Unit 3: Equilibrium Carrier Concentrations

Lecture 3.3: Carrier concentration vs. Fermi level

Mark Lundstrom

lundstro@purdue.edu
Electrical and Computer Engineering
Purdue University
West Lafayette, Indiana USA

Carrier concentrations

Electrons

$$n_0 = N_C \mathcal{F}_{1/2}\left[\left(E_F - E_C\right)/k_B T\right] \text{ m}^{-3}$$

$$N_C = \frac{1}{4}\left(\frac{2m_n^* k_B T}{\pi\hbar^2}\right)^{3/2}$$

nondegenerate:

$$n_0 = N_C e^{\left(E_F - E_C\right)/k_B T}$$

Holes

$$p_0 = N_V \mathcal{F}_{1/2}\left[\left(E_V - E_F\right)/k_B T\right] \text{ m}^{-3}$$

$$N_V = \frac{1}{4}\left(\frac{2m_p^* k_B T}{\pi\hbar^2}\right)^{3/2}$$

nondegenerate:

$$p_0 = N_V e^{\left(E_V - E_F\right)/k_B T}$$

Electron concentration

Electrons

$$n_0 = N_C \mathcal{F}_{1/2}\left[\left(E_F - E_C\right)/k_B T\right] \text{ m}^{-3}$$

$$N_C = \frac{1}{4}\left(\frac{2m_D^* k_B T}{\pi\hbar^2}\right)^{3/2}$$

nondegenerate:

$$n_0 = N_C e^{\left(E_F - E_C\right)/k_B T}$$

For Si at T = 300K:

$m_D^* = 1.182 m_0$ (DOS effective mass)

$$N_C = 3.23 \times 10^{19} \text{ cm}^{-3}$$

Hole concentration

For Si at T = 300K:

$m_p^* = 0.81 m_0$ (DOS effective mass)

$N_V = 1.83 \times 10^{19}$ cm^{-3}

Holes

$$p_0 = N_V \mathcal{F}_{1/2}\left[\left(E_V - E_F\right)/k_B T\right] \text{ m}^{-3}$$

$$N_V = \frac{1}{4}\left(\frac{2 m_p^* k_B T}{\pi \hbar^2}\right)^{3/2}$$

nondegenerate:

$$p_0 = N_V e^{\left(E_V - E_F\right)/k_B T}$$

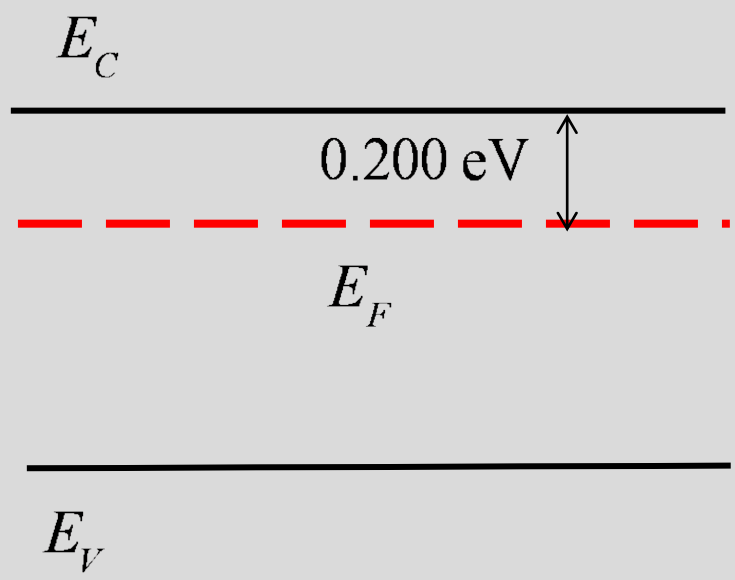
Fermi level and electron concentration

Given the Fermi level, we can deduce the **electron** and hole concentrations.

