Primer on Semiconductors

Unit 5: The Semiconductor Equations

Lecture 5.2: Energy band diagrams

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The semiconductor equations

$$\frac{\partial p}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_p}{q} \right) + G_p - R_p$$

$$\frac{\partial n}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_n}{-q} \right) + G_n - R_n$$

$$\nabla \bullet \left(K_S \varepsilon_0 \vec{\mathcal{E}} \right) = \rho$$

Three equations in three unknowns:

$$p(\vec{r}), n(\vec{r}), V(\vec{r})$$

$$\vec{J}_p = pq\mu_p \vec{\mathcal{E}} - qD_p \vec{\nabla} p$$

$$\vec{J}_n = nq\mu_n \vec{\mathcal{E}} + qD_n \vec{\nabla} n$$

$$\rho = q\left(p - n + N_D^+ - N_A^- \right)$$

$$\vec{\mathcal{E}}(\vec{r}) = \nabla V(\vec{r})$$

Energy band diagrams

An energy band diagram is a plot of the bottom of the conduction band and the top of the valence band vs. position.

Energy band diagrams are a powerful tool for understanding semiconductor devices because they provide **qualitative solutions to the semiconductor equations**.

Kroemer's lemma of proven ignorance

"Whenever I teach my semiconductor device physics course, one of the central messages I try to get across early is the importance of energy band diagrams. I often put this in the form of "Kroemer's lemma of proven ignorance:

> If, in discussing a semiconductor problem, you cannot draw an **Energy Band Diagram**, this shows that **you** don't know what you are talking about."

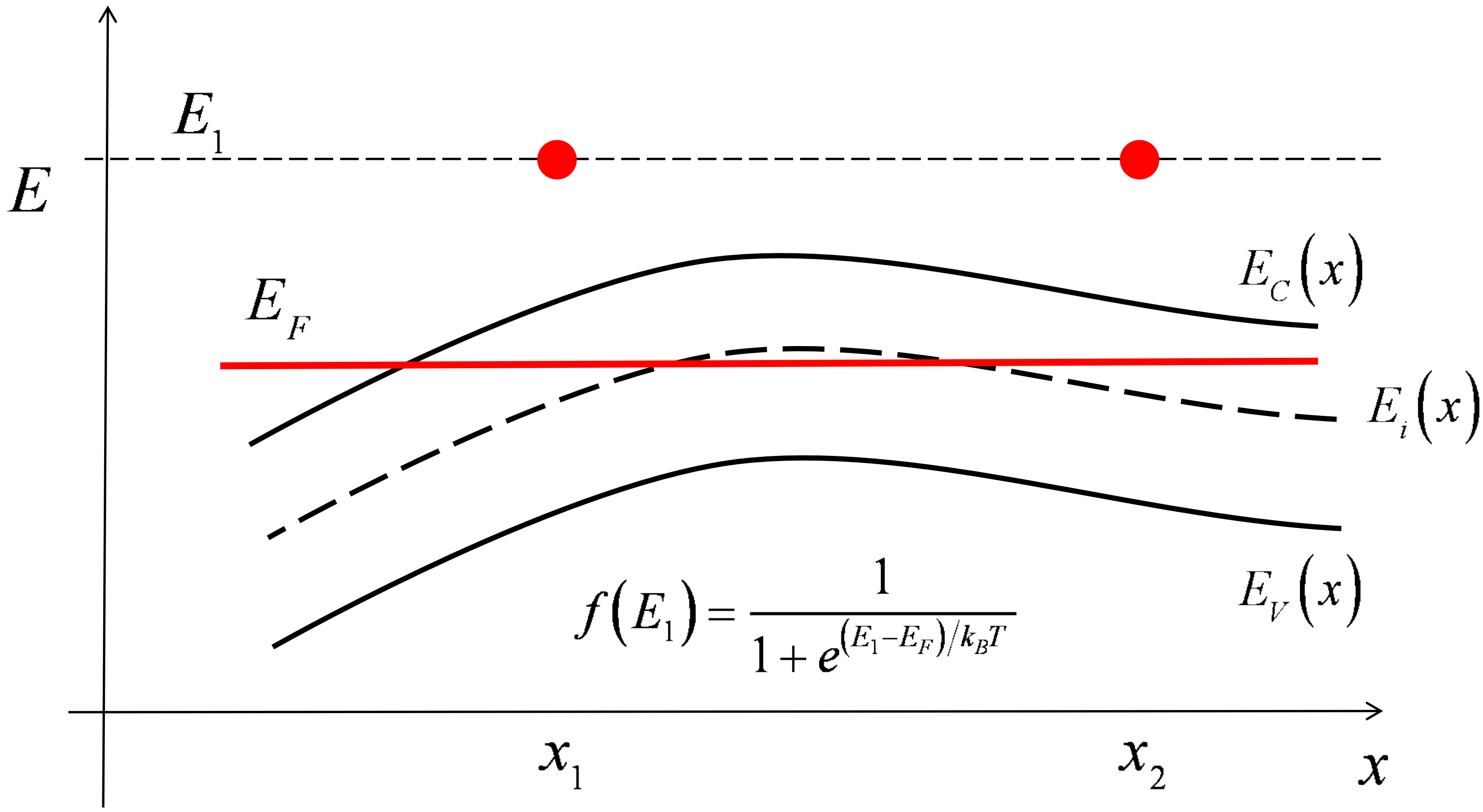
(Nobel Lecture, 2000)

Kroemer's corollary

If you can draw one, but don't, then **your audience** won't know what you are talking about."

(Nobel Lecture, 2000)

An important principle (in equilibrium)



$$f(E_1) = \frac{1}{1 + e^{(E_1 - E_F)/k_B T}}$$

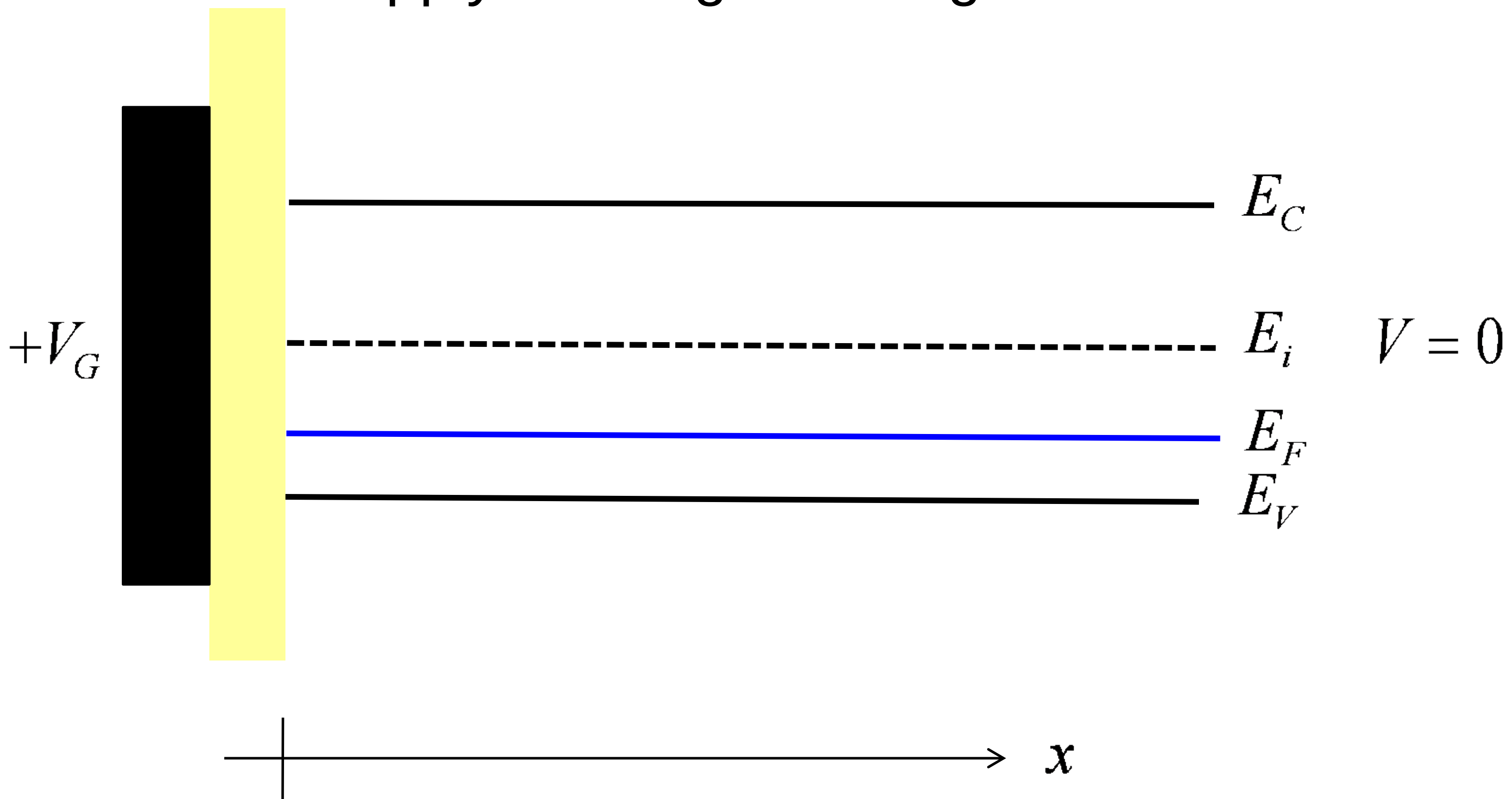
The Fermi level in equilibrium

The Fermi level is constant in equilibrium.

$$J_n = n\mu_n \frac{dF_n}{dx} = 0 = n\mu_n \frac{dE_F}{dx} \rightarrow E_F \text{ is constant}$$

Band bending

What happens when we apply a voltage to the gate?



