Applied Electronics

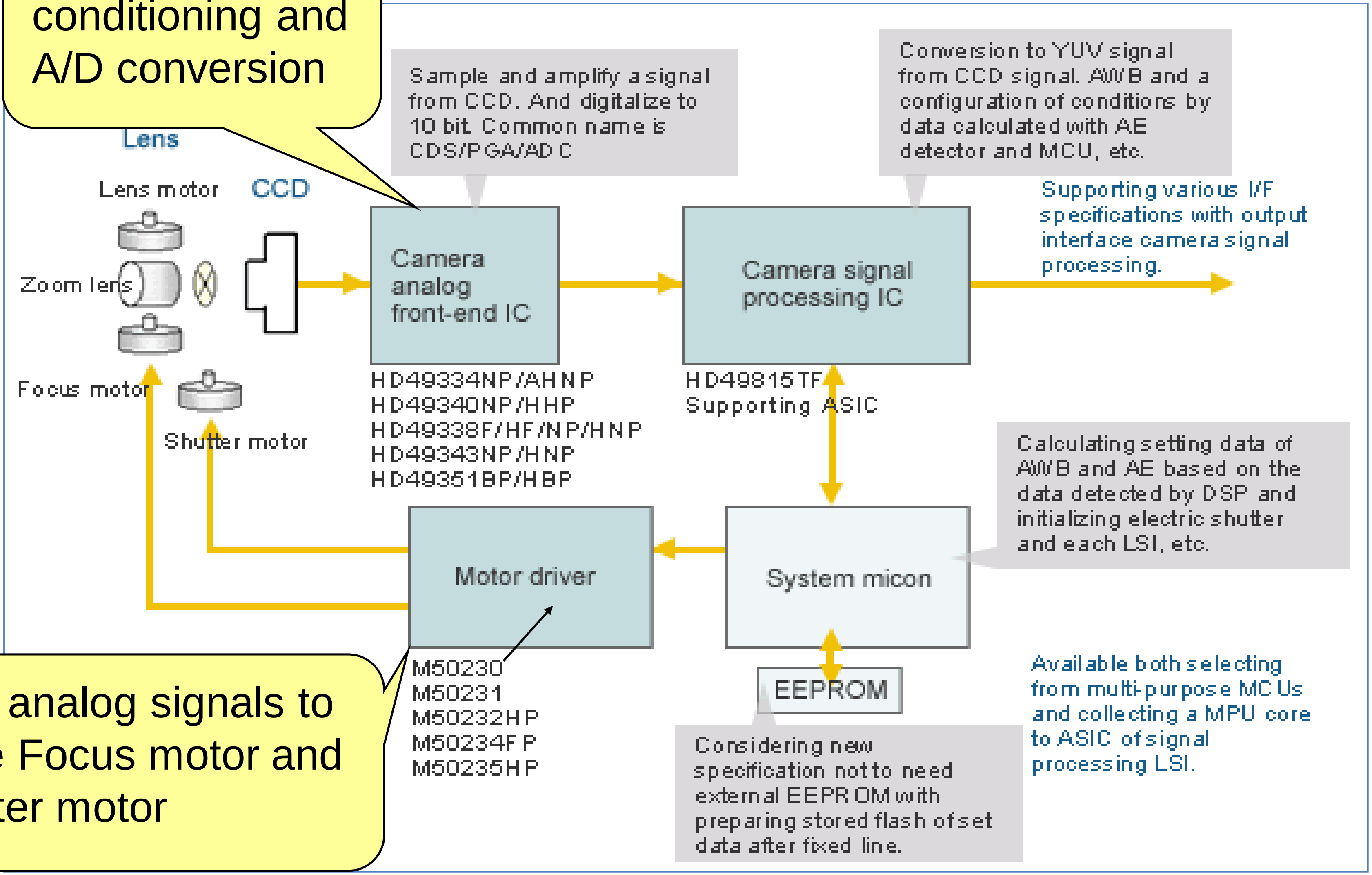
Introduction to A/D and D/A

Analog vs Digital

- Analog signals
  - Continuous in time and amplitude domains
- Digital signals
  - Discrete in time (sampling)
  - Discrete in amplitude (quantization)
- Electronic systems migrate towards digital
- Is there a “loss of information” when A → D ?
  - Quantitative analysis, define relevant parameters
  - How to keep under control information loss

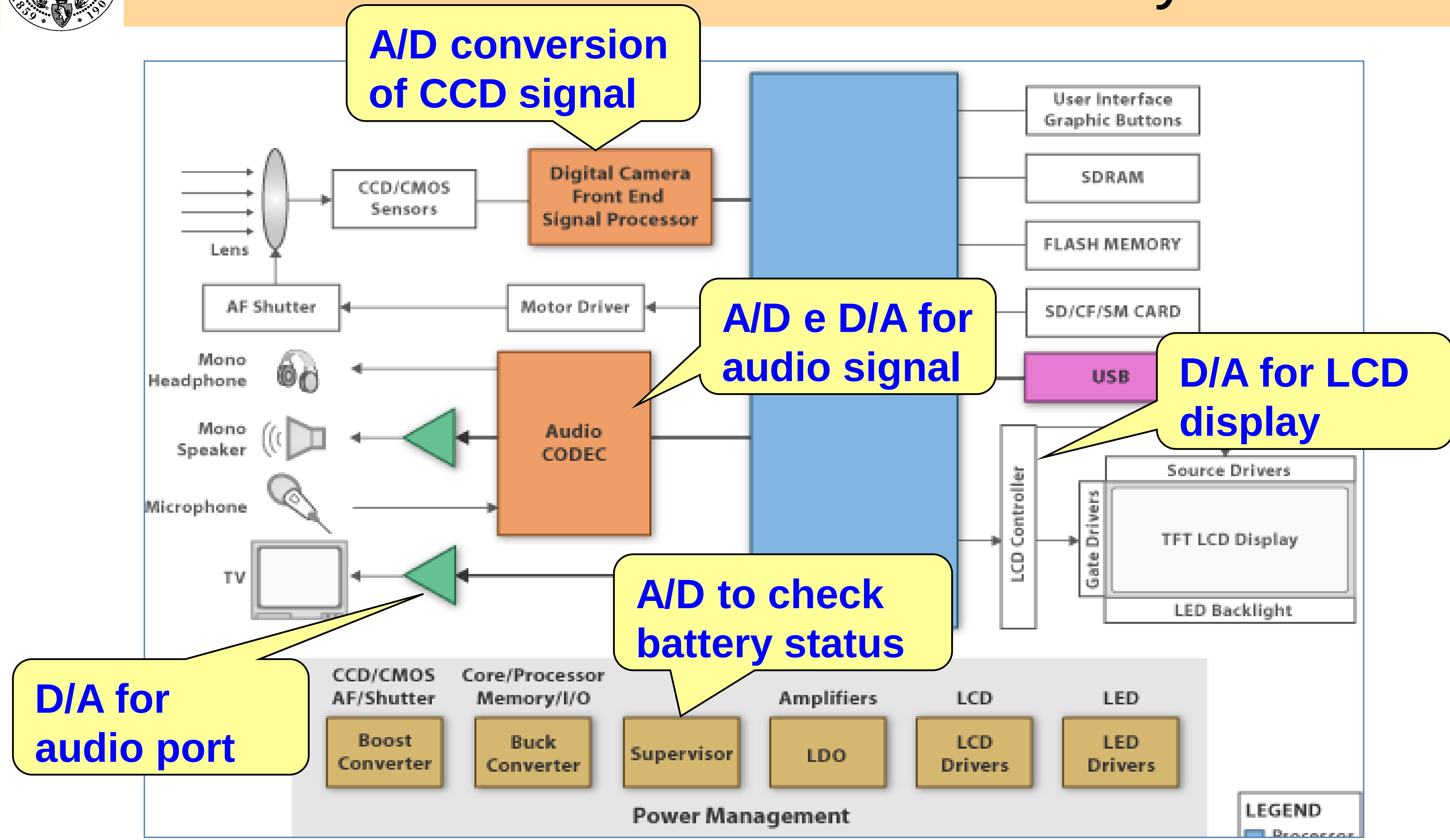
A/D and D/A in a Digital Camera

Signal conditioning and A/D conversion

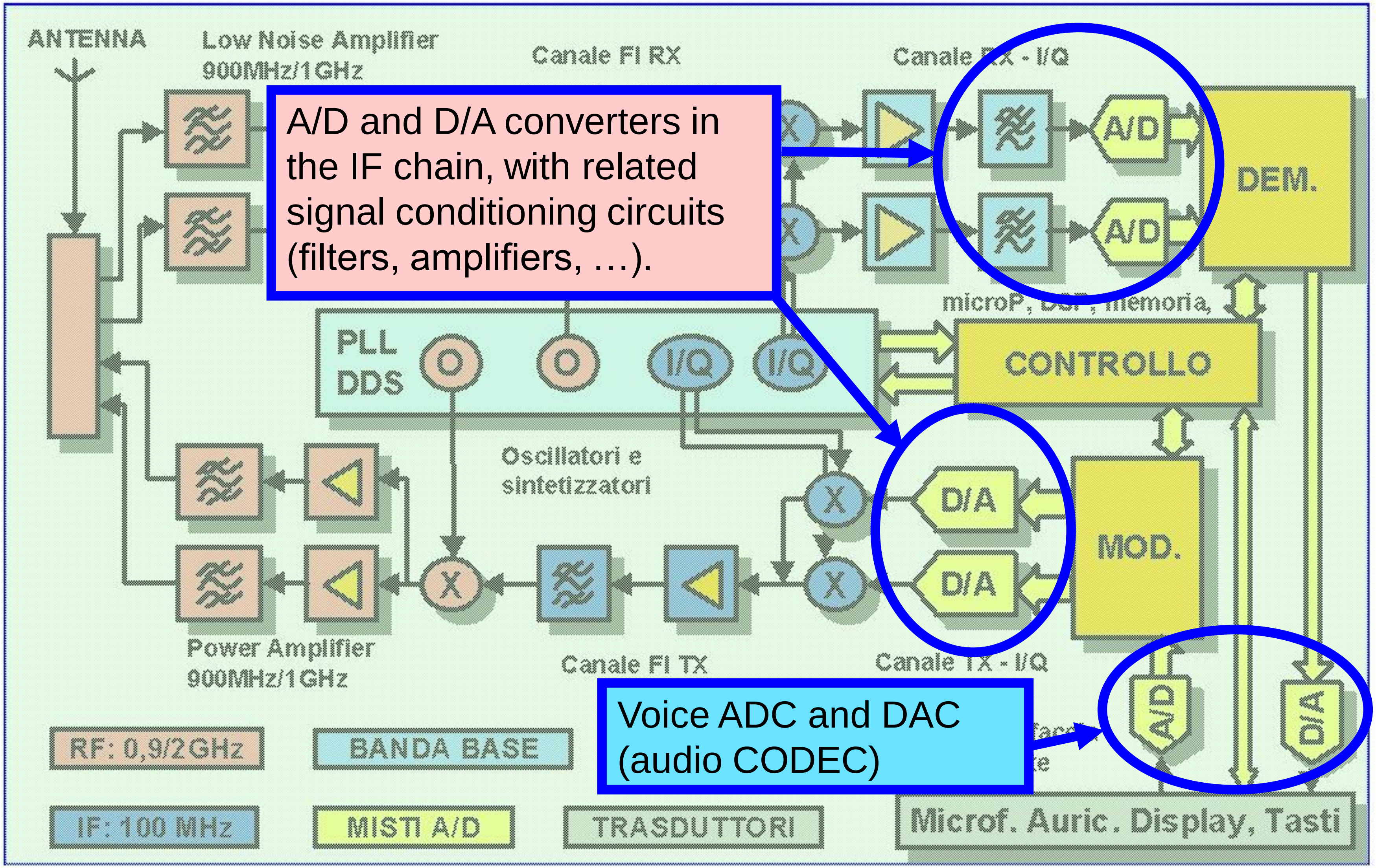


D/A: analog signals to drive Focus motor and shutter motor

A/D and D/A in an Audio/Video System



A/D and D/A in a Cell Phone

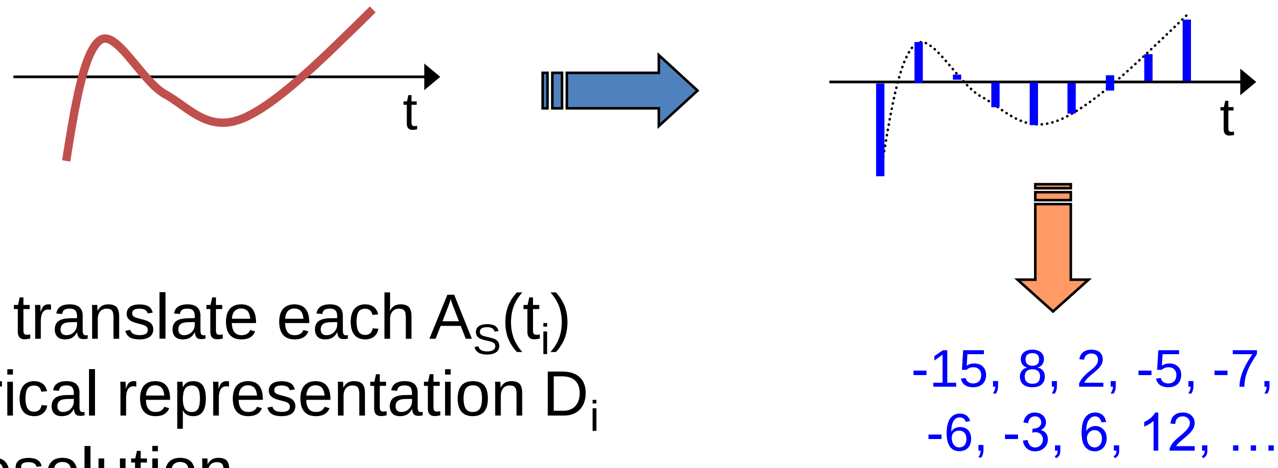


A/D and D/A converters in the IF chain, with related signal conditioning circuits (filters, amplifiers, …).

