Primer on Semiconductors

Unit 5: The Semiconductor Equations

Lecture 5.4: Minority carrier diffusion equation

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Semiconductor equations

$$\frac{\partial p}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_p}{q} \right) + G_p - R_p$$

$$\frac{\partial n}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_n}{-q} \right) + G_n - R_n$$

$$\nabla \bullet \left(K_S \varepsilon_0 \vec{\mathcal{E}} \right) = \rho$$

Although not as fundamental as Maxwell's equations, these equations are the starting point for the analysis of most semiconductor devices.

Solving the semiconductor equations

$$\frac{\partial p}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_p}{q} \right) + G_p - R_p$$

$$\frac{\partial n}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_n}{-q} \right) + G_n - R_n$$

$$\nabla \bullet \left(K_S \varepsilon_0 \vec{\mathcal{E}} \right) = \rho$$

1) Direct, numerical solutions
2) Qualitative solutions with energy band diagrams
3) Simplify the equations, then solve analytically

Outline of the lecture

Analyzing semiconductor problems involving minority carriers usually comes down to solving the **minority carrier diffusion equation** (MCDE), a simplification of the semiconductor equations.

Minority carrier devices include solar cells, bipolar transistors, and light-emitting diodes.

In this lecture, I will discuss several examples, which illustrate solving the MCDE for several common situations.

Minority carrier diffusion equation

$$n = n_0 = N_D$$

(N-type semiconductor in low level injection)

$$\frac{\partial p}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_p}{q} \right) + G_p - R_p$$

(hole continuity equation)

$$\frac{\partial p}{\partial t} = -\frac{d}{dx}\left(\frac{J_{px}}{q} \right) + G_L - R_p$$

(1D, generation by light)

$$\frac{\partial \Delta p}{\partial t} = -\frac{d}{dx}\left(\frac{-qD_p \, d\Delta p / dx}{q} \right) + G_L - \frac{\Delta p}{\tau_p}$$

(low-level injection, no electric field)

$$\frac{\partial \Delta p}{\partial t} = D_p \frac{d^2 \Delta p}{dx^2} - \frac{\Delta p}{\tau_p} + G_L$$

(D_p spatially uniform)

Minority carrier diffusion equation

$$\boxed{\frac{\partial \Delta p(x,t)}{\partial t} = D_p \frac{d^2 \Delta p(x,t)}{dx^2} - \frac{\Delta p(x,t)}{\tau_p} + G_L}$$

To use the MCDE to solve a problem, **first check** to be sure that the simplifying assumptions needed to derive the MCDE from the continuity equation apply.

Then simplify the MCDE (if possible), **specify the initial condition** (if necessary) and the **two boundary conditions** (if necessary).

Reminder: Low level injection

N-type semiconductor:

$$n(x,t) \approx n_0 = N_D$$

$$p(x,t) \approx \Delta p(x,t) >> p_0 = n_i^2 / n_0$$

$$\boxed{\frac{\partial \Delta p(x,t)}{\partial t} = D_p \frac{d^2 \Delta p(x,t)}{dx^2} - \frac{\Delta p(x,t)}{\tau_p} + G_L}$$

P-type semiconductor:

$$p(x,t) \approx p_0 = N_A$$

$$n(x,t) \approx \Delta n(x,t) >> n_0 = n_i^2 / p_0$$

$$\boxed{\frac{\partial \Delta n(x,t)}{\partial t} = D_n \frac{d^2 \Delta n(x,t)}{dx^2} - \frac{\Delta n(x,t)}{\tau_n} + G_L}$$

Example: MCDE for electrons in Si

P-type Si at T = 300 K

$$N_A = 10^{17}\ cm^{-3} = p_0$$

$$\mu_n = 300\ \mathrm{cm}^2/\mathrm{V\text{-}s}$$

$$D_n = \left(\frac{k_B T}{q}\right)\mu_n = 7.8\ \mathrm{cm}^2/\mathrm{s}$$

$$\tau_n = 10^{-6}\ \mathrm{s}$$

$$L_n = \sqrt{D_n \tau_n} = 28\ \mu\mathrm{m}$$

MCDE for electrons

$$\boxed{\frac{\partial \Delta n(x,t)}{\partial t} = D_n \frac{d^2 \Delta n(x,t)}{dx^2} - \frac{\Delta n(x,t)}{\tau_n} + G_L}$$

"diffusion length"

Example 1: Steady-state, uniform illumination

Steady-state, uniform generation (no spatial variation)

$$G_L = 10^{20}\ \text{cm}^{-3}\text{s}^{-1}$$

Solve for Δn and for the QFL's.

