Applied Electronics

C2 – Transmission Line Models

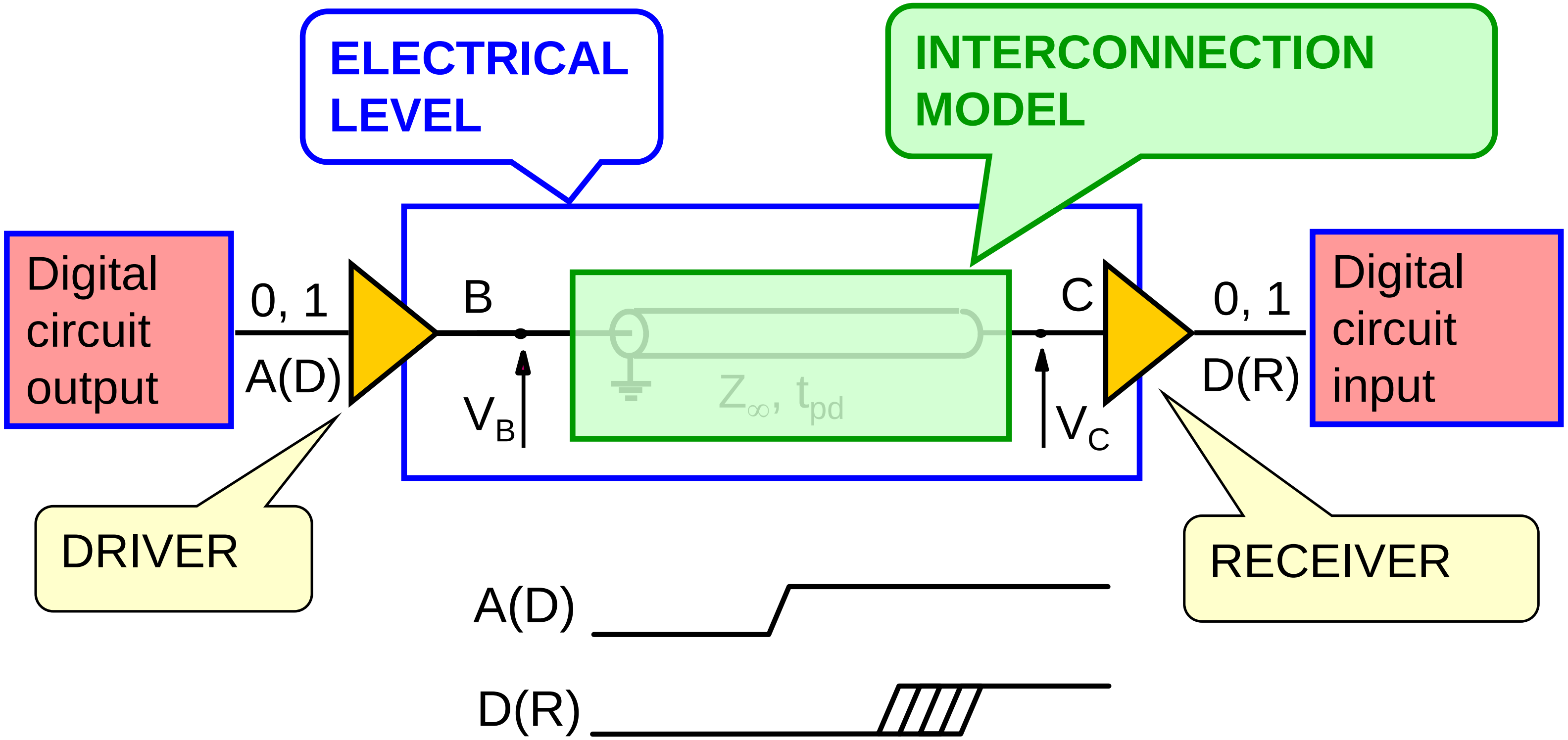
- Transmission lines
- Propagation and reflections
- Evaluation transmission time and skew

Lecture C2: Transmission Line Models

- Transmission lines
- Propagation of digital signals
- Reflections
- Driving conditions
- Transmission time, t_{TX}, and skew, t_K

- References
  - D. Del Corso: Interconnections for high-speed… lectures 1, 2

Interconnection system



From B to C (voltages): additive noise, variable delay, ...

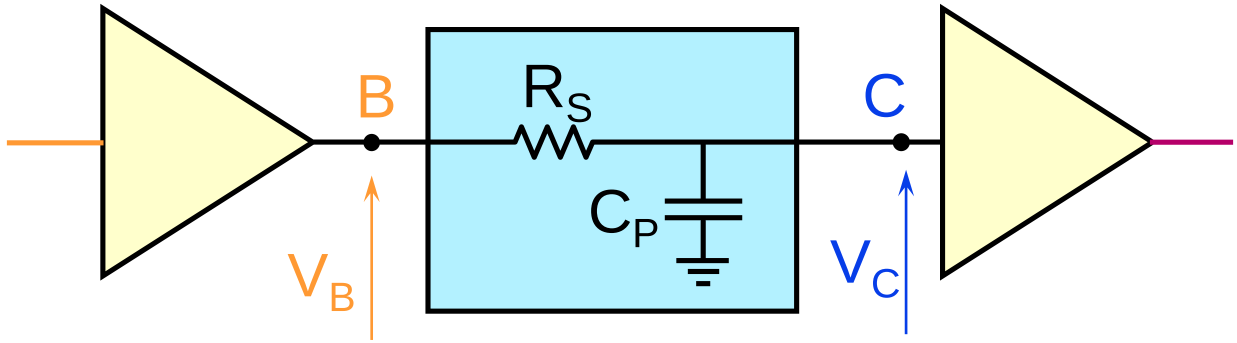
Effects from A to D (logical variables): delay, delay variations (skew).

Recall the parameters

- Transmission time, t_{TX}
  - Delay after which is detected the change in the logic state
- With an RC model, t_{TX} depends on
  - Driver start and end levels (V_H, V_L)
  - Receiver threshold, V_T, which is between V_{IH} and V_{IL}
  - Driver output resistance (R_O)
  - Equivalent receiver input capacitance (C_I)
- Skew time, t_K
  - Variations of t_{TX}
  - Linked to the variations of the parameters that determine t_{TX}

RC model of interconnection

- Interconnection modeled with an RC cell
  - Same models for drivers and receivers, with R_O, R_I, C_I
  - Interconnection with R series, R_S, and C parallel, C_P



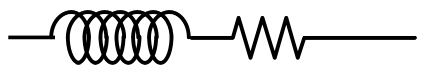
- Behavior
  - First order low pass cell
  - Exponential step response
- Objective: determine the evolution in time of V_B and V_C

Lumped parameter models

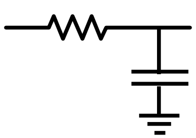
- DC analysis: equipotential conductors

V1 = V2, but….

