# Field and Service Robotics Technical Project: Snake-like Underwater Robot

Antonio Pone Mat: P38000107
Francesco Schetter Mat: P38000128

Prof. Fabio Ruggiero

## 1 Introduction

Hyper-redundant mechanism (HRMs), also known as snake-like robot, take inspiration from the nature; it is of our interest due to its capability to move in challenging environment both underwater and ground scenario.
The high number of DOFs of snake robot gives them the ability to moves in irregular environments making them more suitable than the classic wheeled or legged robots; underwater exploration is one of the most challenging field of interests.
Like most of the mobile robot, one big problem of this kind of robot is the power efficiency and supply; one aspect that affects more the efficiency is the locomotion. Higher agility and maneuverability are related to smaller size of the robot; the particular aspect is that this robot don't use propellers but propels himself mimicking the movement of a snake.
This technical project re-implements the studies based on the paper [1] which presents a solution to the modeling problem that results in a closed form, avoiding the numerical evaluation of drag effects. [1] Consider both the linear and the nonlinear drag forces (resistive fluid forces), the added mass effect (reactive fluid forces), the fluid moments and current effect in an analytical simplified form; the mentioned paper, at the end, provides the kinematic, dynamic and hydrodynamic model.
This model can be suitable both for land and underwater is only necessary to put the fluid parameters to 0 and change the fluid friction into the ground friction.
Since in this report we are interested in planar movements the buoyancy effect is neglected; will be shown the model of the whole system and simulation in cases of lateral and eel-like movements. Simulations and scripts

are made in MATLAB-Simulink 2023a. All the code and simulation results are available at the following Github repository: Snake-like underwater robot

# 2 Modeling

In this section will be shown the kinematic, dynamic and hydrodynamic model of the system.

## 2.1 Notation and defined symbols

| Symbols | Description |
|---|---|
| $n$ | Number of links |
| $l$ | Half length of the link |
| $a$ | Radius of the link |
| $m$ | Mass of each link |
| $\theta_i$ | Angle of each link to respect of global frame |
| $\phi_i$ | Joint angle |
| $p_{x/y}$ | Coordinates of CoM in global frame |
| $X/Y$ | Coordinates of each link in global frame |
| $u$ | Actuator torque input |
| $h_{x/y}$ | Joint constraint forces |
| $\tau$ | Fluid torque |
| $f_{x/y}$ | Fluid force |
| $Re$ | Reynolds number |
| $\rho$ | Density of the fluid |
| $C_{f/D}$ | Drag coefficients in $x, y$ directions of motion |
| $C_A$ | Added mass coefficient |
| $C_M$ | Added inertia coefficients |
| $V_{x/y}$ | Current velocity components |

To simplify the model expression the following vector and matrices are defined:

$$A = \begin{bmatrix} 1 & 1 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 1 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & -1 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 1 & -1 \end{bmatrix} \tag{1}$$

where: $A, D \in \mathbb{R}^{(n-1)\times n}$

$$e = \begin{bmatrix} 1 & \dots & 1 \end{bmatrix}^T \in \mathbb{R}^n \qquad E = \begin{bmatrix} e & 0_{n\times 1} \\ 0_{n\times 1} & e \end{bmatrix} \in \mathbb{R}^{2n\times 2} \tag{2}$$

$$sin(\theta) = \left[ \begin{array}{ccc} sin(\theta_1) & \dots & sin(\theta_n) \end{array} \right]^T \in \mathbb{R}^n \quad \mathbb{S}_\theta = diag(sin(\theta)) \in \mathbb{R}^{n \times n} \tag{3}$$

$$cos(\theta) = \left[ \begin{array}{ccc} cos(\theta_1) & \dots & cos(\theta_n) \end{array} \right]^T \in \mathbb{R}^n \quad \mathbb{C}_\theta = diag(cos(\theta)) \in \mathbb{R}^{n \times n} \tag{4}$$

$$sgn(\theta) = \left[ \begin{array}{ccc} sgn(\theta_1) & \dots & sgn(\theta_n) \end{array} \right]^T \in \mathbb{R}^n \tag{5}$$

$$\dot{\theta}^2 = \left[ \begin{array}{ccc} \dot{\theta}_1^2 & \dots & \dot{\theta}_n^2 \end{array} \right]^T \in \mathbb{R}^n \tag{6}$$

$$J = jI_n \qquad L = lI_n \qquad M = mI_n \tag{7}$$

$$K = A^T(DD^T)^{-1}D \quad H = (I_n - \frac{1}{n}ee^T)^{-1}K^T \quad V = A^T(DD^T)^{-1}A \tag{8}$$

Let's define also the rotation matrix to the global frame $(G)$ from the link $i$

$$R^G_{link,i} = \left[ \begin{array}{cc} cos(\theta_i) & -sin(\theta_i) \\ sin(\theta_i) & cos(\theta_i) \end{array} \right] \tag{9}$$

