# MATH 265 Homework 2

Due Sep 12

- Please scan your work and upload it to Gradescope by **the end of Sep 12**.

## 1 Non-Graded Questions

Textbook Section 1.2: Questions 6, 13, 18
Textbook Section 1.3: Questions 3, 11, 12

## 2 Graded Questions

1. (2 points) Use the principle of mathematical induction to prove that

$$\frac{1}{\sqrt[3]{1}} + \frac{1}{\sqrt[3]{2}} + \ldots + \frac{1}{\sqrt[3]{n}} \geq n^{\frac{2}{3}}$$

for all $n \in \mathbb{N}$.

2. (2 points) Let $x_0 = 3$ and for a given $x_n$, suppose

$$x_{n+1} = \frac{1}{8}x_n^2 + 2$$

for all $n \in \mathbb{N} \cup \{0\}$. Prove by induction that $x_n < x_{n+1} < 4$ for all $n \in \mathbb{N} \cup \{0\}$.

3. (3 points) The Fibonacci numbers are defined by the recurrence relation

$$F_1 = 1, \quad F_2 = 1,$$

and

$$F_n = F_{n-1} + F_{n-2}$$

for $n \geq 3$. Prove using induction that

$$F_{m+k} = F_{m-1}F_k + F_mF_{k+1}$$

for all $k, m \in \mathbb{N}$ with $m \geq 2$.

4. (3 points) A real number $z$ is said to be *algebraic* if for some $n \geq 1$, there are integers $a_0, a_1, \ldots, a_n$ with $a_n \neq 0$, such that

$$a_nz^n + a_{n-1}z^{n-1} + \ldots + a_1z + a_0 = 0.$$

Prove that the set of all algebraic numbers is countably infinite.

**Hint** : For every positive integer $N$ there are only finitely many equations with $n + |a_0| + |a_1| + \ldots + |a_n| = N$.