Voice ADC and DAC (audio CODEC)

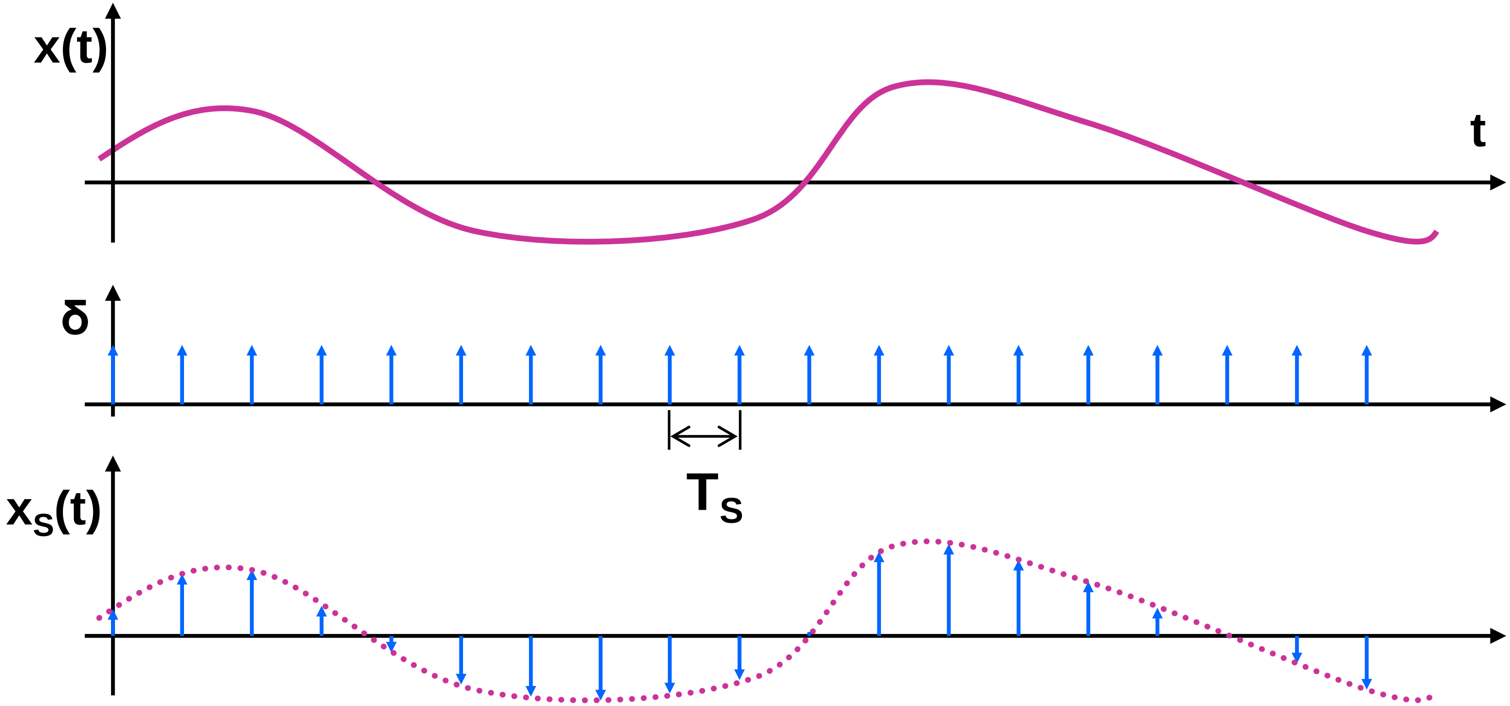
Sampling and Quantization

- A/D conversion has two steps
  - Sampling: replace the time continuous analog signal A(t) with samples $A_S(t_i)$ representing the signal at specific times, t_i



  - Quantization: translate each $A_S(t_i)$ into its numerical representation D_i with a finite resolution

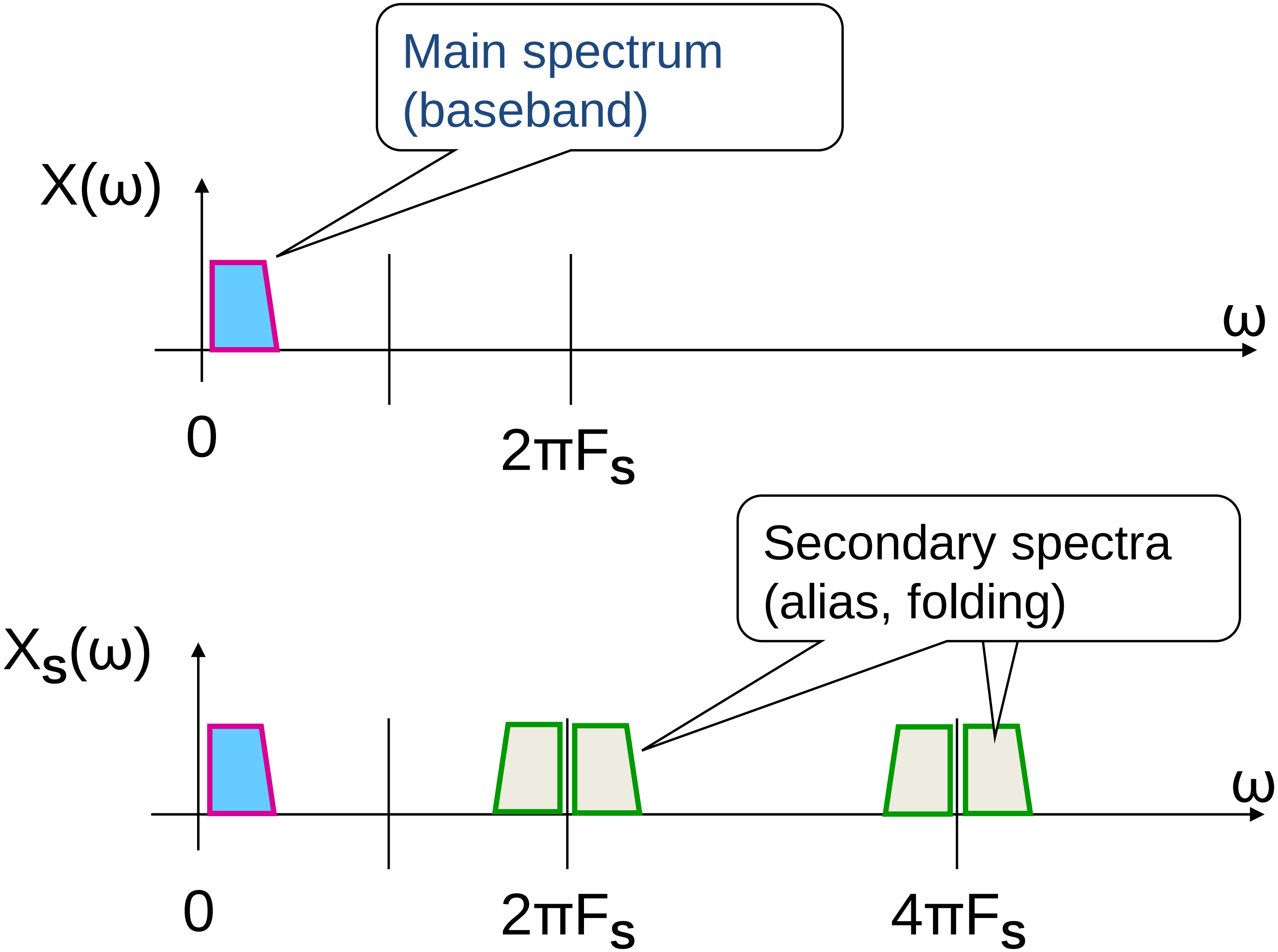
Sampling in the Time Domain



$$x_s(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$F_s = 1/T_s$ is the sampling frequency

Sampling in the Frequency Domain

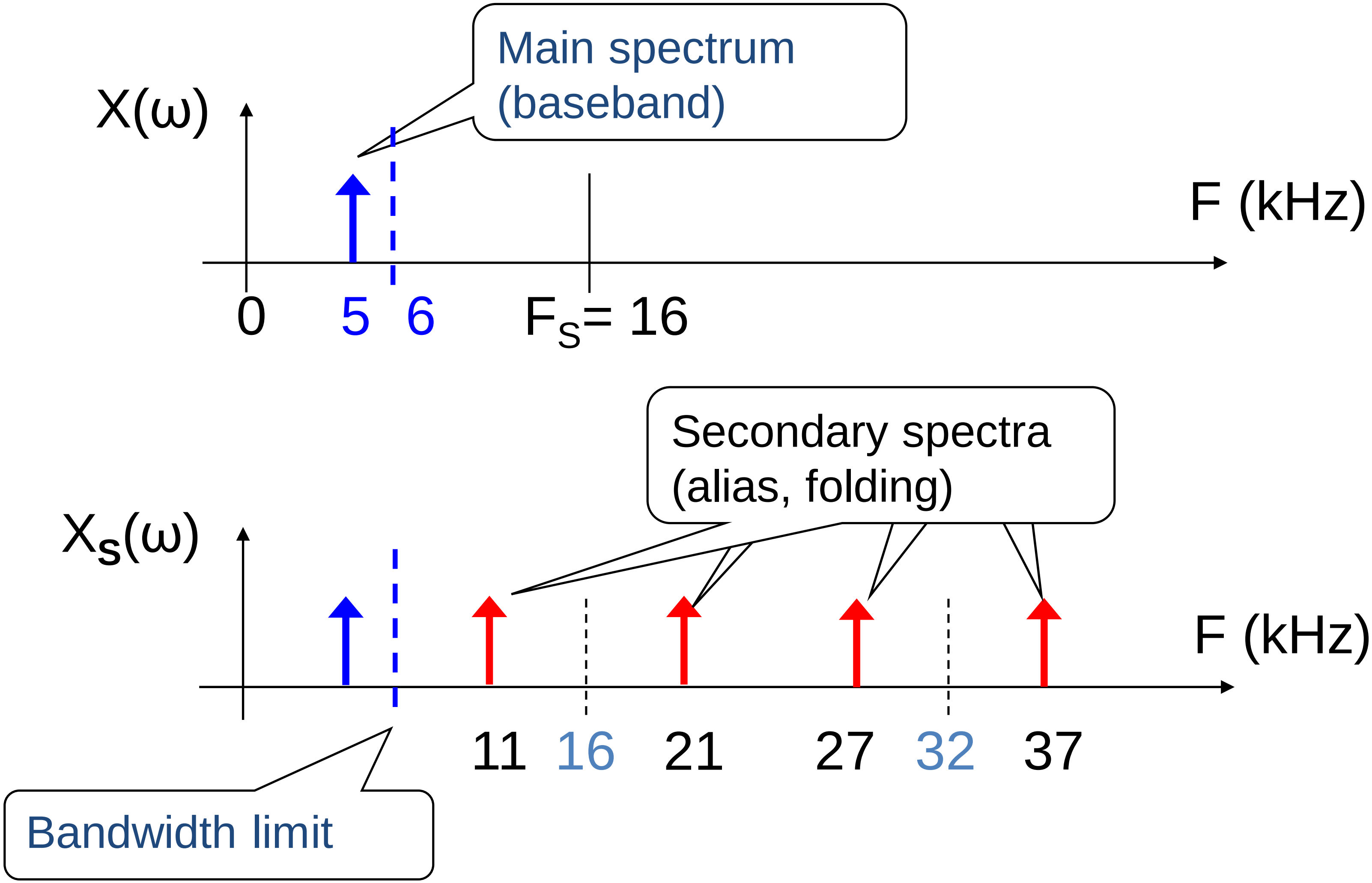


Sampled Sine Signal

- Continuous sine signal of frequency F_a
  - Has a single spectral line at F_a frequency

