Unit 4: Carrier Transport, Recombination, and Generation

Lecture 4.2: Current from the nanoscale to macroscale

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Carrier transport

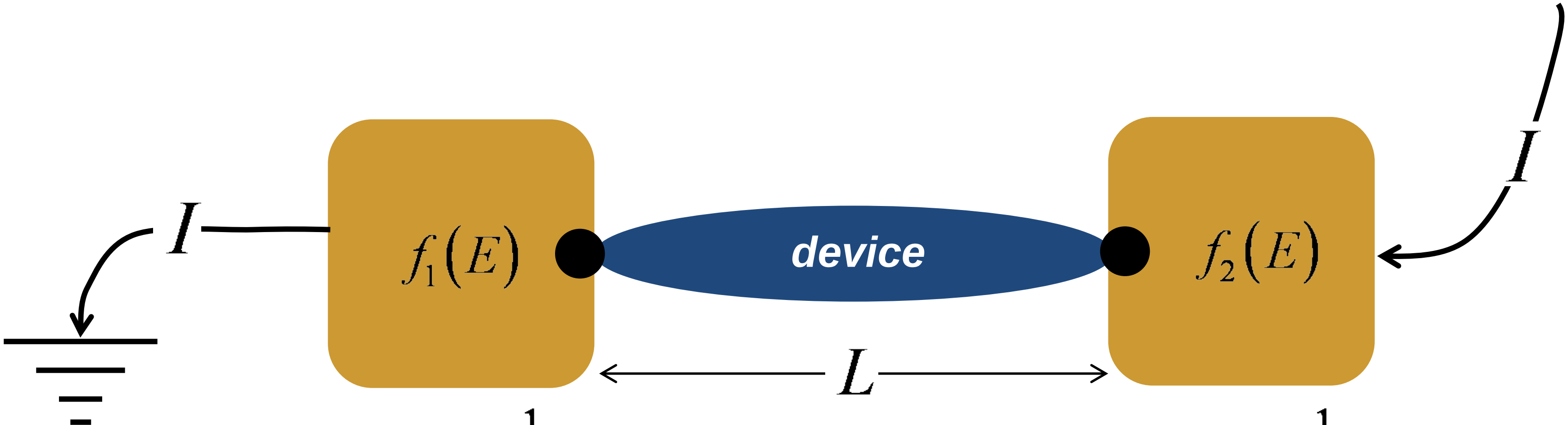
Our goals in this lecture are:

1) To understand the current in a nano-device and the important concept of **quantized conductance**.

2) To use the Landauer Approach to derive a current equation for bulk (i.e. large) semiconductors.

Comment: Traditional semiconductor device physics is based on a current equation for bulk semiconductors. Nano-devices are increasingly important, but the macroscopic current equation continues to provide the overall framework for understanding semiconductor devices.

Landauer approach to current flow



$$f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T}} \qquad f_2(E) = \frac{1}{1 + e^{(E - E_{F2})/k_B T}}$$

$$I = \frac{2q}{h} \int \mathcal{T}(E) M(E) \left(f_1 - f_2\right) dE$$

Can be used to describe the current in small and large devices and in short to long devices.

Comment

We will restrict our attention to small applied voltages (or small electric fields in bulk semiconductors).

Things become more complicated at high electric fields, but we can often describe the effects by modifying the small electric field results.

Fermi window (small bias)

$$I = \frac{2q}{h}\int \mathcal{T}(E)M(E)\left(\underline{f_1(E) - f_2(E)}\right)dE$$

$$f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T}} = f_0(E)$$

$$\delta E_F = -qV$$

$$f_2(E) \approx f_1(E) + \frac{\partial f_1}{\partial E_F}\delta E_F$$

$$\boxed{f_1(E) - f_2(E) = \left(-\frac{\partial f_0}{\partial E}\right)(qV)}$$

$$f_2(E) \approx f_1(E) + \left(-\frac{\partial f_1}{\partial E}\right)\delta E_F$$

$$f_2(E) - f_1(E) \approx -\left(-\frac{\partial f_1}{\partial E}\right)\delta E_F$$

