

# COMP 4190 Assignment 1 Answer

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## Problem 1

For my first question, I have two functions with different purposes.

- **TransformWord**

- This function performs the BFS.
- Starting from the begin word, it performs BFS by comparing the current word with the words in the word list.
- If the current word differs by a single letter, the new word is inserted into the queue and the sequence counter is incremented by 1.
- The visited word is added to a set to prevent circular loops.
- The loop continues until the end word is reached or the queue is empty.

- **CheckAdjacentWords**

- This function will compare two strings and check whether those two strings differ by 1 letter or not

## Problem 2

For the second question, I also have two different functions.

- **CountProvinces**

- This function will iterate through the isConnected array and check whether the current city has been visited or no
- If the city hasn't been visited, we will visit that city and find the connection through DFS, by calling the DFS function
- for unvisited city, we will increment the province count by 1

- **DFS**

- This function will iterate through one of the isConnected entry
- if indeed, the city is connected ( $==1$ ) with another city and that city hasn't been visited previously
- we will recursively go into that city and repeat the same process (this is the DFS process)
- Otherwise, we will go into the next iteration of the isConnected

## Problem 3

For the third question, I also have two different functions.

- **HikeDijkstra**

- This is where the Dijkstra's algorithm is being used.
- It will have two main data structures,
  - \* queue[(row, col, current effort)], where current effort is the min effort to reach the cell thus far
  - \* distance[][], to record each node effort thus far (this is how we use the Dijkstra algorithm, where we will always compare with the lower distance)

Two conditions for the loop to stop:

- \* the queue is empty
- \* or, we have reached the end of the grid
- Starting from the top left grid, for each node, it will find all the possible neighbors
- for each of those neighbors, we will calculate the height of the neighbor with the current node
- get the maximum absolute difference with the previous height
- and we would only append to the queue only if the new max difference is lower than the last recorded distance (this is where the Dijkstra's algorithm is happening)

- **FindNeighbors**

- Given a specific entry (row, col), this function will calculate the neighbors for that specific (row, col)

## Problem 4

- **FindGoal**

- this is where the iterative deepening is being handled
- This function is using a while loop, it will keep calling the DepthGoalSearch until we found the result or until the maximum depth is reached
- first, it will start with curr max depth = 0, the loop will keep incrementing the current max depth until it reaches the max depth
- if the current max depth has reached the max depth but result hasn't been found, it will return -1
- Otherwise, it will keep trying curr max depth + 1

- **DepthGoalSearch**

- This is where the DFS is happening
- if the current node is equal to the goal, we return the current depth
- if current depth is curr max depth, we return -1 (iterative deepening will be handled in the FindGoal function)
- if children doesn't exist, return -1
- else, it will use a depth first search, iterating through all its children first

## Problem 5

- (a) The state  $dp[i][j]$  represents the minimum number of edit operations to transform  $s$  into  $t$ . For each value in  $i$  and  $j$ , the table will save the previous minimum comparison between prefixes of  $s[1..i]$  and  $t[1..j]$ , and  $dp[m][n]$  will get the minimum value of the edit operations between the two strings.

For recurrence, where  $i \geq 1, j \geq 1$ :

- $dp[i - 1][j] + 1 \rightarrow$  this specify the delete operation, where last character of  $s_i$  is deleted from  $s[1..i]$ , to transform  $s[1..i - 1]$  into  $t[1..j]$
- $dp[i][j - 1] + 1 \rightarrow$  this specify the insert operation, where we would like to insert character  $t_j$  into string  $s$ , transforming  $s[1..i]$  into  $t[1..j - 1]$ .
- $dp[i - 1][j - 1] + l \rightarrow$  this specify the substitution operation,
  - where if  $s_i = t_j$  we won't add 1, so cost is 0
  - otherwise, if  $s_i \neq t_j$ , we will add 1 to substitute the value

- (b) The time complexity is  $O(m \times n)$ .

Since the two strings  $s$  and  $t$  have lengths  $m$  and  $n$ ,  $i$  ranges from 0 to  $m$ , and  $j$  ranges from 0 to  $n$  the dp table has  $(m + 1)(n + 1)$  cells.

Futhermore, the calculation of each dp entry using the three recurrences (insertion, deletion, substitution), all of those operations are just accessing the array, thus this can be considered to be  $O(1)$  as the time complexity.

Thus, since each entry is filled using constant number of operations  $O(1)$  and each result is being stored in an  $(m + 1)(n + 1)$  array. Therefore, the time complexity is  $O(m \times n)$ .

- (c) The memory/ space complexity of this DP implementation is  $O(m \times n)$ .