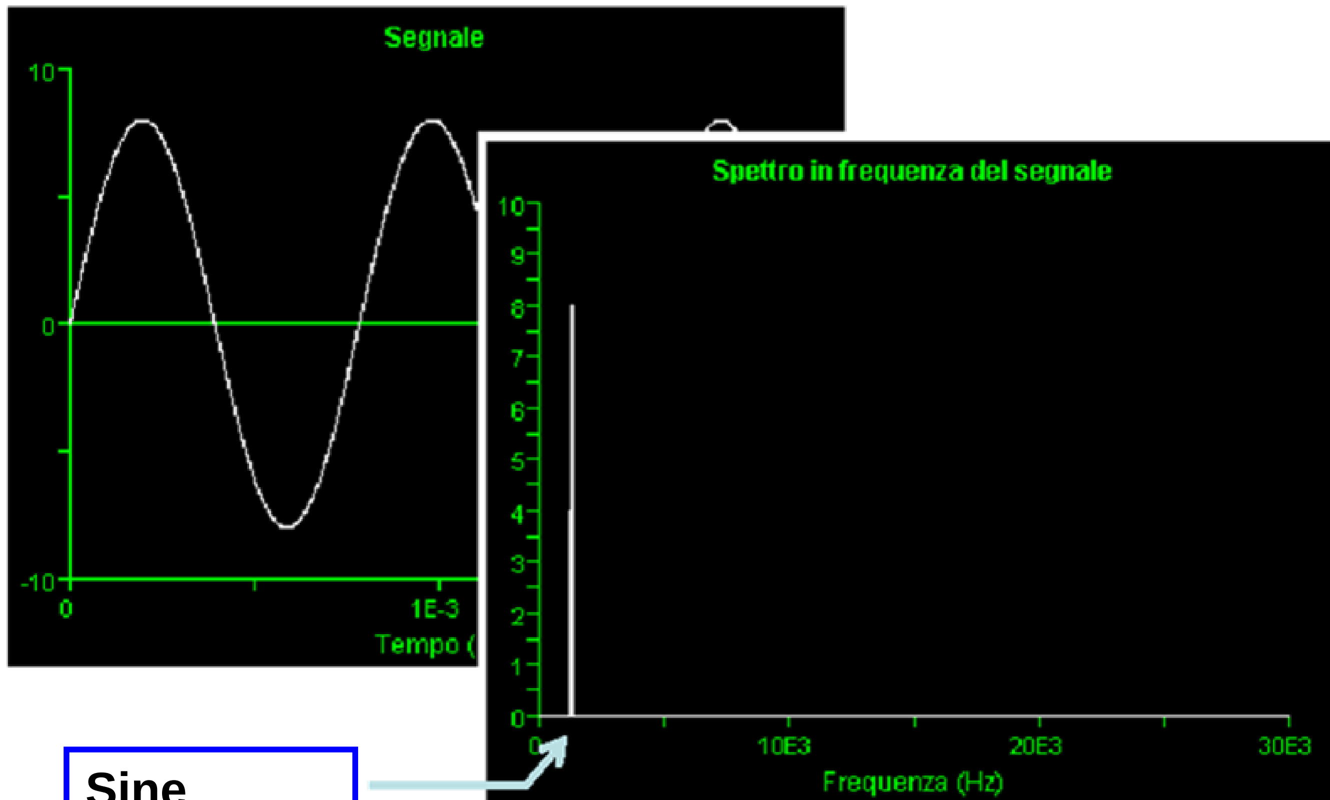
- Continuous sine signal of frequency F_a sampled with F_s
  - Spectral line replicated around K F_s
- Example
  - Signal F_a = 5 kHz sampled at F_s = 16 kHz
    - F_{a1a} = 16 kHz – 5 kHz = 11 kHz, F_{a1b} = 16 kHz + 5 kHz = 21 kHz
    - F_{a2a} = 32 kHz – 5 kHz = 27 kHz, F_{a2b} = 32 kHz + 5 kHz = 37 kHz
    - F_{a3a} = 48 kHz – 5 kHz = 43 kHz, F_{a3b} = 48 kHz + 5 kHz = 53 kHz
    - ……
- With 6 kHz bandwidth, all components are outside band

Numerical Example



Continuous Signal

- Time domain
  - ♦ Continuous sine wave
- Freq. domain
  - ♦ One spectral line

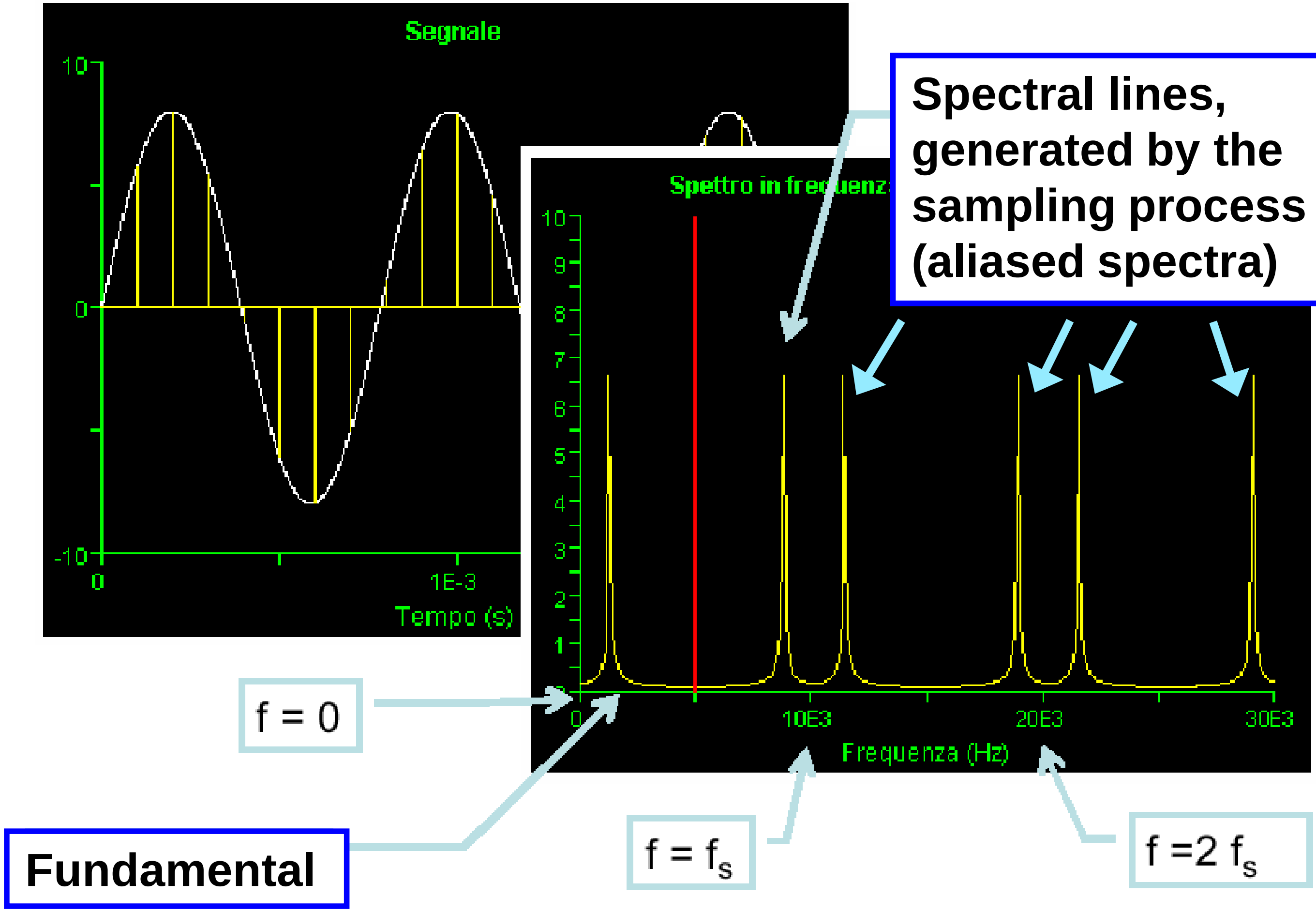


Sine signal

Sampled Signal

- Replicas around K F_S

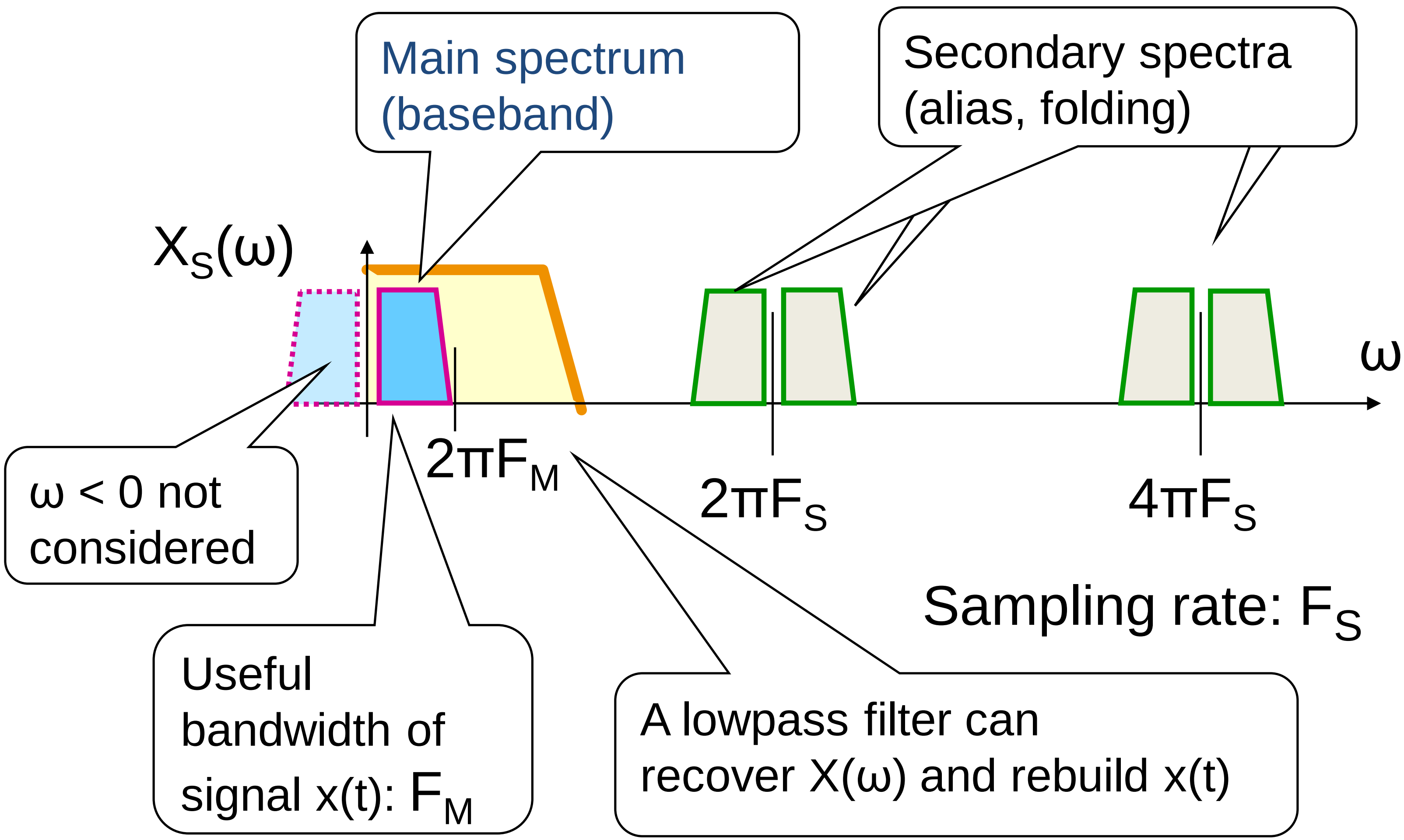
$$X_s(\omega) = \frac{1}{T_s} \sum_{-\infty}^{\infty} X\left(\omega - n\frac{2\pi}{T_s}\right)$$



