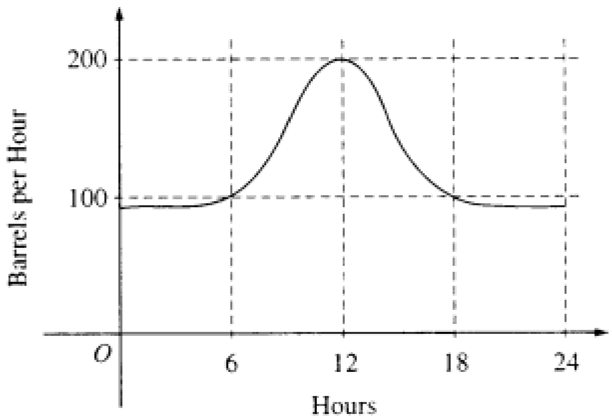
AP Classroom Problems

Aiden Rosenberg

October 25, 2022 A.D

3.01

1. 

The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

$$\boxed{\int_0^{24} B(t)\,dt \approx 3,000}$$

t (hours)	4	7	12	15
$R(t)$ (liters per hour)	6.5	6.2	5.9	5.6

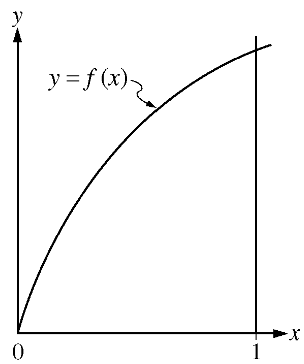
2. A tank contains 50 liters of oil at time $t = 4$ hours. Oil is being pumped into the tank at a rate $R(t)$ where $R(t)$ is measured in liters per hour, and t is measured in hours. Selected values of $R(t)$ are given in the table above. Using a right Riemann sum with three subintervals and data from the table, what is the approximation of the number of liters of oil that are in the tank at time $t = 15$ hours?

$$\int_4^{15} R(t)\,dt + R(0)$$

$$\approx\Big((7-4)\cdot R(7) + (12-7)\cdot R(12) + (15-12)\cdot R(17)\Big) + 50$$

$$= 64.9 + 50$$

$$= \boxed{114.9 \text{ liters}}$$



3. A left Riemann sum, a right Riemann sum, and a trapezoidal sum are used to approximate the value of $\int_0^1 f(x)\,dx$, each using the same number of sub intervals. The graph of the function f is shown in the figure above. Which of the sums give an underestimate of the value of $\int_0^1 f(x)\,dx$?

   (a) When $f(x)$ is concave down both left and trapezoidal sums are underestimates.

$$\boxed{\text{I and III only}}$$

4. Let f be the function given by $f(x) = 9^x$. If four subintervals of equal length are used, what is the value of the right Riemann sum approximation for $\int_0^2 f(x)\,dx$?

   (a) $\Delta x = 0.5$

$$\text{RHS}_4 = 0.5 \cdot \big(A(0.5) + A(1) + A(1.5) + A(2)\big)$$
$$\text{RHS}_4 = 0.5 \cdot \big(3 + 9 + 27 + 81\big)$$
$$\text{RHS}_4 = 0.5 \cdot \big(120\big)$$
$$\boxed{\text{RHS}_4 = 60}$$

x	0	0.5	1	1.5	2	2.5	3
$f(x)$	0	4	10	18	28	40	54

5. The table above gives selected values for a continuous function f. If f is increasing over the closed interval $[0, 3]$, which of the following could be the value of $\int_0^3 f(x)\,dx$?

   (a) $\text{LHS}_6 = 0.5\big(0 + 4 + 10 + 18 + 28 + 40\big) = 50$
   (b) $\text{RHS}_6 = 0.5\big(4 + 10 + 18 + 28 + 40 + 54\big) = 77$

$$\text{LHS}_6 < \int_0^3 f(x)\,dx < \ \text{RHS}_6$$
$$50 < \int_0^3 f(x)\,dx < 77$$
$$\boxed{62 \textit{ Satisfies the necessary conditions}}$$

x	2	3	5	8	13
$f(x)$	6	-2	-1	3	9

6. The function f is continuous on the closed interval $[2, 13]$ and has values as shown in the table above. Using the intervals $[2, 3]$, $[3, 5]$, $[5, 8]$, and $[8, 13]$ what is the approximation of $\int_2^{13} f(x)\,dx$ obtained from a left Riemann sum?

$$\text{LHS}_4 = f(2)\cdot(1) + f(3)\cdot(2) + f(5)\cdot(3) + f(8)\cdot(5)$$
$$\text{LHS}_4 = 6 + (-2)\cdot(2) + (-1)\cdot(3) + (3)\cdot(5)$$
$$\boxed{\text{LHS}_4 = 14}$$

x	0	a^2	$3a$	$6a$	$7a$
$f(x)$	1	-1	-3	-7	-9

7. The continuous function f is decreasing for all x. Selected values of f are given in the table above, where a is a constant with $0 < a < 3$. Let R be the right Riemann sum approximation for $\int_0^{7a} f(x)\,dx$ using the four subintervals indicated by the data in the table. Which of the following statements is true?

   (a)
   $$\boxed{R = (a^2 - 0)\cdot(-1) + (3a - a^2)\cdot(-3) + (6a - 3a)\cdot(-7) + (7a - 6a)\cdot(-9)}$$

   (b)
   $$\boxed{R \text{ is an underestimate for } \int_0^{7a} f(x)\,dx}$$

8. Which of the following is the midpoint Riemann sum approximation of $\int_4^6 \sqrt{x^3+1}\,dx$ using 4 subintervals of equal width?

