# *Introduction to Probability, Second Edition*

Fifthist

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# Preface

This book is an unofficial solution manual for the exercises in *Introduction to Probability, Second Edition* by Joseph Blitzstein and Jessica Hwang.

# Chapter 1

# Probability and Counting

## 1.1 Counting

### 1.1.1

**Intuition**

Imagine eleven empty slots to place the letters into.

How many ways are there to place the four $I$-s into the slots? For each placement of $I$s, can we figure out the number of ways to place the remaining letters into the 7 empty slots?

**Solution**

We have one $M$, four $I$-s, four $S$-s, and two $P$-s. There are $\binom{11}{4}$ ways to place the $I$-s, $\binom{7}{4}$ ways to place $S$-s, $\binom{3}{2}$ ways to place the $P$-s, and $\binom{1}{1}$ ways to place the $M$.

$$\binom{11}{4} \times \binom{7}{4} \times \binom{3}{2} \times \binom{1}{1}$$

### 1.1.2

a. **Intuition**

If the first digit can't be 0 or 1, how many choices are we left with for the first digit? For each choice of first digit, how many choices do we have for the remaining six digits?

**Solution**

If the first digit can't be 0 or 1, we have eight choices for the first digit - 2 to 9. The remaining six digits can be anything from 0 to 9. Hence, the solution is

$$8 \times 10^6$$

b. **Intuition**

How many phone numbers start with 911?

Can we use the answer from the previous part to find the desired quantity?

**Solution**

We can subtract the number of phone numbers that start with 911 from the total number of phone numbers we found in the previous part.

If a phone number starts with 911, it has ten choices for each of the remaining four digits.

$$8 \times 10^6 - 10^4$$

### 1.1.3

a. **Intuition**

How many choices of restaurants does Fred have on Monday?

Once Fred attends a restaurant on Monday, how many choices of restaurants does he have for the remainder of the week?

**Solution**

Fred has 10 choices for Monday, 9 choices for Tuesday, 8 choices for Wednesday, 7 choices for Thursday and 6 choices for Friday.

$$10 \times 9 \times 8 \times 7 \times 6$$

b. **Intution**

We are told that Fred will not attend a restaurant he went to the previous day, but can he go to a restaurant he went to two or more days ago?

**Solution**

For the first restaurant, Fred has 10 choices. For all subsequent days, Fred has 9 choices, since the only restriction is that he doesn't want to eat at the restaurant he ate at the previous day.

$$10 \times 9^4$$

### 1.1.4

a. **Intuition**

How many matches are there in a *round-robin* tournament?

How many outcomes are possible for each match?

**Solution**

There are $\binom{n}{2}$ matches.

For a given match, there are two outcomes. Each match has two possible outcomes. We can use the multiplication rule to count the total possible outcomes.

$$2^{\binom{n}{2}}$$

b. **Intuition**

How many opponents will every player play against?

How many times will a given pair of players face each other?

**Solution**

Since every player plays every other player exactly once, the number of games is the number of ways to pair up $n$ people.

$$\binom{n}{2}$$

### 1.1.5

a. **Intuition**

How many players are left by the end of a round compared to the number of players at the start of the round?

How many rounds need to pass for a single player to be left standing?

**Solution**

By the end of each round, half of the players participating in the round are eliminated. So, the problem reduces to finding out how many times the number of players can be halved before a single player is left.

The number of times $N$ can be divided by two is

$$\log_2 N$$

b. **Intuition**

Suppose there are $N_r$ players at the start of round $r$. If every player plays exactly one game, how many games will be played in round $r$?

**Solution**

The number of games in a given round is $\frac{N_r}{2}$. We can sum up these values for all the rounds.

$$\begin{aligned} f(N) &= \frac{N}{2} + \frac{N}{4} + \frac{N}{8} + \cdots + \frac{N}{2^{\log_2 N}} \\ &= N \sum_{i=0}^{\log_2 N} \frac{1}{2^i} \\ &= N \times \frac{N-1}{N} \\ &= N - 1 \end{aligned} \tag{1.1}$$

c. **Intuition**

How many players need to be eliminated before the tournament is over?

How many players are eliminated as a result of a single match?

**Solution**

Tournament is over when a single player is left. Hece, $N-1$ players need to be eliminated. As a result of a match, exactly one player is eliminated. Hence, the number of matches needed to eliminate $N-1$ people is

$$N - 1$$