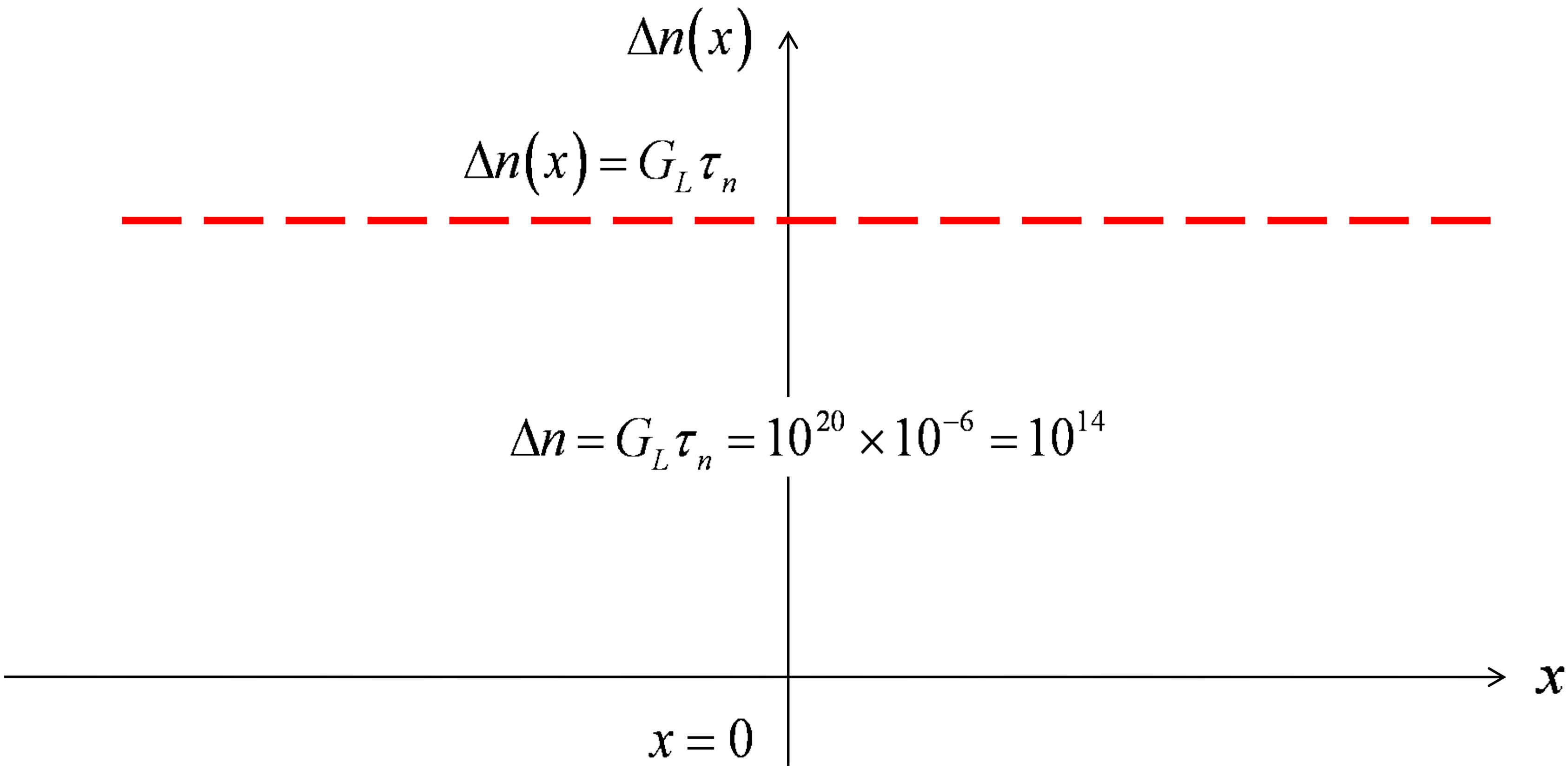
1) Simplify the MCDE
2) Solve the MCDE for *Δn*
3) Deduce F_n from *Δn*

$$\frac{\partial \Delta n}{\partial t} = D_n \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{\tau_n} + G_L$$

$$0 = 0 - \frac{\Delta n}{\tau_n} + G_L$$

$$\boxed{\Delta n = G_L \tau_n}$$

Example 1: Solution

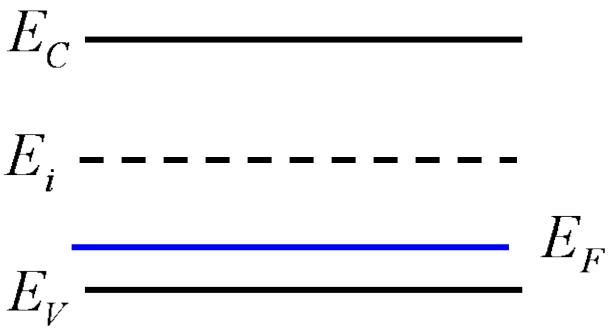


Steady-state, uniform generation, no spatial variation

Example 1: Equilibrium Fermi level

P-type / equilibrium

$$n_0 = \frac{n_i^2}{p_0} = 10^3 \quad \text{cm}^{-3}$$



$$p_0 = 10^{17} \quad \text{cm}^{-3}$$

$$p_0 = n_i e^{(E_i - E_F)/k_B T}$$

$$10^{17} = 10^{10} e^{(E_i - E_F)/k_B T}$$

$$E_F = E_i - 0.41 \text{ eV}$$

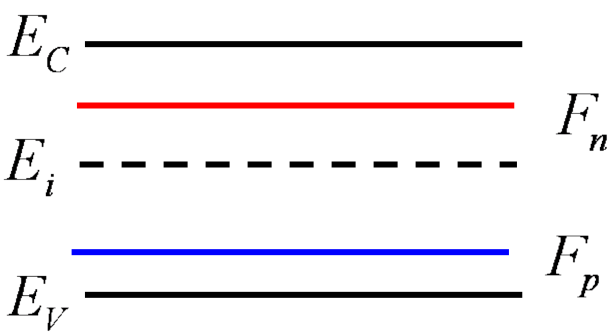
Example 1: Quasi-Fermi levels

P-type / **out of equilibri**um

$$F_p = E_i - 0.41\ \text{eV}$$

$$\Delta n = 10^{14}\ \ \text{cm}^{-3} >> n_0$$

$$n \approx \Delta n = n_i e^{(F_n - E_i)/k_B T}$$



$$10^{14} = 10^{10} e^{(F_n - E_i)/k_B T}$$

$$F_n = E_i + 0.24\ \text{eV}$$

$$p_0 = 10^{17}\ \ \text{cm}^{-3}$$

Steady-state, uniform generation, no spatial variation

Example 2: Transient decay to equilibrium

Now turn off the light.
Transient, no generation, no spatial variation

Solve for Δn(t) and for the QFL's.