   (a) $\Delta x = \frac{b-a}{n} = \frac{6-4}{4} = \frac{1}{2}$
   (b) $f(x) = \sqrt{x^3+1}$

$$\text{MRAM}_4 = \frac{1}{2}\big(f(4.25) + f(4.75) + f(5.25) + f(5.75)\big)$$
$$\boxed{\text{MRAM}_4 = \frac{1}{2}\left(\sqrt{4.25^3+1} + \sqrt{4.75^3+1} + \sqrt{5.25^3+1} + \sqrt{5.75^3+1}\right)}$$

x	0	25	30	50
$f(x)$	4	6	8	12

9. The values of a continuous function f for selected values of x are given in the table above. What is the value of the left Riemann sum approximation to $\int_0^5 f(x)\,dx$ using the subintervals $[0,25]$ $[25,30]$ and $[30,50]$?

$$\text{LHS}_3 = f(0)\cdot(25) + f(25)\cdot(5) + f(30)\cdot(20)$$
$$\text{LHS}_3 = 4\cdot(25) + 6\cdot(5) + 8\cdot(20)$$
$$\boxed{\text{LHS}_4 = 290}$$

x	0	1	2	3	4	5	6
$f(x)$	0	5	2	-1	-2	0	3

10. The function f is continuous on the closed interval $[0,6]$ and has values as shown in the table above. Using the intervals $[0,2]$, $[2,4]$, and $[4,6]$, what is the approximation of $\int_0^6 f(x)\,dx$ obtained from a midpoint Riemann sum?

$$\text{MRAM}_3 = 2\big(f(1) + f(3) + f(5)\big)$$
$$\text{MRAM}_3 = 2\big(5 + (-1) + 0\big)$$
$$\boxed{\text{MRAM}_3 = 8}$$

3.02

1. Let f and g be continuous functions such that $\int_0^{10} f(x)\,dx = 21$, $\int_0^{10} \frac{1}{2}g(x)\,dx = 8$, and $\int_3^{10}(f(x) - g(x))\,dx = 2$. What is the value of $\int_0^3(f(x) - g(x))\,dx$?

(a)
$$\int_0^{10} \frac{1}{2}g(x)\,dx = 8 \implies \int_0^{10} g(x)\,dx = 16$$

(b)
$$\int_0^{10}(f(x) - g(x))\,dx = \int_0^{10} f(x)\,dx - \int_0^{10} g(x)\,dx = 21 - 16 = 5$$
$$\int_0^{10}(f(x) - g(x)) = \int_0^{3}(f(x) - g(x))\,dx + \int_3^{10}(f(x) - g(x))\,dx$$
$$\implies 2 + \int_0^{3}(f(x) - g(x))\,dx = 5$$
$$\implies \boxed{\int_0^{3}(f(x) - g(x))\,dx = 3}$$

2. Let f and g be continuous functions for $a \leq x \leq b$. If $a < c < b$, $\int_a^b f(x)\,dx = P$,$\int_c^b f(x)\,dx = Q$, $\int_a^b g(x)\,dx = R$, and $\int_c^b g(x)\,dx = S$, then $\int_a^c (f(x) - g(x))\,dx =$

   (a) $\int_a^c f(x)\,dx = \int_a^b f(x)\,dx - \int_c^b f(x)\,dx = P - Q$

   (b) $\int_a^c g(x)\,dx = \int_a^b g(x)\,dx - \int_c^b g(x)\,dx = R - S$

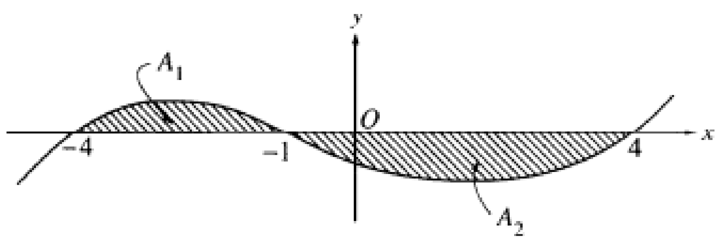
$$\int_a^c (f(x) - g(x))\,dx = \int_a^c f(x)\,dx - \int_a^c g(x)\,dx$$
$$\Longrightarrow \big(P - Q\big) - \big(R - S\big)$$
$$\Longrightarrow \boxed{P - Q - R + S}$$

3. Let f and g be continuous functions such that $\int_0^6 f(x)\,dx = 9$, $\int_3^6 f(x)dx = 5$, and $\int_3^0 g(x)dx = -7$. What is the value of $\int_0^3 (\frac{1}{2}f(x) - 3g(x))dx$?

   (a) $\int_0^6 f(x)\,dx = \int_0^3 f(x)\,dx + \int_3^6 f(x)\,dx \Longrightarrow \int_0^3 f(x)\,dx = 4$

   (b) $\int_3^0 g(x)dx = -7 \Longrightarrow \int_0^3 g(x)dx = 7$

$$\frac{1}{2}\int_0^3 f(x)\,dx - 3\int_0^3 g(x))dx = \frac{1}{2} \cdot 4 - 3 \cdot 7 = \boxed{-19}$$



4. The graph of $y = f(x)$ is shown in the figure above. If A_1 and A_2 are positive numbers that represent the areas of the shaded regions, then in terms of A_1 and A_2, $\int_{-4}^4 f(x)\,dx - 2\int_{-1}^4 f(x)dx =$

   (a) $A_1 = \int_{-4}^{-1} f(x)\,dx$

   (b) $A_2 = \int_{-1}^{2} f(x)\,dx$

   (c) $A_1 - A_2 = \int_{-4}^{4} f(x)\,dx$

$$\int_{-4}^4 f(x)\,dx - 2\int_{-1}^4 f(x)dx = (A_1 - A_2) - 2(-A_2) = \boxed{A_1 + A_2}$$

5. Let f and g have continuous first and second derivatives everywhere. If $f(x) \leq g(x)$ for all real x, which of the following must be true?

   I. $f'(x) \leq g'(x)$ for $x \in \mathbb{R}$

   II. $f''(x) \leq g''(x)$ for $x \in \mathbb{R}$

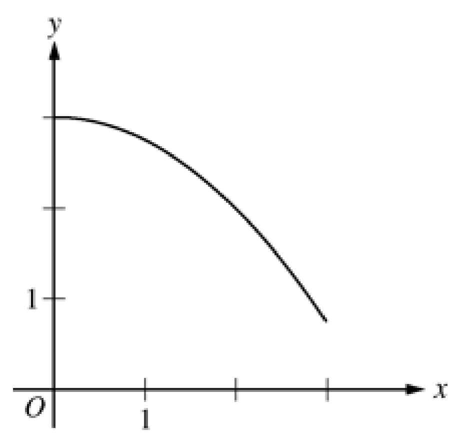
   III. $\int_0^1 f(x)\,dx \leq \int_0^1 g(x)\,dx$

$$\boxed{\text{III only}}$$

6. The function f is defined by $f(x) = \begin{cases} 2 & \text{for } x < 3 \\ x - 1 & \text{for } x \geq 3 \end{cases}$ What is the value of $\int_1^5 f(x)\,dx$?

$$\int_1^3 2\,dx + \int_3^5 (x-1)\,dx$$

$$2x\Big|_1^3 + \left[\frac{x^2}{2} - x\right]_3^5 = (6-2) + \left(\left(\frac{25}{2} - 5\right) - \left(\frac{9}{2} - 3\right)\right) = 4 + 6 = \boxed{10}$$

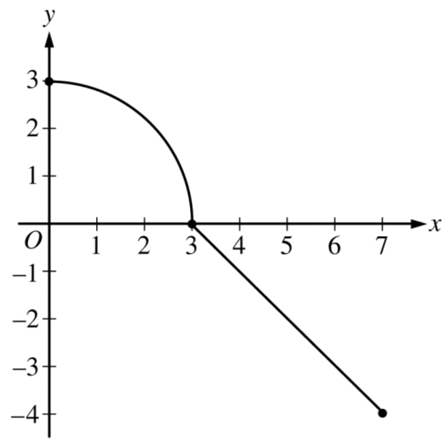


Graph of f

7.