$$n_0 = N_C \exp\left(\frac{\left(E_F - E_C\right)}{k_B T}\right)$$

$$N_C = 3.23 \times 10^{19} \text{ cm}^{-3}$$

(silicon at 300 K)



$$n_0 = N_C \exp\left(\frac{-0.200}{0.026}\right)$$

$$= N_C \exp\left(-7.69\right)$$

$$= N_C \times 4.56 \times 10^{-4}$$

$$= 1.47 \times 10^{16} \text{ cm}^{-3}$$

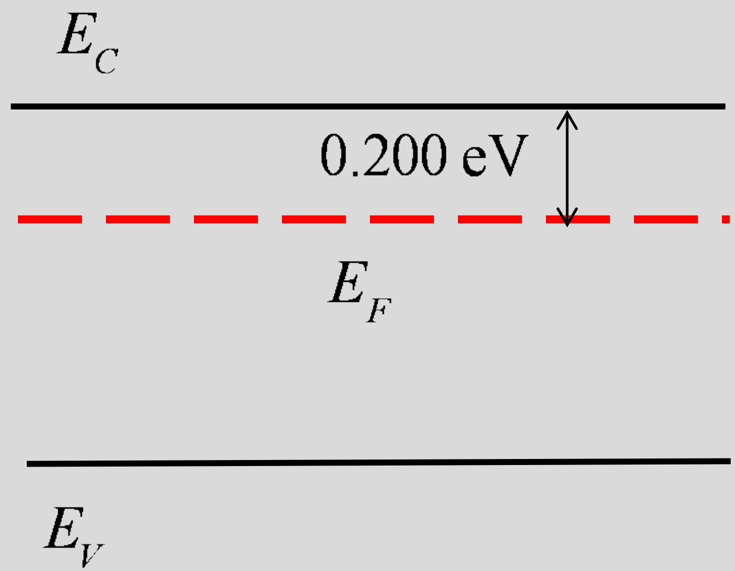
Fermi level and hole concentration

Given the Fermi level, we can deduce the electron and **hole** concentrations.

$$p_0 = N_V \exp\left(\frac{\left(E_V - E_F\right)}{k_B T}\right)$$

$$N_V = 1.83 \times 10^{19}\ \text{cm}^{-3}$$

(silicon at 300 K)



$$p_0 = N_V \exp\left(-\frac{1.11 - 0.200}{0.026}\right)$$

$$= N_V \exp\left(-35.0\right)$$

$$= N_V \times 6.31 \times 10^{-16}$$

$$= 1.14 \times 10^{4}\ \text{cm}^{-3}$$

From carrier concentration to Fermi level

$$n_0 = N_C e^{(E_F - E_C)/k_B T}$$

$$p_0 = N_V e^{(E_V - E_F)/k_B T}$$

If we are given *n*:

$$E_F = E_C + k_B T \ln\left(\frac{n_0}{N_C}\right)$$

If we are given *p*:

$$E_F = E_V - k_B T \ln\left(\frac{p_0}{N_V}\right)$$

np product

The equilibrium product of the electron and hole concentrations is a **very important** quantity for a semiconductor.

np product

$$n_0 p_0 = N_C e^{(E_F - E_C)/k_B T} N_V e^{(E_V - E_F)/k_B T}$$

$$n_0 p_0 = N_C N_V e^{(E_V - E_C)/k_B T}$$

$$n_0 p_0 = N_C N_V e^{-E_G/k_B T} = n_i^2$$

$$\boxed{n_i = \sqrt{N_C N_V} e^{-E_G/2k_B T}}$$

$$n_0 = N_C e^{(E_F - E_C)/k_B T}$$

$$p_0 = N_V e^{(E_V - E_F)/k_B T}$$

$$N_C = 2\left[\frac{\left(m_n^* k_B T\right)}{2\pi\hbar^2}\right]^{3/2}$$

$$N_V = 2\left[\frac{\left(m_p^* k_B T\right)}{2\pi\hbar^2}\right]^{3/2}$$

np product

$$n_0 p_0 = n_i^2$$

$$n_i = \sqrt{N_C N_V} e^{-E_G/2k_BT}$$

- Independent of Fermi level (for nondegenerate semiconductor)
- Depends exponentially on band gap
- Depends exponentially on temperature
- For Si at 300 K

