Primer on Semiconductors

Unit 4: Carrier Transport, Recombination, and Generation

Lecture 4.3: Drift-diffusion equation

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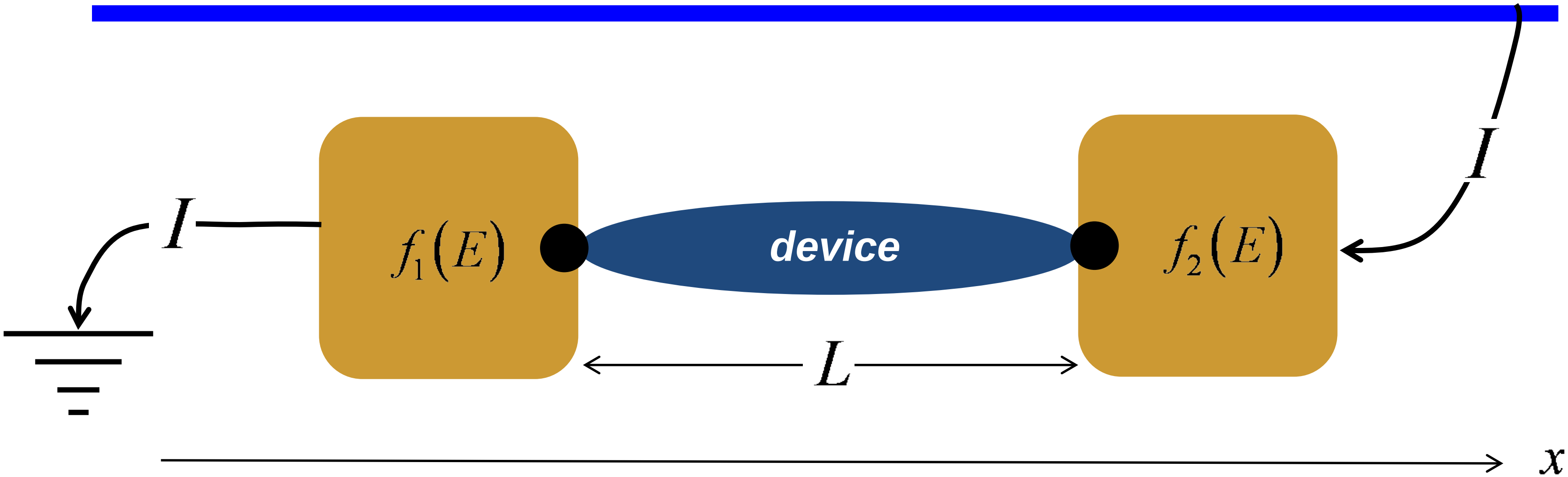
Review



$$I = \frac{2q}{h}\int \mathcal{T}(E)M(E)(f_1 - f_2)dE$$

$$J_x = n\mu_n \frac{dF_n}{dx}$$

For small devices, *M* is countable

For short devices, $\mathcal{T}$= 1, ballistic.

A special case for large and long devices.

Current equation for bulk semiconductors

$$\boxed{J_x = n\mu_n \frac{dF_n}{dx}}$$

$$n = N_C e^{(F_n - E_C)/k_B T}$$

$$E_C(x) = E_{ref} - qV(x)$$

$$F_n = E_C + k_B T \ln\left(\frac{n}{N_C}\right)$$

$$\frac{dE_C}{dx} = -q\frac{dV}{dx} = q\mathcal{E}$$

$$\frac{dF_n}{dx} = \frac{dE_C}{dx} + k_B T \frac{1}{n}\frac{dn}{dx}$$

$$k_B T \mu_n = qD_n \qquad D_n/\mu_n = k_B T/q$$

$$J_x = n\mu_n \frac{dE_C}{dx} + k_B T \mu_n \frac{dn}{dx}$$

$$\boxed{J_x = nq\mu_n\mathcal{E} + qD_n \frac{dn}{dx}}$$

The drift-diffusion equation

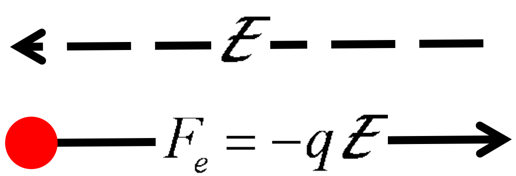
$$\boxed{J_x = nq\mu_n\mathcal{E} + qD_n\frac{dn}{dx}}$$

$$D_n/\mu_n = k_BT/q$$

(Einstein relation)

$$J_{drift} = nq\mu_n\mathcal{E}$$

current due to **drift** in an electric field



μ_n $\quad \dfrac{\mathrm{m}^2}{\mathrm{V\text{-}s}}$ "mobility"

