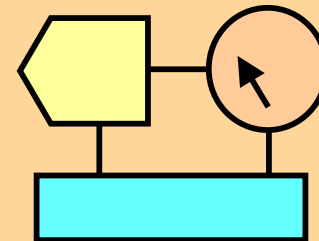




Applied Electronics

Ce – Exercises with Interconnections

- Delays and skew with RC model
- Transmission lines, reflections
- Cycle speed
 - Serial clock tolerance
 - Bypass capacitors





Ce: Exercises with Interconnections

- Delays and skews with RC models
 - ♦ Ce1, Ce2
- Transmission line models
 - ♦ Ce3, Ce4
- Time Diagrams, IWS
 - ♦ Ce5, Ce6
- Examples of exam exercises
 - ♦ Ce7, Ce8

Exercise Ce4: Time Diagrams

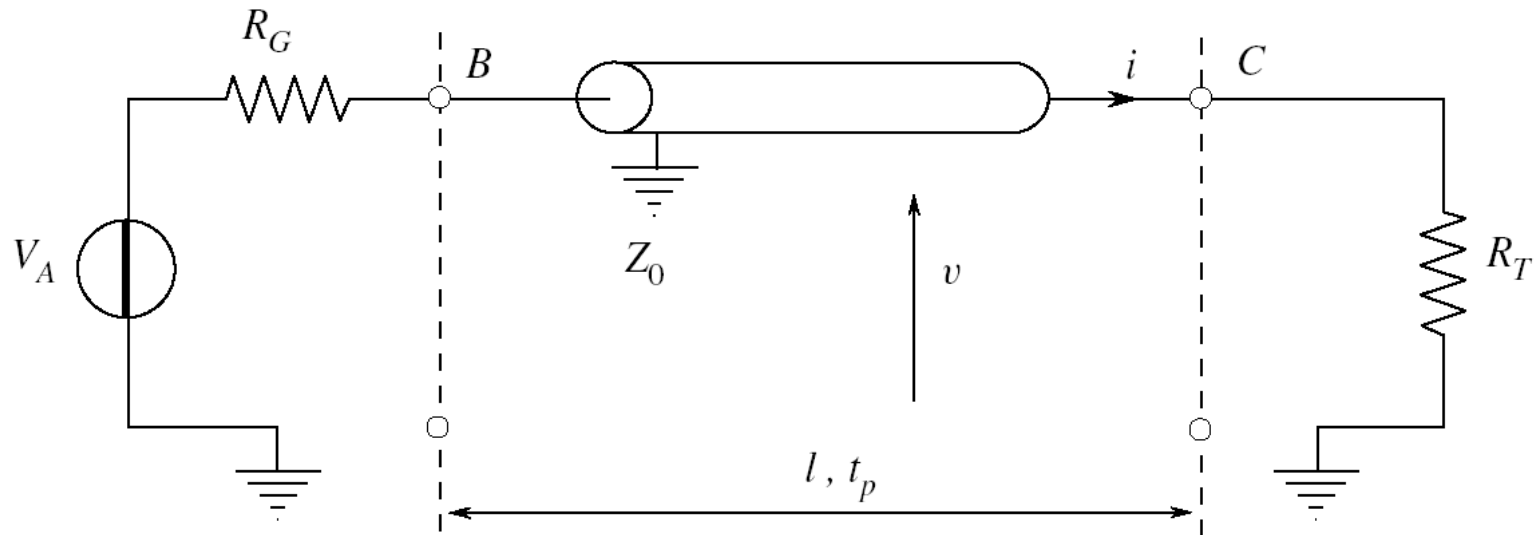
- Draw for $4 t_p$, using the lattice diagram technique, the time evolution of V_B and V_C for a V_A switch from 0 V to 5 V.

Parameters:

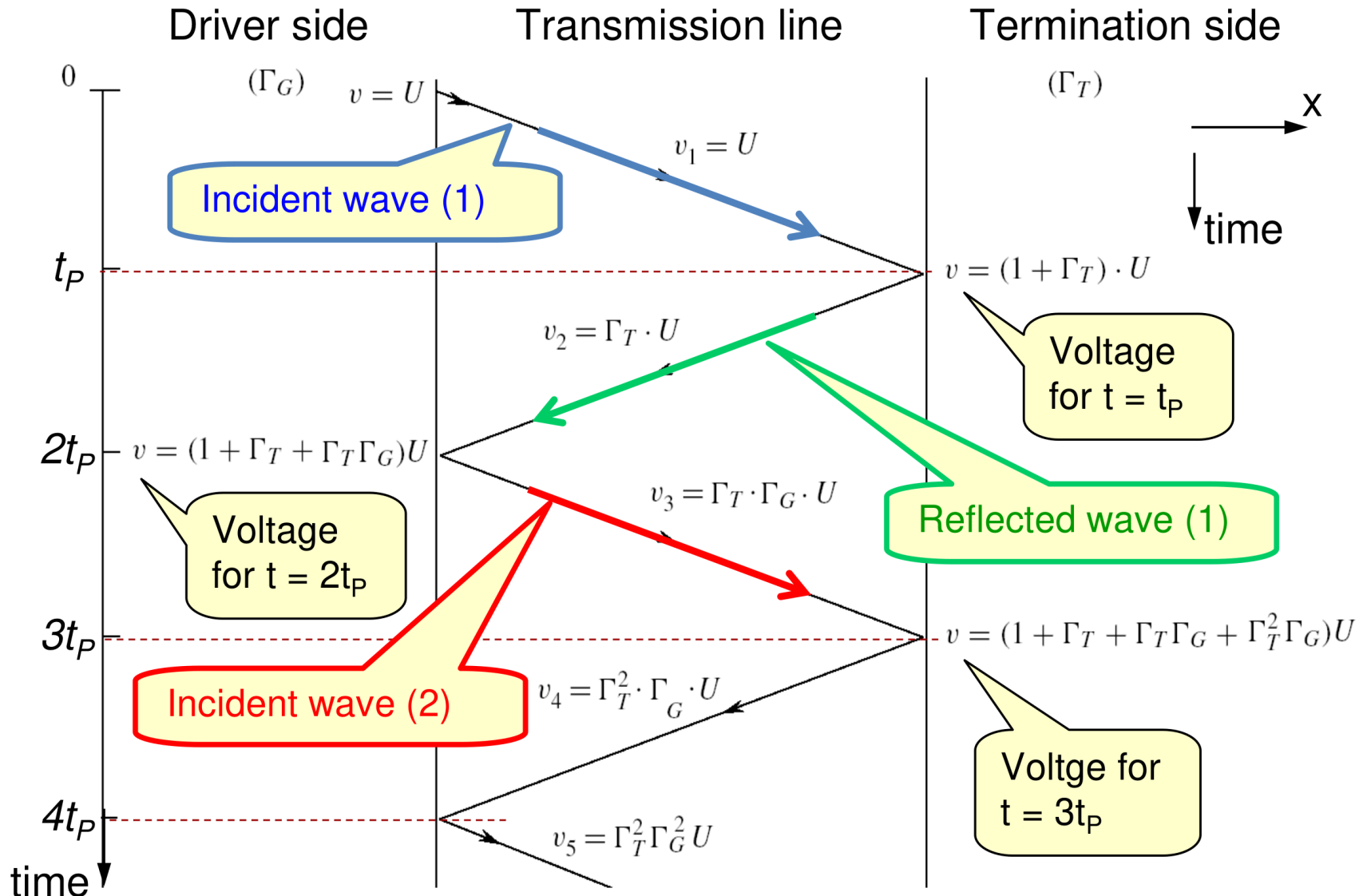
$$R_G = 50 \, \Omega, \quad R_T = \infty, \quad Z_\infty = 50 \, \Omega,$$

