Primer on Semiconductors

Unit 5: The Semiconductor Equations

Lecture 5.5: Unit 5 Recap

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The unknowns

3 unknowns

$$p(\vec{r}), n(\vec{r}), V(\vec{r})$$

or

$$F_p(\vec{r}), F_n(\vec{r}), V(\vec{r})$$

We need to formulate 3 equations in 3 unknowns.

Continuity equation

$$\boxed{\frac{\partial p}{\partial t} = -\nabla \bullet \frac{\vec{J}_p}{q} + G_p - R_p}$$

$$\vec{J}_p = pq\mu_p \vec{\mathcal{E}} - qD_p \vec{\nabla} p$$

optical generation
or impact ionization

Radiative,
Auger, or defect-
assisted

Need an equation for the electric field

Electrostatics

$$\oint \vec{D} \bullet d\vec{S} = Q \quad \longleftrightarrow \quad \nabla \bullet \vec{D} = \rho(x)$$

Gauss's Law Poisson equation

$$\vec{D} = K_S \varepsilon_0 \vec{\mathcal{E}}$$

$$\nabla \bullet \left(K_S \varepsilon_0 \vec{\mathcal{E}} \right) = \rho(\vec{r})$$

$$\rho(\vec{r}) = q\left[p(\vec{r}) - n(\vec{r}) + N_D^+(\vec{r}) - N_A^-(\vec{r}) \right]$$

The "semiconductor equations"

$$\frac{\partial p}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_p}{q} \right) + G_p - R_p$$

$$\frac{\partial n}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_n}{-q} \right) + G_n - R_n$$

$$\nabla \bullet \left(K_S \varepsilon_0 \vec{\mathcal{E}} \right) = \rho$$

Three equations in three unknowns:

$$p(\vec{r}), n(\vec{r}), V(\vec{r})$$

or

$$F_p(\vec{r}), F_n(\vec{r}), V(\vec{r})$$

$$\vec{J}_p = pq\mu_p \vec{\mathcal{E}} - qD_p \vec{\nabla} p = p\mu_p \vec{\nabla} F_p$$

$$\vec{J}_n = nq\mu_n \vec{\mathcal{E}} + qD_n \vec{\nabla} n = n\mu_n \vec{\nabla} F_n$$

$$\rho = q\left(p - n + N_D^+ - N_A^- \right)$$

$$\vec{\mathcal{E}}(\vec{r}) = \nabla V(\vec{r})$$

Energy band diagrams

An energy band diagram is a plot of the bottom of the conduction band and the top of the valence band vs. position.

An energy band diagram is a powerful tool for understanding semiconductor devices because they provide **qualitative solutions to the semiconductor equations.**

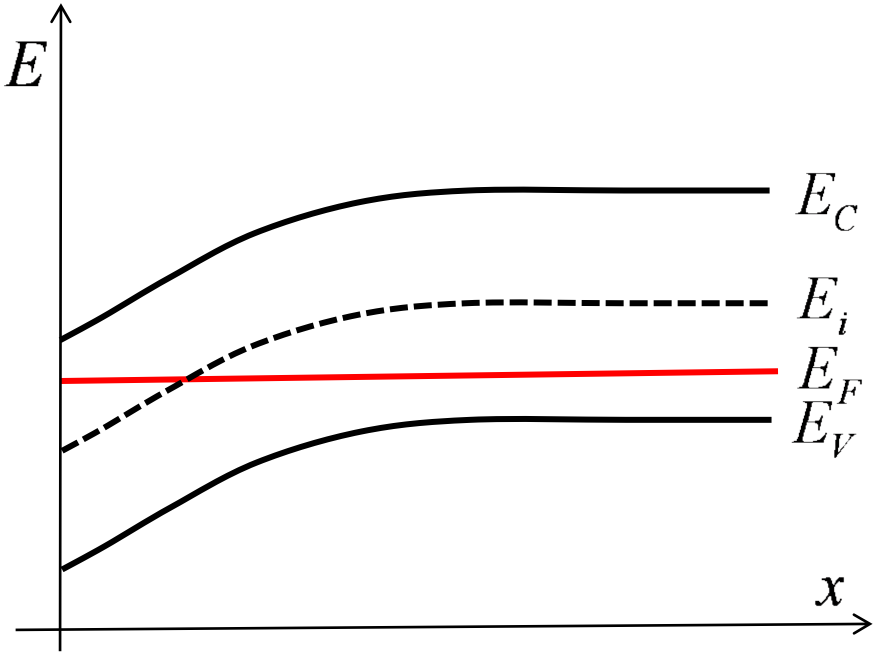
An important principle

The Fermi level is constant in equilibrium.