1) Simplify the MCDE
2) Solve the MCDE for *Δn*
3) Deduce F_n from *Δn*

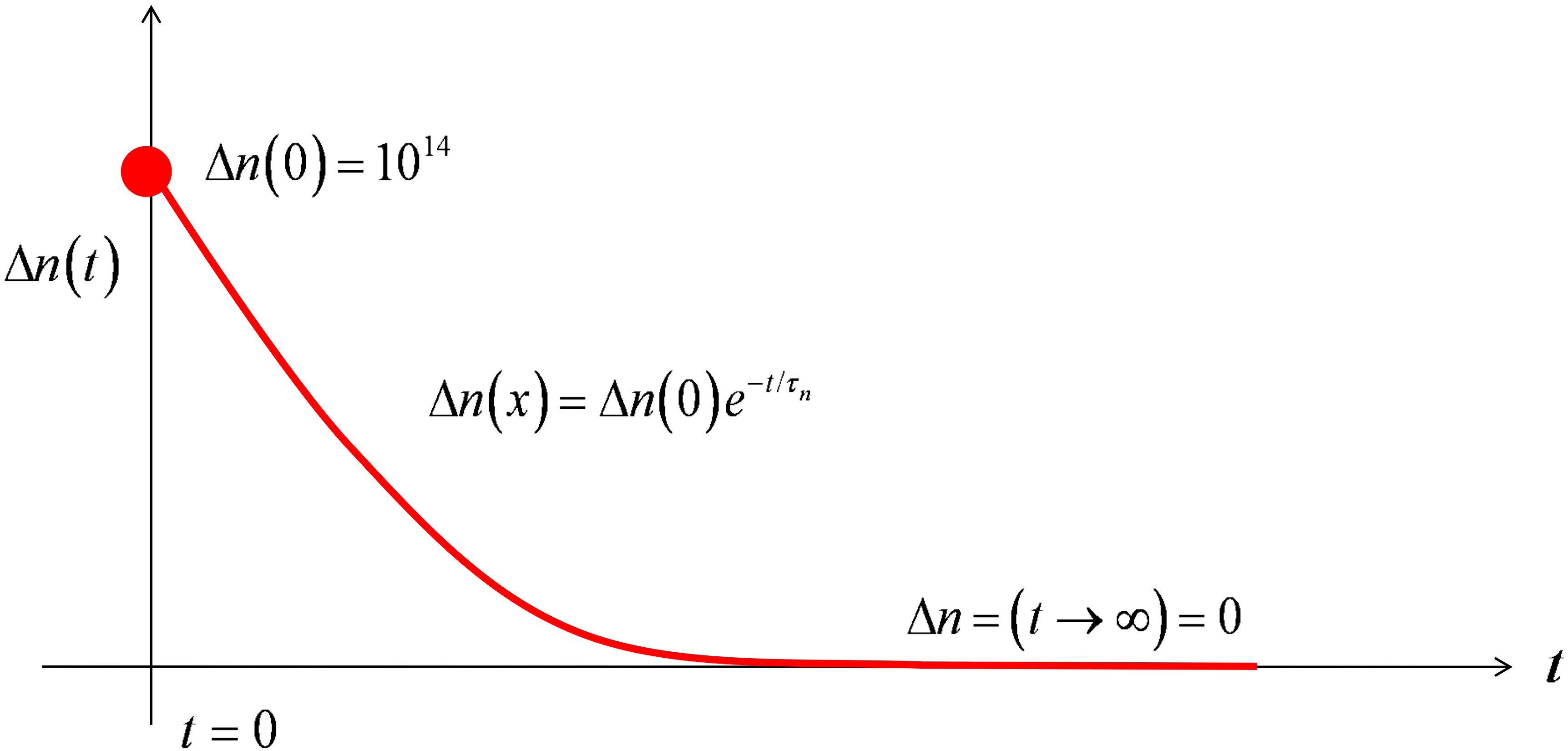
$$\frac{\partial \Delta n(x,t)}{\partial t} = D_n \frac{d^2 \Delta n(x,t)}{dx^2} - \frac{\Delta n(x,t)}{\tau_n} + G_L$$