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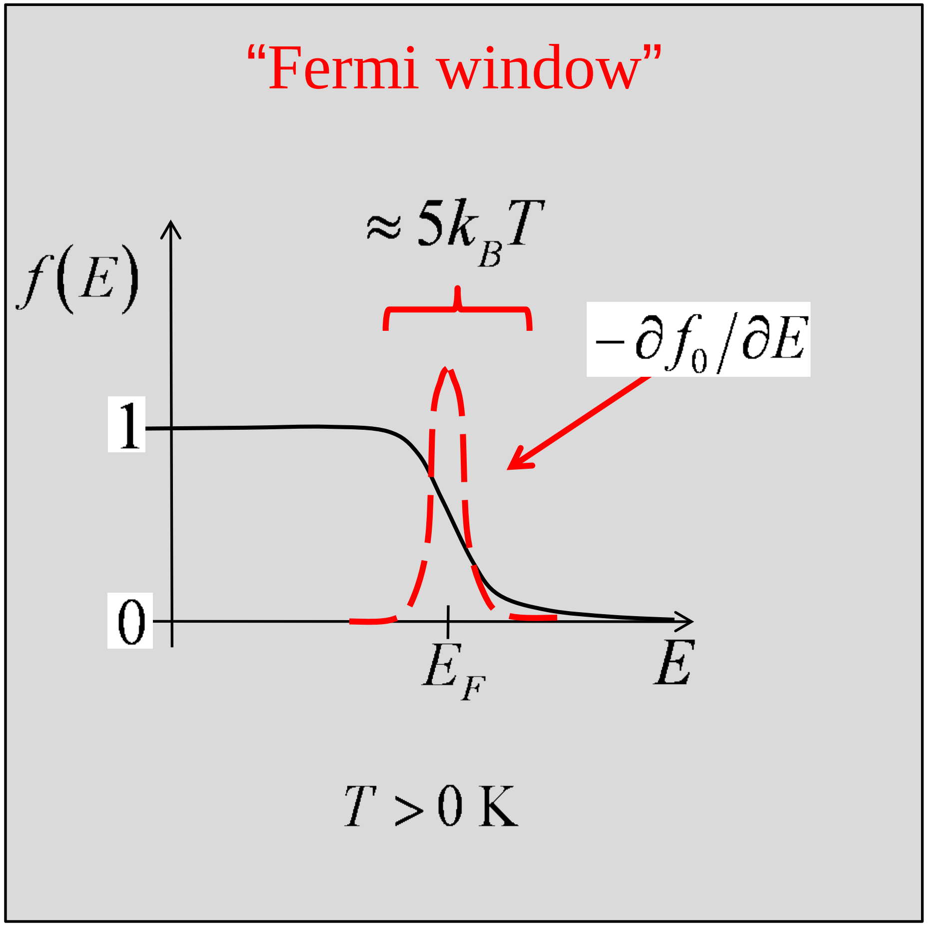
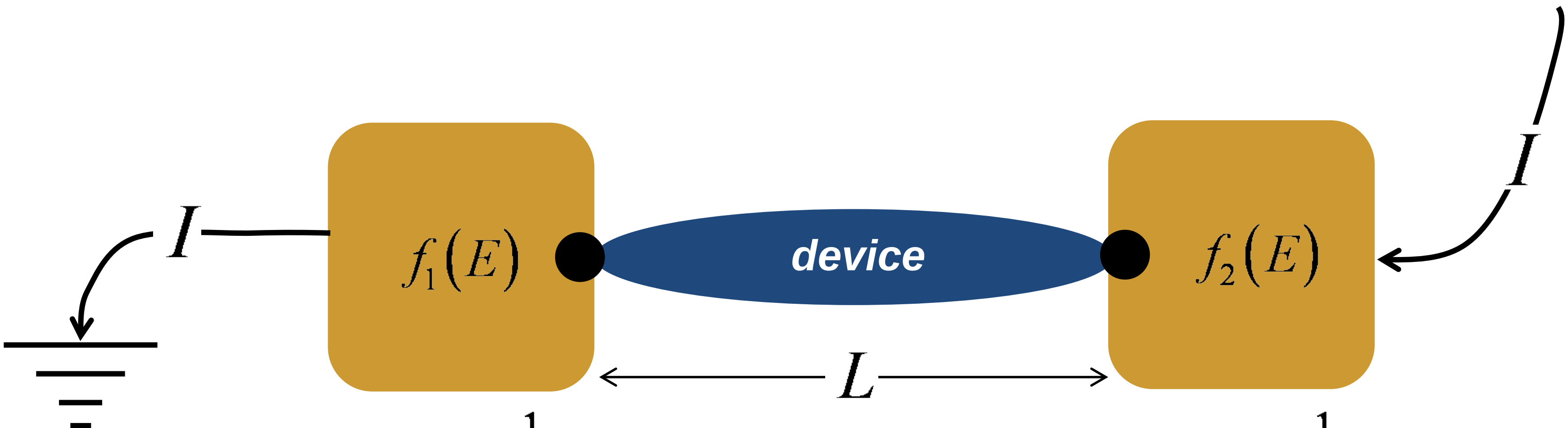
Fermi window: small bias



