EC2 SIMPLE SUPPORTED RECTANGULAR BEAM DESIGN

Beam design

Design main reinforcement

Adjacent slab deep: $t = 160 \ \boldsymbol{mm}$

Beam dimension: $b = 230 \ mm$

> $h = 350 \ mm$ $l \coloneqq 6300 \ mm$

Beam

 $f_{ck} = 20 \frac{N}{mm^2}$ $\gamma_c = 1.5$ Concrete class: C20/25

 $f_{yk} = 500 \frac{N}{mm^2} \qquad \gamma_s = 1.15$ Steel class: B500

 $a_1 := 300 \ mm$ $a_2 := 300 \ mm$ Length of supports:

 $l_{eff} = l + min\left(\frac{a_1 + a_2}{2}, h\right) = \left(6.6 \cdot 10^3\right) \$ mm Effective length:

Load and effects

Selfweight of beam: $SW_b = h \cdot b \cdot 25 \frac{kN}{m^3} = 2.013 \frac{kN}{m}$

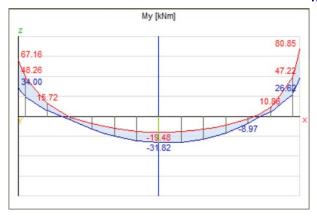
Selfweight of covering: $SW_c = 1.5 \frac{kN}{m}$

 $\begin{array}{ll} g_k \!\coloneqq\! 3.513 \, \frac{\textbf{k} N}{\textbf{m}} & \gamma_g \!\coloneqq\! 1.35 & g_d \!\coloneqq\! \gamma_g \!\cdot\! g_k \!=\! 4.743 \, \frac{\textbf{k} N}{\textbf{m}} \\ q_k \!\coloneqq\! 2 \, \frac{\textbf{k} N}{\textbf{m}} & \gamma_q \!\coloneqq\! 1.5 & q_{d\triangle} \!\coloneqq\! \gamma_q \!\cdot\! q_k \!=\! 3 \, \frac{\textbf{k} N}{\textbf{m}} \end{array}$ Permanent load:

Live load:

 $p_{ed} = g_d + q_d = 7.743 \frac{kN}{m}$ Load combination for ULS analysis:

 $p_{qp} \coloneqq g_k + \psi_2 \cdot q_k = 4.113 \frac{kN}{m}$ Load combination for SLS analysis:



Design reinforcement at mid-span

Bending moment:
$$M_{ed} = 31.82 \ kN \cdot m$$

Concrete cover:
$$Cc = 25 \ mm$$
 $\eta = 10 \ mm$

Assume effective depth:
$$d = min(0.9 \cdot h, h - 50 \text{ mm}) = 300 \text{ mm}$$

$$d' = 50 \ mm$$

Design strength:
$$f_{cd} \coloneqq \frac{f_{ck}}{\gamma_c} = 13.333 \frac{N}{mm^2}$$
 $f_{yd} \coloneqq \frac{f_{yk}}{\gamma_s} = 434.783 \frac{N}{mm^2}$

Compatibility for plastic analysis

Compressed part:
$$\xi'_{co} := \frac{560}{700 - f_{vol}} = 2.111$$

Tensield part:
$$\xi_{co} \coloneqq \frac{560}{700 + f_{vd}} = 0.493 \qquad x_{co} \coloneqq \xi_{co} \cdot d = 148.046 \ \textit{mm} \qquad \alpha \coloneqq 1$$

Optimal bending moment:
$$M_u := x_{co} \cdot b \cdot \alpha \cdot f_{cd} \cdot \left(d - \frac{x_{co}}{2}\right) = 102.595 \ \textit{kN} \cdot \textit{m}$$

Because of Mu greater than Med, we don't need compression steel.

$$M_{ed} = x_c \cdot b \cdot \alpha \cdot f_{cd} \cdot \left(d - \frac{x_c}{2}\right) \qquad x_c \coloneqq d - \sqrt{d^2 - \frac{2 \cdot M_{ed}}{b \cdot f_{cd}}} = 36.85 \ \textit{mm}$$

