Primer on Semiconductors

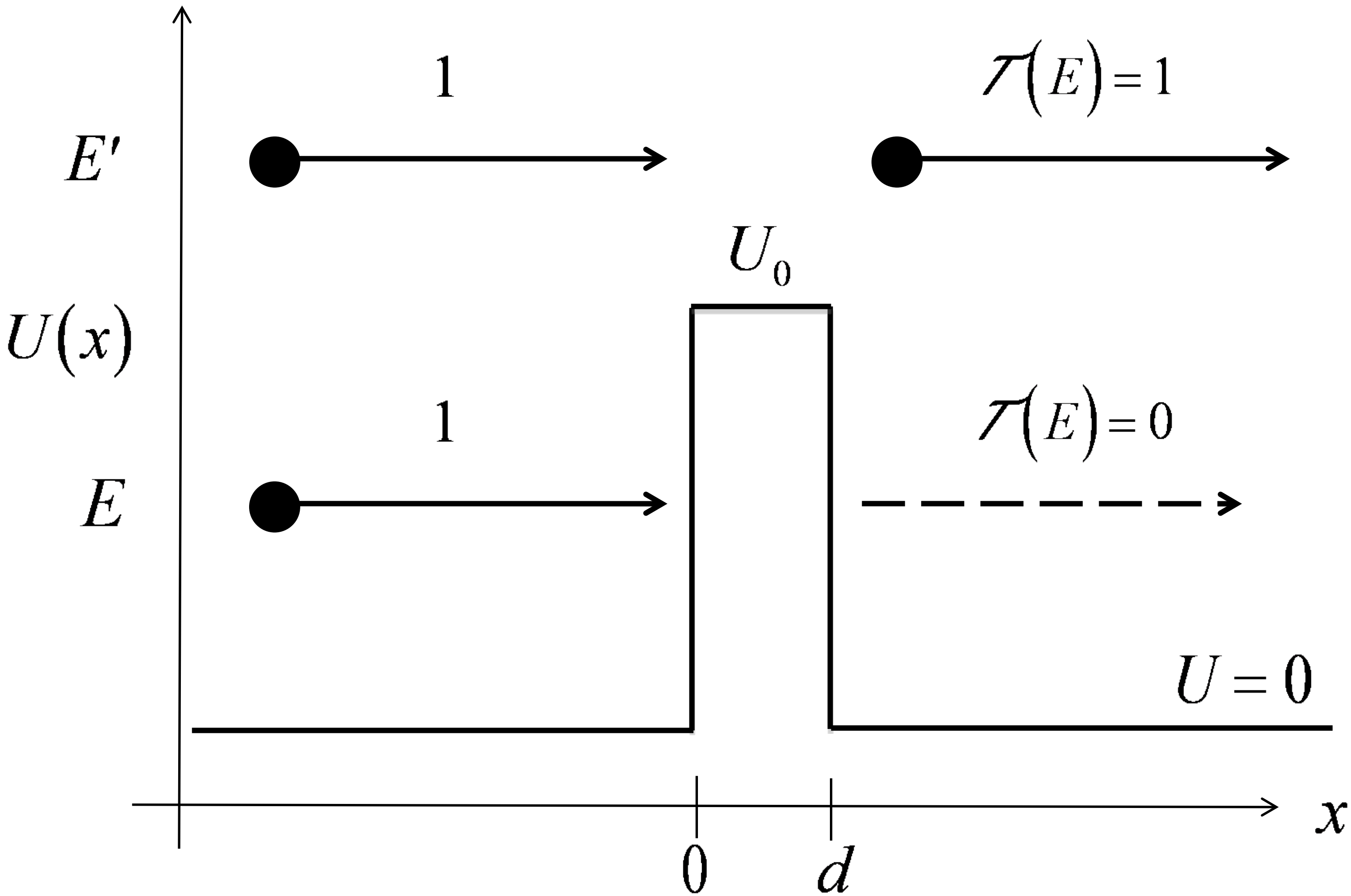
Unit 2: Quantum Mechanics

Lecture 2.3: Quantum tunneling and reflection

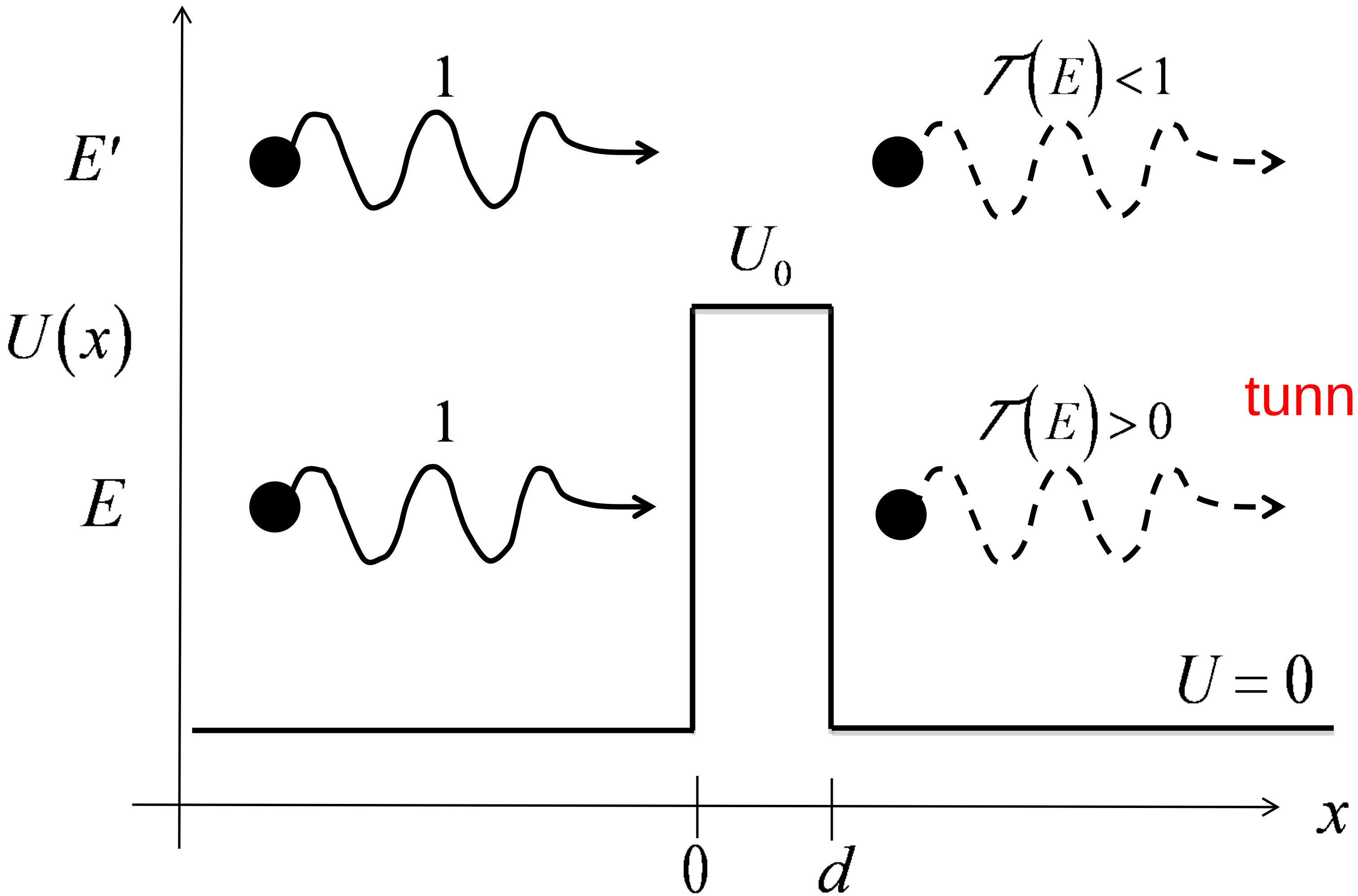
Mark Lundstrom

lundstro@purdue.edu
Electrical and Computer Engineering
Purdue University
West Lafayette, Indiana USA

Classical vs. Quantum



Classical vs. Quantum



Solutions to the wave equation

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}\left[E - U_0\right]\psi(x) = 0$$

$$E > U_0$$

$$k^2 = \frac{2m}{\hbar^2}\left[E - U_0\right]$$

$$\frac{d^2\psi(x)}{dx^2} + k^2\psi(x) = 0$$

$$\psi(x) = Ae^{+ikx} + Be^{-ikx}$$

Solutions to the wave equation

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}\left[E - U_0\right]\psi(x) = 0$$