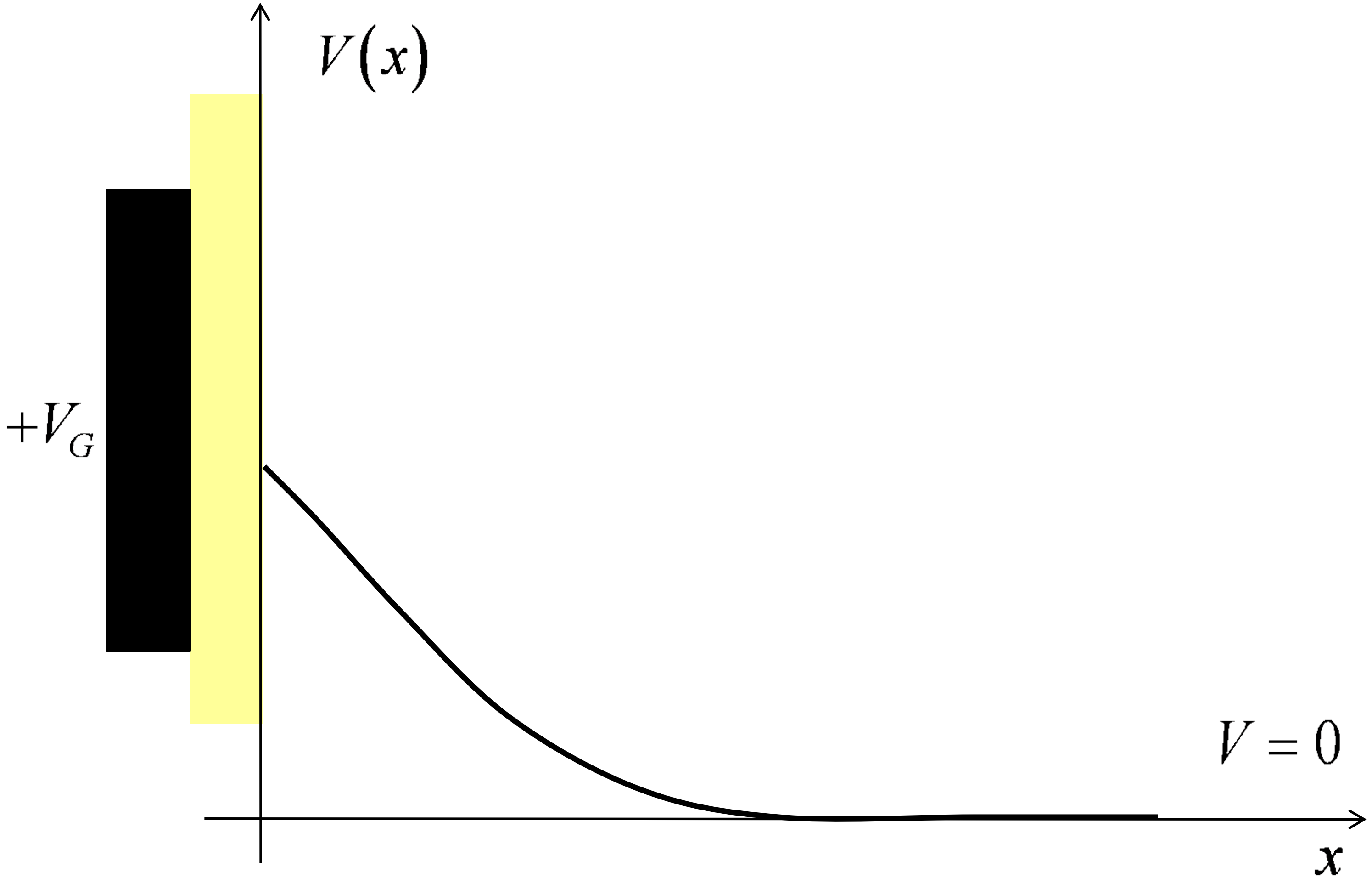
Voltage and electron potential energy

$$E = -qV$$

$+V$

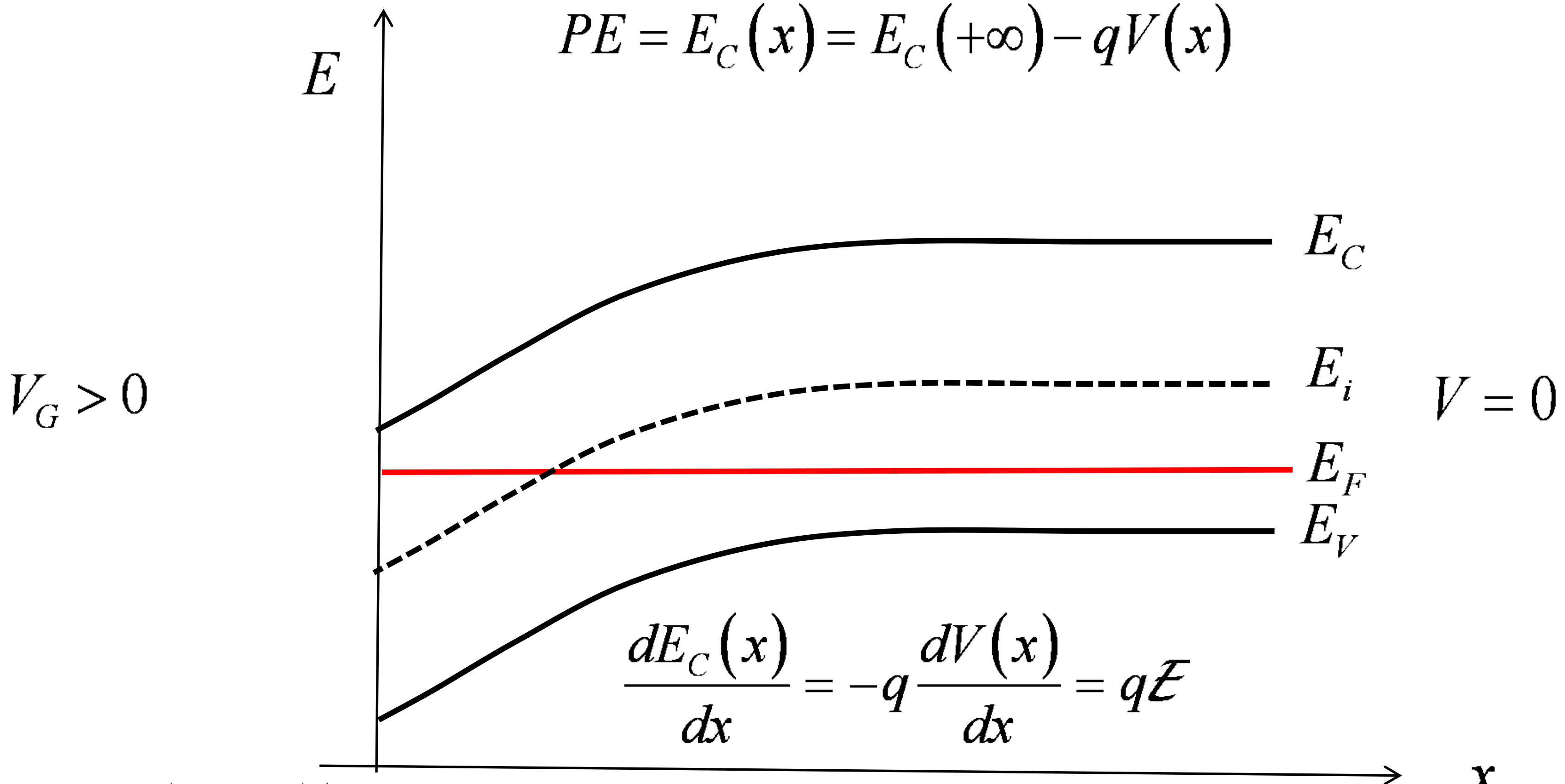
A positive potential **lowers** the energy of an electron.

