1. $f = (\neg p \vee q) \wedge (q \to (\neg r \wedge \neg p)) \wedge (p \vee r)$

p	q	r	$q \to \neg r$	$q \to \neg p$	$\neg p \vee q$	$p \vee r$	
T	T	T	F	F	T	T	
T	T	F	T	F	T	T	
T	F	T	T	T	F	T	
T	F	F	T	T	F	T	
F	T	T	F	T	T	T	
F	T	F	T	T	T	F	
F	F	T	T	T	T	T	0
F	F	F	T	T	T	F	

I know
this is illegal
and technically works
but yeah

The problem is satisfiable. the model of the formula is $\{\neg p, \neg q, r\}$. All interpretation are false for the formula if it is not $\{\neg p; \neg q, r\}$. Ex:

$I_f(\{p, q, r\}) = F$.

2. $\{b \wedge (c \vee a),\ a \to b,\ \neg(b \vee c)\}$

The set is unsatisfiable. There is no model where all the formulas have to be true.
Ex: $b = T$, $a = T$, $c = T$, $\neg(b \vee c) = \neg(T \vee T) = F$

$\neg T \to F$
$F \to F$
T

3. if $U \models A$ and $U \models (\neg A \vee B)$ then $U \models B$ is true.

$A = T$ $\quad U \models A = T \quad U \models (\neg T \vee B)$
$U \models (F \vee B)$
Therefore
$U \models B$

Since to prove it we must assume the if statement is true.

4. Assume A is unsatisfiable, and B is satisfiable
$\neg C \to [(B \vee D) \to A]$ is valid.
Prove / Show C is can not be unsatisfiable

• Since B is satisfiable, we know a model exist where the interpretation is true
$\neg C \to [(T \vee D) \to A]$
• Since A is unsatisfiable
$\neg C \to [T \to F]$
$\neg C \to F$

If C is unsatisfiable, that means there would be a model where c is not true then
$\neg F \to F$
$\neg F \to F$
$T \to F$ would not because

T F ¬T F

$A \to B = \neg A \vee B$, and in our problem that would mean $\neg T \vee F = F \vee F$ which means that $\neg C \to [(B \vee D) \to A]$ is not valid as to be valid it must have all models that are true. Therefore, C cannot be unsatisfiable.

5. Suppose $(A \wedge B) \to C$ is unsatisfiable. ~~that~~ means all models are false

$(A \wedge B) \to C$ is unsatisfiable ~~so~~ it can only be false. Thus for it to be false $(A \wedge B)$ must be true all the time. If A was satisfiable, there would be models where A is false, making $(A \wedge B) \to C$ be satisfiable which breaks our given.

So A must be valid so all models of A are true, to result in $(A \wedge B) \to C$ to have all models that are false, aka unsatisfiable.