Lecture Exercise 1

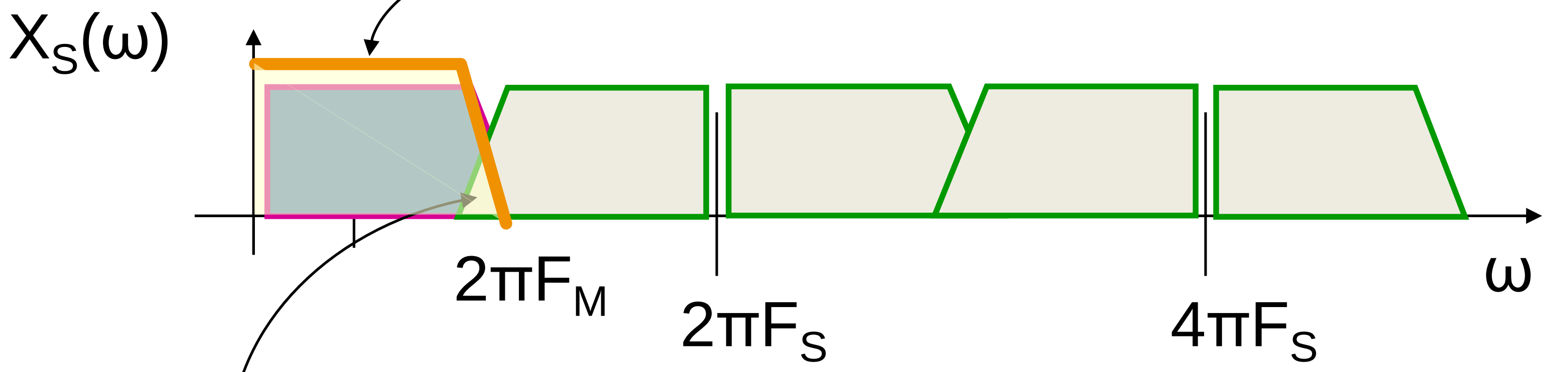
- Draw, in the 0 – 100 kHz range, the spectrum of:
  - 10 kHz sine signal
  - 10 kHz sine signal, F_S = 40 kS/s
  - 10 kHz sine signal, F_S = 40 kS/s + low-pass filter cutoff 15 kHz
  - 10 kHz sine signal, F_S = 18 kS/s
  - 10 kHz sine signal, F_S = 18 kS/s + low-pass filter cutoff 15 kHz
  - 25 kHz sine signal, F_S = 40 kS/s
  - 25 kHz sine signal, F_S = 40 kS/s + low-pass filter cutoff 30 kHz
- Compare the spectra and discuss the differences
  - Point out spurious components caused by aliasing

$x(t)$ Recovery – Spaced Aliases



$x(t)$ Recovery – Overlapping Aliases

No lowpass filter can recover X(ω) and rebuild x(t)



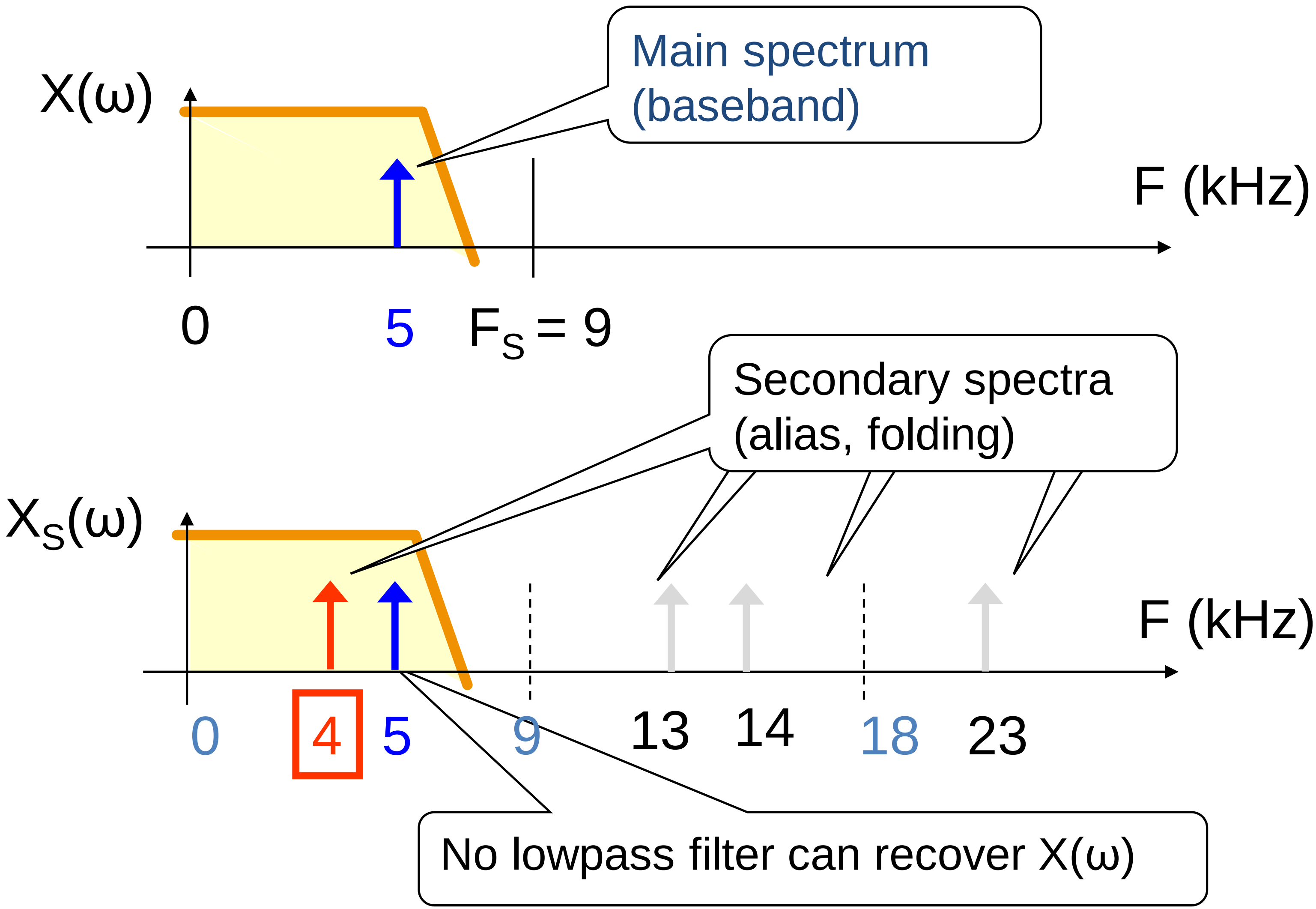
Overlap area
no recovery possible

$F_M > F_S/2$
Inband aliasing noise

Numerical Example

- F_a = 5 kHz (signal useful bandwidth: 6 kHz)
- F_s = 9 kHz (sampling rate)
  - ♦ F_s lower than twice the signal bandwidth
- First alias (two sidebands)
  - ♦ F_{a1a} = 9 kHz – 5 kHz = 4 kHz, F_{a1b} = 9 kHz + 5 kHz = 14 kHz
  - ♦ F_{a1a} is within the signal bandwidth
    Cannot be removed by filtering
  - ♦ Sampling creates a 4 kHz component which was not in the original input signal
  - ♦ F_{a1b} (and higher) is out of band and does not generate problems

Numerical Example – Graph

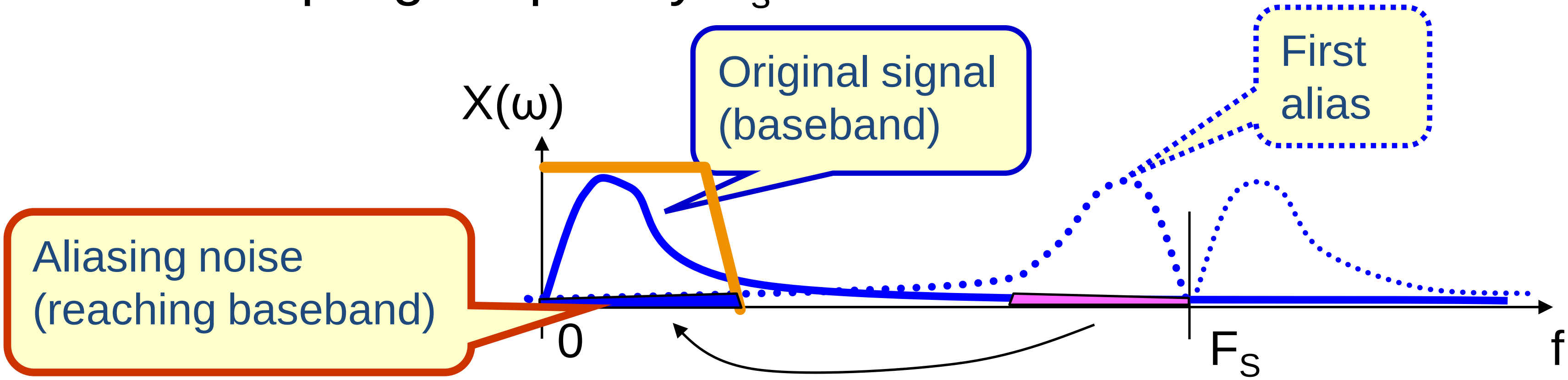


What Is the Actual Limit?

- The Nyquist-Shannon theorem
  - A signal must be sampled at least 2 x BANDWIDTH (BW)
  - Example: a 1 GHz carrier, 100 kHz BW signal can be safely sampled at > 200 kS/s
- Less stringent specs for A/D converters
  - Min sampling rate depends on signal bandwidth, not the carrier
- Tight specs for the sampling circuit
  - Sampling jitter related to carrier, not to bandwidth
- Keep a margin: sample at K x 2 x BW
  - K → oversampling ratio

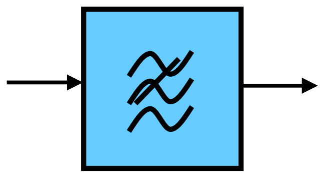
Aliasing for Real Signals

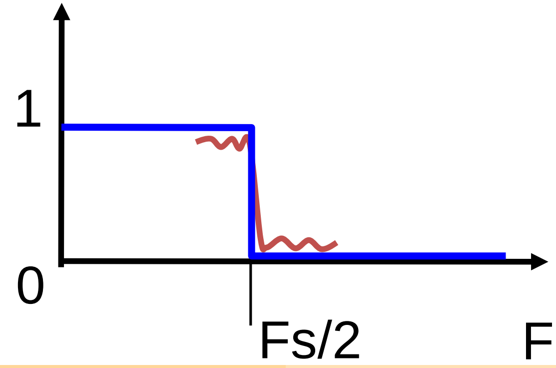
- Actual signals are not frequency-limited
  - Always some residual HF (over-Nyquist) components
  - HF signals are folded to baseband by sampling
    → aliasing NOISE
- The amount of aliasing noise is related to
  - Input signal frequency spectrum (modified by anti-alias filter)
  - Sampling frequency F_s



Anti Aliasing Lowpass Filter

- To avoid information loss, signals must be sampled at least twice the bandwidth (Nyquist)
  - Valid for limited BW signals
- To get a limited BW signal
  - Add a low-pass anti-aliasing filter
  - Real filters have ripple and finite attenuation
    - Always some energy above Fs/2



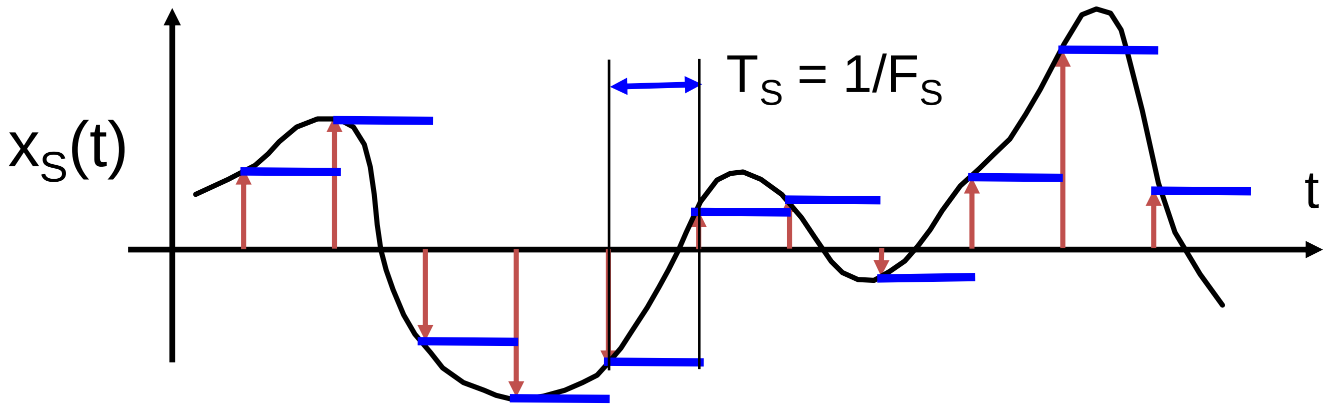


Oversampling

- Sampling only slightly above the Nyquist limit requires steep antialiasing filters → expensive!
- Option: sampling rate much higher than the Nyquist limit → oversampling
  - E.g., 1 MS/s of a 3 kHz audio signal has aliases at 2, 3, … MHz
- Relaxed specifications on the anti-alias input filter, but higher bit rate (more samples/s)
  - Higher bit rate requires faster digital processing
  - Bit rate can be reduced by digital filters
- Complexity moved from analog to digital domain
  - Easier & cheaper

