HW7

① D = The set of all ints
R = {R1}
R1 = <
p maps to R1

$V_I(\forall x \exists y\, p(x,y)) = T$ since every int x, we can find a int y such that $x<y$

$V_I(\exists y \forall x\, p(x,y)) = F$ because you can't find a int y that's greater than itself and all other ints

Therefore $\forall x \exists y\, p(x,y) \not\equiv \exists y \forall x\, p(x,y)$

② $\underbrace{[\forall x p(x) \vee \exists x q(x)]}_{A} \rightarrow \underbrace{\exists x [p(x) \vee q(x)]}_{B}$

Proof by contradiction:
$V_I(A \rightarrow B) = F$, iff $V_I(A) = T$, $V_I(B) = F$

$V_I(\exists x p(x) \vee q(x)) = F$, means
$V_I(\exists x p(x)) = F$ and $V_I(\exists x q(x)) = F$
since $V_I(\exists x p(x)) = F$, then
$V_I(\forall x p(x)) = F$

Therefore $[\forall x p(x) \vee \exists x q(x)] \rightarrow \exists x [p(x) \vee q(x)]$ is not valid

N - Norwegian
D - Dane
S - Sneers

U - Unfashionable
← predicates

(3) a. $\exists x . \text{Norwegian}(x) \land \text{unfashionable}(x)$

b. $\forall x . \text{Dane}(x) \rightarrow \text{Norwegian}(x)$

c. $\exists x . \text{Sneers}(x, \text{Andrej})$

d. $\exists x . \text{Dane}(x) \land \text{Sneers}(x, \text{Andrej})$

e. $\forall x . N(x) \rightarrow S(x, \text{andrej})$

f. $\forall x \exists y . N(x) \rightarrow S(x, y)$

g. $\forall x . N(x) \land U(x) \rightarrow D(\text{Bjørnson}) \land S(x, \text{Bjørnson})$

h. $\exists x \forall y . N(y) \rightarrow S(y, x)$

i. $\exists x \forall y . U(x) \land D(x) \land ((N(y) \land U(y)) \rightarrow S(x, y))$

j. $\exists x \forall y . D(x) \land U(x) \land ((N(y) \land U(y) \land S(y, \text{Andrej})) \rightarrow S(x, y))$

(4) a. (i) 2 (ii) 1 (iii) 3

b. (i) 1 (ii) 3 (iii) 3 (iv) 2

c. (i) 1 (ii) 1 (iii) 3 (iv) 3

d. (i) 1 (ii) 1 (iii) 3 (iv) 1

11 32

C. (i) 1 (ii) 1 (iii) 3 (iv) 2