

# Statue in the Headlights

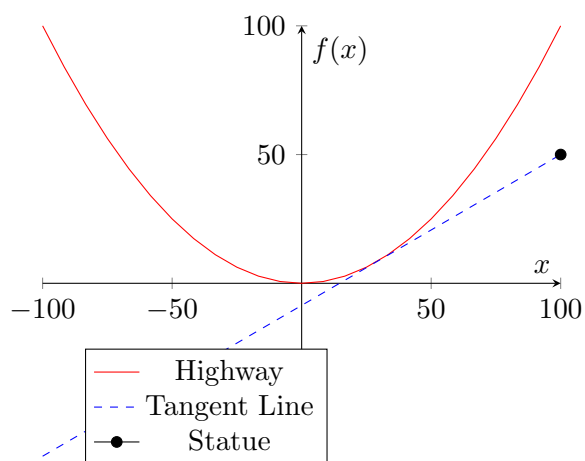
Aiden Rosenberg

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## 1 Discription

A car is traveling at night along a highway shaped like a parabola with its vertex at the origin. The car starts at a point 100 miles west and 100 miles north of the origin and travels in an easterly direction. There is a statue located 100 miles east and 50 miles north. At what point on the highway will the car's headlights illuminate the statue?

### 1.1 Graphical Analysis



## 1.2 Algebraic Solution

$$f(x) = ax^2$$

$$f(-100) = 100 = a(-100)^2 \implies a = \frac{1}{100}f(x) = \frac{x^2}{100}$$

$$f'(x) = \frac{dy}{dx}\left(\frac{x^2}{100}\right) = \frac{x}{50}$$

$$y - 50 = \frac{x}{50}(x - 100)$$

$$y = \frac{x^2}{50} - 2x + 50$$

$$\frac{x^2}{50} - 2x + 50 = \frac{x^2}{100}$$

$$0 = x^2 - 200x + 5000$$

$$x = \frac{200 \pm \sqrt{(200)^2 - 4(5000)}}{2}$$

$$\implies x = -50(\sqrt{2} - 2) \approx 29.28$$

$\implies x = 50(\sqrt{2} + 2) \approx 170.71$  *This solution is only true when the taillights face the point*

$$f(x)_{x=-50(\sqrt{2}-2)} = -50(2\sqrt{2} - 3) \approx 8.57$$

## 1.3 Solution

When the car is approximately 29.28 metres north and 8.57 metres east of the origin the car's headlights will illuminate the statues.