MATH 265 HW6

Hanzhang Yin

Oct/7/2024

Question 1

Proof. Let $\epsilon > 0$, Consider:

$$N = \max\left\{1, \left\lceil \frac{1}{\left(\frac{\sqrt{2}}{\sqrt{2}+\epsilon}\right)^2 - 1} \right\rceil\right\} \in \mathbb{N}$$

Then for $n > N$:

$$n > \frac{1}{\left(\frac{\sqrt{2}}{\sqrt{2}+\epsilon}\right)^2 - 1}$$

$$\Rightarrow \frac{1}{n} < \left(\frac{\sqrt{2}}{\sqrt{2}+\epsilon}\right)^2 - 1 \Rightarrow 1 + \frac{1}{n} < \left(\frac{\sqrt{2}}{\sqrt{2}+\epsilon}\right)^2$$

$$\Rightarrow \sqrt{1+\frac{1}{n}} < \frac{\sqrt{2}}{\sqrt{2}+\epsilon} \Rightarrow \frac{\sqrt{2}}{\sqrt{1+\frac{1}{n}}} < \sqrt{2}+\epsilon$$

$$\Rightarrow \left|\frac{\sqrt{2}}{\sqrt{1+\frac{1}{n}}} - \sqrt{2}\right| < \epsilon \Rightarrow \left|\frac{\sqrt{2n}}{\sqrt{n+1}} - \sqrt{2}\right| < \epsilon$$

Hence, by definition, $\lim_{n\to\infty} \frac{\sqrt{2n}}{\sqrt{n+1}} = \sqrt{2}$ $\square$

Question 2

Proof. Let $\epsilon > 0$, (Note: $\frac{1}{n^2} - 1 < \frac{-1}{n^2}$), Consider:

$$N = \max\left\{1, \left\lceil \frac{1}{n^2} - 1 \right\rceil\right\} \in \mathbb{N}$$

Then for $n > N$:

$\square$