Unit 4: Carrier Transport, Recombination, and Generation

Lecture 4.1: The Landauer approach

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Carrier transport

The flow of charge carriers produces currents, which can be controlled to produce electronic devices.

Our goal is to understand the flow of charge carriers.

In learning chemistry, we begin with the simplest atom – the hydrogen atom and then to proceed to more complex atoms, molecules, compounds, etc.

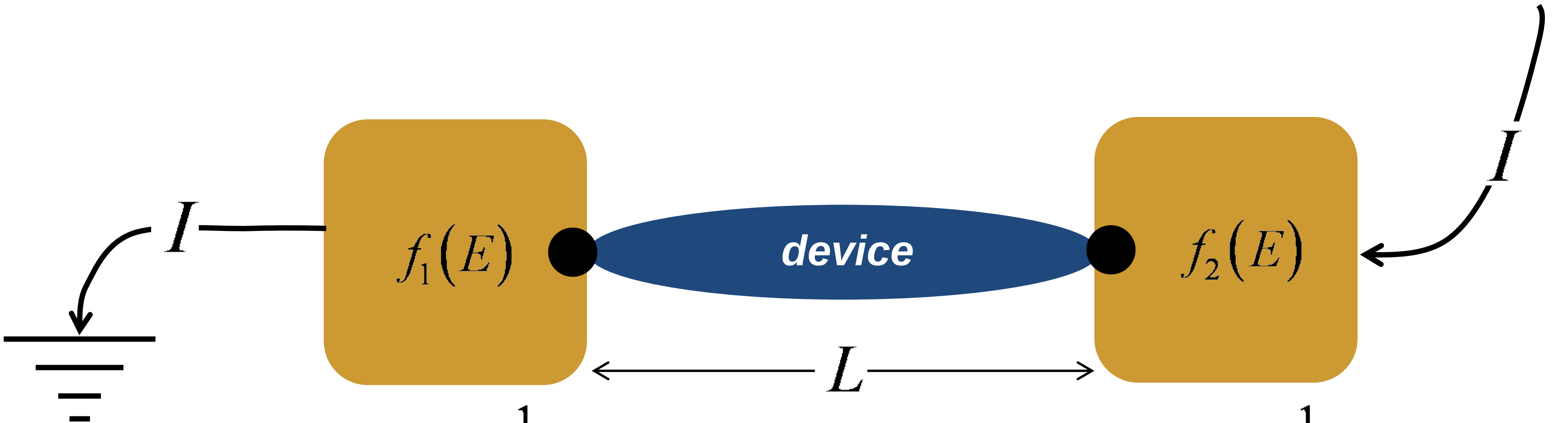
We will take a similar approach - begin by understanding the current in a small, nanodevice, and then extend that understanding to large, bulk semiconductors.

Comments

Our approach will be very simple, intuitive, and descriptive. Those who want a deeper understanding, should consult:

Supriyo Datta, *Lessons from Nanoelectronics*, 2nd Ed., Part A: Basic Concepts, World Scientific Publishing Co., Singapore, 2017.

Current in a nano device



$$f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T}} \qquad f_2(E) = \frac{1}{1 + e^{(E - E_{F2})/k_B T}}$$

How does the current that flows in contact 2, depend on the voltages on the two contacts?

Current

$$I = \frac{2q}{h}\int \mathcal{T}(E)M(E)\big(f_1(E) - f_2(E)\big)dE$$

Fundamental constants

Transmission:

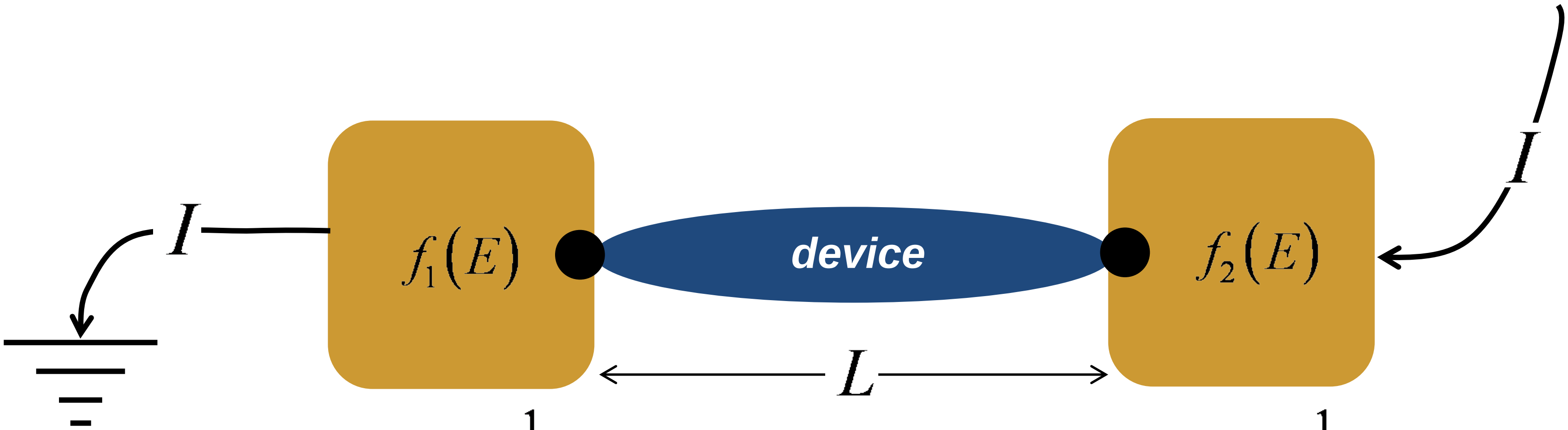
$0 < \mathcal{T}(E) < 1$

No. of Channels

Ideal "Landauer" contacts

Can be derived from rigorous transport theory (the Boltzmann equation), but this expression is intuitively easy to understand.

