$$T_{\text{convention 1}} = \frac{2}{3}\begin{bmatrix} \cos(\widehat{\omega}t) & \cos\left(\widehat{\omega}t - \frac{2\pi}{3}\right) & \cos\left(\widehat{\omega}t + \frac{2\pi}{3}\right) \\ -\sin(\widehat{\omega}t) & -\sin\left(\widehat{\omega}t - \frac{2\pi}{3}\right) & -\sin\left(\widehat{\omega}t + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$T_{\text{convention 2}} = \frac{2}{3}\begin{bmatrix} \sin(\widehat{\omega}t) & \sin\left(\widehat{\omega}t - \frac{2\pi}{3}\right) & \sin\left(\widehat{\omega}t + \frac{2\pi}{3}\right) \\ \cos(\widehat{\omega}t) & \cos\left(\widehat{\omega}t - \frac{2\pi}{3}\right) & \cos\left(\widehat{\omega}t + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Convention 1

Let's first assume that:

$$\begin{bmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \end{bmatrix} = \begin{bmatrix} V\cos(\omega t) \\ V\cos\left(\omega t - \frac{2\pi}{3}\right) \\ V\cos\left(\omega t + \frac{2\pi}{3}\right) \end{bmatrix}$$

$$\begin{bmatrix} v_d(t) \\ v_q(t) \\ v_0(t) \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos(\hat{\omega} t) & \cos\left(\hat{\omega} t - \frac{2\pi}{3}\right) & \cos\left(\hat{\omega} t + \frac{2\pi}{3}\right) \\ -\sin(\hat{\omega} t) & -\sin\left(\hat{\omega} t - \frac{2\pi}{3}\right) & -\sin\left(\hat{\omega} t + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} V\cos(\omega t) \\ V\cos\left(\omega t - \frac{2\pi}{3}\right) \\ V\cos\left(\omega t + \frac{2\pi}{3}\right) \end{bmatrix} = \begin{bmatrix} V\cos(\omega t - \hat{\omega} t) \\ V\sin(\omega t - \hat{\omega} t) \\ 0 \end{bmatrix}$$

Let's now assume that:

$$\begin{bmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \end{bmatrix} = \begin{bmatrix} V\sin(\omega t) \\ V\sin\left(\omega t - \frac{2\pi}{3}\right) \\ V\sin\left(\omega t + \frac{2\pi}{3}\right) \end{bmatrix}$$

$$\begin{bmatrix} v_d(t) \\ v_q(t) \\ v_0(t) \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos(\hat{\omega} t) & \cos\left(\hat{\omega} t - \frac{2\pi}{3}\right) & \cos\left(\hat{\omega} t + \frac{2\pi}{3}\right) \\ -\sin(\hat{\omega} t) & -\sin\left(\hat{\omega} t - \frac{2\pi}{3}\right) & -\sin\left(\hat{\omega} t + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} V\sin(\omega t) \\ V\sin\left(\omega t - \frac{2\pi}{3}\right) \\ V\sin\left(\omega t + \frac{2\pi}{3}\right) \end{bmatrix}$$

$$= \begin{bmatrix} V\sin(\omega t - \hat{\omega} t) \\ -V\cos(\omega t - \hat{\omega} t) \\ 0 \end{bmatrix} = \begin{bmatrix} V\cos\left(\omega t - \hat{\omega} t - \frac{\pi}{2}\right) \\ V\sin\left(\omega t - \hat{\omega} t - \frac{\pi}{2}\right) \\ 0 \end{bmatrix}$$

Convention 2

Let's first assume that:

$$\begin{bmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \end{bmatrix} = \begin{bmatrix} V\cos(\omega t) \\ V\cos\left(\omega t - \frac{2\pi}{3}\right) \\ V\cos\left(\omega t + \frac{2\pi}{3}\right) \end{bmatrix}$$

$$\begin{bmatrix} v_d(t) \\ v_q(t) \\ v_0(t) \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \sin(\widehat{\omega} t) & \sin\left(\widehat{\omega} t - \frac{2\pi}{3}\right) & \sin\left(\widehat{\omega} t + \frac{2\pi}{3}\right) \\ \cos(\widehat{\omega} t) & \cos\left(\widehat{\omega} t - \frac{2\pi}{3}\right) & \cos\left(\widehat{\omega} t + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} V\cos(\omega t) \\ V\cos\left(\omega t - \frac{2\pi}{3}\right) \\ V\cos\left(\omega t + \frac{2\pi}{3}\right) \end{bmatrix} = \begin{bmatrix} -V\sin(\omega t - \widehat{\omega} t) \\ V\cos(\omega t - \widehat{\omega} t) \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} V\cos\left(\omega t - \widehat{\omega} t - \frac{\pi}{2}\right) \\ V\sin\left(\omega t - \widehat{\omega} t - \frac{\pi}{2}\right) \\ 0 \end{bmatrix}$$