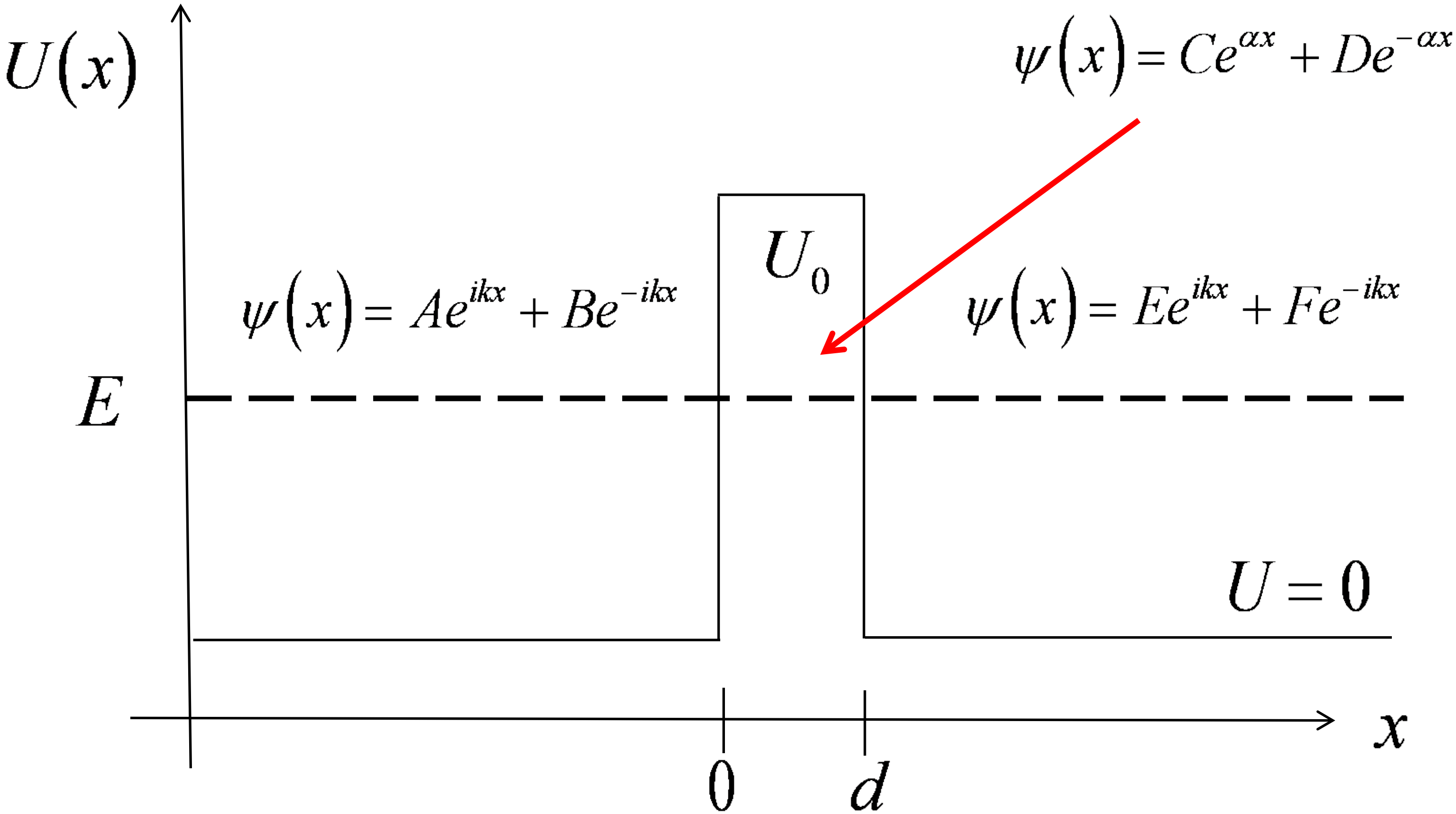
$E < U_0$

$$\alpha^2 = \frac{2m}{\hbar^2}\left[U_0 - E\right]$$

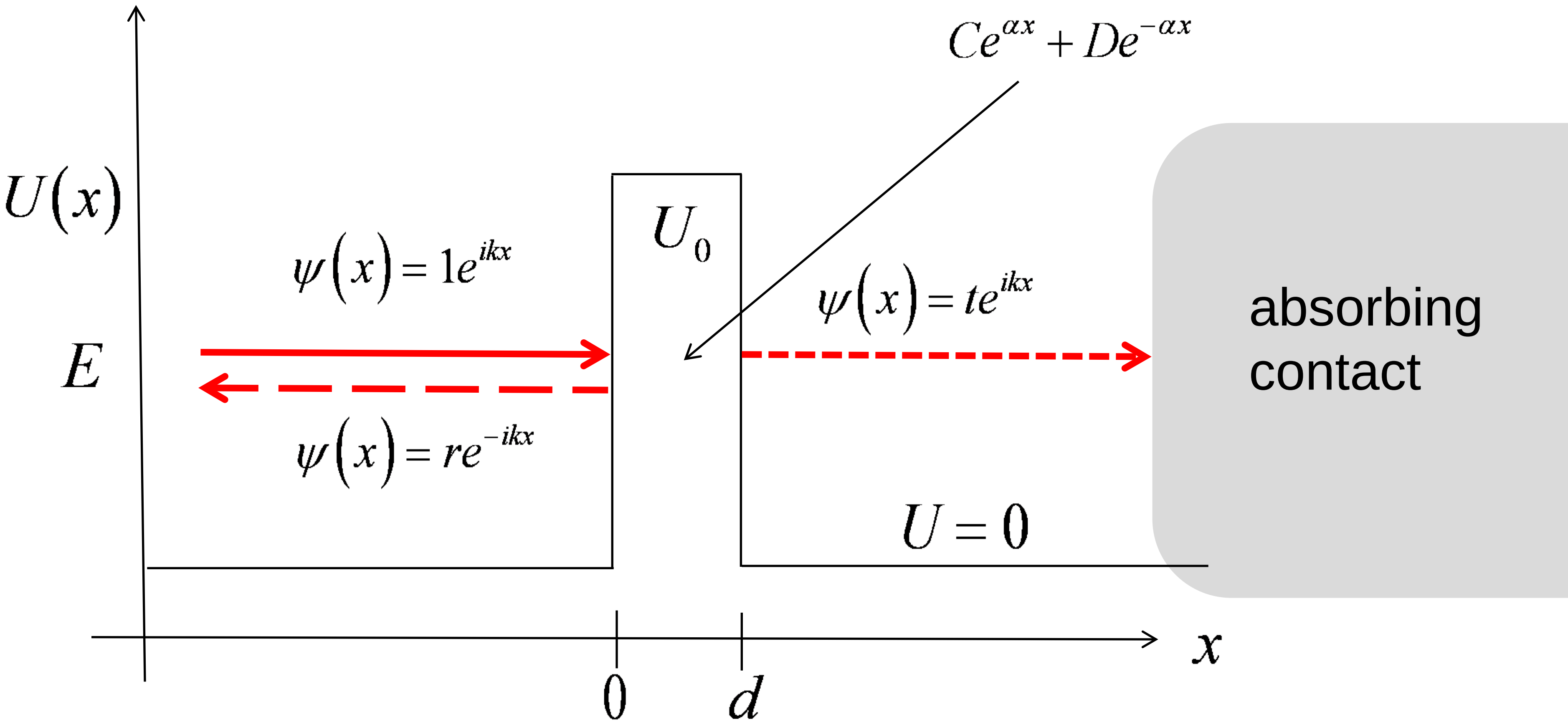
$$\frac{d^2\psi(x)}{dx^2} - \alpha^2\psi(x) = 0$$

$$\psi(x) = Ce^{\alpha x} + De^{-\alpha x}$$

Tunneling ($E < U_0$)



