MATH 265 HW2

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Question 1

Proof. First lets check the base case. For $n = 1$:

$$\frac{1}{\sqrt{1}} = 1 > \frac{1^{\frac{3}{2}}}{3} = \frac{1}{3}$$

Thus, the base case holds.
For forming up the inductive hypothesis, assume the statement is true for somme $n = k$,

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{k}} \geq \frac{k^{\frac{3}{2}}}{3}$$

Now for the inductive step, we need to show the statement holds for $n = k + 1$,

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}} \geq \frac{(k+1)^{\frac{3}{2}}}{3}$$

From inductive hypothesis, we can add $\frac{1}{\sqrt{k+1}}$ both side:

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}} \geq \frac{(k)^{\frac{3}{2}}}{3} + \frac{1}{\sqrt{k+1}}$$

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