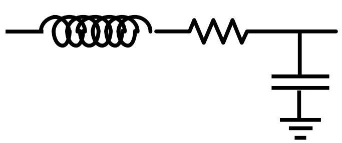
- Real conductors have resistance and inductance



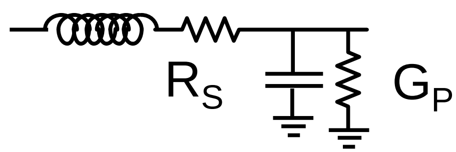
- There are parasitic capacitances and leakage resistances towards ground



R-C



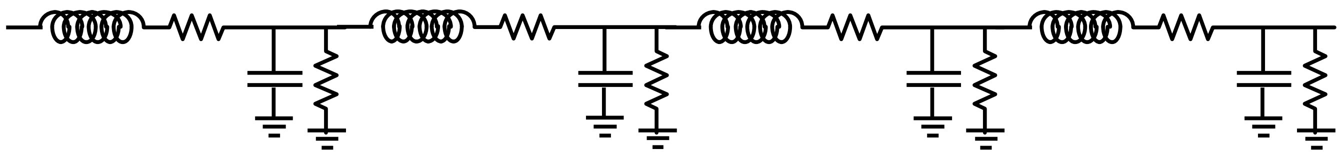
R-L-C



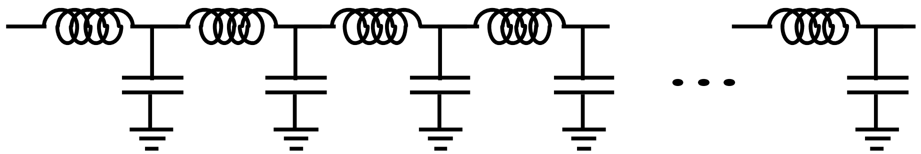
G_P-R_S-L-C

Other lumped parameter models

- R, L, and C are distributed along the entire conductor



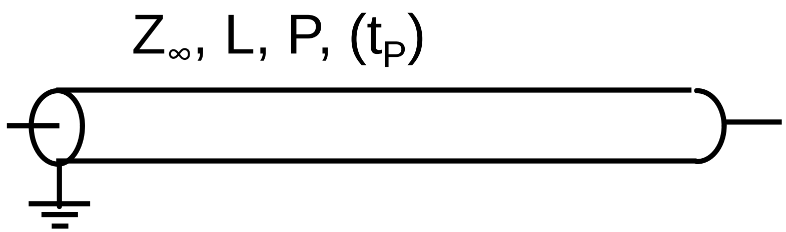
- If R_S and G_P are zero → connection without losses



- Increasing the number of cells, we obtain a transmission line (without losses)

Transmission line

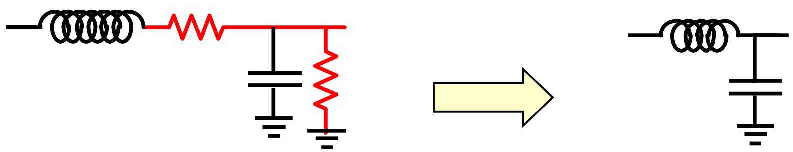
- Parameters of a transmission line
  - Z_∞: characteristic impedance
  - L: length
  - P: propagation speed
  - t_P: propagation time



- In steady state (dc) it is a direct link (single node) for a lossless line

Examples of transmission lines

- Transmission lines without losses
  - R series are null → perfect conductor
  - Parallel G are null → perfect insulator
  - Base cell



- Can be considered lossless lines
  - Printed circuit (PCB) tracks
  - Cables (coaxial, flat cables, twisted pair, ...)
- Are considered lines with losses (not covered here)
  - Connections within the integrated circuits
    R_S, G_P not zero → losses!

Lossless line parameters

- Physical parameters (dimensions and materials)
  - L_U: unitary inductance
  - C_U: unitary capacitance
  - L: length
- Electrical parameters
  - Z_∞: characteristic impedance (typical range: 10 Ω ... 1000 Ω)
  - P: propagation speed (0.6–0.8 c → 18–24 cm/ns)
  - Propagation time t_P = L / P
    - Time needed by the electrical signal to move along the conductor

$$Z_\infty = \sqrt{\frac{L_U}{C_U}}$$

$$P = \frac{1}{\sqrt{L_U C_U}}$$

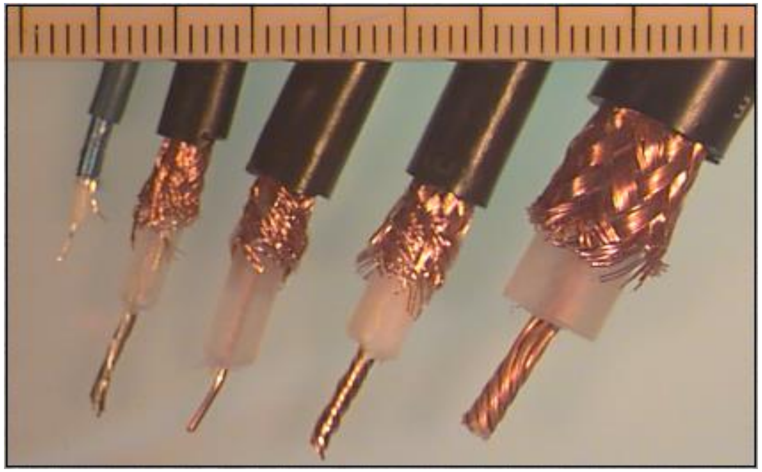
Transmission line models

- The transmission line model is more accurate than R L C models
- Use the line model when the conductor cannot be considered equipotential, i.e.
  - ♦ Transition times (rise/fall, t_R/t_F) are much smaller than the propagation time, t_P
    - Typical P = 18–24 cm/ns, hence for L = 10 cm → t_P = 0.4–0.6 ns
  - ♦ Long links & signals with fast transitions
    - Matters the steepness of the fronts, not the frequency
- In the following we use only simplified models (no losses)

Various types of cables/lines

Coaxial cables

- $Z_\infty \rightarrow$ 47....100 Ω

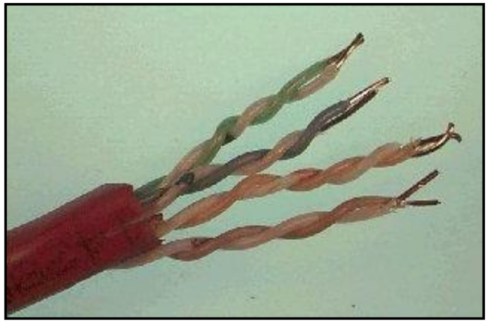


Flat cables

- $Z_\infty \rightarrow$ 100...1000 Ω

Twisted pair

- $Z_\infty \rightarrow$ 100…600 Ω



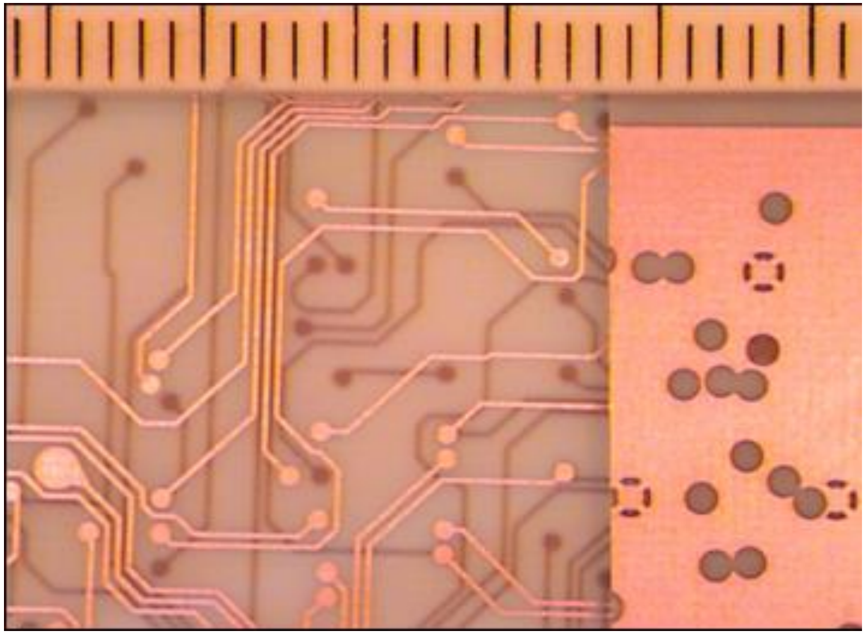
- Speed P = 0.6–0.8 c (18–24 cm/ns)

