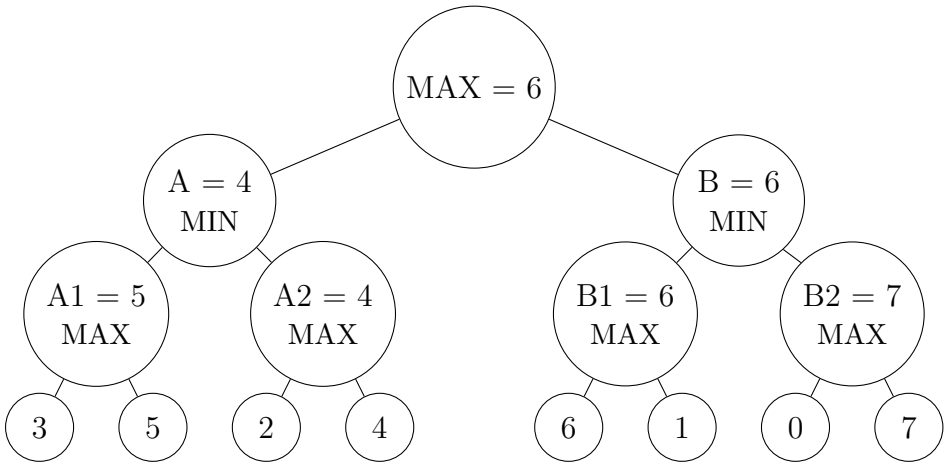
COMP 4190 Assignment 2 Answer

Fidelio Ciandy, 7934456

Problem 4

(a)



(b) - Based on my computation, MAX should choose the right side at the root.
- 6 is the payoff MAX can guarantee despite MIN's action
- minimax corresponds to a worst-case guarantee for MAX because just below MAX, there's MIN which forces MAX to choose the best out of the worst (MAX's worst case). Thus, a worst-case guarantee.

(c)

Problem 5

(a)

$$\sum_X \sum_Y P(X = x, Y = y) = 0.10 + 0.20 + 0.10 + 0.15 + 0.25 + 0.20 = 1$$

(b)

$$P(X = 0) = 0.10 + 0.20 + 0.10 = 0.40$$

$$P(X = 1) = 0.15 + 0.25 + 0.20 = 0.60$$

$$P(Y = 0) = 0.10 + 0.15 = 0.25$$

$$P(Y = 1) = 0.20 + 0.25 = 0.45$$

$$P(Y = 2) = 0.10 + 0.20 = 0.30$$

(c) Conditional probability formula:

$$P(X \mid Y) = \frac{P(X \cap Y)}{P(Y)}$$

Bayes' theorem:

$$P(X \mid Y) = \frac{P(X)\,P(Y \mid X)}{P(Y)}$$

$$P(X = 1 \mid Y = 1) = \frac{0.6(0.25/0.6)}{0.45} = 0.56$$

$$P(Y = 2 \mid X = 1) = \frac{0.3(0.20/0.3)}{0.6} = 0.33$$

$$P(X = 0 \mid Y = 0) = \frac{0.10}{0.25} = 0.40$$

(d)

$$P(X = 0 \mid Y = 1) = \frac{0.2}{0.45} = 0.44$$

$$P(X = 1 \mid Y = 1) = \frac{0.25}{0.45} = 0.56$$

$$P(Y = 0 \mid X = 0) = \frac{0.1}{0.4} = 0.25$$

$$P(Y = 1 \mid X = 0) = \frac{0.2}{0.4} = 0.5$$

$$P(Y = 2 \mid X = 0) = \frac{0.1}{0.4} = 0.25$$

(e) Conditional probability identity:
if $P(A|B) = P(A)$ and $P(B|A) = P(B)$, then events A and B are independent.
Seeing from $P(X = 0 \mid Y = 1) = \frac{0.2}{0.45} = 0.44$ and $P(X = 0) = 0.10 + 0.20 + 0.10 = 0.40$
$0.44 \neq 0.40$, thus, X and Y are not independent.

(f) 1. Joint probability $P(X, Y) \rightarrow$ the probability that both happens at the same time. In other words, the probability that the AI model correctly predicts "object" at a specific sensor. For instance, $P(X = 1, Y = 1)$, meaning that probability of AI model predict "yes" and the object actually present at Y = 1 is 0.2

2. Marginal probability $P_x(X) \rightarrow$ the probability of X alone, ignoring Y. In other words, the probability that the object present or not present, independent of Y.

3. Conditional probability $P(X|Y) \rightarrow$ the probability of X given that Y already happened. In other words, the probability of the AI model predicts "object present" or not given an object has been detected previously at Y. For instance, $P(X = 1|Y = 1)$, is the probability that AI predicts an "object present" given object detected by sensor 1.