Main Datas

Width of the roof: $l := 13m$ Roof angle: $\alpha := 45deg$ Rafter spacing: $s_r := 1.1m$ Height of wall: $h_w := 1.0m$

Number of rafters: $n := 8$ Timber grade: C24 Collar postion: h_0/h =0,4

Partial factor for timber members: $\gamma_t := 1.3$ Modification factor: $k_{mod} := 0.9$

1. Material properties

1.1 Timber properties

$f_{m.k} := 24\frac{N}{mm^2}$ $f_{t.0.k} := 14\frac{N}{mm^2}$ $f_{t.90.k} := 0.4\frac{N}{mm^2}$ $f_{c.0.k} := 21\frac{N}{mm^2}$ $f_{c.90.k} := 5.3\frac{N}{mm^2}$ $f_{v.k} := 2.5\frac{N}{mm^2}$

$f_{m.d} := \frac{f_{m.k}}{\gamma_t}\cdot k_{mod} = 16.615\frac{N}{mm^2}$ $f_{t.0.d} := \frac{f_{t.0.k}}{\gamma_t}\cdot k_{mod} = 9.692\cdot\frac{N}{mm^2}$ $f_{t.90.d} := \frac{f_{t.90.k}}{\gamma_t}\cdot k_{mod} = 0.277\cdot\frac{N}{mm^2}$

$f_{c.0.d} := \frac{f_{c.0.k}}{\gamma_t}\cdot k_{mod} = 14.538\frac{N}{mm^2}$ $f_{c.90.d} := \frac{f_{c.90.k}}{\gamma_t}\cdot k_{mod} = 3.669\cdot\frac{N}{mm^2}$ $f_{v.d} := \frac{f_{v.k}}{\gamma_t}\cdot k_{mod} = 1.731\cdot\frac{N}{mm^2}$

$E_{0.05} := 7.4\frac{kN}{mm^2}$ $\rho_k := 350\frac{N}{mm^3}$

1.2 Steel properties

Nail yield moment: $f_{u.k} := 340\frac{N}{mm^2}$ Bolt yield moment: $f_{u1.k} := 240\frac{N}{mm^2}$

Partial factor for steel connection: $\gamma_{connection} := 1.1$

2. Loads and effects

2.1 Self weight analysis

rafter section

Layers	Self weight		width	depth	S.W.
	kN/m3	kN/m2	cm	cm	kN/m
Roof tile		0,6	110		0,66
Tile batten	7		110	2,4	0,62
Counter batten	7		4,8	2,4	0,01
Lagging	0,2		110	30	0,07
				Total	1,35

collar section

Layers	Self weight		width	depth	S.W.
	kN/m3	kN/m2	cm	cm	kN/m
Batten	7		110	2,4	0,18
Lagging	0,2		110	30	0,07
				Total	0,25

2.2 Snow load

Ground level: $A := 300$ Characteristic value of snow load on the ground: $s_k := 0.25 \cdot \left(1 + \frac{A}{100}\right) \cdot \frac{kN}{m^2} = 1 \cdot \frac{kN}{m^2}$

Exposure coefficient: $C_e := 1$ Thermal coefficient: $C_t := 1$ Shape coefficient: $\mu_1 := 0.8 \cdot \frac{(60 - \alpha)}{30} = 0.4$

The Characteristic snow load on 1 rafter: $s_d := s_r \cdot \mu_1 \cdot C_e \cdot C_t \cdot s_k = 0.44 \cdot \frac{kN}{m}$

2.3 Wind load

The width of the building (width of roof): $d := 1$ Longitudinal length of the building: $b := n \cdot s_r = 8.8\,m$

Height of the roof: $h := \tan(\alpha) \cdot \frac{1}{2} = 6.5\,m$ Height of the building: $z := h + 3m + h_w = 10.5\,m$

Reference peak pressure: $q_{ref} := 0.25 \frac{kN}{m^2}$ Exposure factor (from diagram): $c_{e.z} := 1.67$