$$J_{diff} = qD_n\frac{dn}{dx}$$

current due to **diffusion** in a concentration gradient

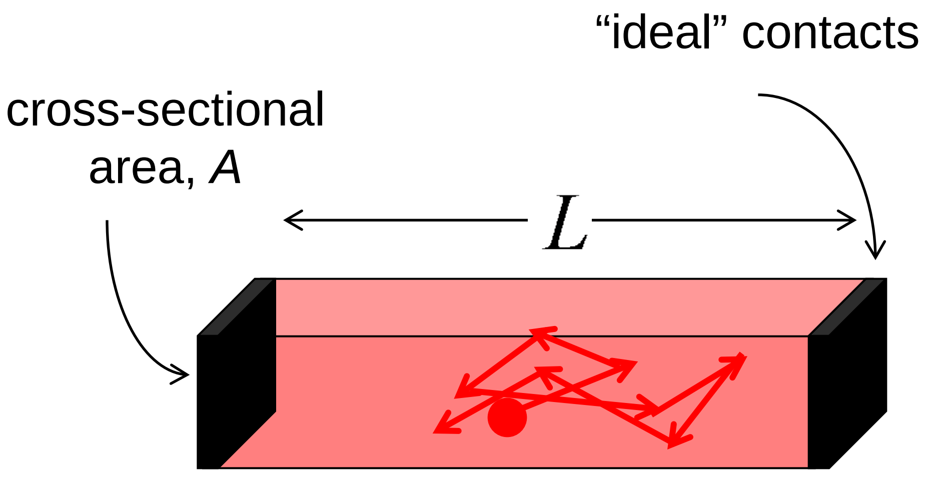
$$D_n/\mu_n = k_BT/q$$

D_n $\quad \dfrac{\mathrm{m}^2}{\mathrm{s}}$ "diffusion coefficient"

Drift and Diffusion

In the next few slides, we will briefly discuss the drift and diffusion currents separately.

Semiconductor in equilibrium



uniform n-type
semiconductor

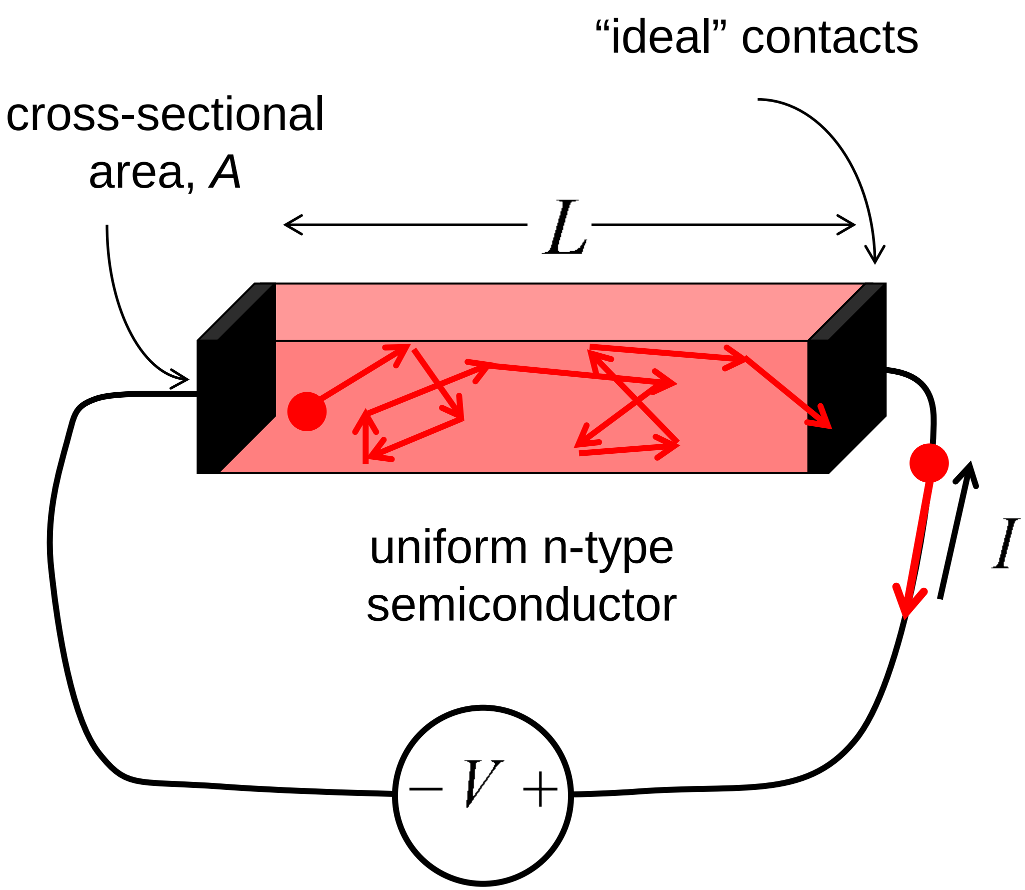
$$\langle KE \rangle = \frac{3}{2} k_B T$$

$$\langle KE \rangle = \frac{1}{2} m_n^* \langle \upsilon^2 \rangle$$

$$\sqrt{\langle \upsilon^2 \rangle} = \upsilon_{rms} = \sqrt{\frac{3k_B T}{m_n^*}}$$

