L-filter Design

For a wye-connected load, with a floating neutral point, in Fig. 0.1 the neutral point voltage can be calculated as (2.1). Hence, the voltage across phase ‘a’ can be calculated as (2.2). The maximum voltage across phase ‘a’ is (2.3).

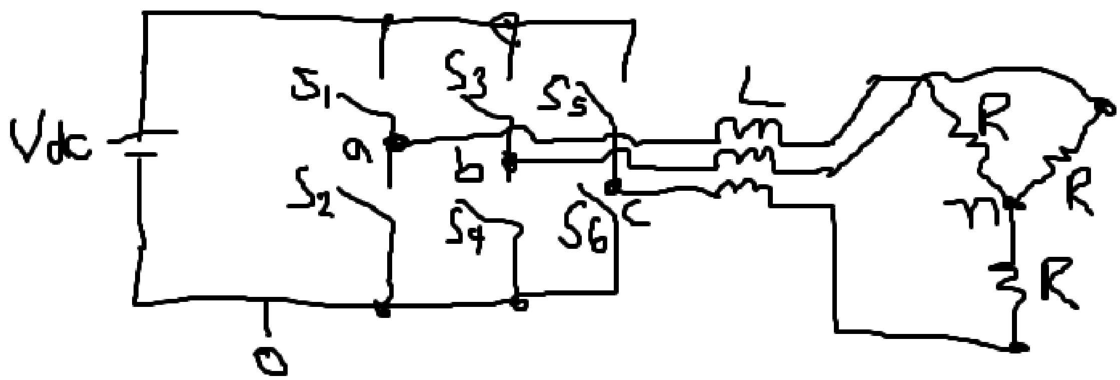


Fig. 0.1: Three-phase inverter with a wye-connected three-phase load.

$$v_n = \frac{1}{3}(v_{ao} + v_{bo} + v_{co}) \tag{2.1}$$

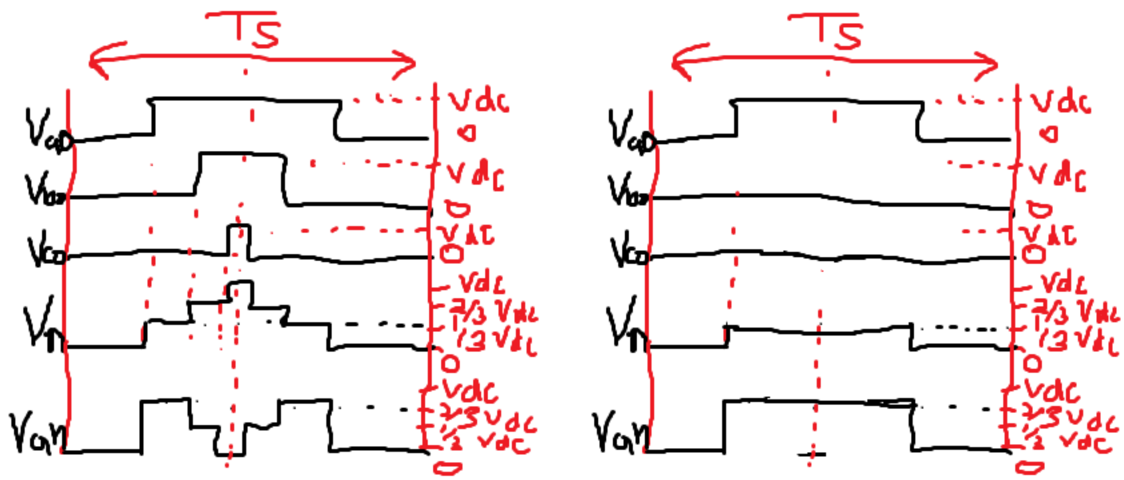
$$v_{an} = v_{ao} - \frac{1}{3}(v_{ao} + v_{bo} + v_{co}) = \frac{2}{3}v_{ao} - \frac{1}{3}v_{bo} - \frac{1}{3}v_{co} = v_{dc}\left(\frac{2}{3}s_a - \frac{1}{3}s_b - \frac{1}{3}s_c\right) \tag{2.2}$$

$$v_{an} = v_{dc}\left(\frac{2}{3}(1) - \frac{1}{3}(0) - \frac{1}{3}(0)\right) = \frac{2}{3}v_{dc} \tag{2.3}$$

The PWM switching waveform for an example voltage vector can be seen in Fig. 0.2(a). From this diagram, we can that the effective switching frequency for the voltage across phase ‘a’, is actual twice the switching frequency (2.4). We can model the phase ‘a’ circuit as Fig. 0.3 from looking at Fig. 0.1. If we perform superposition on the circuit in Fig. 0.3 and look at just the dc component (or square wave) voltages, then the voltage across the inductor will be zero, (as an inductor is represented as a short-circuit for dc circuits) as shown in Fig. 0.4 for the V_a maximum voltage vector. Therefore, the equation for voltage across the resistance will be (2.5) when using the V_a maximum voltage vector. We can now model the effective phase ‘a’ circuit as Fig. 0.5 when using the V_a maximum voltage vector.

$$f_{eff} = 2f_{sw} \tag{2.4}$$

$$V_R = \frac{2}{3}v_{dc} \times \frac{T_{on}}{T_{eff}} \tag{2.5}$$



(a) (b)

Fig. 0.2: (a) PWM switching waveform for an example voltage vector. (b) PWM switching waveform with a maximum voltage vector for phase ‘a’ (Sa = 1, Sb = 0, Sc = 0).

