Measuring Atmospheric Humidity using Interferometry of Aircraft Navigational Broadcasts

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7 December 2018

Measuring humidity is expensive and difficult. This project, proposed by the Met Office, explores a method of measuring humidity using radio interferometry on ADS-B navigational broadcasts from commercial air traffic. The key component in measuring the humidity is obtaining a value of the apparent position of the aircraft due to refractive bending of the ADS-B. The angle of arrival $\hat{B} \cdot \hat{S}$ was measured to an accuracy $\pm 0.14138\%$, where the majority of this error comes from uncertainty in the baseline measurement. The results show a promising future for this method of measuring humidity, but a lot more testing must be done.

1 Introduction

Humidity can vary greatly in short time and space scales. Figure 1 shows an example given by the Australian Bureau of meteorology. The graph shows that relative humidity changes unpredictably and quickly, which means forecasting for this atmospheric property is almost impossible. It is very expensive and difficult to even make a direct humidity measurement. In the UK, there are 6 weather balloon sites and a limited number of on-ground weather stations that could make the measurements, which is simply not enough for large scale humidity models. This method proposed by the Met Office uses the refractive bending of aircraft navigational broadcasts to make a measure on humidity, which could be groundbreaking for forecasting models. This project will do initial investigations into this method, determining if it works and laying a foundation for future work to build from.

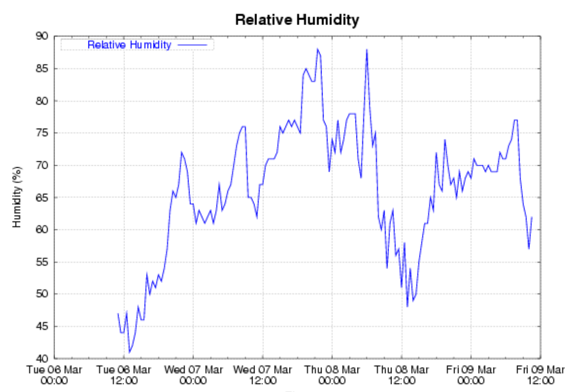

Figure 1: A relative humidity vs time graph published by the Australian Government Bureau of Meteorology [2]

2 Theory

The theory behind this method lies in three areas. Firstly, the ADS-B broadcasts, then some atmospheric physics and lastly two antenna interferometry. Through combining these areas, a method to measure humidity can be created.

2.1 ADS-B

Automatic Dependent Surveillance Broadcasts (ADS-B) is a surveillance technology in which an aircraft determines its position via satellite navigation and periodically broadcasts it along with its identification and other bits of information. These broadcasts are emitted in extended squitters at frequency 1090MHz, or wavelength 0.275m [1][4].

2.2 Atmospheric Physics & Electromagnetic Wave Bending

For this project, a vertically stratified atmosphere is assumed as shown in Figure 2.

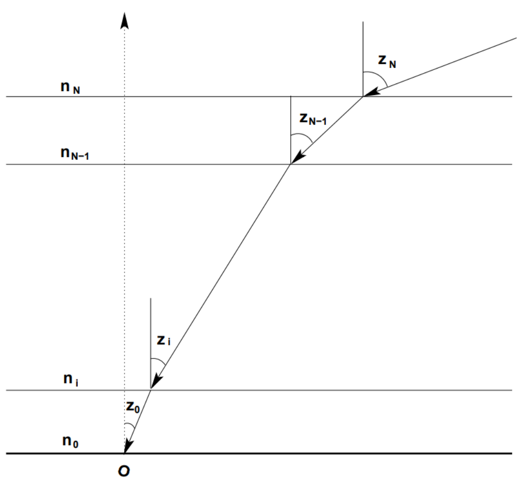

Figure 2: An EM wave propagating (and bending) through a vertically stratified atmosphere [3]

Here n are the refractive indexes and z are the zenith angles. Assuming the aircraft emitting the EM waves is at the radio horizon, defining the angle of refraction $R = z - z_0$ and applying Snells Law (1), the expression (2) can be obtained

$$n_i \sin\theta_i = n_{i+1} \sin\theta_{i+1}, \tag{1}$$

$$R = (n_0 - 1)\tan z_0, \tag{2}$$

The refractive index can be related to the radio refractivity in the expression (3)

$$(n_0 - 1) = 10^{-6} N_0, \tag{3}$$

Where N_0 is the radio refractivity at the observer. This refractivity can be expressed as shown in (4) [5]

$$N_0 = \frac{aP}{T} + \frac{be}{T^2}, \tag{4}$$

Where a and b are constants defined by atmospheric conditions, P is atmospheric pressure, T is temperature and e is the partial pressure of water vapour. This partial pressure of water vapour can be related to the relative humidity by (5)

$$RH = \frac{e}{e_s} \cdot 100, \tag{5}$$

Where RH is the % relative humidity and e_s is the saturation vapour pressure at the temperature T (Pressure at which the water vapour condenses). So if values of the actual position of the aircraft and a value of the apparent position of the aircraft due to refractive bending are obtained, the radio refractivity can be calculated and from this, the relative humidity.

