Linearly Stratified Flow Past Rotating 2D Ellipses

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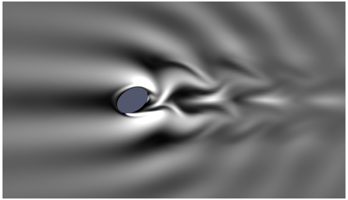

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Motivation

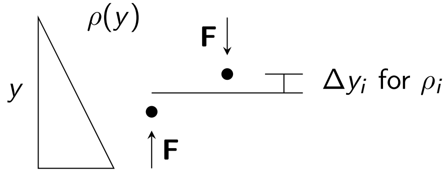


Figure: Restoring force and equilibrium on particle i.

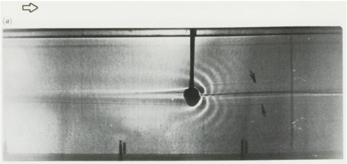


Figure: Lee waves from flow past a cylinder Boyer *et al.* (1989).

- In nature, there exist innumerable flows with varying density, known as stratified flows, such as the ocean.
- The density variation is caused by salt content or temperature variation.
- Stratified mediums exhibit a restoring force due to particles being in the lowest potential energy position when at the same depth as particles of similar density.
- This force creates *internal gravity waves*.
    - Flow past a bluff body creates steady gravity waves known as *lee waves*.
    - Bluff body oscillation forms unsteady waves.

Motivation

- Internal gravity waves and other effects from stratfication affect the flow structures and forces on bodies.
- Stratification is defined by the Densimetric Froude number.
    - Lower Froude number means higher stratification.
- Submersible bodies are approximated by slender bodies, and nonspherical bodies have been neglected in past studies.
- Ortiz-Tarin *et al.* (2019) look at static slender bodies; we introduce rotation.
- We want to better understand the dynamics of spheroidal bodies in stratified flows for design in submersible vehicles.

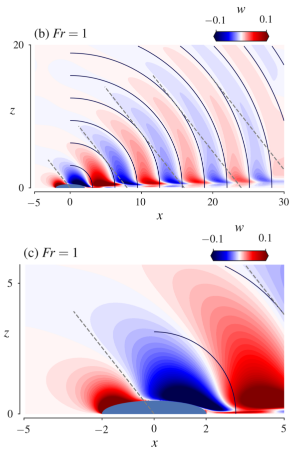


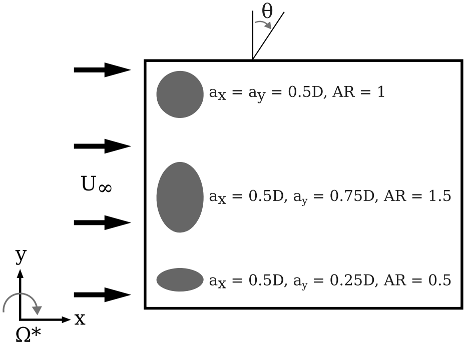
Figure: Internal gravity waves shown in the y-velocity fields Ortiz-Tarin *et al.* (2019).

Equations and Setup

Incompressible Navier-Stokes Equations (1), with Boussinesq approximation and transport equation (2).

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{u} - \frac{1}{Fr^2} (\rho' \; - \; y) \hat{y}, \quad \nabla \cdot \mathbf{u} = 0 \tag{1}$$

$$\frac{\partial \rho'}{\partial t} + \mathbf{u} \cdot \nabla \rho' = \frac{1}{Pr Re} \nabla^2 \rho' \tag{2}$$

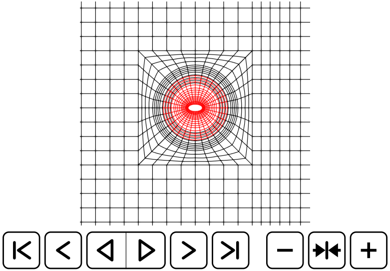


- Density perturbation ρ' from background density ρ_0.
- Thermal diffusivity κ, kinematic viscosity ν
- Aspect ratio $AR = a_y / a_x$, with $\Omega^* = \frac{\Omega D}{2U_\infty}$
- Brunt–Väisälä frequency $N_{BV} = \left(\frac{g}{\rho_0} \rho'_{y,0} \right)^{1/2}$

$$Re = \rho_0 \frac{UD}{\mu}, Pr = \rho_0 \frac{\kappa}{\mu}, Fr^{-2} = \frac{gD^2}{\rho_0 U^2} |\rho'_{y,0}|$$