Electrostatic potential vs. position



Electrostatic potential causes band bending

$$PE = E_C(x) = E_C(+\infty) - qV(x)$$



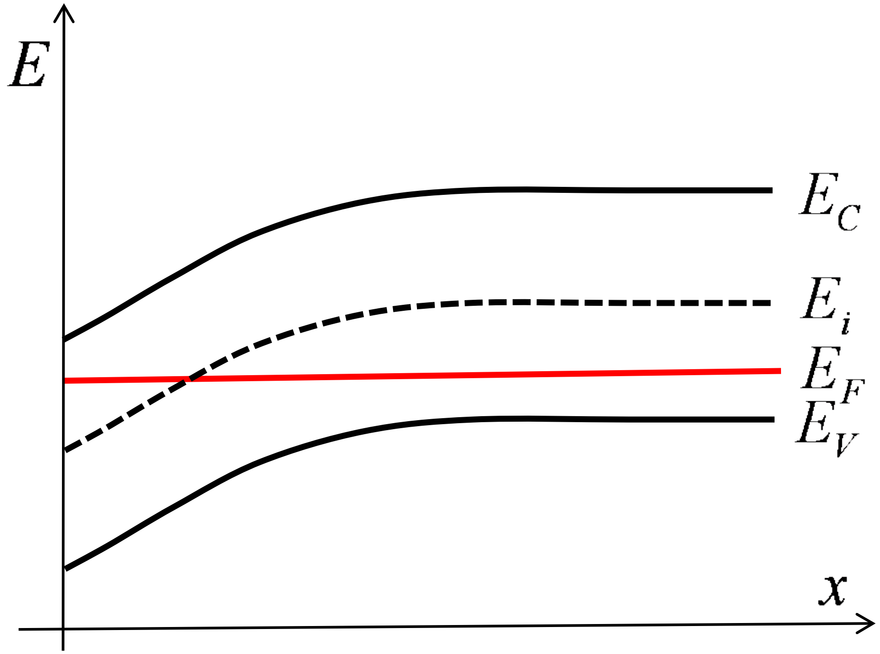
$$\frac{dE_C(x)}{dx} = -q\frac{dV(x)}{dx} = q\mathcal{E}$$

$$n(x) = n_i e^{(E_F - E_i(x))/k_B T}$$

$$p(x) = n_i e^{(E_i(x) - E_F)/k_B T}$$

Band diagrams

Drawing the band diagram



$$\frac{d\mathcal{E}}{dx} = \frac{\rho(x)}{K_S \varepsilon_0}$$

Reading the band diagram

$$V(x) \propto -E_C(x)$$

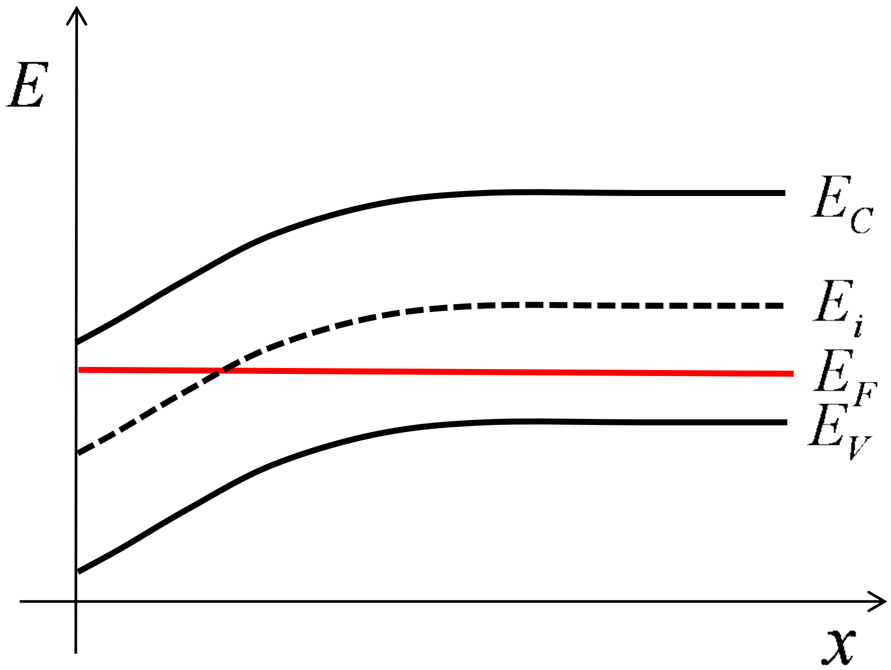
$$\mathcal{E} \propto dE_C(x)/dx$$

$$\log n(x) \propto E_F - E_i(x)$$

$$\log p(x) \propto E_i(x) - E_F$$

$$\rho(x) \propto d^2 E_C / dx^2$$

Practice

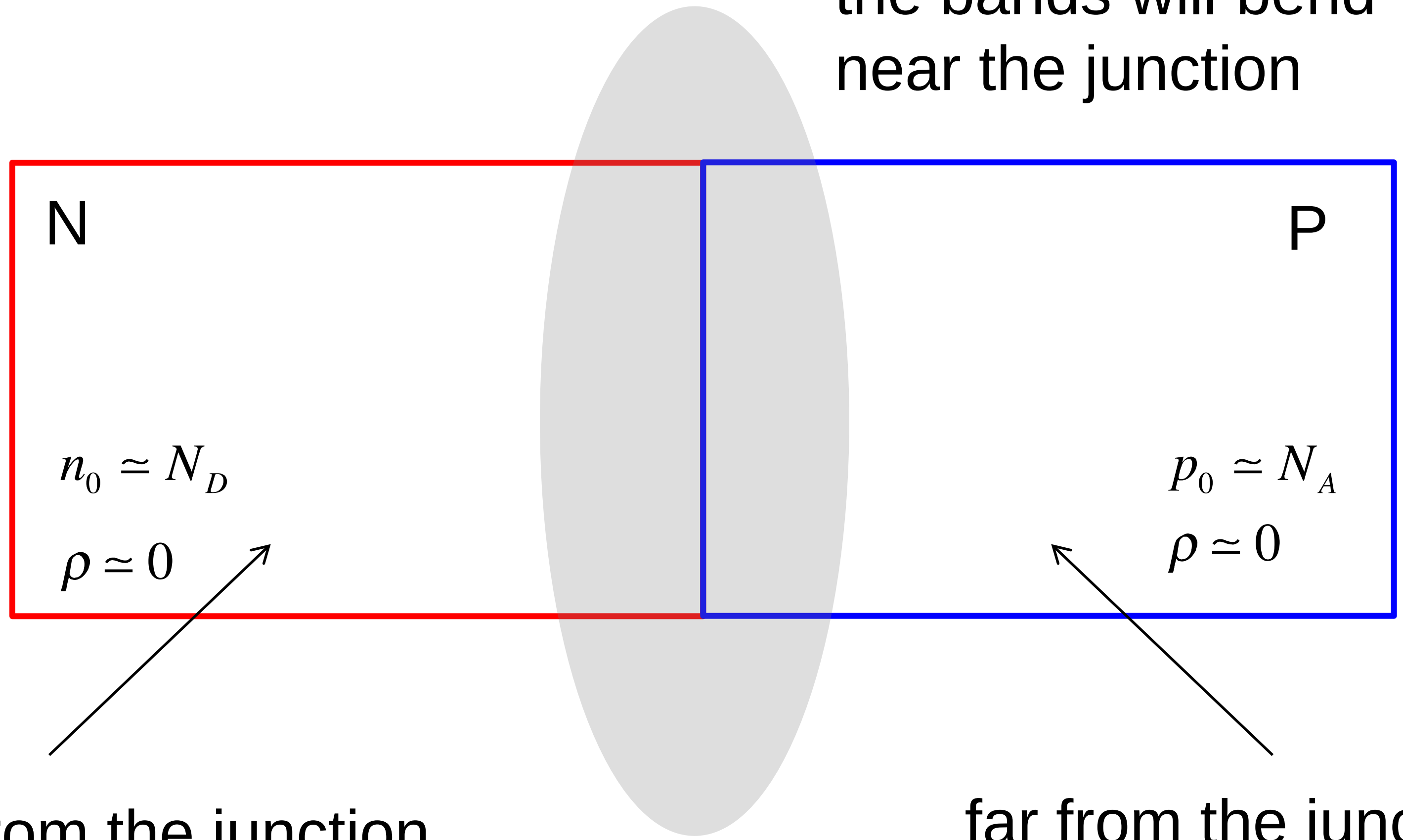


Sketch vs. position:

- Electrostatic potential
- Electric field
- Electron density
- Hole density
- Space charge density

Another example: NP junction in equilibrium

the bands will bend
near the junction



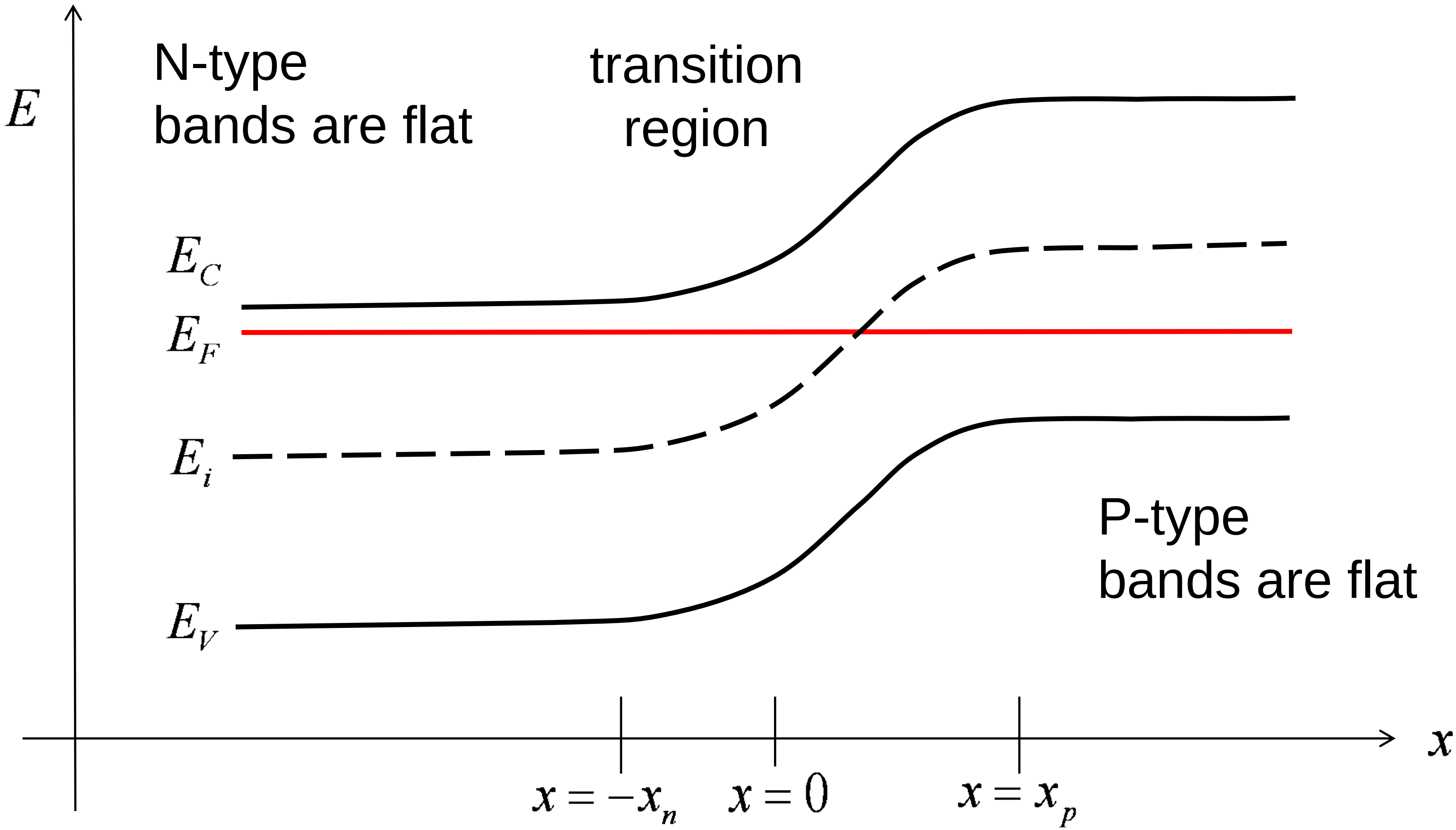
far from the junction,
the bands will be flat

far from the junction,
the bands will be flat

Procedure: Equilibrium energy band diagram

1) Begin with E_F.
2) Draw the E-bands where you know the carrier density then connect the two regions.
3) Then “read” the energy band diagram to obtain the electrostatic potential, electric field, carrier densities, and space charge density vs. position.