What is transmission?

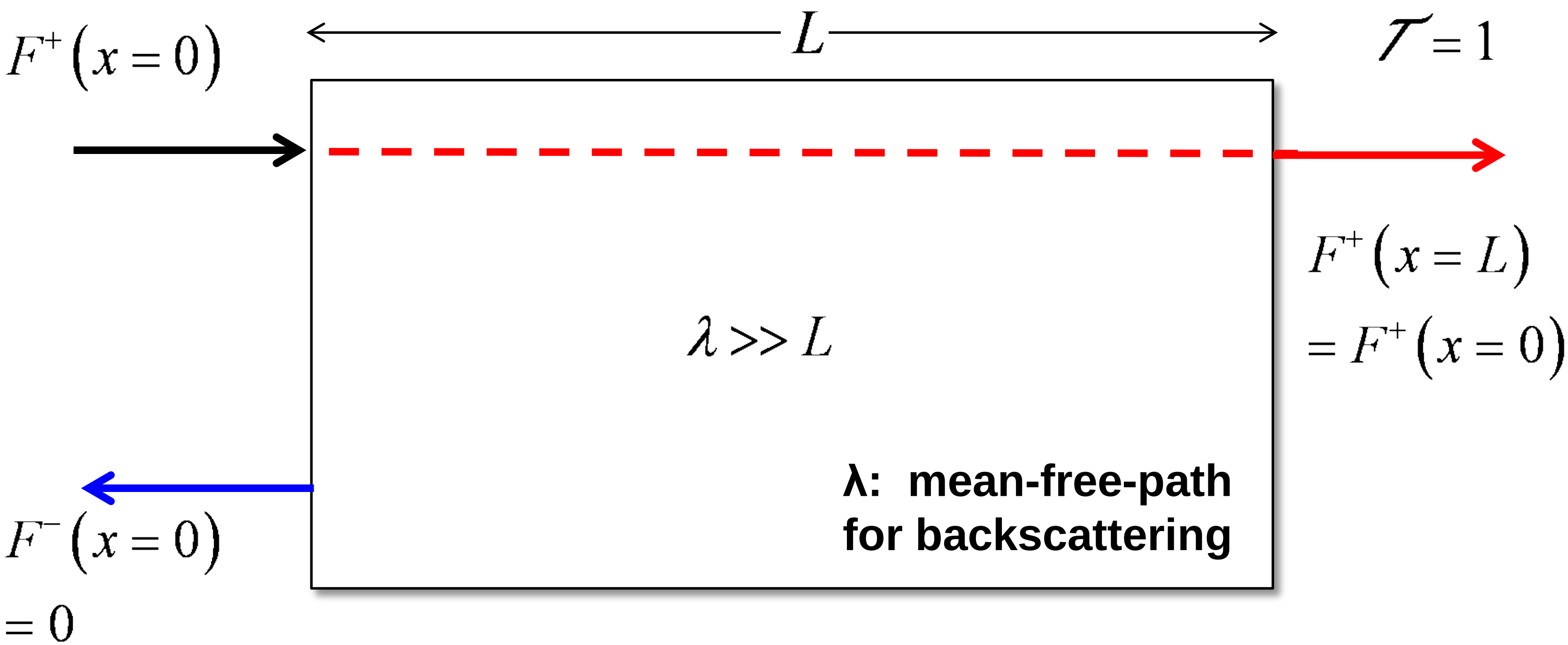


$$f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T}}$$

$$f_2(E) = \frac{1}{1 + e^{(E - E_{F2})/k_B T}}$$

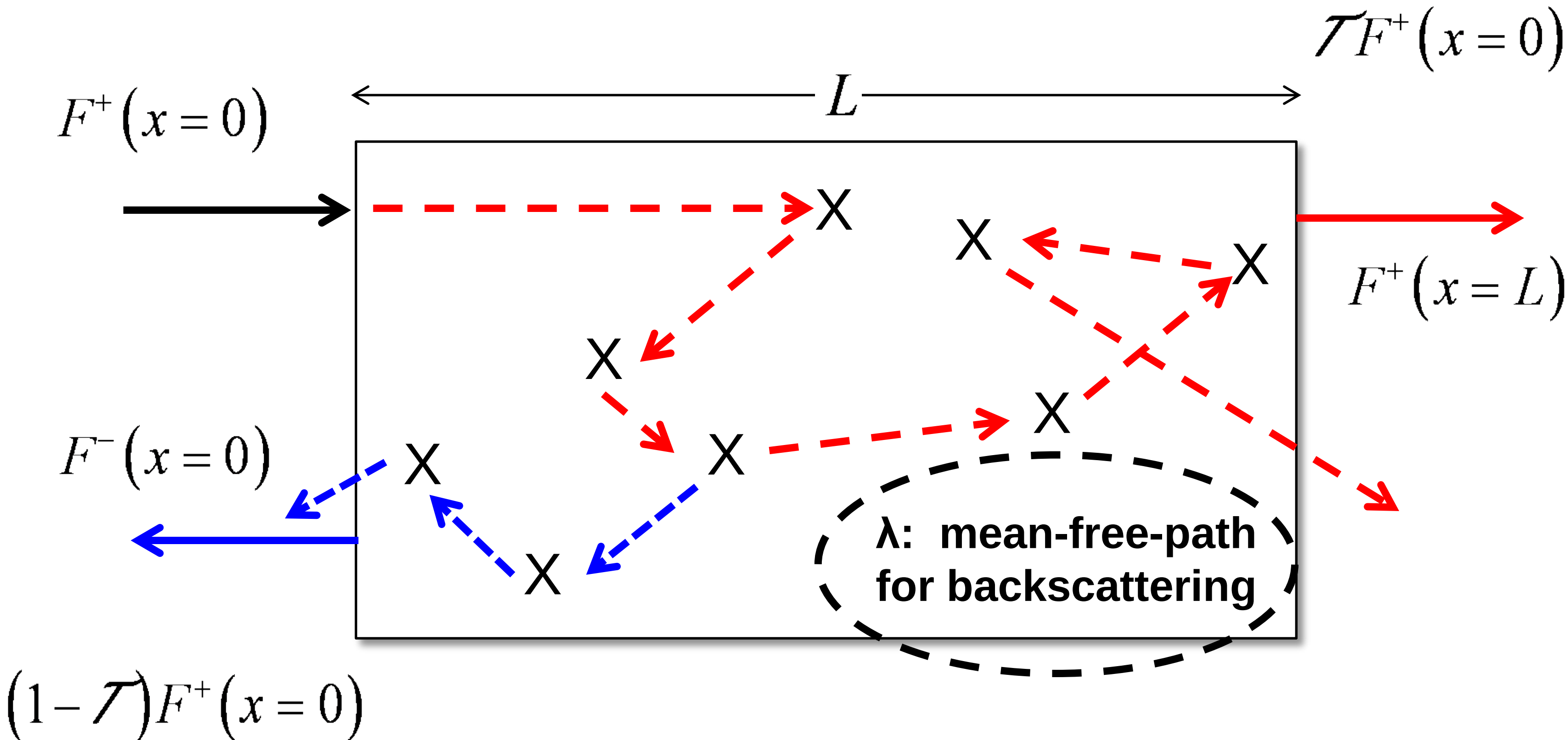
$$I = \frac{2q}{h} \int \mathcal{T}(E) M(E) (f_1 - f_2) dE$$

Transmission (ballistic)