The requirment steel area

Mainbars:
$$\phi = 12 \text{ mm}$$
 Stirrups: $\phi_{sw} = 6 \text{ mm}$

$$A_{sreq} \coloneqq \frac{x_c \cdot b \cdot \alpha \cdot f_{cd}}{f_{ud}} = 259.916 \ \textit{mm}^2 \quad A_{rebars} \coloneqq \frac{\pi \cdot \phi^2}{4} = 113.097 \ \textit{mm}^2 \quad \frac{A_{sreq}}{A_{rebars}} = 2.298 \ \text{mm}^2 \quad \frac{A_{sreq}}{A_{rebars}} = 2.298 \ \text{mm$$

Provide number of rebars: n = 3

Provide area of reinforcement: $A_s := A_{rehars} \cdot n = 339.292 \text{ mm}^2$ (At bottom)

Provide depth:
$$d_{real} = h - Cc - \phi_{sw} - \frac{\phi}{2} - \eta = 303 \ \textit{mm}$$

The real neutral axis:
$$x_{creal} := \frac{A_s \cdot f_{yd}}{b \cdot \alpha \cdot f_{ed}} = 48.104 \ mm$$

$$\frac{x_{creal}}{d_{real}}$$
 = 0.159 Smaller than ξ_{co} := 0.493

 $f_{ctm} = 0.3 \cdot \sqrt[3]{f_{ck}^2} = 2.21$

$$\xi_{co} = 0.493$$

Standard regulation:

$$\begin{split} & \rho_{sl} \coloneqq & \frac{A_s}{b \cdot d_{real}} = 0.005 \\ & \rho_{s1min} \coloneqq \max \left(0.13\% \,, 0.26 \cdot \frac{f_{ctm}}{f_{yk}} \right) = 0.001 \end{split}$$

 $\rho_{slmin} < \rho_{s1} < \rho_{slmax}$

Space between bars

 $\rho_{slmax} := 4\%$

$$\zeta \coloneqq \max(\phi, 20 \ mm) = 20 \ mm$$

$$b_{min} \coloneqq 2 \cdot \left(Cc + \phi_{sw} \right) + n \cdot \phi + \left(n - 1 \right) \cdot \zeta = 138 \ \textbf{mm} \qquad b > b_{min}$$

$$M_{rd}\!:=\!x_{creal}\boldsymbol{\cdot} b\boldsymbol{\cdot} \alpha\boldsymbol{\cdot} f_{cd}\boldsymbol{\cdot} \left(d-\frac{x_{creal}}{2}\right)\!=\!40.707~\textbf{kN}\boldsymbol{\cdot} \textbf{m} \qquad \qquad M_{rd}\!>\!M_{ed}$$

 $\frac{M_{ed}}{M_{ed}} = 0.782$ Performance:

In the moment disgram, we found that at the support where has a huge moment. In this case, we will also design the reinforcement at support. Because the compressed and tensiled zone will opposite to the zone location at midspan, the tensiled reinforcement will be set in the top.

Concrete class:

C20/25 $f_{ck} = 20 \frac{N}{mm^2} \qquad \gamma_c = 1.5$

Steel class:

B500

 $f_{yk} = 500 \frac{N}{mm^2}$ $\gamma_s = 1.15$

Design reinforcement at support

Bending moment:

 $M_{ed} = 80.80 \ kN \cdot m$

Concrete cover:

 $Cc = 25 \, mm$

 $\eta \coloneqq 10 \ \mathbf{mm}$

Assume effective depth:

 $d = min(0.9 \cdot h, h - 50 \ mm) = 300 \ mm$

 $d' = 50 \ mm$

Compatibility for plastic analysis

Compressed part:
$$\xi'_{co} := \frac{560}{700 - f_{yd}} = 2.111$$

Tensield part:
$$\xi_{co} \coloneqq \frac{560}{700 + f_{vd}} = 0.493$$
 $x_{co} \coloneqq \xi_{co} \cdot d = 148.046 \ \textit{mm}$ $\alpha \coloneqq 1$

$$\text{Optimal bending moment:} \quad M_u \coloneqq x_{co} \boldsymbol{\cdot} b \boldsymbol{\cdot} \alpha \boldsymbol{\cdot} f_{cd} \boldsymbol{\cdot} \left(d - \frac{x_{co}}{2} \right) = 102.595 \ \textit{kN} \boldsymbol{\cdot} \textit{m}$$

Because of Mu greater than Med, we don't need compression steel.

$$M_{ed} = x_c \cdot b \cdot \alpha \cdot f_{cd} \cdot \left(d - \frac{x_c}{2}\right) \qquad x_c \coloneqq d - \sqrt{d^2 - \frac{2 \cdot M_{ed}}{b \cdot f_{cd}}} = 106.857 \ \textit{mm}$$

