Introduction to Probability, Second Edition

Fifthist

2020-03-04

Contents

Preface

This book is an unofficial solution manual for the exercises in *Introduction to Probability, Second Edition* by Joseph Blitzstein and Jessica Hwang.

Chapter 1

Probability and Counting

1.1 Counting

1.1.1

Intuition

There are 11 slots to put letters into. We have one M, four I, four S, and two P. Then, there are $\binom{11}{1}$ ways to place the M, $\binom{10}{4}$ ways to place I, $\binom{6}{4}$ ways to place the S, and $\binom{2}{2}$ ways to place the P.

Solution

$$\binom{11}{1} \times \binom{10}{4} \times \binom{6}{4} \times \binom{2}{2}$$

1.1.2

a. **Intuition**

If the first digit can't be 0 or 1, we are left with 8 choices for the first digit. The remaining six digits can by any digits.

Solution

$$8 \times 10^6$$

b. **Intuition**

We can subtract the number of seven digits phone numbers that start with 911 from the total number of phone numbers we found in the previous part.

If a phone number starts with 911, it has ten choices for each of the remaining four digits.

Solution

$$8 \times 10^6 - 10^4$$

1.1.3

a. **Intuition**

Fred has 10 choices for Monday, 9 choices for Tuesday, 8 choices for Wednesday, 7 choices for Thursday and 6 choices for Friday.

Solution

$$10 \times 9 \times 8 \times 7 \times 6$$

b. **Intution**

For the first restaurant, Fred has 10 choices. For all subsequent days, Fred has 9 choices, sinces he doesn't want to eat at the restaurant he ate at the previous day.

Solution

$$10 \times 9^4$$

1.1.4

a. **Intuition**

There are $\binom{n}{2}$ matches in a round-robin setting. For a given match, there are two outcomes. Irrespective of the outcome, the next match also has two possible outcomes. Hence, we can use the multiplication rule to count the total possible outcomes.

Solution

$$2^{\binom{n}{2}}$$

b. **Intuition**

Since every player plays every other player exactly once, the number of games is the number of ways to pair up n people.

Solution

$$\binom{n}{2}$$

1.1.5

a. **Intuition**

By the end of each round, half of the players participating in the round are eliminated. So, the problem reduces to finding out how many times can the number of players be divided by two until a single player is left.

Solution

The number of times N can be divided by two is

$$\log_2 N$$

b. **Intuition**

In a given round, two opponents participate in only that one match. Thus, the number of games in a given round is $\frac{N_r}{2}$ where N_r is the number of players in the r-th round.

Solution

$$\begin{aligned} f(N) &= \frac{N}{2} + \frac{N}{4} + \frac{N}{8} + \cdots + \frac{N}{2^{\log_2 N}} \\ &= N \sum_{i=0}^{\log_2 N} \frac{1}{2^i} \\ &= N \times \frac{N-1}{N} \\ &= N - 1 \end{aligned} \tag{1.1}$$

c. **Intuition**

Tournament is over when a single player is left. Hece, $N-1$ players need to be eliminated. As a result of a match, exactly one player is eliminated. Hence, $N-1$ matches are needed to eliminate $N-1$ people.

Solution

$$N - 1$$