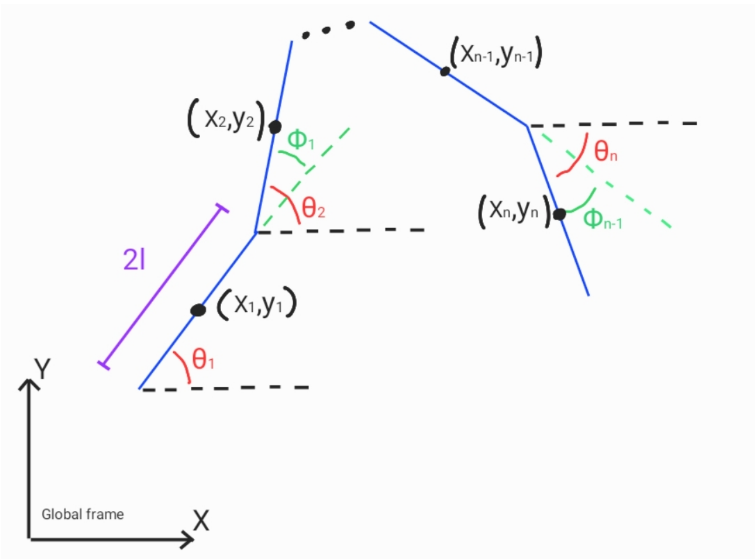



Figura 1

## 2.2 Kinematic

The $\Phi$ angles are the angle of the joint connecting each link

$$\phi_i = \theta_i - \theta_{i-1} \qquad \Phi = \left[ \begin{array}{ccc} \phi_1 & \dots & \phi_{n-1} \end{array} \right] \in \mathbb{R}^{n-1} \tag{10}$$

The total orientation of the snake can be computed as the average of all the angles

$$\bar{\theta} = \frac{1}{n}\sum_{i=1}^{n}\theta_i \tag{11}$$

Than the heading velocity of the center of mass of the robot can be computed as:

$$\bar{v}_t = \dot{p}_x cos(\bar{\theta}) + \dot{p}_y sin(\bar{\theta}) \tag{12}$$

The links are constrained to each other due to the revolute joints so the following is defined:

$$DX + lAcos(\theta) = 0 \qquad DY + lAsin(\theta) = 0 \tag{13}$$

The position of center of mass of each link in the global frame will be:

$$X = -lK^T cos(\theta) + ep_x \qquad Y = -lK^T sin(\theta) + ep_y \tag{14}$$

Deriving 2 times the (14) the following kinematic model can be obtained:

$$\ddot{X} = lH(\mathbb{C}_\theta \dot{\theta}^2 + \mathbb{S}_\theta \ddot{\theta}) \qquad \ddot{Y} = lH(\mathbb{S}_\theta \dot{\theta}^2 - \mathbb{C}_\theta \ddot{\theta}) \tag{15}$$

## 2.3 Hydrodynamic

For this snake underwater robot we will take into account the **drag force** and the **added mass effect** since they both have a non neglectable effect on the motion of the system.
Since the hydrodynamic forces induced by an object moving in underwater are highly non linear, [1] consider a simplified model for this forces, and it adopt the **Morison's equations** to compute the forces between the fluid and the cylindrical links of the snake.
Each link of the robot is considered as an independent segment approximated as a cylinder and the force motion depends only on the transverse motion of the link and we suppose that the fluid motion is not turbolent, this mean $R_e \in \{10^4; 10^5\}$
The following assumption are made:

- The fluid is viscid, incompressible and irrotational in the inertia frame.
- The robot is neutrally buoyant, we assume that the weight of the robot is equal to the buoyancy force
- The current,in the inertial frame, is constant and irrotational.

The fluid forces are function of the current where the current can be characterized by a velocity vector, the current velocity expressed in the global frame is:

$$v_c = \begin{bmatrix} V_{x,i} \\ V_{y,i} \end{bmatrix} \tag{16}$$

We consider the forces acting on each link made by 2 components, the **added mass effect** and the **drag force**, this mean that each link has got 2 drag coefficients ($c_t$ and $c_n$) which describe the force respectively in $x$ and $y$ direction.
The force exerted on each link is given by:

$$f_i^{link,i} = -\hat{C}_A \dot{v}_{r,i}^{link.i} - \hat{C}_D sgn(v_{r,i}^{link,i})(v_{r,i}^{link,i})^2 \tag{17}$$

where $\dot{v}_{r,i}^{link,i} = \ddot{p}_i^{link.i} - \dot{v}_c^{link.i}$ is the relative acceleration of link $i$. The matrix $\hat{C}_A$ and $\hat{C}_D$ depends on the shape of the body and are the following:

$$\hat{C}_D = \begin{bmatrix} c_t & 0 \\ 0 & c_n \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\rho\pi C_f \frac{(b+a)}{2} 2l & 0 \\ 0 & \frac{1}{2}\rho C_D 2a2l \end{bmatrix} \tag{18}$$

$$\hat{C}_A = \begin{bmatrix} \mu_t & 0 \\ 0 & \mu_n \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \rho\pi C_A a^2 2l \end{bmatrix} \tag{19}$$

where $C_D, C_f$ are the drag coefficients and $C_A$ is the added mass coefficient; $x$ component of this last effect is 0 ($\mu_t = 0$) because the added mass effect longitudinally to the surface of the cylinder is neglectable.
Now we have to model the moment of the fluid acting on each link of the snake ($\tau$), in many studies the fluid moments are neglected but in this report, as said in [1], we will consider it to have a more accurate model in terms of hydrodynamic effect and also because they are related to power consume.
We can see in [1] that this fluid torque can be modelled in such way:

$$\tau = -\Lambda_1\ddot{\theta} - \Lambda_2\dot{\theta} - \Lambda_3\dot{\theta}|\dot{\theta}| \tag{20}$$

