

# MATH 265 HW4

Hanzhang Yin

Sep/18/2024

## Question 1

*Proof.* Given  $w$  is a lower bound of  $S$ , by definition:

$$w \leq s, \quad \forall s \in S.$$

Since  $w \in S$ , there exists a  $s \in S$  s.t.  $s = w$ . Then,

$$w \leq s = w \implies w = s.$$

Suppose there exists another lower bound  $v \in \mathbb{R}$  of  $S$  such that  $v > w$ . However, since  $w \in S$ , it must satisfy:

$$v \leq w.$$

we derived a contradiction. Therefore, no such  $v$  exists, and  $w$  must be the g.l.b. of  $S$  □

## Question 2

*Proof.* First, we want to show  $\inf(A) = m$  is an upper bound of  $-A$ .

Let  $\inf(A) = m$ , by definition of infimum,  $m \leq a$  for all  $a \in A$ .

$$\Rightarrow -m \geq -a, \forall a \in A$$

By our definition of set  $-A$ , this means  $-\inf(A)$  is an upper bound of  $-A$ .

Following that, now we can show  $-\inf(A)$  is the l.u.b of  $-A$ :

Assume there exists an upper bound  $u \in -A$  s.t.  $u < -\inf(A) = -m$ . Then:

$$u < -m \rightarrow -u > m$$

Since  $\inf(A) = m$ , for  $\epsilon = -u - m > 0$ , there exists  $a_\epsilon \in A$  s.t.

$$a_\epsilon < m + \epsilon = m + (-u - m) = -u \Rightarrow -a_\epsilon > u$$

Hence,  $u$  is NOT an upper bound of  $-A$ , and  $-m \leq u$  always. This shows that  $-\inf(A) = \sup(-A)$ . □

### Question 3

(a)

*Proof.* Since  $A \cap B \subseteq A$  and  $A \cap B \subseteq B$ , any upper bound of  $A$  or  $B$  is also an upper bound of  $A \cap B$ .

Let  $\sup A = u$ . By definition,  $u$  is the least upper bound of  $A$ , hence  $u$  is an upper bound for  $A \cap B$ . Similarly, let  $\sup B = w$ . Then  $w$  is an upper bound for  $A \cap B$ .

Since  $\sup(A \cap B)$  is the least upper bound of  $A \cap B$ , it must satisfy:

$$\sup(A \cap B) \leq u = \sup A \quad \text{and} \quad \sup(A \cap B) \leq w = \sup B.$$

Therefore,

$$\sup A, \sup B \geq \sup(A \cap B).$$

□

(b)