Numerical Methodology - Temporal Discretization

- Semi-implicit BDF-3/EXT-3 timestepping
  - Time derivative - BDF-3
  - Nonlinear terms - EXT-3
  - Pressure and viscous terms - implicit
- Spatially interpolated boundary conditions at interdomain boundaries (Mittal *et al.*, 2019).
  - Picard iterations
  - M-order temporal extrapolation at boundaries for accuracy
  - Q corrector iterations for stability
- Arbitrary Lagrangian-Eulerian (ALE) formulation to track spinning inner mesh.
- Method allows better shape adaptation.
- Spectral accuracy retained

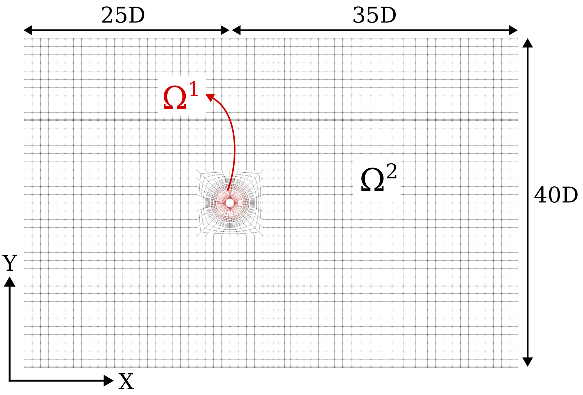


Numerical Methodology - Spatial Discretization

- Spectrally accurate overset grid method
  - Inner mesh (Ω^1 : 512 elements)
  - Outer mesh (Ω^2 : 3200 elements)
- Schwarz-Spectral Element Method (Schwarz-SEM)
- Hexahedral elements
- Symmetric top and bottomr boundaries
- Lagrangle polynomials

Parameters	Values
AR	$0.5, 1.0, 1.5$
Fr^2	$0.01, 0.1, 1.0, 10.0, 100.0, \infty$
Re	120
Pr	1
Ω^*	0.0 ($AR = 1$ only), 1.0

- Polynomial order $N = 11$
- Gauss-Lobatto-Legendre (GLL) nodes



[1]https://nek5000.mcs.anl.gov/

Validation

We validate Schwarz-SEM stratification implementation in two ways. First, by validating nonrotating stratified cases against monodomain stratified cases. Second, by comparing rotating unstratified cases with rotating unstratified cases from Lu *et al.* (2018) (two papers) produced with *Ansys® Fluent* with *Sliding Mesh*. Drag and lift show similarity.

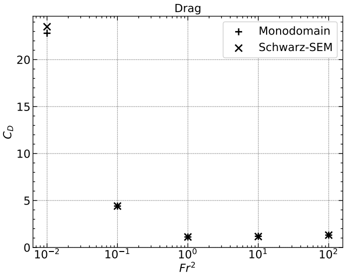


Figure: Comparision of drag against monodomain case.

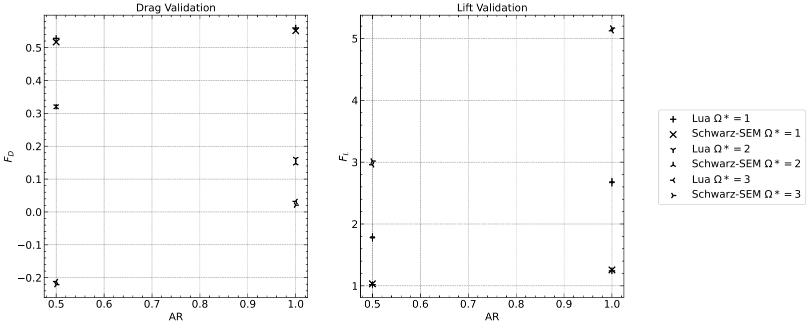
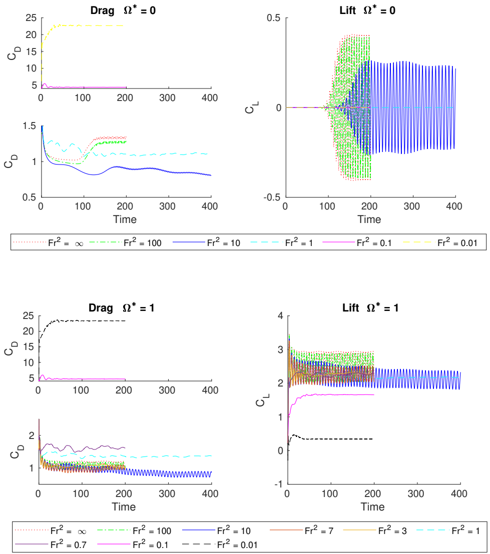