The graph of the function f is shown above for $0 \leq x \leq 3$. Of the following, which has the least value?

$$\boxed{\text{Right Riemann sum approximation of } \int_1^3 f(x)\,dx \text{ with 4 subintervals of equal length}}$$



Graph of f

8.

The graph of the function f, which has a domain of $[0, 7]$, is shown in the figure above. The graph consists of a quarter circle of radius 3 and a segment with slope -1. Let b be a positive number such that $\int_0^b f(x)dx = 0$. What is the value of b?

(a) $\int_0^3 f(x)\,dx = \frac{\pi \cdot r^2}{4} = \frac{9\pi}{4}$

(b) $f(x) = \begin{cases} \sqrt{9-x^2} & \text{for } x < 3 \\ 3 - x & \text{for } x \geq 3 \end{cases}$

$$\int_3^b (3-x)\,dx = -\frac{9\pi}{4}$$

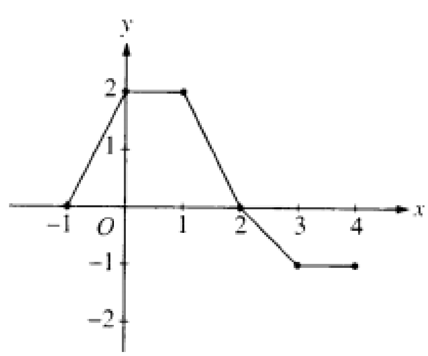
$$\implies -\frac{9\pi}{4} = 3x - \frac{x^2}{2}\bigg|_3^b = 3b - \frac{b^2}{2} - \left(9 - \frac{9}{2}\right)$$

$$\implies 12b - 2b^2 - 36 + 18 = -9\pi$$

$$\implies 12b - 2b^2 - 18 + 9\pi = 0 \implies \boxed{b \approx 6.7599}$$

9. $\int_{-1}^{2} \frac{|x|}{x}$ is

$$\int_{-1}^{2} \frac{|x|}{x} = |x|\bigg|_{-1}^{2} = 2 - 1 = \boxed{1}$$



10.

The graph of a piecewise-linear function f, for $-1 \leq x \leq 4$, is shown above. What is the value of $\int_{-1}^{4} f(x)\,dx$?

(a) $\int_{-1}^{0} f(x)\,dx = (0.5) \cdot (1) \cdot (2) = 1$

(b) $\int_0^1 f(x)\,dx = (1)\cdot 2 = 2$

(c) $\int_1^2 f(x)\,dx = (0.5)\cdot(1)\cdot(2) = 1$

(d) $\int_2^3 f(x)\,dx = (0.5)\cdot(1)\cdot(-1) = -0.5$

(e) $\int_3^4 f(x)\,dx = (1)\cdot(-1) = -1$

$$\boxed{\int_{-1}^{4} f(x)\,dx = 2.5}$$

3.03

1. If $G(x)$ is an antiderivative for $f(x)$ and $G(2) = -7$, then $G(4) =$

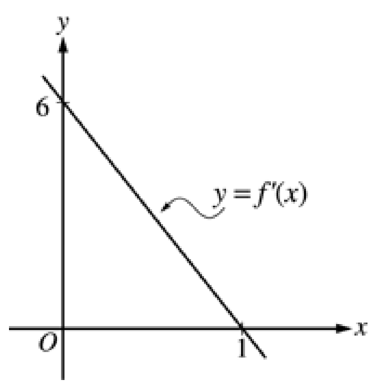
$$G(4) = G(2) + \int_2^4 f(t)\,dt$$

$$\boxed{G(4) = -7 + \int_2^4 f(t)\,dt}$$

2. If the function f is defined by $f(x) = \sqrt{x^3+2}$ and g is an antiderivative of f such that $g(3) = 5$, then $g(1) =$

$$G(1) = G(3) - \int_1^3 f(x)\,dx$$

$$\boxed{G(1) \approx -1.585}$$

3. 

The graph of f', the derivative of f, is the line shown in the figure above. If $f(0) = 5$, then $f(1) =$

$$f(1) = f(0) + \int_0^1 f'(x)\,dx = 5 + (0.1)(6)(3) = \boxed{8}$$

4. $\int_0^1 \sqrt{x}(x+1)\,dx$

$$\int_0^1 \sqrt{x}(x+1)\,dx = \int_0^1 x^{3/2} + x^{1/2}\,dx$$

$$\implies \frac{2x^{5/2}}{5} + \frac{2x^{2/3}}{3}\bigg|_0^1 = \frac{2}{5} + \frac{2}{3} = \boxed{\frac{16}{15}}$$

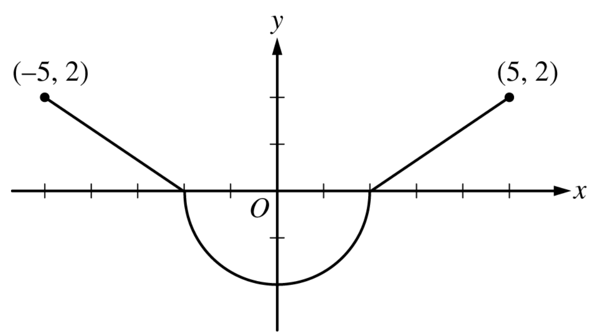
5. What are all values of k for which $\int_{-3}^{k} x^2\,dx = 0$

$$\left.\frac{x^2}{2}\right|_{-3}^{k} = \frac{k^3-27}{2}$$

$$\implies \frac{k^3-27}{2} = 0 \implies \boxed{k=3}$$

6. $\int_1^2 \frac{x-4}{x^2}\,dx$

$$\int_1^2 \frac{x-4}{x^2}\,dx = \int_1^2 (x^{-1}-4x^{-2})\,dx = \ln|x| + \left.\frac{4}{x}\right|_1^2 = \big((\ln 2+2\big) - \big(\ln 1+4\big) = \boxed{\ln 2 - 2}$$