Where the coefficients $\Lambda_1, \Lambda_2, \Lambda_3$ depends on the shape of the body and fluid properties

$$\begin{cases} \Lambda_1 = \lambda_1 I_n & \lambda_1 = \frac{1}{12}\rho\pi C_M(a^2 - b^2)^2 l^3 \\ \Lambda_2 = \lambda_2 I_n & \lambda_2 = \frac{1}{6}\rho\pi C_f(a + b)l^3 \\ \Lambda_3 = \lambda_3 I_n & \lambda_3 = \frac{1}{8}\rho\pi C_f(a + b)l^4 \end{cases} \tag{21}$$

$\lambda_1$ is related to the added mass effect, while $\lambda_2, \lambda_3$ are related to the drag force

At the end the global frame fluid forces on the link are:

$$f = \begin{bmatrix} f_x \\ f_y \end{bmatrix} = \begin{bmatrix} f_{Ax} \\ f_{Ay} \end{bmatrix} + \begin{bmatrix} f_{Dx}^I \\ f_{Dy}^I \end{bmatrix} + \begin{bmatrix} f_{Dx}^{II} \\ f_{Dy}^{II} \end{bmatrix} \tag{22}$$

where $(f_{Ax}, f_{Ay})$ are the added mass forces, the $(f_{Dx}^{I}, f_{Dy}^{I})$ are linear drag forces and $(f_{Dx}^{II}, f_{Dy}^{II})$ are non-linear drag forces, as defined in [1].

### 2.4 Dynamic

In this section we are going to compute the equation of motion for the underwater snake robot.
The forces balance equations acting on each link is given by:

$$m\ddot{X} = D^T h_x + f_x \qquad m\ddot{Y} = D^T h_y + f_y \tag{23}$$

We can compute the acceleration of the center of mass:

$$\begin{bmatrix} \ddot{p}_x \\ \ddot{p}_y \end{bmatrix} = \frac{1}{nm} \begin{bmatrix} e^T & 0_{1\times n} \\ 0_{1\times n} & e^T \end{bmatrix} f \tag{24}$$

We can see that the acceleration of the center of mass of the robot is the sum of the total forces acting on the robot divided by the mass.
The torque balance equation for each link is expressed in matrix form and it is the following:

$$J\ddot{\theta} = D^T u - l\mathbb{S}_\theta A^T h_x + l\mathbb{C}_\theta A^T h_y + \tau \tag{25}$$

As shown in [1], after some substitution the model of underwater snake robot is:

$$\begin{cases} M_\theta \ddot{\theta} + W_\theta \dot{\theta}^2 + V_\theta \dot{\theta} + \Delta_3 |\dot{\theta}| \dot{\theta} + l\mathbb{S}_\theta K d_{Dx} + l\mathbb{C}_\theta K f_{Dy} = D^T u \\ \begin{bmatrix} \ddot{p}_x \\ \ddot{p}_y \end{bmatrix} = \frac{1}{nm} \begin{bmatrix} e^T & 0_{1\times n} \\ 0_{1\times n} & e^T \end{bmatrix} f \end{cases} \tag{26}$$

Where the matrix $M_\theta, W_\theta, V_\theta$ are defined as [1].

## 3 Simulation

All the variables we need are initialized with an "init-function" called `init_snake` loaded in Simulink model.
The objective of the simulation is study 2 type of gait for snake robot in different scenarios:

- **Lateral undulation**
- **Eel-like motion**

For gait visualization, an external MATLAB script will be used.