ballistic transport: $\mathcal{T} = 1$

Transmission (diffusive)



"diffusive transport" $\mathcal{T} << 1$

Transmission

$$\mathcal{T}(E) = \frac{\lambda(E)}{\lambda(E) + L}$$

λ is the "mean-free-path for backscattering"

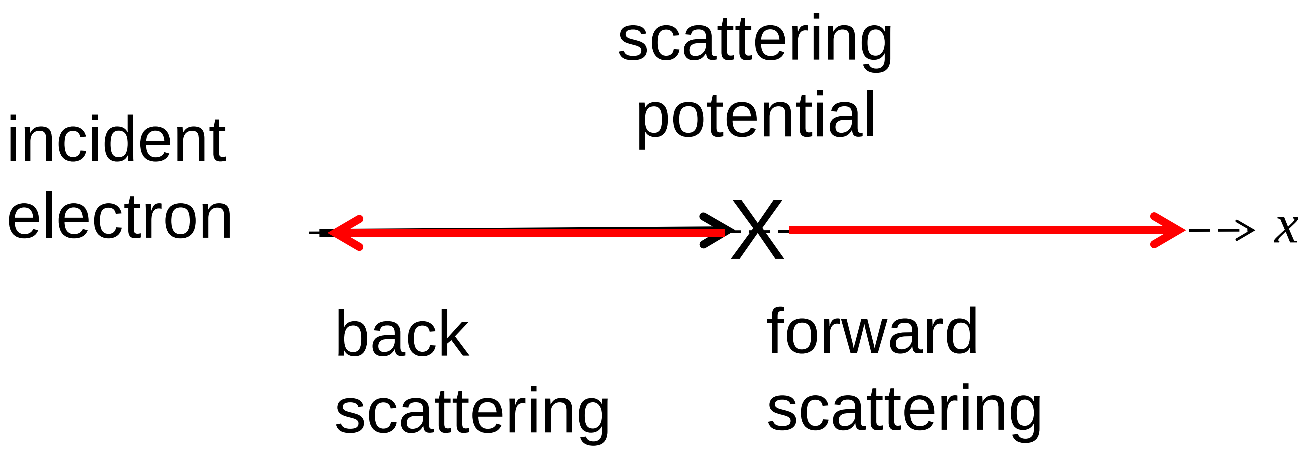
This expression can be derived with relatively few assumptions.

