MATH 265 Homework 8

Due Nov 14

Instructions:

- Please scan your work and upload it to Gradescope by **the end of Nov 14**.

1 Non-Graded Questions

Textbook Section 3.5: Questions 2, 3, 5, 8
Textbook Section 3.6: Questions 1, 3, 6
Textbook Section 3.7: Questions 2, 4, 9, 12, 15
Textbook Section 4.1: Questions 1, 3, 6, 8, 11a, 16, 17

2 Graded Questions

1. (2 points) Let (x_n) be a sequence such that $|x_{n+1} - x_n| \leq 2^{-n}$ for all $n \in \mathbb{N}$. Prove that (x_n) is a Cauchy sequence.

2. (2 points) Consider the series:
$$\sum_{n=0}^{\infty} \frac{x^n}{n!}.$$
Prove that for any real number x this series is convergent.

3. (3 points) Let $E \subset [0,1]$ be the set of all real numbers $0 \leq x \leq 1$ such that a decimal representation of x has at least one entry which is a 7.
   (a) Draw a 'rough' sketch of what the set E looks like on a number line between 0 and 1.
   (b) Prove that $\frac{1}{2}$ is a cluster point of E. (In fact, every number between 0 and 1 is a cluster point of E.)

4. (3 points) Let
$$f(x) = \frac{x^4 - 2x^3 + x^2 - 1}{3x^6 + x^3 + 1}.$$
Use an $\epsilon - \delta$ argument to show that
$$\lim_{x \to -1} f(x) = 1.$$