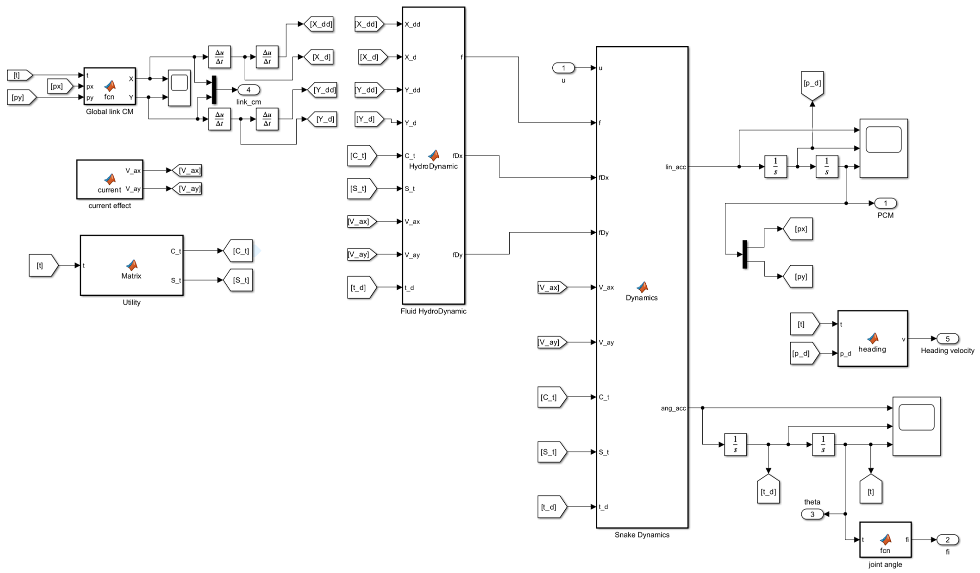



Figura 2: Dynamic model of snake robot with hydrodynamic effects

## 3.1 Lateral undulation

Lateral undulation is the fastest and most common form of snake locomotion; the lateral undulation is realized by creating continuous body waves that are propagated backwards from head to tail, so the joint of the snake robot has follow the sinusoidal reference:

$$\phi_i^* = \alpha sin(\omega t + (i-1)\beta) + \gamma \tag{27}$$

where $\alpha$ is the amplitude of the wave, $\omega$ is the angular frequency of the sinusoidal join motion, $\beta$ determines the phase shift between the sequential joints, and $\gamma$ is the joint offset used to control the direction of the motion.

## 3.2 Eel-like motion

Eel-like motion is performed with lateral axial undulations with increasing amplitude from nose to tail. The equation for the eel-like motion is made by controlling each joint of the snake with the following reference signal:

$$\phi_i^* = \alpha \left(\frac{n-i}{n+1}\right) sin(\omega t + (i-1)\beta) + \gamma \tag{28}$$

where the parameter $\alpha \left(\frac{n-i}{n+1}\right)$ corresponds to the increasing amplitude.

## 3.3 Controller

For this simulation a simple **PD** controller is used, so the control input $u$ is chosen in the following way:

$$u_i = K_{p,i}(\phi_i^* - \phi_i) + K_{d,i}\frac{d}{dt}(\phi_i^* - \phi_i) \tag{29}$$

Where $K_{p,i}, K_{d,i} > 0$ [1]

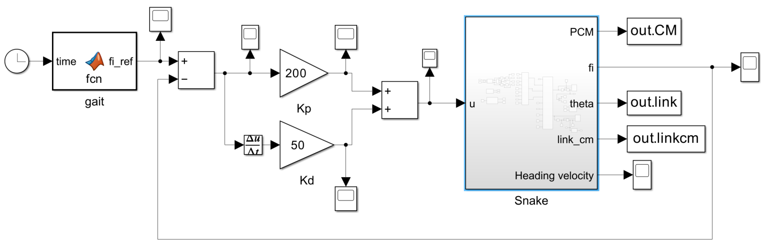



Figura 3: PD controller for snake robot

## 3.4 Parameters

| Parameter | Value |
|---|---|
| $n$ | 7 |
| $m$ | 0.55 $Kg$ |
| $a$ | 0.0375 $m$ |
| $l$ | 0.0625 $m$ |
| $R_e$ | $10^5$ |
| $\rho$ | 1000 $Kg/m^3$ |
| $C_f$ | 0.01 |
| $C_D$ | 1 |
| $C_A$ | 1 |
| $C_M$ | 1 |
| $\alpha$ | $\frac{\pi}{6}$ |
| $\beta$ | $\frac{\pi}{6}$ |
| $\gamma$ | 0 |
| $\omega$ | $\frac{7}{18}\pi\ rad/s$ |
| $K_p$ | 200 |
| $K_d$ | 50 |

Differently from [1] we considered the case of perfect cylindrical link so the parameters are set as the table above; values of radius and length are taken from [2]. Since we made the assumption of neutrally buoyancy we compute the mass of each link in this way:

$$mg = \rho\Delta g \Rightarrow m = \rho\Delta \Rightarrow m = 1000\pi a^2 2l \tag{30}$$

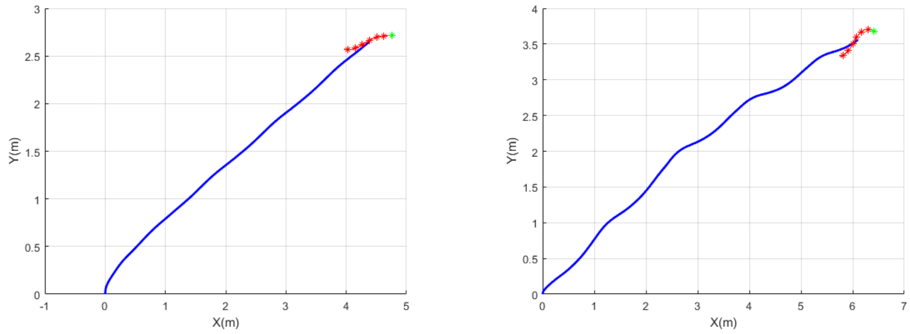
where $\Delta$ is the volume of the link.
For the simulation MATLAB/Simulink-2023a with $25s$ seconds of simulation time and "auto" solver; "absolute" and "relative" tolerance equal to $10^{-4}$ as [1].