2.3 Two Antenna Radio Interferometry

To calculate the apparent position of the aircraft, two antenna interferometry can be used. The basic set up of these antennas are shown in Figure 3. Here, two antennas are separated by a baseline B(m). The EM waves approach the antennas along a constant line of phase with angle of arrival θ, which causes a geometrical time delay between antenna 1 and antenna 2 receiving the signal. This geometrical time delay can be expressed by simple trigonometry shown in (6),

$$\tau_g = \frac{B}{c}\cos\theta, \tag{6}$$

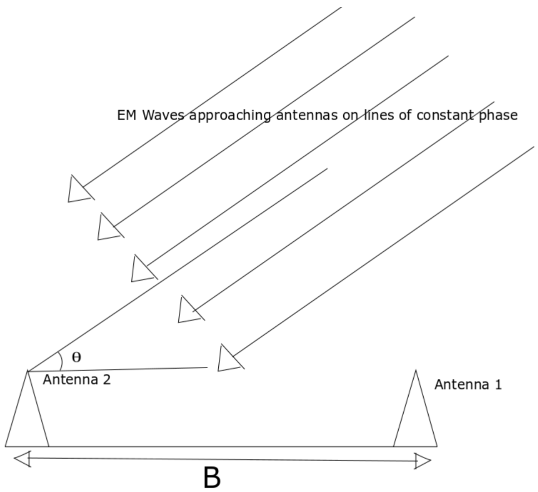

Figure 3: Two Antenna Interferometer with EM waves approaching from a distance source.

Where c is the speed of light. Each antenna, upon receiving the radio waves, will induce voltages which can be expressed as (7),

$$V_1 = V\cos(\omega t) \;\; and \;\; V_2 = V\cos(\omega(t-\tau_g)), \quad (7)$$

Where because the 2nd antenna receives the radio wave with a time difference τ_g, the induced voltage will be out of phase. To find the phase, the induced voltages are multiplied together as shown in (8)

$$V_1V_2 = V^2\cos(\omega t)\cos(\omega(t-\tau_g)), \quad (8)$$

Expanding this gives

$$V_1V_2 = \frac{V^2}{2}(\cos(2\omega t + \omega\tau_g) + \cos(\omega\tau_g)), \quad (9)$$

Taking the infinite time average of (9) makes the $\cos(\omega\tau_g)) = 0$ so the equation becomes (10)

$$< V_1V_2 >= \frac{V^2}{2}\cos(\omega t), \quad (10)$$

Where $\omega\tau_g$ is the phase between the two antennas induced voltage. Plugging this back in to (6) and using $\omega = \frac{2\pi c}{\lambda}$ gives (10)

$$\phi = \omega\tau_g = \frac{2\pi}{\lambda}B\cos\theta, \quad (11)$$

This equation gives a relation between the phase and the angle of arrival of the EM waves. However this angle of arrival $\cos\theta$ only holds for the 2 dimensional approach taken in deriving (11). To obtain an expression that can be used in this case, the curvature of the earth needs to be taken into account.

The $\cos\theta$ is replaced by $\vec{B}\cdot\vec{S}$ where B is the direction of the baseline and S is the direction to the plane. This can become $B(\hat{B}\cdot\hat{S})$ where $\hat{S} = \frac{\vec{S}}{|S|}$. Using spherical trigonometry and due to our baseline facing a 45 deg angle which lets $B_{NS} \approx B_{EW} \approx \frac{B}{\sqrt{2}}$, this yields a much more complex version of $\cos\theta$ shown in (12).

$$\hat{B}\cdot\hat{S} = \frac{\frac{1}{\sqrt{2}}(-r_p\sin\theta_p\cos\Delta\lambda_p\cos\theta + r_p\sin\theta_p\sin\Delta\lambda_p + r_p\cos\theta_p\sin\theta_0)}{\sqrt{r_p^2 - r_0^2 - 2r_pr_0(\sin\theta_p\sin\theta_0\cos\Delta\lambda_p + \cos\theta_p\cos\theta_0)}}, \quad (12)$$

Where r_p and r_0 are the distances to the plane and to the interferometer from the centre of the earth, θ_p and θ_0 are the angles associated with these distances and $\Delta\lambda_p$ is defined as $\lambda_p - \lambda_0$ which is the change in elevation angle from the interferometer to the plane. This equation is trivial for a computer to solve and $\hat{B}\cdot\hat{S}$ is the angle of arrival of the ADS-B wave.