$$P = 0,8 \, c, \quad l = 20 \, \text{cm}$$

- Repeat the calculation with $R_G = 270 \, \Omega$ and $R_G = 15 \, \Omega$



Exercise Ce4: Time Diagrams



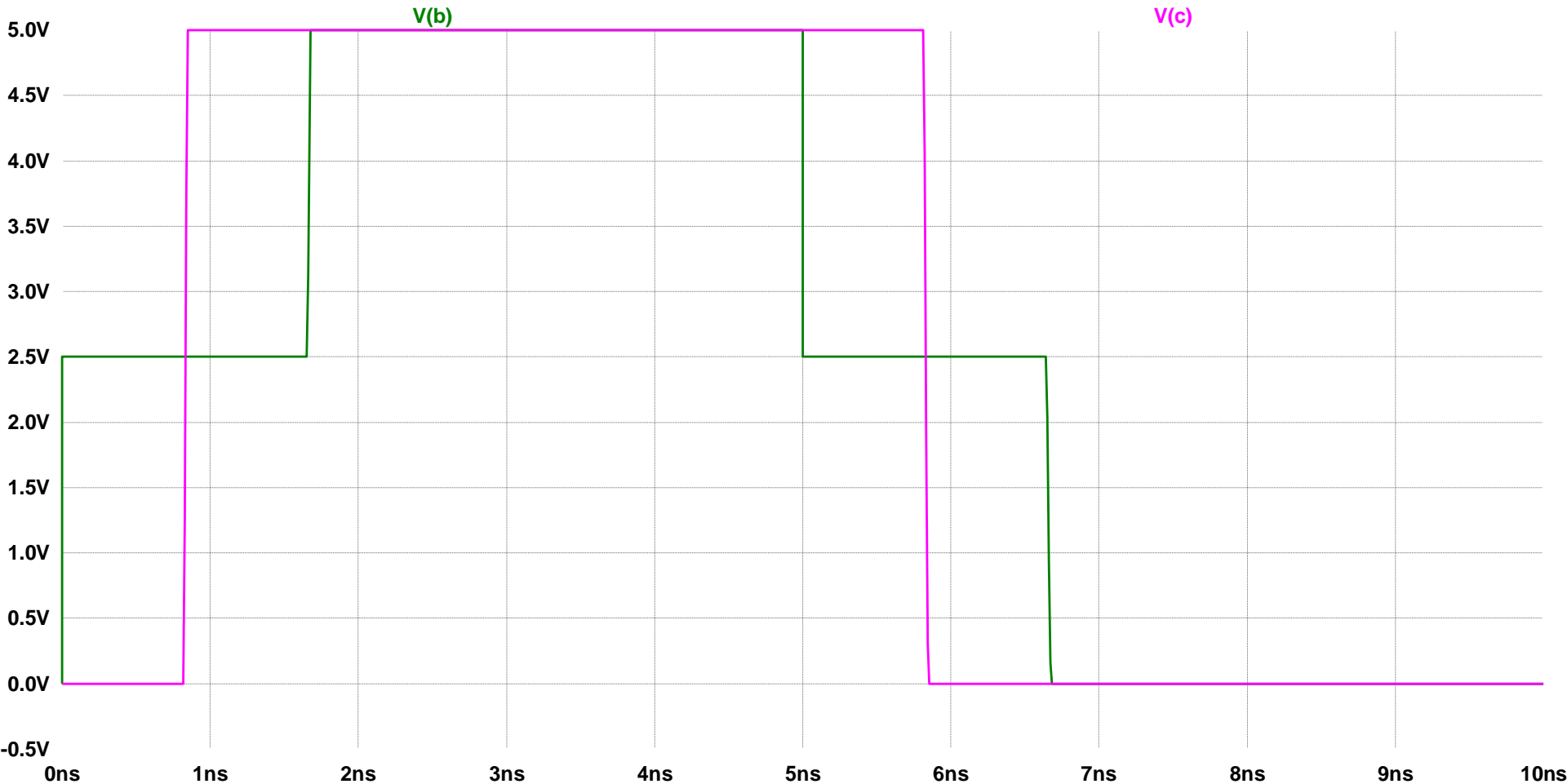


Exercise Ce4: Time Diagrams $R_G = 50 \, \Omega$

- $\Gamma_G = \frac{R_G - Z_\infty}{R_G + Z_\infty} = \frac{50 \, \Omega - 50 \, \Omega}{50 \, \Omega + 50 \, \Omega} = 0, \quad \Gamma_T = \frac{\infty - 50 \, \Omega}{\infty + 50 \, \Omega} = 1$
- $t_P = \frac{l}{P} = \frac{0.2 \, \text{m}}{0.8 \cdot 3 \cdot 10^8 \, \text{m/s}} = 0.83 \, \text{ns}$
- $V_B(0) = \frac{Z_\infty}{R_G + Z_\infty} V_A = \frac{50 \, \Omega}{50 \, \Omega + 50 \, \Omega} 5 \, \text{V} = 2.5 \, \text{V}$
- $V_C(t_P) = (1 + \Gamma_T) V_B(0) = (1 + 1) \cdot 2.5 \, \text{V} = 5 \, \text{V}$
- $V_B(2 t_P) = (1 + \Gamma_T + \Gamma_T \Gamma_G) V_B(0) =$
 $(1 + 1 + 1 \cdot 0) \cdot 2.5 \, \text{V} = 5 \, \text{V}$
- $V_C(3 t_P) = (1 + \Gamma_T + \Gamma_T \Gamma_G + \Gamma_T^2 \Gamma_G) V_B(0) =$
 $(1 + 1 + 1 \cdot 0 + 1^2 \cdot 0) \cdot 2.5 \, \text{V} = 5 \, \text{V}$
- $V_B(4 t_P) = (1 + \Gamma_T + \Gamma_T \Gamma_G + \Gamma_T^2 \Gamma_G + \Gamma_T^2 \Gamma_G^2) V_B(0) =$
 $(1 + 1 + 1 \cdot 0 + 1^2 \cdot 0 + 1^2 \cdot 0^2) \cdot 2.5 \, \text{V} = 5 \, \text{V}$