The requirment steel area

Mainbars:
$$\phi = 12 \ \textit{mm}$$
 Stirrups: $\phi_{sw} = 6 \ \textit{mm}$

$$A_{sreq} := \frac{x_c \cdot b \cdot \alpha \cdot f_{cd}}{f_{vd}} = 753.695 \ \textit{mm}^2 \quad A_{rebars} := \frac{\pi \cdot \phi^2}{4} = 113.097 \ \textit{mm}^2 \quad \frac{A_{sreq}}{A_{rebars}} = 6.664 \ \text{m}^2$$

Provide number of rebars: n = 7

Provide area of reinforcement: $A_s := A_{rebars} \cdot n = 791.681 \text{ mm}^2$ (At the top)

Provide depth: $d_{real} \coloneqq h - Cc - \phi_{sw} - \frac{\phi}{2} - \eta = 303 \ \textit{mm}$

The real neutral axis: $x_{creal} := \frac{A_s \cdot f_{yd}}{b \cdot \alpha \cdot f_{cd}} = 112.242 \ \textit{mm}$

Plasticity check: $\frac{x_{creal}}{d_{real}} = 0.37$ Smaller than $\xi_{co} = 0.493$

Standard regulation:

$$\begin{split} \rho_{sl} \coloneqq & \frac{A_s}{b \cdot d_{real}} = 0.011 & f_{ctm} \coloneqq 0.3 \cdot \sqrt[3]{f_{ck}}^2 = 2.21 \\ \rho_{s1min} \coloneqq & \max \left(0.13\%, 0.26 \cdot \frac{f_{ctm}}{f_{yk}} \right) = 0.001 \\ \rho_{slmax} \coloneqq & 4\% & \rho_{slmin} < \rho_{s1} < \rho_{slmax} \end{split}$$

Space between bars

$$\zeta \coloneqq \max(\phi, 20 \ mm) = 20 \ mm$$

$$b_{min} \coloneqq 2 \cdot \left(Cc + \phi_{sw} \right) + n \cdot \phi + \left(n - 1 \right) \cdot \zeta = 266 \ \textbf{mm} \qquad b < b_{min}$$

We should arrange the steel rebars in 2 rows. Number at first row is 4, at second row is 3

$$M_{rd}\!\coloneqq\!x_{creal}\!\cdot\!b\cdot\!\alpha\cdot\!f_{cd}\!\cdot\!\left(d-\frac{x_{creal}}{2}\right)\!=\!83.945~\textbf{kN}\cdot\textbf{m} \qquad \qquad M_{rd}\!>\!M_{ed}$$

Performance:
$$\frac{M_{ed}}{M_{rd}} = 0.963$$

Shear design

$$V_{ed} = 50.09 \text{ kN}$$

Then we will figure out the VRdc d = 303

Size factor:
$$K = min\left(1 + \sqrt[1]{\frac{200}{d}}, 2.0\right) = 1.66$$

Smallest width: $b_w = 230 \ mm$

$$\begin{split} & \text{Reinforcement ratio:} \qquad \rho_{\rho} \coloneqq min \bigg(\frac{A_s}{b_w \cdot d}, 0.02 \bigg) = 0.011 \\ & \nu_{min} \coloneqq 0.035 \cdot K^{\frac{3}{2}} \cdot f_{ck}^{-\frac{1}{2}} = 0.31 \\ & V_{Rdc} \coloneqq \max \left(\frac{0.18}{\gamma_c} \cdot K \cdot \left(100 \cdot \rho_{\rho} \cdot f_{ck} \right)^{\frac{1}{3}} \cdot b_w \cdot d, \nu_{min} \right) = 3.684 \cdot 10^4 \end{split}$$

$$V_{Rdc} \coloneqq \max \left(\frac{0.18}{\gamma_c} \cdot K \cdot \left(100 \cdot \rho_\rho \cdot f_{ck} \right)^{\frac{1}{3}} \cdot b_w \cdot d, \nu_{min} \right) = 3.684 \cdot 10^4 \qquad N$$

$$V_{Rdc} = 27.77 \text{ kN}$$

$$V_{Rdc} \! < \! V_{ed}$$
 Shear reinforcement is necessary

Then we will figure out the VRdcmax

 $Z = 0.9 \cdot d = 272.7 \ mm$ Distance between Fc and Fs

Performance factor of the concrete
$$\nu \coloneqq 0.6 \cdot \left(1 - \frac{f_{ck}}{250}\right) = 0.552$$

$$V_{Rdmax} := \alpha_{cw} \cdot b_w \cdot Z \cdot \nu \cdot f_{cd} \cdot 0.5 \ mm = 230.813 \ kN$$