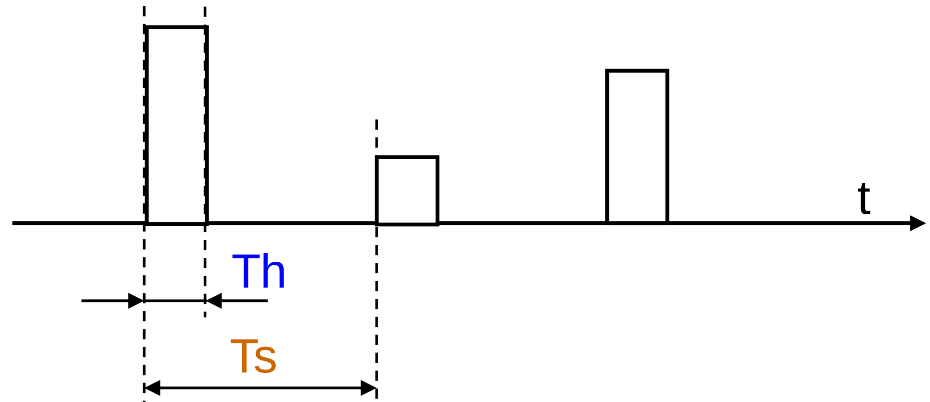
Sample/Hold Module

- The A/D converter operates on each sample
  - Input signal (sample) must be constant during conversion
- Two operations required
  - Sample: read the analog signal value at a specific time
  - Hold: keep that value for some time
  - Sample/Hold (or Track/Hold) unit



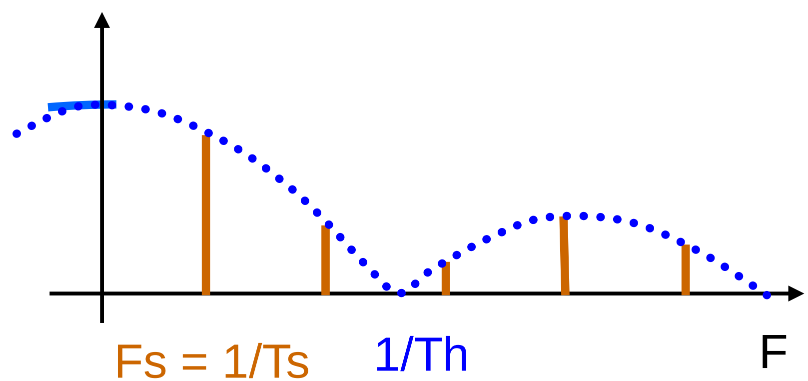
Spectrum of Signals After Hold

- Narrow T_H pulses
  → wider spectrum envelope



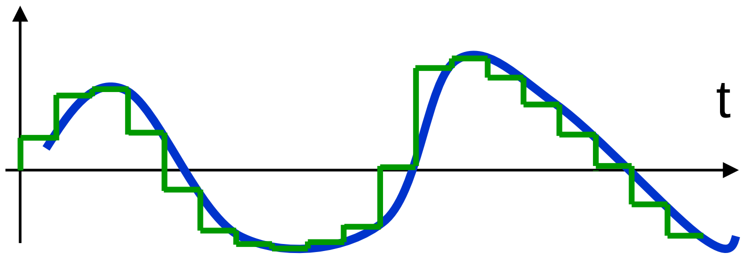
- For $T_H = 0$ (delta)
  → flat envelope

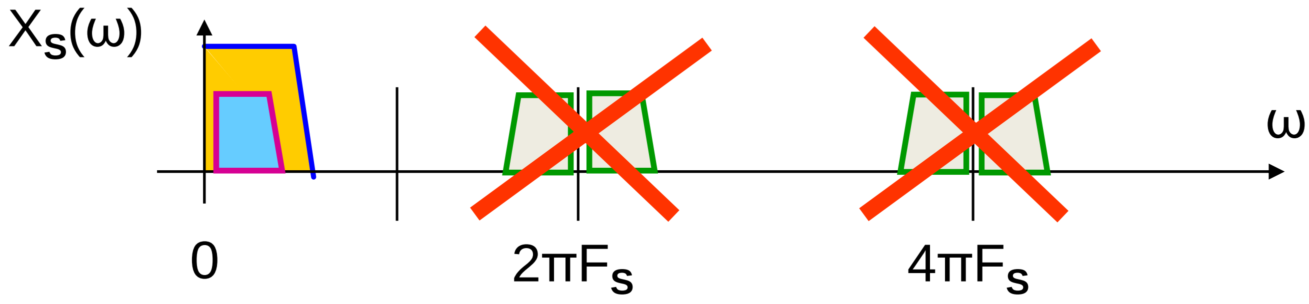
- For $T_H = T_S$ (hold till next sample)
  → the envelope is 0 at $F = F_S$



Reconstruction Filter

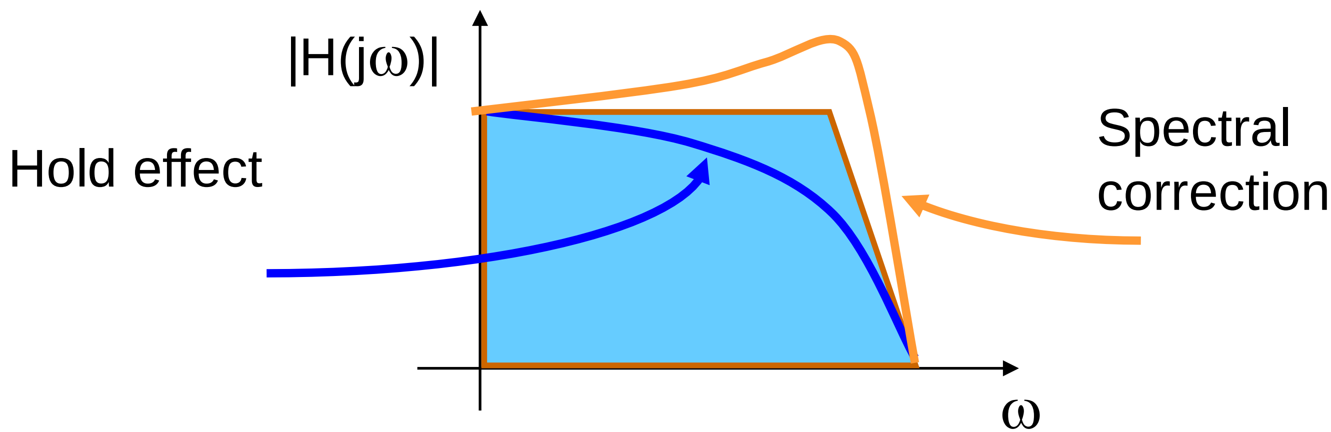
- DAC delivers samples → secondary spectra (aliases)
- Aliases must be removed to get a continuous signal
- Need a low-pass reconstruction filter





Corrected Reconstruction Filter

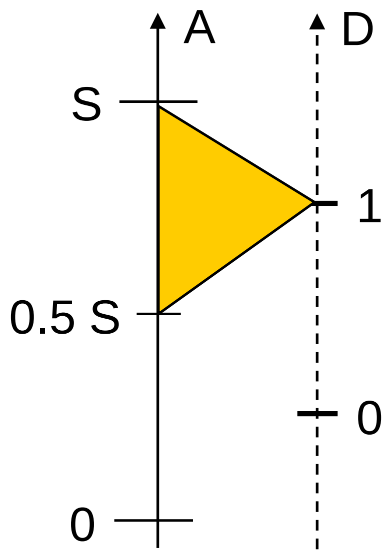
- The reconstruction filter should correct spectral distortion caused by Hold
  - Peaking at high band limit



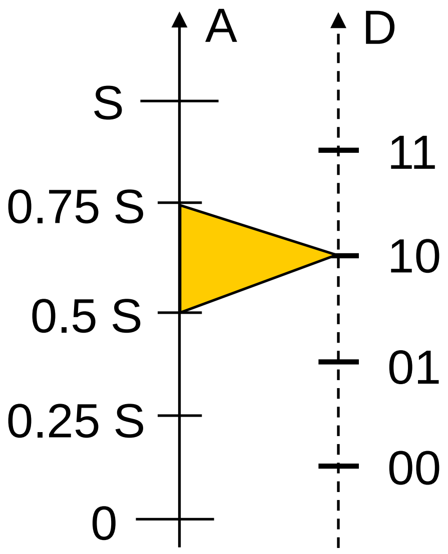
$$H_2(\omega) = H_0(\omega)/H_1(\omega) = e^{j\omega T/2}\left[\frac{\omega}{2\sin(\omega T/2)}\right]$$

Examples: 1- and 2-Bit Quantization

- 1 bit → 2 D values
- Divide A in two intervals
- Each interval corresponds to a digital value (0, 1)

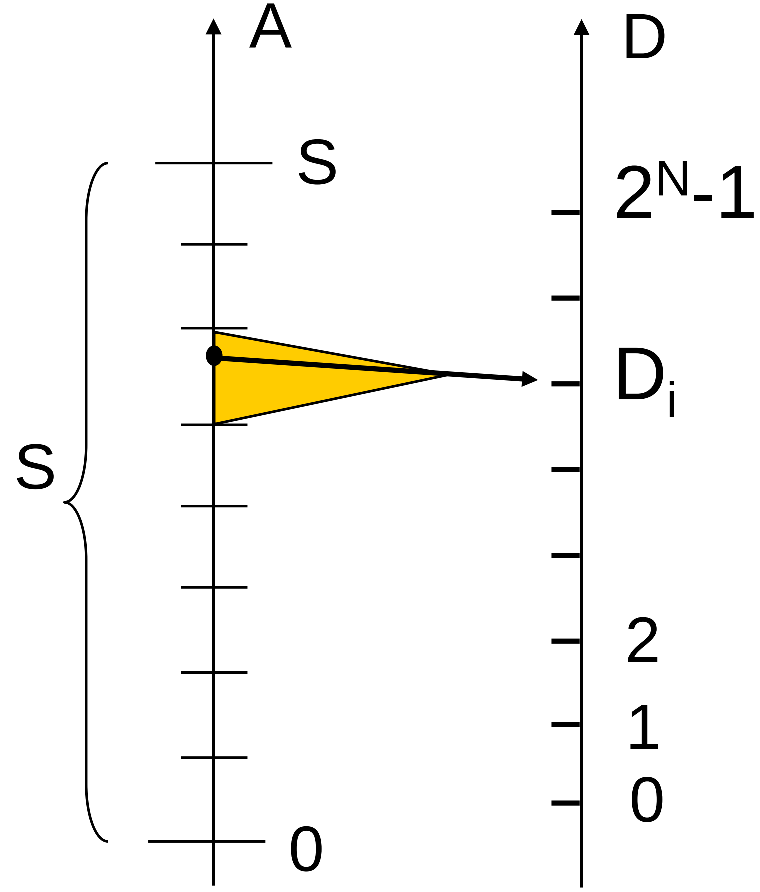


- 2 bit → 4 D values
- Divide A in four intervals
- Each interval maps to a digital value (00, 01, 10, 11)



Quantization (1)

- Analog signal can have any value in the input range (0 … S)
- Digital signal is a sequence of numbers
  - Usually binary with N bits
  - 2^N possible values (0 ... $2^N – 1$)
- D_i defines the signal interval, not the exact value
  - The difference is the quantization error ε_q