$$W_F(E) = \left(-\frac{\partial f_0}{\partial E}\right)$$

$$\int W_F(E)\,dE = 1$$

Current for a small voltage difference



$$f_1(E) = \frac{1}{1+e^{(E-E_{F1})/k_BT}} \qquad f_2(E) = \frac{1}{1+e^{(E-E_{F2})/k_BT}}$$

$$I = \frac{2q}{h}\int \mathcal{T}(E)M(E)(f_1 - f_2)\,dE$$

$$f_1(E) - f_2(E) \rightarrow \left(-\frac{\partial f_1}{\partial E}\right)(qV) \Rightarrow I = GV$$

Small bias conductance

$$I = GV \qquad \text{A}$$

$$G = \frac{2q^2}{h} \int \mathcal{T}(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE \qquad \text{S}$$

Conductance at T = 0 K

$$G = \frac{2q^2}{h}\int \mathcal{T}(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE \quad \text{S}$$

$T = 0\ \text{K}$:

$$\left(-\frac{\partial f_0}{\partial E}\right) = \delta(E_F)$$

$$\boxed{G(T = 0\ \text{K}) = \frac{2q^2}{h}\mathcal{T}(E_F)M(E_F)}$$

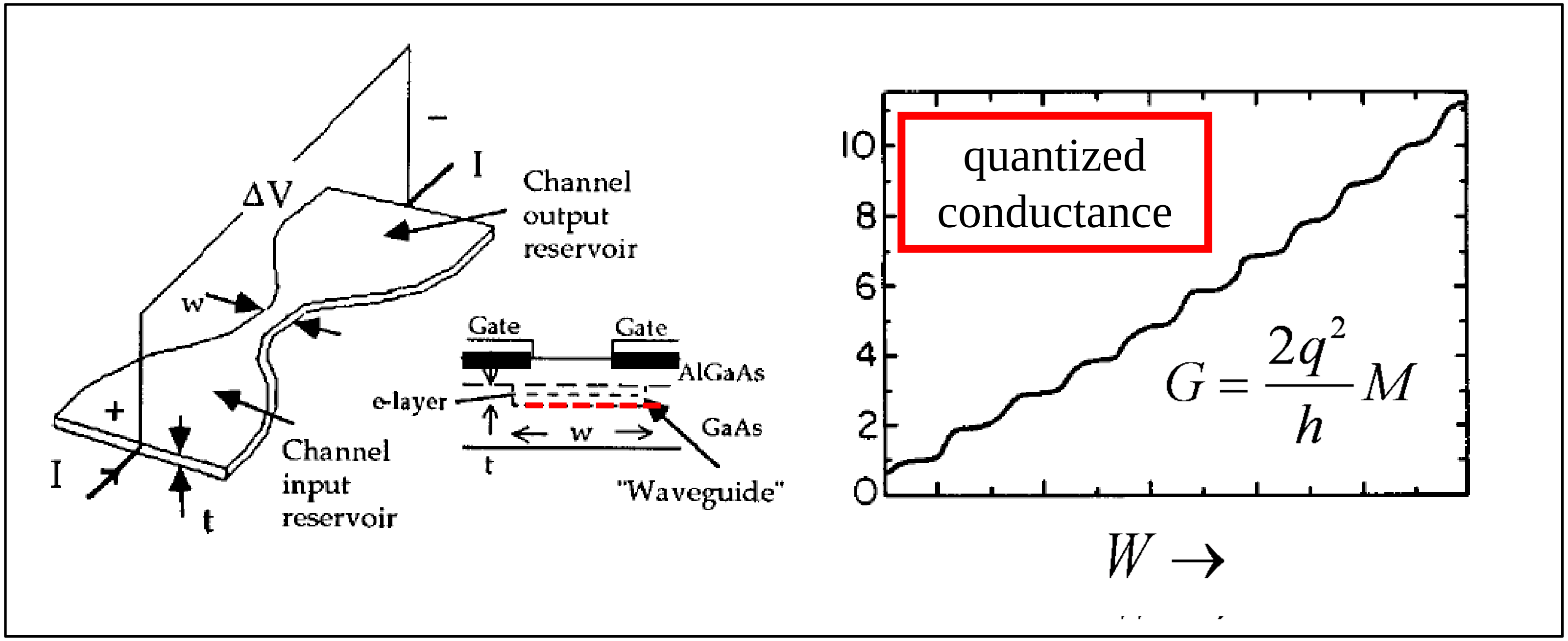
2D conductor

$$M(E) = W g_v \frac{\sqrt{2m^* E}}{\pi\hbar}$$

For large *W*, *M* is ~ W

For small *W*, *M* comes in discrete units.

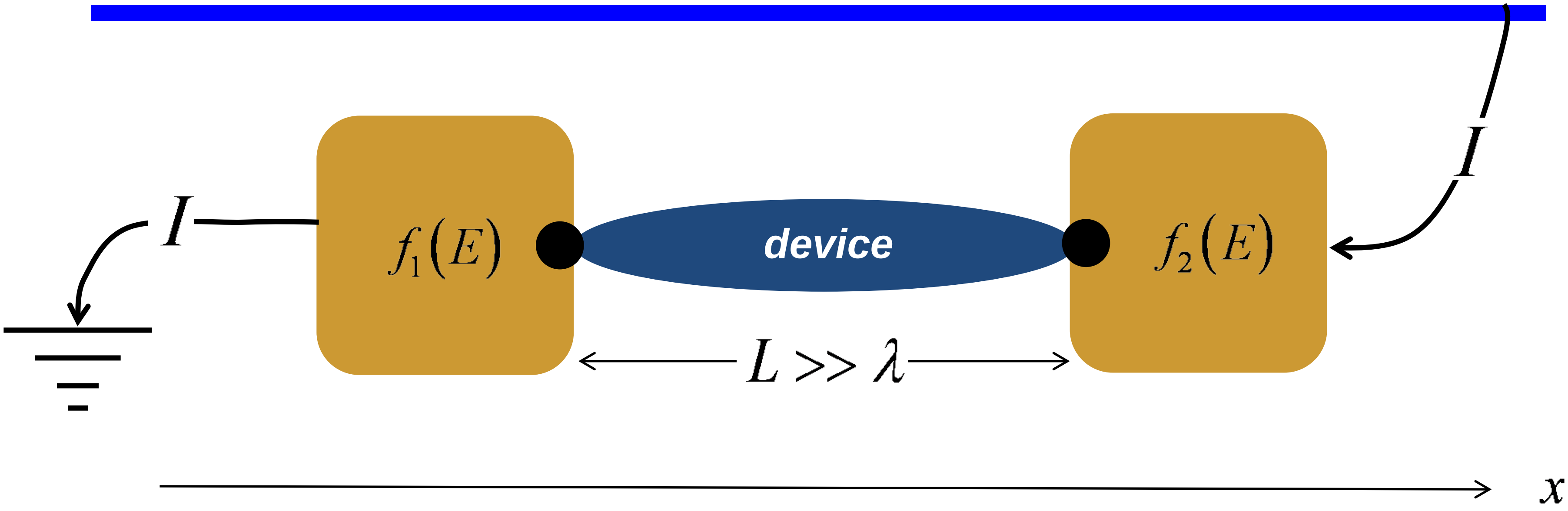
Quantized conductance



Ohm's Law (1827)

