```
subs := {omega0 → Sqrt[k / m], zeta → c / (2 * Sqrt[m * k]), x0 → -g * m / k}

xt = A * Exp[-zeta * omega0 * t] * Sin[Sqrt[1 - zeta^2] * omega0 * t + phi] - g / omega0^2
```

$$-\frac{g}{omega0^2} + A\, e^{-omega0\, t\, zeta}\, Sin\left[phi + omega0\, t\, \sqrt{1 - zeta^2}\,\right]$$

```
vt = FullSimplify[D[xt, t]]
```

$$A\, e^{-omega0\, t\, zeta}\, omega0$$
$$\left(\sqrt{1 - zeta^2}\, Cos\left[phi + omega0\, t\, \sqrt{1 - zeta^2}\,\right] - zeta\, Sin\left[phi + omega0\, t\, \sqrt{1 - zeta^2}\,\right]\right)$$

```
energy = (m * g * x + k * x^2 / 2 + m * v^2 / 2 == m * g * (x0 - A) + k * (x0 - A)^2 / 2) /. subs
```

$$\frac{m\, v^2}{2} + g\, m\, x + \frac{k\, x^2}{2} == g\, m\left(-A - \frac{g\, m}{k}\right) + \frac{1}{2}\, k\left(-A - \frac{g\, m}{k}\right)^2$$

```
Solve[energy, A]
```

$$\left\{\left\{A \to -\frac{\sqrt{g^2\, m^2 + k\, m\, v^2 + 2\, g\, k\, m\, x + k^2\, x^2}}{k}\right\}, \left\{A \to \frac{\sqrt{g^2\, m^2 + k\, m\, v^2 + 2\, g\, k\, m\, x + k^2\, x^2}}{k}\right\}\right\}$$

```
amplitude =
```
$$\frac{\sqrt{k\, m\, v^2 + (g\, m + k\, x)^2}}{k}$$

$$\frac{\sqrt{k\, m\, v^2 + (g\, m + k\, x)^2}}{k}$$

```
getphi[A_, isUp_] := If[isUp, ArcSin[(x + g / omega0^2) / A], π - ArcSin[(x + g / omega0^2) / A]]
```

```
experiment := {g → 9.8, m → 3, k → 2, c → 0.3}

initials := {x → -5, v → 3}

amplitude /. experiment /. initials
```
10.3726

```
getphi[amplitude /. experiment /. initials, True] /. subs /. experiment /. initials
```
1.20871

```
Minimize[{((xt + 5)^2 + (vt - 3)^2) /. subs /. experiment /. {t → 0}, A > 0}, {A, phi}]
```
{0., {A → 10.6008, phi → 1.15558}}

```
{xt == -5, vt == 3, A > 0} /. subs /. experiment /. initials
```

$$\left\{-14.7 + A\, e^{-0.05\,t}\, \text{Sin}[\text{phi} + 0.814964\,t] \;{==}\; -5,\right.$$

$$\left.\sqrt{\frac{2}{3}}\; A\, e^{-0.05\,t}\,(0.998123\,\text{Cos}[\text{phi} + 0.814964\,t] - 0.0612372\,\text{Sin}[\text{phi} + 0.814964\,t]) \;{==}\; 3,\; A > 0\right\}$$

```
Solve[{xt == -5, vt == 3, A > 0, phi ≥ 0, phi < 2 * π} /. subs /. experiment /. initials /. {t → 0},
 {A, phi}]
```

$$\{\{A \to 10.6008,\; \text{phi} \to 1.15558\}\}$$

```
xt /. subs /. {t → 0}
```

$$-\frac{g\,m}{k} + A\,\text{Sin}[\text{phi}]$$

```
vt /. subs /. {t → 0}
```

$$A\,\sqrt{\frac{k}{m}}\;\left(\sqrt{1 - \frac{c^2}{4\,k\,m}}\;\text{Cos}[\text{phi}] - \frac{c\,\text{Sin}[\text{phi}]}{2\,\sqrt{k\,m}}\right)$$

```
Solve[{xt == x, vt == v, A > 0, phi ≥ 0, phi < 2 * π} /. subs /. experiment /. {t → 0}, {A, phi}]
$Aborted
```

```
xt /. subs /. experiment /. {A → 10.60077410113466`, phi → 1.155576128610492`, t → 0}
-5.
```

```
vt /. subs /. experiment /. {A → 10.60077410113466`, phi → 1.155576128610492`, t → 0}
3.
```

```
xt /. subs /. {t → 0}
```

