

Project 2 Proposal

Hanzhang Yin

November 8, 2024

In this project, we will test our algorithms using examples from physics and mathematics, including univariate polynomial equations, a system of polynomial equations, and a non-polynomial equation for testing Newton's, Bisection and the secant methods for root approximations. Each example includes a brief explanation of its scientific background and the potential challenges it presents for numerical methods.

Univariate Polynomial Equations

Polynomial 1: Lagrange Points in Orbital Mechanics

In celestial mechanics, Lagrange points are positions where a small body can maintain its position relative to two larger bodies due to gravitational forces. The L_1 Lagrange point is found by solving a quintic polynomial:

$$x^5 - (3 + \mu)x^4 + (3 + 2\mu)x^3 - \mu x^2 + 2\mu x - \mu = 0,$$

where x is the normalized distance and μ is the mass ratio of the two bodies. This example is chosen for its complexity and the potential for numerical issues when μ approaches zero, leading to multiple or closely spaced roots.

Degeneration: As $\mu \rightarrow 0$, the polynomial simplifies, which we will connect to the original equation by analyzing numerical stability and how root multiplicity affects convergence.

Polynomial 2: Ideal Gas Law For Real Gas

The modified ideal gas law for real gases can be expressed as:

$$PV^3 - (Pb + RT)V^2 + aV - ab = 0,$$

where P is pressure, T is temperature, R is the gas constant, and a , b are gas-specific constants. This cubic polynomial can exhibit different root structures under varying conditions, such as near the critical point where multiple real roots merge.

Degeneration: Approaching the **critical point** might render multiple real roots converging, challenging numerical solvers with potential ill-conditioned equations.

System of Polynomial Equations

Example: Coupled Spring-Mass System

In a physics-based spring-mass system with two masses connected by springs, the equations of motion can be represented as:

$$k_1x_1 - k_2(x_2 - x_1) = m_1a_1,$$

$$k_2(x_2 - x_1) + k_3x_2 = m_2a_2,$$

where k_1, k_2 , and k_3 are spring constants, x_1 and x_2 are displacements, and a_1 and a_2 are accelerations. This example is straightforward and allow testing for potential singular behaviors when k_2 is small or approaches zero.

Non-Polynomial Equation

Example: Sinusoidal Motion in Spring Displacement

The motion of a mass-spring system can be described by:

$$x(t) = 5 \sin(2t),$$

or for finding specific points ($x = 3$ in this case):

$$5 \sin(2t) - 3 = 0.$$

This non-polynomial equation will be used to test Newton's method and the secant method. Solving for t involves periodic behavior, which can pose challenges for convergence due to the trigonometric nature of the function and the need for good initial guesses.

Anticipated Challenges and Degeneracies

The Lagrange polynomial and the modified ideal gas law equation present cases with potential singularities and closely spaced roots, which can slow down the convergence. The non-polynomial trigonometric equation may experience convergence issues if the initial guesses are far from the actual roots due to the oscillatory behavior (For Newton's Method Especially).