Energy band diagram



Now, “read” the e-band diagram

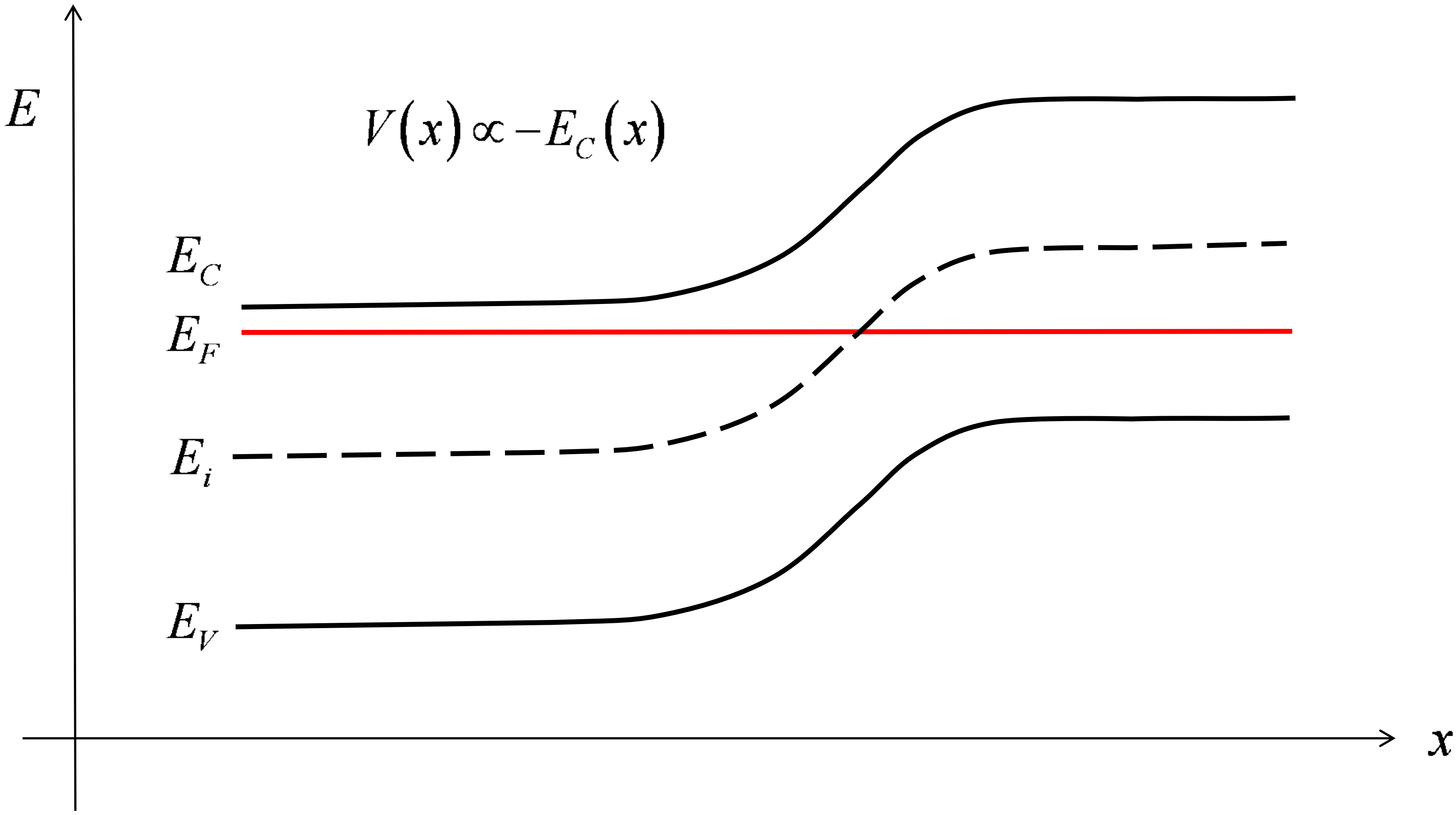
1) Electrostatic potential vs. position

2) Electric field vs. position

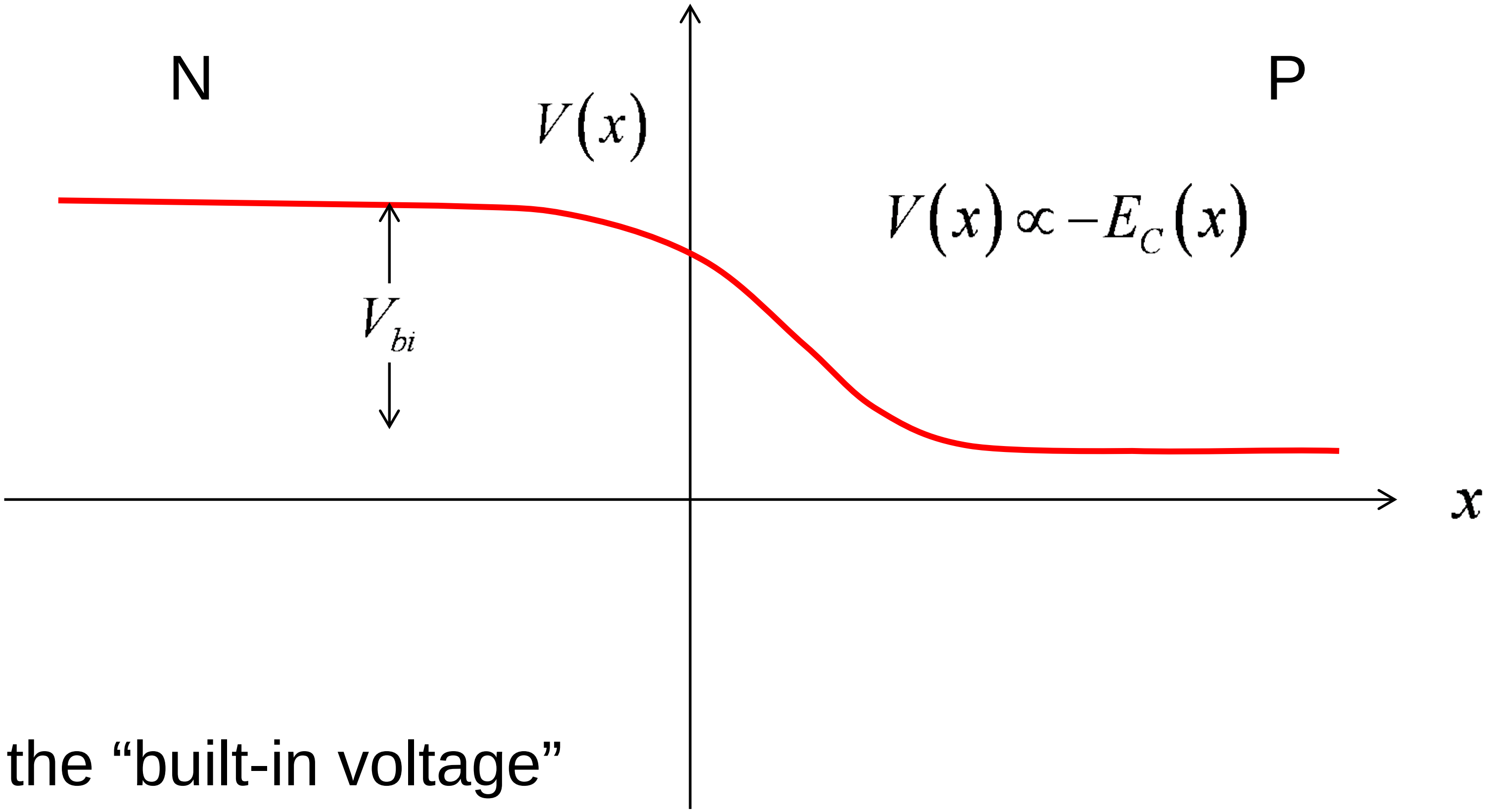
3) Electron and hole densities vs. position

4) Space-charge density vs. position

Electrostatic potential?

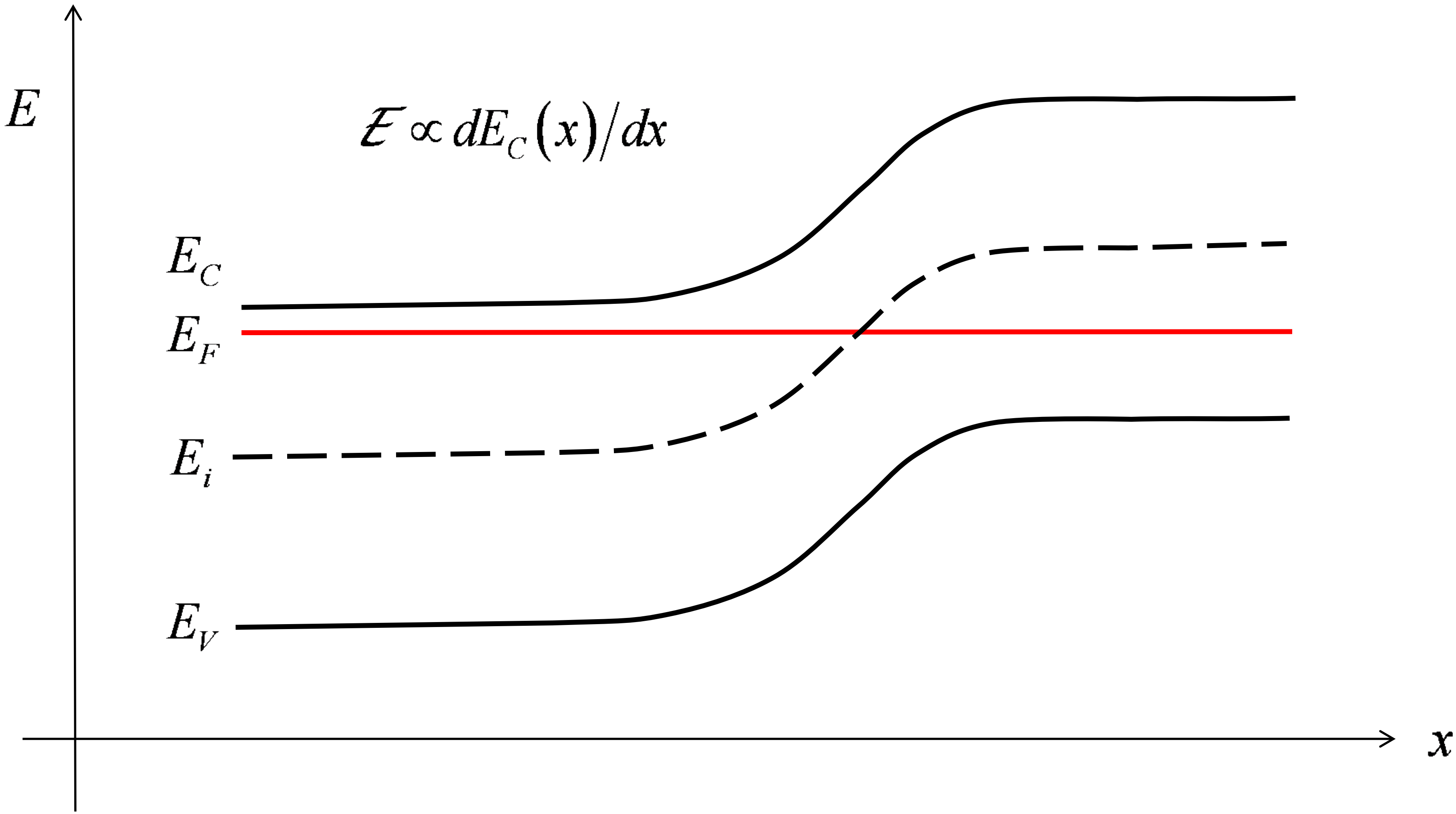


Electrostatics: *V*(*x*)

