

algoritmi bidirezionali

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1 Ottimizzazione sugli ultimi livelli

1.1 FlowFordFulkerson

1.2 DoBfs

Algorithm 1 Ricerca del flusso massimo

Require: rete (G, u, s, t) **Ensure:** valore del flusso massimo

```
1: vuotiSouce  $\leftarrow$  pila di nodi
2: vuotiSink  $\leftarrow$  pila di nodi
3: fMax  $\leftarrow$  0
4: vuotiSouce.push(s)
5: vuotiSink.push(t)
6: while TRUE do
7:    $(f, n) \leftarrow \text{DoBfs}(G, \text{vuotiSource}, \text{vuotiSink})$ 
8:   if  $f = 0$  then
9:     break
10:  end if
11:  vuotiSouce.Clear()
12:  vuotiSink.Clear()
13:   $n.\text{flussoPassante} \leftarrow n.\text{flussoPassante} + f$ 
14:  momSource  $\leftarrow n$ 
15:  momSink  $\leftarrow n$ 
16:  while momSource  $\neq s$  do
17:    momSource.previousEdge.addFlow(f)
18:    if  $u_f(\text{momSource.previousEdge}) < 0 \vee f(\text{momSource.previousEdge}) < 0$  then
19:      vuotiSource.Clear()
20:      flowError  $\leftarrow \text{GetFlow}(s, n).\text{flussoPassante}$ 
21:      mom  $\leftarrow n$ 
22:      while mom  $\neq \text{momSource}$  do
23:         $\text{mom.flussoPassante} \leftarrow \text{mom.flussoPassante} - \text{flowError}$ 
24:        mom.PreviousEdge.addFlow(flowError)
25:        mom  $\leftarrow \text{mom.previousNode}$ 
26:      end while
27:      vuotiSource.push(momSource)
28:      momSource.valid  $\leftarrow$  false
29:       $f \leftarrow f + \text{flowError}$ 
30:    else if  $u_f(\text{momSource.previousEdge}) = 0$  then
31:      momSource.valid  $\leftarrow$  false
32:      vuotiSource.push(momSource)
33:    end if
34:     $\text{momSource.flussoPassante} \leftarrow \text{momSource.flussoPassante} - f$ 
35:    momSource  $\leftarrow \text{momSource.previousNode}$ 
36:  end while
```

```

37: while momSink  $\neq$  t do
38:   momSink.nextEdge.addFlow(f)
39:   if  $u_f(\textit{momSink.nextEdge}) < 0 \vee f(\textit{momSink.nextEdge}) < 0$  then
40:     vuotiSink.Clear()
41:     flowError  $\leftarrow$  GetFlow(t, n).flussoPassante
42:     mom  $\leftarrow$  n
43:     while mom  $\neq$  momSink do
44:       mom.flussoPassante  $\leftarrow$  mom.flussoPassante - flowError
45:       mom.nextEdge.addFlow(flowError)
46:       mom  $\leftarrow$  mom.nextNode
47:     end while
48:     mom  $\leftarrow$  n
49:     while mom  $\neq$  s do
50:       mom.flussoPassante  $\leftarrow$  mom.flussoPassante - flowError
51:       mom.PreviousEdge.addFlow(flowError)
52:       mom  $\leftarrow$  mom.previousNode
53:     end while
54:     vuotiSink.Push(momSink)
55:     momSink.valid  $\leftarrow$  false
56:     f  $\leftarrow$  f + flowError
57:   else if  $u_f(\textit{momSink.nextEdge}) = 0$  then
58:     momSink.valid  $\leftarrow$  false
59:     vuotiSource.push(momSource)
60:   end if
61:   momSink.flussoPassante  $\leftarrow$  momSink.flussoPassante - f
62:   momSink  $\leftarrow$  momSink.nextNode
63: end while
64: fMax  $\leftarrow$  fMax + f
65: end while
66: return fMax

```

Algorithm 2 DoBfs con ottimizzazione sugli ultimi livelli

Require: rete (G, u, s, t) , $noCapsSource$, $noCapsSink$, cioè pile di nodi contenenti nodi non più raggiungibili attraverso il cammino trovato

Ensure: valore del flusso inviabile, nodo appartenente LastSinkNodes, cioè tutti i nodi che sono intermedi che fanno da ponte tra le due ricerche.

```
1:  $codaSource \leftarrow$  coda di nodi vuota
2:  $codaSink \leftarrow$  coda di nodi vuota
3:  $codaEdgeSink \leftarrow$  coda di archi vuota
4:  $codaEdgeSource \leftarrow$  coda di archi vuota
5: if  $\neg noCapsSource.isEmpty$  then
6:    $p \leftarrow$  null
7:    $repaired \leftarrow$  true
8:   while  $\neg noCapsSource.isEmpty$  do
9:      $noCapSource \leftarrow noCapsSource.pop()$ 
10:     $GetFlow(p, noCapSource)$ 
11:     $p \leftarrow noCapSource$ 
12:     $Repair(noCapSource)$ 
13:    if non riesco a riparare  $noCapSource$  then
14:       $noCapsSource.Push(noCapSource)$ 
15:       $repaired \leftarrow$  false
16:      break
17:    end if
18:  end while
```