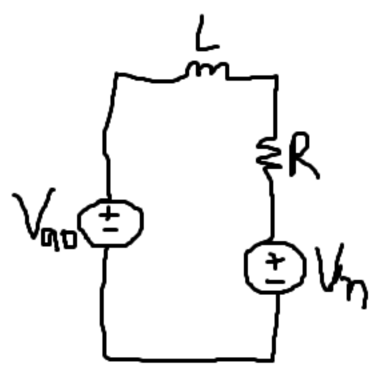


Fig. 0.3: Effective circuit for phase ‘a’.

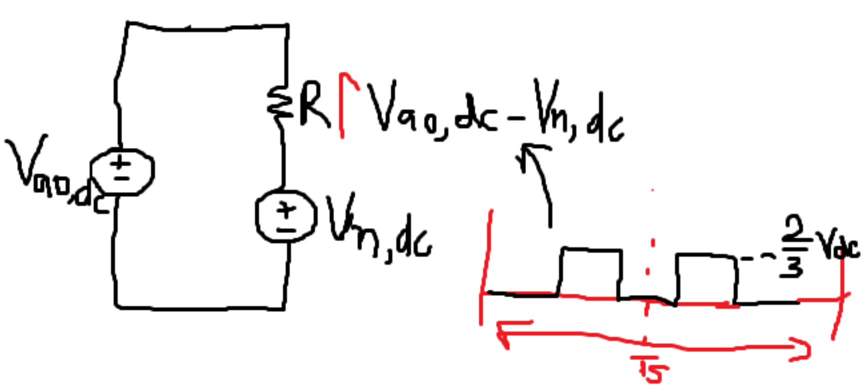


Fig. 0.4: Superposition effective circuit for phase ‘a’ looking at just the dc component (or square wave) voltages when using V_a maximum voltage vector (1,0,0).

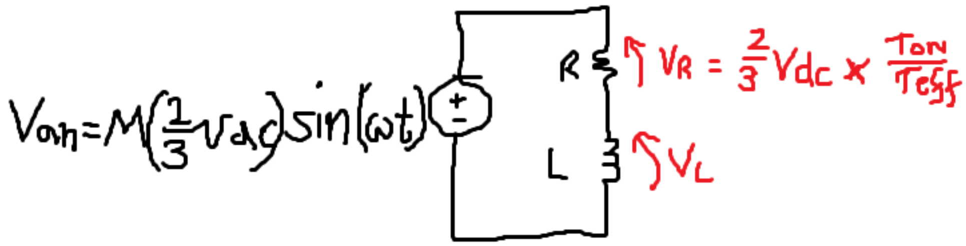


Fig. 0.5: Effective phase ‘a’ circuit when using the maximum V_a voltage vector (1,0,0).

From Fig. 0.5, the voltage across the inductor during the T_{on} period is (2.6), where M is the maximum modulation index for the average voltage across the RL load. We can rearrange (2.5) to solve for the maximum current ripple to get (2.6). We can also obtain an equation for V_{an} when using the maximum V_a voltage vector (1,0,0) in (2.7). If we rearrange this, then we can obtain (2.8). Lastly, we can substitute (2.8) into (2.6) to get (2.9). We can then perform then derivative of (2.10) to obtain the local maxima, which are calculated as (2.11). Therefore, to calculate the maximum current ripple we should use (2.11).

$$V_{L,\text{on}} = V_{an,\text{on}} - V_{R,\text{on}} = \frac{2}{3} v_{dc} M \sin(\omega t) - \frac{2}{3} v_{dc} = L\frac{di}{dt} = L\frac{\Delta i_{Lpp}}{T_{on}} \tag{2.5}$$

$$\Delta i_{Lpp} = \frac{\frac{2}{3} v_{dc} T_{\text{on}} (1 - M\,sin(\omega t))}{L} \tag{2.6}$$

$$V_{an} = \frac{2}{3} v_{dc} M \sin(\omega t) = \frac{2}{3} v_{dc} \frac{T_{\text{on}}}{T_{\text{eff}}} \tag{2.7}$$

$$T_{\text{on}} = \frac{M \sin(\omega t)}{f_{\text{eff}}} = \frac{M \sin(\omega t)}{2 f_{\text{sw}}} \tag{2.8}$$

$$\Delta i_{Lpp} = \frac{\frac{2}{3} v_{dc} M \sin(\omega t)(1 - M\,sin(\omega t))}{2 f_{\text{sw}} L} \tag{2.9}$$

$$\frac{d\Delta i_{Lpp}}{dt} = \frac{d}{dt}\left(\frac{\frac{2}{3} v_{dc} M \sin(\omega t)(1 - M\,sin(\omega t))}{2 f_{\text{sw}} L}\right) = 0 \tag{2.10}$$

$$\frac{d(\Delta i_{Lpp})}{d(\omega t)} = 0\ at \begin{cases} \omega t = sin^{-1}(\frac{1}{2M}) & 0.5 < M < 1 \\ \omega t = \frac{\pi}{2} & 0 < M < 0.5 \end{cases} \tag{2.11}$$

$$\Delta i_{Lpp,\text{max}} = \begin{cases} \frac{\frac{2}{3} v_{dc}}{8 f_{\text{sw}} L} \quad @\ \omega t = \sin^{-1}\left(\frac{1}{2M}\right) & 0.5 < M < 1 \\ \frac{\frac{2}{3} v_{dc} M(1-M)}{2 f_{\text{sw}} L} \quad @\ \omega t = \frac{\pi}{2} & 0 < M < 0.5 \end{cases} \tag{2.12}$$