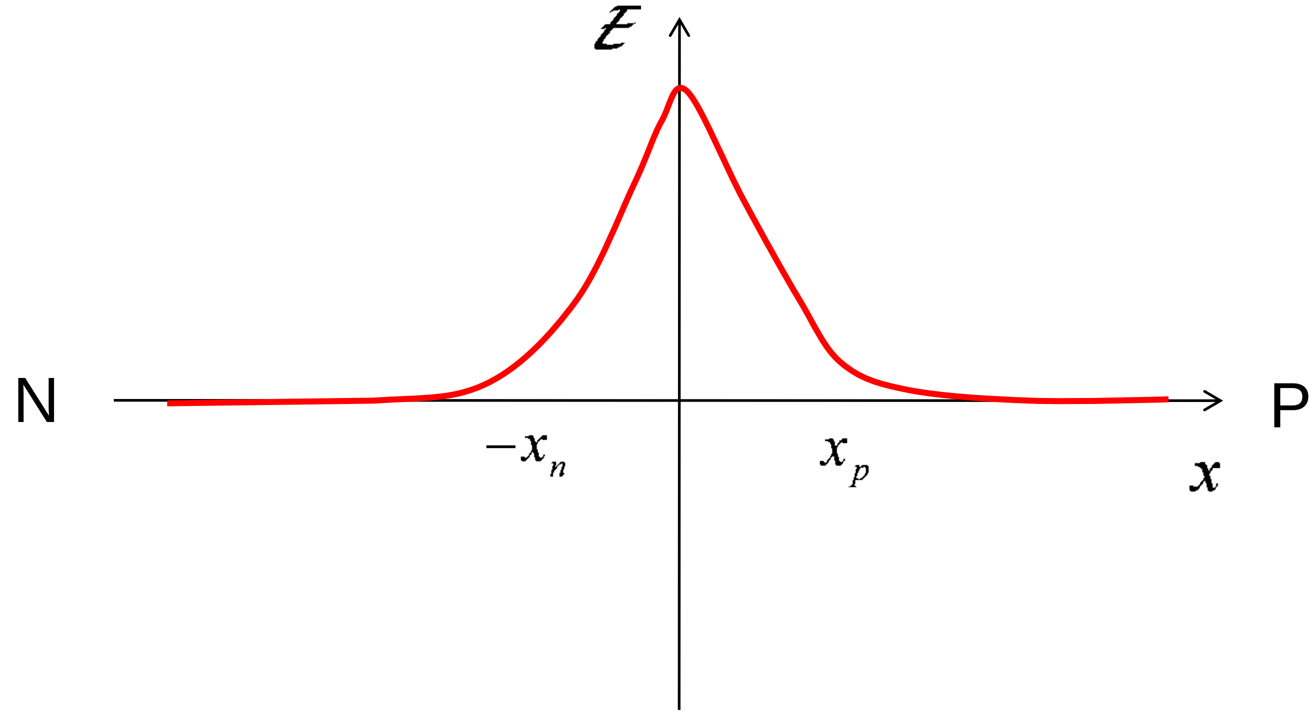


V_{bi} is the "built-in voltage"

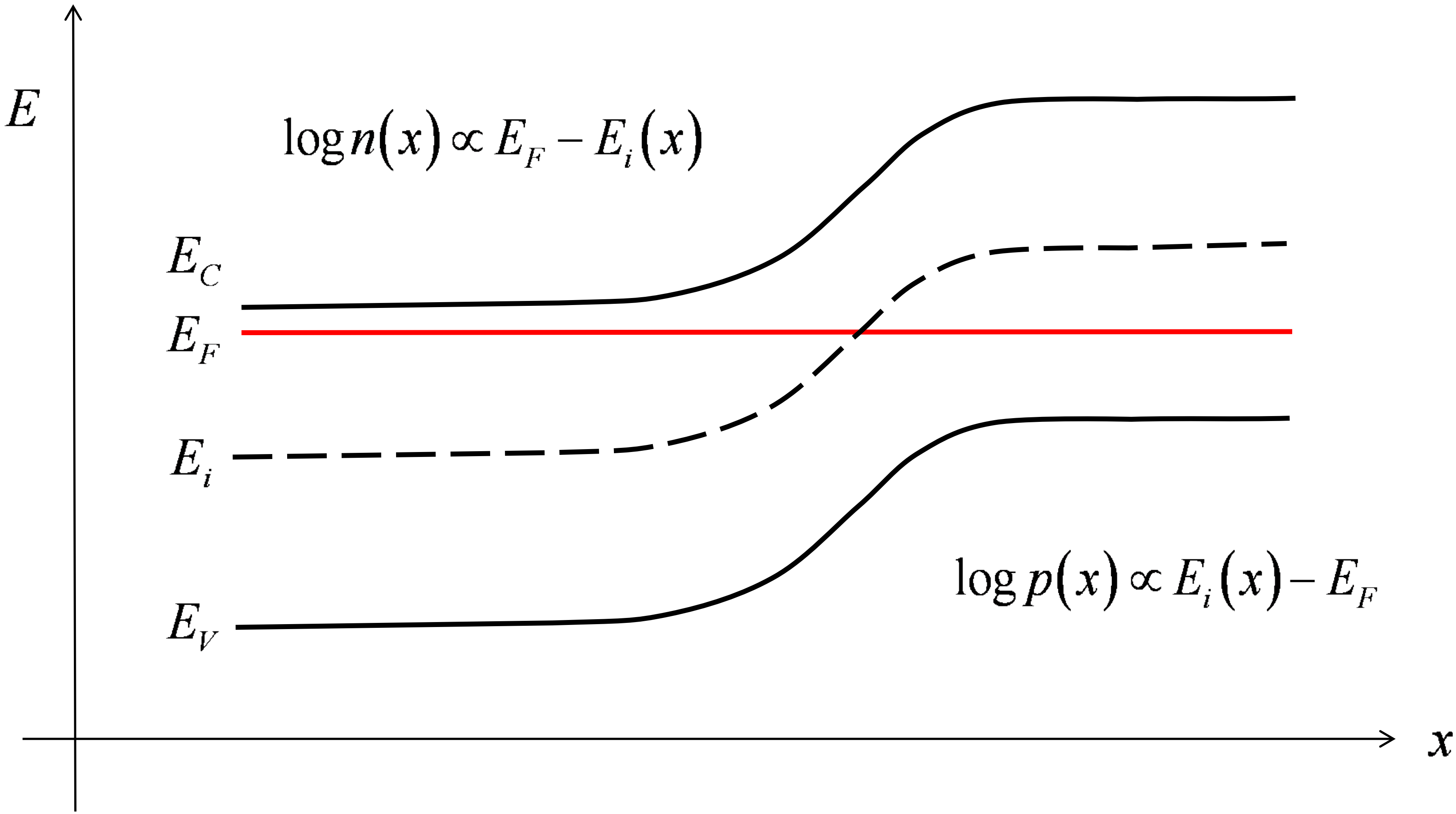
Electric field?