Exercise Ce4: Time Diagrams $R_G = 50 \Omega$



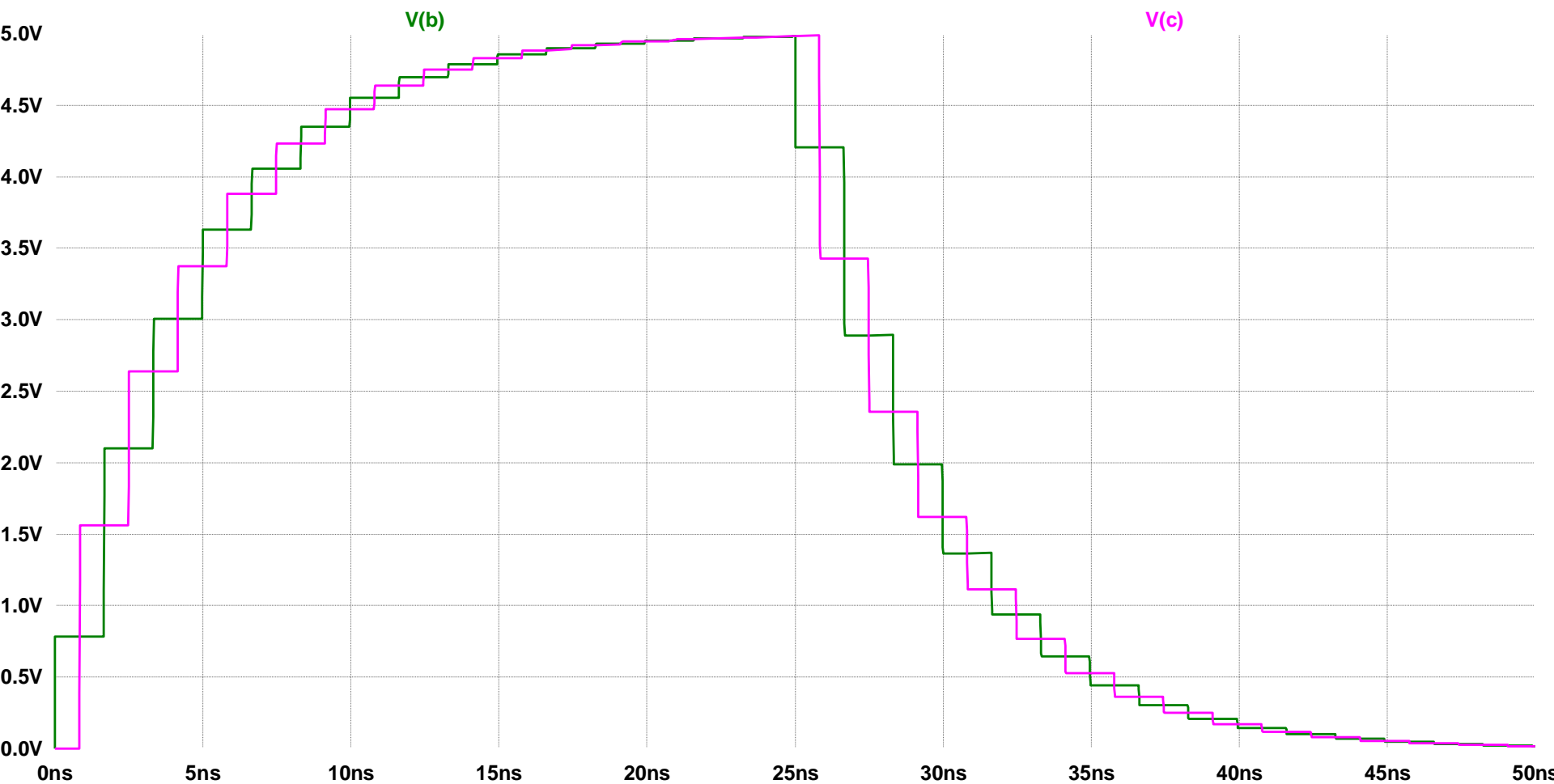


Exercise Ce4: Time Diagrams $R_G = 270 \, \Omega$

- $\Gamma_G = \frac{R_G - Z_\infty}{R_G + Z_\infty} = \frac{270 \, \Omega - 50 \, \Omega}{270 \, \Omega + 50 \, \Omega} = 0.69, \quad \Gamma_T = \frac{\infty - 50 \, \Omega}{\infty + 50 \, \Omega} = 1$
- $t_P = \frac{l}{P} = \frac{0.2 \, \text{m}}{0.8 \cdot 3 \cdot 10^8 \, \text{m/s}} = 0.83 \, \text{ns}$
- $V_B(0) = \frac{Z_\infty}{R_G + Z_\infty} V_A = \frac{50 \, \Omega}{270 \, \Omega + 50 \, \Omega} 5 \, \text{V} = 0.78 \, \text{V}$
- $V_C(t_P) = (1 + \Gamma_T) V_B(0) = (1 + 1) \cdot 0.78 \, \text{V} = 1.56 \, \text{V}$
- $V_B(2 t_P) = (1 + \Gamma_T + \Gamma_T \Gamma_G) V_B(0) =$
 $(1 + 1 + 1 \cdot 0.69) \cdot 0.78 \, \text{V} = 2.10 \, \text{V}$
- $V_C(3 t_P) = (1 + \Gamma_T + \Gamma_T \Gamma_G + \Gamma_T^2 \Gamma_G) V_B(0) =$
 $(1 + 1 + 1 \cdot 0.69 + 1^2 \cdot 0.69) \cdot 0.78 \, \text{V} = 2.64 \, \text{V}$
- $V_B(4 t_P) = (1 + \Gamma_T + \Gamma_T \Gamma_G + \Gamma_T^2 \Gamma_G + \Gamma_T^2 \Gamma_G^2) V_B(0) =$
 $(1 + 1 + 1 \cdot 0.69 + 1^2 \cdot 0.69 + 1^2 \cdot 0.69^2) \cdot 0.78 \, \text{V} = 3.01 \, \text{V}$