Tunneling



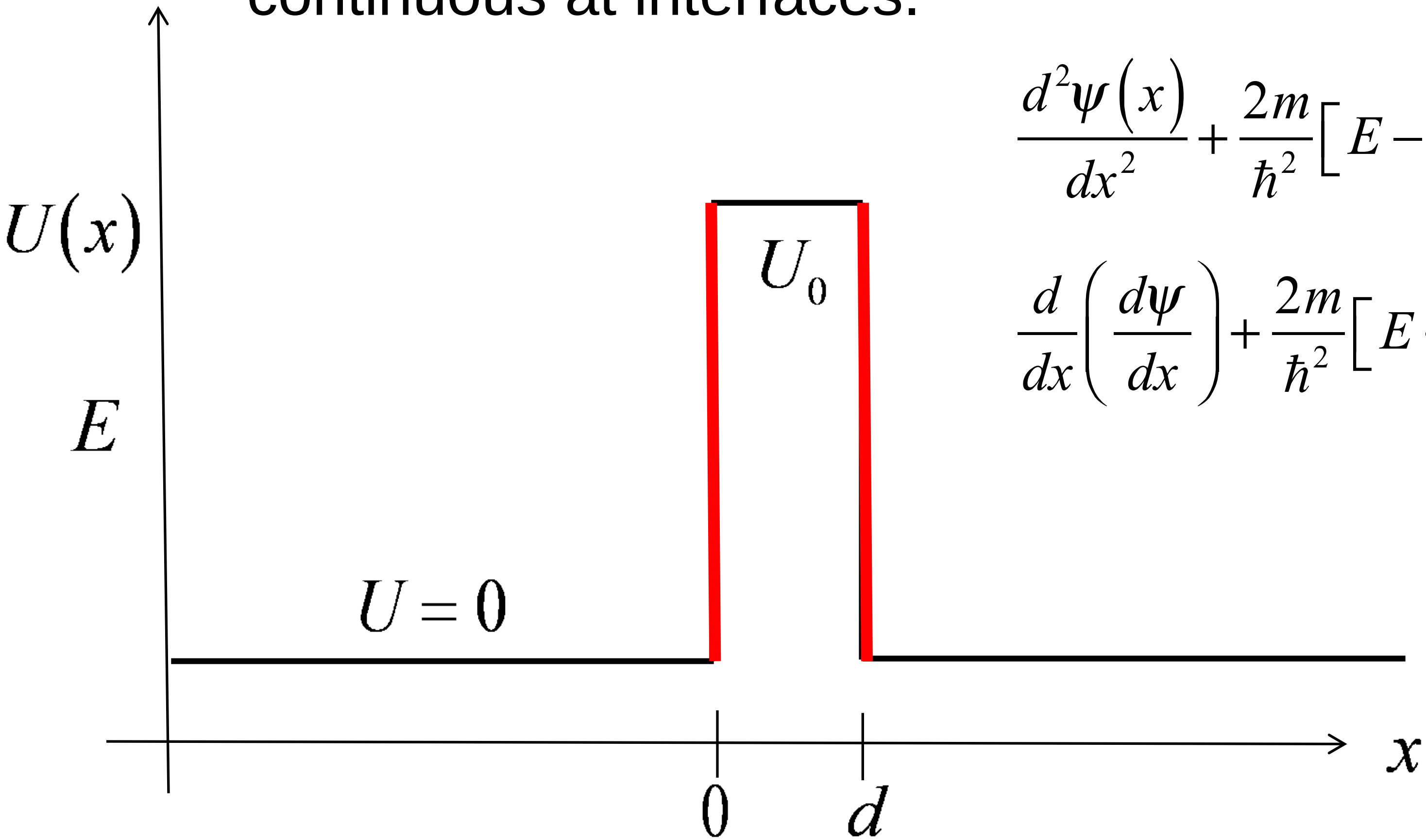
$Ce^{\alpha x} + De^{-\alpha x}$
$U(x)$
U_0
$\psi(x) = 1e^{ikx}$
$\psi(x) = te^{ikx}$
absorbing contact
E
$\psi(x) = re^{-ikx}$
$U = 0$
x
0
d

Boundary conditions

The wave function and its derivative must be continuous at interfaces.

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}\left[E - U_0\right]\psi(x) = 0$$