3 Method

Combining all of the theory, there are the ADS-B broadcasts which gives the exact GPS position of the aircraft but also serve as the electromagnetic wave that the interferometry is performed on. The two antenna interferometry leads to measuring the phase by multiplying the signals from the two antennas together. This leads to a calculation for the angle of arrival $\hat{B}\cdot\hat{S}$ (apparent position) of the ADS-B through (11). And finally there is atmospheric physics which allows relative humidity to be calculated once the apparent and actual positions of the aircraft are known (2) (4) (5) [5].

The set up of the equipment is detailed in Figure 4. There are 1090MHz filters attached to the antennas to ensure there is much reduced interfer-

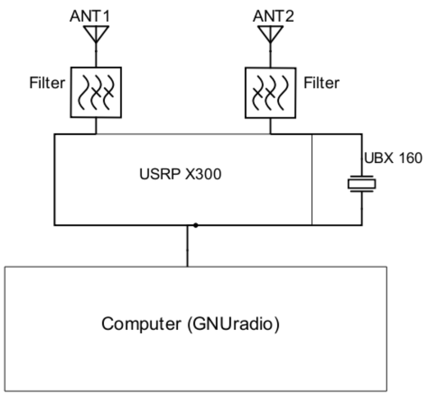

Figure 4: The schematic of the equipment used for the radio interferometry.

ence from other radio broadcasts. The USRP X300 box is a software defined radio, which converts the signal from both antennas simultaneously into a bit-stream for the interferometer to interpret. The UBX 160 box is responsible for aligning the phase, so it ensures the USRP X300 box is converting the same EM wave on both antenna ports at the same time. The bit-stream is then sent along a gigabit Ethernet cable to a powerful computer which uses GNUradio to perform the interferometry and also interpret the signal as an ADS-B broadcast, decoding the data within.

The antennas that were used are 1090MHz specific vertical antennas given by the Met Office. These antennas measured approximately 1.5m long. As part of the experiment, two tripod rigs were built to hold these antennas 6ft (1.83m) off the ground. This is so they could avoid any ground level obstruction and ”see over” walls and fences.

The most important part of determing whether this method will work for measuring humidity is to test how accurately the apparent position of the aircraft can be measured. This will be detailed in the next section.

4 Uncertainty and Anticipated Error

The uncertainty in the angle of arrival can be found from (13).

$$\phi = \frac{2\pi}{\lambda} B \cos\theta, \tag{13}$$

Taking the differential w.r.t θ yields a term $\sin\theta$, but $\theta = \Delta\theta << 1$ so the small angle approximation can be used (13)

$$\Delta\phi = \frac{2\pi}{\lambda} B \Delta\theta, \tag{14}$$

This can be extended to $\hat{B} \cdot \hat{S}$ trivially. Therefore the uncertainty in the angle of arrival (apparent position) is given by (15)

$$\Delta(\hat{B} \cdot \hat{S}) = \frac{\lambda}{2\pi B} \Delta\phi, \tag{15}$$

However the baseline also has an uncertainty in measurement. This is taken to be $\pm 0.01m$. Therefore the longer the baseline B, the more accurate the angle of arrival will be able to be measured to. However this is a purely theoretical approach. There is a lot of room for error in the experimental set up, the conditions on the day and in the data processing.

Experimentally, there is a fine balance between increasing the baseline to reduce the uncertainty and having to increase the cable length which increases uncertainty. Signals have to travel along cables from the antenna to the USRP X300 box. The longer the cable the longer it takes for the signal to travel and the more room for error to occur. This is mitigated by keeping the baseline between 10-30λs in length (2.75-8.25m) and by using high speed copper cables.

Conditions on the day, for example cloud density, temperature and wind speeds, could have an effect on the measurements taken, giving rise to uncertainty. If it is a windy day, the antennas could be blown and shifted slightly in the wind. This would cause a slight change in phase measurements, which would give rise to more uncertainty in the readings. Although weather conditions would only cause a minimal effect on uncertainty readings, in order to create an optimal environment to test how accurately the angle of arrival can be measured, data was only be collected on clear days.

There is a huge amount of data being sent to the computer to be analysed, therefore a very powerful computer is needed to process this data. Initially, the computer was being saturated with data and the majority of the ADS-B broadcasts had to be dropped. This was mitigated by dumping all the raw data into a .dat file as hexadecimal during observation, and running the interferometry on this data afterwards.