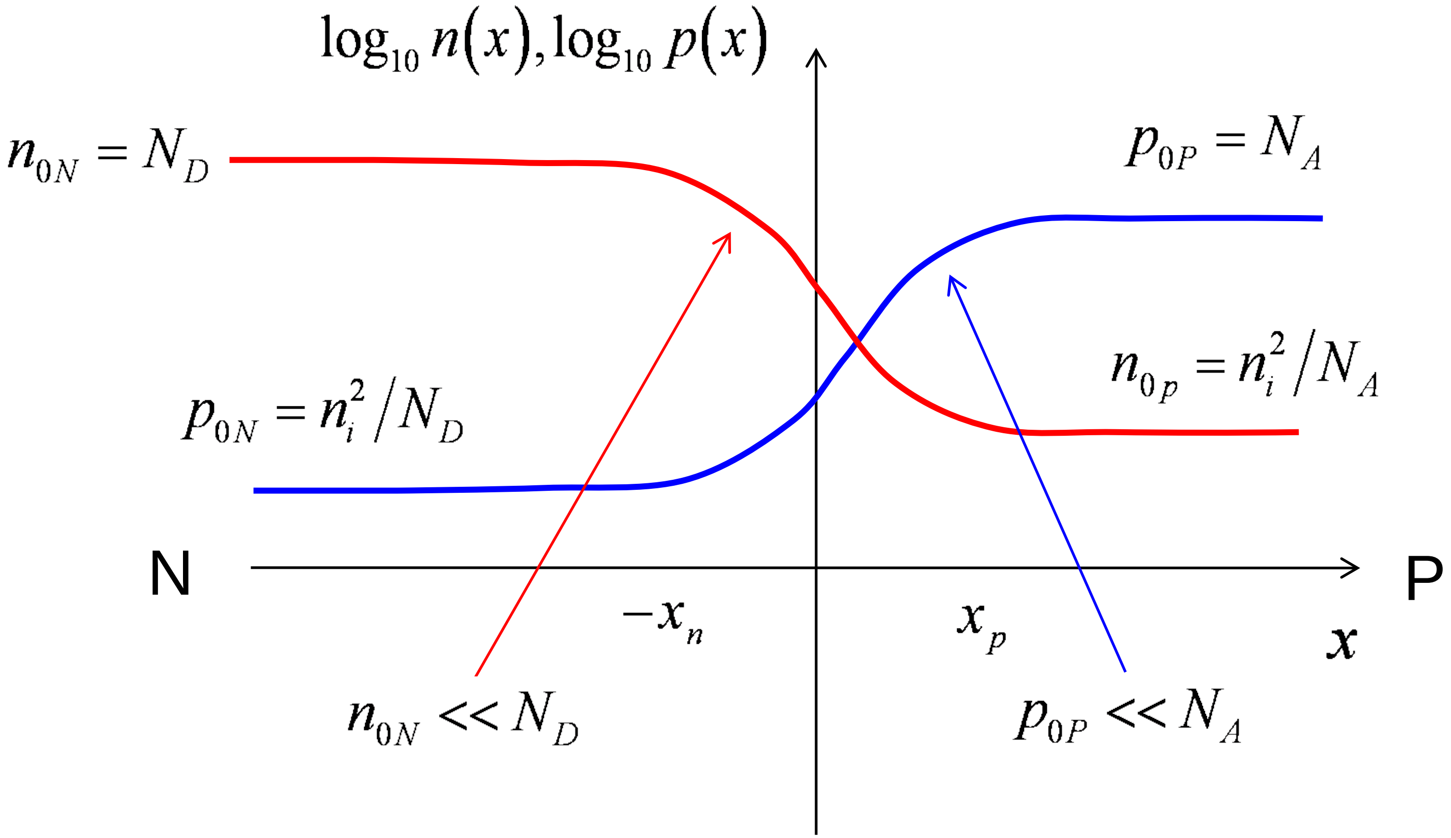
Electric field: $\mathcal{E}$ (x)



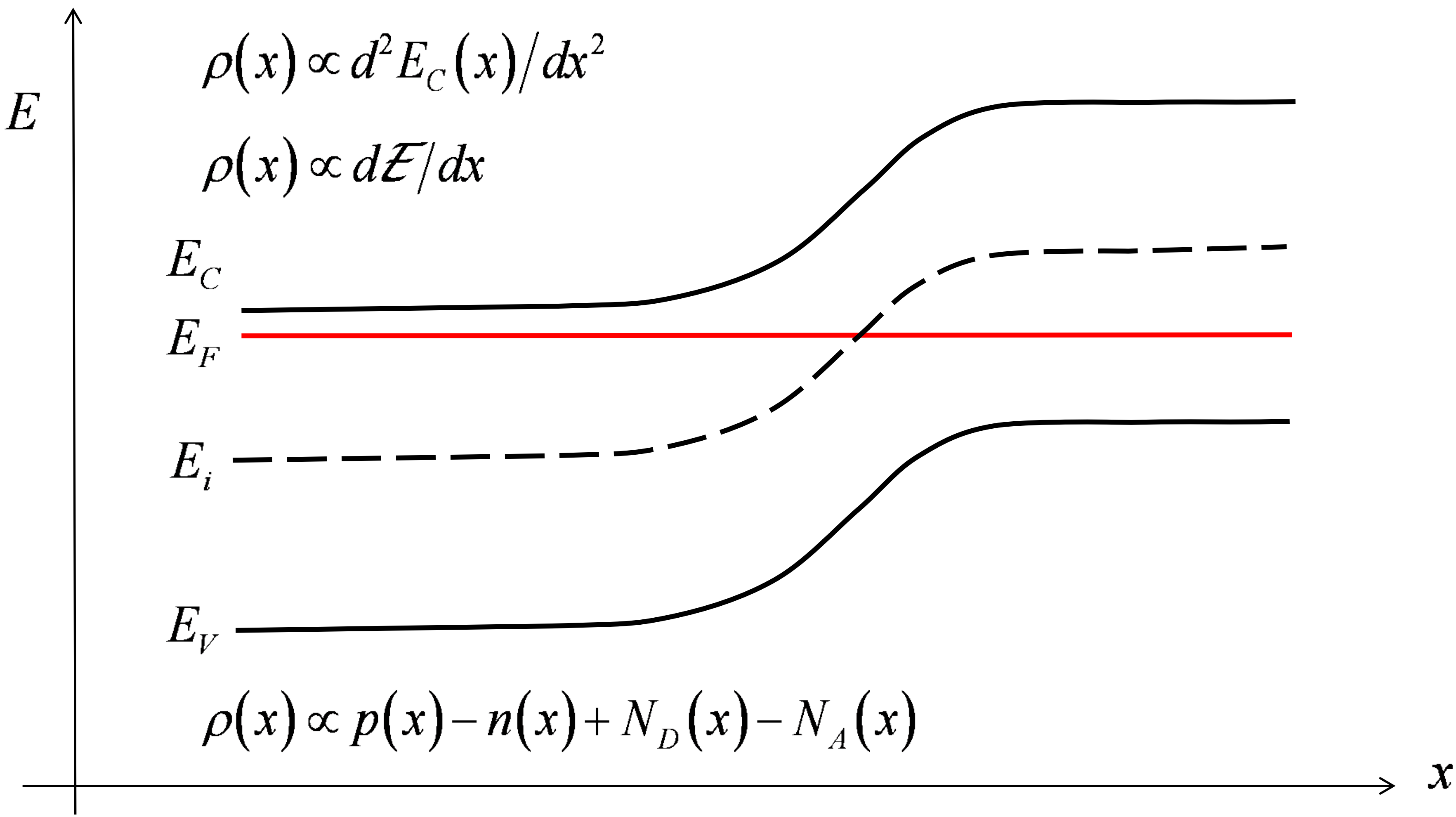
Carrier densities?



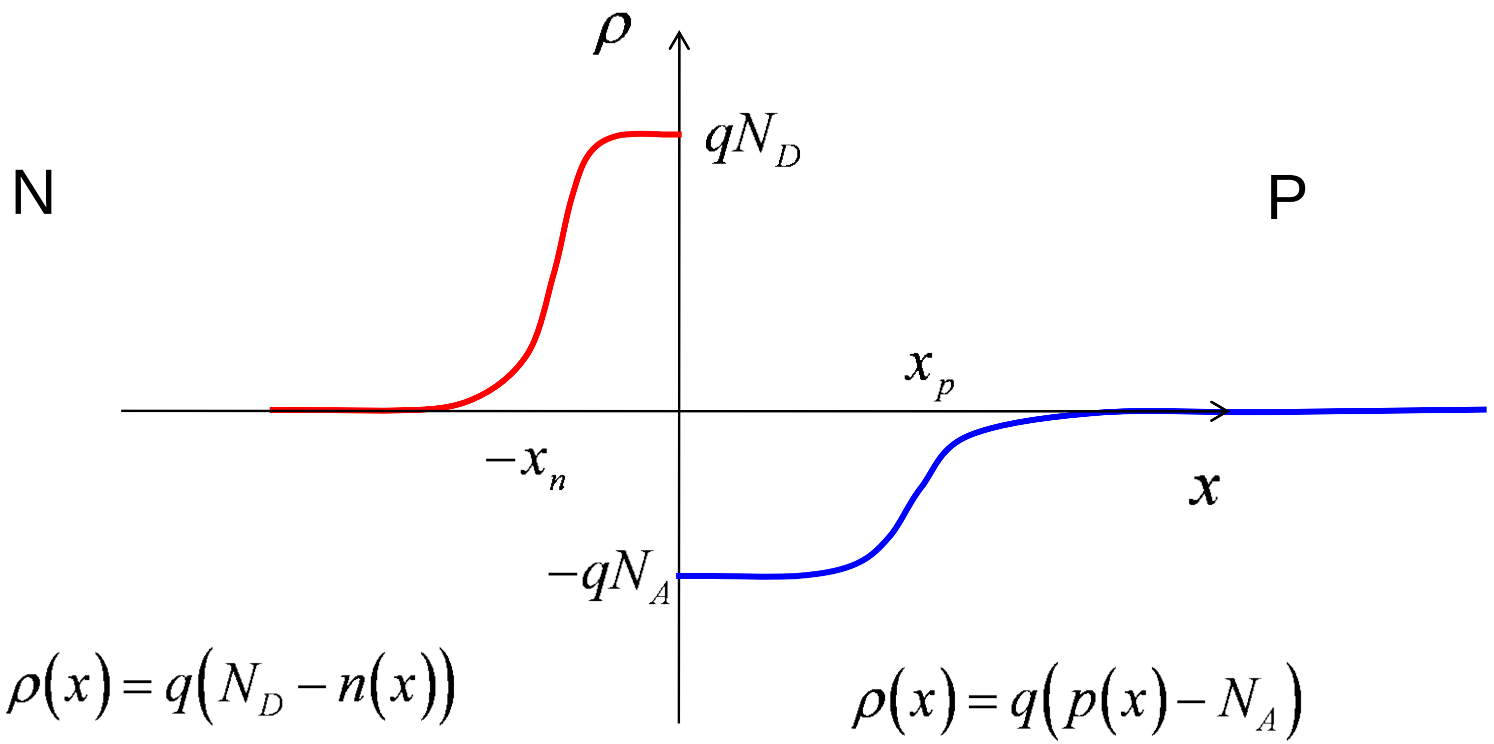
Carrier densities vs. *x*



Space charge?