1) Diffusive: $L >> \lambda \quad \mathcal{T} = \frac{\lambda}{L} << 1$

2) Ballistic: $L << \lambda \quad \mathcal{T} = 1$

$$\lambda(E) \neq \upsilon(E)\tau(E) = \Lambda$$

MFP for **backscattering** in 1D



If we assume that the scattering is ***isotropic*** (equal probability of scattering forward or back) then average time between **backscattering** events is 2τ.

$$\lambda(E) = 2\upsilon(E)\tau(E) \qquad \left\{\Lambda(E) = \upsilon(E)\tau(E)\right\}$$

MFP and diffusion coefficient

Recall Fick's Law of Diffusion: $F = -D\dfrac{dc}{dx} \quad \dfrac{\#}{\text{cm}^2\text{-s}}$

Concentration of diffusing particles: $c \quad \dfrac{1}{\text{cm}^3}$

Diffusion coefficient: $D \quad \dfrac{\text{cm}^2}{s}$

It turns out that there is a simple relation between the MFP and the diffusion coefficient: $D = \dfrac{\upsilon_T \lambda_0}{2}$

MFP and diffusion coefficient

$$\boxed{D = \frac{\upsilon_T \lambda_0}{2}}$$

The MFP, λ_0, is assumed to be independent of energy.

Nondegenerate conditions are assumed.

$$\upsilon_T = \sqrt{\frac{2k_B T}{\pi m^*}}$$

The uni-directional thermal velocity is the average velocity of electrons travelling on the +x, or –x, etc. direction in equilibrium.

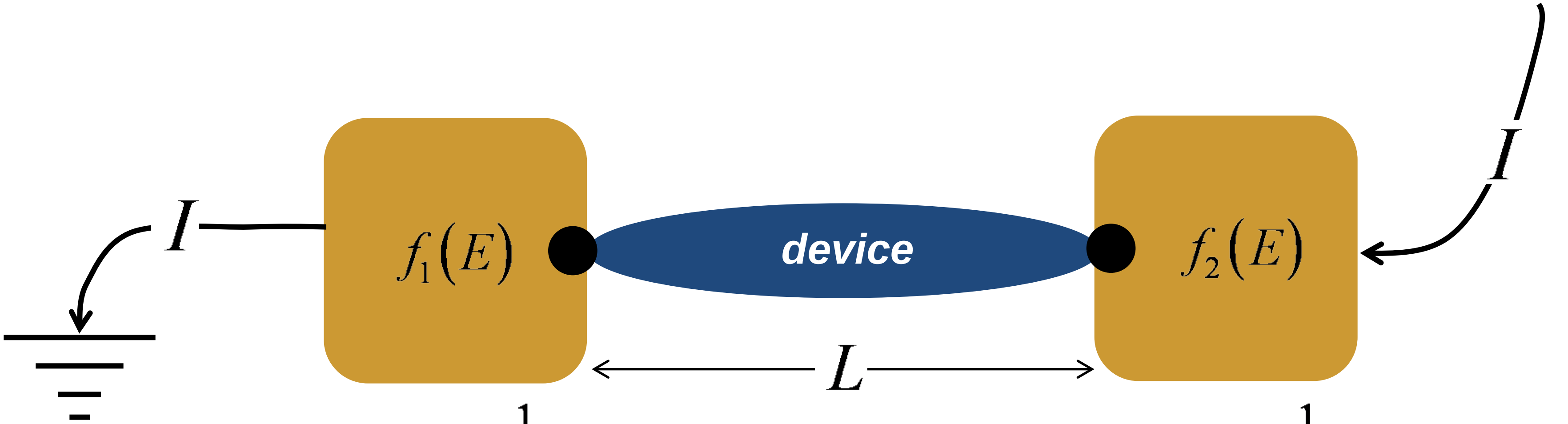
MFP and diffusion coefficient

$$\boxed{D = \frac{\upsilon_T \lambda_0}{2}}$$

This is a useful formula because is allows us to deduce the MFP from a measured *D*.

For more discussion, see Sec. 6.2 in Mark Lundstrom and Changwook Jeong, *Near-Equilibrium Transport*, World Scientific, 2013.

What is a channel?