Exercise Ce4: Time Diagrams $R_G = 270\ \Omega$



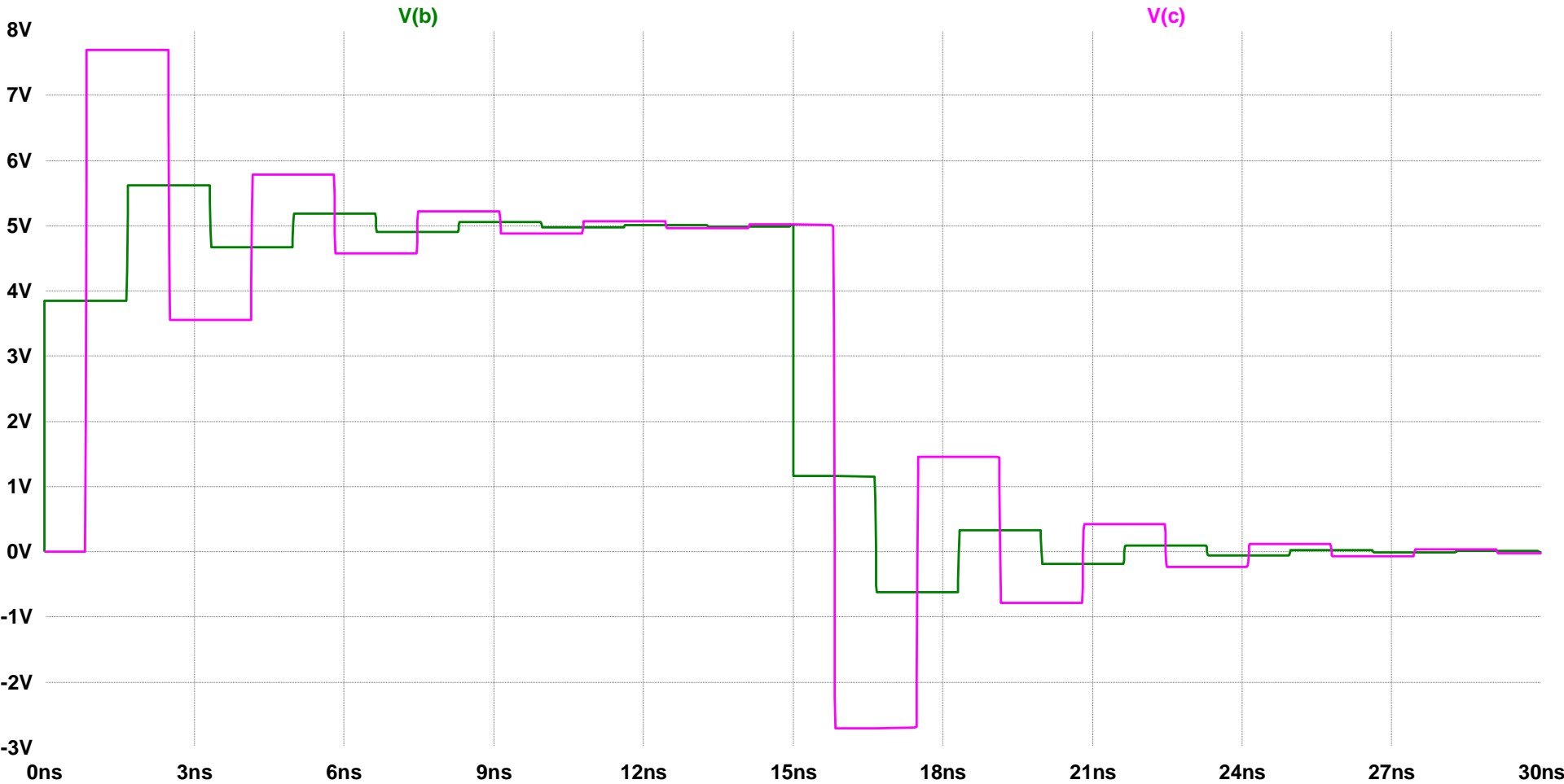


Exercise Ce4: Time Diagrams $R_G = 15 \Omega$

- $\Gamma_G = \frac{R_G - Z_\infty}{R_G + Z_\infty} = \frac{15 \Omega - 50 \Omega}{15 \Omega + 50 \Omega} = -0.54, \quad \Gamma_T = \frac{\infty - 50 \Omega}{\infty + 50 \Omega} = 1$
- $t_P = \frac{l}{P} = \frac{0.2 \text{ m}}{0.8 \cdot 3 \cdot 10^8 \text{ m/s}} = 0.83 \text{ ns}$
- $V_B(0) = \frac{Z_\infty}{R_G + Z_\infty} V_A = \frac{50 \Omega}{15 \Omega + 50 \Omega} 5 \text{ V} = 3.85 \text{ V}$
- $V_C(t_P) = (1 + \Gamma_T) V_B(0) = (1 + 1) \cdot 3.85 \text{ V} = 7.70 \text{ V}$
- $V_B(2 t_P) = (1 + \Gamma_T + \Gamma_T \Gamma_G) V_B(0) =$
 $[1 + 1 + 1 \cdot (-0.54)] \cdot 3.85 \text{ V} = 5.62 \text{ V}$
- $V_C(3 t_P) = (1 + \Gamma_T + \Gamma_T \Gamma_G + \Gamma_T^2 \Gamma_G) V_B(0) =$
 $[1 + 1 + 1 \cdot (-0.54) + 1^2 \cdot (-0.54)] \cdot 3.85 \text{ V} = 3.54 \text{ V}$
- $V_B(4 t_P) = (1 + \Gamma_T + \Gamma_T \Gamma_G + \Gamma_T^2 \Gamma_G + \Gamma_T^2 \Gamma_G^2) V_B(0) =$
 $[1 + 1 + 1 \cdot (-0.54) + 1^2 \cdot (-0.54) + 1^2 \cdot (-0.54)^2]$



Exercise Ce4: Time Diagrams $R_G = 15\ \Omega$



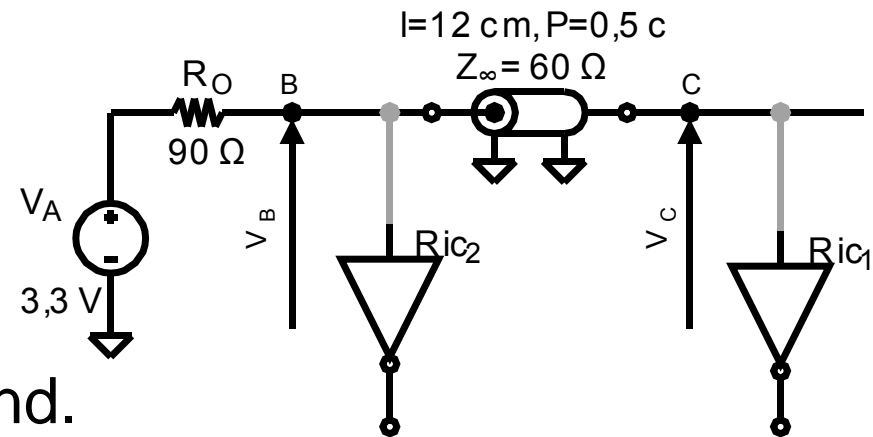
Exercise Ce7

- A driver powered at 3.3 V and with $R_0 = 90\ \Omega$ drives a connection with $Z_\infty = 60\ \Omega$, propagation speed $p = 0.5\text{ c}$, length 12 cm , open at remote end.

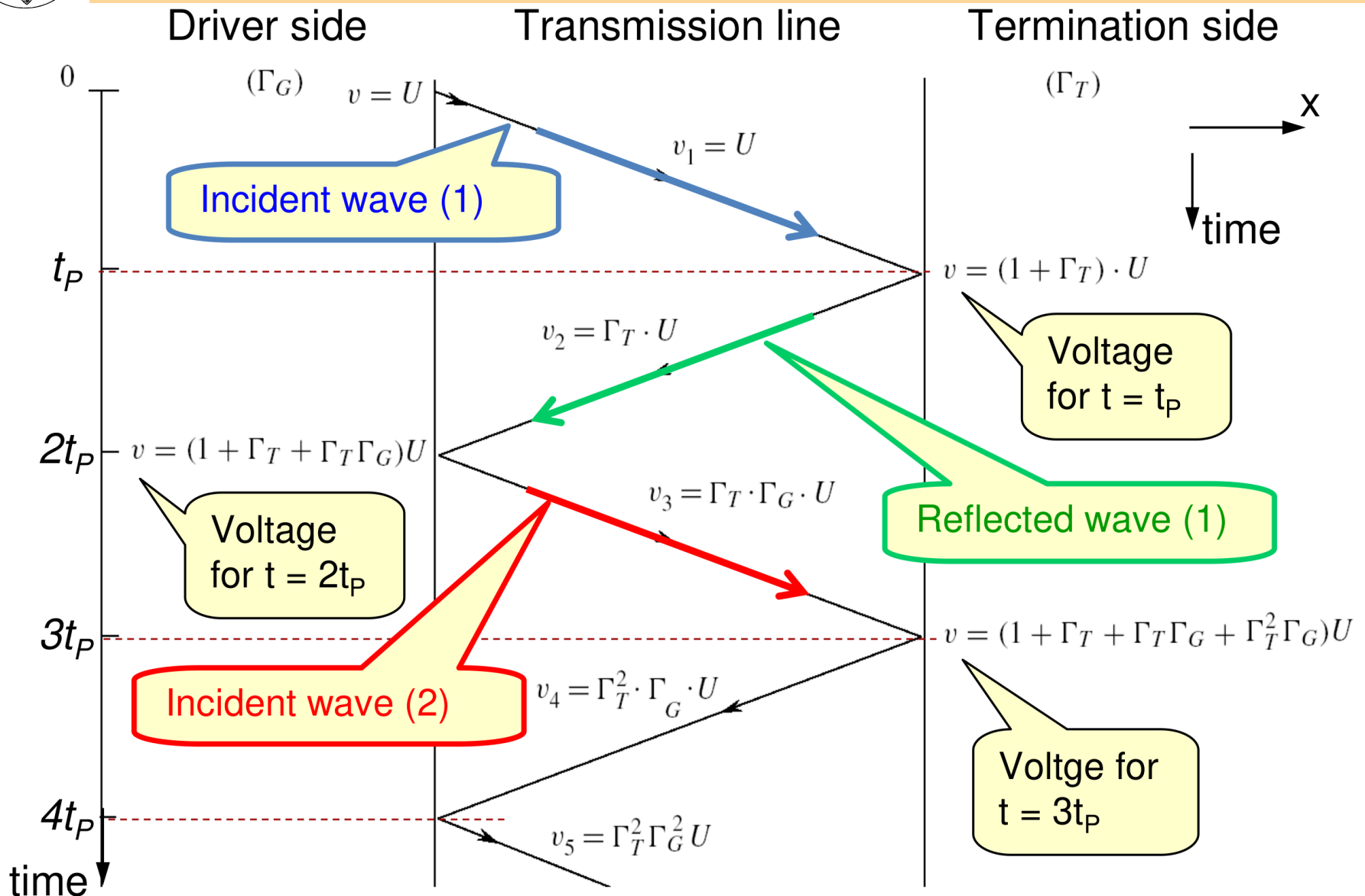
The receivers are CMOS with

$V_{IL} = 0.8\text{ V}$ and $V_{IH} = 2\text{ V}$. For the transition $L \rightarrow H$:

- Determine the amplitude of the first step and the propagation time
- Determine the minimum and maximum transmission times for receivers located on the driver side and termination side
- Qualitatively draw the waveform at the termination if the input capacitance of the receiver connected there is 10 pF .