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19:  if  $\neg noCapsSink.isEmpty \wedge repaired$  then
20:    for all  $n \in LastSinkNodes$   $| n.valid$  do
21:       $GetFlow(noCapSource, n)$  {da n cerco di retrocedere verso noCap-
      Source, aggiornando ricorsivamente le informazioni dei nodi in modo oppor-
      tuno (soprattutto per quanto riguarda n)}
22:      if  $GetFlow$  ha trovato un percorso  $\wedge n.flussoPassante \neq 0$  then
23:        if  $edge.reversed$  then
24:          return  $(\min(n.flussoPassante, f(edge)), n)$ 
25:        else
26:          return  $(\min(n.flussoPassante, u_f(edge)), n)$ 
27:        end if
28:      end if
29:    end for
30:  end if
31:  if  $\neg repaired$  then
32:    if  $noCapSource = s$  then
33:       $codaSource.enqueue(noCapSource)$ 
34:    else if  $noCapSource \in LastSinkNodes$  then
35:       $codaSource \leftarrow LastSourceNodes$  {nodi collegati ai nodi di LastSin-
      kNodes}
36:    else
37:      for all  $n \in V(G) | n.sourceSide \wedge n.label + 1 = noCapSource.label$ 
      do
38:         $codaSource.enqueue(n)$ 
39:      end for
40:      for all  $n \in V(G) | n.SourceSide \wedge n.label \geq noCapSource.label$  do
41:         $n.reset()$ 
42:      end for
43:    end if
44:  end if
45: end if

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46: if  $\neg noCapsSink.isEmpty$  then
47:    $repaired \leftarrow \text{true}$ 
48:    $p \leftarrow \text{null}$ 
49:   while  $\neg noCapsSink.isEmpty$  do
50:      $noCapsSink \leftarrow noCapsSink.pop()$ 
51:      $GetFlow(p, noCapSink)$ 
52:      $p \leftarrow noCapSink$ 
53:      $Repair(noCapSink)$ 
54:     if non riesco a riparare  $noCapSink$  then
55:        $noCapsSink.push(p)$ 
56:        $repaired \leftarrow \text{false}$ 
57:       break
58:     end if
59:   end while
60:   if  $repaired \wedge noCapsSource.isEmpty$  then
61:     for all  $n \in LastSinkNodes|n.valid$  do {nodo di confine valido}
62:       if  $n.previousEdge.reversed$  then
63:          $sourceFlow \leftarrow \min(n.previousNode.inFlow, f(n.previousEdge))$ 
64:       else
65:          $sourceFlow \leftarrow \min(n.previousNode.inFlow, u_f(n.previousEdge))$ 
66:       end if
67:        $GetFlow(p, n)$ 
68:       if è stato trovato un percorso tra  $p$  ed  $n \wedge n.flussoPassante \neq 0 \wedge$ 
 $sourceFlow > 0$  then
69:         return  $(\min(n.flussoPassante, sourceFlow), n)$ 
70:       end if
71:     end for
72:   end if
73:   if  $\neg repaired$  then
74:     if  $noCapSink = t$  then
75:        $codaSink.enqueue(noCapSink)$ 
76:     else
77:       for all  $n \in V(G)|n.label + 1 = noCapSink.label$  do
78:          $codaSink.enqueue(n)$ 
79:       end for
80:       for all  $n \in N(G)|\neg n.sourceSide \wedge n.label \geq noCapSink.label$  do
81:          $n.reset()$ 
82:       end for
83:     end if
84:   end if
85: end if