$$n_i = 1.0 \times 10^{10} \text{ cm}^{-3}$$

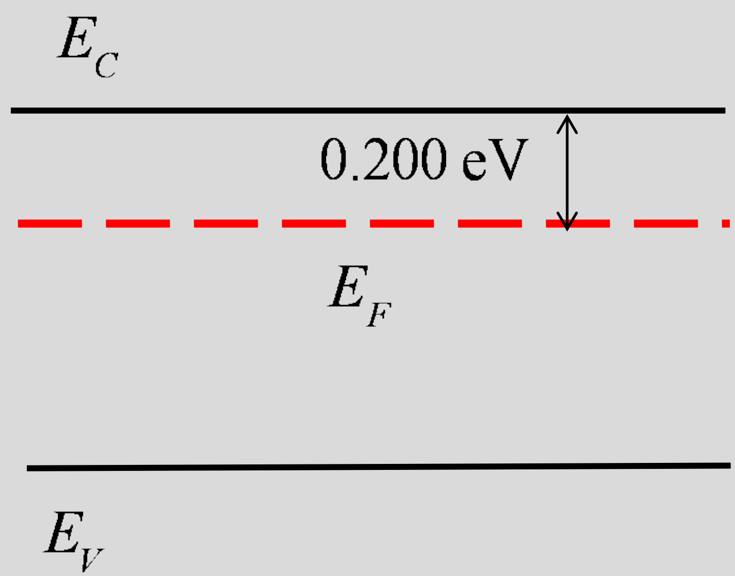
Recall: Fermi level and hole concentration

Given the Fermi level, we can deduce the electron and **hole** concentrations.

$$p_0 = N_V \exp\left(\frac{(E_V - E_F)}{k_B T}\right)$$

$$N_V = 1.83 \times 10^{19} \text{ cm}^{-3}$$

(silicon)



$$p_0 = N_V \exp\left(-\frac{1.11 - 0.200}{0.026}\right)$$

$$= N_V \exp(-35.0)$$

$$= N_V \times 6.31 \times 10^{-16}$$

$$= 1.14 \times 10^{4} \text{ cm}^{-3}$$

Another way

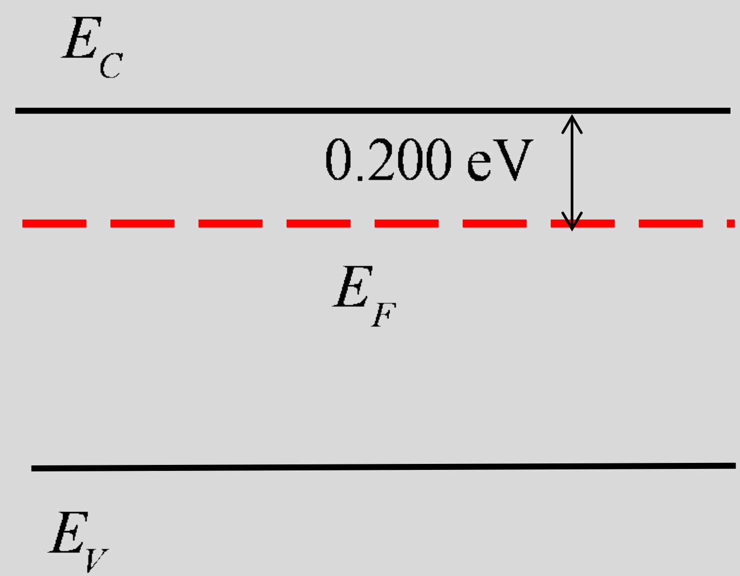
Find the electron concentration first.