The starting point for drawing energy band diagrams.

Band diagrams

Drawing the band diagram



Reading the band diagram

$$V(x) \propto -E_C(x)$$

$$\mathcal{E} \propto dE_C(x)/dx$$

$$\log n(x) \propto E_F - E_i(x)$$

$$\log p(x) \propto E_i(x) - E_F$$

$$\rho(x) \propto d^2 E_C / dx^2$$

Drawing equilibrium band diagrams

1) Begin with E_F

2) Draw the E-bands where you know the carrier density

3) Then add the rest

Equilibrium vs. non-equilibrium

equilibrium	non-equilibrium
$n_0 = n_i e^{(E_F - E_i)/k_B T}$	$n = n_i e^{(F_n - E_i)/k_B T}$
$p_0 = n_i e^{(E_i - E_F)/k_B T}$	$p = n_i e^{(E_i - F_p)/k_B T}$
$n_0 p_0 = n_i^2$	$np = n_i^2 e^{(F_n - F_p)/k_B T}$
$f_0 = \dfrac{1}{1 + e^{(E - E_F)/k_B T}}$	$f_c = \dfrac{1}{1 + e^{(E - F_n)/k_B T}}$
	$1 - f_v = 1 - \dfrac{1}{1 + e^{(E - F_p)/k_B T}}$

Solving the semiconductor equations

$$\frac{\partial p}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_p}{q} \right) + G_p - R_p$$

$$\frac{\partial n}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_n}{-q} \right) + G_n - R_n$$

$$\nabla \bullet \left(K_S \varepsilon_0 \vec{\mathcal{E}} \right) = \rho$$

For problems that focus on minority carriers in low-level injections, these equations can be simplified, so that we only need to solve the minority carrier diffusion equation (MCDE).

Example: Minority hole diffusion equation

$$n = n_0 = N_D$$

(N-type semiconductor in low level injection)

$$\frac{\partial p}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_p}{q} \right) + G_p - R_p$$

(hole continuity equation)

$$\frac{\partial p}{\partial t} = -\frac{d}{dx}\left(\frac{J_{px}}{q} \right) + G_L - R_p$$

(1D, generation by light)

$$\frac{\partial \Delta p}{\partial t} = -\frac{d}{dx}\left(\frac{-qD_p \, d\Delta p/dx}{q} \right) + G_L - \frac{\Delta p}{\tau_p}$$

(low-level injection, no electric field)

$$\frac{\partial \Delta p}{\partial t} = D_p \frac{d^2 \Delta p}{dx^2} - \frac{\Delta p}{\tau_p} + G_L$$

(D_p spatially uniform)

General procedure

1) Write down the MCDE

N-type $$\frac{\partial \Delta p(x,t)}{\partial t} = D_p \frac{d^2 \Delta p(x,t)}{dx^2} - \frac{\Delta p(x,t)}{\tau_p} + G_L$$

P-type $$\frac{\partial \Delta n(x,t)}{\partial t} = D_n \frac{d^2 \Delta n(x,t)}{dx^2} - \frac{\Delta n(x,t)}{\tau_n} + G_L$$

2) Simplify the MCDE for the specific problem

3) Solve the MCDE for the excess minority carrier density

4) Deduce QFL from the excess minority carrier density

General features of MCDE solutions

Transient solutions goes as exp[-t/τ_n]

For long regions, steady-state spatial solutions go as exp[-x/L_n] in a long region

For short regions, steady-state solutions are linear.

Summary

$$\frac{\partial p}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_p}{q} \right) + G_p - R_p$$

$$\frac{\partial n}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_n}{-q} \right) + G_n - R_n$$

$$\nabla \bullet \left(K_S \varepsilon_0 \vec{\mathcal{E}} \right) = \rho$$

$$\vec{J}_p = p\mu_p \vec{\nabla} F_p = pq\mu_p \vec{\mathcal{E}} - qD_p \vec{\nabla} p$$

$$\vec{J}_n = n\mu_n \vec{\nabla} F_n = nq\mu_n \vec{\mathcal{E}} + qD_n \vec{\nabla} n$$

$$\rho = q\left(p - n + N_D^+ - N_A^- \right)$$

$$\vec{\mathcal{E}}\left(\vec{r} \right) = \nabla V\left(\vec{r} \right)$$

1) Direct, numerical solutions
2) Qualitative solutions with energy band diagrams
3) Simplified, analytical solutions.

Vocabulary

Built-in potential
Continuity equation
Diffusion length
Einstein relation
Energy band diagram
Gauss's Law
Low level injection
Minority carrier diffusion equation
Poisson equation
Relative dielectric constant
Quasi-Fermi levels
Quasi-neutrality
Semiconductor equations