7. Graph of f'

The graph of f', the derivative of a function f, consists of two line segments and a semicircle, as shown in the figure above. If $f(2)=1$, then $f(-5)=$

(a) $-\int_{-5}^{2} f'(x)\,dx = \int_{-5}^{-2} f'(x)\,dx + \int_{-2}^{2} f'(x)\,dx$

(b) $\int_{-5}^{-2} f'(x)\,dx = (0.5)(3)(2) = 3$

(c) $\int_{-2}^{2} f'(x)\,dx = -2\pi$

$$f(-5) = f(2) - \int_{-5}^{2} f'(x)\,dx = 1 - \big(3-2\pi\big) = \boxed{2\pi - 2}$$

8. If n is a known positive integer, for what value of k is $\int_1^k x^{n-1}\,dx = \frac{1}{n}$?

$$\int_1^k x^{n-1}\,dx = \left.\frac{x^n}{n}\right|_1^k = \frac{k^n}{n} - \frac{1^n}{n}$$

$$\frac{k^n-1}{n} = \frac{1}{n} \implies k^n = 2$$

$$\implies \boxed{k = 2^{1/n}}$$

9. If $g(x) = x^2-3x+4$ and $f(x) = g'(x)$, then $\int_1^3 f(x)\,dx =$

(a) $g(3) = 3^2 - 3(3) + 4 = 4$

(b) $g(1) = 1 - 3 + 4 = 2$

$$\int_1^3 f(x)\,dx = \int_1^3 g'(x)\,dx = g(3) - g(1) = 4 - 2 = \boxed{2}$$

10. Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$. If $F(1) = 0$, then $F(9) =$

$$F(9) = F(1) + \int_1^9 \frac{(\ln x)^2}{x}\,dx$$

$$\boxed{F(9) \approx 5.827}$$

3.04

1. If $\frac{dy}{dx} = -10e^{-t/2}$ and $y(0) = 20$, what is the value of $y(6)$?

$$y(6) = y(0) + \int_0^6 -10e^{-t/2}\,dt$$

$$y(6) = y(0) + 20e^{-t/2}\Big|_0^6$$

$$y(6) = 20 + (20e^{-3} - 20) = \boxed{20e^{-3}}$$

2. Let f be a differentiable function such that $f(1) = \pi$ and $f'(x) = \sqrt{x^3 + 6}$. What is the value of $f(5)$?

$$f(5) = f(1) + \int_1^5 f'(x)\,dx$$

$$f(5) = \pi + \int_1^5 \sqrt{x^3 + 6}\,dx$$

$$\boxed{f(5) \approx 27.814}$$

x	0	1	2	3
$f(x)$	5	2	3	6
$f'(x)$	-3	1	3	4

3. The derivative of the function f is continuous on the closed interval $[0, 4]$. Values of f and f' for selected values of x are given in the table above. If $\int_0^4 f'(t)\,dt = 8$, then $f(4) =$

$$\int_0^4 f'(t)\,dt = f(4) - f(0) = 8$$

$$f(4) = 8 + f(0) = \boxed{13}$$

4. $\int_0^x \sin t\,dt =$

$$\int_0^x \sin t\,dt = -\cos(t)\Big|_0^x = -\cos(x) + \cos(0) = \boxed{1 - \cos(x)}$$

5. $\int_1^2 \frac{dx}{2x+1} =$

   (a) Let $u = 2x+1 \Longrightarrow du = 2dx$

$$\frac{1}{2}\int_3^5 \frac{1}{u}\,du = \frac{\ln u}{2}\bigg|_3^5 = \boxed{\frac{1}{2}(\ln 5 - \ln 3)}$$

6. If $\int_0^k (2kx - x^2)\,dx = 18$, then $k =$

$$\int_0^k (2kx - x^2)\,dx = kx^2 - \frac{x^3}{3}\bigg|_0^k = k^3 - \frac{k^3}{3} = \frac{2k^3}{3}$$

$$18 = \frac{2k^3}{3} \Longrightarrow 27 = k^3$$

$$\boxed{k = 3}$$

7. If the function f has a continuous derivative on $[0, c]$, then $\int_0^c f'(x)\,dx =$

$$\boxed{f(x) - f(0)}$$

8. Let g be a differentiable function such that $g(10) = 2e$ and $g'(x) = 5e^{-\sqrt{x}}$. What is the value of $g(2)$?

$$g(2) = g(10) - \int_2^{10} g'(x)\,dx$$

$$\boxed{g(2) \approx 1.329}$$

x	0	1	2	3
$f(x)$	4	9	12	10
$f'(x)$	5	4	1	-6

9. Selected values of the twice-differentiable function f and its derivative f' are given in the table above. What is the value of $\int_0^3 f'(x)\,dx$?

$$\int_0^3 f'(x)\,dx = f(3) - f(0) = 10 - 4 = \boxed{6}$$

10. Let g be a differentiable function such that $g(4) = 0.325$ and $g'(x) = \frac{1}{x}e^{-x}(\cos(\frac{x}{100}))$. What is the value of $g(1)$?

$$g(1) = g(4) - \int_1^4 g'(x)\,dx$$

$$\boxed{g(1) \approx 0.109}$$

x	0	2	4	6
$f(x)$	-22	-6	2	2
$f'(x)$	10	6	2	-2

11. Selected values of the twice-differentiable function f and its derivative f' are given in the table above. What is the value of $\int_0^6 f'(x)\,dx$?

$$\int_0^6 f'(x)\,dx = f(6) - f(0) = 2 - (-22) = \boxed{24}$$

12. Let f be a continuous function on the closed interval $[0,2]$. If $2 \le f(x) \le 4$, then the greatest possible value of $\int_0^2 f(x)\,dx$ is

$$\int_0^2 2\,dx = 2x\Big|_0^2 = \boxed{8}$$

13. $\int_1^4 |x-3|\,dx =$

   (a) Let $f(x) = |x-3|$

$$f(x) = \begin{cases} 3-x & \text{for } x < 3 \\ x-3 & \text{for } x \ge 3 \end{cases}$$

$$\int_1^4 |x-3|\,dx = \int_1^3 (3-x)\,dx + \int_3^4 (x-3)\,dx$$

$$\int_1^4 |x-3|\,dx = 3x - \frac{x^2}{2}\Big|_1^3 + \frac{x^2}{2} - 3x\Big|_3^4$$

$$\Longrightarrow \Big((9-4.5)-(3-0.5)\Big)+\Big((8-12)-(4.5-9)\Big)$$

$$\Longrightarrow (2) + (0.5) = \boxed{2.5}$$

3.05

1. If $0 \le b \le 2$, for what value of b is $\int_0^b \cos(e^x)\,dx$ a minimum?

$$F(x) = \int_0^x \cos(e^t)\,dt$$

$$F'(x) = \cos(e^x)$$

$$F'(x) < 0 \text{ when } x \in [0.451, 1.550]$$

$$F'(x) > 0 \text{ when } x \in [0, 0.451] \text{ and when } x \in [1.550, 2]$$

$$\boxed{\text{When } x \approx 1.550 \Longrightarrow F(x) \text{ has a local minimum}}$$

