Primer on Semiconductors

Unit 5: The Semiconductor Equations

Lecture 5.1: Mathematical formulation

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Solving semiconductor problems

For a uniform semiconductor in equilibrium, we can assume space-charge neutrality:

$$\rho = q\left(p_0 - n_0 + N_D^+ - N_A^-\right) = 0$$

We also know that in equilibrium:

$$n_0 p_0 = n_i^2$$

In general, neither one of the above equations is valid, so how do we solve for the electron and hole concentrations and for the electric field too?

Equilibrium vs. non-equilibrium

equilibrium	non-equilibrium
$n_0 = n_i e^{(E_F - E_i)/k_B T}$	$n = n_i e^{(F_n - E_i)/k_B T}$
$p_0 = n_i e^{(E_i - E_F)/k_B T}$	$p = n_i e^{(E_i - F_p)/k_B T}$
$n_0 p_0 = n_i^2$	$np \neq n_i^2$
$f_0 = \dfrac{1}{1 + e^{(E - E_F)/k_B T}}$	$f_c = \dfrac{1}{1 + e^{(E - F_n)/k_B T}}$
	$1 - f_v = 1 - \dfrac{1}{1 + e^{(E - F_p)/k_B T}}$

The unknowns

$$p(\vec{r}) = n_i e^{\left(E_i(\vec{r}) - F_p(\vec{r})\right)/k_B T}$$

$$n(\vec{r}) = n_i e^{\left(F_n(\vec{r}) - E_i(\vec{r})\right)/k_B T}$$

$$E_i(\vec{r}) = E_i^{ref} - qV(\vec{r})$$

3 unknowns

$$p(\vec{r}), n(\vec{r}), V(\vec{r})$$

or

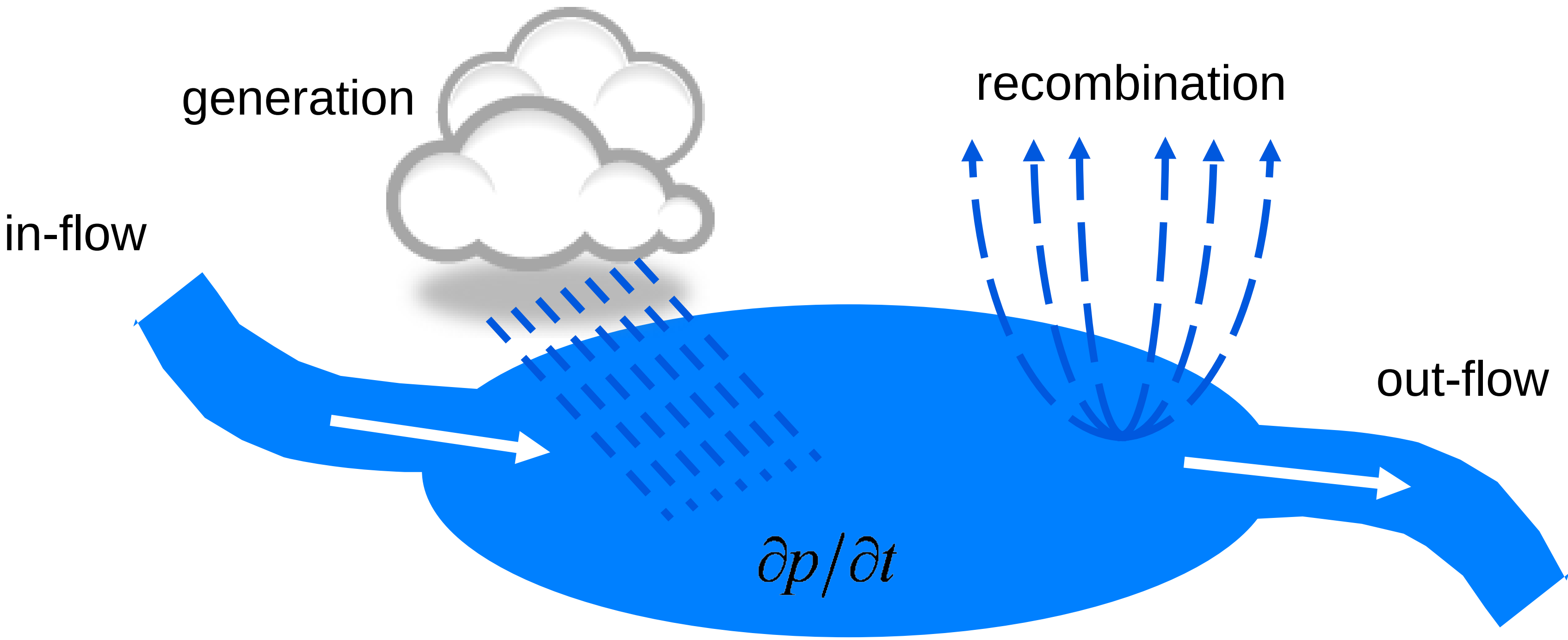
$$F_p(\vec{r}), F_n(\vec{r}), V(\vec{r})$$