# 4 Results

In this section we will discuss the results of the simulation where we considered different scenario for each type of motion; this study is focused on planar motion, this mean we did not take into account the vertical motion thanks to the assumption that the buoyancy effect and weight of the snake are equivalent and opposite. We will analize 4 cases:

- `full_hydro` in this scenario we consider added mass effect, linear and non-linear drag effect and current effect
- `no_current` this scenario is the same of the previous one but without current effect
- `linear_drag` in this case we consider only added mass effect and linear drag effect
- `no_hydro` in this scenario no hydrodynamic effect will be considered

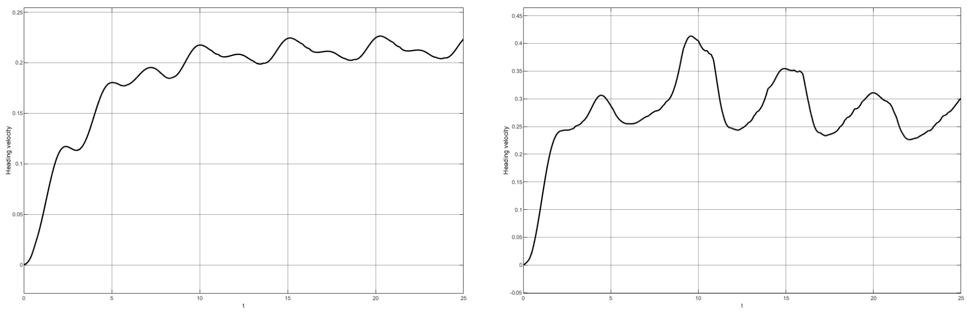
## 4.1 Full Hydro




((a)) Eel-like ((b)) Lateral undulation

Figura 4: Center of mass

We can see that the **Eel-like** motion is slower than the **Lateral** one since the center of mass of the snake travelled less distance in the same time interval but instead of the **Lateral**, a little bit straighter; to confirm this conclusion the heading velocity plots are shown below:

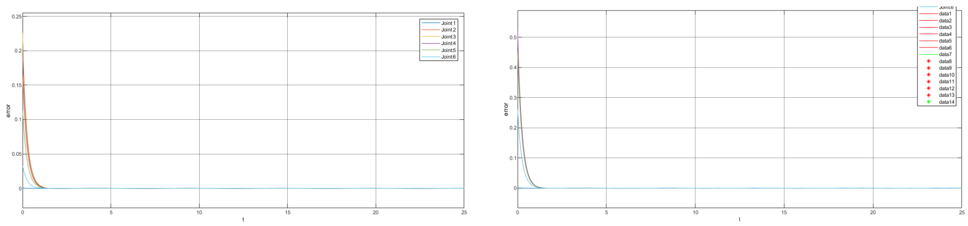



((a)) Eel-like ((b)) Lateral undulation

Figura 5: Heading velocity

This case is really important because we can see how the hydrodynamic effect acts on a submerged body; since we know that the hydrodynamic effect are related to the relative motion between body and fluid as we can see in this case the hydrodynamic effect is significant in the $y$ direction and it is higher when the motion is higher because it depends on the body velocity.

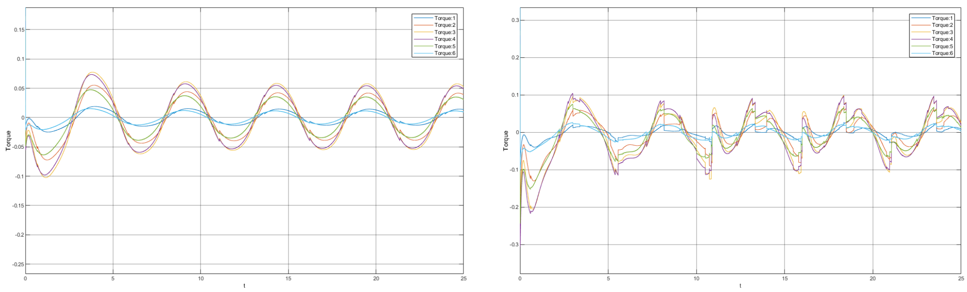
From a control view point will be shown the **joint error** and **torque**.



((a)) Eel-like ((b)) Lateral undulation

Figura 6: Joint error

We can see the joint error goes to 0 in less than $2.5s$ for both gait this mean that a **PD** controller is enough for this system and this also mean that the joints of the snake are following the desired reference signals.

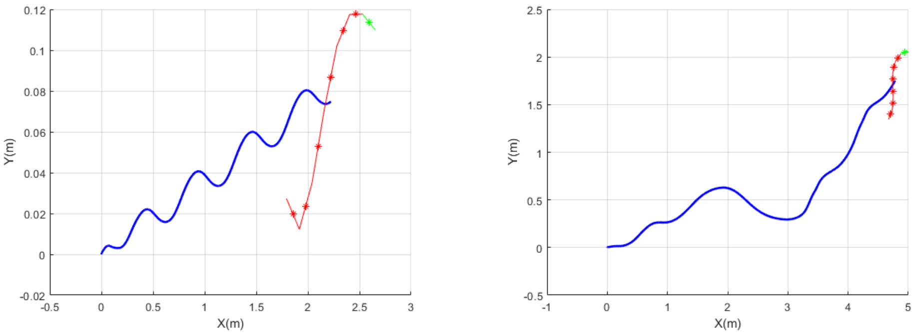


((a)) Eel-like ((b)) Lateral undulation

Figura 7: Torque input

We can see the control inputs for **Eel-like** has maximum values less than the control input for the **Lateral**. The lateral control input are more distorted than the Eel-like because this kind of motion has higher joint angle variation. In conclusion the **Eel-like** motion is more suitable for environment with some obstacles (for example rock or small space in the bottom of the sea) because the motion of the snake is almost linear but is slower; while the **Lateral** motion is good for a free obstacles environment (like open sea) because is faster and in a free obstacles environment is not really important the accuracy of the path.
Snake-like underwater robot full-hydro video Eel-like
Snake-like underwater robot full-hydro video Lateral undulation

