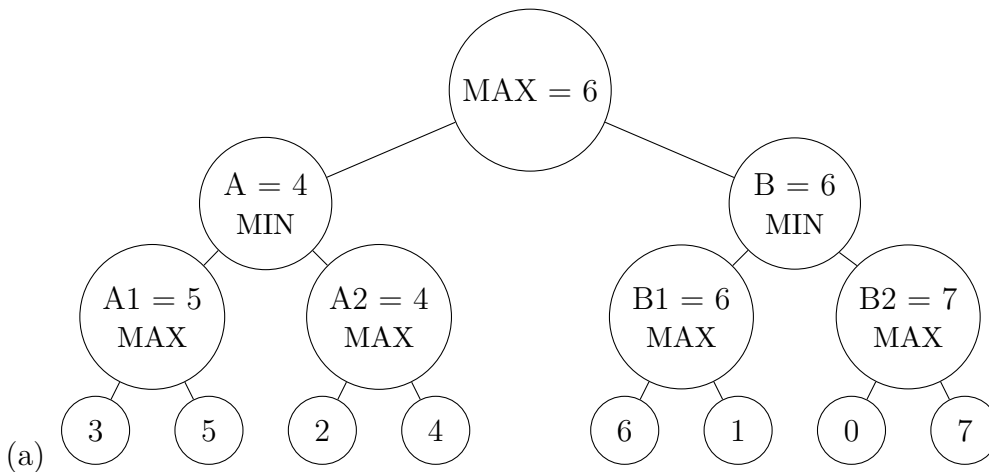# COMP 4190 Assignment 2 Answer

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Problem 4

(a)

(b) - Based on my computation, MAX should choose the right side at the root.
- 6 is the payoff MAX can guarantee despite MIN's action
- minimax corresponds to a worst-case guarantee for MAX because just below MAX, there's MIN which forces MAX to choose the best out of the worst (MAX's worst case). Thus, a worst-case guarantee.

(c)

Problem 5

(a)

$$\sum_X \sum_Y P(X = x, Y = y) = 0.10 + 0.20 + 0.10 + 0.15 + 0.25 + 0.20 = 1$$

(b)

$P(X = 0) = 0.10 + 0.20 + 0.10 = 0.40$

$P(X = 1) = 0.15 + 0.25 + 0.20 = 0.60$

$P(Y = 0) = 0.10 + 0.15 = 0.25$

$P(Y = 1) = 0.20 + 0.25 = 0.45$

$P(Y = 2) = 0.10 + 0.20 = 0.30$

(c) Conditional probability formula:

$$P(X \mid Y) = \frac{P(X \cap Y)}{P(Y)}$$

Bayes' theorem:

$$P(X \mid Y) = \frac{P(X)\,P(Y \mid X)}{P(Y)}$$

$$P(X = 1 \mid Y = 1) = \frac{0.6(0.25/0.6)}{0.45} = 0.56$$

$$P(Y = 2 \mid X = 1) = \frac{0.3(0.20/0.3)}{0.6} = 0.33$$

$$P(X = 0 \mid Y = 0) = \frac{0.10}{0.25} = 0.40$$

(d)

$$P(X = 0 \mid Y = 1) = \frac{0.2}{0.45} = 0.44$$

$$P(X = 1 \mid Y = 1) = \frac{0.25}{0.45} = 0.56$$

$$P(Y = 0 \mid X = 0) = \frac{0.1}{0.4} = 0.25$$

$$P(Y = 1 \mid X = 0) = \frac{0.2}{0.4} = 0.5$$

$$P(Y = 2 \mid X = 0) = \frac{0.1}{0.4} = 0.25$$

(e) Conditional probability identity:
   if $P(A|B) = P(A)$ and $P(B|A) = P(B)$, then events A and B are independent.
   Seeing from $P(X = 0 \mid Y = 1) = \frac{0.2}{0.45} = 0.44$ and $P(X = 0) = 0.10 + 0.20 + 0.10 = 0.40$
   $0.44 \neq 0.40$, thus, X and Y are not independent.

(f) 1. Joint probability $P(X, Y) \rightarrow$ the probability that both happens at the same time. In other words, the probability that the AI model correctly predicts "object" at a specific sensor. For instance, $P(X = 1, Y = 1)$, meaning that probability of AI model predict "yes" and the object actually present at Y = 1 is 0.2

   2. Marginal probability $P_x(X) \rightarrow$ the probability of X alone, ignoring Y. In other words, the probability that the object present or not present, independent of Y.

   3. Conditional probability $P(X|Y) \rightarrow$ the probability of X given that Y already happened. In other words, the probability of the AI model predicts "object present" or not given an object has been detected previously at Y. For instance, $P(X = 1|Y = 1)$, is the probability that AI predicts an "object present" given object detected by sensor 1.

Problem 6

(a) Suppose generate 1000 samples

    (1) Expected samples from each pair (X,Y) out of 1000 samples

        (I) $(X = 0, Y = 0) = 1000 * 0.1 = 100$

        (II) $(X = 0, Y = 1) = 1000 * 0.2 = 200$

        (III) $(X = 0, Y = 2) = 1000 * 0.1 = 100$

        (IV) $(X = 1, Y = 0) = 1000 * 0.15 = 150$

        (V) $(X = 1, Y = 1) = 1000 * 0.25 = 250$

        (VI) $(X = 1, Y = 2) = 1000 * 0.2 = 200$

    (2) Samples where X = 0. Y = 1

        (I) $X = 0 \rightarrow 1000 * (0.1 + 0.2 + 0.1) = 400$

        (II) $Y = 1 \rightarrow 1000 * (0.2 + 0.25) = 450$

(b) Simulate sampling,

    (I) by placing 1000 cards in a box. Each card has label (X, Y), where the number of each card match the expected samples from (a)

    (II) mix those 1000 cards

    (III) draw one card at random from the box. This represents one sample from this box

(c) The reason why this simulates to conditional probability $P(X|Y)$ is because, by getting the value of Y and restrict the sample only by using that same value of Y. Meaning that we randomly selects only from the reduced set of samples using that same value of Y. Thus, this relates to the definition of conditional probability, where the probability of getting X from this reduced set represents $P(X|Y)$, which is the probability of X, given that Y has occured.

(d) Suppose that we generate more and more sample from this distribution. Note the formula of standard deviation. Low standard deviation means that data is consistent. On the other hand, high standard deviation means data has high variability. Since standard deviation is inversely propotional from N, thus, as sample N increases, the standard deviation decreases. Thus, data is more consistent, and should approach the probabilities in the table.

Problem 7