Lastly, the antennas must be placed in clear view of the skies. Any obstruction could cause a change in the phase profile of the ADS-B. Although flights following the same flight path should have the same phase profile as they cross the sky, this change in phase will cause incorrect measurements for the angle of arrival of the aircraft.

5 Results & Discussion

Two sets of data were taken. The first was on the University of Exeters Physics Building slab block roof and the second was taken further up the Physics tower. For both sets the baseline was set up to face ≈ 45 deg, looking out towards London and over the channel into France.

5.1 Slab Block Roof Data

The trajectories of different aircrafts compared to the University of Exeter is shown in Figure 5. The lower

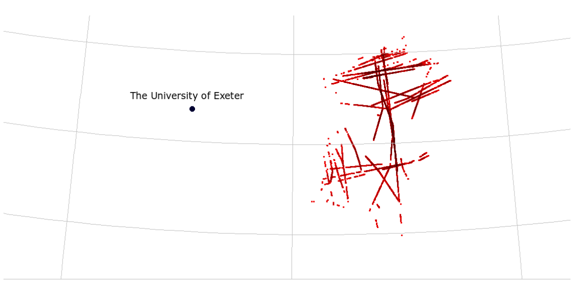

Figure 5: The flight paths of different aircrafts received and analysed by the interferometer from data from the slab block roof.

trajectories on the graph are crossing the channel, flying over Guernsey and Brittany. The trajectories further up are in the UK, around London where there is a large volume of air traffic due to Heathrow and Gatwick airports. The phase vs angle of arrival graph shown in Figure 6 is extremely messy, from equation (11), a straight line graph that wraps from $-\pi$ to π was expected. Although a slight straight line trend

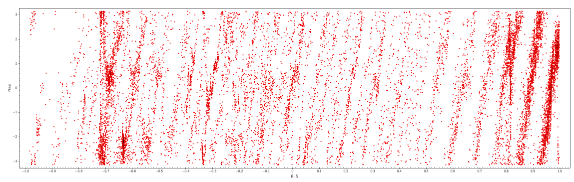

Figure 6: Phase vs angle of arrival graph for results taken on the slab block roof.

is seen, the data is far too scattered to be analysed properly. These results were due to two main sources of error. On the day, there was high wind and this caused the vertical antennas to sway during the observation. Also, the slab block roof isn't high enough to avoid a significant amount of reflection and refraction from the floor and other buildings. This effected the phase readings and caused the scattering of data points away from the expected straight lines.

5.2 Physics Tower Data

The flight paths of the aircrafts is shown in Figure 8 Again these trajectories look out East - South-east from Exeter, seeing from London to Brittany. The phase vs angle of arrival graph for this set of data is shown in Figure 9.

This data is much more linear and conforms to the line expected from (11). Unwrapping this phase gives Figure 9.

This phase unwraps to a straight line graph of gradient ≈ 167.274. The theoretical gradient of this line is given by $m = \frac{2\pi B}{\lambda}$. Using B = 7.32m, the theoretical gradient is = 167.162. The experiment closely fitting to the theory is a sign that these results are good and that this method can work.

The average uncertainty in $\Delta\phi = 0.01922$. Extending this to an uncertainty in $\Delta\theta$ using (15) gives

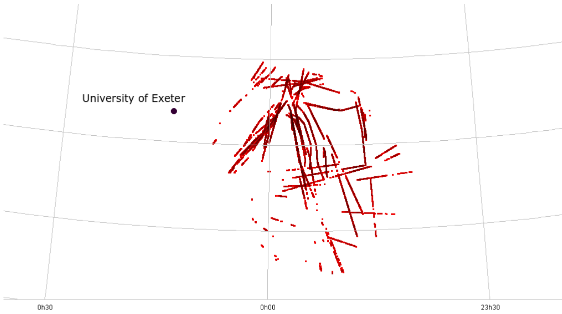

Figure 7: The flight paths of different aircrafts received and analysed by the interferometer from data from the physics tower.

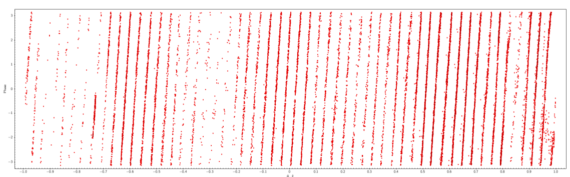

Figure 8: Phase vs angle of arrival graph for results taken on the slab block roof.