$$\upsilon_{rms} \approx 10^7 \text{ cm/s}$$

Semiconductor under bias



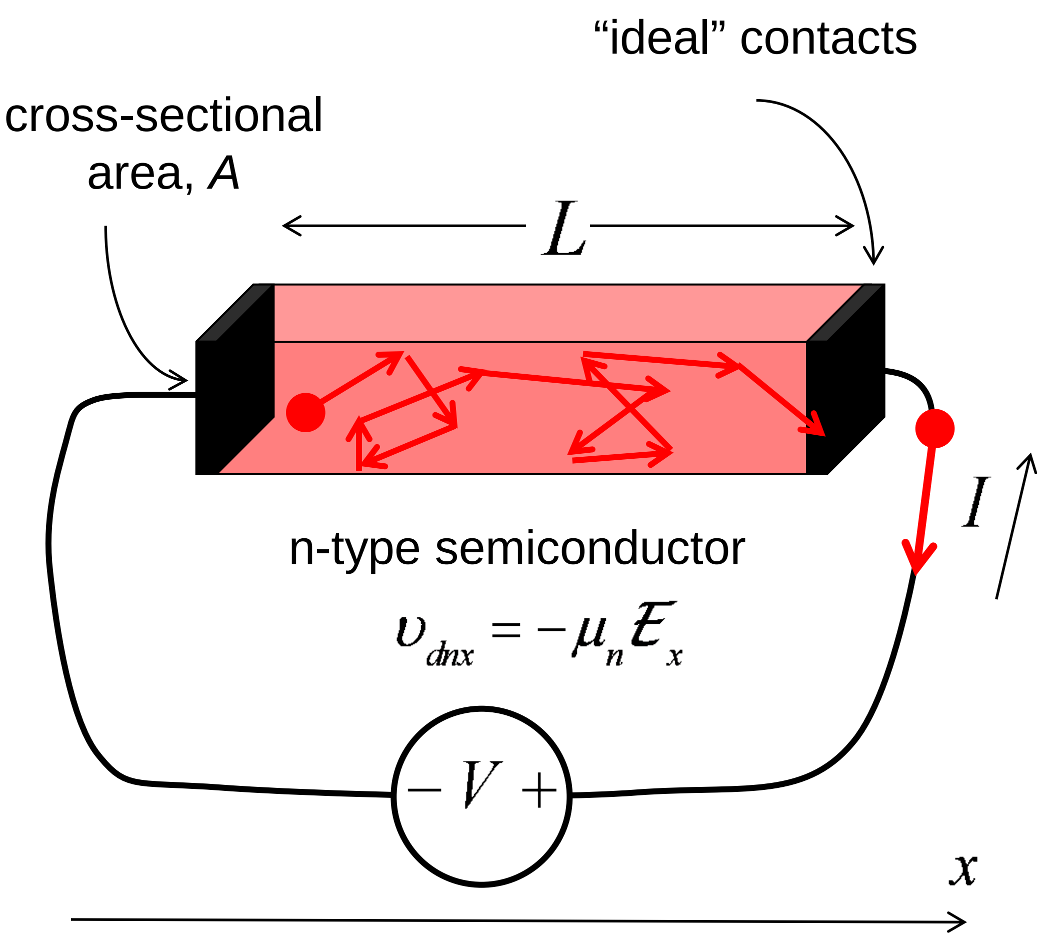
1) random walk with a small bias from left to right

2) assume that electrons "drift" to the right at an average velocity, υ_d

$$\upsilon_{dnx} = -\mu_n \mathcal{E}_x$$

3) what is *I* ?

