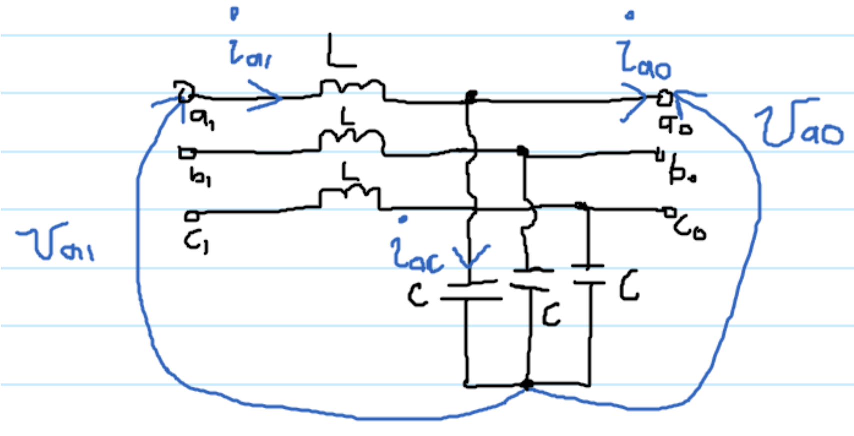
Conventional LC Filter Inverter Control

Current Control Loop

$$\text{KVL} \rightarrow v_{abc_1} = L\frac{di_{abc_1}}{dt} + v_{abc_o} \tag{1}$$



We could rearrange (1) to get our plant transfer function in the *abc* domain. However, our PI control block is conventionally performed in the *dq*0 domain (not the *abc* domain). This is because the *dq*0 transformation can transform three sinusoidal current or voltage references into two dc reference signals, and PI controllers need dc signals so that the integral term can iterate the error to zero during steady-state conditions. Therefore, we must transform (1) into the *dq*0 domain via transformation *T*.

Now we know that multiplying a variable in the *abc* domain by *T* will yield that variable in the *dq*0 domain (2).

$$v_{dq0} = Tv_{abc} \tag{2}$$

Therefore, if we multiply (1) by *T*, then we get (3). This can be simplified to (4) using the identity in (2).

$$Tv_{abc_1} = T\left[L\frac{di_{abc_1}}{dt}\right] + Tv_{abc_o} \tag{3}$$

$$v_{dq0_1} = T\left[L\frac{di_{abc_1}}{dt}\right] + v_{dq0_o} \tag{4}$$

Note that in (4), we can't simplify the middle term as there are multiple variables inside of the brackets. Since we know that $T^{-1}T = 1$, we can create (5) from (4).

$$v_{dq0_1} = T\left[LT^{-1}T\frac{di_{abc_1}}{dt}\right] + v_{dq0_o} \tag{5}$$

To solve, the $T\frac{di_{abc_1}}{dt}$ term, we should rearrange the product rule (6) to obtain (7). Using (7), we can solve for the $T\frac{di_{abc_1}}{dt}$ term in (8).

$$\frac{d(xy)}{dt} = x\frac{dy}{dt} + \frac{dx}{dt}y \tag{6}$$

$$x\frac{dy}{dt} = \frac{d(xy)}{dt} - \frac{dx}{dt}y \tag{7}$$

$$T\frac{di_{abc_1}}{dt} = \frac{d(Ti_{abc_1})}{dt} - \frac{dT}{dt}i_{abc_1} = \frac{d\left(i_{dq0_1}\right)}{dt} - \frac{dT}{dt}i_{abc_1} \tag{8}$$

From (8), we can substitute in $T^{-1}T$ as the last term in still in the *abc* domain to get (9). The final term in (9) has a $\frac{dT}{dt}T^{-1}$ term, which needs simplification. We can solve for this term in (10), which yields (11).