$$\frac{d}{dx}\left(\frac{d\psi}{dx}\right) + \frac{2m}{\hbar^2}\left[E - U_0\right]\psi(x) = 0$$



Lundstrom: 2018

Boundary conditions

$$\psi\left(0^-\right) = \psi\left(0^+\right)$$

$$\psi'\left(0^-\right) = \psi'\left(0^+\right)$$

$$1 + r = C + D$$

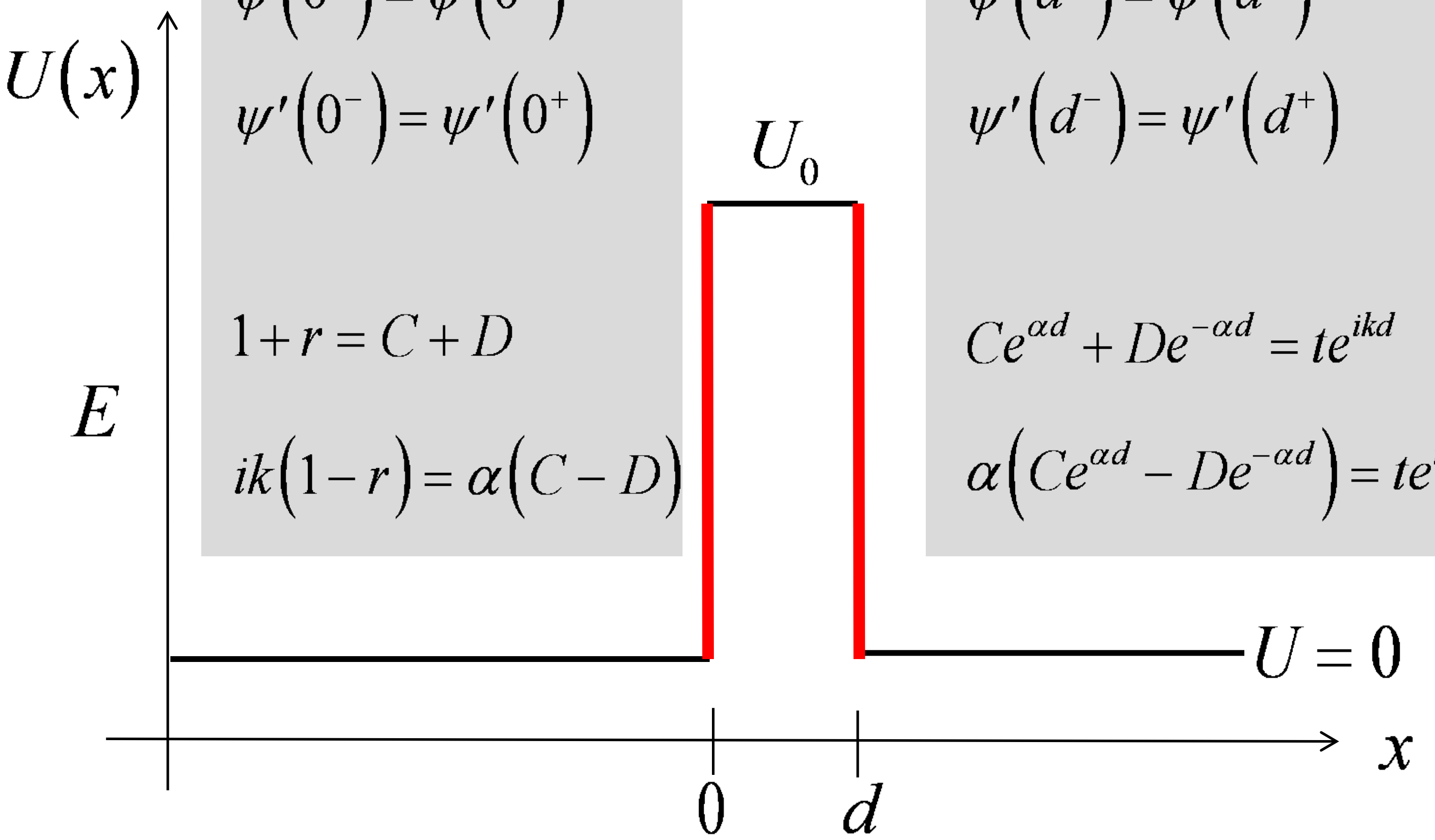
$$ik\left(1 - r\right) = \alpha\left(C - D\right)$$