$$n_0 = 1.47 \times 10^{16} \text{ cm}^{-3}$$

Then use

$$n_0 p_0 = n_i^2$$

$$n_i = \sqrt{N_C N_V} e^{-E_G/2k_BT} \neq 1.0 \times 10^{10} \text{ cm}^{-3}$$



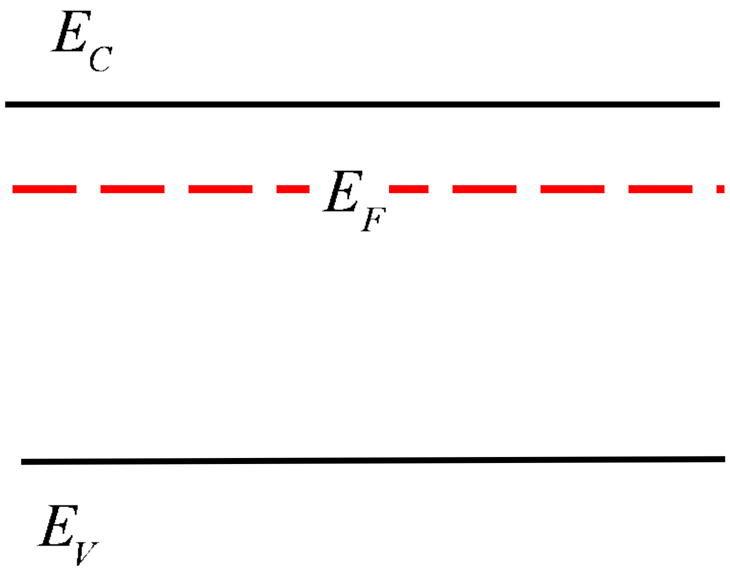
$$p_0 = n_i^2 / n_0$$

$$= \left(10^{10}\right)^2 / 1.47 \times 10^{16}$$

$$= 0.68 \times 10^3 \text{ cm}^{-3}$$

$$\boxed{\left(\text{vs. } 1.14 \times 10^4 \text{ cm}^{-1}\right)}$$

E-band diagram for N-type semiconductor

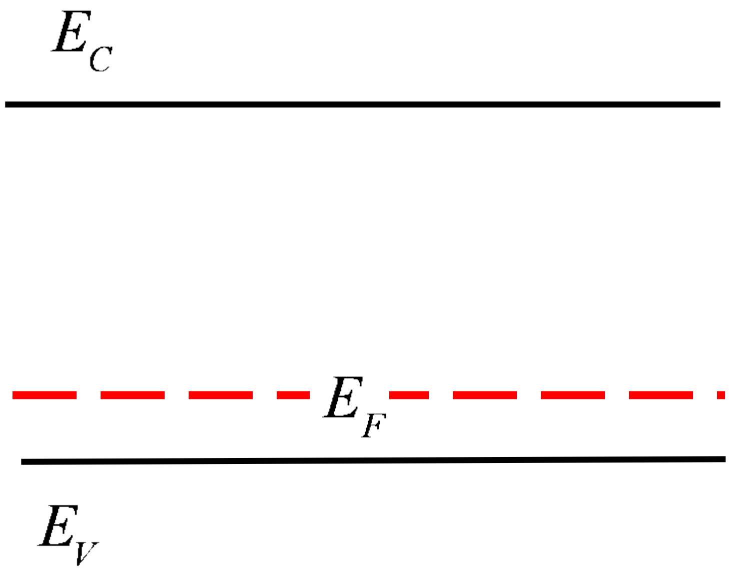


Fermi level is closer to the conduction band than to the valence band.

Electron concentration is greater than the hole concentration. $n_0 > p_0$

But the *np* product does not change. $n_0 p_0 = n_i^2$

E-band diagram for P-type semiconductor

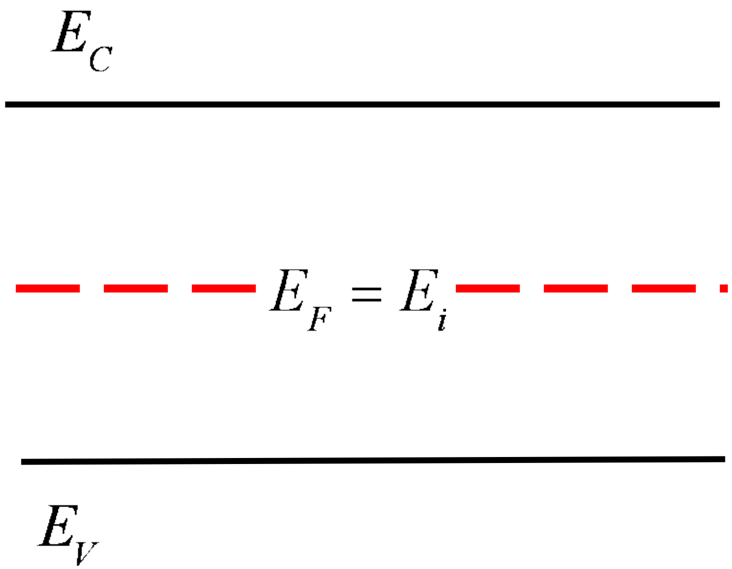


Fermi level is closer to the valence band than to the conduction band.

Hole concentration is greater than the electron concentration. $p_0 > n_0$

But the *np* product does not change. $n_0 p_0 = n_i^2$

Intrinsic semiconductor



Fermi level is near the middle of the gap.

Hole concentration is equal to the electron concentration. $p_0 = n_0$

The *np* product is still the same. $n_0 p_0 = n_i^2$

Exactly where is the intrinsic Fermi level?