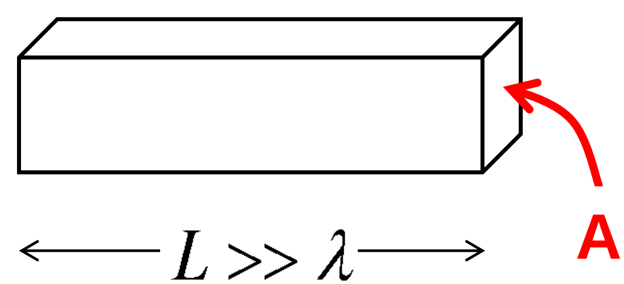
D. Holcomb, *American J. Physics*, **67**, pp. 278-297 1999.
Data from: B. J. van Wees, et al., *Phys. Rev. Lett*. **60**, 848851, 1988.

Landauer Approach to bulk transport



We seek an equation for the current density in the +x direction.

In 3D: $J_x = -I/A$



Current equation in the bulk

$$G = \frac{2q^2}{h}\int \mathcal{T}(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE \qquad \mathcal{T}(E) = \frac{\lambda(E)}{\lambda(E)+L} \rightarrow \frac{\lambda(E)}{L}$$

diffusive

$$J_x = -I/A = \left\{\frac{2q^2}{h}\int \frac{\lambda(E)}{L}\frac{M(E)}{A}\left(-\frac{\partial f_0}{\partial E}\right)dE\right\}V \qquad \frac{qV}{L} = -\frac{dF_n}{dx}$$

$$J_x = \sigma_n \frac{d(F_n/q)}{dx} \qquad \sigma_n = \frac{2q^2}{h}\int \lambda(E)\big(M(E)/A\big)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

Current equation (3D bulk)

$$J_x = \sigma_n \frac{d(F_n/q)}{dx}$$

σ_n: Conductivity (S/m) $\rho_n = 1/\sigma_n$: Resistivity (Ohm-m)

$$\sigma_n \equiv nq\mu_n$$

n: electron density
q: magnitude of the electronic charge
μ_n: mobility m^2/V-s

$$\boxed{J_x = n\mu_n \frac{dF_n}{dx}}$$

The quasi-Fermi level or electrochemical potential

"electrochemical potential"

$$J_x = \sigma_n \frac{d\left(F_n/q\right)}{dx}$$

"quasi-Fermi level"

The quantity, F_n, is analogous to the Fermi level, but it is defined out of equilibrium and reduces to the Fermi level in equilibrium.

$$n_0 = N_C e^{\left(E_F - E_C\right)/k_B T} \rightarrow n = N_C e^{\left(F_n - E_C\right)/k_B T}$$

In equilibrium, the current is zero: $J_x = 0 = \sigma_n \frac{d\left(F_n/q\right)}{dx} \rightarrow \frac{dE_F}{dx} = 0$

The Fermi level is constant, independent of position, in equilibrium.

The mobility

$$J_x = \sigma_n \frac{d\left(F_n/q\right)}{dx} \qquad\qquad J_x = n\mu_n \frac{dF_n}{dx}$$

The fundamental quantity is the **conductivity** – mobility is a derived quantity.

To relate the mobility to material properties, we should begin by evaluating the conductivity.