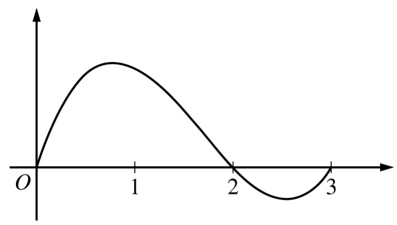
2. Let g be a function with first derivative given by $g'(x) = \int_0^x e^{-t^2}\,dt$. Which of the following must be true on the interval $0 < x < 2$?

   (a) $g'(x) = \int_0^x e^{-t^2}\,dt \Longrightarrow g'(x) > 0$ when $x \in [0, 2] \Longrightarrow g'(x) > 0$ when $x \in [0, 2]$

   (b) $g''(x) = \frac{d}{dx}\left(\int_0^x e^{-t^2}\,dt\right) = e^{-x^2} \Longrightarrow g''(x) > 0$ when $x \in [0, 2]$

   (c) Note that $e^x > 0$ for all $x \in \mathbb{R}$

$$\boxed{g \text{ is increasing, and the graph of } g \text{ is concave up.}}$$



3. Graph of f

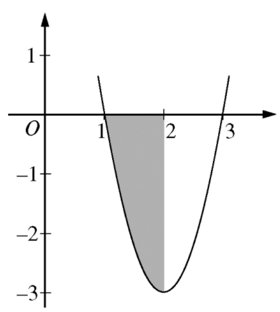
The graph of the differentiable function f is shown in the figure above. Let h be the function defined by $h(x) = \int_0^x f(t)\,dt$. Which of the following correctly orders $h(2)$, $h'(2)$, and $h''(2)$?

   (a) $h(2) = \int_2^x f(t)\,dt > 2$ *Area of the graph*

   (b) $h'(2) = 0$ *Point of the function*

   (c) $h''(2) < 0$ *Slope of the tangent line*

$$\boxed{h''(2) < h'(2) < h(2)}$$



4. Graph of f

The figure above shows the graph of the function f. If $g(x) = \int_1^x f(t)\,dt$ and the shaded region has an area of 2, what is the value of $g(2)$?

$$\boxed{g(2) = \int_1^2 f(t)\,dt = -2}$$

5. If is the function given by $f(x) = \int_4^{2x} \sqrt{t^2 - t}\, dt$, then $f'(2) =$

$$F'(x) = \frac{d}{dx}\left(\int_4^{2x} \sqrt{t^2 - t}\, dt\right) = \frac{d}{dx}\Big(F(2x) - f(4)\Big)$$

$$F'(x) = 2F'(2x) = 2\sqrt{4x^2 - 2x}$$

$$F'(2) = 2\sqrt{4(2)^2 - 2(2)} = \boxed{2\sqrt{12}}$$

6. $\frac{d}{dx}\left(\int_0^{x^2} \sin(t^3)\, dt\right) =$

$$F'(x) = \frac{d}{dx}\left(\int_0^{x^2} \sin(t^3)\, dt\right) = \frac{d}{dx}\Big(F(x^2) - F(4)\Big)$$

$$F'(x) = 2x \cdot F'(x^3) = \boxed{2x \sin(x^6)}$$

7. $\frac{d}{dx}\left(\int_0^{x} \sqrt{1 + t^2}\, dt\right) =$

$$F(x) = \frac{d}{dx}\left(\int_0^{x} \sqrt{1 + t^2}\, dt\right) = \frac{d}{dx}\Big(F(x) - F(0)\Big)$$

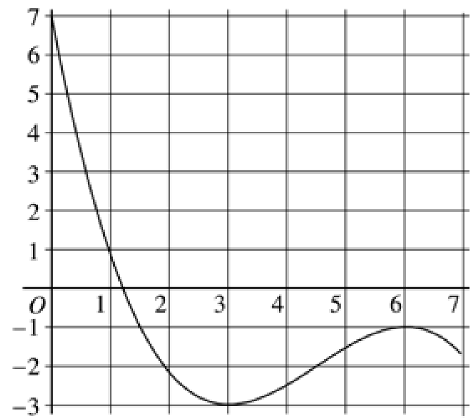
$$F'(x) = \boxed{\sqrt{1 + x^2}}$$

8. If $F(x) = \int_0^{x} \sqrt{t^3 + 1}$, then $F'(2) =$

$$F'(x) = \frac{d}{dx}\left(\int_0^{x} \sqrt{t^3 + 1}\, dt\right) = \frac{d}{dx}\Big(F(x) - F(0)\Big)$$

$$F'(x) = f(x) = \sqrt{x^3 + 1}$$

$$F'(2) = \sqrt{(2)^3 + 1} = \boxed{3}$$



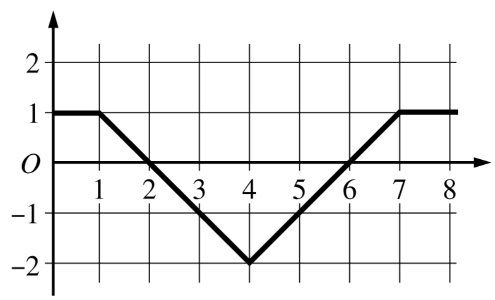
9.

Graph of f

The graph of the function f shown in the figure above has horizontal tangents at $x = 3$ and $x = 6$. If $g(x) = \int_0^{2x} f(t)\,dt$ what is the value of $g'(3)$?

$$g'(x) = \frac{d}{dx}\left(\int_0^{2x} f(t)\,dt\right) = \frac{d}{dx}\Big(g(2x) - g(0)\Big)$$

$$g'(x) = 2 \cdot f(2x) = \boxed{-2}$$



10.