Drift current and drift velocity



$$I = -Q/t_t$$

$$Q = -qnAL$$

$$t_t = L/\upsilon_{dnx}$$

$$I = nq\upsilon_{dnx} A$$

$$J_{nx} = -nq\upsilon_{dnx} \text{ A/m}^2$$

$$J_{px} = +pq\upsilon_{dnx} \text{ A/m}^2$$

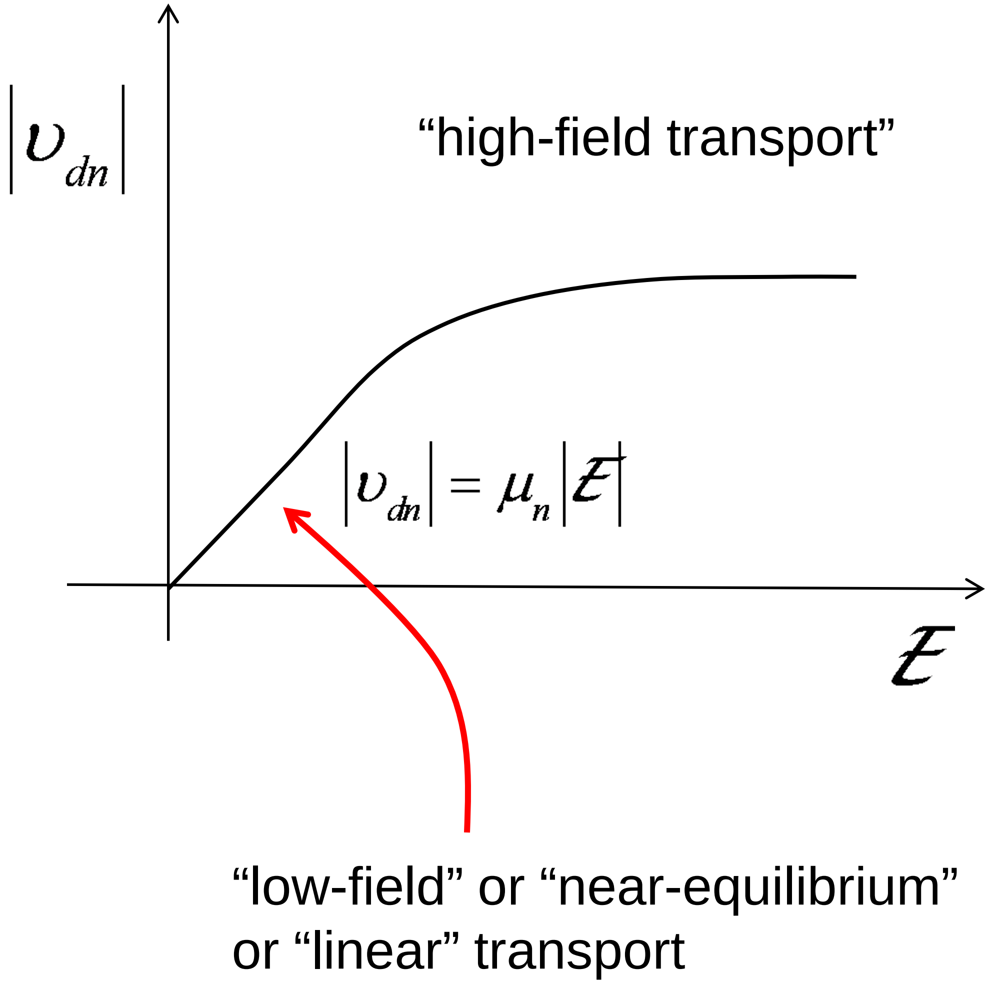
Lundstrom: 2018

Velocity and electric field

$$\upsilon_{dn} = -\mu_n \mathcal{E}$$

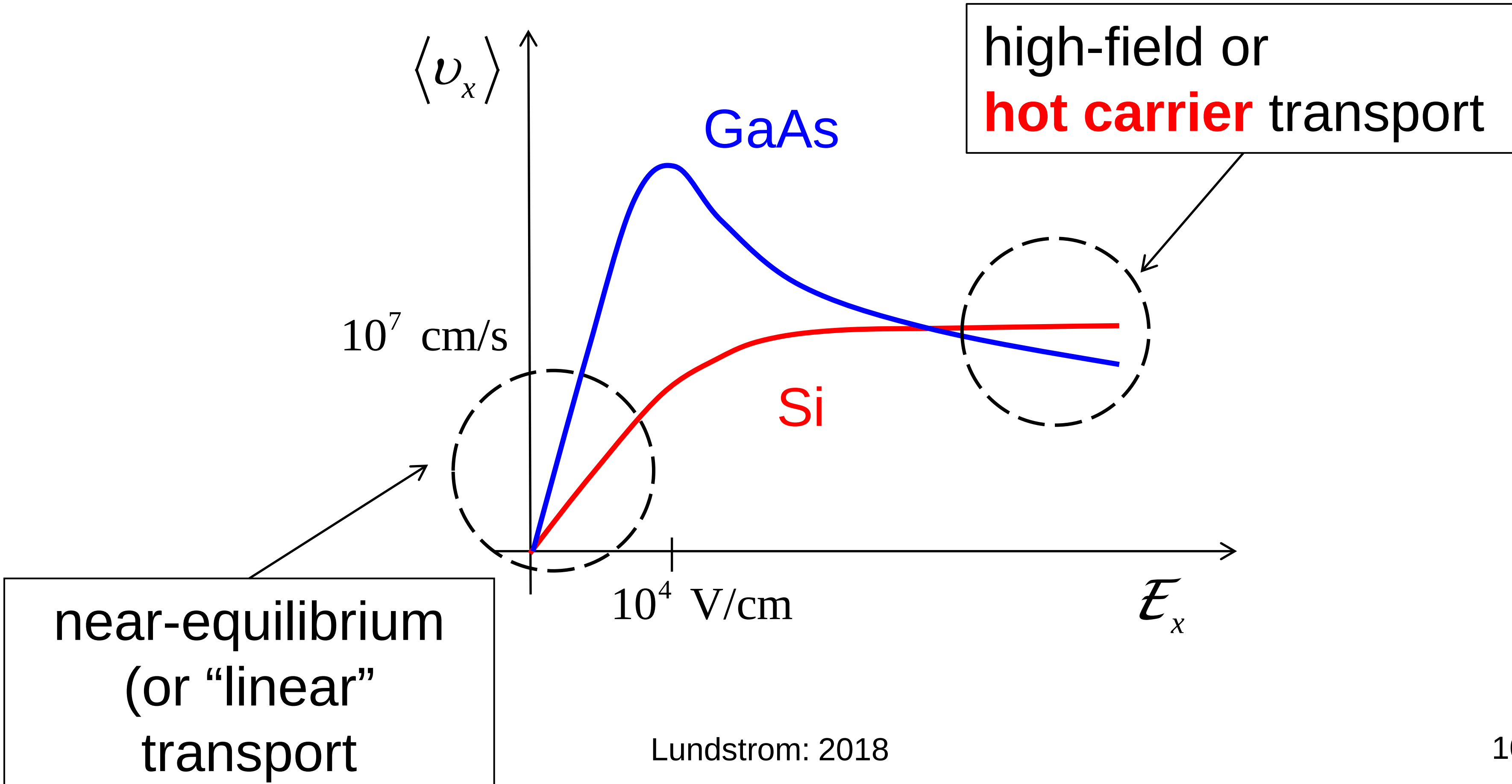
$$\mu_n \equiv \left(\frac{\upsilon_T \lambda_0}{2\left(k_B T / q\right)} \right) \text{ cm}^2/\text{V-s}$$

$$\mu_n = \left(\frac{q\tau}{m_n^*} \right) \text{cm}^2/\text{V-s}$$



Velocity vs. electric field

(***bulk*** semiconductors assumed)