$$-\frac{g\,m}{k} + A\,\text{Sin}[\text{phi}]$$

FullSimplify[vt /. subs /. {t → 0}]

$$A \sqrt{\frac{k}{m}} \left(\sqrt{1 - \frac{c^2}{4\,k\,m}} \; \text{Cos[phi]} - \frac{c\,\text{Sin[phi]}}{2\,\sqrt{k\,m}} \right)$$

Solve$\left[\left\{-\dfrac{g\,m}{k} + A\,\text{Sin[phi]} == x,\right.\right.$

$$\left. A\,\sqrt{\frac{k}{m}} \left(\sqrt{1 - \frac{c^2}{4\,k\,m}} \; \text{Cos[phi]} - \frac{c\,\text{Sin[phi]}}{2\,\sqrt{k\,m}} \right) == v, \; g \geq 0, \; k > 0, \; m > 0, \; c \geq 0 \right\}, \; \{A, \, \text{phi}\}\right]$$

FullSimplify$\left[\text{Solve}\left[-g * m / k + \dfrac{\sqrt{k\,m\,v^2 + (g\,m + k\,x)^2}}{k} * \text{Sin[phi]} == x, \; \text{phi}\right]\right]$

$$\left\{\left\{\text{phi} \to \text{ConditionalExpression}\left[\pi - \text{ArcSin}\left[\frac{g\,m + k\,x}{\sqrt{k\,m\,v^2 + (g\,m + k\,x)^2}}\right] + 2\,\pi\,C[1], \; C[1] \in \text{Integers}\right]\right\},\right.$$

$$\left.\left\{\text{phi} \to \text{ConditionalExpression}\left[\text{ArcSin}\left[\frac{g\,m + k\,x}{\sqrt{k\,m\,v^2 + (g\,m + k\,x)^2}}\right] + 2\,\pi\,C[1], \; C[1] \in \text{Integers}\right]\right\}\right\}$$

$\left\{\left\{\text{phi} \to \text{ConditionalExpression}\left[\pi - \text{ArcSin}\left[\dfrac{g\,m + k\,x}{\sqrt{k\,m\,v^2 + (g\,m + k\,x)^2}}\right] + 2\,\pi\,C[1], \; C[1] \in \text{Integers}\right]\right\},\right.$

$\left\{\text{phi} \to \text{ConditionalExpression}\left[\text{ArcSin}\left[\dfrac{g\,m + k\,x}{\sqrt{k\,m\,v^2 + (g\,m + k\,x)^2}}\right] + 2\,\pi\,C[1],\right.\right.$
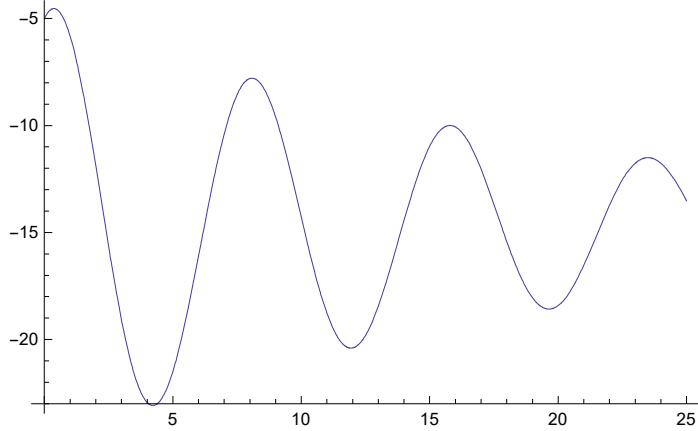
$\left.\left.\left. C[1] \in \text{Integers}\right]\right\}\right\}$ /. experiment /. initials