Tracks / lines on printed circuit

- PCB tracks, with ground planes

- Characteristic impedance
  $Z_\infty \rightarrow 10...300\ \Omega$

- Propagation speed
  P = 0.6–0.8 c
  (18–24 cm/ns)



What influences the parameters

- Z_∞ and P depend on C and L
- C and L depend on physical characteristics (dimensions, materials, …)
- Narrow tracks (small W)
  - L increases ($L_U \propto \ln(1 / W)$)
  - C decreases ($C_U \propto 1 / \ln(1 / W)$)
    - Z_∞ increases
- Wide tracks (large W)
  - L decreases, C increases
    - Z_∞ decreases
  - Same effect if adding capacitance to inputs/outputs

$$Z_\infty = \sqrt{\frac{L_U}{C_U}}$$

$$P = \frac{1}{\sqrt{L_U C_U}}$$

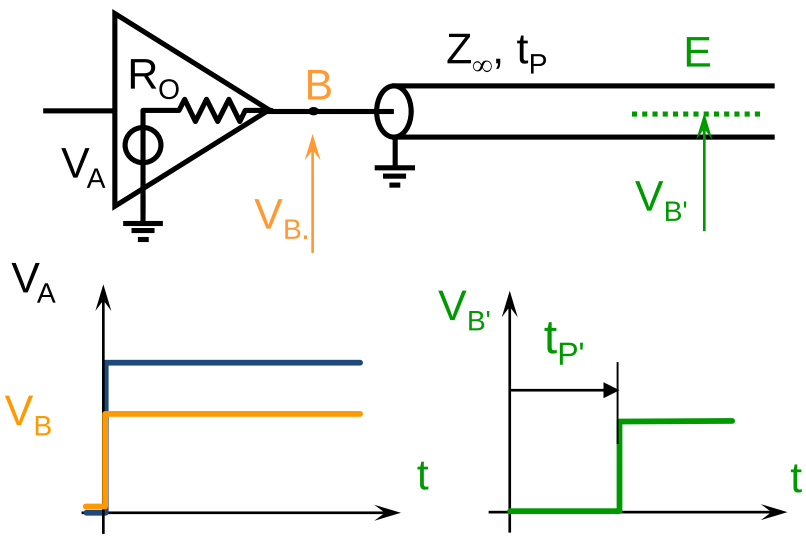
Lecture C2: Transmission Line Models

- Propagation of digital signals
- Transmission line models
- Reflections
- Driving conditions
- Transmission time t_{TX}
- Skew t_K

Line driven with voltage step

- Digital signal (0→1): voltage step 0 V → V_A
- Linear model for driver, lossless line
  - The step $V_B(0)$ on the beginning of the line is given by the partition of V_A on R_O and Z_∞

$$V_B(0) = \frac{Z_\infty}{R_O + Z_\infty} V_A$$



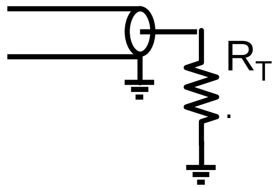
- The first step
  - Moves along the conductor without distortions
  - Needs t = $t_{P'}$ to move between two points of the line

Incident wave and reflected wave

- The step propagates along the line, from the generator to the load, from left to right:
  - wave PROGRESSIVE or INCIDENT (V_{P1})
- At any point on the line
  - $V(t) / I(t) = Z_\infty$ → if Z_∞ is constant, V and I do not change
- Conductor discontinuity and loads
  - If Z_∞ varies ($Z_{\infty 1}$ → $Z_{\infty 2}$), V and I change to match $V / I = Z_{\infty 2}$
  - The variations generate a reflected (regressive) wave, which travels back, from right to left
  - The reflected wave behaves like the incident wave (can have further reflections, ...)

At termination

- Line terminated on R_T (remote side)
  - At the ends of R_T, $V / I = R_T$
- The step reaches the termination at t_P
  - t_P is the propagation time or flight time
  - It depends on P (speed) and L (length)



- If $R_T = Z_\infty$, V / I does not change (no discontinuity)
  - The termination absorbs all the progressive wave energy
  - No reflected waves
- If $R_T \neq Z_\infty$, V / I must change
  - R_T does not absorb all the progressive wave energy
  - Generates a reflected wave moving towards the driver

Reflection coefficient (Γ)

- Is defined the reflection coefficient Γ_T

$$\Gamma_T = \frac{R_T - Z_\infty}{R_T + Z_\infty}$$

- An incident (progressive) wave V_P determines at the termination R_T a reflected wave V_R of amplitude
  - $V_R = \Gamma_T V_P$
- In general, the wave reflected by an impedance discontinuity from Z_∞ to Z_1 has an amplitude V_R:
  - $V_R = \Gamma_1 V_P$, with $\Gamma_1 = (Z_1 - Z_\infty) / (Z_1 + Z_\infty)$
- The total voltage at any point on the line is
  - Algebraic sum of the incident wave and the reflected wave

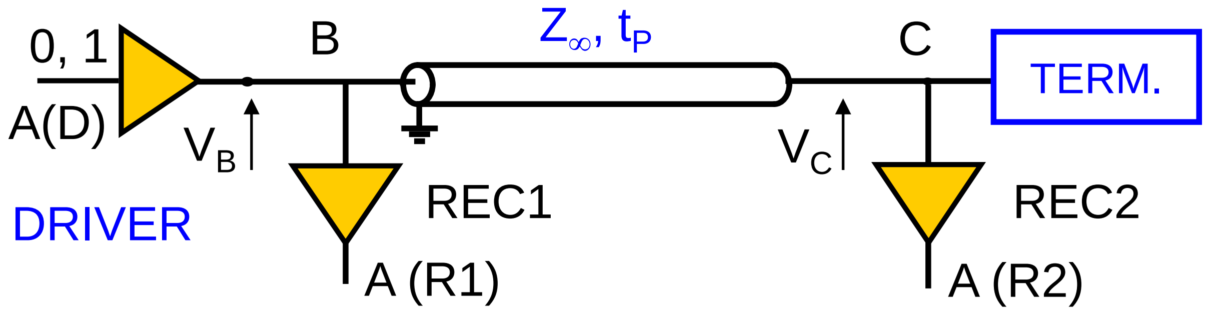
Values of the reflection coefficient

$$\Gamma_T = \frac{R_T - Z_\infty}{R_T + Z_\infty}$$

- Closed line on $R_T = Z_\infty$ ($\Gamma_T = 0$)
  - No discontinuity, hence no reflected waves
  - All incident energy is dissipated on the termination resistor
- Line open: $R_T \rightarrow \infty$ ($\Gamma_T = 1$)
  - Null total current, hence reflected wave with $I_R = -I_I$
  - Voltage of reflected wave equal to the incident one
  - Total voltage at termination doubles
- Line shorted: $R_T = 0\ \Omega$ ($\Gamma_T = -1$)
  - Null total voltage at termination
  - Reflected wave voltage $V_R = -V_I$