Drift current

$$\upsilon_{dn} = -\mu_n \mathcal{E}$$

$$\upsilon_{dp} = +\mu_p \mathcal{E}$$

$$J_n = -nq\upsilon_{dn} \text{ A/m}^2$$

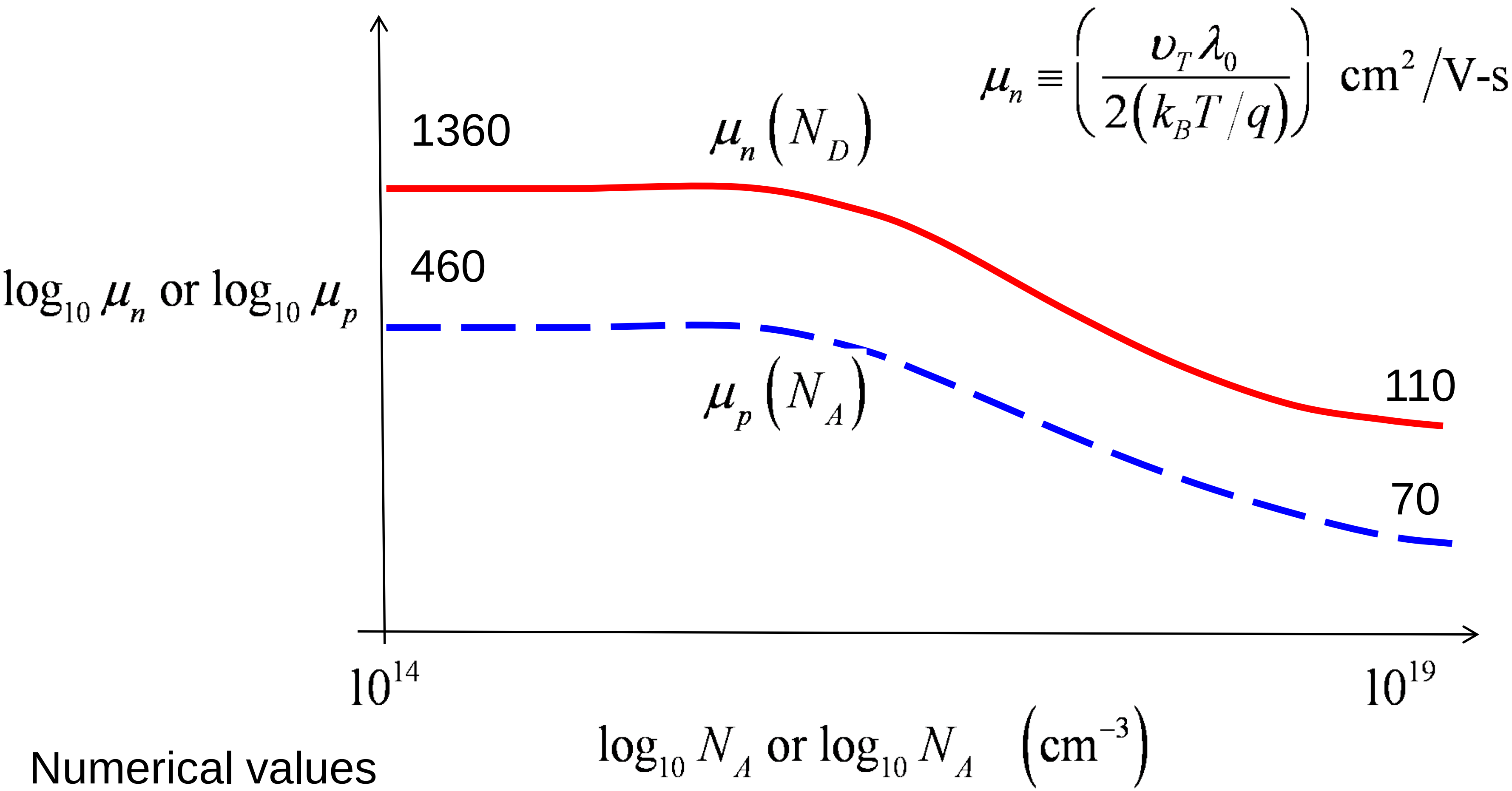
$$J_p = pq\upsilon_{dp} \text{ A/m}^2$$

$$\boxed{\begin{aligned} J_n &= nq\mu_n \mathcal{E} \text{ A/m}^2 \\ J_p &= pq\mu_p \mathcal{E} \text{ A/m}^2 \end{aligned}}$$

To describe high-field transport:

$$\mu_n, \mu_p \rightarrow \mu_n(\mathcal{E}), \mu_p(\mathcal{E})$$

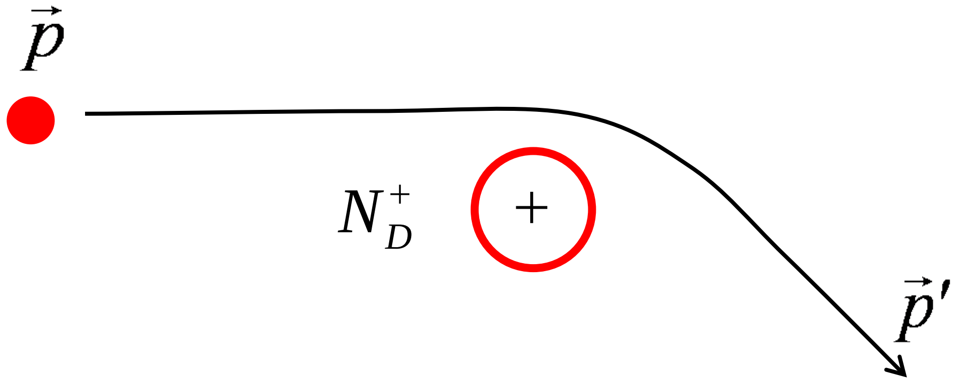
Mobility vs. doping



Numerical values are for Si at 300 K.

