# WORKSHEET ON L'HOPITAL'S RULE AND IMPROPER INTEGRALS

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1. $\lim_{x\to 0} \frac{1-\cos x}{x+x^2} \overset{\text{H}}{=} \lim_{x\to 0} \frac{\sin x}{1+2x} = \boxed{0}$

2. $\lim_{x\to 0} \frac{\sqrt{1+x}-1-\frac{x}{2}}{x^2} \overset{\text{H}}{=} \lim_{x\to 0} \frac{\frac{1}{2}(1+x)^{-1/2}-\frac{1}{2}}{2x} \overset{\text{H}}{=} \lim_{x\to 0} \frac{-\frac{1}{4}(1+x)^{-3/2}}{2} = \boxed{-\frac{1}{8}}$

3. $\lim_{x\to 1} \left(\frac{1}{\ln x} - \frac{1}{x-1}\right) = \lim_{x\to 1} \frac{x-1-\ln x}{x\ln x - \ln x} \overset{\text{H}}{=} \lim_{x\to 1} \frac{1-x^{-1}}{\ln x + 1 - x^{-1}} \overset{\text{H}}{=} \lim_{x\to 1} \frac{x^{-2}}{x^{-1}+x^{-}2} = \lim_{x\to 1} \frac{1}{x+1} = \boxed{\frac{1}{2}}$

4. $\lim_{\theta\to \frac{\pi}{2}} \frac{1-\sin\theta}{1+\cos 2\theta} \overset{\text{H}}{=} \lim_{\theta\to \frac{\pi}{2}} \frac{-\cos\theta}{-2\sin 2\theta} \overset{\text{H}}{=} \lim_{\theta\to \frac{\pi}{2}} \frac{\sin\theta}{-4\cos 2\theta} = \boxed{\frac{1}{4}}$

5. $\lim_{x\to 0} \frac{\cos x - 1}{e^x - x - 1} \overset{\text{H}}{=} \lim_{x\to 0} \frac{-\sin x}{e^x - 1} \overset{\text{H}}{=} \lim_{x\to 0} \frac{-\cos x}{e^x} = \boxed{-1}$

6. $a = \lim_{x\to 0} (e^x+1)^{1/x} \Longrightarrow \ln a = \lim_{x\to 0} \frac{\ln(e^x+1)}{x} \overset{\text{H}}{=} \lim_{x\to 0} \frac{e^x+1}{e^x+x} = 2 \Longrightarrow a = \boxed{e^2}$

7. $\lim_{x\to \frac{\pi}{2}^-} \left(\cos x\right)^{\overbrace{\cos x}^{0}} = \boxed{1}$