We need to formulate 3 equations in 3 unknowns.

First equation: Continuity equation for holes

$$\frac{\partial p}{\partial t} = \text{in-flow - out-flow} + G - R$$

$$\frac{\partial p}{\partial t} = -\nabla \bullet \frac{\vec{J}_p}{q} + G_p - R_p$$



One equation in 3 unknowns

$$\frac{\partial p}{\partial t} = -\nabla \bullet \frac{\vec{J}_p}{q} + G_p - R_p$$

$$\vec{J}_p = pq\mu_p \vec{\mathcal{E}} - qD_p \vec{\nabla} p$$

optical generation
or impact ionization

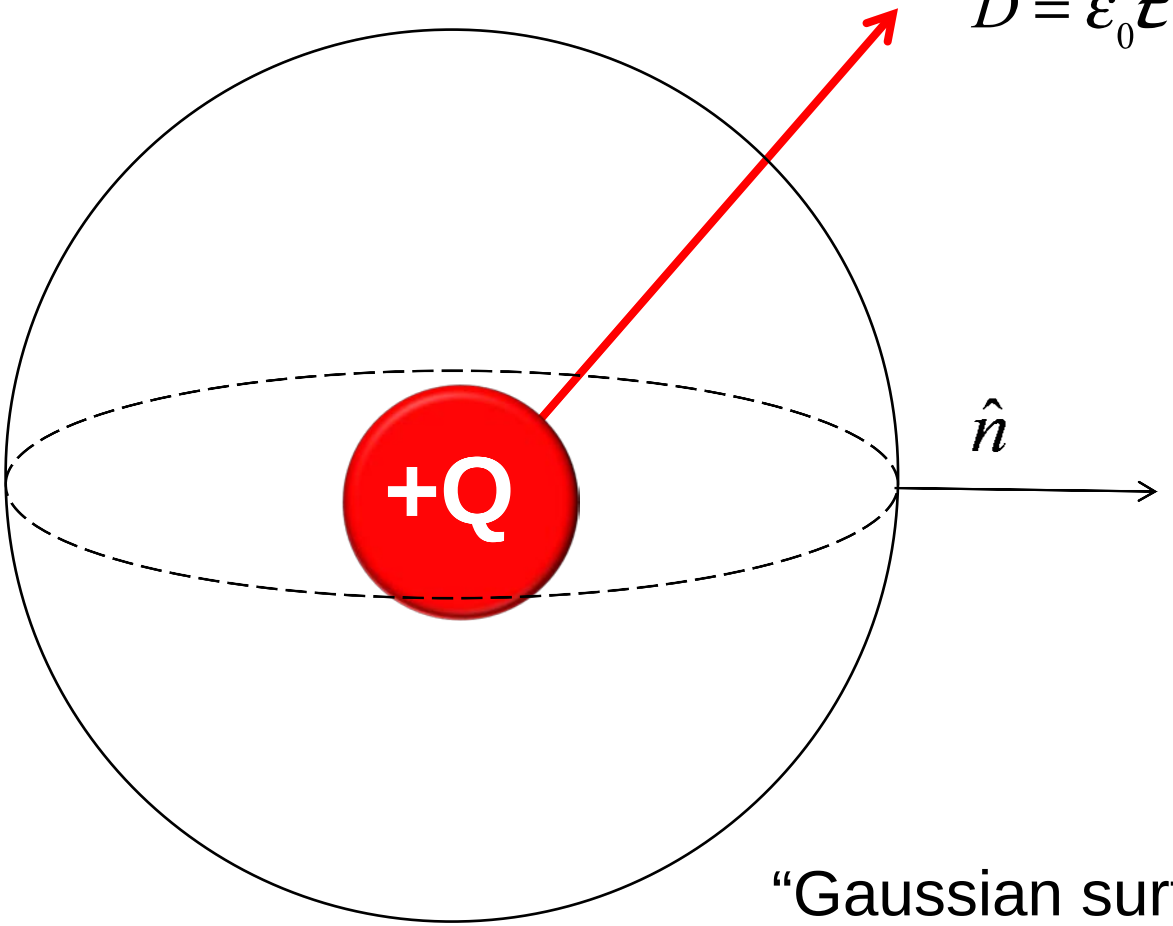
Radiative, Auger
or defect-assisted
processes