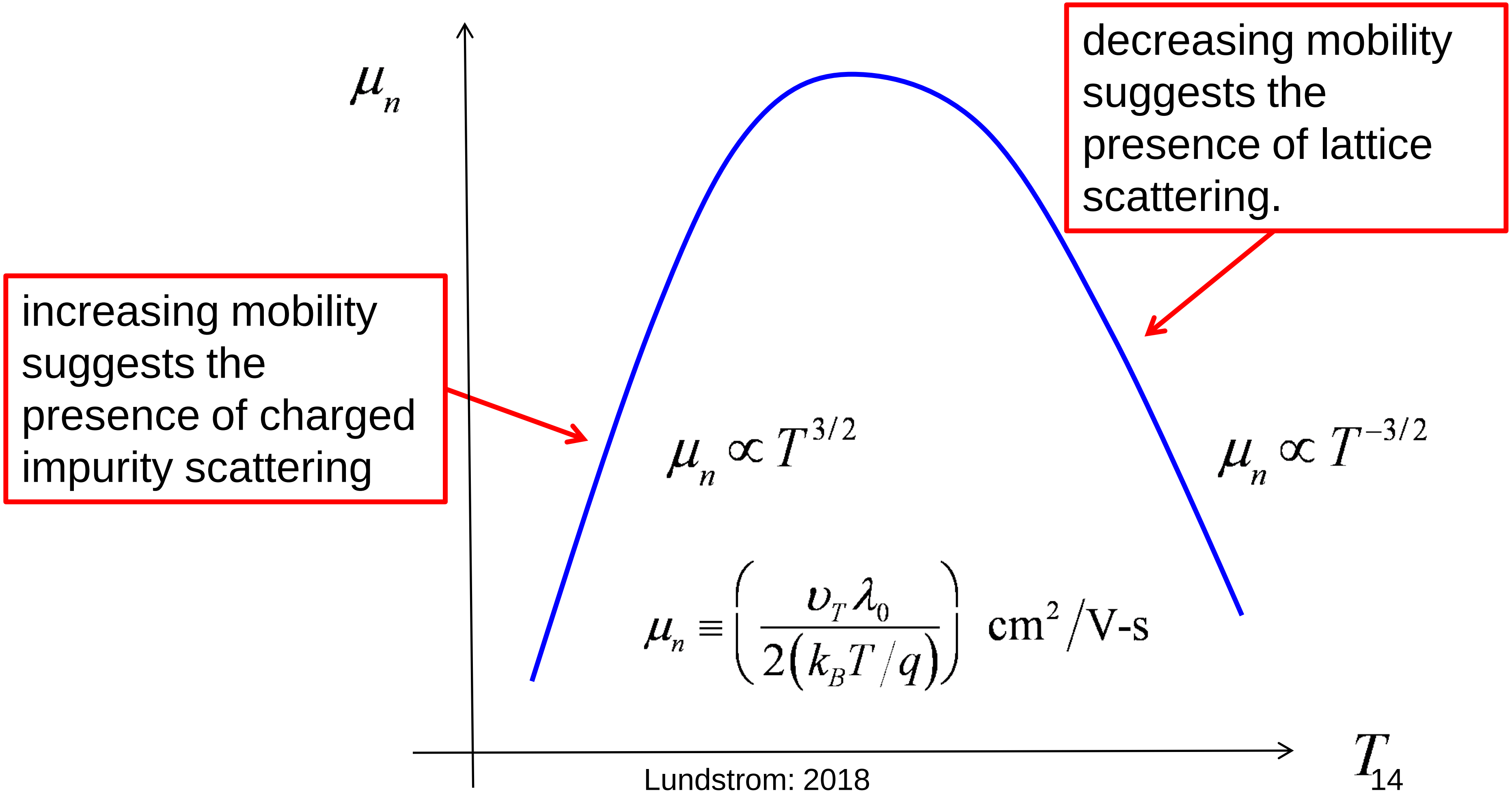
Ionized impurity scattering

electrons in N-type material



Donors provide electrons to the conduction band, but ionized can “scatter” those electrons.

Mobility vs. temperature



Drift current, conductivity, resistivity

$$J_n = nq\mu_n \mathcal{E} \text{ A/m}^2 \qquad J_n = \sigma_n \mathcal{E} \text{ A/m}^2 \qquad \sigma_n = nq\mu_n \ (\text{units?})$$