The geometry is suitable for shear reinforcement $V_{Rdmax} > V_{ed}$

Design of shear links Assume $\phi_{sw} = 6 \ mm$

$$A_{sw} = 2 \cdot \frac{\pi \cdot \phi_{sw}^2}{4} = 56.549 \ mm^2$$

Design the space of shear links

$$S_{req} \coloneqq \frac{A_{sw} \boldsymbol{\cdot} f_{yd}}{V_{ed}} \boldsymbol{\cdot} Z \boldsymbol{\cdot} 1.0 = 133.853 \ \boldsymbol{mm}$$

$$S_{max1} = 0.75 \cdot d = 227.25 \ mm$$

$$b_w = 230 \ mm - 2 \cdot 25 \ mm = 180 \ mm$$

Minimun shear reinforcement ρ_w

$$\rho_w \coloneqq \frac{A_{sw}}{S_{max1} \cdot b_w} = 0.001$$

$$f_{ck} = 20$$

$$f_{uk} = 500$$

$$\rho_{wmin} = 0.08 \cdot \frac{\left(\sqrt{f_{ck}}\right)}{f_{yk}} = 7.155 \cdot 10^{-4}$$

$$\alpha_c = 1.0$$

$$\rho_{wmax} \coloneqq 0.5 \cdot \frac{\alpha_c \cdot \nu \cdot f_{cd}}{\left(1 - \cos\left(0.5 \ \pi\right)\right) \cdot f_{yd}} = 0.008$$

$$S_{min} \coloneqq \frac{A_{sw}}{\rho_{wmax} \cdot b_w} = 37.117 \ \textit{mm} \qquad \qquad S_{max2} \coloneqq \frac{A_{sw}}{\rho_{wmin} \cdot b_w} = 439.051 \ \textit{mm}$$

$$S_{max2} := \frac{A_{sw}}{\rho_{max0} \cdot b_{m}} = 439.051 \ mm$$

$$S_{max} := min(S_{max1}, S_{max2}) = 227.25$$
 mm

Stirrups space in the middle span

$$S_{provide} = 220 \ mm$$

$$S_{provide} > S_{min}$$

Stirrups space at the support

$$S_{provide} := 10 \cdot \left(\left(\frac{S_{req}}{10} \right) - 0.5 \ \textit{mm} \right) = 128.853 \ \textit{mm}$$

$$S_{provide} = 120 \ mm$$

SLS analysis at middle span

$$M_{qp} \coloneqq 21.21 \ \mathbf{kN \cdot m}$$

Uncrack cross section properties SS1

$$h := 350 \ mm$$
 $b := 230 \ mm$ $d := 303 \ mm$ $A_s := 339.292 \ mm^2$

$$E_s$$
 effective modolus for steel which is 200

$$E_s \coloneqq 200 \; \frac{\textit{kN}}{\textit{mm}^2}$$

$$E_c \coloneqq 30 \; \frac{\textit{kN}}{\textit{mm}^2}$$

effective modolus for concrete C20/25 which is 30

$$E_c = 30 \; \frac{\mathbf{kN}}{\mathbf{mm}^2}$$

$$\alpha_e \coloneqq \frac{E_s}{E_{ceff}} = 22.54$$

$$f_{ctm} \coloneqq 2.2 \, rac{N}{mm^2}$$

Homogenious concrete area

$$A_{i1} := h \cdot b + (\alpha_e - 1) \cdot A_s = (8.781 \cdot 10^4) \ mm^2$$

Neutral Axis

$$x_{i1} \coloneqq \frac{h \cdot b \cdot \frac{h}{2} + \left(\alpha_e - 1\right) \cdot \left(A_s \cdot d\right)}{A_{i1}} = 185.653 \ \textit{mm}$$

Inertia

$$I_{i1} \coloneqq \frac{b \cdot {x_{i1}}^3}{3} + \frac{b \cdot \left(h - x_{i1}\right)^3}{3} + \left(\alpha_e - 1\right) \cdot \left(A_s \cdot \left(d - x_{i1}\right)^2\right) = \left(9.315 \cdot 10^8\right) \ \textit{mm}^4$$