Shape coefficients:

in case of θ=0° $c_{pe.G} := 0.7$ $c_{pe.H} := 0.6$ $c_{pe.I} := -0.2$ $c_{pe.J} := -0.3$

in case of θ=90° $c_{pe.D} := \mathrm{linterp}\left[\begin{pmatrix} 0.25 \\ 1 \end{pmatrix}, \begin{pmatrix} 0.7 \\ 0.8 \end{pmatrix}, \frac{h}{d}\right] = 0.733$ $c_{pe.E} := -\mathrm{linterp}\left[\begin{pmatrix} 0.25 \\ 1 \end{pmatrix}, \begin{pmatrix} 0.3 \\ 0.5 \end{pmatrix}, \frac{h}{d}\right] = -0.367$

Wind load values:

Surface load on the roof	*Uniformly distributed load on the rafter*
$w_{d.G} := q_{ref} \cdot c_{e.z} \cdot c_{pe.G} = 0.292 \cdot \frac{kN}{m^2}$	$w_{d.G} := w_{d.G} \cdot s_r = 0.321 \cdot \frac{kN}{m}$
$w_{d.H} := q_{ref} \cdot c_{e.z} \cdot c_{pe.H} = 0.25 \cdot \frac{kN}{m^2}$	$w_{d.H} := w_{d.H} \cdot s_r = 0.276 \cdot \frac{kN}{m}$
$w_{d.I} := q_{ref} \cdot c_{e.z} \cdot c_{pe.I} = -0.084 \cdot \frac{kN}{m^2}$	$w_{d.I} := w_{d.I} \cdot s_r = -0.092 \cdot \frac{kN}{m}$
$w_{d.J} := q_{ref} \cdot c_{e.z} \cdot c_{pe.J} = -0.125 \cdot \frac{kN}{m^2}$	$w_{d.J} := w_{d.J} \cdot s_r = -0.138 \cdot \frac{kN}{m}$

3. Structural analysis

3.1 Rafter design

Buckling length

Length of the rafter: $s := \frac{\frac{1}{2}}{\cos(\alpha)} = 9.192\,m$ Height of the section above the collar: $h_0 := h \cdot 0.4 = 2.6\,m$

Length of the rafter section above the collar: $s_0 := \frac{h_0}{\sin(\alpha)} = 3.677\,m$ Rest of the rafter length: $s_u := s - s_0 = 5.515\,m$

The buckling length of the rafter: $l_0 := \begin{cases} 0.8 \cdot s & \text{if } s_u < 0.7 \cdot s \\ s & \text{if } s_u \geq 0.7 \cdot s \end{cases} = 7.354\,m$

Cross section properties

Width of the cross section: $b_r := 150mm$ Depth of the cross section: $h_r := 200mm$

Cross section area: $A_r := b_r \cdot h_r = 300 \cdot cm^2$ Cross section inertia: $I_{y.r} := \frac{b_r \cdot h_r^3}{12} = 1 \times 10^4 \cdot cm^4$

Radius of gyration: $i_{y.r} := \sqrt{\frac{I_{y.r}}{A_r}} = 5.774 \cdot cm$ Slenderness: $\lambda_y := \frac{l_0}{i_{y.r}} = 127.373$

Euler critical stress: $\sigma_{c.crit.y} := \pi^2 \cdot \frac{E_{0.05}}{\lambda_y^2} = 4.502 \cdot \frac{N}{mm^2}$ Relative slenderness: $\lambda_{rel.y} := \sqrt{\frac{f_{c.0.k}}{\sigma_{c.crit.y}}} = 2.16$

Structural analysis

Modification factors: $\beta_c := 0.2$ $k_m := 0.7$ $k_y := 0.5 \cdot \left[1 + \beta_c \cdot (\lambda_{rel.y} - 0.5) + \lambda_{rel.y}^2\right] = 2.998$

$$k_{c.y} := \frac{1}{k_y + \sqrt{k_y^2 - \lambda_{rel.y}^2}} = 0.197$$

The internal forces (from AxisVM): $N_{Ed.r} := 28.57kN$ $M_{Ed.r} := 7.42kNm$

The axial and bending stress: $\sigma_{c.0.d} := \frac{N_{Ed.r}}{A_r} = 0.952 \cdot \frac{N}{mm^2}$ $\sigma_{m.y.d} := \frac{M_{Ed.r}}{I_{y.r}} \cdot \frac{h_r}{2} = 7.42 \cdot \frac{N}{mm^2}$