Landauer derivation of the mobility

$$\sigma_n = \frac{2q^2}{h}\int \lambda(E)\left(M(E)/A\right)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

$$\lambda(E) = \lambda_0 \qquad M(E)/A = \frac{m^*}{2\pi\hbar^2}\left(E - E_C\right) \qquad f_0(E) = \frac{1}{1+e^{\left(E-E_F\right)/k_BT}}$$

$$\sigma_n = \frac{2q^2}{h}\lambda_0\left(\frac{m^* k_BT}{2\pi\hbar^2}\right)e^{\left(E_F-E_C\right)/k_BT}$$ (nondegenerate)

$$n_0 = \frac{1}{4}\left(\frac{2m_n^* k_BT}{\pi\hbar^2}\right)^{3/2} e^{\left(E_F-E_C\right)/k_BT}$$

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Mobility

$$\sigma_n = n_0 q \left(\frac{\upsilon_T \lambda_0}{2(k_B T/q)} \right) = n_0 q \mu_n$$

$$\boxed{\mu_n \equiv \left(\frac{\upsilon_T \lambda_0}{2(k_B T/q)} \right) \ \mathrm{cm^2/V\text{-}s}}$$

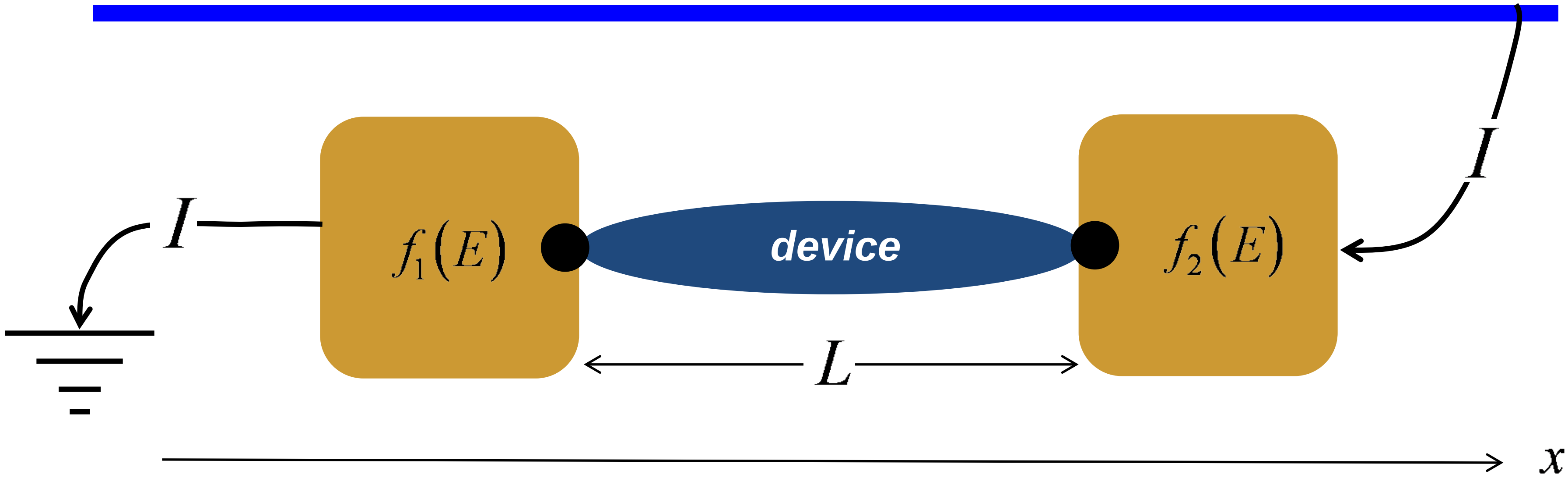
$$\upsilon_T = \sqrt{\frac{2k_B T}{\pi m^*}} \ \mathrm{cm/s}$$

uni-directional thermal velocity

Note: $$(k_B T/q)\mu_n = \left(\frac{\upsilon_T \lambda_0}{2} \right) = D_n \ \mathrm{cm^2/s}$$

(the diffusion coefficient discussed earlier.)

Summary



$$I = \frac{2q}{h}\int \mathcal{T}(E) M(E)(f_1 - f_2)\,dE$$

$$J_x = n\mu_n \frac{dF_n}{dx}$$

For small devices, *M* is countable

For short devices, $\mathcal{T}$= 1, ballistic.

A special case for large and long devices.