$$T\frac{di_{abc_1}}{dt} = \frac{d\left(i_{dq0_1}\right)}{dt} - \frac{dT}{dt}T^{-1}Ti_{abc_1} = \frac{d\left(i_{dq0_1}\right)}{dt} - \frac{dT}{dt}T^{-1}i_{dq0_1} \tag{9}$$

$$\frac{dT}{dt}T^{-1} = \frac{d}{dt}\left(\frac{2}{3}\begin{bmatrix} \cos(\omega t) & \cos\left(\omega t - \frac{2\pi}{3}\right) & \cos\left(\omega t + \frac{2\pi}{3}\right) \\ -\sin(\omega t) & -\sin\left(\omega t - \frac{2\pi}{3}\right) & -\sin\left(\omega t + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\right)\left(\frac{2}{3}\begin{bmatrix} \cos(\omega t) & \cos\left(\omega t - \frac{2\pi}{3}\right) & \cos\left(\omega t + \frac{2\pi}{3}\right) \\ -\sin(\omega t) & -\sin\left(\omega t - \frac{2\pi}{3}\right) & -\sin\left(\omega t + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\right)^{-1} = \begin{bmatrix} 0 & \omega & 0 \\ -\omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{10}$$

$$T\frac{di_{abc_1}}{dt} = \frac{d\left(i_{dq0_1}\right)}{dt} - \begin{bmatrix} 0 & \omega & 0 \\ -\omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} i_{dq0_1} \tag{11}$$

Lastly, since the inductance *L* is a scalar value we get (12). Substituting (11-12) into (5) gives (13).

$$TLT^{-1} = LTT^{-1} = L \tag{12}$$

$$v_{dq0_1} = L\frac{di_{dq0_1}}{dt} - L\begin{bmatrix} 0 & \omega & 0 \\ -\omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} i_{dq0_1} + v_{dq0_o} \tag{13}$$

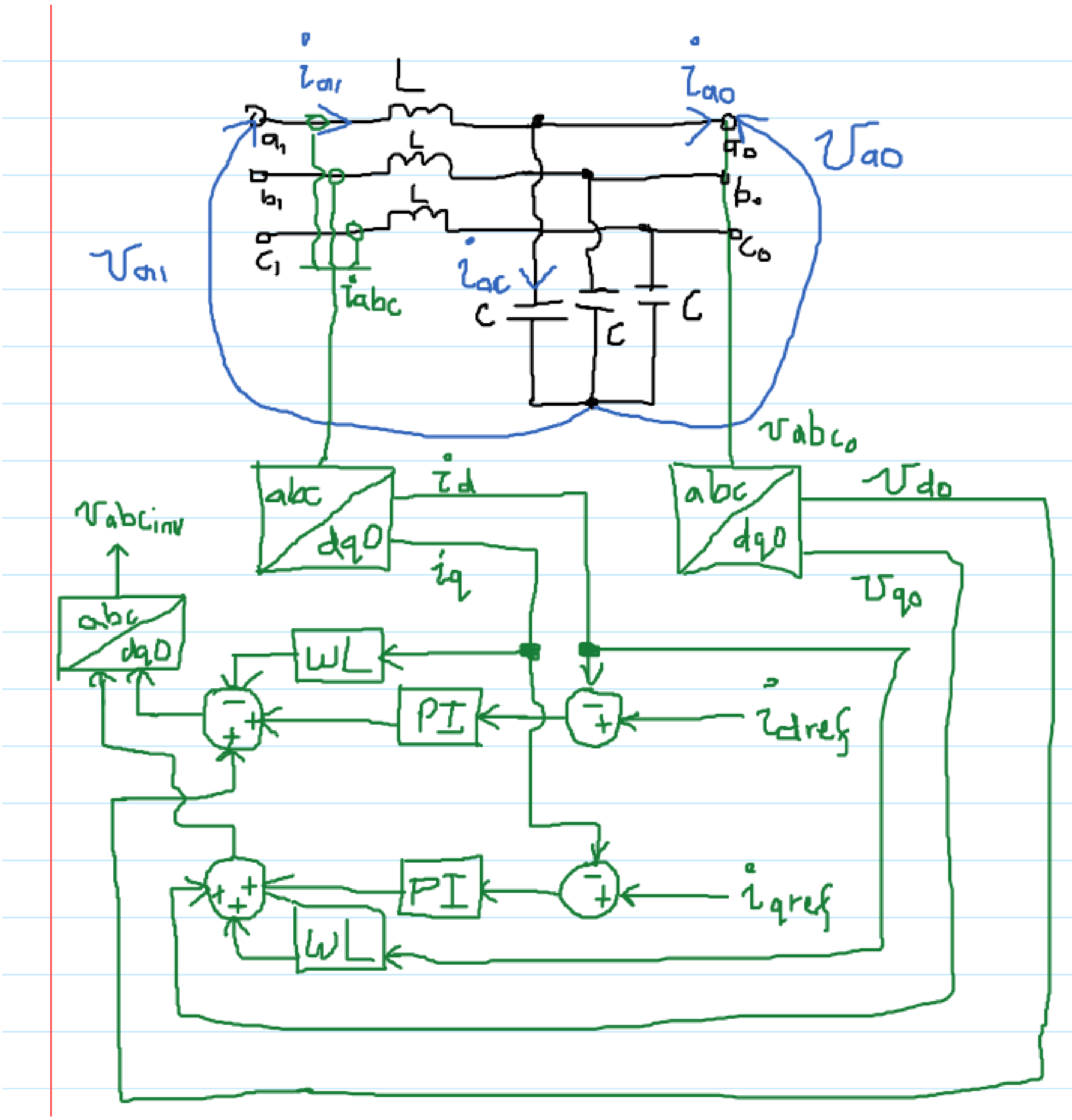
Therefore, the final equation for the input voltage in the *dq*0 domain is (14).