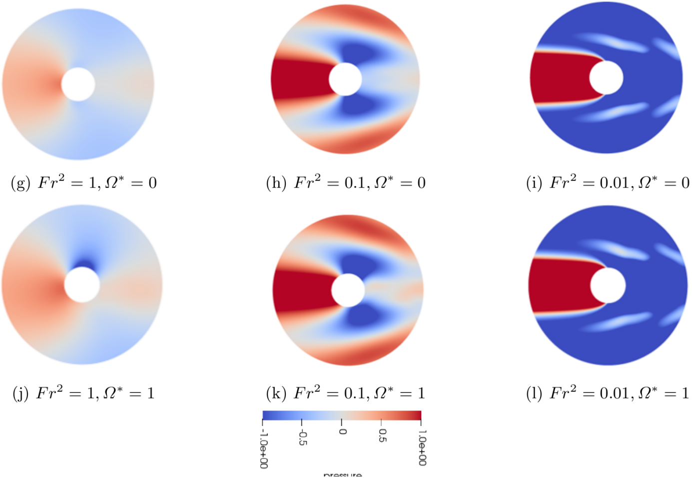
Figure: Comparison of drag and lift in unstratified flow against Lu *et al.* (2018) finite-volume results at $Re = 200$

Circle Drag and Lift Time Series



- The plots to the left show time series data for nonspinning and spinning cyclinders.
- We observe the presence of different flow regimes
  - A regime in which stratification dominates and there is little oscillation in lift
  - A regime in which vortex-shedding dominates and there is substantial oscillation in lift.
- The increase in drag with stratification increase if a result of upstream blocking.

Circle Drag and Lift



(g) $Fr^2 = 1, \Omega^* = 0$ (h) $Fr^2 = 0.1, \Omega^* = 0$ (i) $Fr^2 = 0.01, \Omega^* = 0$

(j) $Fr^2 = 1, \Omega^* = 1$ (k) $Fr^2 = 0.1, \Omega^* = 1$ (l) $Fr^2 = 0.01, \Omega^* = 1$

Figure: Pressure distribution around 2D cylinders. High pressure region in front of the body at high stratification is upstream blocking.

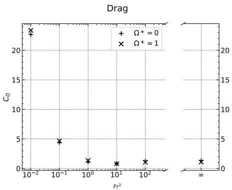


Figure: Mean Drag Coefficients

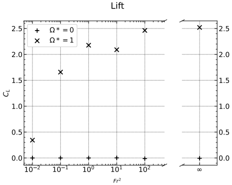
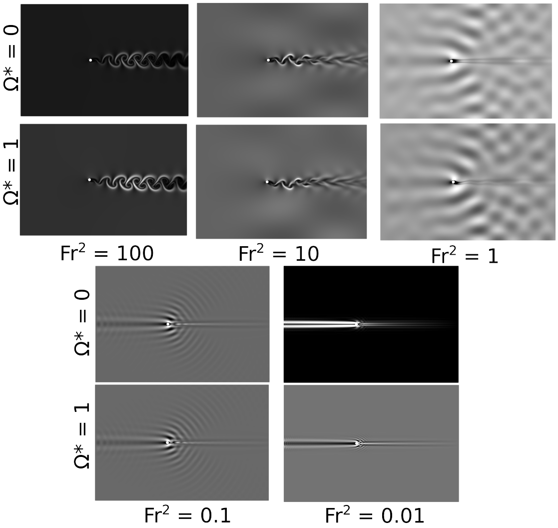


Figure: Mean Lift Coefficients

Circle Schlieren Plots

- The following plots are Schlieren plots calculated with density gradient $|\nabla\rho|$
- We observe the presence of different flow regimes
  - A regime in which stratification dominates
  - A regime in which vortex-shedding dominates
- There exists a transitional regime at $Fr^2 \approx 10$