Exercise Ce7





Exercise Ce7: a)

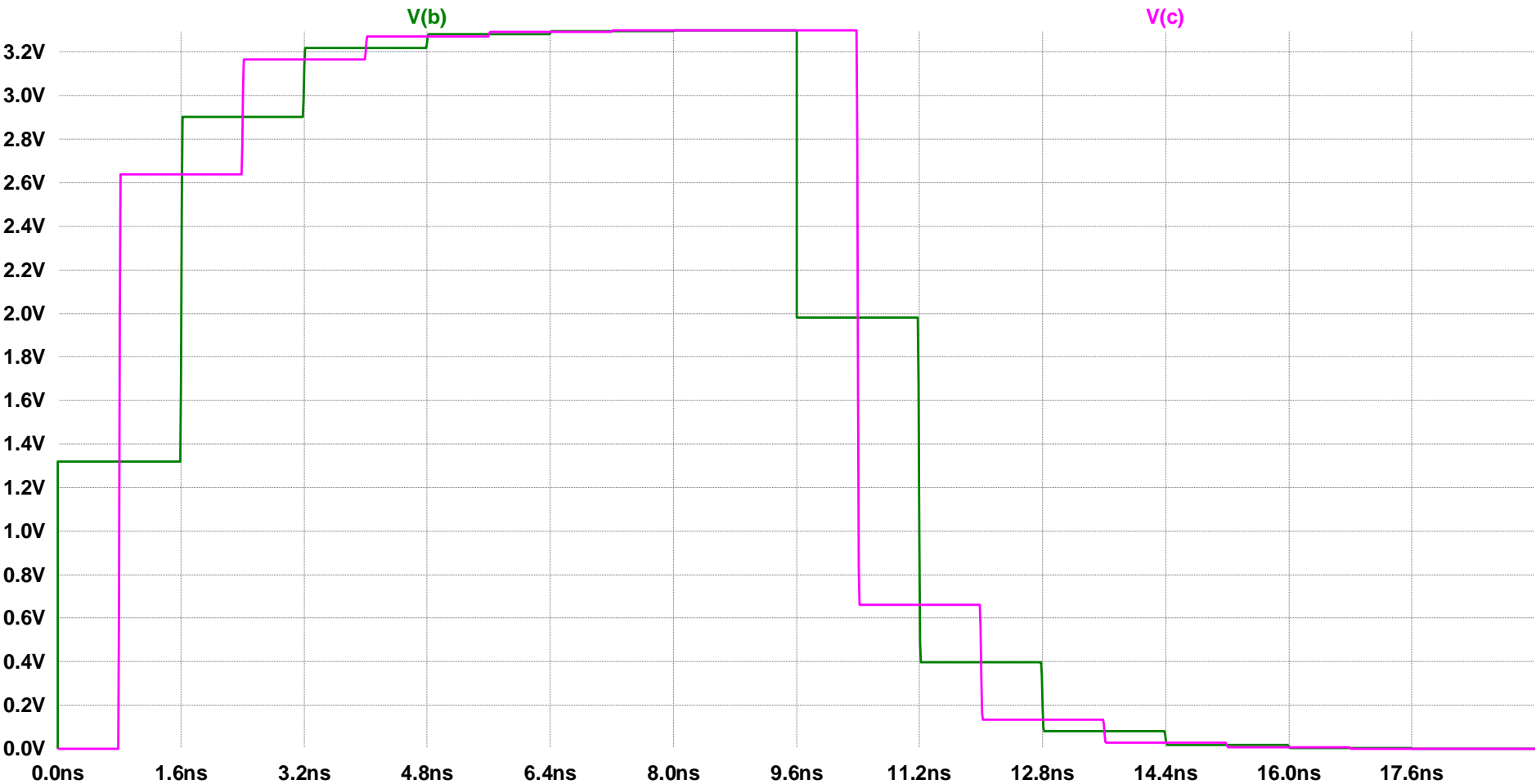
- $V_B(0) = \frac{Z_\infty}{R_D + Z_\infty} V_A = \frac{60 \, \Omega}{90 \, \Omega + 60 \, \Omega} 3.3 \, \text{V} = 1.32 \, \text{V}$
- $t_p = \frac{l}{p} = \frac{0.12 \, \text{m}}{0.5 \cdot 3 \cdot 10^8 \, \text{m/s}} = 0.8 \, \text{ns}$

Exercise Ce7: b)

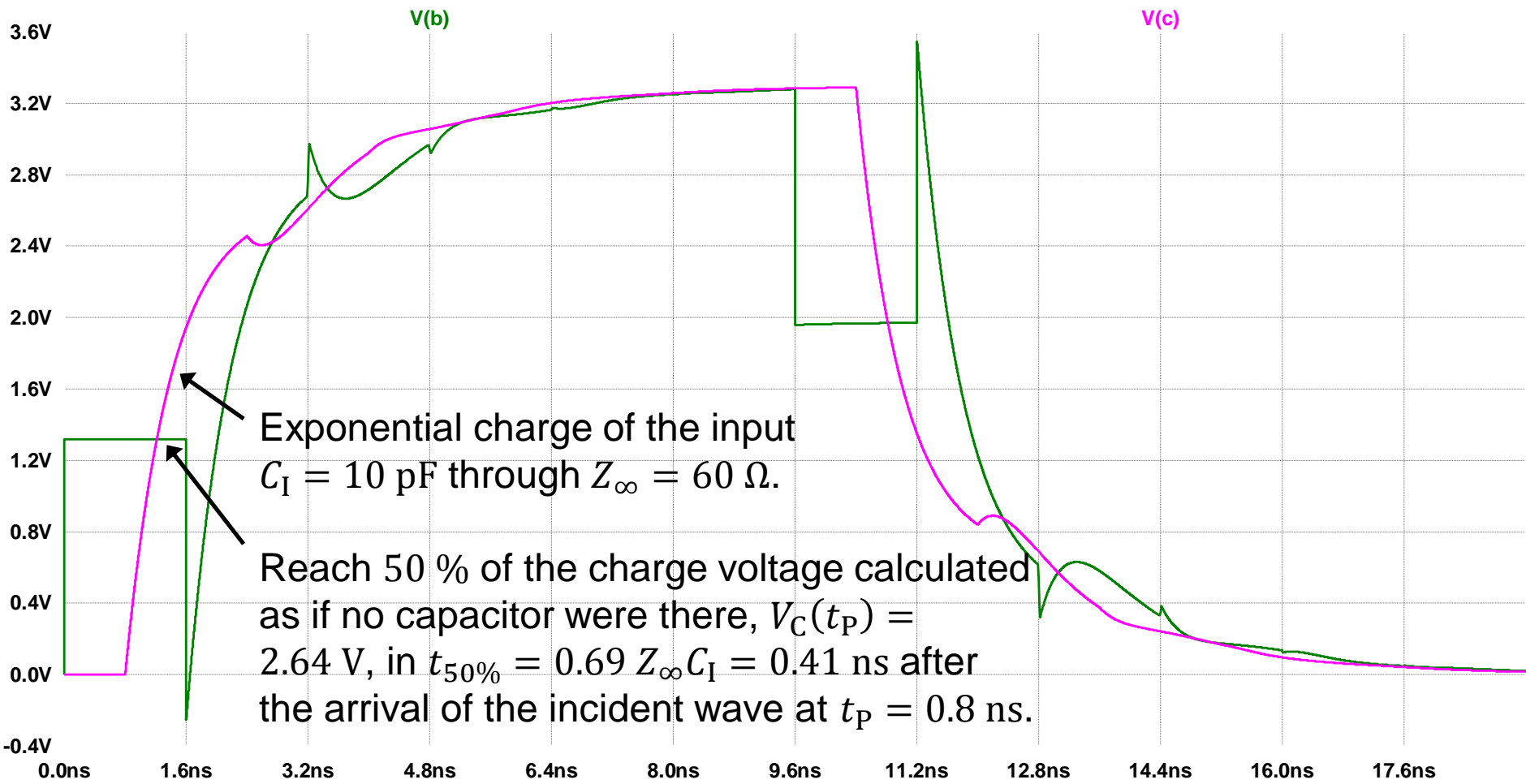
- $\Gamma_D = \frac{R_O - Z_\infty}{R_O + Z_\infty} = \frac{90 \Omega - 60 \Omega}{90 \Omega + 60 \Omega} = 0,2, \quad \Gamma_T = \frac{\infty - 60 \Omega}{\infty + 60 \Omega} = 1$
- $V_B(0) = \frac{Z_\infty}{R_D + Z_\infty} V_A = \frac{60 \Omega}{90 \Omega + 60 \Omega} 3.3 \text{ V} = 1.32 \text{ V}$
 - ♦ $V_B(0) > V_{IL} \rightarrow t_{TX_{min}}^B = 0 \text{ s}; \quad V_B(0) < V_{IH}$
- $V_B(2 t_P) = (1 + \Gamma_T + \Gamma_T \Gamma_G) V_B(0) =$
 $(1 + 1 + 1 \cdot 0.2) \cdot 1.32 \text{ V} = 2.9 \text{ V}$
 - ♦ $V_B(2 t_P) > V_{IH} \rightarrow t_{TX_{max}}^B = 2 t_P = 2 \cdot 0.8 \text{ ns} = 1.6 \text{ ns}$
- $V_C(t_P) = (1 + \Gamma_T) V_B(0) = (1 + 1) \cdot 1.32 \text{ V} = 2.64 \text{ V}$
 - ♦ $V_C(t_P) > V_{IL}$ and $V_C(t_P) > V_{IH} \rightarrow t_{TX_{min}}^C = t_{TX_{max}}^C = t_P = 0.8 \text{ ns}$