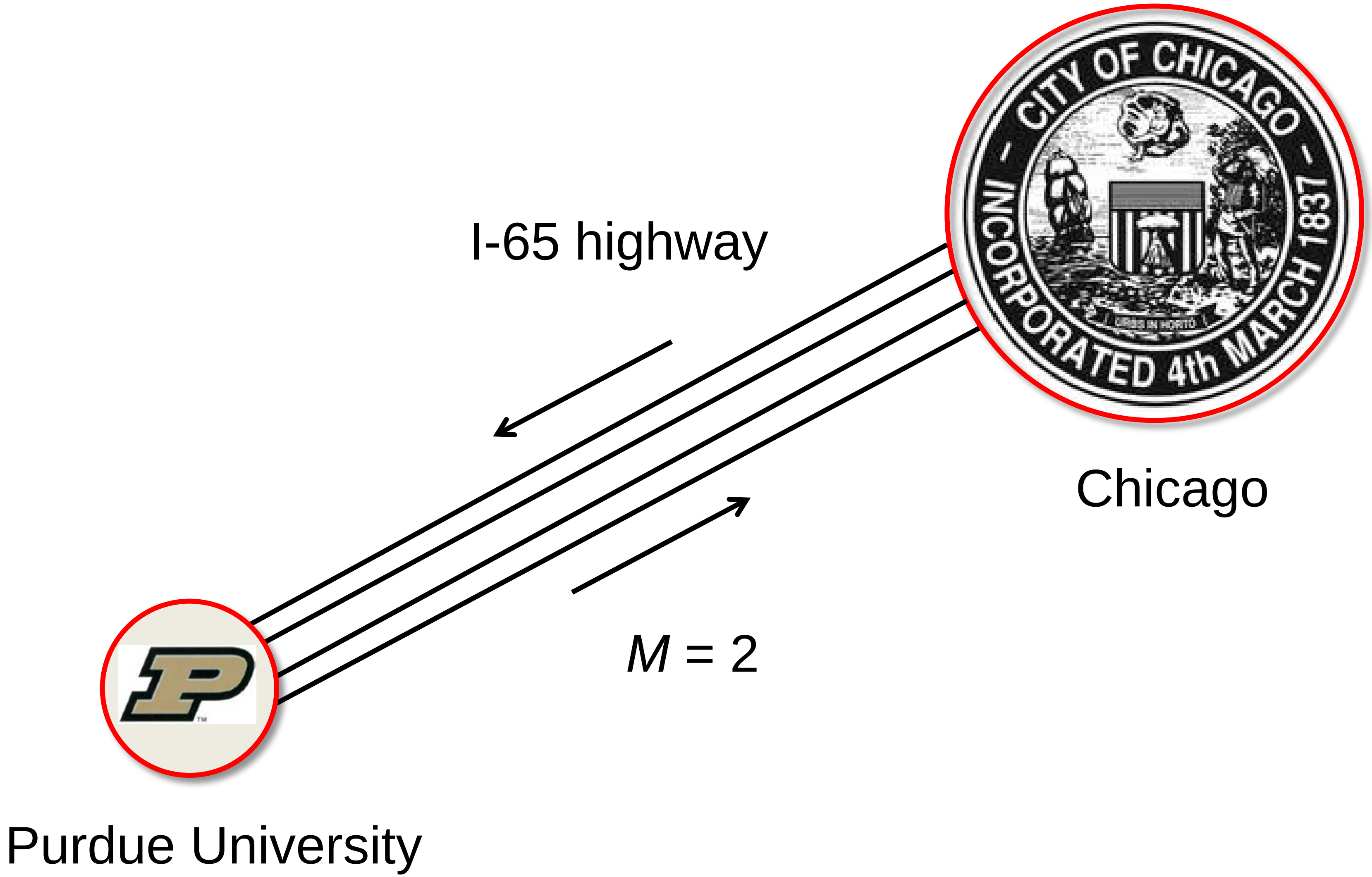


$$f_1(E) = \frac{1}{1+e^{(E-E_{F1})/k_BT}} \qquad f_2(E) = \frac{1}{1+e^{(E-E_{F2})/k_BT}}$$

$$I = \frac{2q}{h}\int \mathcal{T}(E)M(E)(f_1 - f_2)\,dE$$

(channels are also called “modes”)

Channels are like lanes on a highway

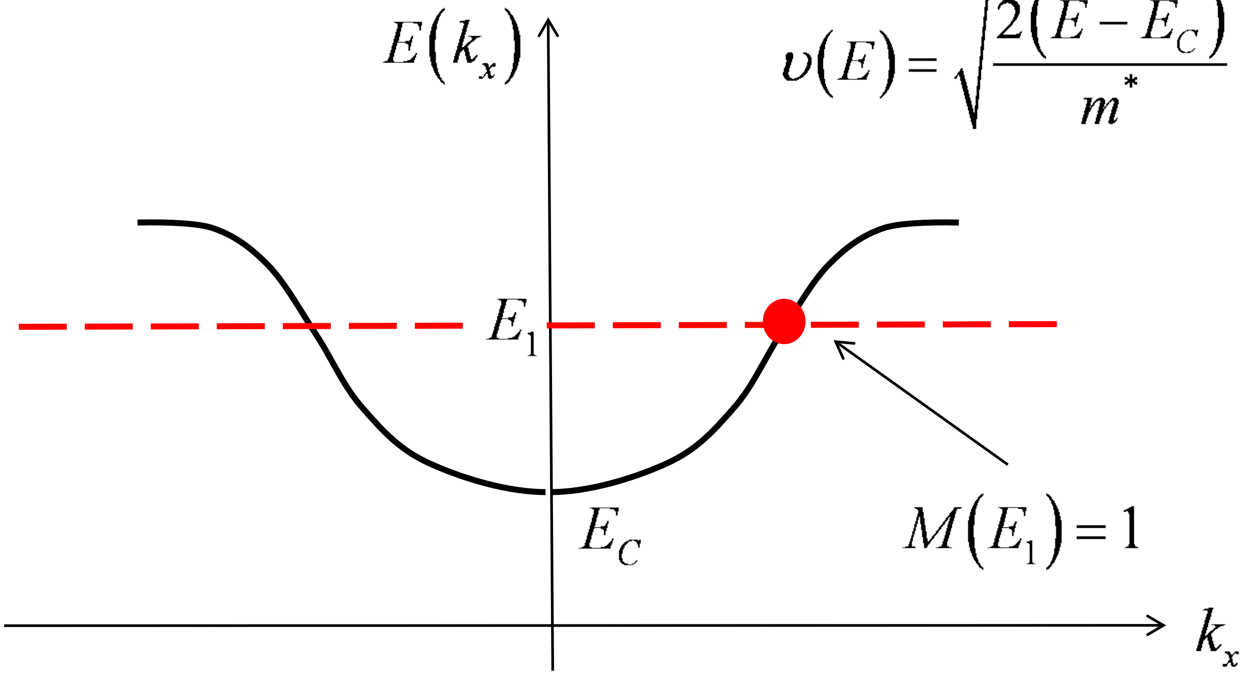


Channels (modes) from *E*(*k*)

A channel is a state with a velocity.

