## algoritmi bidirezionali

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February 18, 2022

- 1 Ottimizzazione sugli ultimi livelli
- 1.1 FlowFordFulkerson
- 1.2 DoBfs

## Algorithm 1 Ricerca del flusso massimo

```
Require: rete (G, u, s, t)
Ensure: valore del flusso massimo
 1: vuotiSouce \leftarrow pila di nodi
 2: vuotiSink \leftarrow pila di nodi
 3: fMax \leftarrow 0
 4: vuotiSouce.push(s)
 5: vuotiSink.push(t)
 6: while TRUE do
      (f, n) \leftarrow \text{DoBfs}(G, \text{vuotiSource}, \text{vuotiSink})
 7:
      if f = 0 then
         break
 9:
      end if
10:
      vuotiSouce.Clear()
11:
12:
      vuotiSink.Clear()
      n.flussoPassante \leftarrow n.flussoPassante + f
13:
      momSource \leftarrow n
14:
      momSink \leftarrow n
15:
16:
      while momSource \neq s do
17:
         momSource.previousEdge.addFlow(f)
         if u_f(momSource.previousEdge) < 0 \lor f(momSource.previousEdge)
18:
    )<0 then
           vuotiSource.Clear()
19:
            flowError \leftarrow GetFlow(s, n).flussoPassante
20:
           mom \leftarrow n
21:
           while mom \neq momsource do
22:
              mom.flussoPassante \leftarrow mom.flussoPassante - flowError
23:
              mom. Previous Edge. add Flow (flow Error)
24:
25:
              mom \leftarrow mom.previousNode
26:
           end while
           vuotiSource.push(momSource)
27:
           momSource.valid \leftarrow false
28:
            f \leftarrow f + flowError
29:
         else if u_f(momSource.previousEdge) = 0 then
30:
31:
           momSource.valid \leftarrow false
            vuotiSource.push(momSource)
32:
         end if
33:
         momSource.flussoPassante \leftarrow momSource.flussoPassante-f
34:
         momSource \leftarrow momSource.previousNode
35:
      end while
36:
```

```
while momSink \neq t do
37:
38:
         momSink.nextEdge.addFlow(f)
         if u_f(momSink.nextEdge) < 0 \lor f(momSink.nextEdge) < 0 then
39:
           vuotiSink.Clear()
40:
           flowError \leftarrow GetFlow(t, n).flussoPassante
41:
           mom \leftarrow n
42:
           while mom \neq momsink do
43:
              mom.flussoPassante <br/> \leftarrow mom.flussoPassante - flowError
44:
              mom.nextEdge.addFlow(flowError)
45:
              mom \leftarrow mom.\text{nextNode}
46:
           end while
47:
48:
           mom \leftarrow n
           while mom \neq s \ \mathbf{do}
49:
              mom.flussoPassante <br/> \leftarrow mom.flussoPassante - flowError
50:
              mom. Previous Edge. add Flow (flow Error)
51:
              mom \leftarrow mom.previousNode
52:
53:
           end while
           vuotiSink.Push(momSink)
54:
           momSink.valid \leftarrow false
55:
56:
           f \leftarrow f + flowError
57:
         else if u_f(momSink.nextEdge) = 0 then
           momSink.valid \leftarrow \mathit{false}
58:
           vuotiSource.push(momSource)
59:
         end if
60:
         momSink.flussoPassante \leftarrow momSink.flussoPassante -f
61:
         momSink \leftarrow momSink.\text{nextNode}
62:
      end while
63:
64:
      fMax \leftarrow fMax + f
65: end while
66: return fMax
```

## Algorithm 2 DoBfs con ottimizzazione sugli ultimi livelli

**Require:** rete (G, u, s, t), noCapsSource, noCapsSink, cioè pile di nodi contenenti nodi non più raggiungibili attraverso il cammino trovato

