MATH 265 Homework 1

Due Sep 5

Instructions:

Homeworks are split into two parts. The first consists of easier questions designed to reinforce your understanding of definitions and results discussed in class. These **will not** be graded, but are **very much worth** your time to complete. The second part will often consist of more conceptual questions for which you will be asked to submit formal proofs, which **will** be graded. Your submitted homework only needs to contain write-ups of the graded questions.

- Please scan your work as a single PDF file and upload it to Gradescope by **the end of Jan. 27**.
- You should justify all claims in your work unless they have been mentioned in class or another homework problem.

1 Non-Graded Questions

Textbook Section 1.1: Questions 1, 4, 6, 7, 11, 17, 19

2 Graded Questions

1. (2 points) Find an explicit bijection from the set $\mathbb{N} = \{1, 2, 3, \dots\}$ to the set $\{1, 4, 7, 10, 13, \dots\}$ of positive integers equivalent to 1 mod 3. Prove that your defined function is a bijection.
2. (2 points) Find an explicit bijection from the set $\{x \in \mathbb{R} : 0 < x < 1\}$ to the set $\mathbb{R}_{>0} = \{x \in \mathbb{R} : x > 0\}$. Prove that your defined function is indeed a bijection.
3. (3 points) Find an explicit bijection from the set $\{x \in \mathbb{R} : 0 \leq x < 1\}$ to the set $\mathbb{R}_{>0} = \{x \in \mathbb{R} : x > 0\}$. Prove that your defined function is indeed a bijection.

   Hint The function you come up here will not end up being continuous (of course, we'll rigorously define continuity later in the semester). See if you can produce a "slight" modification of your answer to the previous question which will work. A key question to answer is " Where should this bijection send 0?"
4. (3 points) Prove that for any sets A, B, C, if $f : A \to B$ and $g : B \to C$ are both surjective, then their composition $g \circ f$ is also surjective. Similarly, prove that the composition of injective functions is injective.