

MATH 265 Homework 3

Due Sep 19

Instructions:

- Please scan your work and upload it to Gradescope by **the end of Sep 19**.

1 Non-Graded Questions

Textbook Section 2.1: Questions 1, 4, 10, 13, 18

Textbook Section 2.2: Questions 1, 2, 3, 10, 18

2 Graded Questions

1. (3 points) Suppose that for $a, b \in \mathbb{R}$ we have $a \geq b - \varepsilon$ for all $\varepsilon > 0$. Prove that $a \geq b$.
2. (3 points) For $a \in \mathbb{R}$, prove that $(-a)^2 = a^2$.

Hint Recall that $-a$ is the additive inverse of a . Apply algebraic properties of \mathbb{R} to show that

$$a^2 - b^2 = (a + b)(a - b),$$

and then put $b = -a$.

3. (4 points) Prove that

$$\left[\frac{1}{2}(a + b) \right]^2 \leq \frac{1}{2}(a^2 + b^2),$$

for all $a, b \in \mathbb{R}$. Show that equality holds iff $a = b$.