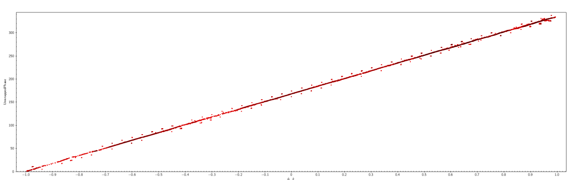

Figure 9: Unwrapped Phase vs angle of arrival graph for results taken on the slab block roof.

$\Delta\theta = 0.00015$. However, the uncertainty in the baseline needs to be taken into account. Using the baseline of $7.32 \pm 0.01m$, the % uncertainty of B is $\pm 0.13661\%$. Using that the phase wraps from $-\pi$ to π, the maximal value of the % uncertainty in the phase $\Delta\phi = 0.00477\%$. This shows that the uncertainty in angle of arrival measurements is dominated by the uncertainty in the baseline. This does not take into account any uncertainties arising from the equipment used or conditions on the day.

Comparing the actual position (GPS position) and apparent position (interferometry position) of the aircraft is a useful way of seeing if this method will work.

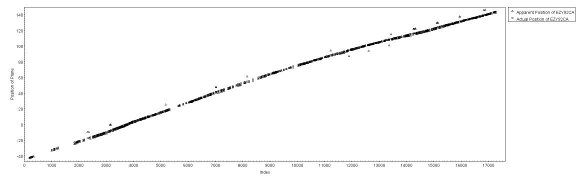

Figure 10: Position vs time graph showing plots of the apparent and actual flight path of EZY92CA.

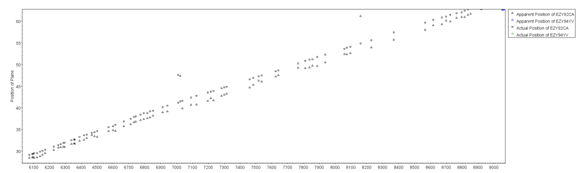

Figure 11: Zoomed in version of Figure 10.

Figure 10 and Figure 11 show the flight path of EZY92CA on a graph of position vs time. From the zoomed in graph Figure 11, the apparent position of the aircraft always lags behind the actual position of the aircraft. This can be consolidated using another set of plots for different aircraft.

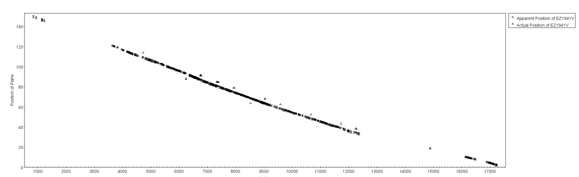

Figure 12: Position vs time graph showing plots of the apparent and actual flight path of EZY92CA.

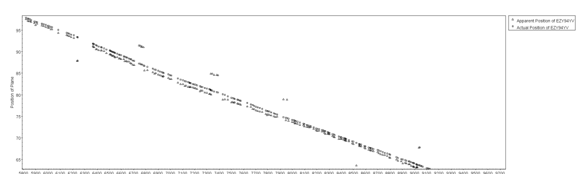

Figure 13: Zoomed in version of Figure 12.

Again, Figure 12 and Figure 13 show that the apparent position is always slightly behind the actual position. One possible reason for this would be that the plane does not broadcast its GPS position as soon as it is updated. Even with a time step $\Delta t = 0.1s$ between receiving a GPS update and broadcasting it, a plane travelling at cruising altitude speed would travel $\approx 20m$. Any otherwise caused difference between the apparent and actual position of the plane is caused by refractive bending.

6 Conclusions

The results gathered by this project show that the method of measuring humidity proposed by the Met Office has a promising future. Using concepts from atmospheric physics and radio interferometry, it was possible to measure the angle of arrival $\hat{B} \cdot \hat{S}$ to an accuracy $\pm 0.00477\%$, however there was a large baseline uncertainty of $\pm 0.13661\%$ which dominates the end uncertainty. Reducing measurement error in the baseline would cause the overall uncertainty to lower significantly.

In future experiments, the vertical antennas should be looked to be set up on a vertical baseline instead of a horizontal baseline. This would improve uncertainty in the phase measurements and allow for more straightforward data interpretation.

A more powerful computer with more storage space would be needed to collect a longer sample of data, to make sure planes observed reached the radio horizon. Only planes at the radio horizon can be used to measure the refractive bending and therefore the relative humidity (2). The next step for this project is to take these improvements into account and directly measure a value for humidity. Then, if more interferometry stations are set up in the surrounding area, it would be possible to cross reference humidity readings and create a real time, 3D model of humidity.

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