$$\{\{\text{phi} \to \text{ConditionalExpression}[1.93288 + 2\,\pi\,C[1], \; C[1] \in \text{Integers}]\},$$
$$\{\text{phi} \to \text{ConditionalExpression}[1.20871 + 2\,\pi\,C[1], \; C[1] \in \text{Integers}]\}\}$$

```
Plot[xt /. subs /. experiment /.
   {A -> 10.372559954032564, phi -> 1.2087096232338574`}, {t, 0, 25}]
```



```
(x''[t] + 2 * zeta * omega0 * x'[t] + omega0^2 * x[t] == -g) /. subs
```

$$\frac{k\,x[t]}{m} + \frac{c\,\sqrt{\frac{k}{m}}\;x'[t]}{\sqrt{k\,m}} + x''[t] == -g$$

```
DSolve[{(x''[t] + 2 * zeta * omega0 * x'[t] + omega0^2 * x[t] == -g) /. subs}, x[t], t]
```

$$\left\{\left\{x[t] \rightarrow -\frac{g\,m}{k} + e^{\frac{\left(-c\sqrt{\frac{k}{m}}\,m-\sqrt{k}\,\sqrt{m}\,\sqrt{c^2-4\,k\,m}\right)t}{2\,m\,\sqrt{k\,m}}}\,C[1] + e^{\frac{\left(-c\sqrt{\frac{k}{m}}\,m+\sqrt{k}\,\sqrt{m}\,\sqrt{c^2-4\,k\,m}\right)t}{2\,m\,\sqrt{k\,m}}}\,C[2]\right\}\right\}$$

```
DSolve[{(x''[t] + 2 * zeta * omega0 * x'[t] + omega0^2 * x[t] == -g) /. subs,
   x'[0] == v1, x[0] == x1}, x[t], t] // FullSimplify
```

$$\left\{\left\{x[t] \to \frac{1}{2\,k^{3/2}\,\sqrt{c^2-4\,k\,m}}\,e^{-\frac{\left(c\sqrt{\frac{k}{m}}\,\sqrt{m}+\sqrt{k}\,\sqrt{c^2-4\,k\,m}\right)t}{2\sqrt{m}\,\sqrt{k\,m}}}\left(c\left(-1+e^{\frac{\sqrt{k\,m}\,\sqrt{c^2-4\,k\,m}\,t}{\sqrt{k}\,m^{3/2}}}\right)\sqrt{\frac{k}{m}}\,\sqrt{m}\,(g\,m+k\,x1)+\right.\right.\right.$$

$$\sqrt{k}\left(\left(1+e^{\frac{\sqrt{k\,m}\,\sqrt{c^2-4\,k\,m}\,t}{\sqrt{k}\,m^{3/2}}}-2\,e^{\frac{\left(c\sqrt{\frac{k}{m}}\,\sqrt{m}+\sqrt{k}\,\sqrt{c^2-4\,k\,m}\right)t}{2\sqrt{m}\,\sqrt{k\,m}}}\right)g\,m\,\sqrt{c^2-4\,k\,m}+\right.$$

$$\left.\left.\left.2\left(-1+e^{\frac{\sqrt{k\,m}\,\sqrt{c^2-4\,k\,m}\,t}{\sqrt{k}\,m^{3/2}}}\right)\sqrt{k}\,\sqrt{m}\,\sqrt{k\,m}\,v1+\left(1+e^{\frac{\sqrt{k\,m}\,\sqrt{c^2-4\,k\,m}\,t}{\sqrt{k}\,m^{3/2}}}\right)k\,\sqrt{c^2-4\,k\,m}\,x1\right)\right)\right\}\right\}$$

$$\left(\frac{1}{2\,k^{3/2}\,\sqrt{c^2-4\,k\,m}}\,e^{-\frac{\left(c\sqrt{\frac{k}{m}}\,\sqrt{m}+\sqrt{k}\,\sqrt{c^2-4\,k\,m}\right)t}{2\sqrt{m}\,\sqrt{k\,m}}}\right.$$