$\boxed{\frac{\sigma_{c.0.d}}{k_{c.y} \cdot f_{c.0.d}} + k_m \cdot \frac{\sigma_{m.y.d}}{f_{m.d}} = 0.645}$ <1 The cross section is **OK**

3.2 Collar design

$b_c := 150mm$ $h_c := 150mm$ $l_{collar} := 2 \cdot \frac{h_0}{\tan(\alpha)} = 5.2\,m$

$A_c := b_c \cdot h_c = 225 \cdot cm^2$ $I_{y.c} := \frac{b_c \cdot h_c^3}{12} = 4.219 \times 10^3 \cdot cm^4$ $i_{y.c} := \sqrt{\frac{I_{y.c}}{A_c}} = 4.33 \cdot cm$ $\lambda_y := \frac{l_{collar}}{i_{y.c}} = 120.089$

$\sigma_{c.crit.y} := \pi^2 \cdot \frac{E_{0.05}}{\lambda_y^2} = 5.064 \cdot \frac{N}{mm^2}$ $\lambda_{rel.y} := \sqrt{\frac{f_{c.0.k}}{\sigma_{c.crit.y}}} = 2.036$ $\beta_c := 0.2$ $k_m := 0.7$

$k_y := 0.5 \cdot \left[1 + \beta_c \cdot (\lambda_{rel.y} - 0.5) + \lambda_{rel.y}^2\right] = 2.727$ $k_{c.y} := \frac{1}{k_y + \sqrt{k_y^2 - \lambda_{rel.y}^2}} = 0.22$

$N_{Ed.c} := 14.82kN$ $M_{Ed.c} := 1.48kNm$

$\sigma_{c.0.d} := \frac{N_{Ed.c}}{A_c} = 0.659 \cdot \frac{N}{mm^2}$ $\sigma_{m.y.d} := \frac{M_{Ed.c}}{I_{y.c}} \cdot \frac{h_c}{2} = 2.631 \cdot \frac{N}{mm^2}$

$\boxed{\frac{\sigma_{c.0.d}}{k_{c.y} \cdot f_{c.0.d}} + \frac{\sigma_{m.y.d}}{f_{m.d}} = 0.364}$ <1 The cross section is **OK**

4. Connection design

4.1 Connection between rafter and wall plate (connection A)

Geometry parameters: $v := \frac{h_r}{4} = 50 \cdot mm$ $c := \frac{v}{\sin(\alpha)} = 70.711 \cdot mm$

Bearing resistance: $\boxed{R_{d.y} := c \cdot b_r \cdot f_{c.90.d} = 38.918 \cdot kN}$ > $E_{d.y} := 24.12kN$ The cross section is **OK**

Angle steel analysis

Screw diameter: $d_{as} := 10mm$ Number of screws: $n_{screw} := 7$ Screw length in timber member: $t_2 := 8cm$

Thickness of angle steel: $t_1 := 2.5mm$ Area of angle steel: $A_{as} := 2.8973cm^2$

Effective area of angle steel: $A_{eff} := 2A_{as} - 2 \cdot d_{as} \cdot t_1 = 5.295 \cdot cm^2$

Resistance of angle steel: $R_{anglesteel} := \frac{f_{u.k}}{\gamma_{connection}} \cdot A_{eff} = 163.651 \cdot kN$

Resistance of bearing: $R_{bearing} := n_{screw} \cdot t_1 \cdot d_{as} \cdot \frac{f_{u.k}}{\gamma_{connection}} = 54.091 \cdot kN$

Effective bolt diameter: $d_{eff} := 0.9 \cdot d_{as} = 9 \cdot mm$

Bolt yield moment: $M_{y.k} := 0.3 \cdot f_{u1.k} \cdot d_{eff}^{2.6} = 2.18 \times 10^4$ Nmm $M_{y.d} := \frac{M_{y.k}}{\gamma_{connection}} \cdot N \cdot mm = 0.02 \cdot kNm$

Characteristic embedment strength parallel to grain: $f_{h.0.k} := 0.082(1mm - 0.01 \cdot d_{as}) \cdot \rho_k = 25.83 \cdot \frac{N}{mm^2}$