Phase Averaged Drag and Lift

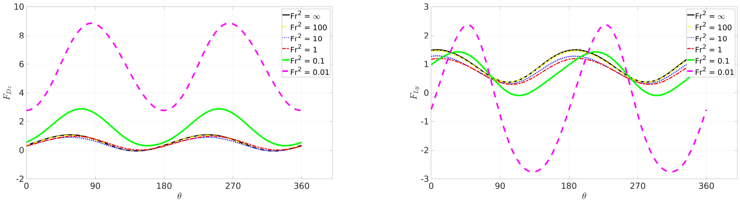


Figure: Phase-averaged drag and lift for $AR = 0.5$

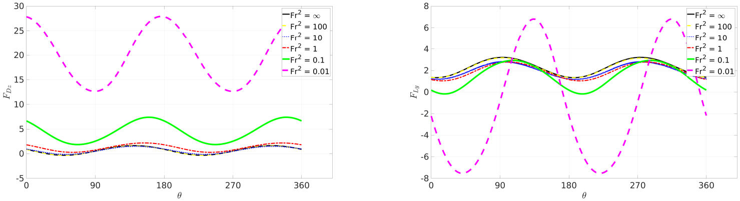
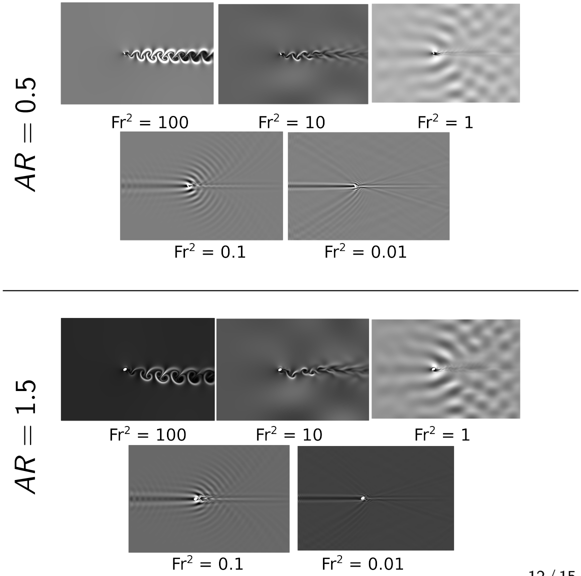


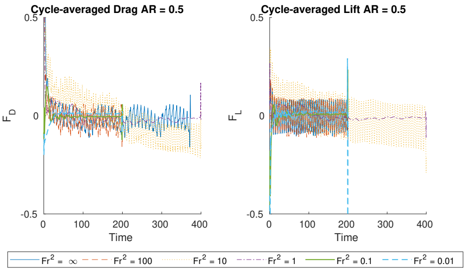
Figure: Phase-averaged drag and lift for $AR = 1.5$

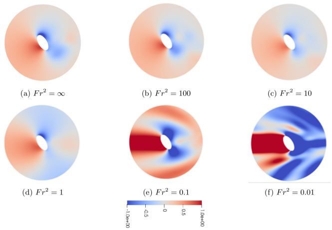
Ellipse Schlieren Plots

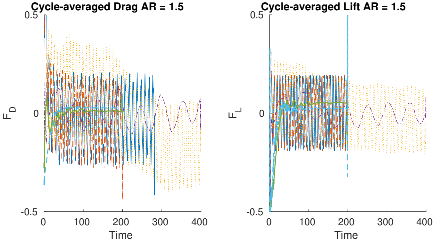
- Same large-scale flow structures as in the circle cases are present
- Unsteady internal gravity waves manifest themselves in the cases of $Fr^2 = 0.1, 0.01$
- Internal gravity wave reflection off domain boundaries is present in the $Fr^2 = 0.01$ case

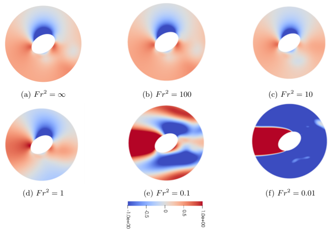


Cycle-averaged Drag and Lift



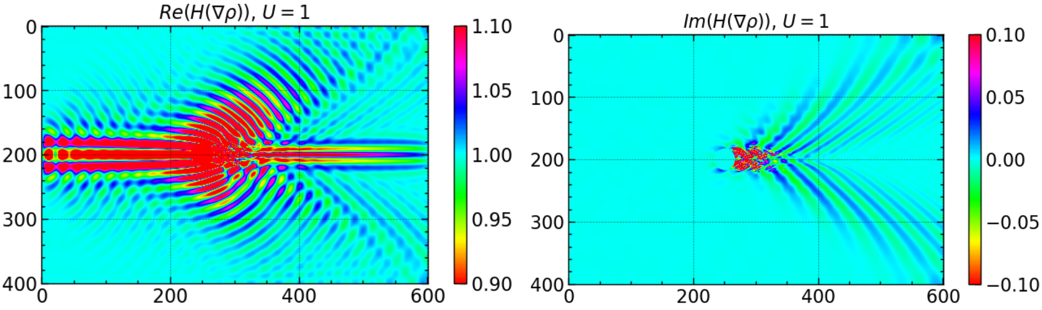






Deeper Analysis

- Applying Hilbert transform to extract analtyic signal using FFT
- Follow the procedure outlined by Mercier *et al.* (2008)
- $\mathfrak{H}(|\nabla\rho|) = \mathfrak{F}^{-1}(2H(\mathfrak{F}(|\nabla\rho|)))$
- $Fr^2 = 0.1, AR = 1.5$ case



Acknowledgements

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References

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