MATH 265 Homework 5

Due Oct 3

Instructions:

- Please scan your work and upload it to Gradescope by **the end of Oct 3**.

1 Non-Graded Questions

Textbook Section 2.4: Questions 1, 4, 7, 13, 18
Textbook Section 2.5: Questions 3, 10, 14

2 Graded Questions

1. (3 points) Consider the set
$$S = \left\{ (-1)^n \left(1 - \frac{1}{n} : n \in \mathbb{Z}_+ \right) \right\},$$
where $\mathbb{Z}_+$ is a set of positive integers.
   (a) Show that 1 is an upper bound for S.
   (b) Show that if M is an upper bound for S, then $M \geq 1$.
   (c) Conclude that $\sup S = 1$.

2. (2 points) Let A, B be subsets of $\mathbb{R}$. Prove that $\sup(A + B) = \sup A + \sup B$.

3. (2 points) Show that if $x \in \mathbb{R}$ with $x > 1$, then the set $\{x^n : n \in \mathbb{Z}_+\}$ is not bounded from above.

4. (3 points) Construct a family of half-open intervals of the form $I_n = [a_n, b_n)$ such that both of the following properties hold.
$$(1, 4) = \bigcup_{n=1}^{\infty} I_n, \quad [2, 3] = \bigcap_{n=1}^{\infty} I_n.$$
Prove that both properties are satisfied by the family you've chosen.