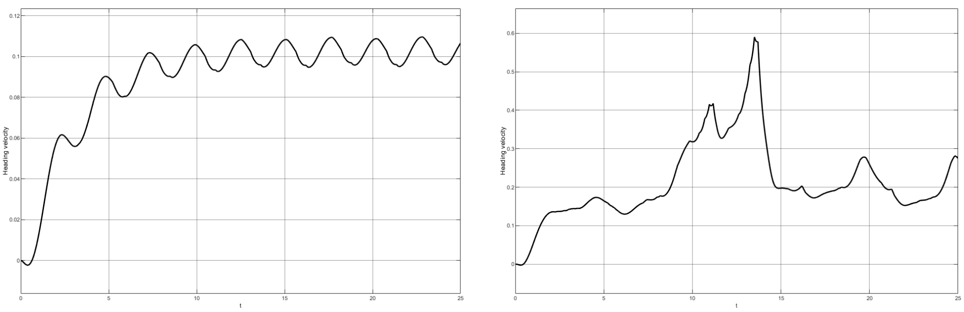
## 4.2 No Current




((a)) Eel-like ((b)) Lateral undulation

Figura 8: Center of mass

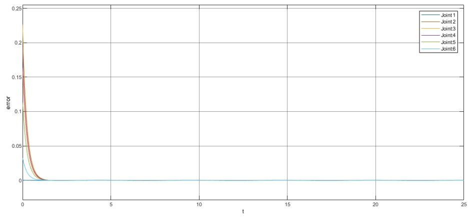
We can see that the previous conclusion was right, the **Eel-like** is slower than the **Lateral** but in this case is more obvious, we can suppose that the current of the previous case helped the snake during the travel (because the current was in the same direction of the motion). We can also see that the **Eel-like** in this case did not follow a straight line but has got some oscillation, while the **Lateral** has the same oscillation of the previous case but with a deviation. We can conclude that the current in some ways helps the snake for travel distance (if the current is in the same direction of motion) and helps also to stabilize the snake



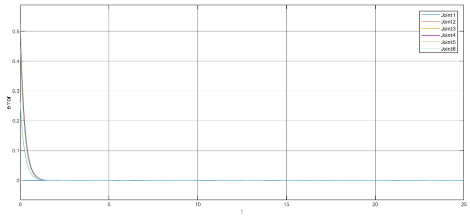
((a)) Eel-like ((b)) Lateral undulation

Figura 9: Heading velocity

This plots show better the difference in speed between the two kind of motion, in particular the **Eel-like** is slower with a speed oscillating around 0.1, so the snake is travelling at a "constant" speed we can say. The **Lateral** instead has a higher value for the speed with a peak when the "deviation" happened, but this speed is varying a lot during the travel.



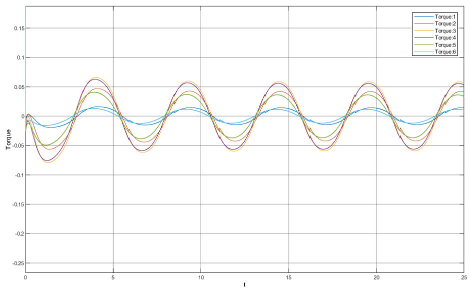
((a)) Eel-like



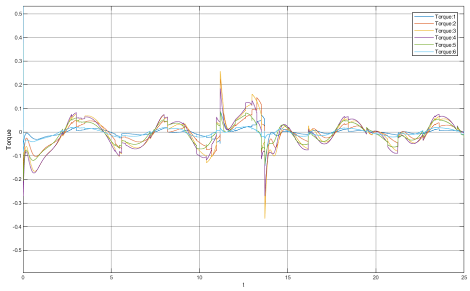
((b)) Lateral undulation

Figura 10: Joint error

Also in this case the joint error goes to 0 in a relative short time, so from a control view point the error has non significant changes to the respect of the first case



