COMP 4190 Assignment 1 Answer

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Problem 1

Problem 2

Problem 3

Problem 4

Problem 5

Problem 6

(a) $\nabla_x f(x) = \frac{1}{2}(A + A^T)x - b$
Since A is symmetric, thus, $A = A^T$
$\nabla_x f(x) = Ax - b$
(b) $x^* : \nabla f(x^*) = 0$
$Ax^* - b = 0$
$Ax^* = b$
$A^{-1}(Ax^*) = A^{-1}b$
$Ix^* = A^{-1}b$
$x^* = A^{-1}b$
(c)

Problem 7

(a) $\nabla f(x) = 2(x - 2)$
(b) $x_{k+1} = x_k - \alpha \nabla f(X_k)$
$x_{k+1} = x_k - \alpha 2(x - 2)$
(c) $x^* : \nabla f(x^*) = 0$
So, $x^* = 2$
(d) if step size α is too large: oscillations (no convergence)
if step size α is too small: slow convergence
(e) $x_{k+1} = x_k - \alpha 2(x - 2)$

$= x_k - 2\alpha x_k + 4\alpha$
$= (1-2\alpha)x_k + 4\alpha$
Example, let $(1-2\alpha) = b$
$x_1 = x_0 b + 4\alpha$

$x_2 = x_1 b + 4\alpha$
$= (x_0 b + 4\alpha)b + 4\alpha$
$= x_0 b^2 + 4\alpha b + 4\alpha$
$= x_0 b^2 + 4\alpha(b+1)$

$x_3 = x_2 b + 4\alpha$
$= (x_0 b^2 + 4\alpha(b+1))b + 4\alpha$
$= (x_0 b^2 + 4\alpha b + 4\alpha)b + 4\alpha$
$= x_0 b^3 + 4\alpha b^2 + 4\alpha b + 4\alpha$
$= x_0 b^3 + 4\alpha(b^2 + b + 1)$

So, in general for x_k
$x_k = x_0 b^k + 4\alpha(b^k + b^{k-1} + ... + 1)$
$(b^k + b^{k-1} + ... + 1)$ is a geometric sum
Thus,
$x_k = x_0 b^k + 4\alpha \left(\frac{1-b^k}{1-b} \right)$
$= x_0 b^k + 4\alpha \left(\frac{1-b^k}{2\alpha} \right)$
$= x_0 b^k + 2(1-b^k)$
$= x_0 b^k + 2 - 2b^k$
$= (x_0 - 2)b^k + 2$
$= (1-2\alpha)^k (x_0 - 2) + 2 \qquad$ ... sub in $b = (1-2\alpha)$

x_k converges to $x^* == x_k$ converges to 2, since $x^* = 2$
Thus, $x_k - 2$ converges to 0
In other word, $(1-2\alpha)^k (x_0 - 2) \to 0$
Furthermore, $x_0 \neq x^* \to x_0 - 2 \neq 0$
Therefore, $(1-2\alpha)^k \to 0$