Crack moment

$$M_{cr}\!\coloneqq\!\frac{f_{ctm}\!\cdot\!I_{i1}}{h\!-\!x_{i1}}\!=\!12.47~\textit{kN}\cdot\!\textit{m} \qquad \qquad \frac{M_{cr}}{M_{qp}}\!=\!0.588 \qquad \text{Cracked section}$$

Geometrical properties for craked analysis SS2

Neutral Axis

$$x_{i2} \coloneqq x_{i2} = \frac{\frac{1}{2} \cdot x_{i2}^{2} \cdot b + \alpha_{e} \cdot A_{s} \cdot d}{x_{i2} \cdot b + \alpha_{e} \cdot A_{s}} \xrightarrow{solve, x_{i2}} \begin{bmatrix} 112.54135350539928277 \cdot mm \\ -179.04164888152074089 \cdot mm \end{bmatrix} = \begin{bmatrix} 112.541 \\ -179.042 \end{bmatrix} mm$$

 $x_{i2} = 112.541 \ mm$

Inertia

$$I_{i2} := \frac{b \cdot x_{i2}^{3}}{3} + \alpha_{e} \cdot A_{s} \cdot (d - x_{i2})^{2} = (3.867 \cdot 10^{8}) \ \textit{mm}^{4}$$

Significant stress value

$$\text{Concrete stress} \qquad \sigma_c\!\coloneqq\!\frac{M_{qp}}{I_{i2}}\!\cdot\!x_{i2}\!=\!6.173\;\frac{\textit{N}}{\textit{mm}^2}$$

Steel stress
$$\sigma_s \coloneqq \alpha_e \cdot \frac{M_{qp}}{I_{i2}} \cdot \left(d - x_{i2}\right) = 235.465 \; \frac{N}{mm^2}$$

Check for the deformation

According to uncracked analysis:

$$k_1 \coloneqq \frac{M_{qp}}{E_{ceff} \cdot I_{i1}} = \left(2.566 \cdot 10^{-6}\right) \frac{1}{\textit{mm}} \qquad e_1 \coloneqq \frac{5}{384} \cdot \frac{p_{qp} \cdot l_{eff}^{-4}}{E_{ceff} \cdot I_{i1}} = 12.294 \; \textit{mm}$$

According to cracked analysis:

$$k_2 \coloneqq \frac{M_{qp}}{E_{ceff} \cdot I_{i2}} = \left(6.182 \cdot 10^{-6}\right) \frac{1}{\textit{mm}} \qquad e_2 \coloneqq \frac{5}{384} \cdot \frac{p_{qp} \cdot l_{eff}^{}}{E_{ceff} \cdot I_{i2}} = 29.616 \; \textit{mm}$$

Combination factor:

$$\zeta'\!\coloneqq\!1-0.5 \bullet\! \left(\! \frac{M_{cr}}{M_{qp}}\!\right)^2 =\! 0.827$$

Calculated deformations:

$$k_{EC} := \zeta' \cdot k_2 + (1 - \zeta') \cdot k_1 = (5.557 \cdot 10^{-6}) \frac{1}{mm}$$

$$e_{EC} := \zeta' \cdot e_2 + (1 - \zeta') \cdot e_1 = 26.622 \ mm$$

Verification:

$$e_{max} \coloneqq \frac{l_{eff}}{200} = 33 \, \, mm \qquad \qquad e_{max} > e_{EC} \qquad \qquad Ok$$

Crack width analysis:

Steel strain: $\varepsilon_s = \frac{\sigma_s}{E_s} = 0.001$

 $\text{Concrete strain:} \qquad \varepsilon_c \coloneqq \frac{\overset{E_s}{f_{ctm}}}{\overset{E_{ceff}}{E_{ceff}}} = 2.479 \cdot 10^{-4}$

Tension stiffening of concrete:

Effective concrete area for tension stiffening

$$\begin{split} &h_{c.eff}\coloneqq min\left(2.5\boldsymbol{\cdot}\left(h-d\right),\frac{\left(h-x_{i2}\right)}{3},\frac{h}{2}\right) = 79.153~\textit{mm} \\ &A_{ceff}\coloneqq h_{c.eff}\boldsymbol{\cdot}b = \left(1.821\boldsymbol{\cdot}10^4\right)~\textit{mm}^2 \qquad \qquad \varepsilon_{sc}\coloneqq \frac{f_{ctm}\boldsymbol{\cdot}A_{ceff}}{E_s\boldsymbol{\cdot}A_s} = 5.902\boldsymbol{\cdot}10^{-4} \\ &\rho_{\rho.eff}\coloneqq \frac{A_s}{A_{ceff}} = 0.019 \end{split}$$