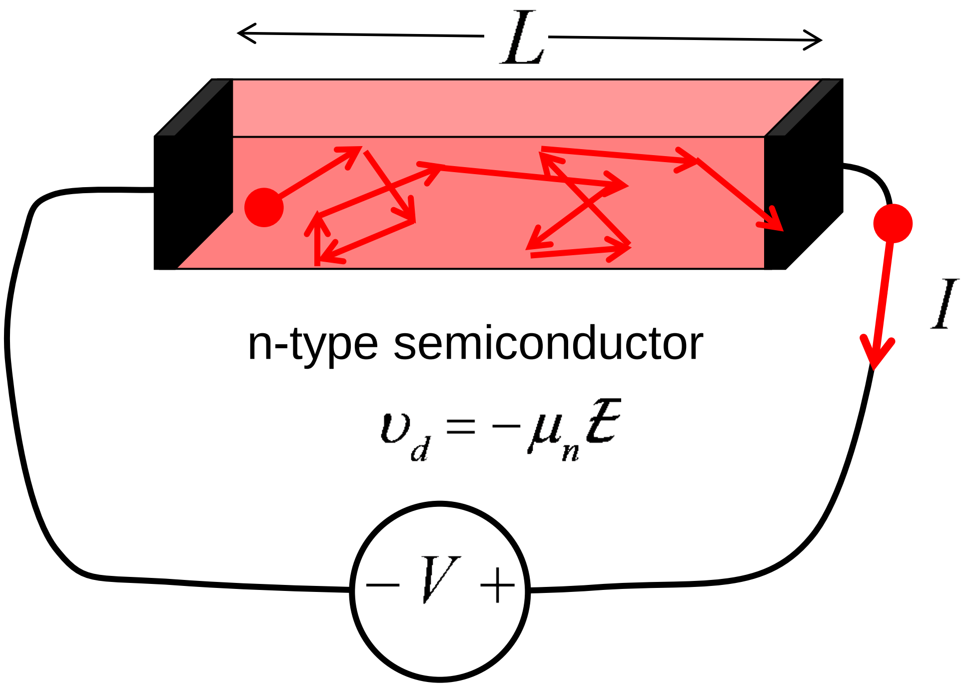
$$J_p = pq\mu_p \mathcal{E} \text{ A/m}^2 \qquad J_p = \sigma_p \mathcal{E} \text{ A/m}^2 \qquad \sigma_p = pq\mu_p$$

$$J_{tot} = J_n + J_p = \left(\sigma_n + \sigma_p\right)\mathcal{E} = \sigma \mathcal{E} \text{ A/m}^2$$

$$J_{tot} = \sigma \mathcal{E} \text{ A/m}^2$$

$$\rho = \frac{1}{\sigma} = \frac{1}{\sigma_n + \sigma_p} = \frac{1}{nq\mu_n + pq\mu_p} \ \Omega\text{-cm}$$

Resistance



$$R = \rho_n \frac{L}{A}$$

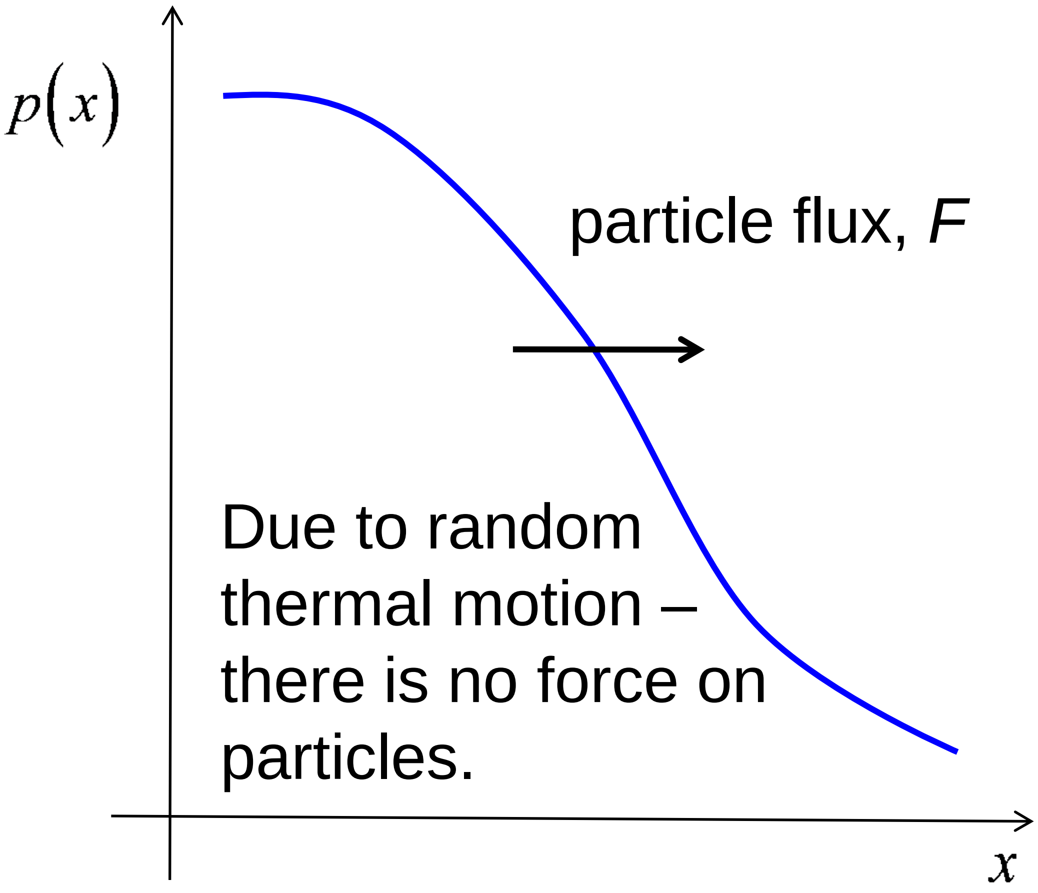
$$J_n = \sigma_n \mathcal{E} \text{ A/m}^2$$

$$I = AJ_n = \sigma_n A \mathcal{E} \text{ Amps}$$

$$I = \sigma_n A \frac{V}{L}$$

$$I = \left(\sigma_n \frac{A}{L} \right) V = GV = \frac{1}{R} V$$

Fick's Law of diffusion



$$F = \frac{J_p}{q} = -D\frac{dp}{dx} \qquad \frac{\#}{\text{cm}^2\text{-s}}$$

$$D \qquad \text{cm}^2/\text{s}$$

(Adolph Fick, 1855)