Modification factor: $k_{90} := 1.35 + 0.015 \cdot d_{as} = 1.5$

Characteristic embedment strength in roof angle: $f_{h.\alpha.k} := \frac{f_{h.0.k}}{k_{90} \cdot \sin(\alpha)^2 + \cos(\alpha)^2} = 20.664 \cdot \frac{N}{mm^2}$

Design value of embedment strength in roof angle: $f_{h.\alpha.d} := \frac{f_{h.\alpha.k}}{\gamma_t} \cdot k_{mod} = 14.306 \cdot \frac{N}{mm^2}$

Resistance of 1 bolt: $R_d := \min\left[\left(\sqrt{2} - 1\right) \cdot f_{h.\alpha.d} \cdot t_2 \cdot d_{as}, 1.1 \cdot \sqrt{2 \cdot M_{y.d} \cdot f_{h.\alpha.d} \cdot d_{as}}\right] = 2.619 \cdot kN$

Resistance of the connection: $\boxed{R_{d.x} := \min\left(R_{anglesteel}, R_{bearing}, R_d \cdot n_{screw}\right) = 18.334 \cdot kN}$ > $E_{d.x} := 17.92kN$

The connection is **OK**

4.2 Connection between rafter and collar (connection B)

4.2.1 Connection resistance by bolts

$t_1 := 50mm$ $t_2 := 250mm$ $d_{bolt} := 14mm$ $\alpha_1 := \operatorname{atan}\left(\frac{V_{Ed.c}}{N_{Ed.c}}\right) = 4.399 \cdot deg$ $\alpha_2 := \alpha - \alpha_1 = 40.601 \cdot deg$

$f_{h.\alpha1.k} := \frac{f_{h.0.k}}{k_{90} \cdot \sin(\alpha_1)^2 + \cos(\alpha_1)^2} = 25.754 \cdot \frac{N}{mm^2}$ $M_{y.Rk} := 0.45 \cdot f_{u.k} \cdot d_{bolt}^{2.6} = 1.461 \times 10^5$ Nmm

$f_{h.\alpha2.k} := \frac{f_{h.0.k}}{k_{90} \cdot \sin(\alpha_2)^2 + \cos(\alpha_2)^2} = 21.316 \cdot \frac{N}{mm^2}$ $\beta := \frac{f_{h.\alpha1.k}}{f_{h.\alpha2.k}} = 1.208$

Failure mode resistances

Failure mode g) $F_{v.bolt.g} := f_{h.\alpha1.k} \cdot t_1 \cdot d_{bolt} = 1.803 \times 10^4$

Failure mode h) $F_{v.bolt.h} := 0.5 \cdot f_{h.\alpha1.k} \cdot t_2 \cdot d_{bolt} \cdot \beta = 5.445 \times 10^4$

Failure mode j) $F_{v.bolt.j} := 1.1 \cdot \frac{f_{h.\alpha1.k} \cdot t_1 \cdot d_{bolt}}{2 + \beta} \left[\sqrt{2 \cdot \beta \cdot (1 + \beta) + \frac{4 \cdot \beta \cdot \left(2 + \beta \cdot M_{y.Rk}\right)}{f_{h.\alpha1.k} \cdot d_{bolt} \cdot t_1^2}} \right] = 1.549 \times 10^4$

Failure mode k) $F_{v.bolt.k} := 1.1 \cdot \sqrt{\frac{2 \cdot \beta}{1 + \beta}} \cdot \sqrt{2 \cdot M_{y.Rk} \cdot f_{h.\alpha1.k} \cdot d_{bolt}} = 1.181 \times 10^4$

$$F_{v.bolt} := \frac{\min\left(F_{v.bolt.g}, F_{v.bolt.h}, F_{v.bolt.j}, F_{v.bolt.k}\right) \cdot N}{\gamma_t} \cdot k_{mod} = 8.177 \times 10^3 \, N \qquad E_{bolt} := \sqrt{N_{Ed.c}^2 + V_{Ed.c}^2} = 1.486 \times 10^4 \cdot N$$

The resistance of the connection (double shear): $\boxed{R_{bolt} := 2 \cdot F_{v.bolt} = 16.353 \cdot kN} > E_{bolt} = 14.864 \cdot kN$