$$\left(c\left(-1+e^{\frac{\sqrt{k\,m}\,\sqrt{c^2-4\,k\,m}\,t}{\sqrt{k}\,m^{3/2}}}\right)\sqrt{\frac{k}{m}}\,\sqrt{m}\,(g\,m+k\,x1)+\sqrt{k}\left(\left(1+e^{\frac{\sqrt{k\,m}\,\sqrt{c^2-4\,k\,m}\,t}{\sqrt{k}\,m^{3/2}}}-2\,e^{\frac{\left(c\sqrt{\frac{k}{m}}\,\sqrt{m}+\sqrt{k}\,\sqrt{c^2-4\,k\,m}\right)t}{2\sqrt{m}\,\sqrt{k\,m}}}\right)\right.\right.$$

$$g\,m\,\sqrt{c^2-4\,k\,m}+2\left(-1+e^{\frac{\sqrt{k\,m}\,\sqrt{c^2-4\,k\,m}\,t}{\sqrt{k}\,m^{3/2}}}\right)\sqrt{k}\,\sqrt{m}\,\sqrt{k\,m}\,v1+$$

$$\left.\left.\left.\left(1+e^{\frac{\sqrt{k\,m}\,\sqrt{c^2-4\,k\,m}\,t}{\sqrt{k}\,m^{3/2}}}\right)k\,\sqrt{c^2-4\,k\,m}\,x1\right)\right)\right)\right) \text{ /. experiment /. } \{x1 \to -5, v1 \to 3, t \to 0\}$$

```
-5. + 0. i
```

$$
\text{newsol} := \frac{1}{2\,k^{3/2}\,\sqrt{c^2 - 4\,k\,m}}\;\mathbb{e}^{-\frac{\left(c\sqrt{\frac{k}{m}}\,\sqrt{m}\,+\,\sqrt{k}\,\sqrt{c^2-4\,k\,m}\right)t}{2\,\sqrt{m}\,\sqrt{k\,m}}}
$$

$$
\left(c\left(-1+\mathbb{e}^{\frac{\sqrt{k\,m}\,\sqrt{c^2-4\,k\,m}\,t}{\sqrt{k}\,m^{3/2}}}\right)\sqrt{\frac{k}{m}}\,\sqrt{m}\,(g\,m+k\,x1)+\sqrt{k}\left(\left(1+\mathbb{e}^{\frac{\sqrt{k\,m}\,\sqrt{c^2-4\,k\,m}\,t}{\sqrt{k}\,m^{3/2}}}-2\,\mathbb{e}^{\frac{\left(c\sqrt{\frac{k}{m}}\,\sqrt{m}\,+\,\sqrt{k}\,\sqrt{c^2-4\,k\,m}\right)t}{2\,\sqrt{m}\,\sqrt{k\,m}}}\right)g\,m\right.\right.
$$

$$
\left.\left.\sqrt{c^2-4\,k\,m}\,+2\left(-1+\mathbb{e}^{\frac{\sqrt{k\,m}\,\sqrt{c^2-4\,k\,m}\,t}{\sqrt{k}\,m^{3/2}}}\right)\sqrt{k}\,\sqrt{m}\,\sqrt{k\,m}\,v1+\left(1+\mathbb{e}^{\frac{\sqrt{k\,m}\,\sqrt{c^2-4\,k\,m}\,t}{\sqrt{k}\,m^{3/2}}}\right)k\,\sqrt{c^2-4\,k\,m}\,x1\right)\right)
$$

```
FullSimplify[DSolve[
  {(x''[t] + 2 * zeta * omega0 * x'[t] + omega0^2 * x[t] == -g) /. subs, x'[0] == v1, x[0] == x1},
  x[t], t], {Element[t, Reals], Element[v1, Reals], Element[x1, Reals], Element[m, Reals],
  Element[g, Reals], Element[c, Reals], Element[k, Reals], m > 0, g > 0, c ≥ 0, k > 0}]
```

$$
\left\{\left\{x[t] \rightarrow \frac{1}{2\,k\,\sqrt{c^2-4\,k\,m}}\right.\right.
$$

$$
\mathbb{e}^{-\frac{\left(c+\sqrt{c^2-4\,k\,m}\right)t}{2\,m}}\left(\left(1+\mathbb{e}^{\frac{\sqrt{c^2-4\,k\,m}\,t}{m}}-2\,\mathbb{e}^{\frac{\left(c+\sqrt{c^2-4\,k\,m}\right)t}{2\,m}}\right)g\,m\,\sqrt{c^2-4\,k\,m}\,+c\left(-1+\mathbb{e}^{\frac{\sqrt{c^2-4\,k\,m}\,t}{m}}\right)(g\,m+k\,x1)+\right.
$$

