CSC 279/479 Computational Geometry - homework 1
due: Sep 11th, 11:59pm ET, submit on Gradescope

PROBLEM 1—line with more than two points?

Given n points $(x_1, y_1), \ldots, (x_n, y_n)$ in the plane we want to check whether there exists a line that contains more than 2 of the points. Give an $O(n^2 \log n)$ algorithm for the problem (to make the problem more interesting don't use hashing).

PROBLEM 2—searching for a point.

We are given two strictly increasing arrays of n numbers: $a[1] < a[2] < \cdots < a[n]$ and $b[1] < b[2] < \cdots , b[n]$. Segments $(a[i], 0)$-$(b[i], 1)$ for $i \in n$ partition the strip $(-\infty, \infty) \times [0, 1]$ into $n + 1$ parts. Given a point $p = (x, y)$ in the strip (that is $y \in [0, 1]$) we want to find which part does p belongs to. Show that binary search solves the problem (in $O(\log n)$ time). Write pseudocode for the binary search (include the invariant (assertion) that holds during the execution of your pseudocode).

PROBLEM 3—area of a simple polygon.

Consider a simple polygon P given (in the clockwise order on the boundary of P) by points $(x_0, y_0), \ldots, (x_{n-1}, y_{n-1})$ (for convenience let $x_n := x_0$ and $y_n := y_0$). Which of the following expressions yields the area of the polygon?

$$\frac{1}{2}\Big|\sum_{i=0}^{n}(x_i y_{i+1} - y_i x_{i+1})\Big| \tag{1}$$

$$\frac{1}{2}\sum_{i=0}^{n}\Big|x_i y_{i+1} - y_i x_{i+1}\Big| \tag{2}$$

$$\frac{1}{2}\Big|\sum_{i=0}^{n}(x_i + x_{i+1})(y_{i+1} - y_i)\Big|. \tag{3}$$

PROBLEM 4—left-most point

Consider a convex polygon given (in the clockwise order on the boundary of P) by points $(x_0, y_0), \ldots, (x_{n-1}, y_{n-1})$ (for convenience let $x_n := x_0$ and $y_n := y_0$). Give an $O(\log n)$ algorithm to find the left-most point of P (that is the point with the smallest x coordinate). Write pseudocode for your algorithm.

PROBLEM 5—line maximizing reflecting pairs (CSC 479 only; bonus for CSC 279).

We are given n points $(x_1, y_1), \ldots, (x_n, y_n)$ in the plane. We want to find a line ℓ that maximizes the number of pairs of points that are each other's reflection across ℓ (two points p and q are each other's reflection across ℓ if the segment pq is perpendicular to ℓ and ℓ intersects the segment pq in the middle). Give $O(n^2 \log n)$ algorithm for the problem.