$$R_p \propto np$$

Need equation for the electric field

Need equation for electrons

Gauss's Law



$$\vec{D} = \varepsilon_0 \vec{\mathcal{E}} \qquad \vec{D} = K_S \varepsilon_0 \vec{\mathcal{E}}$$

$$\boxed{\oint \vec{D} \bullet d\vec{S} = Q}$$

"Gaussian surface"

Gauss's Law in 1D

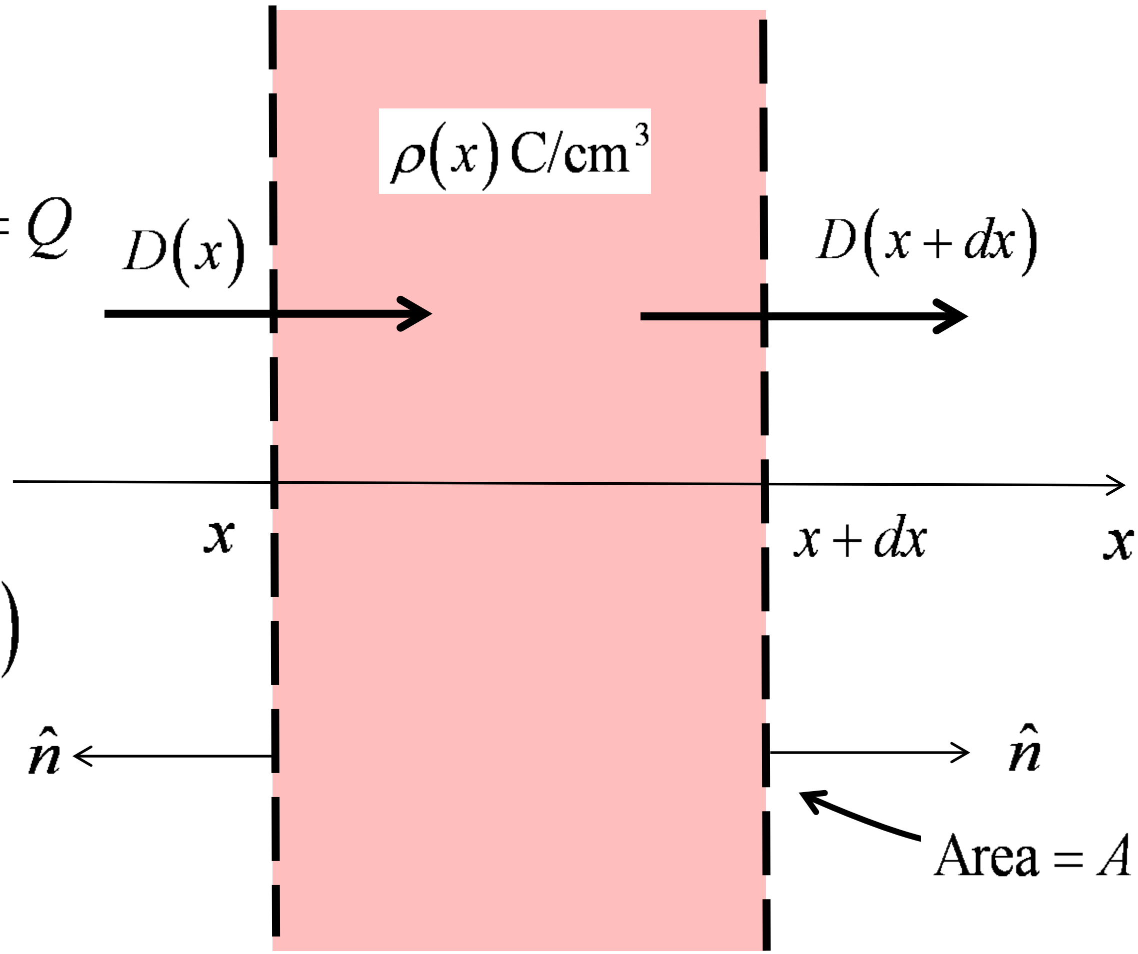
$$\oint \vec{D} \bullet d\vec{S} = Q$$

$$-D(x)A + D(x+dx)A = Q$$

$$Q = \rho(x)A\,dx$$

$$\frac{D(x+dx) - D(x)}{dx} = \rho(x)$$

$$\frac{dD}{dx} = \rho(x)$$



The Poisson equation

$$\oint \vec{D} \bullet d\vec{S} = Q \quad \longleftrightarrow \quad \nabla \bullet \vec{D} = \rho(x)$$

$$D = K_S \varepsilon_0 \mathcal{E}$$

$$\nabla \bullet \left(K_S \varepsilon_0 \vec{\mathcal{E}} \right) = \rho(\vec{r})$$

$$\rho(\vec{r}) = q\left[p(\vec{r}) - n(\vec{r}) + N_D^+(\vec{r}) - N_A^-(\vec{r}) \right]$$

The "semiconductor equations"

$$\frac{\partial p}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_p}{q} \right) + G_p - R_p$$

$$\frac{\partial n}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_n}{-q} \right) + G_n - R_n$$

$$\nabla \bullet \left(K_S \varepsilon_0 \vec{\mathcal{E}} \right) = \rho$$

Three equations in three unknowns:

$$p(\vec{r}), n(\vec{r}), V(\vec{r})$$

$$\vec{J}_p = pq\mu_p \vec{\mathcal{E}} - qD_p \vec{\nabla} p$$

$$\rho = q\left(p - n + N_D^+ - N_A^- \right)$$

$$\vec{J}_n = nq\mu_n \vec{\mathcal{E}} + qD_n \vec{\nabla} n$$

$$\vec{\mathcal{E}}(\vec{r}) = \nabla V(\vec{r})$$

Semiconductor equations

$$\frac{\partial p}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_p}{q} \right) + G_p - R_p$$

$$\frac{\partial n}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_n}{-q} \right) + G_n - R_n$$

$$\nabla \bullet \left(K_S \varepsilon_0 \vec{\mathcal{E}} \right) = \rho$$

Not as fundamental as Maxwell’s equations, but these equations are the starting point for the analysis of most semiconductor devices.