Diffusion currents

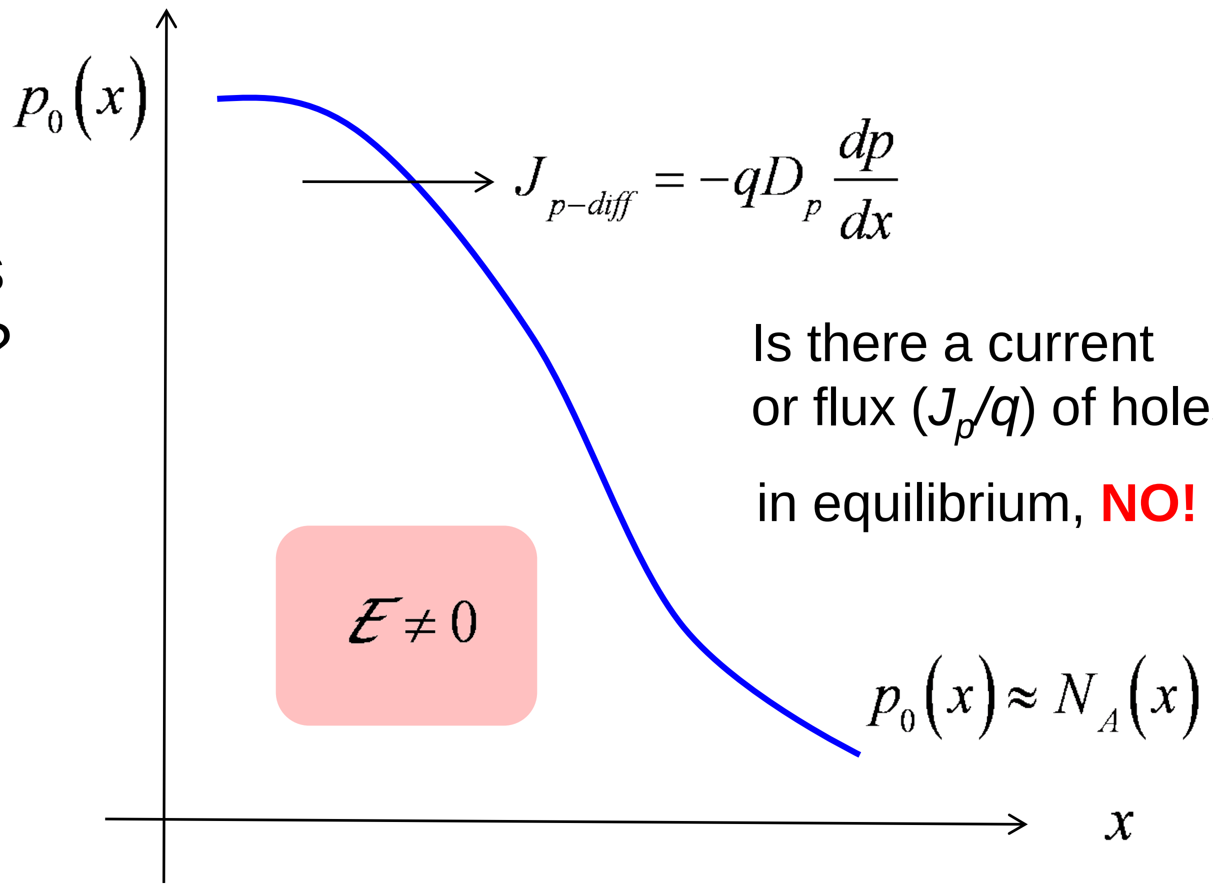
$$J_{p-diff} = -qD_p \frac{dp}{dx} \qquad\qquad J_{n-diff} = +qD_n \frac{dn}{dx}$$

Whenever there is a concentration gradient, there is a diffusion current.

Nonuniformly doped semiconductor in equilibrium

What direction is the electric field?

a) To the right?
b) To the left?



Is there a current or flux (J_p/q) of holes?

in equilibrium, **NO!**

There must be a drift current that exactly cancels the diffusion current.

Summary: Drift- diffusion equation

$$J_{px} = p\mu_p \vec{\nabla} F_p \qquad \vec{J}_p = \vec{J}_{p-drift} + \vec{J}_{p-diff} = pq\mu_p \vec{\mathcal{E}} - qD_p \vec{\nabla} p$$

current = drift current + diffusion current

$$J_{nx} = n\mu_n \vec{\nabla} F_n \qquad \vec{J}_n = \vec{J}_{n-drift} + \vec{J}_{n-diff} = nq\mu_n \vec{\mathcal{E}} + qD_n \vec{\nabla} n$$

total current = electron current + hole current

$$\vec{J} = \vec{J}_p + \vec{J}_n$$

$$\mu_p = \frac{\upsilon_T \lambda_0}{2\left(k_B T / q\right)} = \frac{q\tau}{m_p^*} \qquad D_p / \mu_p = D_n / \mu_n = k_B T / q$$

(Einstein, 1905)