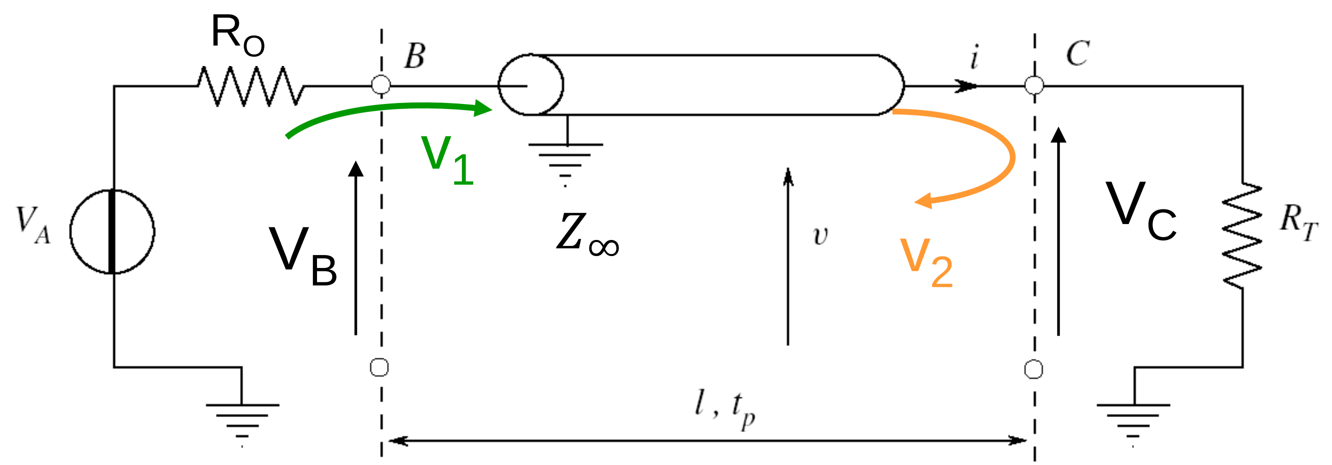
Reference structure

- Driver circuit at far left (near-end)
- Line transmission with parameters Z_∞, t_P
- A termination on far right (far-end)
- Receivers in any position
  - at the driver (near end)
  - at the termination (far end)
  - in-between



Complete equivalent circuit

- The voltage step $V_B(0)$ on the line is the partition of V_A on R_O and Z_∞, and creates the incident wave, v_1
- At the termination R_T, is generated a reflected wave, v_2



$$V_B(0) = \frac{Z_\infty}{R_O + Z_\infty} V_A = v_1 \qquad v_2 = \frac{R_T - Z_\infty}{R_T + Z_\infty} v_1 = \Gamma_T v_1 \qquad V_C = v_1 + v_2$$

First step and first reflection

- The voltage step $V_B(0)$ propagates along the line as incident wave v_1
- v_1 arrives at termination R_T after the propagation time t_P
- At the termination R_T, v_1 generates a reflected wave, v_2
- The reflected wave amplitude, v_2, depends on Γ_T

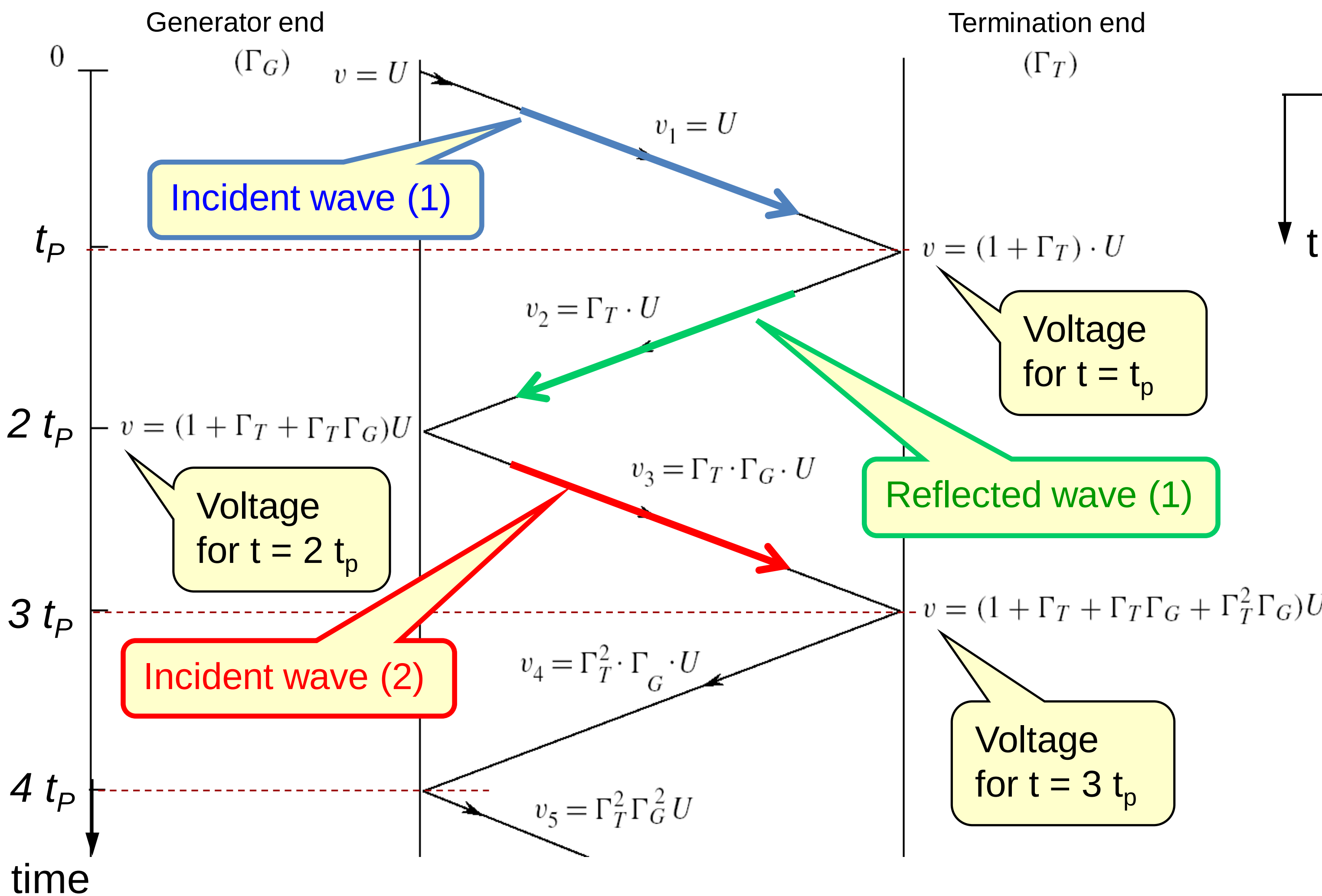
$$v_2 = \frac{R_T - Z_\infty}{R_T + Z_\infty} v_1 = \Gamma_T v_1$$

- The reflected wave v_2 propagates towards the driver
- The termination voltage, V_C, is the algebraic sum $v_1 + v_2$
    - For $t < t_P \rightarrow V_C = 0$
    - For $t > t_P \rightarrow V_C = v_1 + v_2$