Graph of f

The graph of the function f in the figure above consists of four line segments. Let g be the function defined by $g(x) = \int_0^x f(t)\,dt$. Which of the following is an equation of the line tangent to the graph of g at $x = 5$?

$$g'(x) = \frac{d}{dx}\left(\int_0^{x} f(t)\,dt\right) = \frac{d}{dx}\Big(g(x) - g(0)\Big)$$

$$g'(x) = f(x)$$

(a) $g'(5) = -1$

(b) $g(5) = \int_0^5 f(t)\,dt = 1.5 - 3.5 = -2$

$$y = -1(x-5) - 2 = \boxed{3 - x}$$

3.06

1. Which of the following are equivalent to $\int_2^4 \frac{2x+5}{5-x}\,dx$?

$$\begin{array}{r} -2 \\ -x+5 \enclose{longdiv}{2x \quad +5} \\ -2x+10 \\ \hline 15 \end{array}$$

$$\frac{2x+5}{5-x} = \frac{15}{5-x} - 2$$

$$\implies \int_2^4 \left(\frac{15}{5-x} - 2\right) dx$$

(a) Let $u = 5 - x$

(b) $du = -dx$

$$\implies -\int_3^1 \left(\frac{15}{u} - 2\right) du$$

$$\implies \int_1^3 \left(\frac{15}{u} - 2\right) du = 15\ln(3) - 4$$

$$\boxed{\text{II and III only}}$$

2. Which of the following is equivalent to $\int_3^5 x \ln x\,dx$?

(a) $u = \ln(x) \implies du = \frac{1}{x}\,dx$

(b) $dv = x\,dx \implies v = \frac{x^2}{2}$

(c) $\int u\,dv = uv - \int v\,du$

$$\boxed{\implies \frac{1}{2}x^2\ln(x)\Big|_3^5 - \int_3^5 \frac{x}{2}\,dx}$$

3. Let f be the function defined by $f(x) = \int_0^x (2t^3 - 15t^2 + 36t)\,dt$. On which of the following intervals is the graph of $y = f(x)$ concave down?

(a) $f'(x) = \frac{d}{dx}\left(\int_0^x (2t^3 - 15t^2 + 36t)\,dt\right) = 2x^3 - 15x^2 + 36x$

(b) $f''(x) = 6x^2 - 30x + 36 = 6(x-2)(x-3)$

$$\boxed{\text{When } x \in [2,3]\ f''(x) < 0\ \therefore\ f \text{ is concave down.}}$$

x	-4	-3	-2	-1
$f(x)$	0.75	-1.5	-2.25	-1.5
$f'(x)$	-3	-1.5	0	1.5

4. The table above gives values of a function f and its derivative at selected values of x. If f' is continuous on the interval $[-4,-1]$ what is the value of $\int_{-4}^{-1} f'(x)\,dx$?

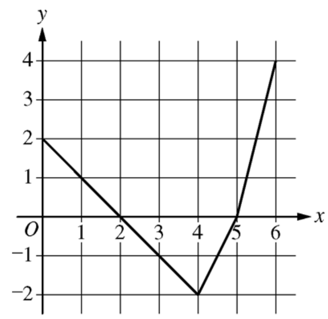
$$\int_{-4}^{-1} f'(x)\,dx = f(-1) - f(-4) = -1.5 - 0.75 = \boxed{-2.25}$$

5. If $f'(x) = \sin\left(\frac{\pi e^x}{2}\right)$ and $f(0) = 1$, then $f(2) =$

$$f(2) = f(0) + \int_0^2 f'(x)\,dx$$

$$f(2) = f(0) + \int_0^2 \sin\left(\frac{\pi e^x}{2}\right)dx$$

$$\boxed{f(2) \approx 1.157}$$

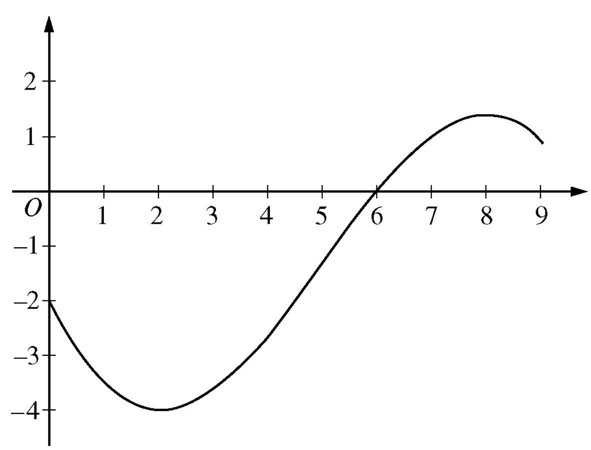
6. 

Graph of f

The graph of the function f, shown above, consists of three line segments. If the function g is an antiderivative of f such that $g(2) = 5$, for how many values of c, where $0 \le c \le 6$, does $g(c) = 3$?

(a) $g(0) = f(2) - \int_0^2 f(t)\,dt = 5 - 2 = 3$
(b) $g(4) = f(2) + \int_2^4 f(t)\,dt = 5 + (-2) = 3$
(c) $g(5) = f(2) + \int_2^5 f(t)\,dt = 5 + (-2) - 1 = 2$
(d) $g(6) = f(2) + \int_2^6 f(t)\,dt = 5 + (-3) + 2 = 4$
(e) Since g is continuous for $[5,6]$ and $3 \in [f(5), f(6)]$ than there is a $c \in [5,6]$ such that $f(c) = 3$

$$\boxed{\text{three}}$$

7. 

Graph of f

The graph of a differentiable function f is shown above. If $h(x) = \int_0^x f(t)\,dt$, which of the following is true?

(a) $h(6) = \int_0^6 f(t)\,dt < 0$

(b) $h'(6) = f(6) = 0$

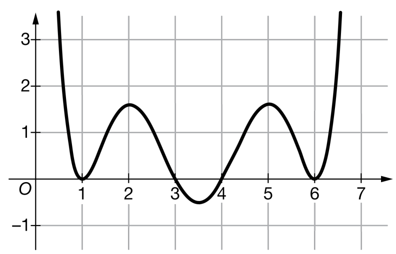
(c) $h''(6) = f'(6) > 0$

$$\boxed{h(6) < h'(6) < h''(6)}$$

8. Let f be the function given by $f(x) = \int_{10}^{x}(-t^2+2t+3)\,dt$. On what intervals is f increasing?

(a) $f'(x) = \frac{d}{dx}\left(\int_{10}^{x}(-t^2+2t+3)\,dt\right) = -x^2+2x+3 = -(x-3)(x+1)$

$$\boxed{\text{When } x \in [-1,3]\ f'(x) > 0 \therefore f \text{ is increasing.}}$$

9. 