The intrinsic Fermi level

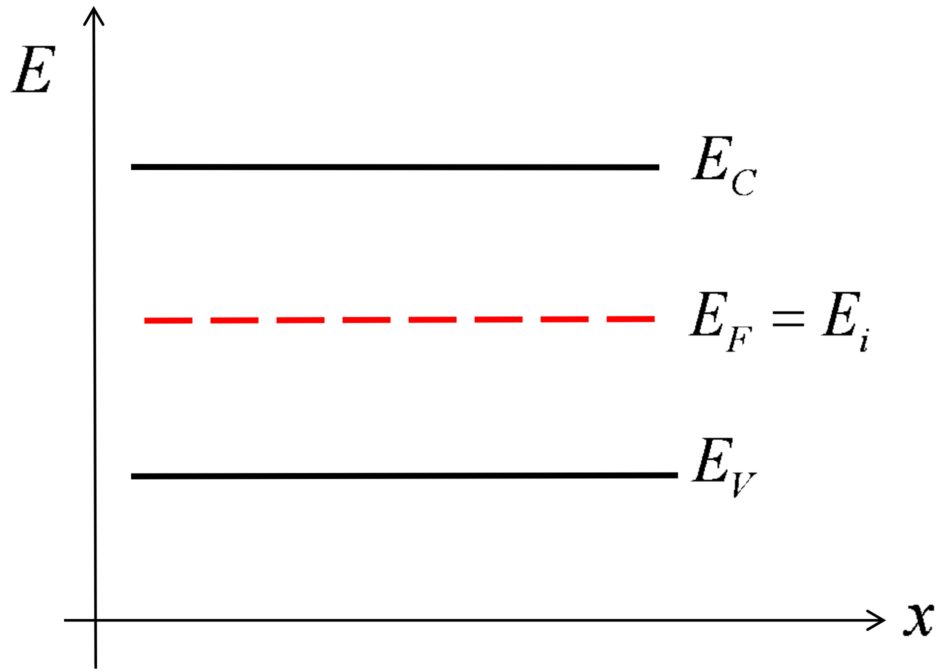
$$n_0 = N_C e^{(E_F - E_C)/k_B T}$$

$$p_0 = N_V e^{(E_V - E_F)/k_B T}$$

$$n_0 = p_0 = n_i \qquad E_F = E_i$$

$$N_C e^{(E_i - E_C)/k_B T} = N_V e^{(E_V - E_i)/k_B T}$$

$$\boxed{E_i = \frac{E_C + E_V}{2} + \frac{k_B T}{2} \ln\left(\frac{N_V}{N_C}\right)}$$



The intrinsic level: Silicon

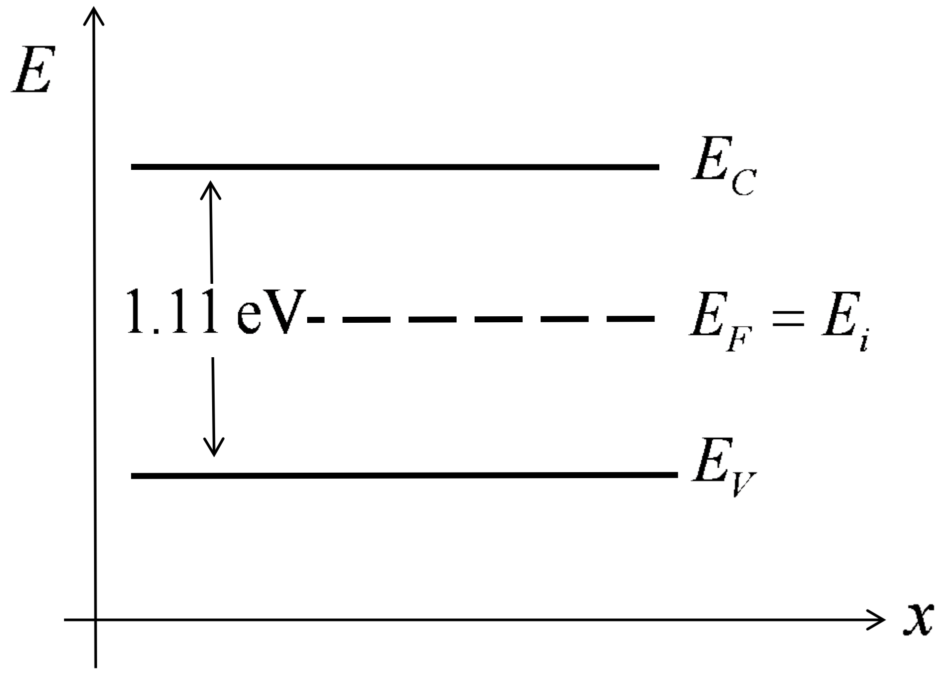
$$\boxed{E_i = \frac{E_C + E_V}{2} + \frac{k_B T}{2} \ln\left(\frac{N_V}{N_C}\right)}$$