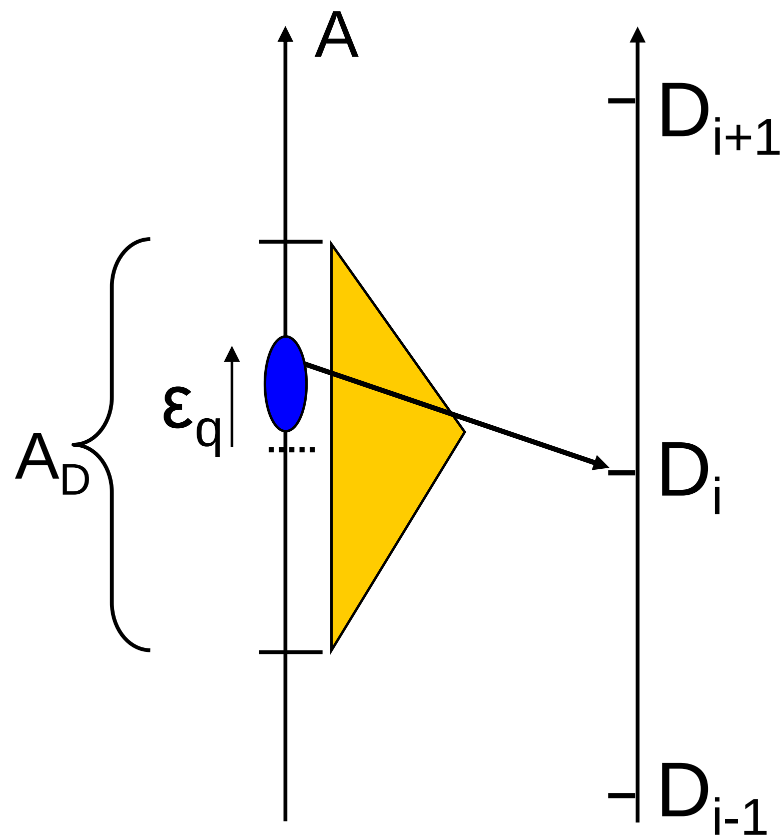
Quantization (2)

- For a range $0 \ldots S$ divided in 2^N intervals, the maximum representation range of A with D_i is

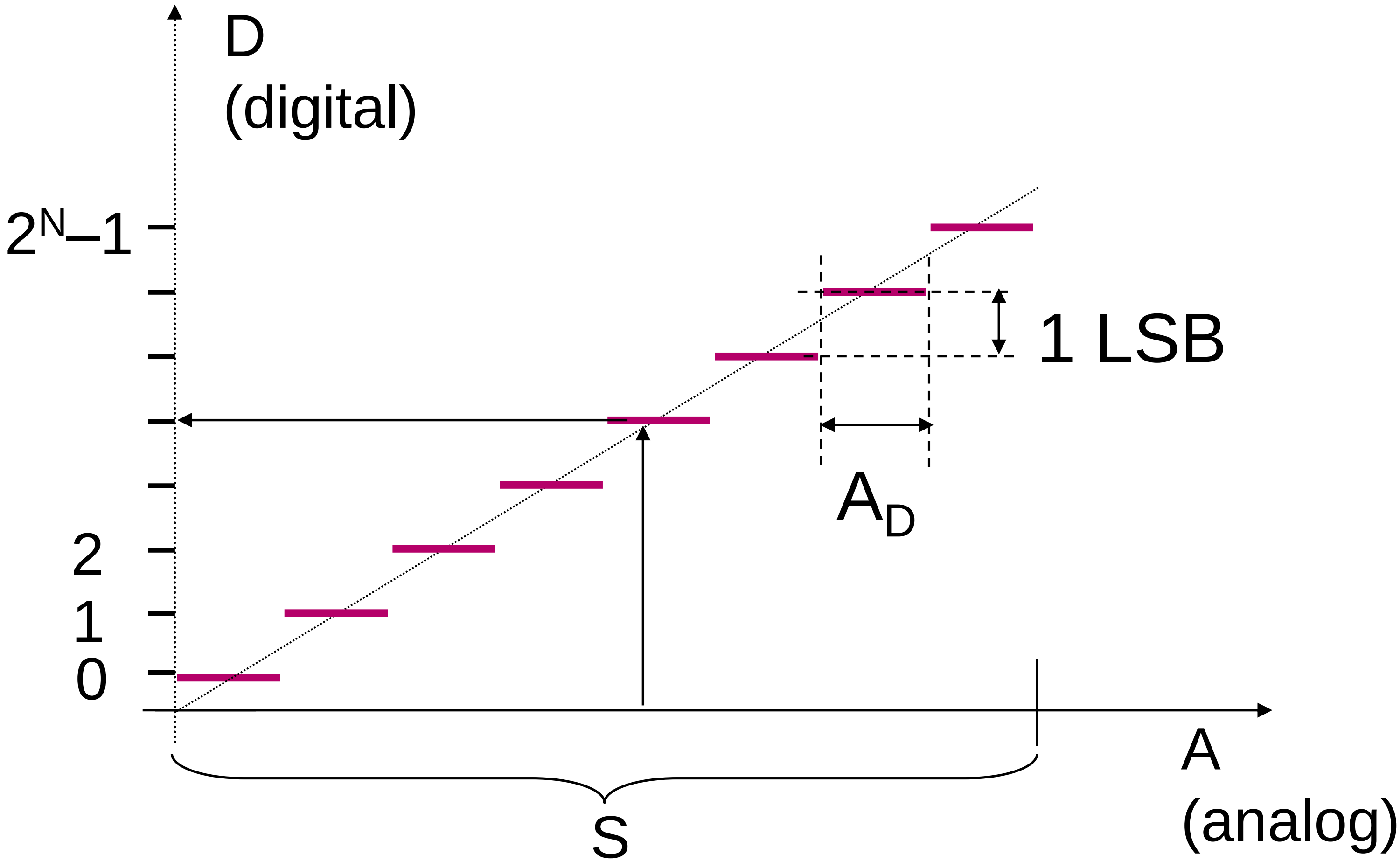
$$A_\mathrm{D} = \frac{S}{2^N} = LSB$$

- The max quantization error $\mathcal{E}_\mathrm{q}$ is

$$|\mathcal{E}_\mathrm{q}| \le \frac{A_\mathrm{D}}{2} = \frac{S}{2^{N+1}}$$

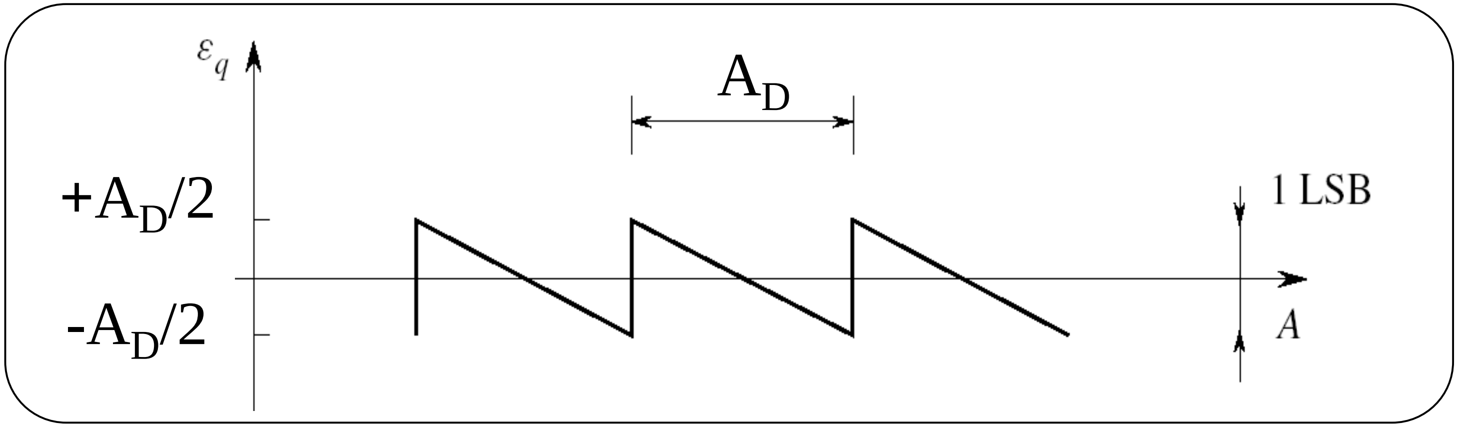


With XY Representation



Quantization Error $\mathcal{E}_\mathrm{q}$

- Same amplitude for all quantization intervals A_D
- $A_\mathrm{D} = \frac{S}{2^N} = 1\ \mathrm{LSB}$
- $\mathcal{E}_\mathrm{q}$ varies within $\pm\frac{A_\mathrm{D}}{2}$ $(1/2\ \mathrm{LSB})$

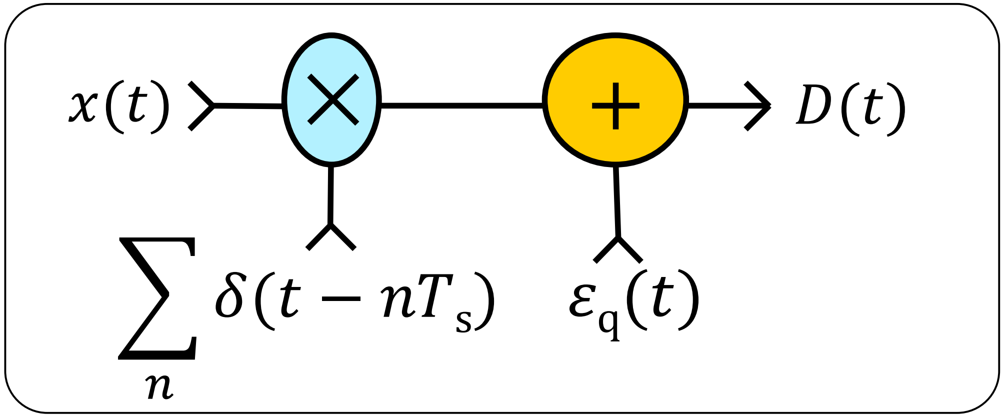


- Maximum value of $\mathcal{E}_\mathrm{q}$:

$$|\varepsilon_{qm}| = \frac{S}{2^{N+1}}$$

Quantization Noise

- Quantization can be seen as noise added to an ideal A → D conversion process



- Which are the features of this “noise”?
- How to define a signal/(quantization noise) ratio SNR_q?
- Which is the relation with the signal and with N?

Quantization Noise Power

- From the noise amplitude distribution

Variance*

$$\sigma^2_{\varepsilon q} = \int_{-A_d/2}^{+A_d/2} \varepsilon_q^2 \rho(\varepsilon_q)\, d\varepsilon_q$$

- Small A_D
  - Constant amplitude distribution of $\mathcal{E}_\mathrm{q}$: $\rho(\mathcal{E}_\mathrm{q}) = \frac{1}{A_\mathrm{D}}$

$$\sigma^2_{\varepsilon q} = \frac{1}{3}\left(\frac{A_d}{2}\right)^3 \frac{2}{A_d};$$

$$\sigma^2_{\varepsilon q} = \frac{A_d^2}{12}$$

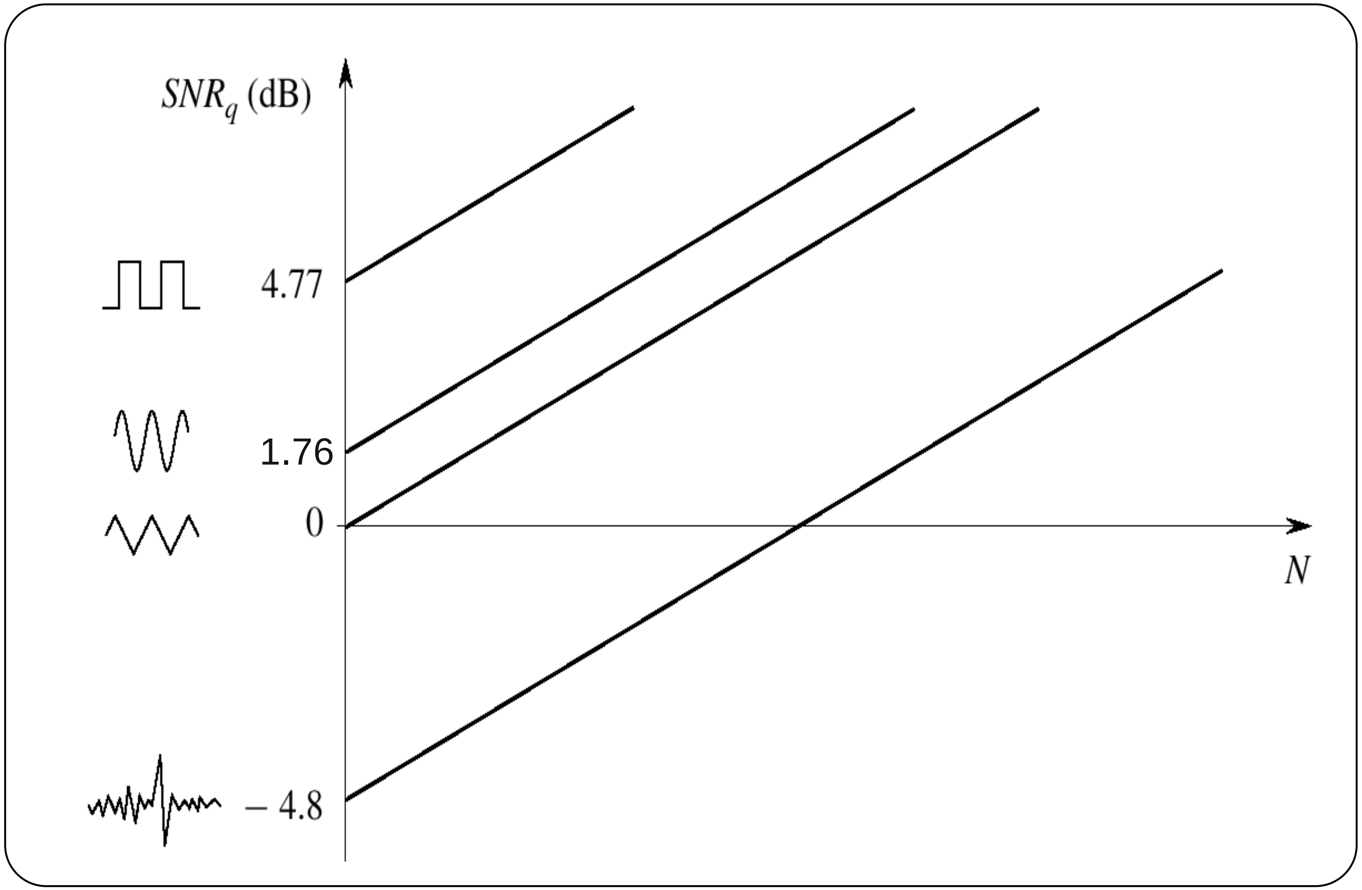
*Variance of random var. X: $\mathrm{Var}(X) = \sigma^2 = \sum(X-\mu)^2/N$, where $\mu = \frac{\sum X}{N}$ is the expected value