$$\psi\left(d^-\right) = \psi\left(d^+\right)$$

$$\psi'\left(d^-\right) = \psi'\left(d^+\right)$$

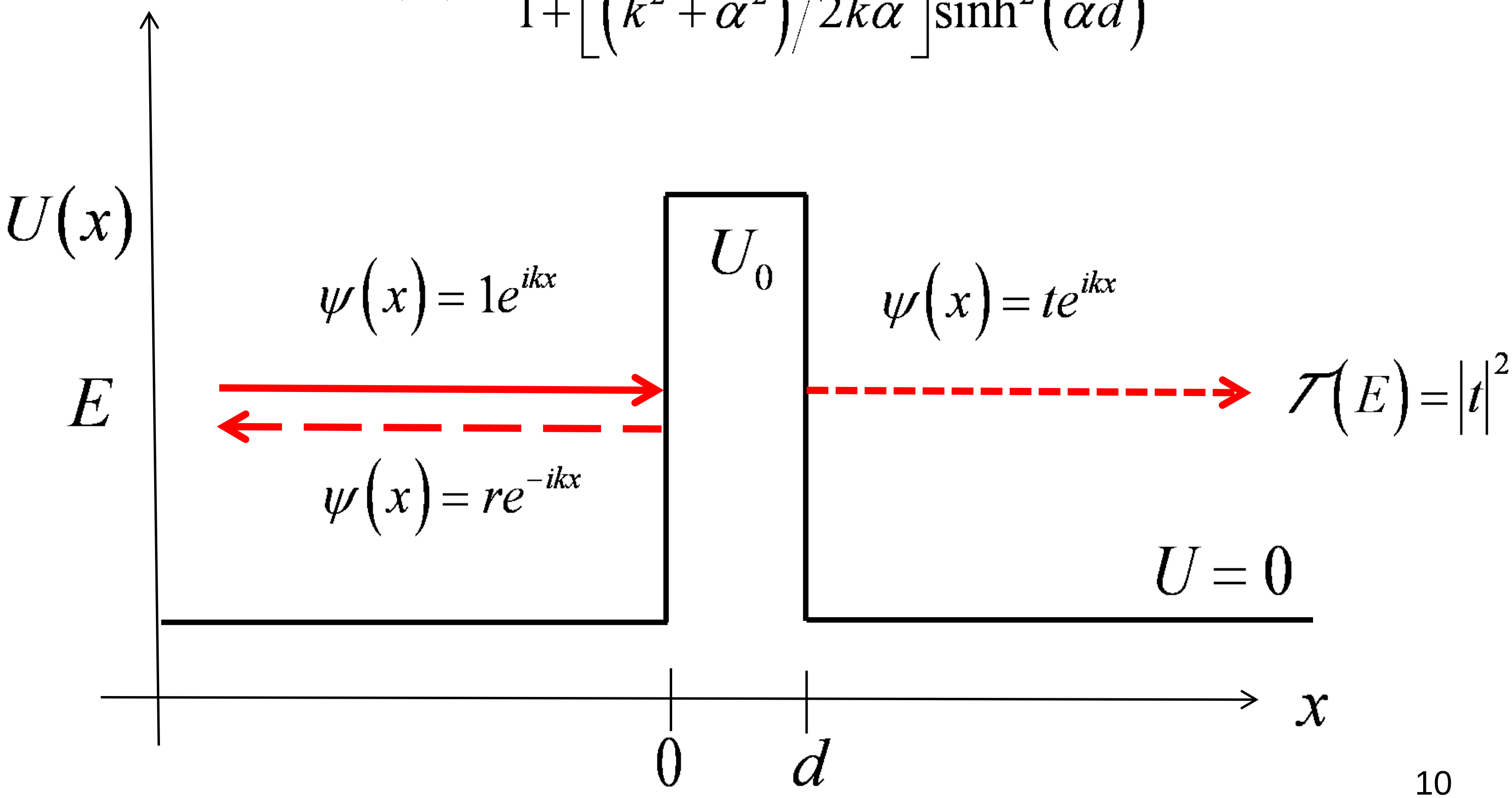
$$Ce^{\alpha d} + De^{-\alpha d} = te^{ikd}$$

$$\alpha\left(Ce^{\alpha d} - De^{-\alpha d}\right) = te^{ikd}$$



Tunneling transmission

$$\mathcal{T}(E) = \frac{1}{1 + \left[\left(k^2 + \alpha^2\right)/2k\alpha\right]\sinh^2(\alpha d)}$$