((a)) Eel-like

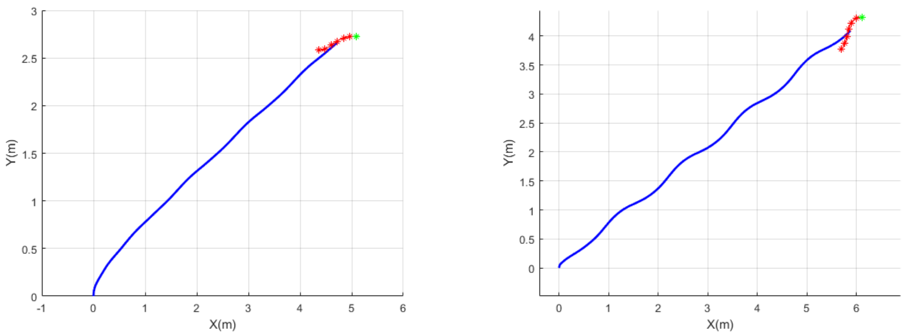


((b)) Lateral undulation

Figura 11: Torque input

Also for the torque the results are very similar to the first case; for the **Lateral** we can see two spikes so we can suppose that the "deviation" is due to drag effects and the controller try to compensate it.
Snake-like underwater robot No-current video Eel-like
Snake-like underwater robot No-current video Lateral undulation

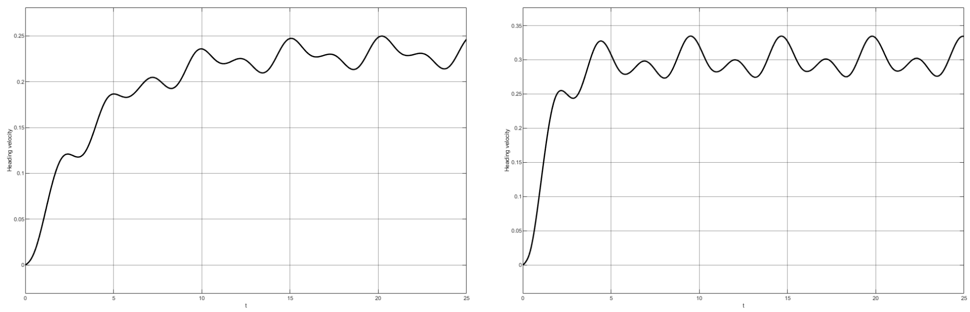
## 4.3 Linear Drag




((a)) Eel-like ((b)) Lateral undulation

Figura 12: Center of mass

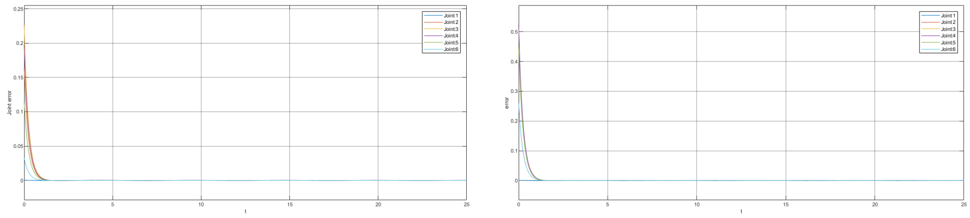
The figure above show that if the non-linear drag effect are neglected more or less the center of mass follow the same path to respect of first case, so we can believe that the non linearity does not affect too much, in terms of path, the system; this can be seen also in the picture below.



((a)) Eel-like ((b)) Lateral undulation

Figura 13: Heading velocity

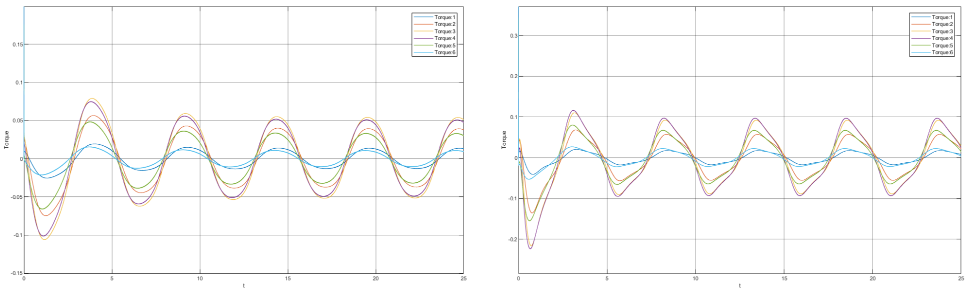
Now the heading velocity show a periodic oscillation along a value in both cases, so we can conclude that the **Eel-like** is not affected, in terms of velocity, by the non-linear effects while the **Lateral** is affected; in fact in the first simulation the heading velocity for **Lateral** motion was varying a lot, in this case has a periodic oscillation



((a)) Eel-like ((b)) Lateral undulation

Figura 14: Joint error

From a view point of controller, we can notice that the presence of only linear terms does not change significantly the controller performance