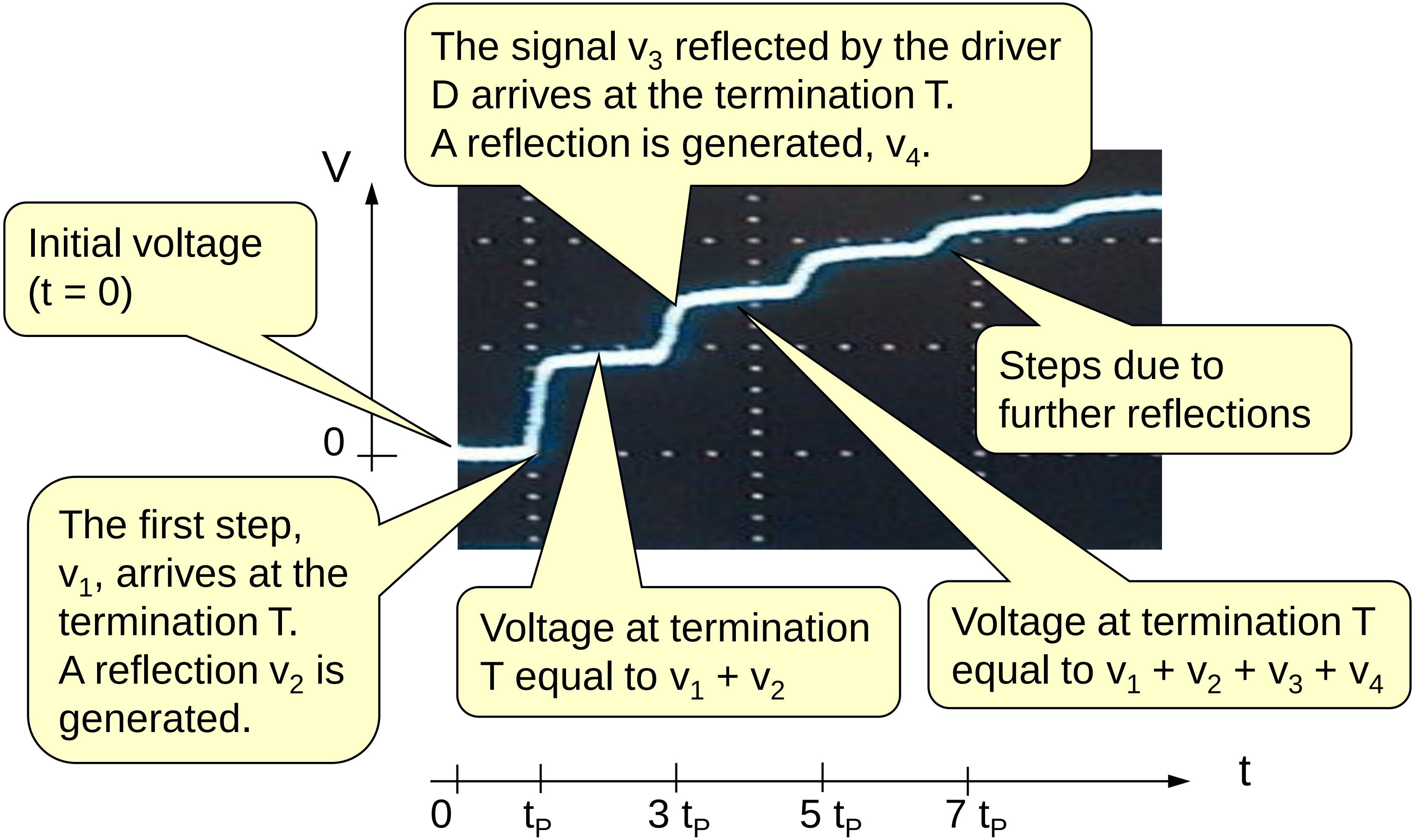
How to represent V(x, t) on the line

- Time evolution of the signal V(t) for $x = x_0$
  - The voltage varies when the initial step and subsequent reflections pass through the observation point
  - Lattice diagram
- Distribution of V(x) along the line at $t = t_0$
  (≈ still picture of the voltage along the line)
  - The incident wave moves from generator to termination
  - Any reflected wave moves in the opposite direction
  - Few information, scarcely used
- Time distribution of V(x) along the line
  - V(x, t) diagrams three-dimensional axes: V, x, t

V(t) in various x on the line: lattice diagram



V(t) at the termination T

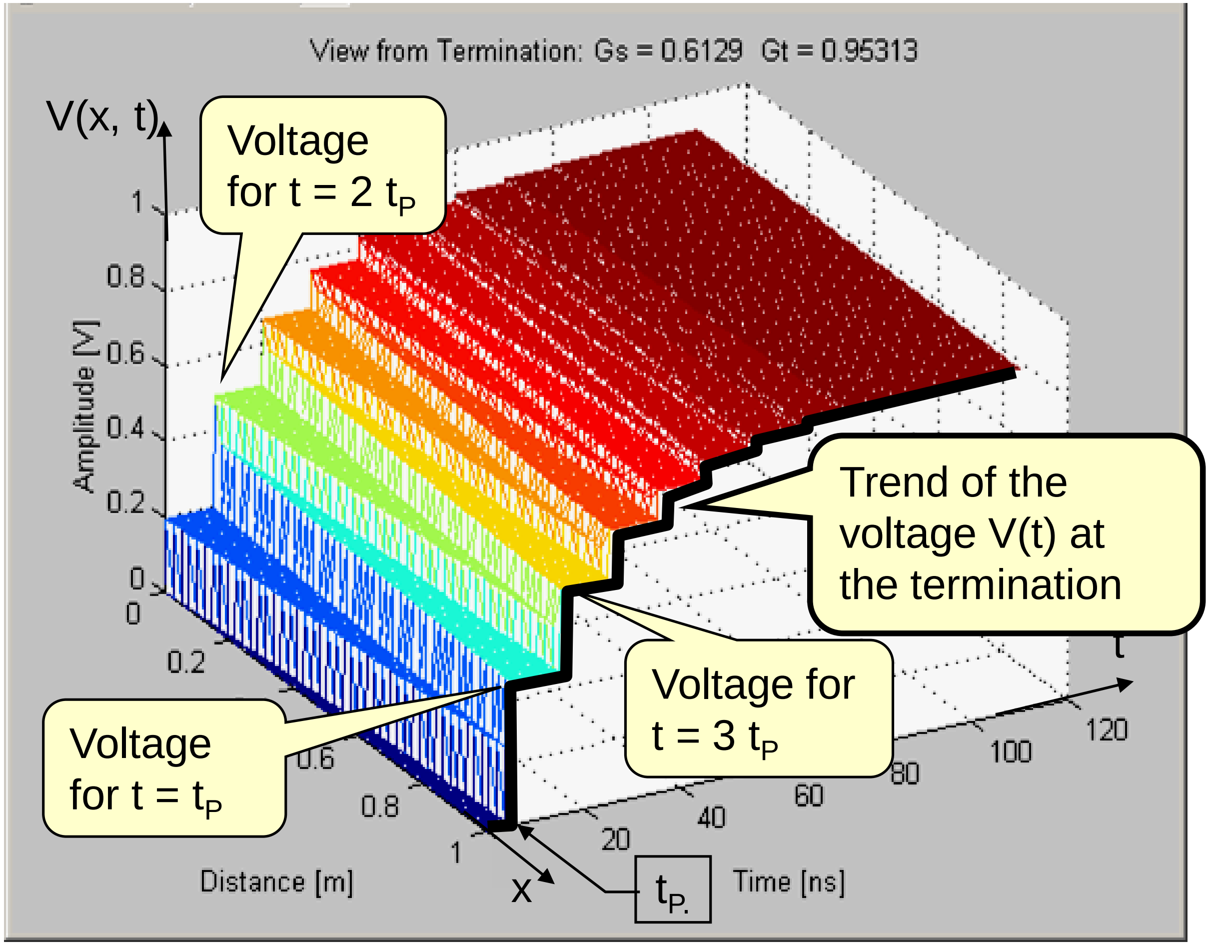


x/t view of the signal on a line

V(t, x) on a transmission line driven by a voltage step:
$\Gamma_T > 0$
$\Gamma_G > 0$

All steps (direct and reflected) have the same polarity.

At each point of the line there is a monotonous staircase.