$$N_C = 3.23 \times 10^{19}\ \text{cm}^{-3}$$

$$N_V = 1.83 \times 10^{19}\ \text{cm}^{-3}$$

$$T = 300\ \text{K}$$

$$E_i = \frac{E_C + E_V}{2} - 0.007\ \text{eV}$$



The intrinsic level is very near the middle of the band gap.

Alternative expression for carrier densities

$$n_0 = N_C e^{(E_F - E_C)/k_B T}$$

$$p_0 = N_V e^{(E_V - E_F)/k_B T}$$

$$n_0 = n_i \, e^{(E_F - E_i)/k_B T}$$

$$p_0 = n_i e^{(E_i - E_F)/k_B T}$$

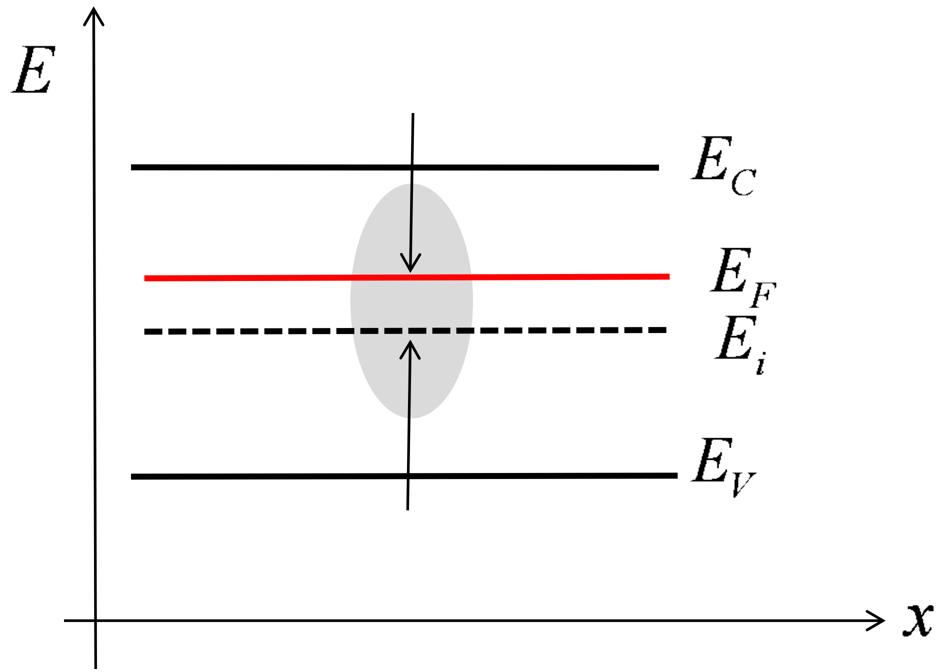
$$n_i = N_C e^{(E_i - E_C)/k_B T} \rightarrow N_C = n_i e^{-(E_i - E_C)/k_B T}$$

$$p_i = N_V e^{(E_V - E_i)/k_B T} \rightarrow N_V = n_i e^{-(E_V - E_i)/k_B T}$$

"Reading" an E-band diagram

$$n_0 = n_i\, e^{(E_F - E_i)/k_B T}$$

$$p_0 = n_i e^{(E_i - E_F)/k_B T}$$



- Fermi level above E_i, n-type
- Fermi level below E_i, p-type

Summary

$$n_0 = N_C e^{(E_F - E_C)/k_B T}$$

$$n_0 = n_i e^{(E_F - E_i)/k_B T}$$

$$p_0 = N_V e^{(E_V - E_F)/k_B T}$$

$$p_0 = n_i e^{(E_i - E_F)/k_B T}$$

$$N_C = \frac{1}{4}\left(\frac{2m_n^* k_B T}{\pi\hbar^2}\right)^{3/2}$$

$$n_i = \sqrt{N_C N_V}\, e^{-E_G/2k_B T}$$

$$N_V = \frac{1}{4}\left(\frac{2m_p^* k_B T}{\pi\hbar^2}\right)^{3/2}$$

$$n_0 p_0 = n_i^2$$