Signal to Quantization Noise Ratio

- Defined as $SNR_\mathrm{q} = \dfrac{\text{Signal Power}}{\text{Quantization Noise } \varepsilon_\mathrm{q} \text{ Power}}$
- Noise power is related to full scale S and bit number N
- Signal power is related to waveform and amplitude
- *Triangular* waves (flat distribution, peak-to-peak = S)
  - $P_\mathrm{s} = S^2/12 \quad \rightarrow \quad SNR_\mathrm{q} = 6\,N\ \mathrm{dB}$
- *Sinusoidal* wave (peak-to-peak = S)
  - $P_\mathrm{s} = S^2/8 \quad \rightarrow \quad SNR\mathrm{q} = (6\,N + 1{,}76)\ \mathrm{dB}$
- *Voice* (Gaussian distribution, $S/2 = 3\sigma$)
  - $P_\mathrm{s} = S^2/36 \quad \rightarrow \quad SNR_\mathrm{q} = (6\,N - 4.77)\ \mathrm{dB}$

SNR_{q} and Number of Bits N

- $SNR_{\mathrm{q}} = (K + 6\ N)$ dB: 1 bit adds 6 dB to SNRq



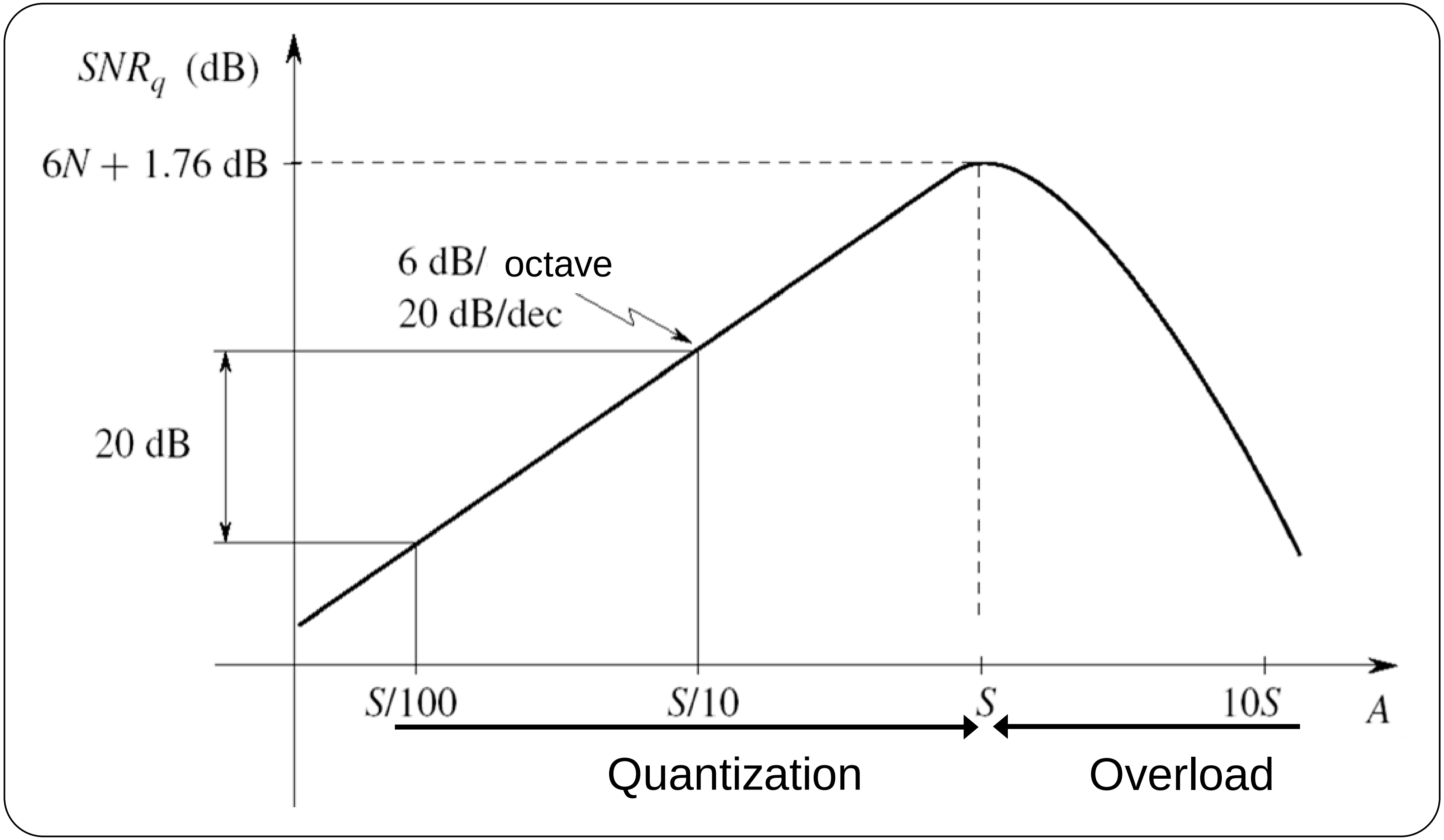
SNR_{q} and Signal Amplitude A

- Previous $SNR\mathrm{q}$ applies only to full-scale (S) signals

- If amplitude $A < S$
  - $SNR\mathrm{q}$ decreases with the signal amplitude (–20 dB/decade or –6 dB/octave)

- If amplitude $A > S$
  - A/D conversion saturates at full scale!
  - Overload condition
  - $SNR\mathrm{q}$ decreases (very quickly) with the increase of the signal amplitude

SNR_{q} vs Signal Amplitude A



Lecture Exercise 2

- Calculate SNR_{q} for *sinusoidal* signals of amplitude $V_{\mathrm{pp}} = S$ for various number of bits N
  - 6 bits $SNR_{\mathrm{q}} =$
  - 8 bits $SNR_{\mathrm{q}} =$
  - 9 bits $SNR_{\mathrm{q}} =$
  - 16 bits $SNR_{\mathrm{q}} =$
- For *sinusoidal* signals of amplitude $V_{\mathrm{pp}} = S/2$:
  - 6 bits $SNR_{\mathrm{q}} =$
  - 9 bits $SNR_{\mathrm{q}} =$

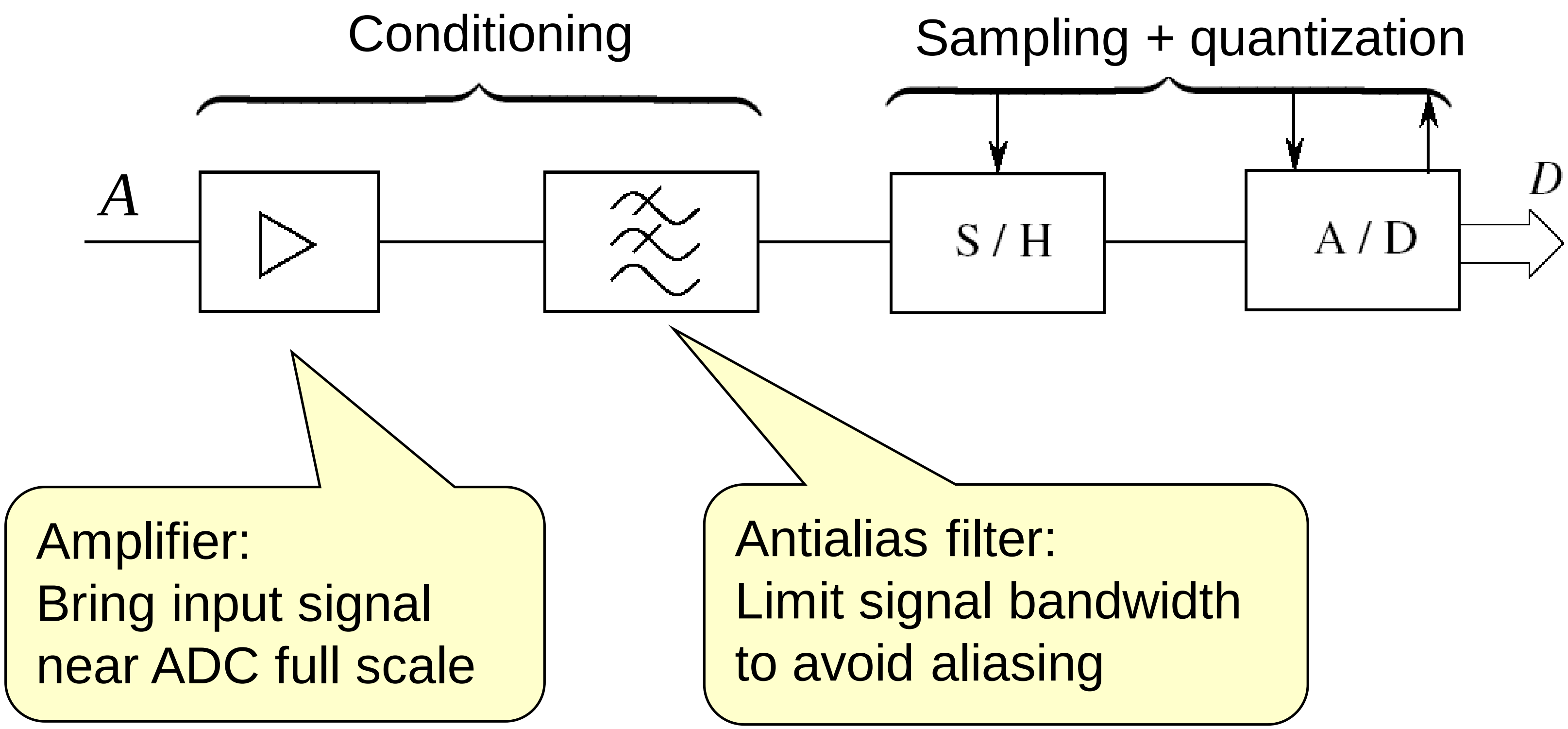
Quantization Noise Spectrum

- Uniform distribution from $0\ \mathrm{Hz}$ up to sampling frequency
  - Spectral power density: $N(f) = \frac{A_{\mathrm{D}}^2}{12 f_S}$
  - If filtered using a band narrower than f_S, the quantization noise power is reduced

A/D Conversion System

- Sampling → Sample/Hold
  - Constraints on signal bandwidth → $f_S > 2\ f_a$
- Quantization → A/D converter
  - Constraints on the signal level → ADC full scale
- Signal conditioning to fit these constraints
  - Amplifier
    - Adapt signal level to ADC full scale
  - Anti-alias filter
    - Limit signal bandwidth
  - Input protection
    - Limit input voltage to avoid damages to the system