Examine solution

$$\mathcal{T}(E) = \frac{1}{1 + \left[\left(k^2 + \alpha^2\right)/2k\alpha\right]\sinh^2(\alpha d)}$$

$$\sinh(\alpha d) = \frac{e^{\alpha d} - e^{-\alpha d}}{2} \approx \frac{e^{\alpha d}}{2}$$

$$\alpha^2 = \frac{2m}{\hbar^2}\left[U_0 - E\right]$$

$$\sinh^2(\alpha d) \approx \frac{e^{2\alpha d}}{4} >> 1$$

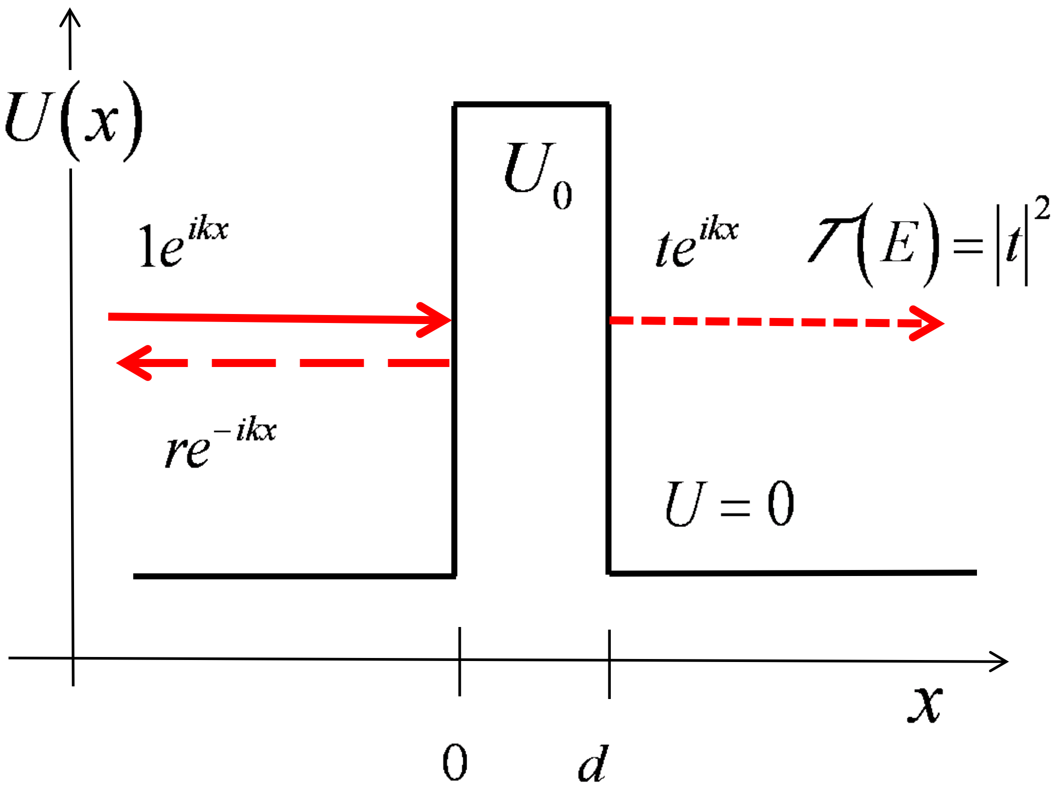
$$k^2 = \frac{2m}{\hbar^2}\left[E\right]$$

$$\mathcal{T}(E) \approx \frac{8k\alpha}{\left(k^2 + \alpha^2\right)e^{2\alpha d}}$$

$$\mathcal{T}(E) \approx 8\sqrt{\left(E/U_0\right)\left(1 - E/U_0\right)}e^{-2\alpha d}$$

Tunneling: conclusions

$$\mathcal{T}(E) \approx \exp\left(-2d\sqrt{2m(U_0 - E)/\hbar^2}\right)$$

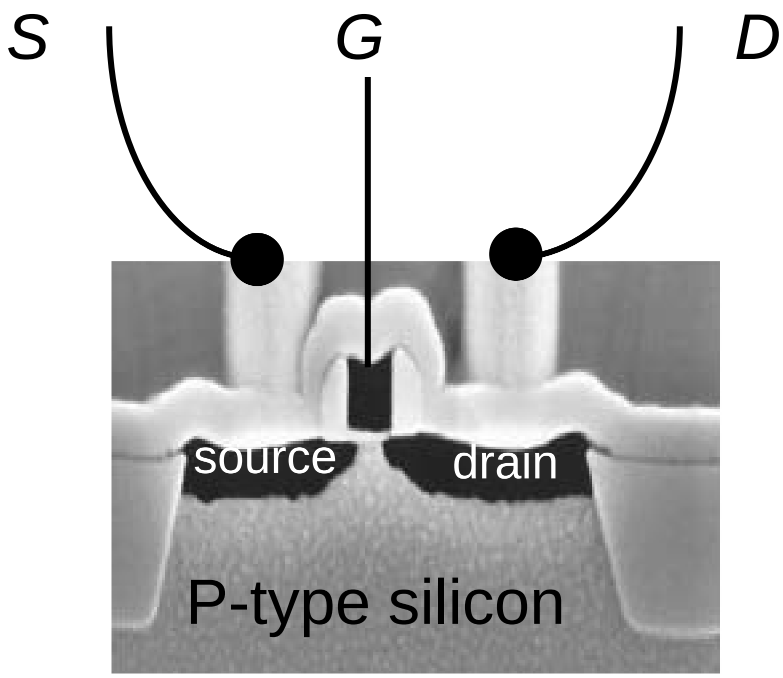


1) Tunneling decreases exponentially with increasing barrier thickness.

