Unit 3: Equilibrium Carrier Concentrations

Lecture 3.4: Carrier concentration vs. doping density

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Carrier concentrations vs. Fermi level

Electrons

$$n_0 = N_C \mathcal{F}_{1/2}\left[\left(E_F - E_C\right)/k_B T\right] \text{ m}^{-3}$$

$$N_C = \frac{1}{4}\left(\frac{2m_n^* k_B T}{\pi \hbar^2}\right)^{3/2}$$

nondegenerate:

$$n_0 = N_C e^{\left(E_F - E_C\right)/k_B T}$$

$$n_0 = n_i e^{\left(E_F - E_i\right)/k_B T}$$

Holes

$$p_0 = N_V \mathcal{F}_{1/2}\left[\left(E_V - E_F\right)/k_B T\right] \text{ m}^{-3}$$

$$N_V = \frac{1}{4}\left(\frac{2m_p^* k_B T}{\pi \hbar^2}\right)^{3/2}$$

nondegenerate:

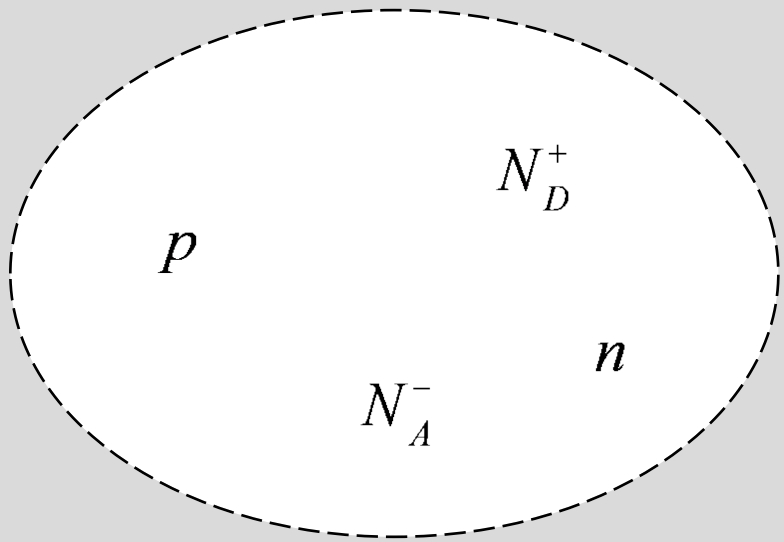
$$p_0 = N_V e^{\left(E_V - E_F\right)/k_B T}$$

$$p_0 = n_i e^{\left(E_i - E_F\right)/k_B T}$$

Space charge density

What is the net charge in this region?

$$\rho = q\left[p - n + N_D^+ - N_A^-\right] \quad \mathrm{C/m^3}$$



bulk, uniform semiconductor

Space charge neutrality

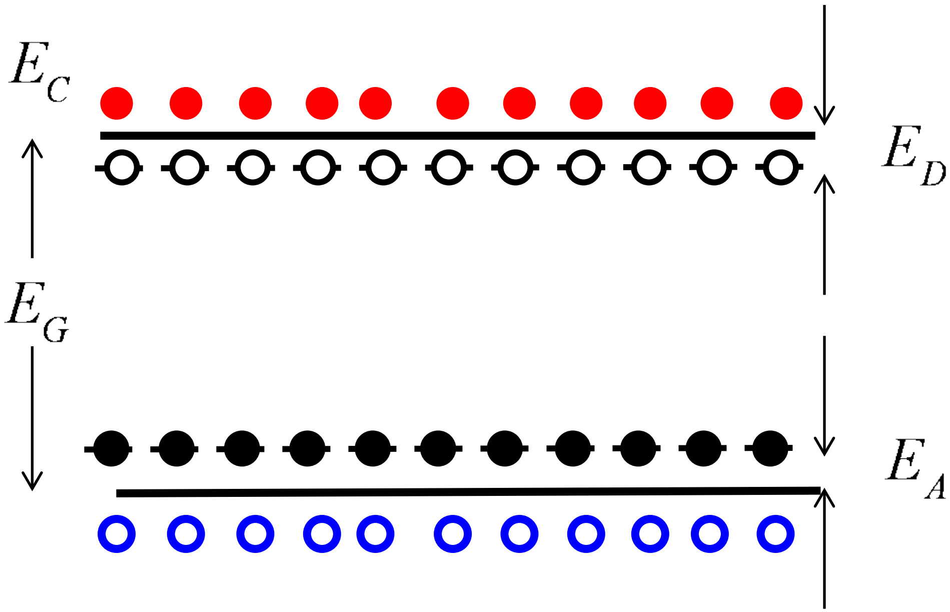
Nature abhors a vacuum. Nature also abhors a charge.