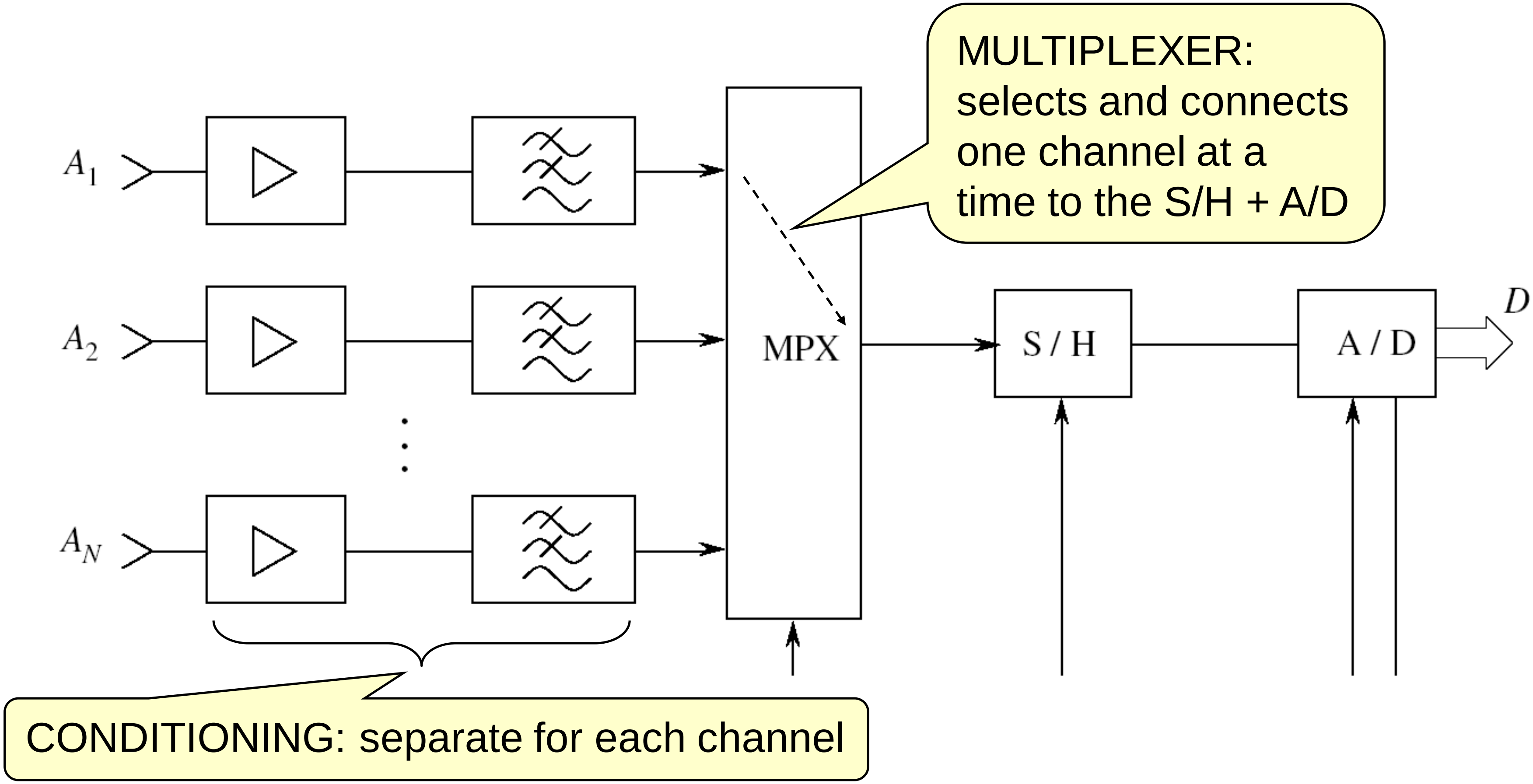
ADC System Block Diagram



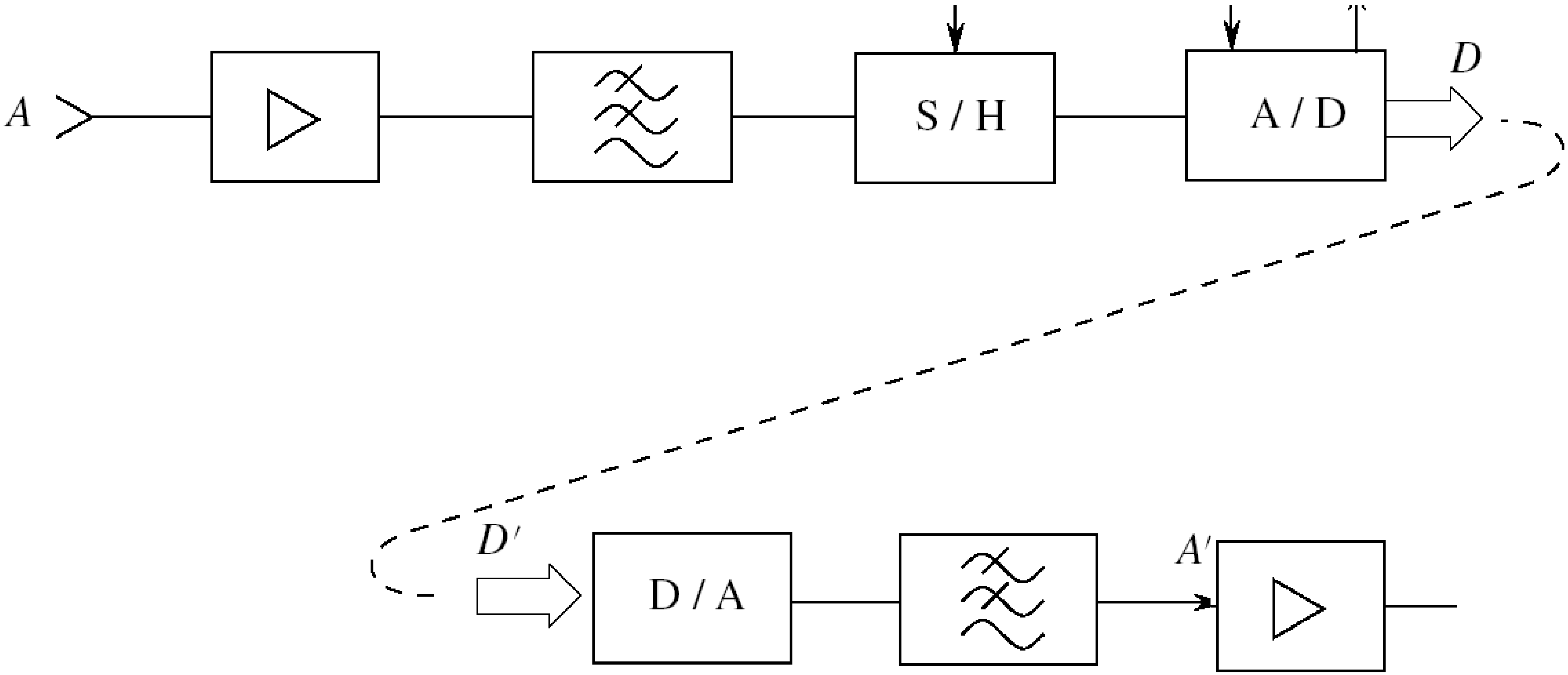
Input protection not shown

Multiple Channel System

- Conditioning specific for each input channel
- Mux connects one input channel at a time to S/H + A/D



Complete A/D – D/A Chain



Total Error

- Each unit introduces errors and noise
  - Amplifier
    - Gain, offset, nonlinearity, band limits
  - Filter
    - Out-of-band signals and noise folded by alias in-band
  - Sample/Hold
    - Sampling jitter
  - A/D converter
    - Quantization error
- Actual accuracy depends on all these elements
  - Not just the bit number N of the A/D

Total Signal-to-Noise Ratio SNR_t

- Key parameter: *total* Signal/Noise ratio, SNR_t
- Caused by
  - Aliasing, quantization, sampling jitter
  - Other errors (amplifier, mux, …)
- Errors are not correlated
  - Find power of single errors
  - Add power of single errors: $P_\mathrm{t} = \sum P_{\mathrm{n}_i}$
  - Evaluate SNR_t

$$\frac{1}{SNR_\mathrm{t}} = -10\lg\left(\sum \frac{P_{\mathrm{n}_i}}{P_\mathrm{S}}\right), \qquad 10\lg\frac{P_\mathrm{S}}{P_{\mathrm{n}_i}} = SNR_i \Longrightarrow \frac{P_{\mathrm{n}_i}}{P_\mathrm{S}} = 10^{-\frac{SNR_i}{10}}$$

Effective Number of Bits: $ENOB$

- SNR_t can be expressed as Equivalent Number Of Bits
  - Computed from SNR_t (measured or evaluated with full-scale sine input signal)
  - $ENOB = \frac{SNR_t - 1.76}{6} = \frac{SNR_t}{6} - 0.3$
  - Includes all noise/error sources (quantization, aliasing, sampling jitter, …)
- Represents the number of *useful* bits of the A/D conversion system

Review Questions

- Why the processing chain includes a low-pass filter?
- What is the function of the amplifier at the input of the conversion chain?
- What determines the aliasing noise and how can lower it?
- How much improves the SNR_{q} if we add 2 bits?
- Describe the relationship between the amplitude of the signal and SNR_{q} is we keep unchanged the full-scale S of the A/D converter.
- Draw the block schematic of a 4-ch. A/D conv. system.
- What parameters give the effective precision of an A/D conversion system?

Exercise 1

- An N-channel acquisition system uses an 8-bit A/D converter that needs 8 μs for a conversion and has an input scale 0 V – 5 V. The sample-and-hold circuit needs 2 μs to acquire and hold stable the input. The frequency band of the input signals is 0 Hz – 5 kHz, and their amplitudes 0 V – 1 V. Each channel uses an amplifier and filter to condition the signal. We need an oversampling factor of 2.5 and aliasing signal-to-noise ratio 3 dB higher than the quantization signal-to-noise ratio.
  - What is the maximum number of input channels?
  - What is the gain and offset of the conditioning amplifier?
  - How many poles needs the antialiasing filter?

Exercise 1: channels

- Minimum sampling time T_S is given by S&H acquisition time and the A/D conversion time
  - $T_S = 2\ \mu s + 8\ \mu s = 10\ \mu s$
- Maximum sampling frequency is
  - $F_S = \frac{1}{T_S} = 100\ \text{kHz}$
- Each channel must be sampled at 2.5 times the Nyquist frequency for a 5 kHz signal band
  - $f_{s_{ch}} = 2 \cdot 5\ \text{kHz} \cdot 2.5 = 25\ \text{kHz}$
- The maximum number of channels is
  - $N = \frac{F_S}{f_{s_{ch}}} = \frac{100\ \text{kHz}}{25\ \text{kHz}} = 4$ channels

Exercise 1: amplifier gain, offset

- The amplifier gain is the ratio between the signal dynamic and A/D input scale
  - $G = \frac{5\ \mathrm{V} - 0\ \mathrm{V}}{1\ \mathrm{V} - 0\ \mathrm{V}} = 5$
- No offset is needed because the average signal offset after amplification is equal to the middle of the A/D input scale.

Exercise 1: antialiasing filter attenuation

- Each channel is sampled at 25 kHz.
- The bandwidth of the signal in each channel is 5 kHz.
- The lowest aliased frequency is thus at
  - 25 kHz – 5 kHz = 20 kHz
- $SNR_{\mathrm{q}} = 6N + 1.76 = 6 \cdot 8 + 1.76 = 49.76\ \mathrm{dB}$
- $SNR_{\mathrm{a}} = 3 + SNR_{\mathrm{q}} = 52.76\ \mathrm{dB}$
- Thus, the antialiasing low-pass filter must start attenuating at 5 kHz (upper signal bandwidth) and reach an attenuation of 52.76 dB at 20 kHz.
  - Between 5 kHz and 20 kHz we have 2 octaves, thus 12 dB/pole
  - For 52.76 dB attenuation we need $52.76/12 = 4.4$ → 5 poles

Exercise 1: antialiasing filter attenuation

- General ways to calculate the number of poles

$$N_\mathrm{P} = \frac{SNR_\mathrm{a}}{20 \cdot \lg \dfrac{f_\mathrm{S} - f_{\mathrm{in}_\mathrm{max}}}{f_{\mathrm{in}_\mathrm{max}}}} = \frac{SNR_\mathrm{a}}{6 \cdot \log_2 \dfrac{f_\mathrm{S} - f_{\mathrm{in}_\mathrm{max}}}{f_{\mathrm{in}_\mathrm{max}}}}$$

$$N_\mathrm{P} = \frac{52.8\ \mathrm{dB}}{20\ \mathrm{dB/dec} \cdot \lg \dfrac{25\ \mathrm{kHz} - 5\ \mathrm{kHz}}{5\ \mathrm{kHz}}} = 4.38 \rightarrow 5\ \mathrm{poles}$$

$$N_\mathrm{P} = \frac{52.8\ \mathrm{dB}}{6\ \mathrm{dB/dec} \cdot \log_2 \dfrac{25\ \mathrm{kHz} - 5\ \mathrm{kHz}}{5\ \mathrm{kHz}}} = 4.4 \rightarrow 5\ \mathrm{poles}$$