The connection is **OK**

4.2.2 Connection resistance by nails

$t_1 := 50mm \quad t_2 := 130mm \quad d_{nail} := 4.6mm \quad \alpha_1 := \operatorname{atan}\left(\frac{V_{Ed.c}}{N_{Ed.c}}\right) = 4.399 \cdot deg \quad \alpha_2 := \alpha - \alpha_1 = 40.601 \cdot deg \quad n_{nail} := 6$

$$f_{h.\alpha1.k} := \frac{f_{h.0.k}}{k_{90} \cdot \sin(\alpha_1)^2 + \cos(\alpha_1)^2} = 25.754 \cdot \frac{N}{mm^2} \qquad M_{y.k} := \frac{270 d_{nail}^{2.6} \cdot N \cdot mm}{\gamma_{connection}} = 0.013 \cdot kNm$$

$$f_{h.\alpha2.k} := \frac{f_{h.0.k}}{k_{90} \cdot \sin(\alpha_2)^2 + \cos(\alpha_2)^2} = 21.316 \cdot \frac{N}{mm^2} \qquad \beta := \frac{f_{h.\alpha1.k}}{f_{h.\alpha2.k}} = 1.208$$

Failure mode resistances:

Failure mode a) $F_{v.nail.a} := f_{h.\alpha1.k} \cdot t_1 \cdot d_{nail} = 5.923 \cdot kN$

Failure mode b) $F_{v.nail.b} := f_{h.\alpha1.k} \cdot t_2 \cdot d_{nail} = 15.401 \cdot kN$

Failure mode c) $F_{v.nail.c} := \frac{f_{h.\alpha1.k} \cdot t_1 \cdot d_{nail}}{1 + \beta} \cdot \left[\sqrt{\beta + 2 \cdot \beta^2 \cdot \left[1 + \frac{t_2}{t_1} + \left(\frac{t_2}{t_1}\right)^2\right] + \beta^3 \cdot \left(\frac{t_2}{t_1}\right)^2} - \left(1 + \frac{t_2}{t_1}\right) \right] = 12.179 \cdot kN$

Failure mode d) $F_{v.nail.d} := 1.1 \cdot \frac{f_{h.\alpha1.k} \cdot t_1 \cdot d_{nail}}{2 + \beta} \cdot \left[\sqrt{2 \cdot \beta \cdot (1 + \beta) + \frac{4 \cdot \beta \cdot (2 + \beta) \cdot M_{y.k}}{f_{h.\alpha1.k} \cdot d_{nail} \cdot t_1^2}} - \beta \right] = 2.527 \cdot kN$

Failure mode e) $F_{v.nail.e} := 1.1 \cdot \frac{f_{h.\alpha1.k} \cdot t_2 \cdot d_{nail}}{1 + 2\beta} \cdot \left[\sqrt{2 \cdot \beta^2 \cdot (1 + \beta) + \frac{4 \cdot \beta \cdot (1 + 2\beta) \cdot M_{y.k}}{f_{h.\alpha1.k} \cdot d_{nail} \cdot t_2^2}} - \beta \right] = 6.704 \cdot kN$

Failure mode f) $F_{v.nail.f} := 1.1 \sqrt{\frac{2 \cdot \beta}{1 + \beta}} \cdot \sqrt{2 \cdot M_{y.k} \cdot f_{h.\alpha1.k} \cdot d_{nail}} = 2.018 \cdot kN$

$$F_{v.nail} := \frac{\min\left(F_{v.nail.a}, F_{v.nail.b}, F_{v.nail.c}, F_{v.nail.d}, F_{v.nail.e}, F_{v.nail.f}\right)}{\gamma_t} \cdot k_{mod} = 1.397 \cdot kN$$

Resistance of the connection: $\boxed{R_{nail} := 2 \cdot n_{nail} \cdot F_{v.nail} = 16.762 \cdot kN} > E_{nail} = 14.864 \cdot kN$ The connection is **OK**

4.3 Connection between the rafters (connection C)

Same method like in the 4.2 section. The connection has to be made by nails.
In this case the connection is in the same plane. The embedment strengths are the the same ($f_{h.\alpha 1.k}$ and $f_{h.\alpha 2.k}$).
The position of the connection C should be near to the zero point of the bending moment diagram, but not the same position like the Connection B. The internal forces of the position C (N and M) should be obtained from the AxisVM.