Exercise Ce7: b)



Exercise Ce7: c)



Exercise Ce8

- A backplane has $L_U = 8 \text{ nH/cm}$, $Z_\infty = 85 \Omega$ without loads, length $l = 48 \text{ cm}$, without terminations, with 24 equidistant connectors. The boards that can be inserted into the connectors have a capacitive load $C_P = 35 \text{ pF}$ each. The system can hold from 2 to 24 boards.

CMOS driver/receiver parameters:

$$V_{AL} = 3.3 \text{ V}; R_O = 95 \Omega; V_{IH} = 2 \text{ V}, V_{IL} = 1 \text{ V}.$$

- a) Calculate the propagation time t_p between the ends, with 2 and 24 boards.
- b) Determine the $t_{TX_{min}}$ between two boards in extreme positions.
- c) Calculate $t_{TX_{max}}$ with 24 boards inserted, driven at one end.
- d) Indicate driver max R_{OH} to operate in IWS for lines driven at one end, with 24 boards inserted.

Exercise Ce8: a)

- $Z_{\infty} = \sqrt{\frac{L_U}{C_U}}, C_U = \frac{L_U}{Z_{\infty}^2} = \frac{8 \cdot 10^{-9} \text{ nH/cm}}{85^2 \Omega^2} = 1.11 \text{ pF/cm}$
- $N = 2$ mounted boards (we assume distributed C_P)
 - ♦ $C_{U_2} = \frac{C_U l + N C_P}{l} = C_U + \frac{N C_P}{l} = 1.11 \text{ pF/cm} + \frac{2 \cdot 35 \text{ pF}}{48 \text{ cm}} = 2.57 \text{ pF/cm}$
 - ♦ $P_2 = \frac{1}{\sqrt{L_U C_U}} = \frac{1}{\sqrt{8 \text{ nH/cm} \cdot 2.57 \text{ pF/cm}}} = 6.97 \cdot 10^9 \text{ cm/s}$
 - ♦ $t_{P_2} = \frac{l}{P} = \frac{48 \text{ cm}}{6.97 \cdot 10^9 \text{ cm/s}} = \mathbf{6.89 \text{ ns}}$
- $N = 24$ mounted boards (we assume distributed C_P)
 - ♦ $C_{U_{24}} = \frac{C_U l + N C_P}{l} = C_U + \frac{N C_P}{l} = 1.11 \text{ pF/cm} + \frac{24 \cdot 35 \text{ pF}}{48 \text{ cm}} = 18.61 \text{ pF/cm}$
 - ♦ $P_{24} = \frac{1}{\sqrt{L_U C_U}} = \frac{1}{\sqrt{8 \text{ nH/cm} \cdot 18.61 \text{ pF/cm}}} = 2.59 \cdot 10^9 \text{ cm/s}$
 - ♦ $t_{P_{24}} = \frac{l}{P} = \frac{48 \text{ cm}}{2.59 \cdot 10^9 \text{ cm/s}} = 18.53 \text{ ns}$



Exercise Ce8: b)

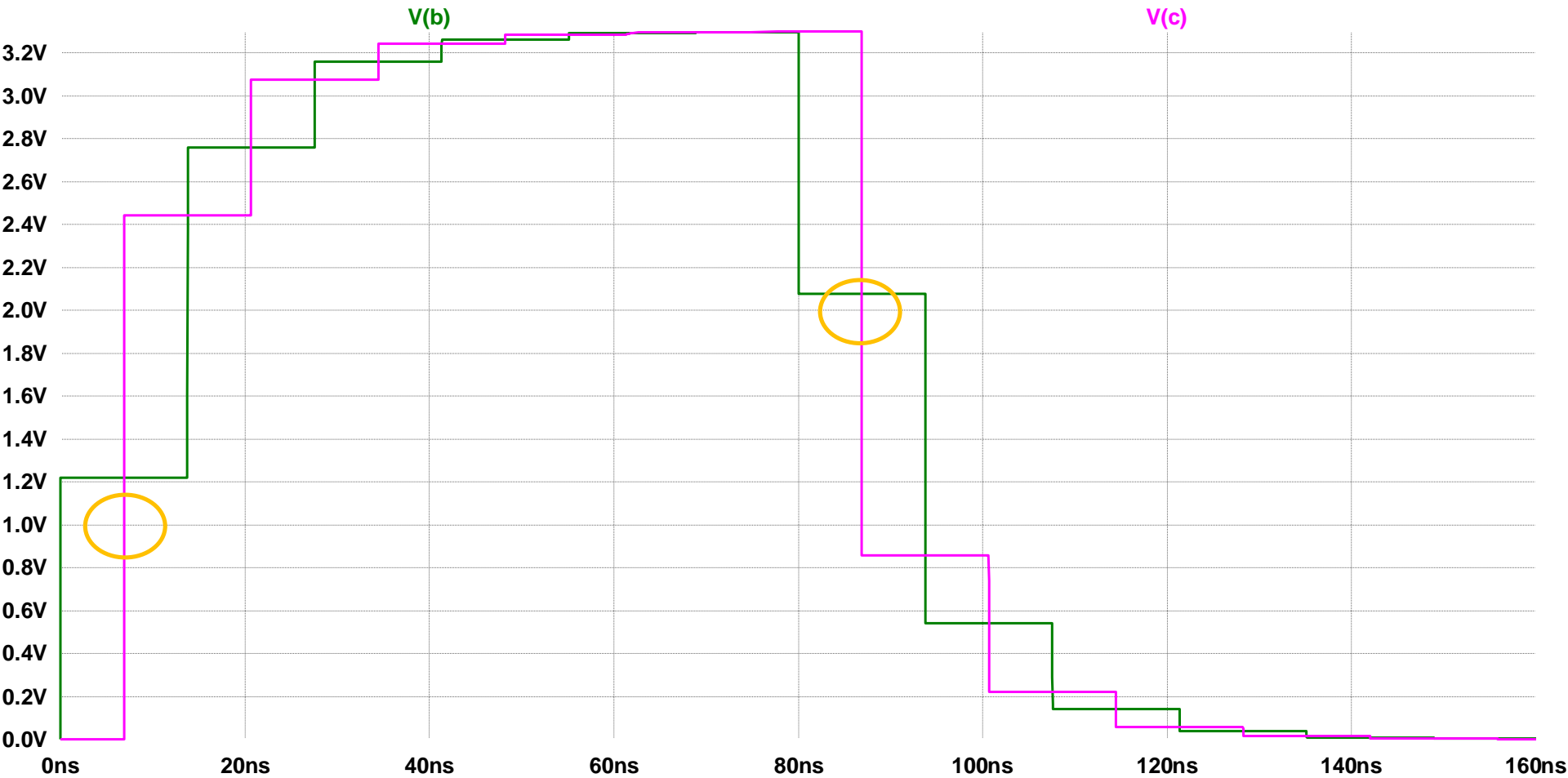
- Transmission time t_{TX} is a multiple of the propagation time, t_P
- From point a), t_P is minimum for $N = 2$ mounted boards
 - ♦ $t_{P_2} = 6.89 \text{ ns}$
 - ♦ $Z_{\infty_2} = \sqrt{\frac{L_U}{C_{U_2}}} = \sqrt{\frac{8 \text{ nH/cm}}{2.57 \text{ pF/cm}}} = 55.8 \Omega$
 - ♦ $\Gamma_{G_2} = \frac{R_0 - Z_{\infty_2}}{R_0 + Z_{\infty_2}} = \frac{95 \Omega - 55.8 \Omega}{95 \Omega + 55.8 \Omega} = 0.26$
 - ♦ $\Gamma_T = 1$ (open termination)