Durability of loads:

Long term load: $k_t = 0.4$

Difference of strain: $\Delta \varepsilon := \varepsilon_s - k_t \cdot \varepsilon_{sc} - k_t \cdot \varepsilon_c = 8.421 \cdot 10^{-4}$

Maximal distance between two neighbouring cracks:

where:

 $\begin{array}{lll} c\coloneqq 25 \ \textit{mm} & \text{Conncrete cover} \\ k_1\coloneqq 0.8 & \text{For ribbed bars} \\ k_2\coloneqq 0.5 & \text{For flexure} \\ \phi\coloneqq 12 \ \textit{mm} & \text{Bar diameter} \end{array}$

$$S_{r.max} = 3.4 c + \frac{0.425 k_1 \cdot k_2 \cdot \phi}{\rho_{o.eff}} = 194.459$$
 mm

Crack width:

$$w_k \coloneqq S_{r.max} \cdot \triangle \varepsilon = 0.164 \ mm$$

$$w_{k,max} = 0.3 \ mm$$

$$w_{k.max} > w_k$$

Ok

SLS analysis at support

$$M_{qp} \coloneqq 49.67 \ \mathbf{kN \cdot m}$$

Uncrack cross section properties SS1

$$h = 350 \text{ mm}$$
 $b = 230 \text{ mm}$ $d = 303 \text{ mm}$ $A_s = 791.681 \text{ mm}^2$

$$E_s$$
 effective modolus for steel which is 200
$$E_s \coloneqq 200 \ \frac{kN}{mm^2}$$
 E_c effective modolus for concrete C20/25 which is 30
$$E_c \coloneqq 30 \ \frac{kN}{mm^2}$$

$$lpha_e \coloneqq rac{E_s}{E_{ceff}} = 22.54$$
 $f_{ctm} \coloneqq 2.2 \, rac{m{mm}^2}{m{mm}^2}$

Homogenious concrete area

$$A_{i1} := h \cdot b + (\alpha_e - 1) \cdot A_s = (9.755 \cdot 10^4) \ mm^2$$

Neutral Axis

$$x_{i1} \coloneqq \frac{h \cdot b \cdot \frac{h}{2} + \left(\alpha_e - 1\right) \cdot \left(A_s \cdot d\right)}{A_{i1}} = 197.375 \ \textit{mm}$$

Inertia

$$I_{i1} \coloneqq \frac{b \cdot {x_{i1}}^3}{3} + \frac{b \cdot \left(h - x_{i1}\right)^3}{3} + \left(\alpha_e - 1\right) \cdot \left(A_s \cdot \left(d - x_{i1}\right)^2\right) = \left(1.052 \cdot 10^9\right) \ \textit{mm}^4$$

Crack moment

$$M_{cr}\!\coloneqq\!\frac{f_{ctm}\!\cdot\! I_{i1}}{h\!-\!x_{i1}}\!=\!15.169~\textbf{\textit{kN}}\!\cdot\!\textbf{\textit{m}} \qquad \qquad \frac{M_{cr}}{M_{qp}}\!=\!0.305 \qquad \text{Cracked section}$$

Geometrical properties for craked analysis SS2

Neutral Axis

 $x_{i2} = 152.709 \ mm$

$$x_{i2.} \coloneqq x_{i2.} = \frac{\frac{1}{2} \cdot x_{i2.}^{2} \cdot b + \alpha_{e} \cdot A_{s} \cdot d}{x_{i2.} \cdot b + \alpha_{e} \cdot A_{s}} \xrightarrow{solve, x_{i2.}} \begin{bmatrix} 152.70945286319035251 \cdot mm \\ -307.8767434083939296 \cdot mm \end{bmatrix} = \begin{bmatrix} 152.709 \\ -307.877 \end{bmatrix} \mathbf{mm}$$

Inertia

$$I_{i2} := \frac{b \cdot x_{i2}^3}{3} + \alpha_e \cdot A_s \cdot (d - x_{i2})^2 = (6.761 \cdot 10^8) \ \boldsymbol{mm}^4$$