Graph of f

The graph of the function f is shown above. Let g be the function defined by $g(x) = \int_1^x f(t)\,dt$. At what values of x in the interval $0.5 < x < 6.5$ does g have a relative maximum?

(a) $g'(x) = f(x)$

(b) Relative maximum: f changes from positive to negative.

$$\boxed{\text{Relative maximum: } x = 3}$$

10. The function h is given by $h(x) = \int_1^x \ln(t\sin t + 5)\,dt$ for $1 \le x \le 7$. On what intervals, if any, is h decreasing?

(a) $h'(x) = \frac{d}{dx}\left(\int_1^x \ln(t\sin t+5)\right) = \ln(x\cdot\sin x + 5)$

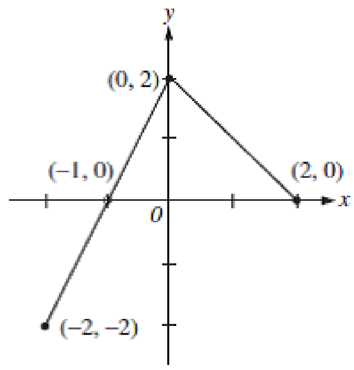
d

$$\boxed{\text{When } x \in [4.323, 5.461]\ h'(x) < 0 \therefore f \text{ is decreasing.}}$$

3.07

1. If F and f are differentiable functions such that $F(x) = \int_0^x f(t)\,dt$, and if $F(a) = -2$ and $F(b) = -2$ where $a < b$, which of the following must be true?

$$\boxed{f(x) = 0 \text{ for some } x \text{ such that } a < x < b.}$$



Graph of f

2.

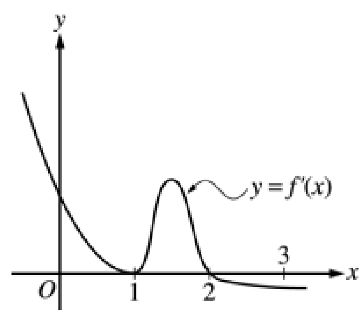
The graph of the function f shown above consists of two line segments. If g is the function defined by $g(x) = \int_0^x f(t)\,dt$, then $g(-1) =$

$$g(-1) = \int_0^{-1} f(t)\,dt = -\int_{-1}^{0} f(t)\,dt = -(0.5)(2)(1) = \boxed{-1}$$

3. The function f is given by $f(x) = \int_1^x \sqrt{t^3+2}\,dt$. What is the average rate of change of f over the interval $[0,3]$?

$$f_{\text{avg}} = \frac{f(3)-f(0)}{3-0} = \frac{\int_1^3 f(x)\,dx - \int_1^0 f(x)\,dx}{3} = \frac{\int_1^3 f(x)\,dx + \int_0^1 f(x)\,dx}{3} = \frac{\int_0^3 f(x)\,dx}{3}$$

$$f_{\text{avg}} = \frac{\int_0^3 f(x)\,dx}{3} \approx \boxed{2.694}$$



4.

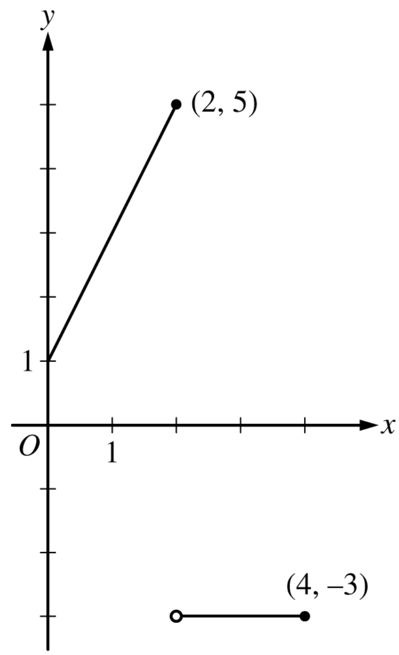
The graph of f', the derivative of the function f, is shown above. If $f(0) = 0$, which of the following must be true?

I. $f(0) > f(1)$: *False since the* $\int_0^1 f'(x)\,dx > 0 \therefore f(1) > f(0)$.

II. $f(2) > f(1)$ *True since the* $\int_1^2 f'(x)\,dx > 0 \therefore f(2) > f(1)$.

III. $f(1) > f(3)$ *False since the* $\int_0^1 f'(x)\,dx < \int_0^3 f'(x)\,dx$

$\boxed{\text{II only}}$



5. Graph of f

The graph of f is shown above for $0 \leq x \leq 4$. What is the value of $\int_0^4 f(x)\,dx$?

$$\int_0^4 f(x)\,dx = \int_0^2 f(x)\,dx + \int_2^4 f(x)\,dx$$

$$\int_0^4 f(x)\,dx = \frac{(1+5)\cdot 2}{2} + (2)(-3) = \boxed{0}$$

6. Let g be the function given by $g(x) = \int_0^x \sin(t^2)\,dt$ for $-1 \leq x \leq 3$. On which of the following intervals is g decreasing?

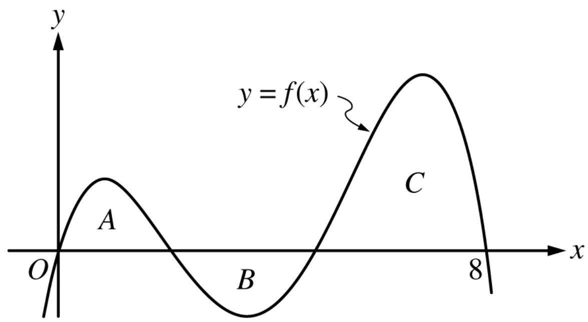
$$g'(x) = \frac{d}{dx}\left(\int_0^x \sin(t^2)\,dt\right) = \sin(x^2)$$

$$\boxed{\text{When } x \in [1.772, 2.507]\ g'(x) < 0\ \therefore\ g \text{ is decreasing.}}$$

7. Let g be the function defined by $g(x) = \int_{-1}^x \frac{t^3-t^2-6t}{\sqrt{t^2+7}}\,dt$ On which of the following intervals is g decreasing?

$$g'(x) = \frac{d}{dx}\left(\int_{-1}^x \frac{t^3-t^2-6t}{\sqrt{t^2+7}}\,dt\right) = \frac{x^3-x^2-6x}{\sqrt{x^2+7}} = \frac{x(x-3)(x+2)}{\sqrt{x^2+7}}$$

$$\boxed{\text{When } x \in [0,3] \text{ and when } x \in (\infty,-2] \implies g'(x) < 0\ \therefore\ g \text{ is decreasing.}}$$

8. 