$$
\left.\left.\left.k\left(2\left(-1+\mathbb{e}^{\frac{\sqrt{c^2-4\,k\,m}\,t}{m}}\right)m\,v1+\left(1+\mathbb{e}^{\frac{\sqrt{c^2-4\,k\,m}\,t}{m}}\right)\sqrt{c^2-4\,k\,m}\,x1\right)\right)\right\}\right\}
$$

$$
\text{news} := \frac{1}{2\,k\,\sqrt{c^2-4\,k\,m}}
$$

$$
\mathbb{e}^{-\frac{\left(c+\sqrt{c^2-4\,k\,m}\right)t}{2\,m}}\left(\left(1+\mathbb{e}^{\frac{\sqrt{c^2-4\,k\,m}\,t}{m}}-2\,\mathbb{e}^{\frac{\left(c+\sqrt{c^2-4\,k\,m}\right)t}{2\,m}}\right)g\,m\,\sqrt{c^2-4\,k\,m}\,+c\left(-1+\mathbb{e}^{\frac{\sqrt{c^2-4\,k\,m}\,t}{m}}\right)(g\,m+k\,x1)+\right.
$$

$$
\left.k\left(2\left(-1+\mathbb{e}^{\frac{\sqrt{c^2-4\,k\,m}\,t}{m}}\right)m\,v1+\left(1+\mathbb{e}^{\frac{\sqrt{c^2-4\,k\,m}\,t}{m}}\right)\sqrt{c^2-4\,k\,m}\,x1\right)\right)
$$

```
news /. experiment /. {x1 → -5, v1 → 3, t → 0}
```

$-5. + 0. \,\mathbb{i}$

Simplify[ExpToTrig[news]]

$$\frac{1}{2\,k\,\sqrt{c^2-4\,k\,m}}\left(\text{Cosh}\left[\frac{\left(c+\sqrt{c^2-4\,k\,m}\right)\,t}{2\,m}\right]-\text{Sinh}\left[\frac{\left(c+\sqrt{c^2-4\,k\,m}\right)\,t}{2\,m}\right]\right)$$

$$\left(-c\,g\,m+g\,m\,\sqrt{c^2-4\,k\,m}-2\,k\,m\,v1-c\,k\,x1+k\,\sqrt{c^2-4\,k\,m}\,x1+\right.$$

$$\left(c\,g\,m+g\,m\,\sqrt{c^2-4\,k\,m}+2\,k\,m\,v1+c\,k\,x1+k\,\sqrt{c^2-4\,k\,m}\,x1\right)\text{Cosh}\left[\frac{\sqrt{c^2-4\,k\,m}\,t}{m}\right]-$$

$$2\,g\,m\,\sqrt{c^2-4\,k\,m}\,\text{Cosh}\left[\frac{\left(c+\sqrt{c^2-4\,k\,m}\right)\,t}{2\,m}\right]+c\,g\,m\,\text{Sinh}\left[\frac{\sqrt{c^2-4\,k\,m}\,t}{m}\right]+g\,m\,\sqrt{c^2-4\,k\,m}$$

$$\text{Sinh}\left[\frac{\sqrt{c^2-4\,k\,m}\,t}{m}\right]+2\,k\,m\,v1\,\text{Sinh}\left[\frac{\sqrt{c^2-4\,k\,m}\,t}{m}\right]+c\,k\,x1\,\text{Sinh}\left[\frac{\sqrt{c^2-4\,k\,m}\,t}{m}\right]+$$

$$\left.k\,\sqrt{c^2-4\,k\,m}\,x1\,\text{Sinh}\left[\frac{\sqrt{c^2-4\,k\,m}\,t}{m}\right]-2\,g\,m\,\sqrt{c^2-4\,k\,m}\,\text{Sinh}\left[\frac{\left(c+\sqrt{c^2-4\,k\,m}\right)\,t}{2\,m}\right]\right)$$