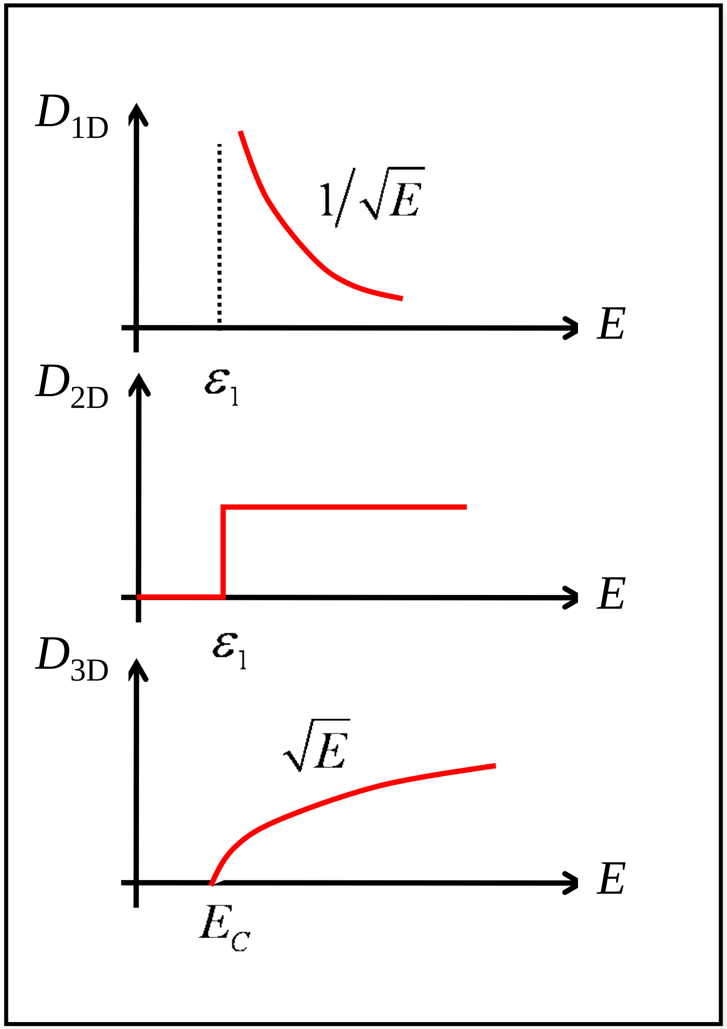
$$M(E) = A\frac{h}{4}\langle \upsilon_x^+(E)\rangle D_{3D}(E)$$

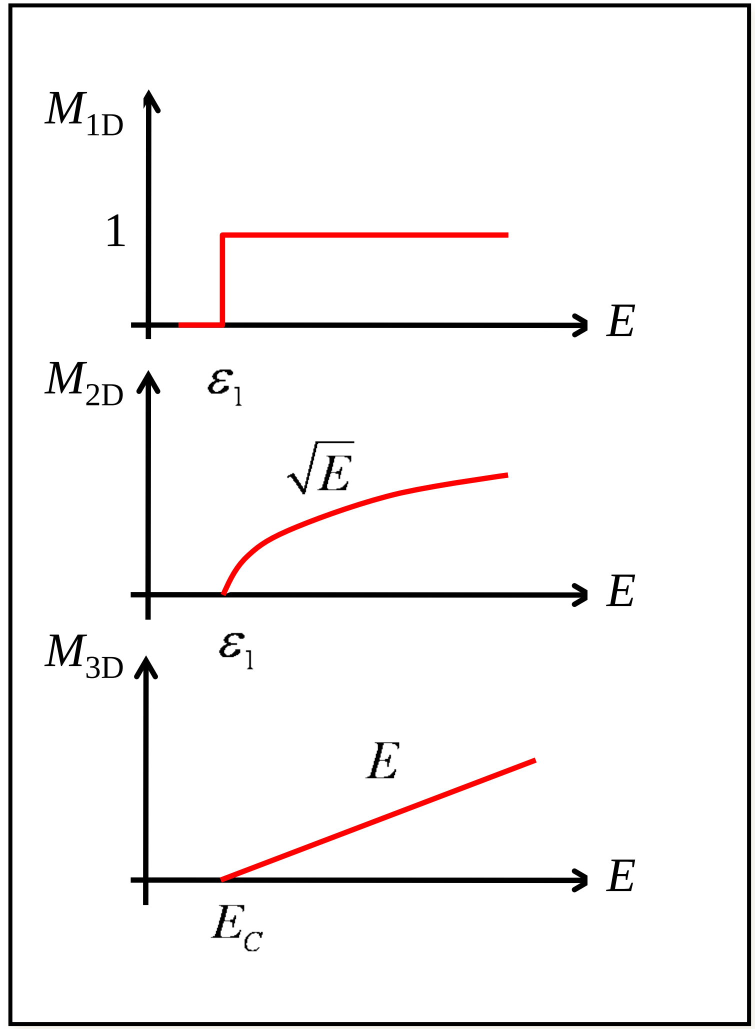
$$\upsilon(E) = \sqrt{\frac{2(E - E_C)}{m^*}}$$



(Easily generalized to arbitrary band structures in 2D and 3D.)

