|---|---|---|
| $f'(x)$ | -0.5 | -0.3 | -0.1 |
| $f(x)$ | 2 | | |

Table 6: $\Delta x = 0.2$

| $x$ | $y$ | $\frac{dy}{dx}$ | $\Delta y$ |
|---|---|---|---|
| 4 | 2 | -0.5 | -0.1 |
| 4.2 | 1.8 | -0.3 | -0.06 |
| 4.4 | 1.84 | -0.1 | |

(a)

(b)

$$y(4.4) \approx 1.84$$

7. The table gives selected values for the derivative of a function $f$ on the interval $-2 \leq x \leq 2$. If $f(-2) = 3$ and Euler's Method with a step-size of 1.5 is used to approximate $f(1)$, what is the resulting approximation?

| $x$ | $f'(x)$ |
|---|---|
| -2 | -0.8 |
| -1.5 | -0.5 |
| -1 | -0.2 |
| -0.5 | 0.4 |
| 0 | 0.9 |
| 0.5 | 1.6 |
| 1 | 2.2 |
| 1.5 | 3 |
| 2 | 3.7 |

Table 7: $\Delta x = 1.5$

| $x$ | $y$ | $\frac{dy}{dx}$ | $\Delta y$ |
|---|---|---|---|
| -2 | 3 | -0.8 | -1.2 |
| -0.5 | 1.8 | 0.4 | 0.6 |
| 1 | 2.4 | 2.2 | |

(a)

(b)

$$f(1) \approx 2.4$$

8. Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = x + 2y$ with the initial condition $f(0) = 1$. Use Euler's Method, starting at $x = 0$ with two steps of equal size to approximate $f(-0.6)$.

Table 8: $\Delta x = 1.5$

| $x$ | $y$ | $\frac{dy}{dx}$ | $\Delta y$ |
|---|---|---|---|
| 0 | 1 | 2 | -0.6 |
| -0.3 | 0.4 | 0.5 | -0.15 |
| -0.6 | 0.25 | | |

(a)

(b)

$$f(-0.6) \approx 0.25$$