FullSimplify[news]

$$\frac{1}{2\,k\,\sqrt{c^2-4\,k\,m}}$$

$$e^{-\frac{\left(c+\sqrt{c^2-4\,k\,m}\right)\,t}{2\,m}}\left(\left(1+e^{\frac{\sqrt{c^2-4\,k\,m}\,t}{m}}-2\,e^{\frac{\left(c+\sqrt{c^2-4\,k\,m}\right)\,t}{2\,m}}\right)g\,m\,\sqrt{c^2-4\,k\,m}+c\left(-1+e^{\frac{\sqrt{c^2-4\,k\,m}\,t}{m}}\right)(g\,m+k\,x1)+\right.$$

$$\left.k\left(2\left(-1+e^{\frac{\sqrt{c^2-4\,k\,m}\,t}{m}}\right)m\,v1+\left(1+e^{\frac{\sqrt{c^2-4\,k\,m}\,t}{m}}\right)\sqrt{c^2-4\,k\,m}\,x1\right)\right)$$

news /. $\left\{\sqrt{c^2-4\,k\,m}\to p,\ x1\to x,\ v1\to v\right\}$

$$\frac{e^{-\frac{(c+p)\,t}{2\,m}}\left(\left(1+e^{\frac{p\,t}{m}}-2\,e^{\frac{(c+p)\,t}{2\,m}}\right)g\,m\,p+c\left(-1+e^{\frac{p\,t}{m}}\right)(g\,m+k\,x)+k\left(2\left(-1+e^{\frac{p\,t}{m}}\right)m\,v+\left(1+e^{\frac{p\,t}{m}}\right)p\,x\right)\right)}{2\,k\,\sqrt{c^2-4\,k\,m}}$$

```
Plot[news /. experiment /. {x1 → -15, v1 → -15}, {t, 0, 25}]
```
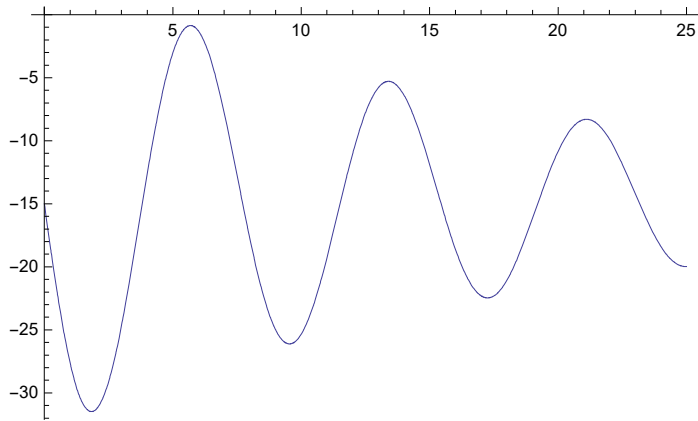


```
Plot[news /. {g → 9.8, m → 2, k → 2, c → 0.1} /. {x1 → -5, v1 → 3}, {t, 0, 100}, PlotRange → All]
```
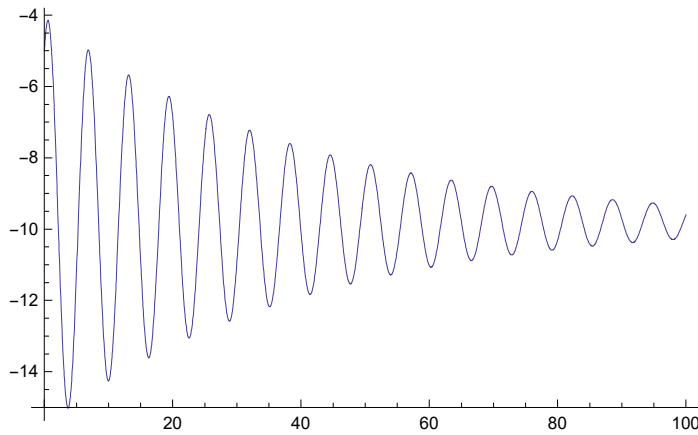


```
news /. {g → 9.8, m → 3, k → 2, c → 4} /. {x1 → -5, v1 → 3} // FullSimplify
```