Mobile charges (electrons and holes) will be attracted to the immobile ionized dopants), so that the net charge is zero.

$$\rho = q\left[p - n + N_D^+ - N_A^- \right] = 0$$

Almost uniform semiconductors will be nearly neutral, but with strong non-uniformities (e.g. PN junctions), there will be a space charge.

Fully ionized dopants



All donors have donated their electrons to the conduction band and are now positively charged.

$$N_D^+ = N_D \qquad N_A^- = N_A$$

All acceptors have accepted an electron from the valence band and are now negatively charged.

Space charge neutrality again

$$\rho = q\left[p - n + N_D^+ - N_A^-\right] = 0$$

Assume **fully ionized dopants** (this will typically be the case for good dopants near and above room temperature).

$$\rho = q\left[p_0 - n_0 + N_D - N_A\right] = 0 \qquad n_0 p_0 = n_i^2$$

Assuming that we know how many dopants we introduced into the semiconductor, these are two equations in two unknowns – *p* and *n*.

Solving for the carrier density

1) charge neutrality: $$p_0 - n_0 + N_D^+ - N_A^- = 0$$

2) Fully ionized dopants: $$p_0 - n_0 + N_D - N_A = 0$$

3) np product: $$n_0 p_0 = n_i^2$$

4) result:
$$\frac{n_i^2}{n_0} - n_0 + N_D - N_A = 0$$

$$p_0 - \frac{n_i^2}{p_0} + N_D - N_A = 0$$

Result: N-type

$$\frac{n_i^2}{n_0} - n_0 + N_D - N_A = 0$$

$$\boxed{n_0 = \frac{N_D - N_A}{2} + \left[\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2\right]^{1/2} \qquad p_0 = \frac{n_i^2}{n_0}}$$

Result: P-type

$$p_0 - \frac{n_i^2}{p_0} + N_D - N_A = 0$$

$$\boxed{p_0 = \frac{N_A - N_D}{2} + \left[\left(\frac{N_A - N_D}{2}\right)^2 + n_i^2\right]^{1/2} \qquad n_0 = \frac{n_i^2}{p_0}}$$

Example 1

Consider Si doped with phosphorus at N_D = 2.00 x 10^{15} cm^{-3}
The temperature is 300 K. What are *n* and *p*?

Recall that at 300 K in Si, n_i = 1.00 x 10^{10} cm^{-3}

Assume that the donors are fully ionized.

$$n_0 = \frac{N_D - N_A}{2} + \left[\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2\right]^{1/2} = \frac{N_D}{2} + \left[\left(\frac{N_D}{2}\right)^2 + n_i^2\right]^{1/2}$$

$$N_D >> n_i \qquad n_0 = N_D = 2.00 \times 10^{15}\ \text{cm}^{-3}$$

$$p_0 = n_i^2 / n_0 \qquad p_0 = \left(10^{10}\right)^2 / 2 \times 10^{15} = 5 \times 10^4\ \ \text{cm}^{-3}$$

Example 2

Consider Si doped with phosphorus at N_D = 2.00 x 10^{15} cm^{-3} and Boron at N_A = 1.00 x 10^{15} cm^{-15}.
The temperature is 300 K. What are *n* and *p*?

$$n_0 = \frac{N_D - N_A}{2} + \left[\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2\right]^{1/2}$$

$$N_D - N_A >> n_i \qquad n_0 = N_D - N_A$$

$$p_0 = n_i^2 / n_0$$

$$\boxed{\begin{aligned} n_0 &= 1.00 \times 10^{15} \text{ cm}^{-3} \\ p_0 &= \left(10^{10}\right)^2 / 1 \times 10^{15} = 1 \times 10^{5} \text{ cm}^{-3} \end{aligned}}$$