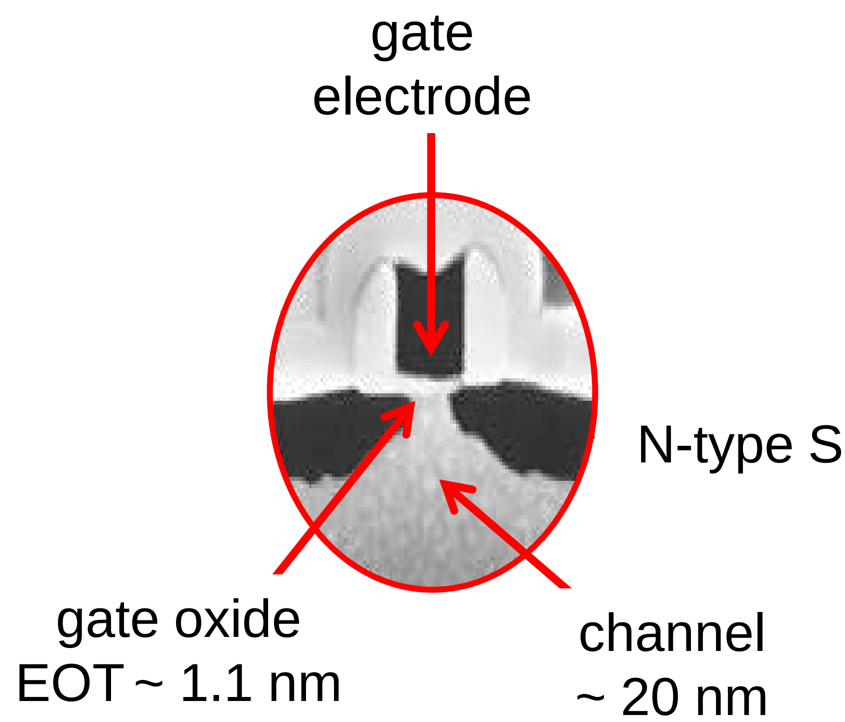
2) Tunneling decreases exponentially with increasing barrier height.

3) Tunneling decreases exponentially with increasing mass.

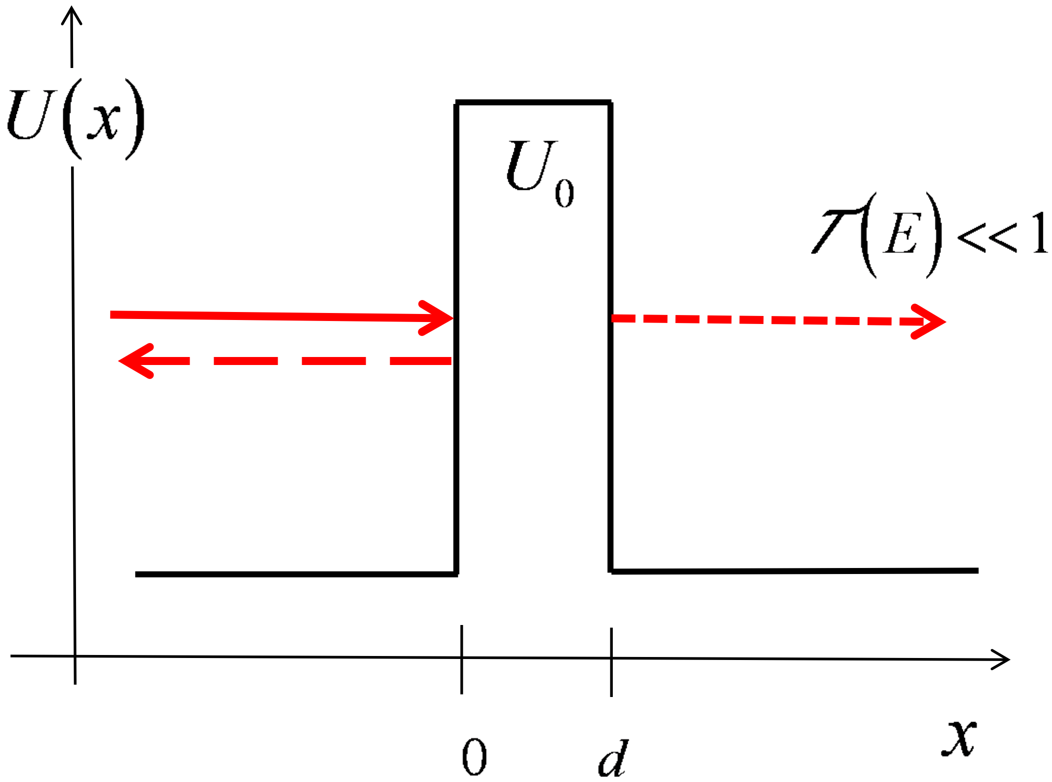
Tunneling in CMOS technology



(Source: Texas Instruments, ~ 2000)