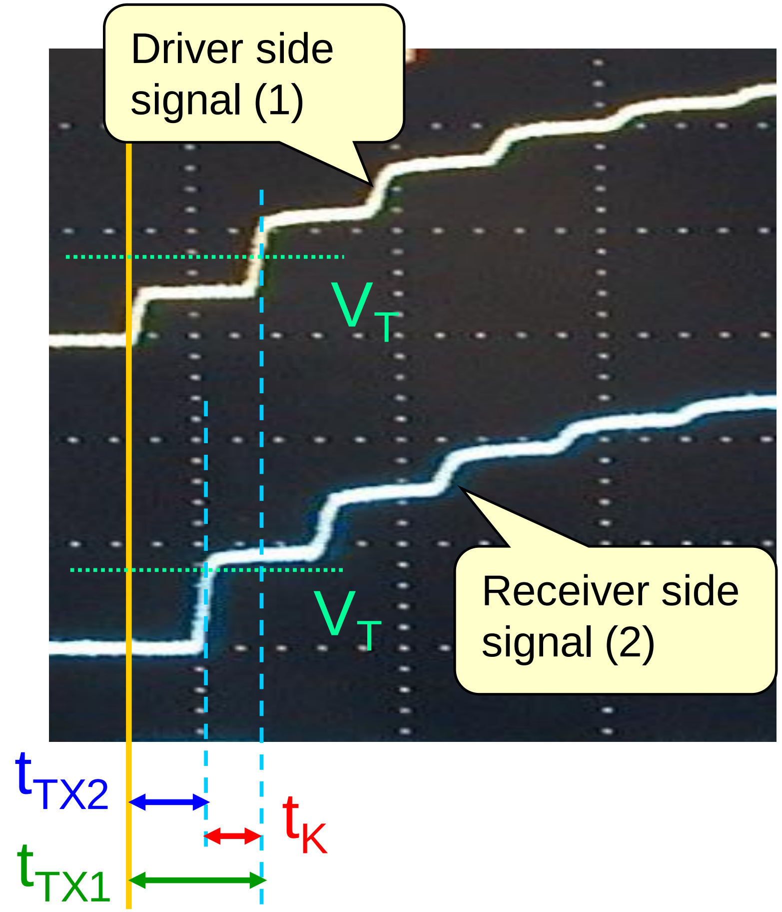


Signal at extremes V(t)

- The signal is obtained by incrementally adding the contributions of the incident and the reflected waves
  - Driver side: discontinuity at 0, 2 t_P, 4 t_P, ...
    - Step propagation
    - Arrival of the waves as they are reflected (right → left)
  - Termination side: discontinuity at t_P, 3 t_P, 5 t_P, ...
    - Arrival of the step
    - Arrival of the subsequent incident waves (left → right)
  - Intermediate points: double the number of discontinuities
    - Incident and reflected waves transit at different times
- Steady state: equipotential line (if without losses)

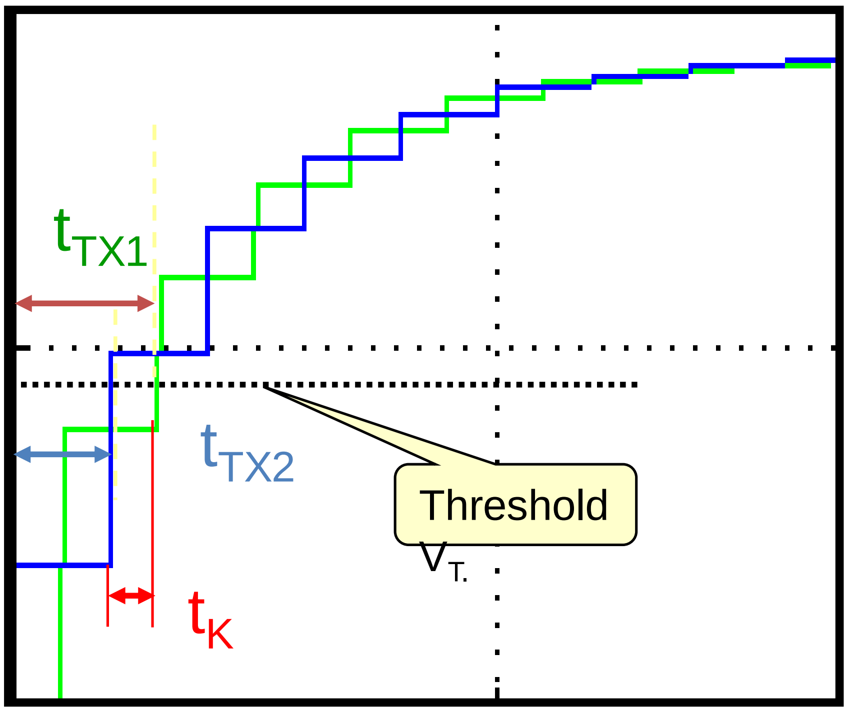
Transmission time and skew

- Same parameters t_{TX} and t_K of the RC model
- Delay for crossing the threshold V_T is the transmission time, t_{TX}
- t_{TX} depends on
  - Driver and receiver parameters
  - Parameters of the interconnexion
  - Operating conditions (reflexions)
  - Position
- t_{TX} variations cause skew, t_K



Overall trend of V_B and V_C

- Example of voltage trend on the driver side and on the termination side
  - Γ is positive at both ends
  - All reflected waves have the same sign
  - Monotonous interlaced voltage steps
  - Different voltages at line ends
  - Voltages equal in the steady state (t → ∞, dc)



Exercise C2.1: reflections

- A step signal is applied to a line terminated on an open circuit with:
  - Driver: R_O = 160 Ω, 5 V power supply
  - Line: Z_∞ = 80 Ω, t_P = 10 ns

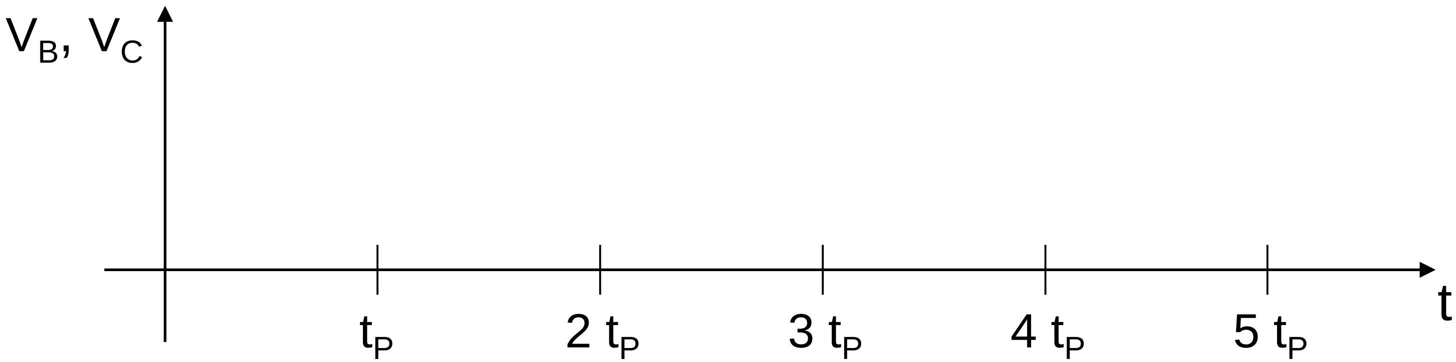
- Plot the time evolution of the signals on the driver and termination ends.