Exercise Ce8: b)

- Two boards at ends B and C
 - ◆ Driver board at the end B
- $V_{B_2}(0) = \frac{Z_{\infty_2}}{R_G + Z_{\infty_2}} V_{AL} = \frac{55.8 \Omega}{95 \Omega + 55.8 \Omega} 3.3 \text{ V} = 1.22 \text{ V}$
- $t_{TX_{min}}^{L \rightarrow H}$ for transition $L \rightarrow H$ to match the minimum threshold, V_{IL}
 - ◆ $V_{B_2}(0) = 1.22 \text{ V}$
 - ◆ $V_{C_2}(t_{P_2}) = (1 + \Gamma_T)V_{B_2}(0) = (1 + 1) \cdot 1.22 \text{ V} = 2.44 \text{ V} > V_{IL}$
- $t_{TX_{min}}^{H \rightarrow L}$ for transition $H \rightarrow L$ to match the maximum, V_{IH}
 - ◆ $V_{B_2}(0) = V_{AL} - \frac{Z_{\infty_2}}{R_G + Z_{\infty_2}} V_{AL} = 3.3 \text{ V} - 1.22 \text{ V} = 2.08 \text{ V}$
 - ◆ $V_{C_2}(t_{P_2}) = (1 + \Gamma_T)[-V_{B_2}(0)] + V_{AL} = 0.86 \text{ V} < V_{IH}$
- $t_{TX_{min}} = t_{P_2} = 6.89 \text{ ns}$



Exercise Ce8: b)





Exercise Ce8: c)

- Transmission time t_{TX} is a multiple of the propagation time, t_P
- From the point a), t_P is maximum for $N = 24$ mounted boards
 - ♦ $t_{P_{24}} = 18.53 \text{ ns}$
 - ♦ $Z_{\infty_{24}} = \sqrt{\frac{L_U}{C_{U_{24}}}} = \sqrt{\frac{8 \text{ nH/cm}}{18.61 \text{ pF/cm}}} = 20.7 \Omega$
 - ♦ $\Gamma_{G_{24}} = \frac{R_O - Z_{\infty_{24}}}{R_O + Z_{\infty_{24}}} = \frac{95 \Omega - 20.7 \Omega}{95 \Omega + 20.7 \Omega} = 0.64$
 - ♦ $\Gamma_T = 1$ (open termination)

Exercise Ce8: c)

- Boards driven from the B end
- The most distant board is mounted at the C end
- $V_{B_{24}}^{L \rightarrow H}(0) = \frac{Z_{\infty_{24}}}{R_{G_{24}} + Z_{\infty_{24}}} V_{AL} = \frac{20.7 \, \Omega}{95 \, \Omega + 20.7 \, \Omega} \cdot 3.3 \, V = 0.59 \, V$
- $t_{TX_{max}}^{L \rightarrow H}$ for $L \rightarrow H$ transition
 - ◆ Comparison with the maximum threshold, $V_{TH} = V_{IH}$
 - ◆ $V_{C_{24}}^{L \rightarrow H}(t_P) = (1 + \Gamma_T) V_{B_{24}}^{L \rightarrow H}(0) = (1 + 1) \cdot 0.59 \, V = 1.18 \, V < V_{IH}$
 - ◆ $V_{C_{24}}^{L \rightarrow H}(3 t_P) = (1 + \Gamma_T + \Gamma_T \Gamma_{G_{24}} + \Gamma_T^2 \Gamma_{G_{24}}) V_{B_{24}}^{L \rightarrow H}(0) =$
 $[1 + 1 + 1 \cdot 0.64 + 1^2 \cdot 0.64] \cdot 0.59 \, V = 1.94 \, V < V_{IH}$
 - ◆ $V_{C_{24}}^{L \rightarrow H}(5 t_P) = (1 + \Gamma_T + \Gamma_T \Gamma_{G_{24}} + \Gamma_T^2 \Gamma_{G_{24}} + \Gamma_T^2 \Gamma_{G_{24}}^2 + \Gamma_T^3 \Gamma_{G_{24}}^2) V_{B_{24}}^{L \rightarrow H}(0) =$
 $[1 + 1 + 1 \cdot 0.64 + 1^2 \cdot 0.64 + 1^2 \cdot 0.64^2 + 1^3 \cdot 0.64^2] \cdot 0.59 \, V =$
 $2.42 \, V > V_{IH}$

Exercise Ce8: c)

- $t_{TX_{max}}^{H \rightarrow L}$ for $H \rightarrow L$ transitions

- ◆ Comparison with the minimum threshold, $V_{TH} = V_{IL}$

- ◆ $V_{C_{24}}^{H \rightarrow L}(t_P) = (1 + \Gamma_T)[-V_{B_{24}}^{L \rightarrow H}(0)] + V_{AL} =$
 $2 \cdot (-0.59 \text{ V}) + 3.3 \text{ V} = 2.12 \text{ V} > V_{IL}$

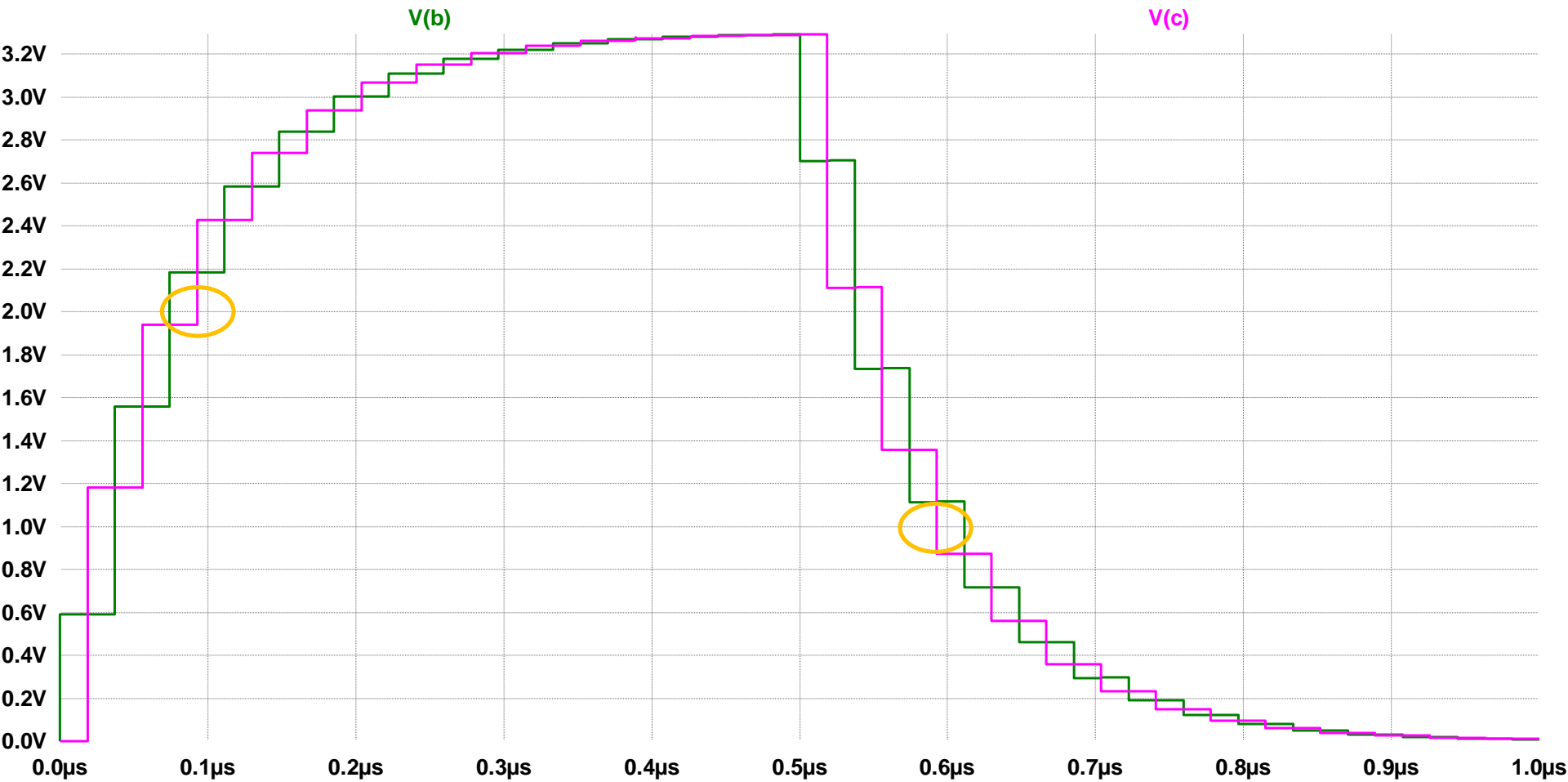
- ◆ $V_{C_{24}}^{H \rightarrow L}(3 t_P) = (1 + \Gamma_T + \Gamma_T \Gamma_{G_{24}} + \Gamma_T^2 \Gamma_{G_{24}})[-V_{B_{24}}^{L \rightarrow H}(0)] + V_{AL} =$
 $[1 + 1 + 1 \cdot 0.64 + 1^2 \cdot 0.64] \cdot (-0.59 \text{ V}) + 3.3 \text{ V} = 1.36 \text{ V} > V_{IL}$