$$\frac{\partial \Delta n}{\partial t} = 0 - \frac{\Delta n}{\tau_n} + 0$$

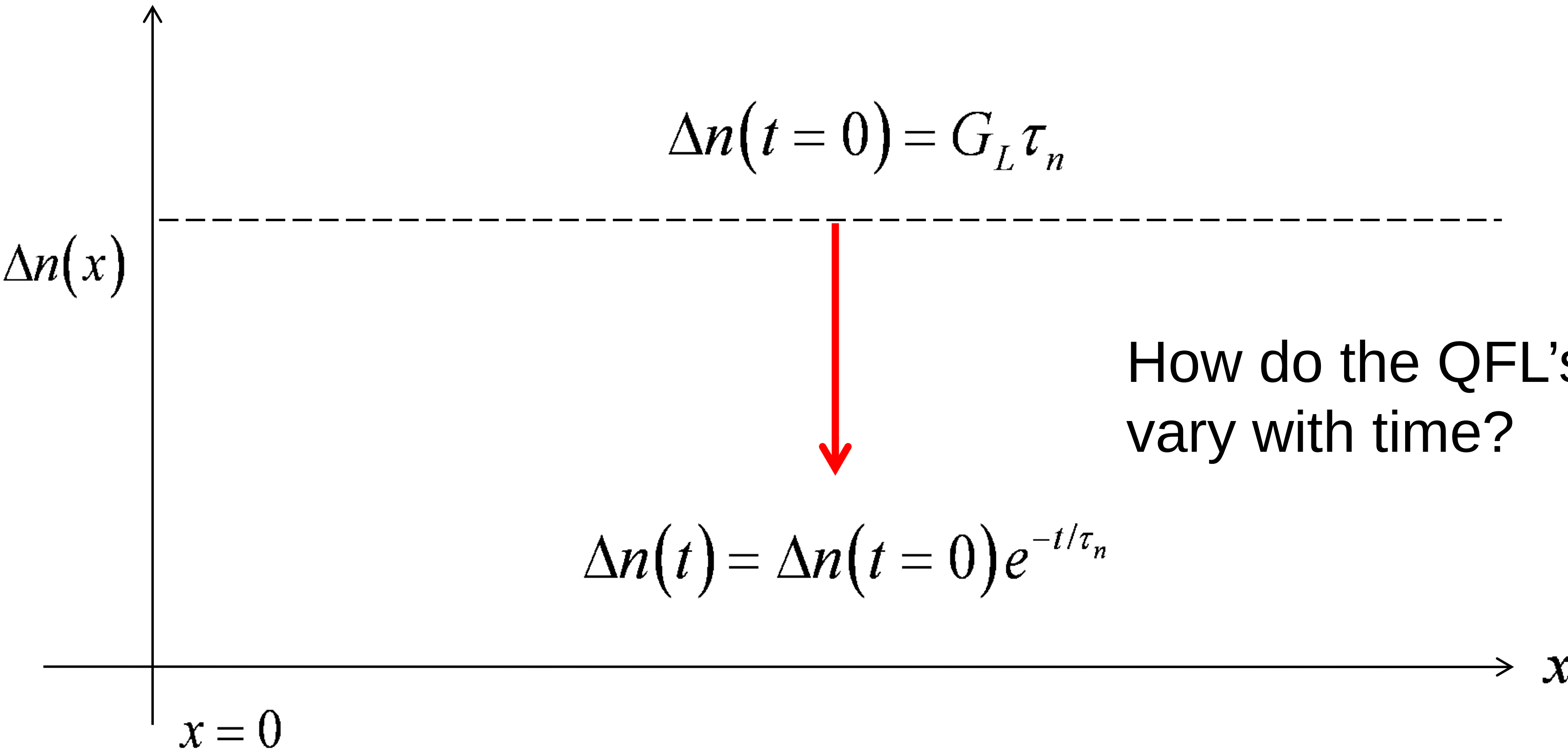
$$\frac{\partial \Delta n}{\partial t} = -\frac{\Delta n}{\tau_p}$$

$$\boxed{\Delta n(t) = \Delta n(0) e^{-t/\tau_n} = 10^{14} e^{-t/\tau_n}}$$

Example 2: Solution



Example 2: Solution



How do the QFL's vary with time?

transient, no generation, no spatial variation

Example 2: Solution

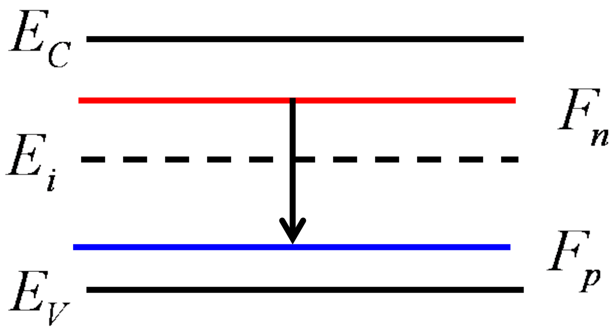
$$F_p = E_i - 0.41\ \text{eV}$$

$$n(t) \approx \Delta n(t) = n_i e^{(F_n(t) - E_i)/k_B T}$$

$$10^{14} e^{-t/\tau_n} = 10^{10} e^{(F_n(t) - E_i)/k_B T}$$

$$F_n(t) = E_i + k_B T \ln(10^4) - k_B T \frac{t}{\tau_n}$$

$$\boxed{F_n(t) = F_n(t=0) - k_B T \frac{t}{\tau_n}}$$



For long times, F_n should approach the equilibrium Fermi level. Explain what is wrong with our answer in the long time limit.

Example 3: SS diffusion in a **long** sample

Steady-state, sample is long (200 micrometers) compared to the diffusion length. No generation.

$$\Delta n(x=0) = 10^{12}\ \text{cm}^{-3}$$
$$\Delta n(x=L) = 0\ \text{cm}^{-3}$$

fixed

Solve for Δn and for the QFL's.

1) Simplify the MCDE
2) Solve the MCDE for *Δn*
3) Deduce F_n from *Δn*

$$\frac{\partial \Delta n}{\partial t} = D_p \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{\tau_n} + G_L$$

$$0 = D_p \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{\tau_n} + 0$$

$$\frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{D_p \tau_n} = 0$$

$$\frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{L_n^2} = 0 \qquad L_n \equiv \sqrt{D_n \tau_n}$$

Example 3: Continued

Steady-state, sample **long** compared to the diffusion length. No generation.