Exercise C2.1a: reflections

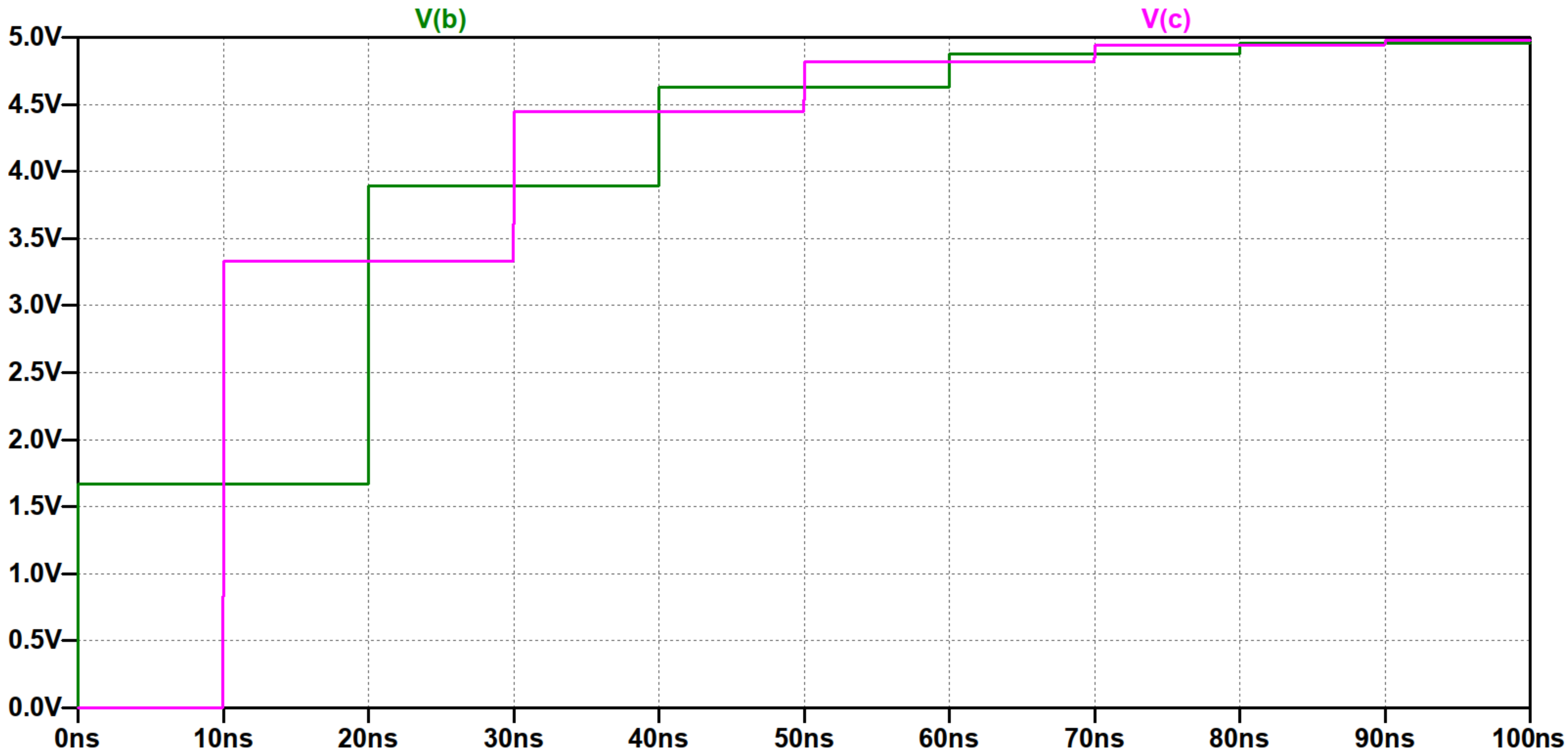
- Reflection coefficients:

- First step width:

- Time evolution:



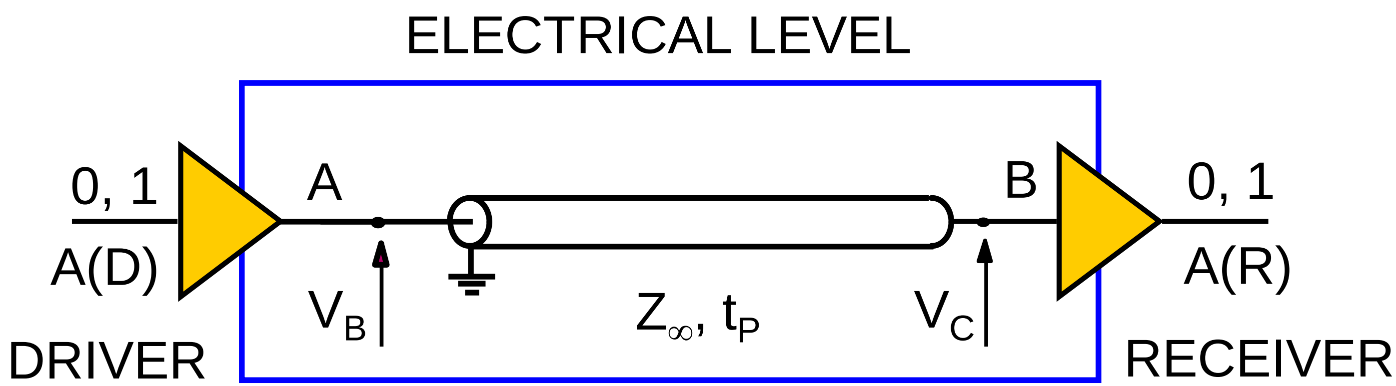
Exercise C2.1a: reflections

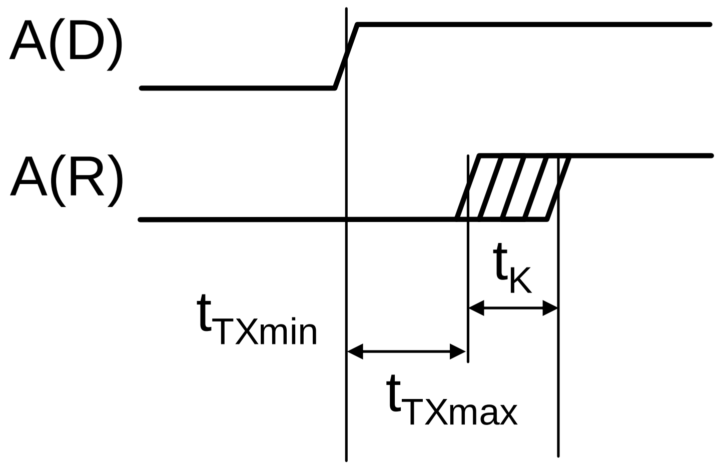


Effects of propagation summary

- The voltage variation propagates from one end to the other in the propagation time t_P
- The receiver recognizes logic state changes when the signal crosses its input threshold V_{TH}. May happen after multiple signal reflections, and introduces a transmission time, t_{TX}, delay between signal generation and detection at the receiver
- This delay can have a variation t_K (skew) due to variations in electrical and physical parameters
- The delays t_{TX} and t_K depend on the electrical (driver, receiver) and physical (line) characteristics

Summary of the parameters





t_{TX} → transmission time
t_{TXmax} → transmission time max
t_{TXmin} → transmission time min