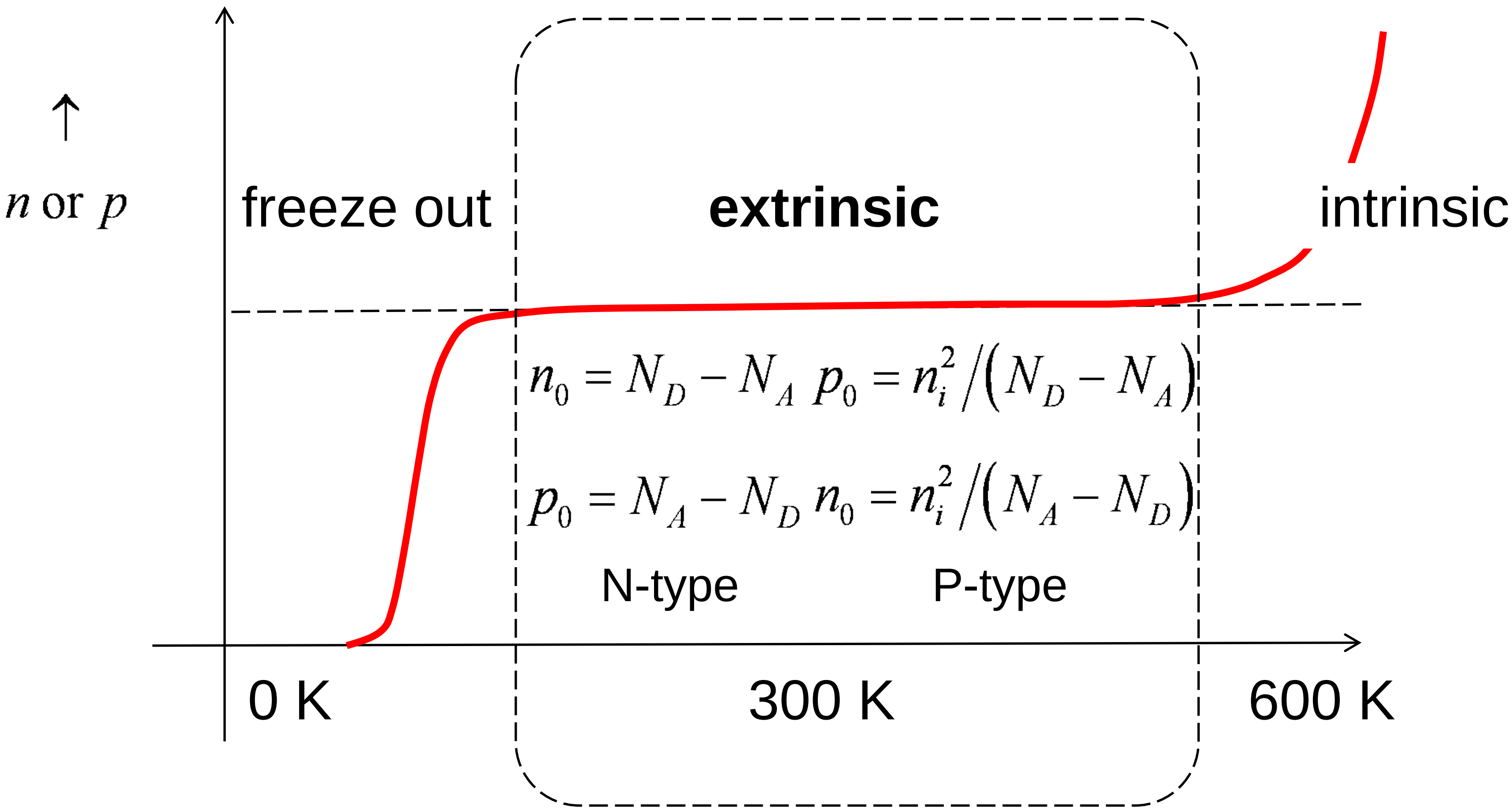
Conclusion

When the net doping density is much greater than the intrinsic carrier concentration and the dopants are fully ionized, then

$$n_0 = N_D - N_A \qquad p_0 = n_i^2 / \left(N_D - N_A\right) \qquad \text{N-type} \quad N_D > N_A$$

$$p_0 = N_A - N_D \qquad n_0 = n_i^2 / \left(N_A - N_D\right) \qquad \text{P-type} \quad N_A > N_D$$

Summary



n or p
freeze out
extrinsic
intrinsic
$n_0 = N_D - N_A \quad p_0 = n_i^2 / \left(N_D - N_A\right)$
$p_0 = N_A - N_D \quad n_0 = n_i^2 / \left(N_A - N_D\right)$
N-type
P-type
0 K
300 K
600 K