$M(E)$ vs. $DOS(E)$ (parabolic bands)





Channels, transmission, and MFP

$$\mathcal{T}(E) = \frac{\lambda(E)}{\lambda(E) + L}$$

1D $\quad M(E) = 1 \qquad \lambda(E) = 2\Lambda(E)$

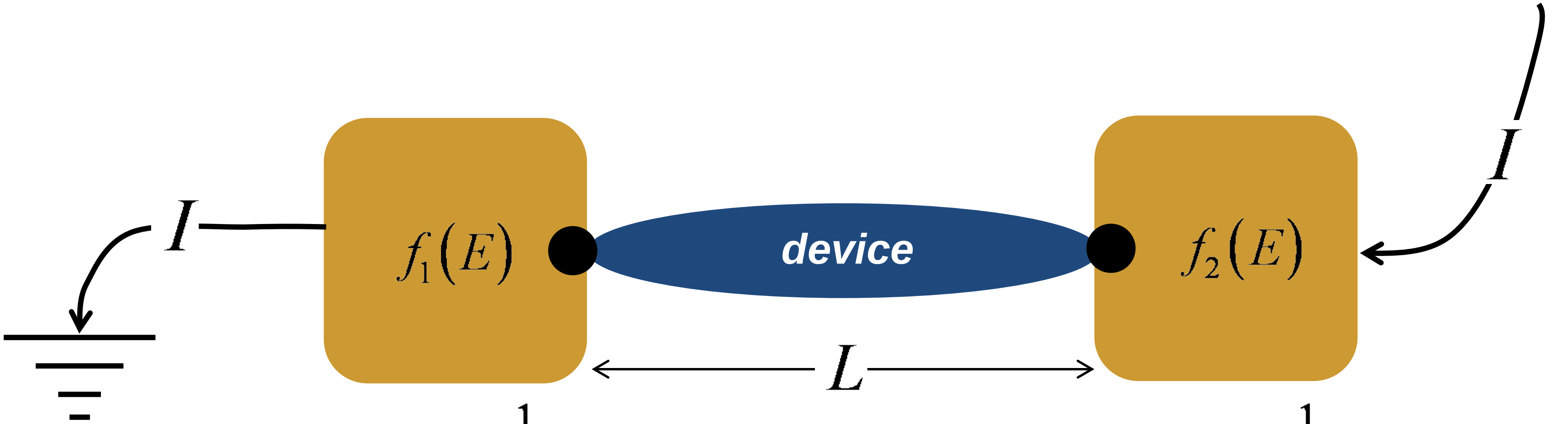
2D $\quad M(E) = W\frac{\sqrt{2m^*(E-E_C)}}{\pi\hbar} \qquad \lambda(E) = \frac{\pi}{2}\Lambda(E)$

3D $\quad M(E) = A\frac{m^*}{2\pi\hbar^2}(E-E_C) \qquad \lambda(E) = \frac{4}{3}\Lambda(E)$

Parabolic bands + large structures with many channels.