- ◆ $V_{C_{24}}^{H \rightarrow L}(5 t_P) = (1 + \Gamma_T + \Gamma_T \Gamma_{G_{24}} + \Gamma_T^2 \Gamma_{G_{24}} + \Gamma_T^2 \Gamma_{G_{24}}^2 + \Gamma_T^3 \Gamma_{G_{24}}^2)[-V_{B_{24}}^{L \rightarrow H}(0)] +$
 $V_{AL} =$
 $[1 + 1 + 1 \cdot 0.64 + 1^2 \cdot 0.64 + 1^2 \cdot 0.64^2 + 1^3 \cdot 0.64^2] \cdot (-0.59 \text{ V}) + 3.3 \text{ V}$
 $= 0.88 \text{ V} < V_{IL}$

- $t_{TX_{max}}^{H \rightarrow L} = 5 t_P = 5 \cdot 18.53 \text{ ns} = 92.7 \text{ ns}$



Exercise Ce8: c)



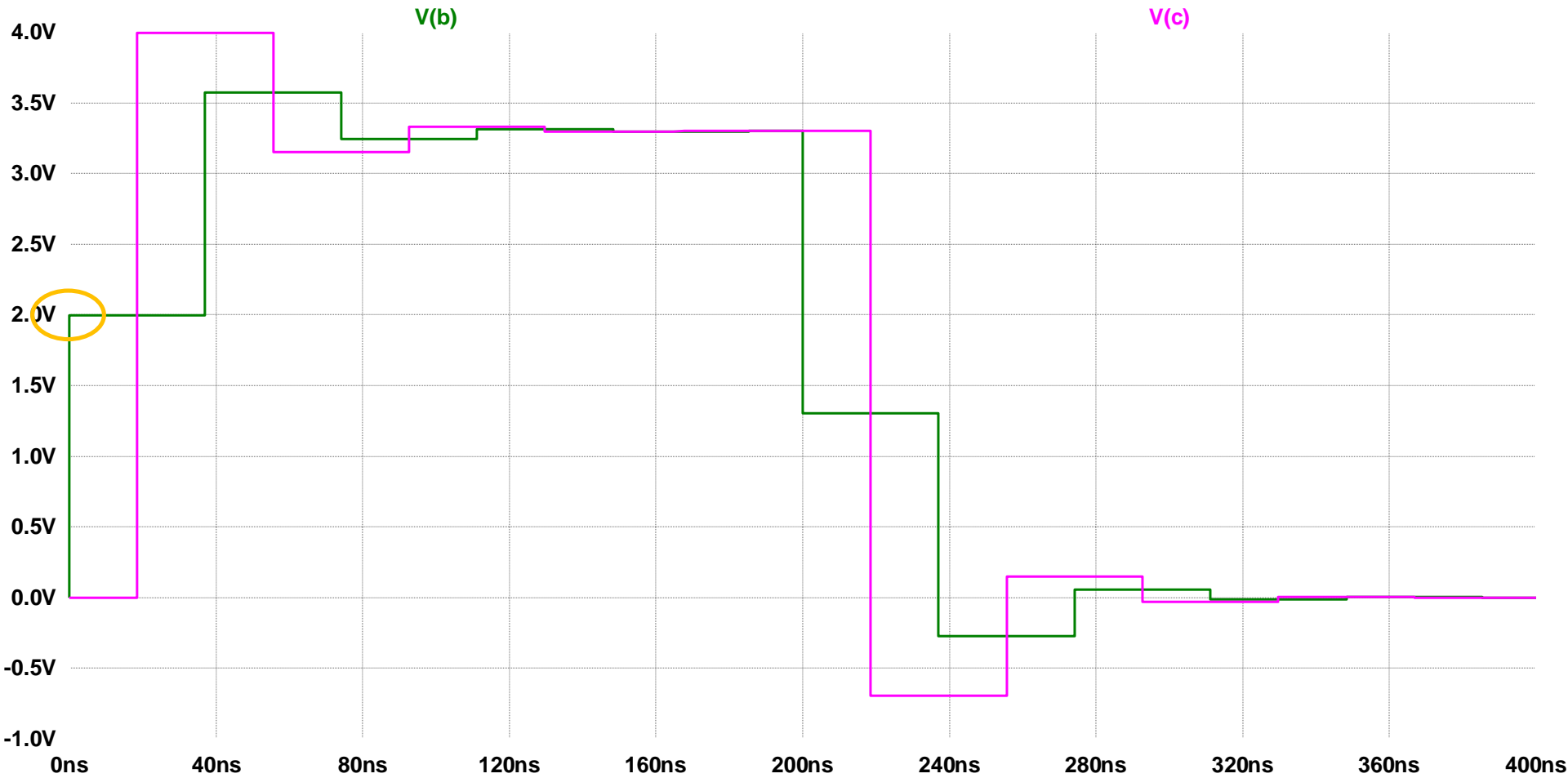


Exercise Ce8: d)

- With 24 boards mounted we know from c)
 - ♦ $Z_{\infty_{24}} = 20.7 \Omega$
- It is required R_{OH} , which applies to the transition $L \rightarrow H$
 - ♦ $V_{B_{24}}^{L \rightarrow H}(0) = \frac{Z_{\infty_{24}}}{R_{OH} + Z_{\infty_{24}}} V_{AL} \geq V_{IH}$ for IWS
 - ♦ $R_{OH} \leq Z_{\infty_{24}} \left(\frac{V_{AL}}{V_{IH}} - 1 \right) = 20.7 \Omega \cdot \left(\frac{3.3 \text{ V}}{2 \text{ V}} - 1 \right) \Rightarrow R_{OH} \leq 13.5 \Omega$



Exercise Ce8: d)





Summary of exercises Ce

- Calculation of delays (transmission time, skew) with RC model.
- Calculation of delays (propagation time, transmission, skew) with line model.
- Behavior under different termination conditions.
- Overall timing and worst-case analysis for synchronous and asynchronous cycles, depending on:
 - ◆ Transmission system parameters (RC or lines)
 - ◆ Component parameters (driver, receiver)
 - ◆ Termination conditions