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86: while  $\neg codaSink.isEmpty \vee \neg codaSource.isEmpty$  do
87:   if  $\neg codaSource.isEmpty \wedge (codaEdgeSource.isEmpty \vee$ 
       $(codaEdgeSink.isEmpty \wedge codaSink.isEmpty \wedge \neg noCapsSink.isEmpty))$ 
      then
88:      $elementSource \leftarrow codaSource.dequeue()$ 
89:     if  $\neg elementSource.sourceSide \wedge \neg elementSource.valid \wedge$ 
         $elementSource.flussoPassante = 0$  then
90:       continue
91:     end if
92:     for all  $e \in \delta(elementSource)$  | il nodo collegato potrà essere es-
        plorato do {dato l'arco  $x$ ,  $(x.NextNode = elementSink \wedge x.Capacity >$ 
         $0 \wedge (x.PreviousNode.InFlow = 0 \vee x.PreviousNode.SourceSide)) \vee$ 
         $(x.PreviousNode = elementSink \wedge x.Flow > 0 \wedge (x.NextNode.InFlow =$ 
         $0 \vee x.NextNode.SourceSide))$ }
93:        $codaEdgeSource.enqueue(e)$ 
94:     end for
95:   end if
96:   if  $\neg codaSink.isEmpty \wedge (codaEdgeSink.isEmpty \vee$ 
       $(codaEdgeSource.isEmpty \wedge codaSource.isEmpty \wedge$ 
       $\neg noCapsSource.isEmpty))$  then
97:      $elementSink \leftarrow codaSink.dequeue()$ 
98:     if  $\neg elementSink.sourceSide \wedge \neg elementSink.valid \wedge$ 
         $elementSink.flussoPassante = 0$  then
99:       continue
100:    end if
101:    for all  $e \in \delta(elementSink)$  | il nodo collegato potrà essere es-
        plorato do {dato l'arco  $x$ ,  $(x.NextNode = elementSink \wedge x.Capacity >$ 
         $0 \wedge (x.PreviousNode.InFlow = 0 \vee x.PreviousNode.SourceSide)) \vee$ 
         $(x.PreviousNode = elementSink \wedge x.Flow > 0 \wedge (x.NextNode.InFlow =$ 
         $0 \vee x.NextNode.SourceSide))$ }
102:       $codaEdgeSink.enqueue(e)$ 
103:    end for
104:  end if

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105:  while ( $\neg codaEdgeSource.isEmpty \vee noCapsSource.isEmpty$ )  $\wedge$ 
      ( $\neg codaEdgeSink.isEmpty \vee noCapSink.isEmpty$ ) do
106:    if  $\neg codaEdgeSource.isEmpty$  then
107:       $e \leftarrow codaEdgeSource.dequeue()$ 
108:       $p \leftarrow e.previousNode$ 
109:       $n \leftarrow e.nextNode$ 
110:      if  $elementSource = p \wedge u_f(e) > 0$  then
111:        if  $n.flussoPassante \neq 0$  then
112:          if  $\neg n.sourceSide$  then
113:             $f \leftarrow \min(n.flussoPassante, p.flussoPassante, u_f(e))$ 
114:            if  $f \neq 0$  then
115:              continue
116:            end if
117:             $n.update(p, edge)$ 
118:             $addLast(n)$ 
119:             $e.reversed \leftarrow false$ 
120:            return  $(f, n)$ 
121:          end if
122:        else
123:           $n.update(p, edge)$ 
124:           $e.reversed \leftarrow false$ 
125:           $codaSource.enqueue(n)$ 
126:        end if
127:      else if  $elementSource = n \wedge f(e) > 0$  then
128:        if  $p.flussoPassante \neq 0$  then
129:          if  $\neg p.sourceSide$  then
130:             $f \leftarrow \min(p.flussoPassante, n.flussoPassante, f(e))$ 
131:            if  $f = 0$  then
132:              continue
133:            end if
134:             $p.upate(n, e)$ 
135:             $addLast(p)$ 
136:             $e.reversed \leftarrow true$ 
137:            return  $(f, p)$ 
138:          end if
139:        else
140:           $p.update(n, e)$ 
141:           $e.reversed \leftarrow true$ 
142:           $codaSource.enqueue(p)$ 
143:        end if
144:      end if
145:    end if

```

```

146:   if  $\neg codaEdgeSink.isEmpty$  then
147:      $e \leftarrow codaEdgeSink.dequeue()$ 
148:      $p \leftarrow e.previousNode$ 
149:      $n \leftarrow e.nextNode$ 
150:     if  $elementSink = n \wedge u_f(e) > 0$  then
151:       if  $p.flussoPassante \neq 0$  then
152:         if  $\neg p.sourceSide$  then continue
153:       else
154:          $f \leftarrow \min(p.flussoPassante, n.flussoPassante, u_f(e))$ 
155:         if  $f = 0$  then
156:           continue
157:         end if
158:          $n.update(p, e)$ 
159:          $e.reversed \leftarrow \text{true}$ 
160:          $addLast(n)$ 
161:         return  $(f, n)$ 
162:       end if
163:     end if
164:      $p.update(n, e)$ 
165:      $e.reversed \leftarrow \text{true}$ 
166:      $codaSink.enqueue(p)$ 
167:   else if  $elementSink = p \wedge f(e) > 0$  then
168:     if  $n.flussoPassante \neq 0$  then
169:       if  $\neg n.sourceSide$  then continue
170:     else
171:        $f \leftarrow \min(p.flussoPassante, n.flussoPassante, f(e))$ 
172:       if  $f = 0$  then
173:         continue
174:       end if
175:        $p.update(n, e)$ 
176:        $e.reversed \leftarrow \text{true}$ 
177:        $addLast(p)$ 
178:       return  $(f, p)$ 
179:     end if
180:   end if
181:    $n.update(p, e)$ 
182:    $e.reversed \leftarrow \text{true}$ 
183:    $codaSink.enqueue(n)$ 
184: end if
185: end if
186: end while
187: end while
188: return  $(0, null)$ 

```