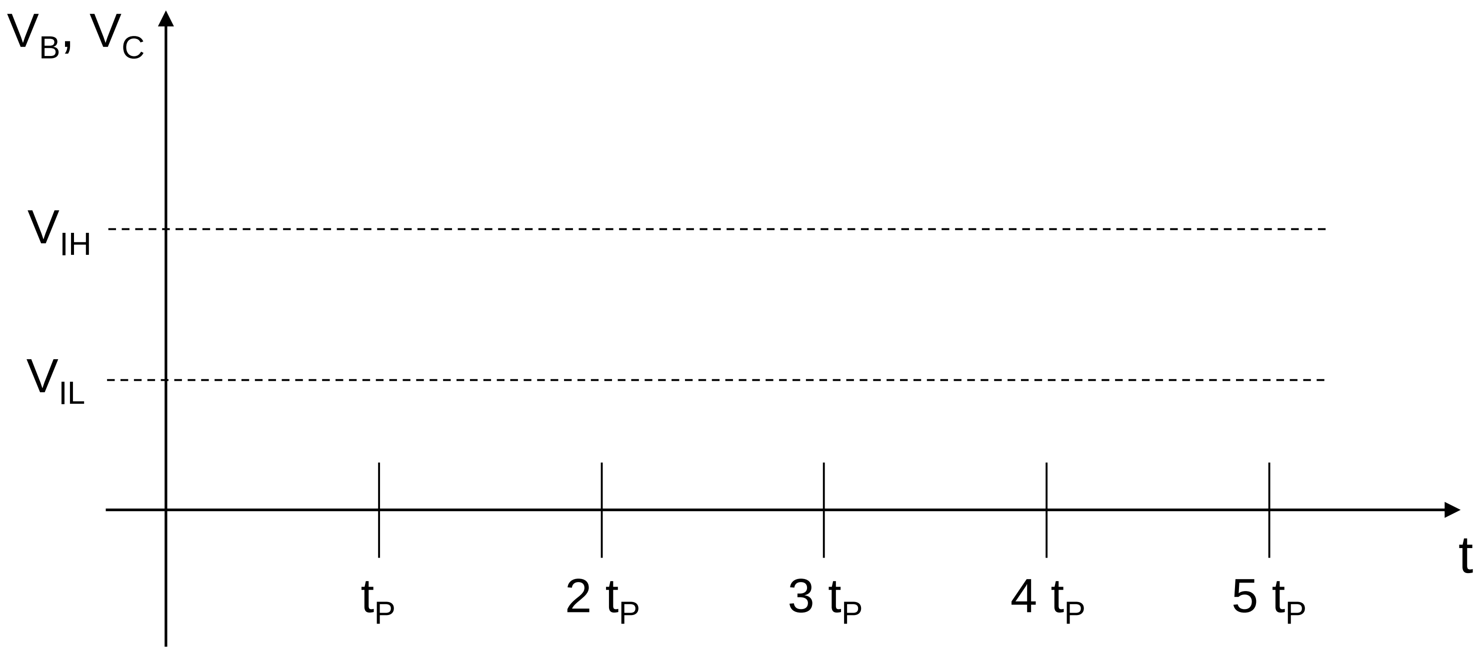
$t_K = t_{TXmax} - t_{TXmin}$ → SKEW

Exercise C2.2: propagation and skew

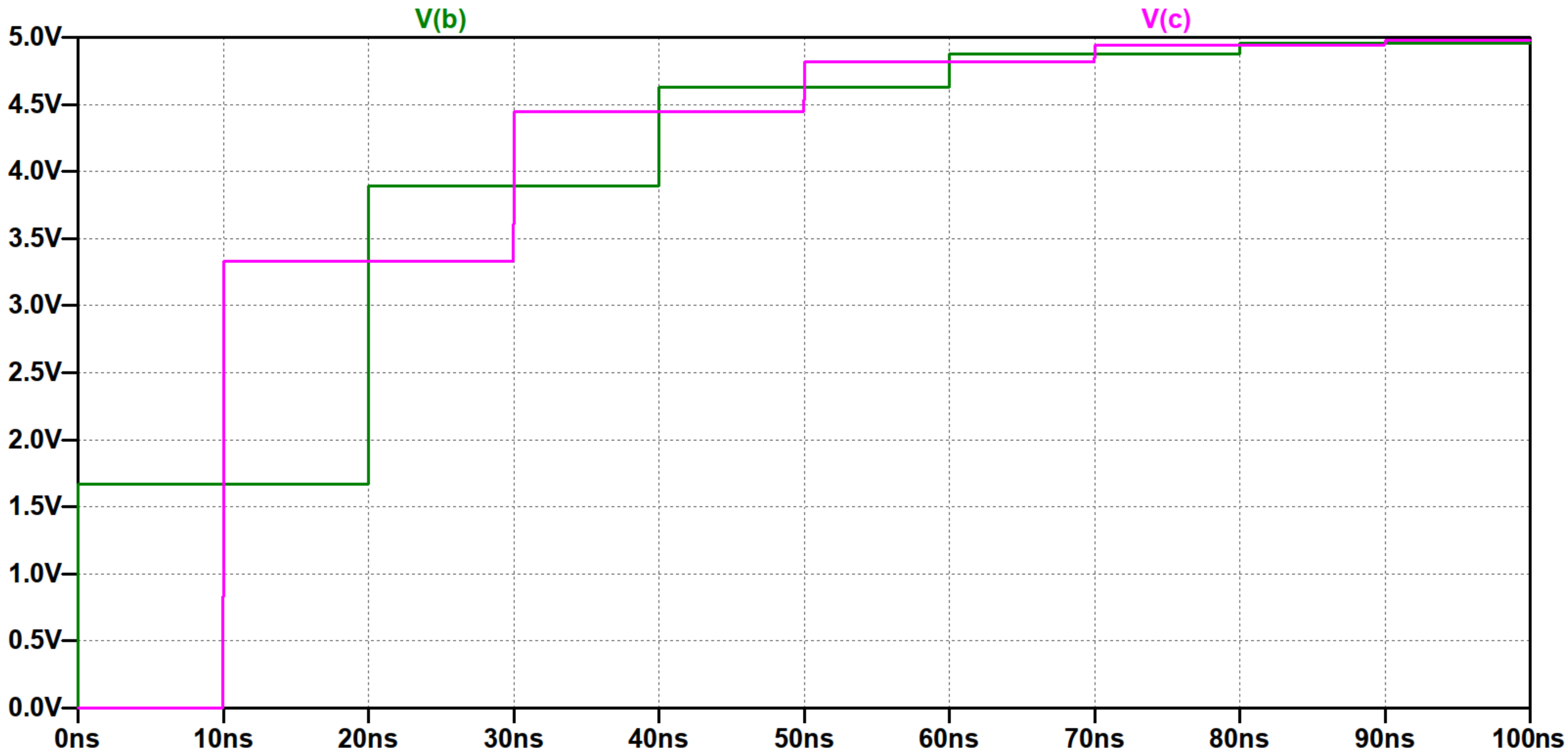
- Connect two receivers to the system of exercise C2.1, one at the output of the driver, and one at the end of the line, with
  - V_{IL} = 1 V, V_{IH} = 3 V
- Evaluate
  - Transmission time t_{TX}
  - Skew t_K

Exercise C2.2a: propagation and skew

- Use the time evolution from exercise C2.1
- Mark the crossing of the thresholds



Exercise C2.2a: propagation and skew



Lecture C2 – final check

- Define the transmission time and skew and indicate which parameters they depend on.
- How propagation time, t_P, and transmission time, t_{TX}, differ for an interconnection over a transmission line?
- Value of the reflection coefficient for terminations
  - Open circuit, short circuit, $R_{T1} = Z_\infty / 2$, $R_{T2} = 2\ Z_\infty$
- The steady state voltage on a lossless transmission line depends on Z_∞?
- First voltage step on a lossless line depends on R_T?
- How could we measure the characteristic impedance Z_∞ of a transmission line?