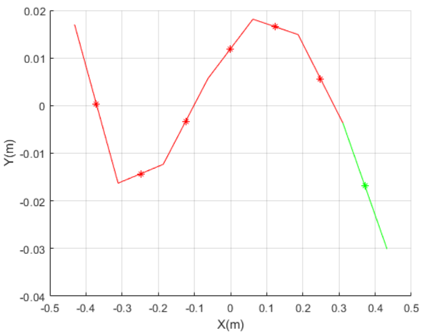


((a)) Eel-like ((b)) Lateral undulation

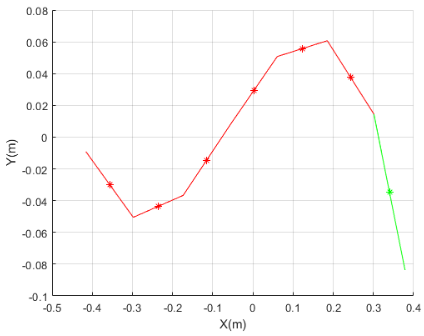
Figura 15: Torque input

Now we have some interesting results, for the **Eel-like** the control input is the same, so also in this case the non linearity absence does not cause significant change while in the **Lateral** we can see that the torque wawes signal are more "clear" than the first case. However the torque values for both motion are the same to respect of the first simulation so, in terms of torque amplitude, the non linearity does not affect significantly. This is a good results because in the real cases is not possible to avoid non-linearity, so we have shown that if we consider a simplified model of the snake robot, avoiding non-linearity, the performance are very similar to the real cases.
Snake-like underwater robot Linear-drag video Eel-like
Snake-like underwater robot Linear-drag video Lateral undulation

## 4.4 No Hydro



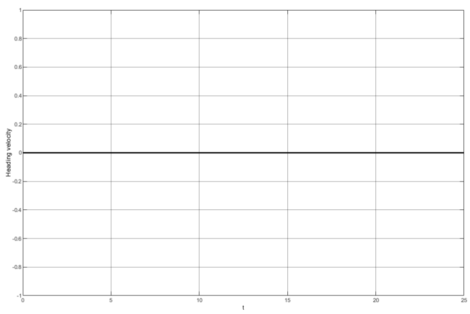

((a)) Eel-like



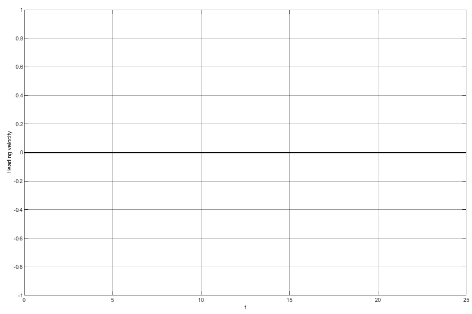

((b)) Lateral undulation

Figura 16: Center of mass

From pictures above we can notice that there is no CoM path for the simulation, this happens because the snake robot is not moving during the time, this conclusion is shown better in the pictures below



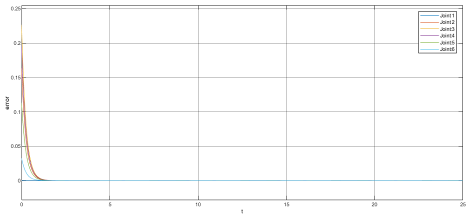
((a)) Eel-like



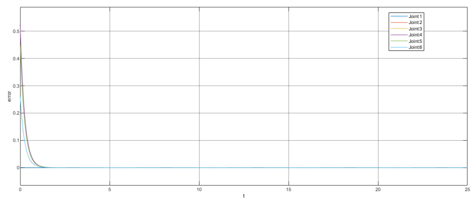
((b)) Lateral undulation

Figura 17: Heading velocity

The heading velocity is 0 during all the time. This happens because this kind of underwater robot is particular, its movement is not done by propellers but is its own body the propeller; with some particular movements the snake push itself in the fluid. Since the hydrodynamic effect are all set to 0 there is no drag coefficients, this mean the robot in not encountering resistance in the water so it can not "push itself". As results its movement is not pushing the robot, also the current is not pushing for same reasons, so the whole system stay in the same position like it is moving in the space.



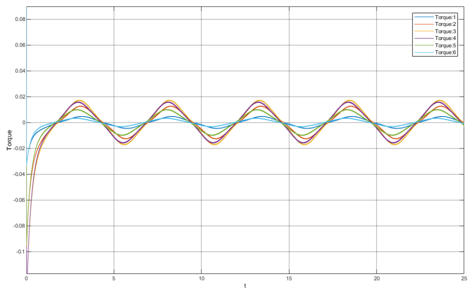
((a)) Eel-like



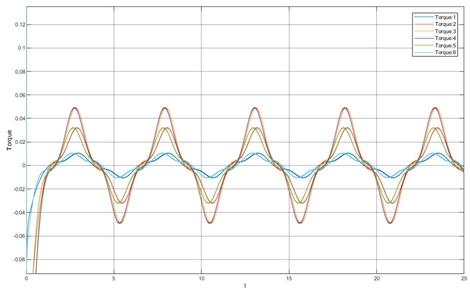
((b)) Lateral undulation

Figura 18: Joint error

Of course the controller does not encounter any problem in following the desired reference signal.



((a)) Eel-like



((b)) Lateral undulation

Figura 19: Torque input

The torque wawes are quite sinusoidal since the controller does not have to compensate for any hydrodynamic effect, so the torque are only moving the snake actuator.
Snake-like underwater robot No-hydro video Eel-like
Snake-like underwater robot No-hydro video Lateral undulation

# 5 References

1 E. Kelasidi, K. Y. Pettersen, J. T. Gravdahl and P. Liljeback: "Modeling of underwater snake robots"

2 Anfan ZHANG, Shugen MA, Bin LI, Minghui WANG, Xian GUO and Yuechao WANG: "Adaptive controller design for underwater snakerobot with unmatched uncertainties"