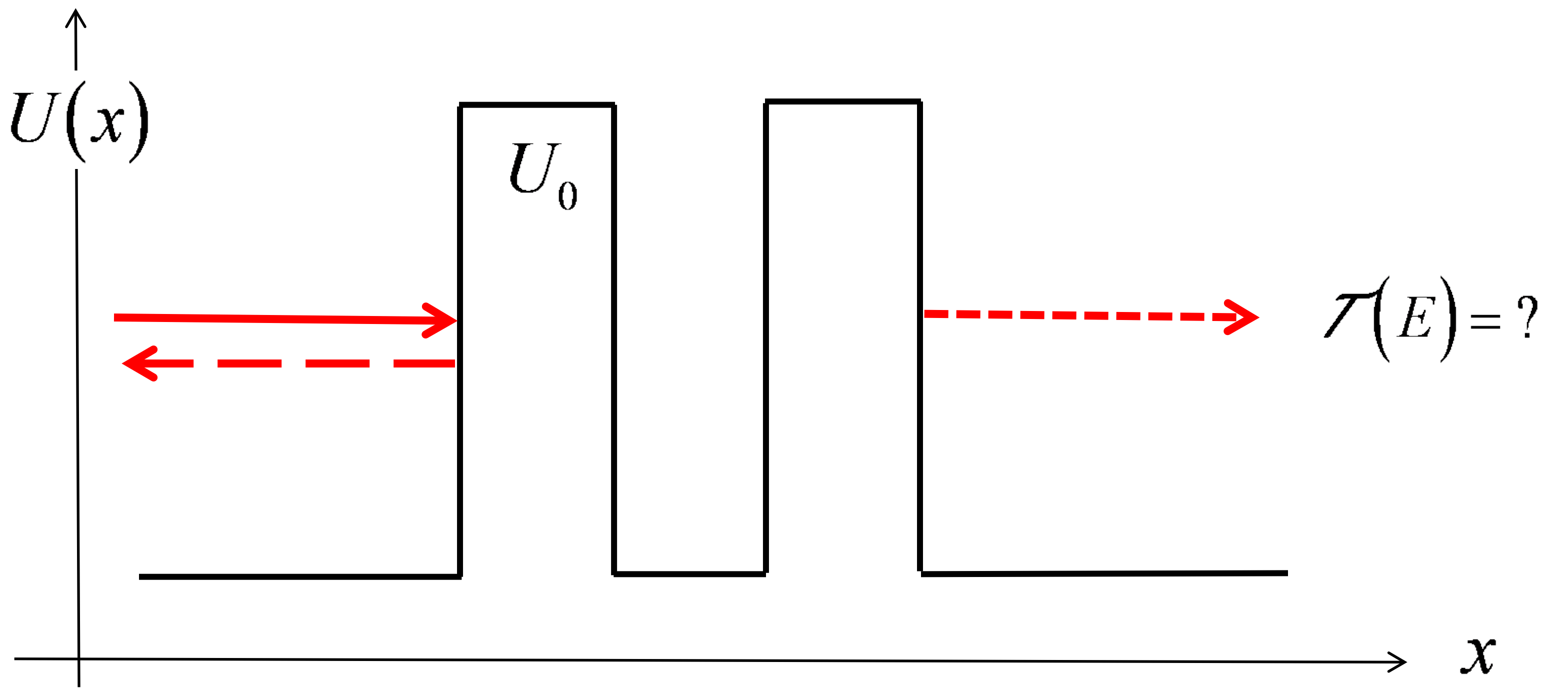


Resonant tunneling



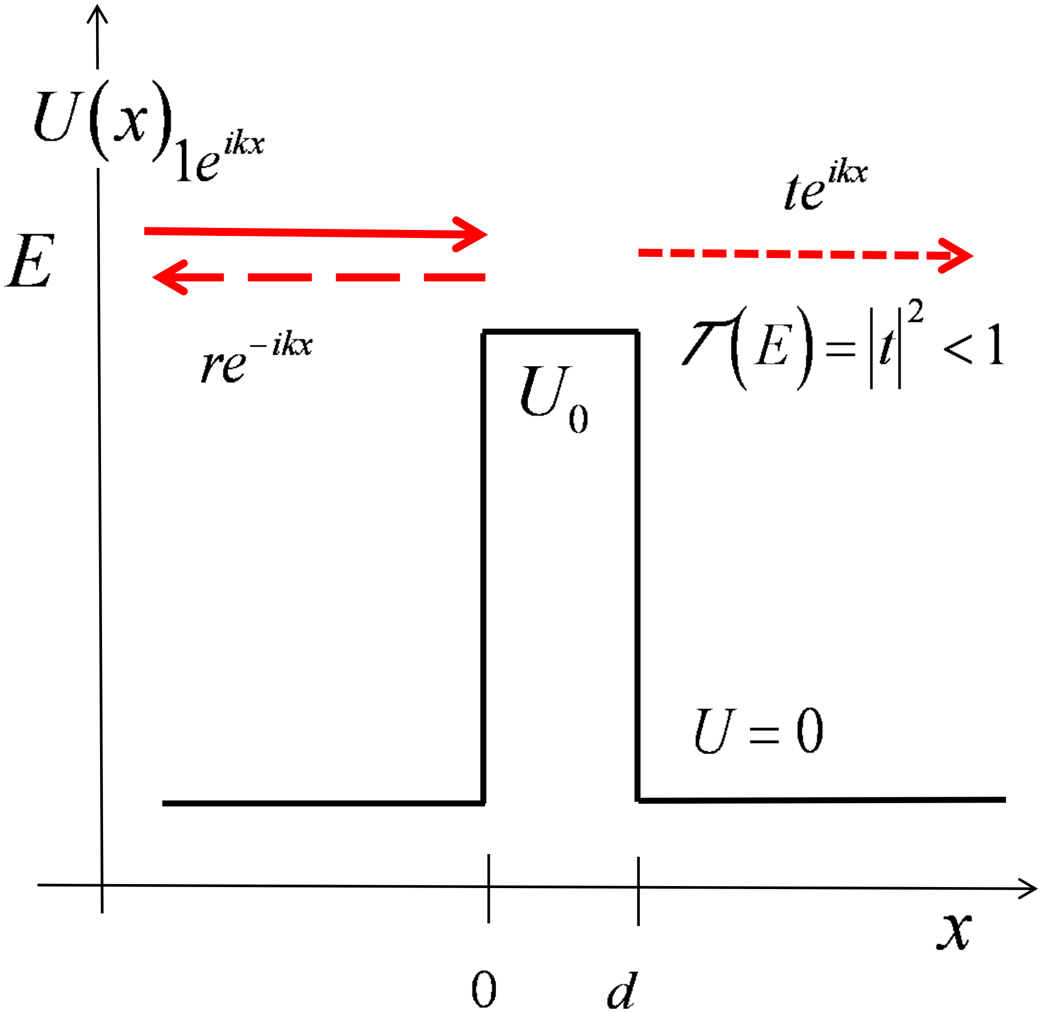
What happens if we put two barriers in series?

Resonant tunneling



Surprisingly, the transmission can be **unity** at a specific energy, the resonant energy, at which all of the multiple reflections add up in phase.

Quantum reflections



The potential must change slowly (on the scale of the electron's wavelength) to treat the electron as a classical particle.

Summary

1) Classical particles can’t get over a barrier unless they have enough energy, but quantum particles can tunnel through.

2) Tunneling decreases exponentially with increasing barrier thickness.

3) Tunneling decreases exponentially with increasing barrier height.

4) Tunneling decreases exponentially with increasing mass.

5) Particles with enough energy to get over the barrier can reflect, if the potential changes rapidly.