

COMP 4190 Assignment 1 Answer

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Problem 1

For my first question, I have two functions with different purposes.

- **TransformWord**

- This function performs the BFS.
- Starting from the begin word, it performs BFS by comparing the current word with the words in the word list.
- If the current word differs by a single letter, the new word is inserted into the queue and the sequence counter is incremented by 1.
- The visited word is added to a set to prevent circular loops.
- The loop continues until the end word is reached or the queue is empty.

- **CheckAdjacentWords**

- This function will compare two strings and check whether those two strings differ by 1 letter or not

Problem 2

Problem 3

Problem 4

Problem 5

Problem 6

(a) $\nabla_x f(x) = \frac{1}{2}(A + A^T)x - b$

Since A is symmetric, thus, $A = A^T$

$$\nabla_x f(x) = Ax - b$$

(b) $x^* : \nabla f(x^*) = 0$

$$Ax^* - b = 0$$

$$Ax^* = b$$

$$A^{-1}(Ax^*) = A^{-1}b$$

$$Ix^* = A^{-1}b$$

$$x^* = A^{-1}b$$

(c)

Problem 7

$$(a) \nabla f(x) = 2(x - 2)$$

$$(b) x_{k+1} = x_k - \alpha \nabla f(X_k)$$

$$x_{k+1} = x_k - \alpha 2(x - 2)$$

$$(c) x^* : \nabla f(x^*) = 0$$

$$\text{So, } x^* = 2$$

(d) if step size α is too large: oscillations (no convergence)

if step size α is too small: slow convergence

$$(e) x_{k+1} = x_k - \alpha 2(x - 2)$$

$$= x_k - 2\alpha x_k + 4\alpha$$

$$= (1 - 2\alpha)x_k + 4\alpha$$

Example, let $(1 - 2\alpha) = b$

$$x_1 = x_0 b + 4\alpha$$

$$x_2 = x_1 b + 4\alpha$$

$$= (x_0 b + 4\alpha)b + 4\alpha$$

$$= x_0 b^2 + 4\alpha b + 4\alpha$$

$$= x_0 b^2 + 4\alpha(b + 1)$$

$$x_3 = x_2 b + 4\alpha$$

$$= (x_0 b^2 + 4\alpha(b + 1))b + 4\alpha$$

$$= (x_0 b^2 + 4\alpha b + 4\alpha)b + 4\alpha$$

$$= x_0 b^3 + 4\alpha b^2 + 4\alpha b + 4\alpha$$

$$= x_0 b^3 + 4\alpha(b^2 + b + 1)$$

So, in general for x_k

$$x_k = x_0 b^k + 4\alpha(b^k + b^{k-1} + \dots + 1)$$

$(b^k + b^{k-1} + \dots + 1)$ is a geometric sum

Thus,

$$x_k = x_0 b^k + 4\alpha \left(\frac{1 - b^k}{1 - b} \right)$$

$$= x_0 b^k + 4\alpha \left(\frac{1 - b^k}{2\alpha} \right)$$

$$= x_0 b^k + 2(1 - b^k)$$

$$= x_0 b^k + 2 - 2b^k$$

$$= (x_0 - 2)b^k + 2$$

$$= (1 - 2\alpha)^k(x_0 - 2) + 2 \quad \dots \text{ sub in } b = (1 - 2\alpha)$$

x_k converges to $x^* \implies x_k$ converges to 2, since $x^* = 2$

Thus, $x_k - 2$ converges to 0

In other word, $(1 - 2\alpha)^k(x_0 - 2) \rightarrow 0$

Furthermore, $x_0 \neq x^* \rightarrow x_0 - 2 \neq 0$

Therefore, $(1 - 2\alpha)^k \rightarrow 0$