The regions A, B, and C in the figure above are bounded by the graph of the function f and the x-axis. The area of region A is 14, the area of region B is 16, and the area of region C is 50. What is the average value of f on the interval $[0, 8]$?

$$\int_0^8 f(x)\,dx = 14 - 16 + 50 = 48$$

$$f_{\text{avg}} = \frac{1}{8-0}\int_0^8 f(x)\,dx = \boxed{6}$$

9. $\frac{d}{dx}\left(\int_0^{x^3} \ln(t^2+1)\,dt\right) =$

$$\frac{d}{dx}\Big(F(x^3) - F(0)\Big)$$

$$F'(x) = 3x^2 \cdot F'(x^3) = \boxed{3x^2\big(\ln(x^6+1)\big)}$$

10. If $\int_1^{10} f(x)\,dx = 4$ and $\int_{10}^{3} f(x)\,dx = 7$ then $\int_1^3 f(x)\,dx$

$$\int_1^{10} f(x)\,dx = \int_1^3 f(x)\,dx + \int_3^{10} f(x)\,dx$$

$$\int_1^{10} f(x)\,dx - \int_3^{10} f(x)\,dx = \int_1^3 (x)\,dx$$

$$\int_1^3 (x)\,dx = 4 - (-7) = \boxed{11}$$

3.08

1. Which of the following is a left Riemann sum approximation of $\displaystyle\int_1^7 (4\ln x + 2)\,dx$ with n subintervals of equal length?

   (a) $\Delta x = \dfrac{7-1}{n} = \dfrac{6}{n}$

   (b) $x_k = a + \Delta x \cdot (k-1) = 1 + \dfrac{6(k-1)}{n}$

$$\Longrightarrow \boxed{\lim_{n\to\infty}\sum_{k=1}^{n}\left[4\ln\left(1+\frac{6(k-1)}{n}\right)+2\right]\cdot\frac{6}{n}}$$

2. Which of the following definite integrals are equal to $\lim_{n\to\infty}\sum_{k=1}^{n}\left(-2+\frac{8k}{n}\right)^3\cdot\frac{8}{n}$

   I. $\int_{-2}^{6} x^3\,dx$: True assuming $\Delta x=\frac{8}{n}$ and $x_k=-2+\frac{8k}{n}$

   II. $\int_{0}^{8}(-2+x)^3\,dx$: True assuming $\Delta x=\frac{8}{n}$ and $x_k=\frac{8k}{n}$

   III. $8\int_{0}^{1}(-2+8x)^3\,dx$: True assuming $\Delta x=\frac{1}{n}$ and $x_k=\frac{k}{n}$

$$\boxed{\text{I, II, and III}}$$

3. Which of the following definite integrals are equal to $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{12k}{n}\cos\left(1+\frac{4k}{n}\right)\cdot\frac{4}{n}$

   (a) $\Delta x=\frac{4}{n}$

   (b) $x_k=\frac{4k}{n}$

$$\Longrightarrow \boxed{\int_0^4 3x\cos(1+x)\,dx}$$

4. Which of the following is a left Riemann sum approximation of $\int_2^8\cos(x^2)\,dx$ with n subintervals of equal length?

   (a) $\Delta x=\frac{8-2}{n}=\frac{6}{n}$

   (b) $x_k=a+\Delta x\cdot(k-1)=2+\frac{6(k-1)}{n}$

$$\Longrightarrow \boxed{\lim_{n\to\infty}\sum_{k=1}^{n}\sin\left(2+\frac{6(k-1)}{n}\right)^2\cdot\frac{6}{n}}$$

5. Which of the following definite integrals are equal to $\lim_{n\to\infty}\sum_{k=1}^{n}\sin\left(-1+\frac{5k}{n}\right)\cdot\frac{5}{n}$

   I. $\int_{-1}^{4}\sin x\,dx$: True assuming $\Delta x=\frac{5}{n}$ and $x_k=-1+\frac{5k}{n}$

   II. $\int_{0}^{5}\sin(-1+x)\,dx$: True assuming $\Delta x=\frac{5}{n}$ and $x_k=\frac{5k}{n}$

III. $5\int_0^1 \sin(-1+5x)\,dx$: True assuming $\Delta x = \dfrac{1}{n}$ and $x_k = \dfrac{k}{n}$

$$\boxed{\text{I, II, and III}}$$

6. Which of the following definite integrals are equal to $\lim\limits_{n\to\infty}\sum_{k=1}^{n}\dfrac{10k}{n}\left(\sqrt{1+\dfrac{5k}{n}}\right)\cdot\dfrac{5}{n}$

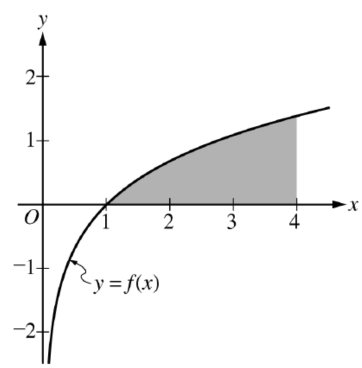
$$\Longrightarrow \boxed{\int_0^5 2x\sqrt{1+x}\,dx}$$

7. Which of the following limits is equal to $\displaystyle\int_2^5 x^2\,dx$

(a) $\Delta x = \dfrac{5-2}{n} = \dfrac{5}{n}$

(b) $x_k = a + \Delta x\cdot k = 2 + \dfrac{5k}{n}$

$$\Longrightarrow \boxed{\lim_{n\to\infty}\sum_{k=1}^{n}\left(2+\frac{3k}{n}\right)^2\cdot\frac{3}{n}}$$



8.

The function f is given by $f(x) = \ln x$. The graph of f is shown above. Which of the following limits is equal to the area of the shaded region?

$$\Longrightarrow \int_1^4 f(x)\,dx = \int_1^4 \ln(x)\,dx$$

(a) $\Delta x = \dfrac{4-2}{n} = \dfrac{3}{n}$

(b) $x_k = a + \Delta x\cdot k = 1 + \dfrac{3k}{n}$

$$\Longrightarrow \boxed{\lim_{n\to\infty}\sum_{k=1}^{n}\ln\left(1+\frac{3k}{n}\right)\cdot\frac{3}{n}}$$