$$\Delta n(x=0) = 10^{12} \text{ cm}^{-3}$$

$$\Delta n(x=L) = 0 \text{ cm}^{-3}$$

fixed

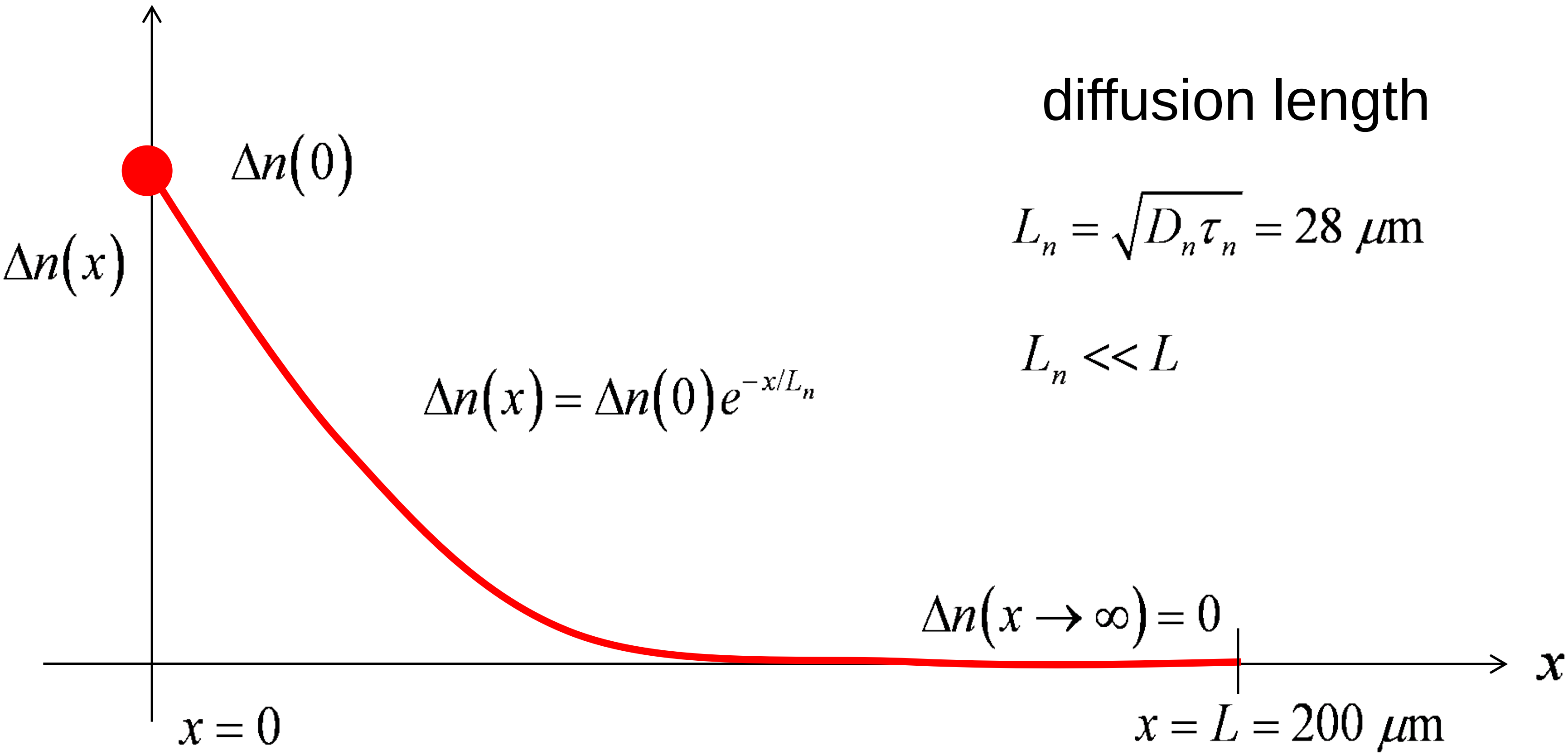
$$\frac{d^2 \Delta n(x)}{dx^2} - \frac{\Delta n(x)}{L_n^2} = 0 \qquad L_n \equiv \sqrt{D_n \tau_n}$$

$$\Delta n(x) = A e^{-x/L_n} + B e^{+x/L_n}$$

$$\Delta n(x) = A e^{-x/L_n}$$

$$\boxed{\Delta n(x) = \Delta n(0) e^{-x/L_n} = 10^{12} e^{-x/L_n}}$$

Example 3: Solution



$$\Delta n(x) = \Delta n(0) e^{-x/L_n}$$

diffusion length

$$L_n = \sqrt{D_n \tau_n} = 28\ \mu\text{m}$$

$$L_n << L$$

Steady-state, sample **long** compared to the diffusion length.

Example 3: Suggested exercise

Draw the energy band diagram with the QFL's. Is there an electron current?

Example 4: SS diffusion in a **short** sample

Steady-state, sample is **5 micrometers long**. No generation.

$$\Delta n(x=0) = 10^{12}\ \text{cm}^{-3}$$

fixed

$$\Delta n(x=5\ \mu\text{m}) = 0$$

1) Simplify the MCDE
2) Solve the MCDE for *Δn*
3) Deduce F_n from *Δn*

$$\frac{\partial \Delta n}{\partial t} = D_p \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{\tau_n} + G_L$$

$$\frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{L_n^2} = 0 \qquad L_n \equiv \sqrt{D_n \tau_n}$$

$$L_n = 28\ \mu\text{m} >> L = 5\ \mu\text{m}$$

$$\frac{d^2 \Delta n}{dx^2} = 0$$

Example 4: Continued

Steady-state, sample is **5 micrometers long**. No generation.