Electrostatics: rho(x)



$$\rho(x) = q\left(N_D - n(x)\right)$$

$$\rho(x) = q\left(p(x) - N_A\right)$$

NP junction electrostatics

Question: How would we actually **calculate** rho(x), $\mathcal{E}(x)$, $V(x)$, $n(x)$, and $p(x)$?

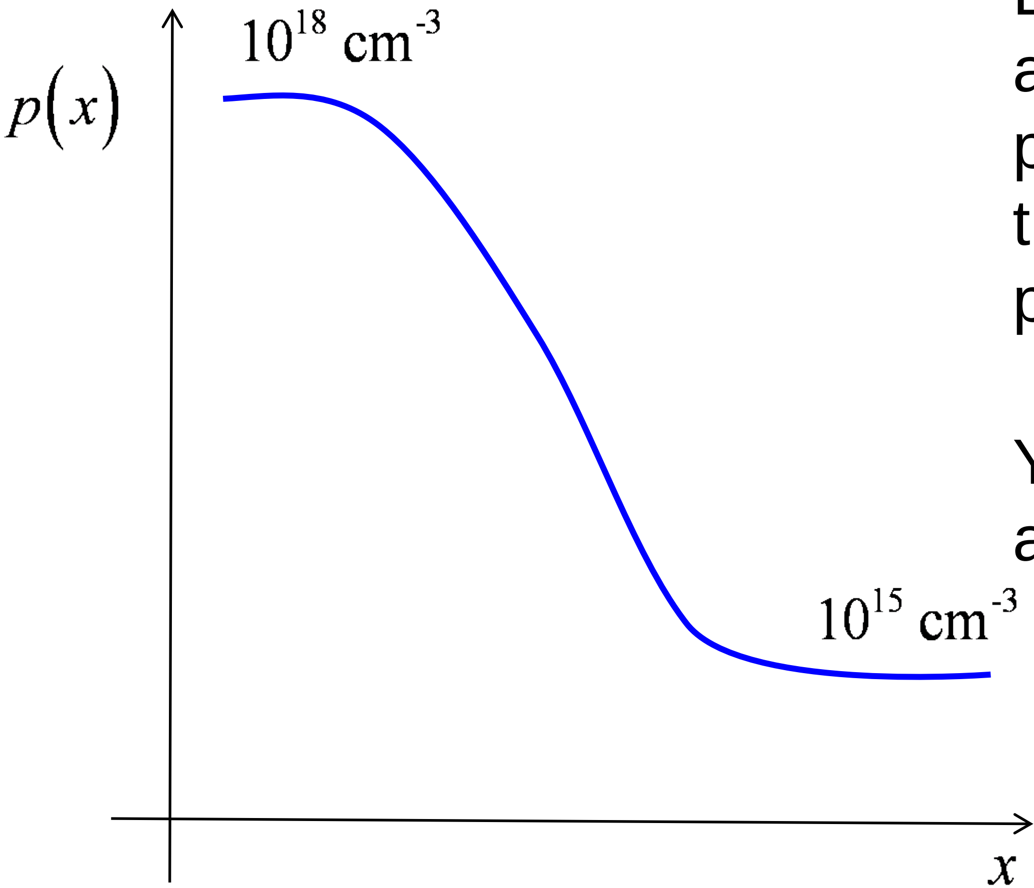
Answer: Solve the semiconductor equations.

$$\frac{\partial p}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_p}{q} \right) + G_p - R_p$$

$$\frac{\partial n}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_n}{-q} \right) + G_n - R_n$$

$$\nabla \bullet \left(K_S \varepsilon_0 \vec{\mathcal{E}} \right) = \rho$$

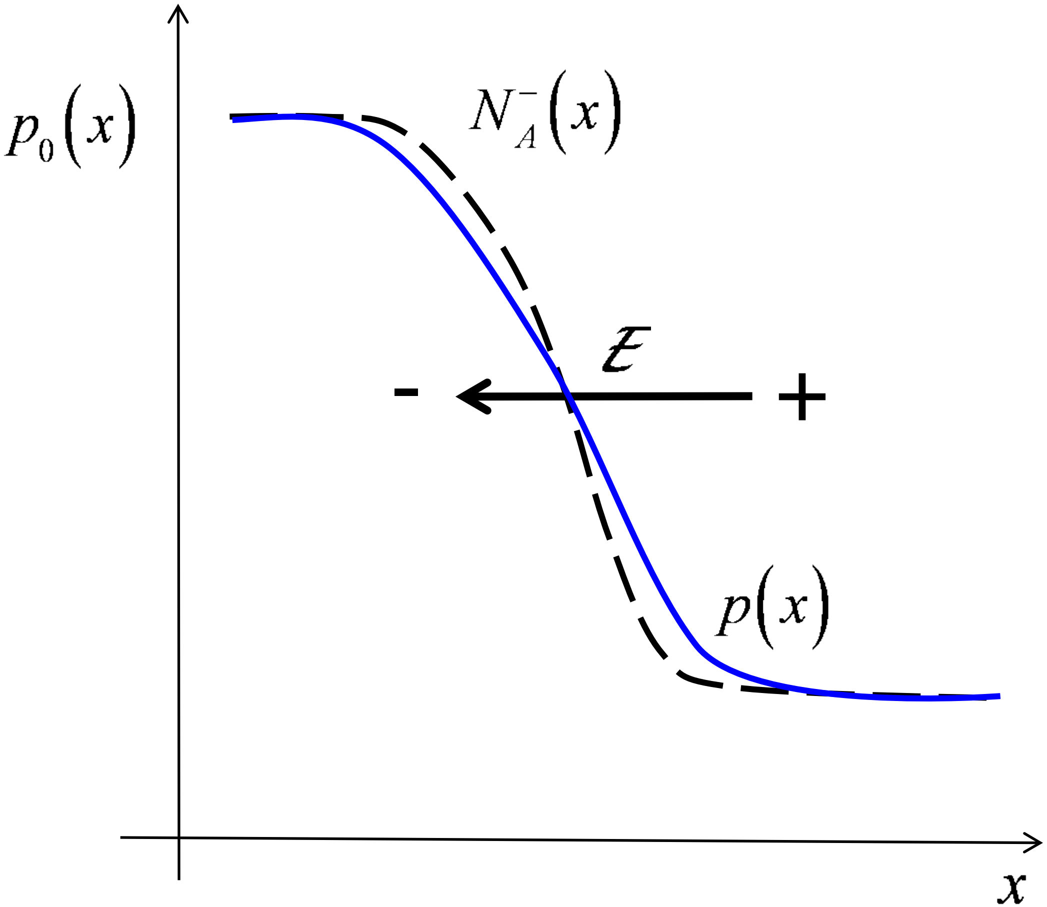
More practice



Draw the e-band diagram and find the electrostatic potential vs. position and the electric field vs. position.

You will find that there is an electric field.

Where does the electric-field come from?



$$p_0(x) \approx N_A^-(x)$$

"quasi-neutral"

The result is a drift current equal and opposite to the diffusion current to that the total current is zero in equilibrium.

Summary

$$\frac{\partial p}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_p}{q} \right) + G_p - R_p$$

$$\frac{\partial n}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_n}{-q} \right) + G_n - R_n$$

$$\nabla \bullet \left(K_S \varepsilon_0 \vec{\mathcal{E}} \right) = \rho$$

Three coupled, nonlinear PDE's in three unknowns:

$$p(\vec{r}),\, n(\vec{r}),\, V(\vec{r})$$

Drawing and then reading an E-band diagram gives us a qualitative solution to these equations.