**Ensure:** valore del flusso inviabile, nodo appartenente LastSinkNodes, cioè tutti i nodi che sono intermedi che fanno da ponte tra le due ricerche.

```
1: codaSource \leftarrow coda di nodi vuota
2: codaSink \leftarrow coda di nodi vuota
3: buffer \leftarrow \text{coda di nodi vuota}
4: \ sinkRepaired \leftarrow false
5: if \neg noCapsSource.isEmpty then
      p \leftarrow \text{null}
      repaired \leftarrow true
7:
      while \neg noCapsSource.isEmpty do
8:
9:
         noCapSource \leftarrow noCapsSource.pop()
10:
         GetFlow(p, noCapSource)
         p \leftarrow noCapSource
11:
         Repair(noCapSource)
12:
         if non riesco a riparare noCapSource then
13:
14:
            noCapsSource.Push(noCapSource)
            repaired \leftarrow \mathit{false}
15:
            break
16:
         end if
17:
      end while
18:
```

```
if \neg noCapsSink.isEmpty \land repaired then
19:
        for all n \in \text{LastSinkNodes} \mid n.valid do
20:
          GetFlow(noCapSource, n) {da n cerco di retrocedere verso noCap-
21:
  Source, aggiornando ricorsivamente le informazioni dei nodi in modo oppor-
  tuno (sopratutto per quanto riguarda n)}
22:
          if GetFlow ha trovato un percorso\land n.flussoPassante \neq 0 then
             if edge.reversed then
23:
               return (\min(n.flussoPassante, f(edge)), n)
24:
25:
               return (\min(n.flussoPassante, u_f(edge)), n)
26:
             end if
27:
          end if
28:
29:
        end for
      end if
30:
      if \neg repaired then
31:
32:
        if noCapSource = s then
          codaSource.enqueue(noCapSource)
33:
        else if noCapSource \in LastSinkNodes then
34:
          codaSource \leftarrow LastSourceNodes {nodi collegati ai nodi di LastSin-
35:
  kNodes}
36:
        else
          for all n \in V(G)|n.sourceSide \land n.label + 1 = noCapSource.label
37:
  do
             codaSource.enqueue(n)
38:
39:
          for all n \in V(G)|n.SourceSide \land n.label \ge noCapSource.label do
40:
             n.reset()
41:
          end for
42:
        end if
43:
      end if
44:
45: end if
```

```
46: if \neg noCapsSink.isEmpty then
      repaired \leftarrow true
47:
      p \leftarrow \text{null}
48:
      while \neg noCapsSink.isEmpty do
49:
50:
         noCapsSink \leftarrow noCapsSink.pop()
         GetFlow(p, noCapSink)
51:
        p \leftarrow noCapSink
52:
         Repair(noCapSink)
53:
        if non riesco a riparare noCapSink then
54:
55:
           noCapsSink.push(p)
           repaired \leftarrow false
56:
           break
57:
         end if
58:
59:
      end while
      if repaired \land noCapsSource.isEmpty then
60:
         for all n \in LastSinkNodes|n.valid do {nodo di confine valido}
61:
           if n.previousEdge.reversed then
62:
              sourceFlow \leftarrow \min(n.previousNode.inFlow, f(n.previousEdge))
63:
           else
64:
65:
              sourceFlow \leftarrow \min(n.previousNode.inFlow, u_f(n.previousEdge))
           end if
66:
           GetFlow(p, n)
67:
           if è stato trovato un percorso tra p ed n \land n.flussoPassante \neq 0 \land
68:
    sourceFlow > 0 then
             return (\min(n.flussoPassante, sourceFlow), n)
69:
           end if
70:
         end for
71:
      end if
72:
73:
      if \neg repaired then
        if noCapSink = t then
74:
           codaSink.enqueue(noCapSink)
75:
76:
           for all n \in V(G)|n.label + 1 = noCapSink.label do
77:
             codaSink.enqueue(n)
78:
           end for
79:
           for all n \in N(G) | \neg n.sourceSide \land n.label \ge noCapSink.label do
80:
81:
           end for
82:
         end if
83:
      end if
84:
85: end if
```

```
while \neg codaSink.isEmpty \lor \neg codaSource.isEmpty do
87:
      if \neg codaSource.isEmpty \land \neg noCapsSource.isEmpty then
         element \leftarrow codaSource.dequeue()
88:
         if \neg element.sourceSide \lor \neg element.valid then