$$\Delta n(x=0) = 10^{12}\ \text{cm}^{-3}$$ fixed

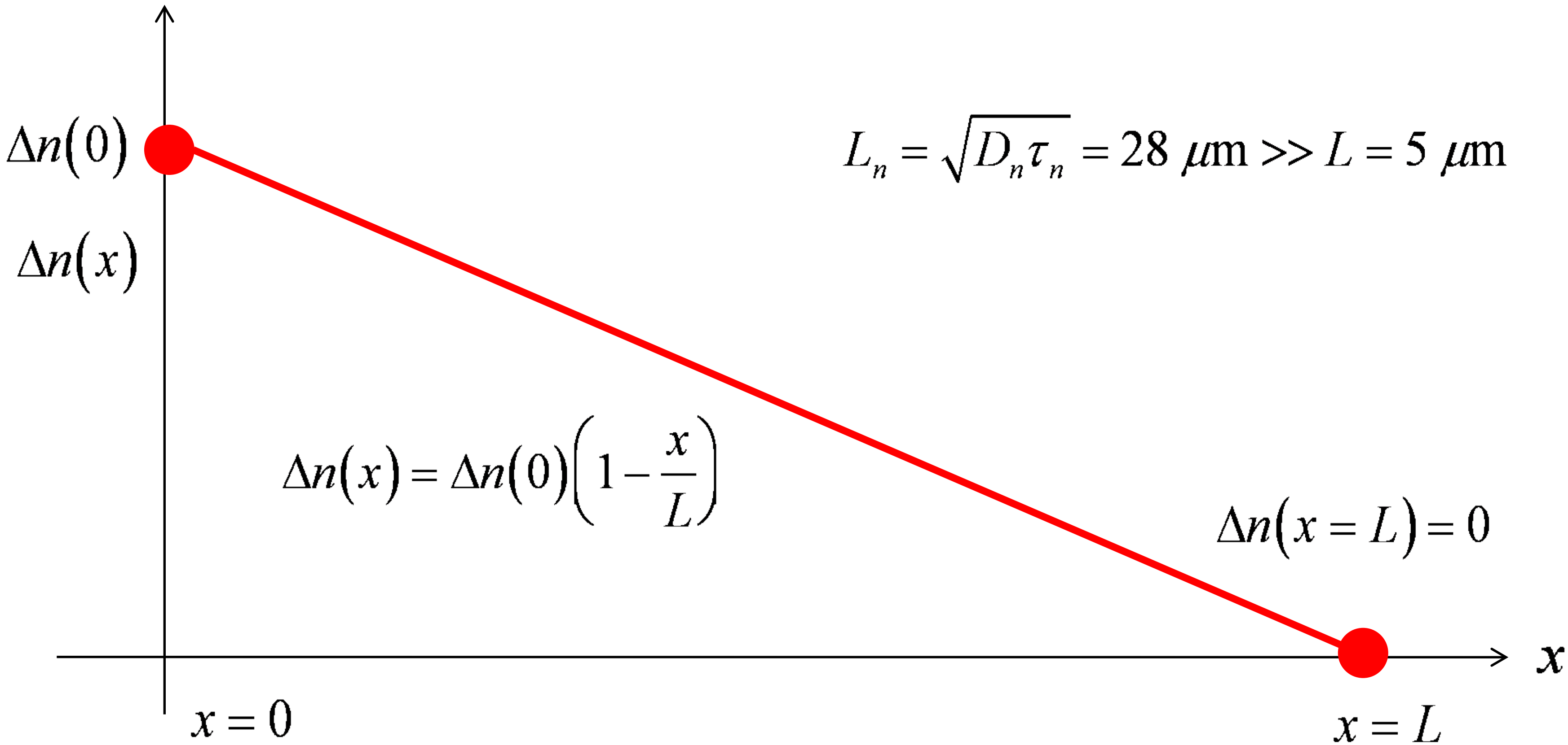
$$\Delta n(x = 5\ \mu\text{m}) = 0$$

$$\frac{d^2 \Delta n(x)}{dx^2} = 0$$

$$\Delta n(x) = Ax + B$$

$$\boxed{\Delta n(x) = \Delta n(0)\left(1 - \frac{x}{L}\right)}$$

Example 4: Solution



Steady-state, sample **short** compared to the diffusion length.

Example 5

Steady-state, sample is **30 micrometers long**. No generation.

$$\Delta n(x=0) = 10^{12}\ \text{cm}^{-3}$$

fixed

$$\Delta n(x = 30\ \mu\text{m}) = 0$$

$$\frac{\partial \Delta n}{\partial t} = D_p \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{\tau_n} + G_L$$

$$0 = D_p \frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{\tau_n} + 0$$

$$\frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{L_n^2} = 0 \qquad L_n \equiv \sqrt{D_n \tau_n}$$

$$L_n = 28\ \mu\text{m} \qquad L = 30\ \mu\text{m}$$

1) Simplify the MCDE
2) Solve the MCDE from *Δn*
3) Deduce F_n from *Δn*

Example 5: Solution

Steady-state, sample is **30 micrometers long**. No generation.

$$\Delta n(x=0) = 10^{12}\ \text{cm}^{-3} \text{ fixed}$$

$$\Delta n(x = 30\ \mu\text{m}) = 0$$

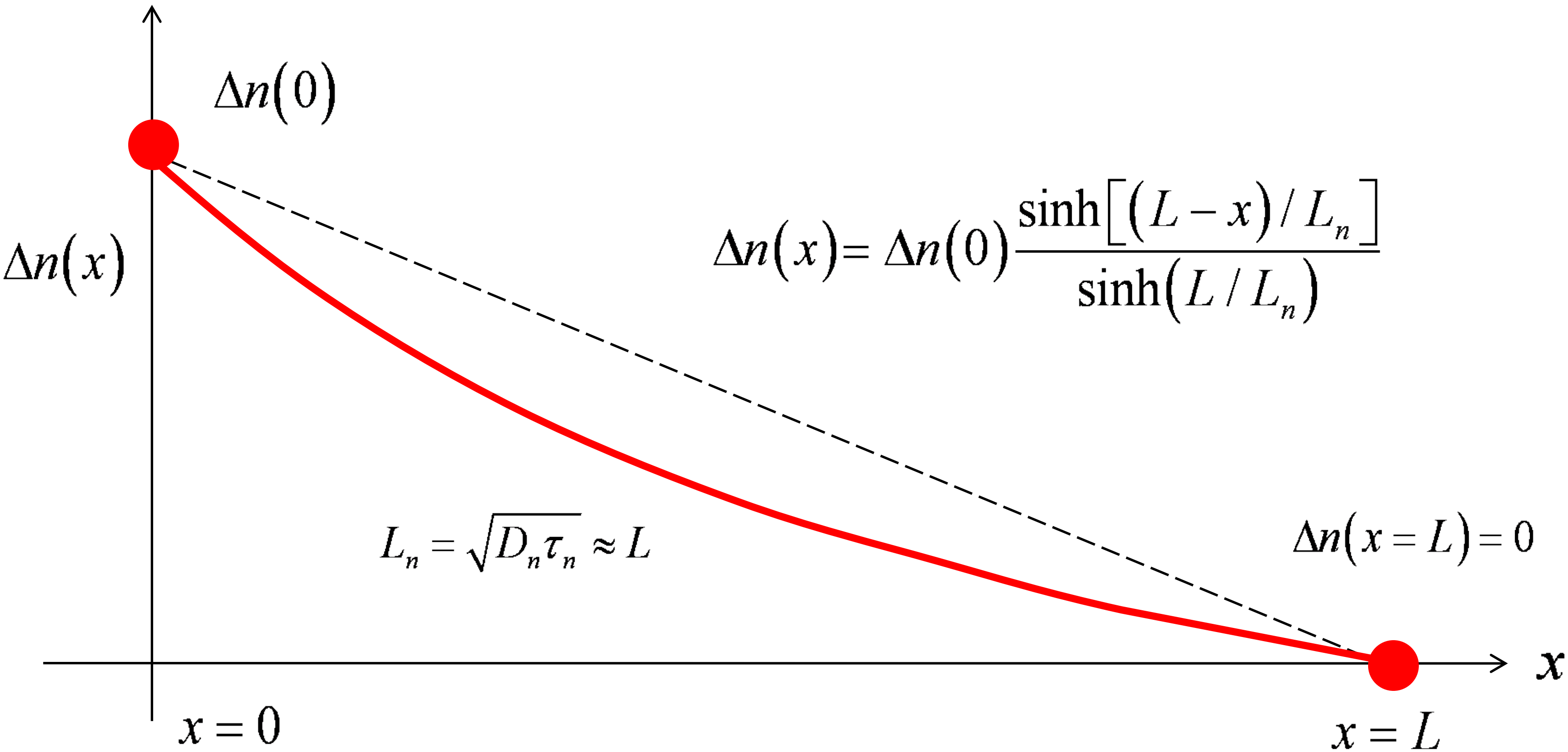
$$\frac{d^2 \Delta n}{dx^2} - \frac{\Delta n}{L_n^2} = 0$$