$$\Lambda(E) = \upsilon(E)\tau(E)$$

Fermi window

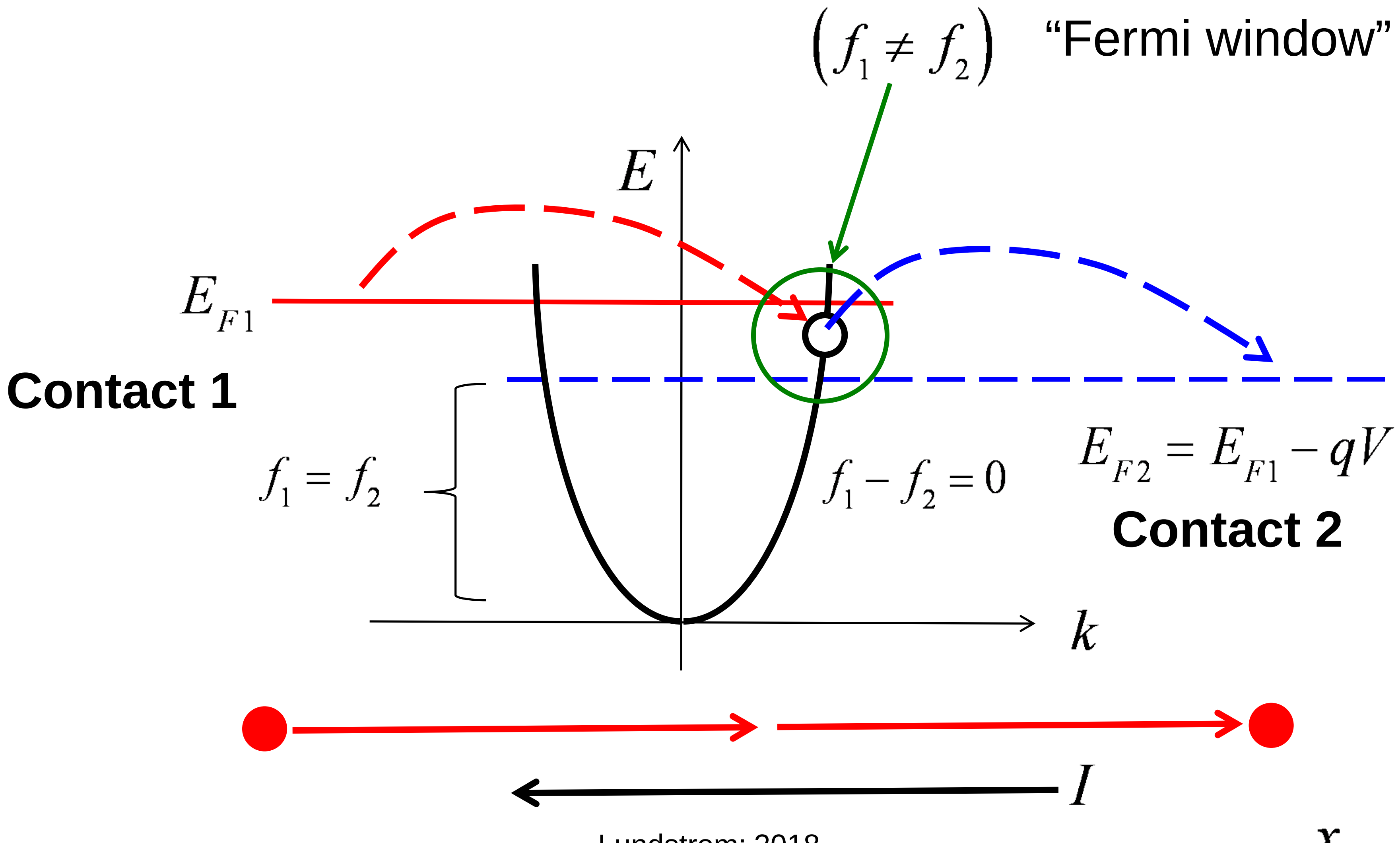


$$f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T}} \qquad f_2(E) = \frac{1}{1 + e^{(E - E_{F2})/k_B T}}$$

$$I = \frac{2q}{h} \int \mathcal{T}(E) M(E) (f_1 - f_2) \, dE$$

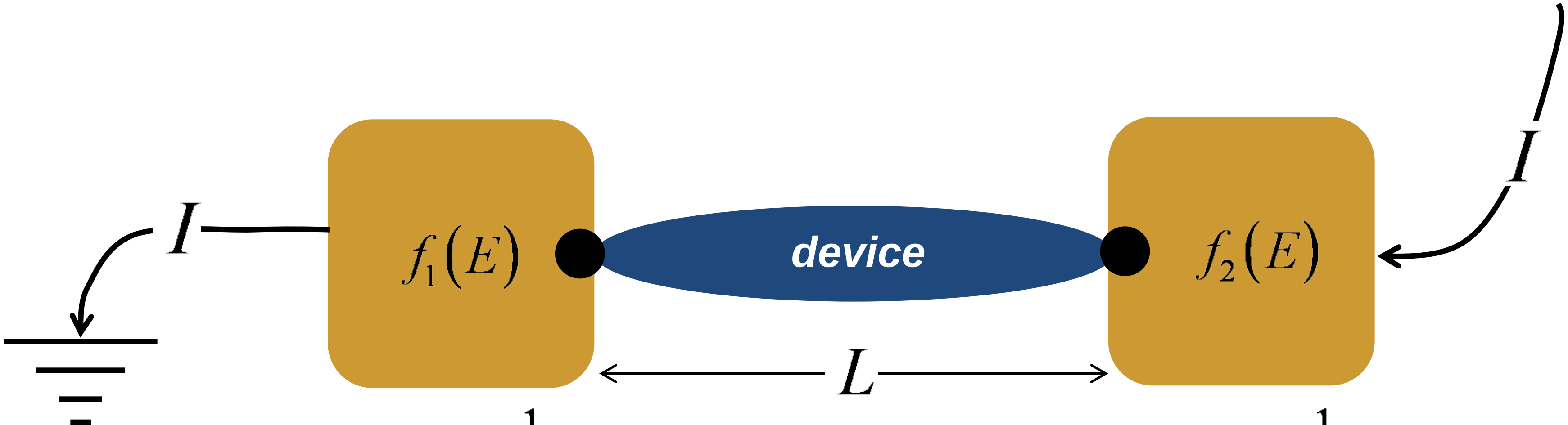
The range of energies over which $(f_1 - f_2) \neq 0$

How current flows



Lundstrom: 2018

Summary



$$f_1(E) = \frac{1}{1 + e^{(E-E_{F1})/k_BT}} \qquad f_2(E) = \frac{1}{1 + e^{(E-E_{F2})/k_BT}}$$

$$I = \frac{2q}{h} \int \mathcal{T}(E) M(E) (f_1 - f_2) dE$$

Can be used to describe the current in small and large devices and in short to long devices.