Convolutions of kernels

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Let $k^*(x) \stackrel{\text{def}}{=} \int_{\mathbb{R}} k(u)k(x-u)\,\mathrm{d}u$.

1. k is uniform $[-1,1]$ kernel. If $k(u) \stackrel{\text{def}}{=} \frac{1}{2}\mathbb{I}(|u|<1)$, then

$$k^*(x) = \frac{1}{4}(2-|x|), \quad -2 \le x \le 2.$$

2. k is triangular kernel on $[-1,1]$, i.e. $k(u) \stackrel{\text{def}}{=} (1-|u|)\mathbb{I}(|u|\le 1)$. Then

$$k^*(x) = \begin{cases} \frac{1}{6}(4-6|x|^2+3|x|^3), & 0 \le |x| \le 1, \\ \frac{1}{6}(8-12|x|+6|x|^2-|x|^3), & 1 < |x| \le 2. \end{cases}$$

3. k is Epanechnikov kernel on $[-1,1]$, i.e. $k(u) = \frac{3}{4}(1-u^2)\mathbb{I}(|u|\le 1)$. Then

$$k^*(x) = \frac{3}{160}(32-40|x|^2+20|x|^3-|x|^5), \quad -2 \le x \le 2.$$

4. k is quartic kernel on $[-1,1]$, i.e. $k(u) \stackrel{\text{def}}{=} \frac{15}{16}(1-u^2)^2\mathbb{I}(|u|<1)$. Then

$$k^*(x) = \frac{225}{256}\left(\frac{256}{315} - \frac{128}{105}|x|^2 + \frac{16}{15}|x|^4 - \frac{8}{15}|x|^5 + \frac{4}{105}|x|^7 - \frac{1}{630}|x|^9\right), \quad -2 \le x \le 2.$$