$$\Delta n(x) = Ae^{-x/L_n} + Be^{+x/L_n}$$

$$\Delta n(0) = A + B = 10^{12}$$

$$\Delta n(L) = Ae^{-L/L_n} + Be^{+L/L_n} = 0$$

Example 5: Solution



Steady-state, sample **neither long nor short** compared to the diffusion length.

Example 6:

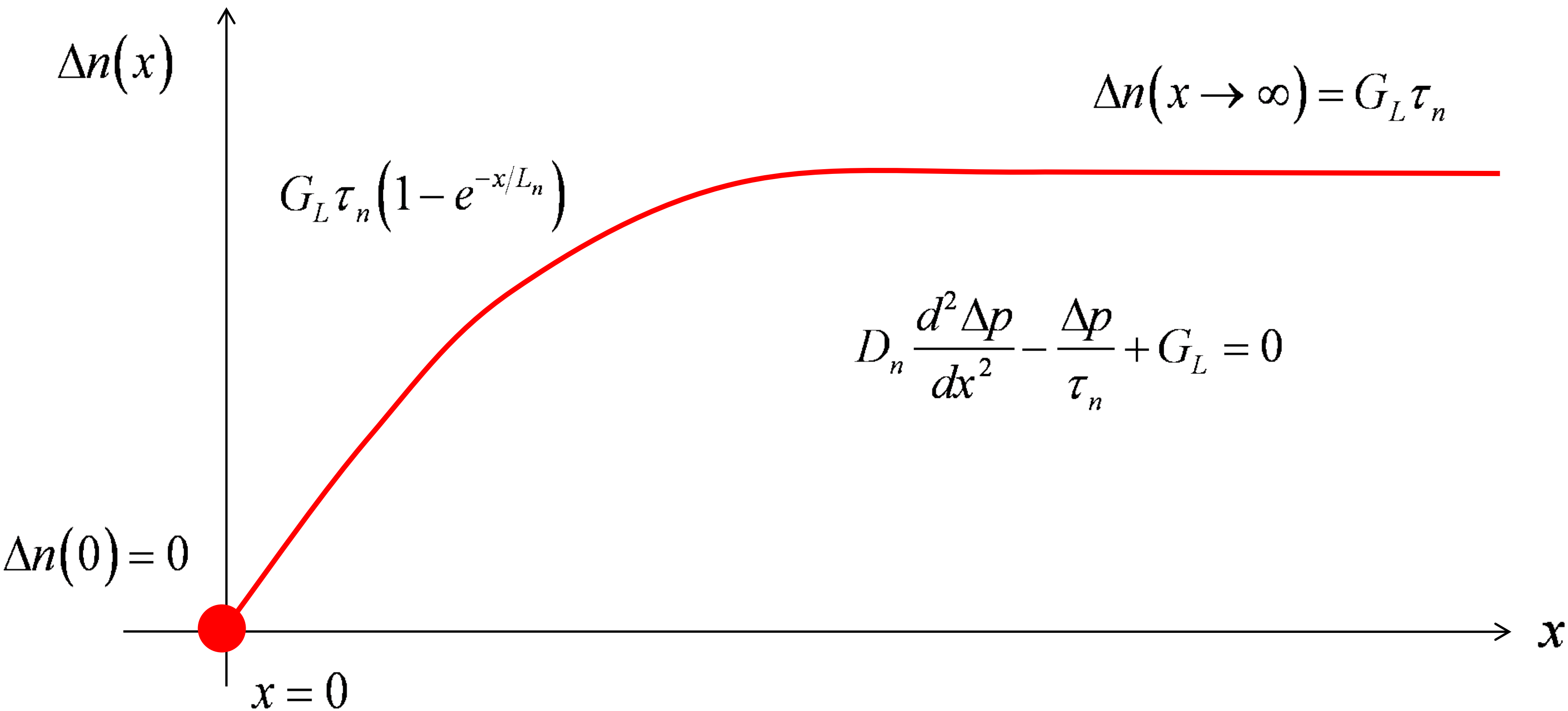
An infinitely long sample is uniformly illuminated with light for a long time. The optical generation rate $G_L = 1 \times 10^{20}\ \text{cm}^{-3}\ \text{sec}^{-1}$. The minority carrier lifetime is 1 microsecond. The surface at $x = 0$ is highly defective, with a high density of R-G centers, so that $\Delta n(0) = 0$.