89:
            continue
90:
         end if
91:
         \textbf{for all} \ \ edge \in element. Edges \ \textbf{do}
92:
           p \leftarrow edge.previousNode
93:
           n \leftarrow edge. \texttt{nextNode}
94:
           if element = p \wedge u_f(edge) > 0 then
95:
              if n.flussoPassante \neq 0 then
96:
97:
                 if n.sourceside then {esplorato da source}
                   continue
98:
                 else
99:
                    f \leftarrow \min(n.flussoPassante, p.flussoPassante, u_f(edge))
100:
101:
                    if f = 0 then
                       continue
102:
                    end if
103:
                    n.update(p, edge)
104:
105:
                    edge.reversed \leftarrow false
106:
                    LastNodesSinkSide.add(n){di conseguenza inserisco tutti i
    nodi collegati direttamente a n che fanno parte di SourceSide in LastN-
    odesSourceSide}
107:
                    return (f, n)
                 end if
108:
               end if
109:
110:
               n.update(p, edge)
               codaSource.enqueue(n)
111:
```

```
else if element = n \wedge f(edge) > 0 then
112:
               if p.flussoPassante \neq 0 then
113:
114:
                 if p.sourceside then
                    continue
115:
                 else
116:
                    f \leftarrow \min(n.flussoPassante, p.flussoPassante, f(edge))
117:
                   if f = 0 then
118:
                      continue
119:
                    end if
120:
                   p.update(n, edge)
121:
122:
                   edge.reversed \gets \mathsf{true}
                   return (f, p)
123:
                 end if
124:
               end if
125:
126:
               p.\mathrm{update}(n,edge)
               edge.reversed \gets \mathsf{true}
127:
               codaSource.enqueue(p)
128:
            end if
129:
          end for
130:
       end if
131:
```

```
if \neg codaSink.isEmpty \land \neg noCapsSink.isEmpty) then
132:
          element \leftarrow codaSink.dequeue()
133:
134:
          if element.sourceSide \lor \neg element.valid then
             continue
135:
          end if
136:
          for all edge \in element.Edges do
137:
138:
             p \leftarrow edge. \texttt{previousNode}
             n \leftarrow edge.\mathtt{nextNode}
139:
             if element = n \wedge u_f(edge) > 0 then
140:
                if p.flussoPassante \neq 0 then
141:
                  if \neg p.sourceSide then
142:
                     continue
143:
                  else
144:
                      f \leftarrow \min(n.flussoPassante, p.flussoPassante, u_f(edge))
145:
                     if f = 0 then
146:
                        continue
147:
                     end if
148:
                     n.\mathrm{update}(p,edge)
149:
                     edge.reversed \leftarrow \! \mathsf{false}
150:
                     return (f, n)
151:
                  end if
152:
                end if
153:
                p.update(n, edge)
154:
                edge.reversed \leftarrow \mathit{false}
155:
                codaSink.enqueue(p)
156:
```

```
else if element = p \land f(edge) > 0 then
157:
158:
              if p.flussoPassante \neq 0 then
                 if \neg p.sourceSide then
159:
                   continue
160:
                 else
161:
                    f \leftarrow \min(p.flussoPassante, n.flussoPassante, f(edge))
162:
                   if f = 0 then
163:
                      continue
164:
165:
                   end if
                   p.update(n, edge)
166:
                   edge.reversed \gets true
167:
                   return (f,p)
168:
                 end if
169:
170:
              end if
              n.update(p, edge)
171:
              edge.reversed \leftarrow \mathsf{true}
172:
173:
              codaSink.enqueue(n)
174:
            end if
         end for
175:
       end if
176:
177: end while
178: return (0, null)
```