$$\begin{bmatrix} v_{d_1} \\ v_{q_1} \end{bmatrix} = \begin{bmatrix} Lsi_{d_1} - \omega Li_{q_1} + v_{d_o} \\ Lsi_{q_1} + \omega Li_{d_1} + v_{q_o} \end{bmatrix} \tag{14}$$

Therefore, from the below figure we can see that we can measure and then feed-forward the $v_{dq_o}$and ωLi_{dq_1}terms. With this architecture, the PI controllers will only compensate for (15). Hence, the transfer plant function that the PI controllers are compensating for is (16). You can use this plant transfer function to design the PI current controllers (if the architecture below is used).

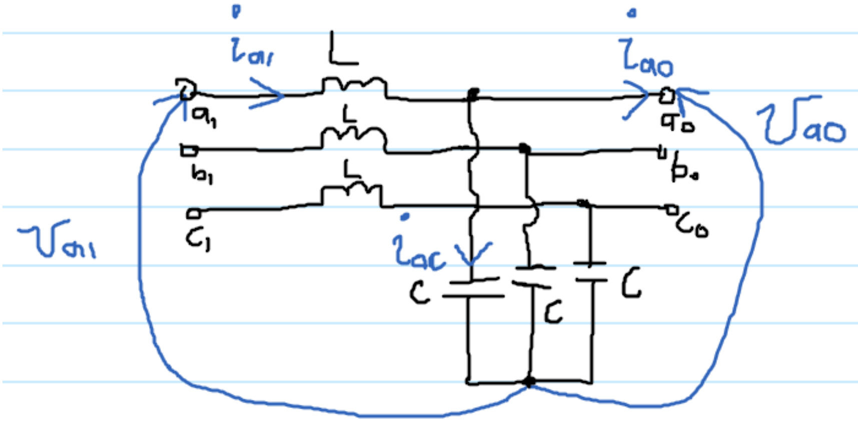
$$\begin{bmatrix} v_{d_{1,\mathrm{PI}}} \\ v_{q_{1,\mathrm{PI}}} \end{bmatrix} = \begin{bmatrix} Lsi_{d_1} \\ Lsi_{q_1} \end{bmatrix} \tag{15}$$

$$\frac{i_{dq_1}}{v_{dq_{1,\mathrm{PI}}}} = \frac{1}{sL} \tag{16}$$



Voltage Control Loop

$$\text{KCL} \rightarrow i_{abc_1} = i_{abc_C} + i_{abc_o} \tag{17}$$



We know that the current through a capacitor is equal to the rate of change of the voltage multiplied by the capacitance, which yields (18).

$$\text{KCL} \rightarrow i_{abc_1} = C\frac{dv_{abc_0}}{dt} + i_{abc_o} \tag{18}$$

We can see that (18) is of the similar form to (1). Hence, from looking at (14) we can find that the equation for the inverter current in the *dq*0 domain is (19).

$$\begin{bmatrix} i_{d_1} \\ i_{q_1} \end{bmatrix} = \begin{bmatrix} Csv_{d_0} - \omega Cv_{q_0} + i_{d_o} \\ Csv_{q_0} + \omega Cv_{d_0} + i_{q_o} \end{bmatrix} \tag{14}$$

You will need to find the open-loop transfer function for the plant and what decoupling (feedforward) is required for the voltage controller.