Find the s.s. excess minority carrier concentration vs. position.

$$D_n \frac{d^2 \Delta p}{dx^2} - \frac{\Delta p}{\tau_n} + G_L = 0$$

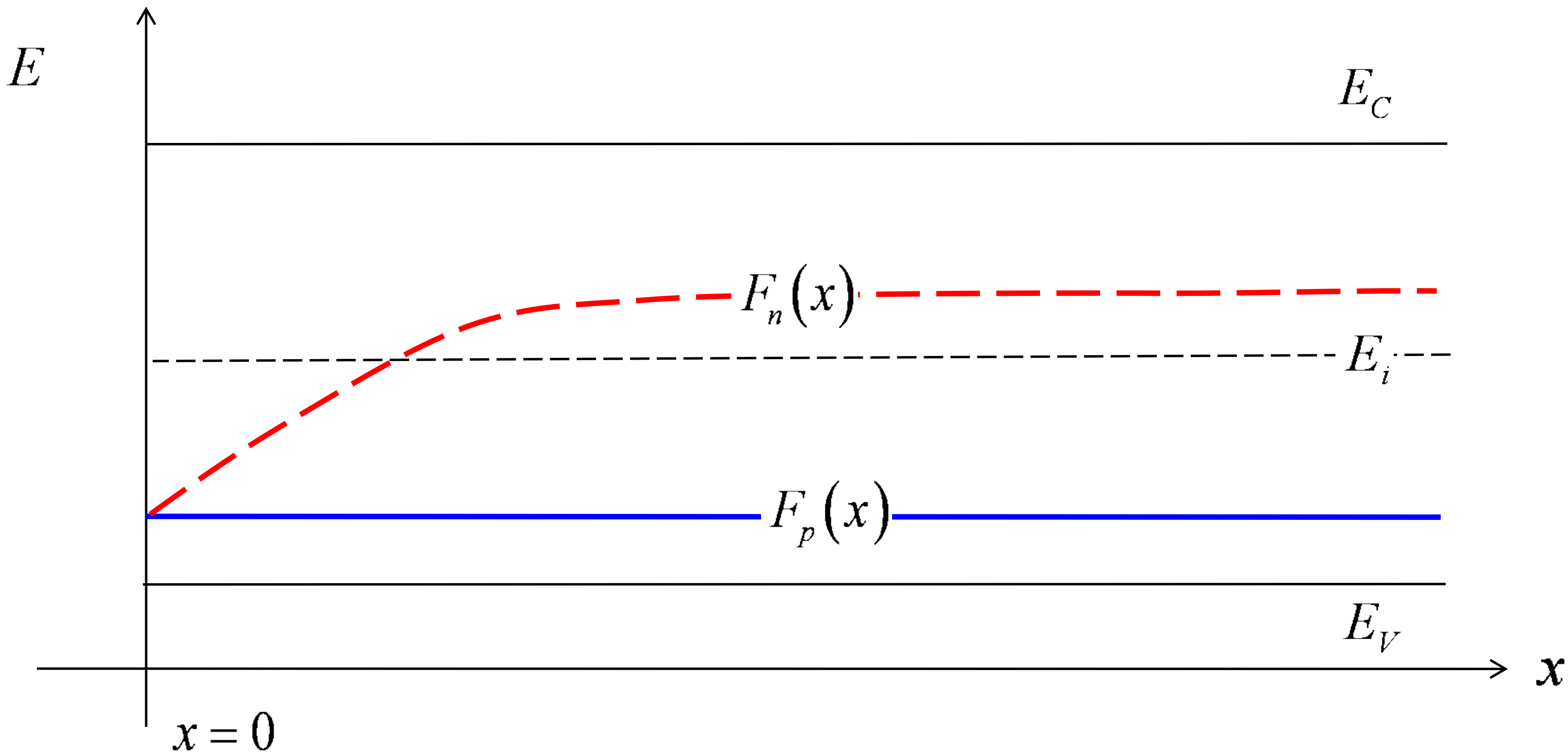
Can we guess the solution?

Example 6: Solution



What does the energy band diagram look like?

Example 6: Energy band diagram



What does a gradient in the QFL mean?

Summary (i)

$$\frac{\partial p}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_p}{q} \right) + G_p - R_p$$

$$\frac{\partial n}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_n}{-q} \right) + G_n - R_n$$

$$\nabla \bullet \left(K_S \varepsilon_0 \vec{\mathcal{E}} \right) = \rho$$

$$\frac{\partial \Delta p(x,t)}{\partial t} = D_p \frac{d^2 \Delta p(x,t)}{dx^2} - \frac{\Delta p(x,t)}{\tau_p} + G_L$$

LL injection in an N-type material (no electric field)

$$\frac{\partial \Delta n(x,t)}{\partial t} = D_n \frac{d^2 \Delta n(x,t)}{dx^2} - \frac{\Delta n(x,t)}{\tau_n} + G_L$$

LL injection in a P-type material (no electric field)

Summary (ii)

$$\frac{\partial \Delta p(x,t)}{\partial t} = D_p \frac{d^2 \Delta p(x,t)}{dx^2} - \frac{\Delta p(x,t)}{\tau_p} + G_L \qquad \frac{\partial \Delta n(x,t)}{\partial t} = D_n \frac{d^2 \Delta n(x,t)}{dx^2} - \frac{\Delta n(x,t)}{\tau_n} + G_L$$

General features of the solutions:

Transient solutions goes as exp[-t/τ_n]

For long regions, steady-state spatial solutions go as exp[-x/L_n] in a long region

For short regions, steady-state solutions are linear.