Let's now assume that:

$$\begin{bmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \end{bmatrix} = \begin{bmatrix} V\sin(\omega t) \\ V\sin\left(\omega t - \frac{2\pi}{3}\right) \\ V\sin\left(\omega t + \frac{2\pi}{3}\right) \end{bmatrix}$$

$$\begin{bmatrix} v_d(t) \\ v_q(t) \\ v_0(t) \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \sin(\widehat{\omega} t) & \sin\left(\widehat{\omega} t - \frac{2\pi}{3}\right) & \sin\left(\widehat{\omega} t + \frac{2\pi}{3}\right) \\ \cos(\widehat{\omega} t) & \cos\left(\widehat{\omega} t - \frac{2\pi}{3}\right) & \cos\left(\widehat{\omega} t + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} V\sin(\omega t) \\ V\sin\left(\omega t - \frac{2\pi}{3}\right) \\ V\sin\left(\omega t + \frac{2\pi}{3}\right) \end{bmatrix} = \begin{bmatrix} V\cos(\omega t - \widehat{\omega} t) \\ V\sin(\omega t - \widehat{\omega} t) \\ 0 \end{bmatrix}$$

Effective of Harmonics on Output of $dq0$ Transformation

Let's assume that:

$$\begin{bmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \end{bmatrix} = \begin{bmatrix} V\sin(\omega t) + V_5\sin(5\omega t + \varphi_5) \\ V\sin\left(\omega t - \frac{2\pi}{3}\right) + V_5\sin\left(5\left(\omega t - \frac{2\pi}{3}\right) + \varphi_5\right) \\ V\sin\left(\omega t + \frac{2\pi}{3}\right) + V_5\sin\left(5\left(\omega t + \frac{2\pi}{3}\right) + \varphi_5\right) \end{bmatrix}$$

$$\begin{bmatrix} v_d(t) \\ v_q(t) \\ v_0(t) \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \sin(\hat{\omega} t) & \sin\left(\hat{\omega} t - \frac{2\pi}{3}\right) & \sin\left(\hat{\omega} t + \frac{2\pi}{3}\right) \\ \cos(\hat{\omega} t) & \cos\left(\hat{\omega} t - \frac{2\pi}{3}\right) & \cos\left(\hat{\omega} t + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} V\sin(\omega t) + V_5\sin(5\omega t + \varphi_5) \\ V\sin\left(\omega t - \frac{2\pi}{3}\right) + V_5\sin\left(5\left(\omega t - \frac{2\pi}{3}\right) + \varphi_5\right) \\ V\sin\left(\omega t + \frac{2\pi}{3}\right) + V_5\sin\left(5\left(\omega t + \frac{2\pi}{3}\right) + \varphi_5\right) \end{bmatrix}$$

$$= \begin{bmatrix} V\cos(\omega t - \hat{\omega} t) - V_5\cos(5\omega t + \hat{\omega} t + \varphi_5) \\ V\sin(\omega t - \hat{\omega} t) + V_5\sin(5\omega t + \hat{\omega} t + \varphi_5) \\ 0 \end{bmatrix}$$

Therefore, when there are harmonics present in the three input voltage signals, the $dq0$ transformation will output the equation below, where $\tilde{v}_d(t)$ and $\tilde{v}_q(t)$ are oscillatory terms that can contain multiple harmonic terms at different frequencies.

$$\begin{bmatrix} v_d(t) \\ v_q(t) \\ v_0(t) \end{bmatrix} = \begin{bmatrix} V\cos(\omega t - \hat{\omega} t) + \tilde{v}_d(t) \\ V\sin(\omega t - \hat{\omega} t) + \tilde{v}_q(t) \\ 0 \end{bmatrix}$$

Therefore, oscillations in the $v_q(t)$ and $v_d(t)$ outputs and hence in the amplitude and frequency estimates will be generated from harmonics in the three input voltage signals. These oscillations in the amplitude and frequency estimates affect the PLL steady-state performance