Same reasoning as the previous part, since the two strings  $s$  and  $t$  have lengths  $m$  and  $n$ ,  $i$  ranges from 0 to  $m$ , and  $j$  ranges from 0 to  $n$  the dp table has  $(m + 1)(n + 1)$  cells. Thus, the memory complexity is  $O(m \times n)$ .

- (d) From the recurrence formula, to fill a row in the DP array, we require only one row from the previous calculation. For instance for  $i=5$ , we only need to access

- $dp[4][5] \rightarrow$  previous row, current column
- $dp[5][4] \rightarrow$  current row, previous column
- $dp[4][4] \rightarrow$  previous row, previous column

Knowing how these calculation work, we could just create two separate array  $prev[]$  and  $curr[]$ . Where  $prev[]$  stores the values of row-1 and  $curr[]$  stores the current row  $i$ . Thus, this approach reduces the space complexity to  $O(n)$ .

## Problem 6

$$(a) \nabla_x f(x) = \frac{1}{2}(A + A^T)x - b$$

Since  $A$  is symmetric, thus,  $A = A^T$

$$\nabla_x f(x) = Ax - b$$

$$(b) x^* : \nabla f(x^*) = 0$$

$$Ax^* - b = 0$$

$$Ax^* = b$$

$$A^{-1}(Ax^*) = A^{-1}b$$

$$Ix^* = A^{-1}b$$

$$x^* = A^{-1}b$$

(c)

## Problem 7

$$(a) \nabla f(x) = 2(x - 2)$$

$$(b) x_{k+1} = x_k - \alpha \nabla f(X_k)$$

$$x_{k+1} = x_k - \alpha 2(x - 2)$$

$$(c) x^* : \nabla f(x^*) = 0$$

$$\text{So, } x^* = 2$$

(d) if step size  $\alpha$  is too large: oscillations (no convergence)

if step size  $\alpha$  is too small: slow convergence

$$(e) x_{k+1} = x_k - \alpha 2(x - 2)$$

$$= x_k - 2\alpha x_k + 4\alpha$$

$$= (1 - 2\alpha)x_k + 4\alpha$$

Example, let  $(1 - 2\alpha) = b$

$$x_1 = x_0 b + 4\alpha$$

$$x_2 = x_1 b + 4\alpha$$

$$= (x_0 b + 4\alpha)b + 4\alpha$$

$$= x_0 b^2 + 4\alpha b + 4\alpha$$

$$= x_0 b^2 + 4\alpha(b + 1)$$

$$x_3 = x_2 b + 4\alpha$$

$$= (x_0 b^2 + 4\alpha(b + 1))b + 4\alpha$$

$$= (x_0 b^2 + 4\alpha b + 4\alpha)b + 4\alpha$$

$$= x_0 b^3 + 4\alpha b^2 + 4\alpha b + 4\alpha$$

$$= x_0 b^3 + 4\alpha(b^2 + b + 1)$$

So, in general for  $x_k$

$$x_k = x_0 b^k + 4\alpha(b^k + b^{k-1} + \dots + 1)$$

$(b^k + b^{k-1} + \dots + 1)$  is a geometric sum

Thus,

$$x_k = x_0 b^k + 4\alpha \left( \frac{1 - b^k}{1 - b} \right)$$

$$= x_0 b^k + 4\alpha \left( \frac{1 - b^k}{2\alpha} \right)$$

$$\begin{aligned}
&= x_0 b^k + 2(1 - b^k) \\
&= x_0 b^k + 2 - 2b^k \\
&= (x_0 - 2)b^k + 2 \\
&= (1 - 2\alpha)^k(x_0 - 2) + 2 \quad \dots \text{ sub in } b = (1 - 2\alpha)
\end{aligned}$$

$x_k$  converges to  $x^* == x_k$  converges to 2, since  $x^* = 2$

Thus,  $x_k - 2$  converges to 0

In other word,  $(1 - 2\alpha)^k(x_0 - 2) \rightarrow 0$

Furthermore,  $x_0 \neq x^* \rightarrow x_0 - 2 \neq 0$

Therefore,  $(1 - 2\alpha)^k \rightarrow 0$

To make,  $(1 - 2\alpha)^k \rightarrow 0$ , we need to make  $|(1 - 2\alpha)| < 1$

Solve for,  $|(1 - 2\alpha)| < 1$

$$-1 < 1 - 2\alpha < 1$$

$$-1 - 1 < 1 - 2 - 1\alpha < 1 - 1$$

$$-2 < -2\alpha < 0$$

$$0 < \alpha < 1$$

Thus, the condition of  $\alpha$  under which it converges to  $x^*$  is  $0 < \alpha < 1$