Discussion: Equilibrium

$$\frac{\partial p}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_p}{q} \right) + G_p - R_p \qquad\qquad 0 = -\nabla \circ \left(\frac{0}{q} \right) + 0 - 0$$

$$\frac{\partial n}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_n}{-q} \right) + G_n - R_n \quad \longrightarrow \quad 0 = -\nabla \circ \left(\frac{0}{-q} \right) + 0 - 0$$

$$\nabla \bullet \left(K_S \varepsilon_0 \vec{\mathcal{E}} \right) = \rho \qquad\qquad \nabla \bullet \left(K_S \varepsilon_0 \vec{\mathcal{E}} \right) = \rho$$

What can we learn from equilibrium?

$$p_0 = n_i e^{\left(E_i - E_F\right)/k_B T} \qquad n_0 = n_i e^{\left(E_F - E_i\right)/k_B T} \qquad n_0 p_0 = n_i^2$$

Reminder: Generation and recombination

$$\frac{\partial p}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_p}{q} \right) + G_p - R_p$$

other generation processes

e.g. optical generation
impact ionization

Band-to-band
SRH
Auger

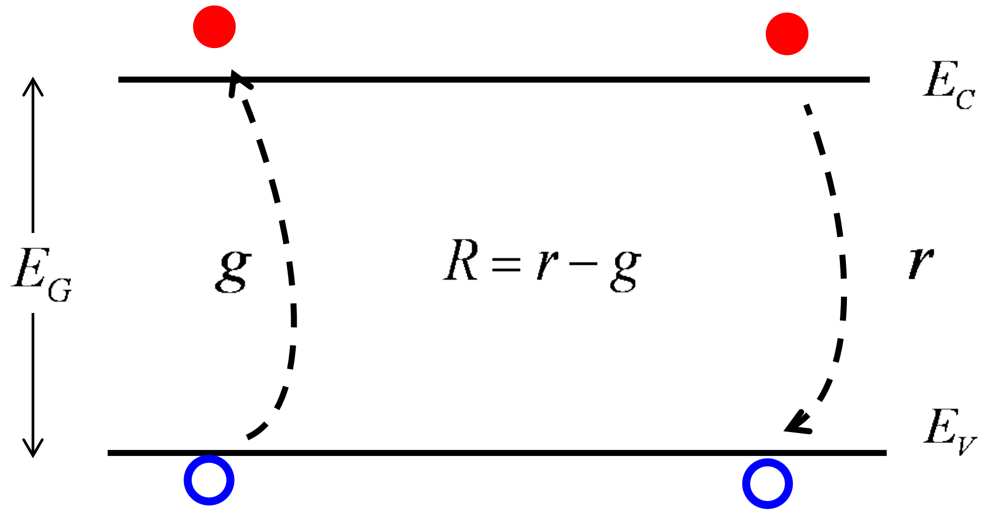
recombination:

$$R_p > 0$$

generation:

$$R_p < 0$$

Discussion



$R_p = R_n = R < 0$

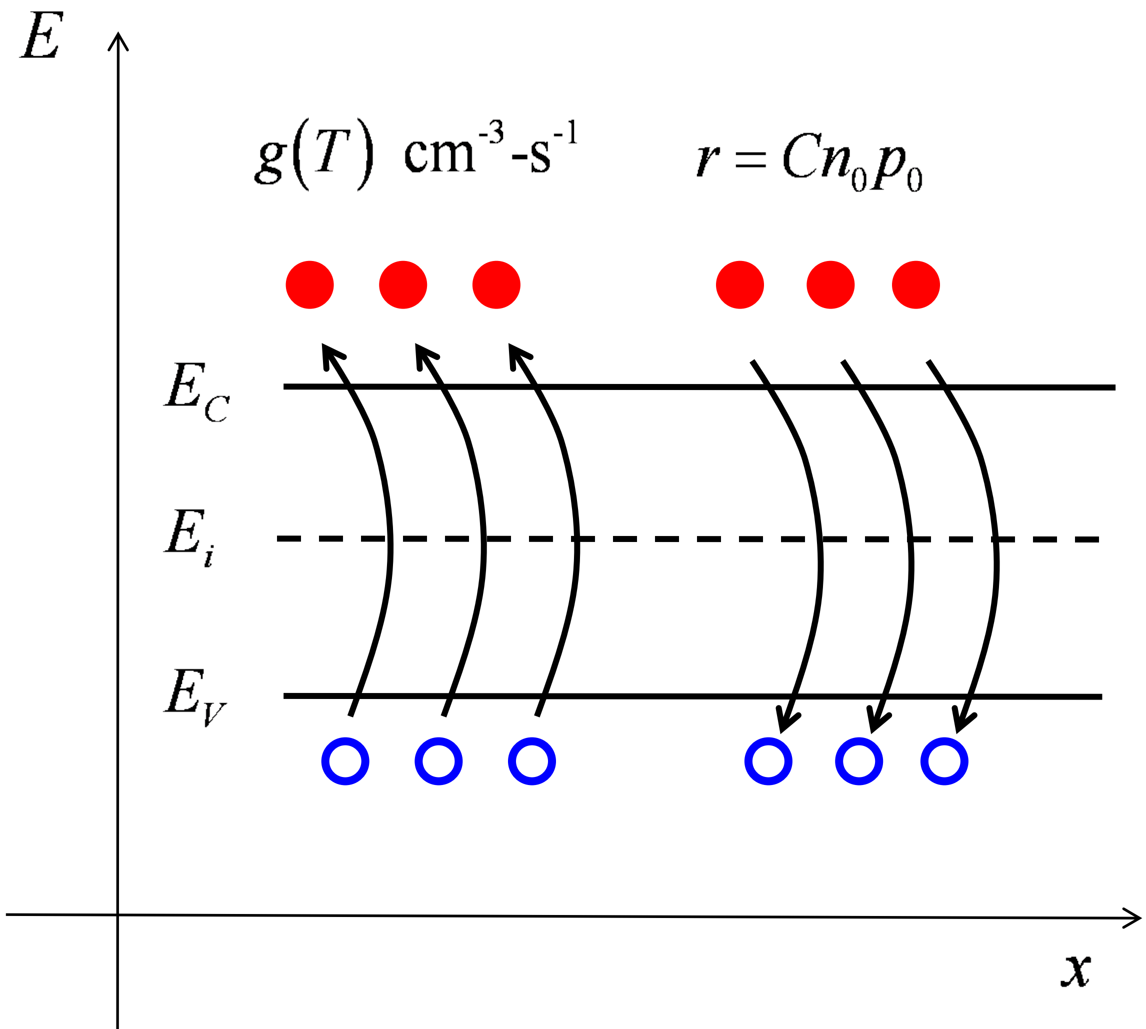
requires energy to break covalent bonds

$R_p = R_n = R > 0$

releases energy

In equilibrium: *R* = 0

Why is $n_0 p_0 = n_i^2$?



In equilibrium:

$$g = r$$

$$n_0 p_0 = \frac{g(T)}{C} = n_i^2(T)$$

$$\boxed{n_0 p_0 = n_i^2(T)}$$

Summary: The semiconductor equations

$$\frac{\partial p}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_p}{q} \right) + G_p - R_p$$

$$\frac{\partial n}{\partial t} = -\nabla \bullet \left(\frac{\vec{J}_n}{-q} \right) + G_n - R_n$$

$$\nabla \bullet \left(K_S \varepsilon_0 \vec{\mathcal{E}} \right) = \rho$$

Three equations in three unknowns:

$$p(\vec{r}),\, n(\vec{r}),\, V(\vec{r})$$

$$\vec{J}_p = pq\mu_p \vec{\mathcal{E}} - qD_p \vec{\nabla} p$$

$$\vec{J}_n = nq\mu_n \vec{\mathcal{E}} + qD_n \vec{\nabla} n$$

$$\rho = q\left(p - n + N_D^+ - N_A^- \right)$$

$$\vec{\mathcal{E}}(\vec{r}) = \nabla V(\vec{r})$$