Significant stress value

Concrete stress
$$\sigma_c \coloneqq \frac{M_{qp}}{I_{i2}} \cdot x_{i2} = 11.219 \; \frac{N}{mm^2}$$

$$\sigma_s\!\coloneqq\!\alpha_e\!\cdot\!\frac{M_{qp}}{I_{i2}}\!\cdot\!\left(d-x_{i2}\right)\!=\!248.873\;\frac{\textit{N}}{\textit{mm}^2}$$

Check for the deformation

According to uncracked analysis:

$$k_1 \coloneqq \frac{M_{qp}}{E_{ceff} \cdot I_{i1}} = \left(5.319 \cdot 10^{-6}\right) \frac{1}{\textit{mm}} \qquad e_1 \coloneqq \frac{5}{384} \cdot \frac{p_{qp} \cdot l_{eff}^{\ \ 4}}{E_{ceff} \cdot I_{i1}} = 10.883 \ \textit{mm}$$

According to cracked analysis:

$$k_2 \coloneqq \frac{M_{qp}}{E_{ceff} \cdot I_{i2}} = \left(8.28 \cdot 10^{-6}\right) \frac{1}{\textit{mm}} \qquad e_2 \coloneqq \frac{5}{384} \cdot \frac{p_{qp} \cdot l_{eff}^{}}{E_{ceff} \cdot I_{i2}} = 16.939 \; \textit{mm}$$

Combination factor:

$$\zeta' \coloneqq 1 - 0.5 \cdot \left(\frac{M_{cr}}{M_{qp}}\right)^2 = 0.953$$

Calculated deformations:

$$k_{EC} := \zeta' \cdot k_2 + (1 - \zeta') \cdot k_1 = (8.142 \cdot 10^{-6}) \frac{1}{mm}$$

$$e_{EC}\!:=\!\zeta'\!\cdot\!e_2\!+\!\left(1\!-\!\zeta'\right)\!\cdot\!e_1\!=\!16.657~\pmb{mm}$$

Verification:

$$e_{max} \coloneqq \frac{l_{eff}}{200} = 33$$
 mm $e_{max} > e_{EC}$ Ok

Crack width analysis:

Steel strain:
$$\varepsilon_s = \frac{\sigma_s}{E_s} = 0.001$$

Concrete strain:
$$\varepsilon_c \coloneqq \frac{f_{ctm}^s}{E_{ceff}} = 2.479 \cdot 10^{-4}$$

Tension stiffening of concrete:

Effective concrete area for tension stiffening

$$\begin{split} &h_{c.eff}\coloneqq min\left(2.5\boldsymbol{\cdot}\left(h-d\right),\frac{\left(h-x_{i2}\right)}{3},\frac{h}{2}\right) = 65.764~\textit{mm} \\ &A_{ceff}\coloneqq h_{c.eff}\boldsymbol{\cdot}b = \left(1.513\boldsymbol{\cdot}10^4\right)~\textit{mm}^2 \qquad \qquad \varepsilon_{sc}\coloneqq \frac{f_{ctm}\boldsymbol{\cdot}A_{ceff}}{E_s\boldsymbol{\cdot}A_s} = 2.102\boldsymbol{\cdot}10^{-4} \\ &\rho_{\rho.eff}\coloneqq \frac{A_s}{A_{ceff}} = 0.052 \end{split}$$

Durability of loads:

Long term load: $k_t = 0.4$

Difference of strain: $\triangle \varepsilon := \varepsilon_s - k_t \cdot \varepsilon_{sc} - k_t \cdot \varepsilon_c = 0.001$

Maximal distance between two neighbouring cracks:

where:

$c \coloneqq 25 \ \boldsymbol{mm}$	Conncrete cover
$k_1 = 0.8$	For ribbed bars
$k_2 = 0.5$	For flexure
$\phi \coloneqq 12 \ mm$	Bar diameter

$$S_{r.max} \coloneqq 3.4 \ c + \frac{0.425 \ k_1 \cdot k_2 \cdot \phi}{\rho_{\rho.eff}} = 123.976 \ \textit{mm}$$

Crack width:

$$w_k \coloneqq S_{r.max} \cdot \triangle \varepsilon = 0.132 \ mm$$

$$w_{k.max} = 0.3 \ \textit{mm}$$
 $w_{k.max} > w_k$ Ok