$$-14.7 + e^{-\frac{1}{3}\left(2+i\sqrt{2}\right)t}\left((4.85 + 10.0409\,i) + (4.85 - 10.0409\,i)\,e^{\frac{2}{3}i\sqrt{2}\,t}\right)$$

```
FullSimplify[DSolve[{ (x''[t] + 2 * zeta * omega0 * x'[t] + omega0^2 * x[t] == -g) /. subs,
  x'[0] == v1, x[0] == x1}, x[t], t], {Element[t, Reals], Element[v1, Reals],
 Element[x1, Reals], Element[m, Reals], Element[g, Reals], Element[c, Reals],
 Element[k, Reals], m > 0, g > 0, c ≥ 0, k > 0, c^2 < 4 * k * m}]
```

$$\left\{\left\{x[t] \to \frac{1}{2\, k^{3/2}\, \sqrt{c^2 - 4\, k\, m}}\right.\right.$$

$$e^{-\frac{\left(c + i\sqrt{-c^2 + 4 k m}\right)t}{2 m}} \left(i\left(1 + e^{\frac{i\sqrt{-c^2 + 4 k m}\, t}{m}} - 2\, e^{\frac{\left(c + i\sqrt{-c^2 + 4 k m}\right)t}{2 m}}\right) g\, m\, \sqrt{k\left(-c^2 + 4 k m\right)} - 2\, k^{3/2}\, m\, v1 + \right.$$

$$2\, e^{\frac{i\sqrt{-c^2 + 4 k m}\, t}{m}} k^{3/2}\, m\, v1 + i\, \sqrt{k^3\left(-c^2 + 4 k m\right)}\, x1 +$$

$$\left.\left.\left.i\, e^{\frac{i\sqrt{-c^2 + 4 k m}\, t}{m}} \sqrt{k^3\left(-c^2 + 4 k m\right)}\, x1 + c\left(-1 + e^{\frac{i\sqrt{-c^2 + 4 k m}\, t}{m}}\right)\sqrt{k}\ (g\, m + k\, x1)\right)\right\}\right\}$$

```
FullSimplify[
```

$$\text{TrigReduce}\left[\text{ExpToTrig}\left[\frac{1}{2\, k^{3/2}\, \sqrt{c^2 - 4 k m}} e^{-\frac{\left(c + i\sqrt{-c^2 + 4 k m}\right)t}{2 m}} \left(i\left(1 + e^{\frac{i\sqrt{-c^2 + 4 k m}\, t}{m}} - 2\, e^{\frac{\left(c + i\sqrt{-c^2 + 4 k m}\right)t}{2 m}}\right) g\right.\right.\right.$$

$$m\, \sqrt{k\left(-c^2 + 4 k m\right)} - 2\, k^{3/2}\, m\, v1 + 2\, e^{\frac{i\sqrt{-c^2 + 4 k m}\, t}{m}} k^{3/2}\, m\, v1 + i\, \sqrt{k^3\left(-c^2 + 4 k m\right)}\, x1 +$$

$$\left.\left.\left.i\, e^{\frac{i\sqrt{-c^2 + 4 k m}\, t}{m}} \sqrt{k^3\left(-c^2 + 4 k m\right)}\, x1 + c\left(-1 + e^{\frac{i\sqrt{-c^2 + 4 k m}\, t}{m}}\right)\sqrt{k}\ (g\, m + k\, x1)\right)\right]\right]\right]
```

$Aborted

```
ExpToTrig[Exp[Sqrt[-1] * x]]
```

```
ExpToTrig[Exp[Sqrt[-1] * x]]
```

$\text{Cos}[x] + i\, \text{Sin}[x]$

```
TraditionalForm[
```

$$\text{Style}\left[\frac{1}{2\, k^{3/2}\, \sqrt{c^2 - 4 k m}} e^{-\frac{\left(c + i\sqrt{-c^2 + 4 k m}\right)t}{2 m}} \left(i\left(1 + e^{\frac{i\sqrt{-c^2 + 4 k m}\, t}{m}} - 2\, e^{\frac{\left(c + i\sqrt{-c^2 + 4 k m}\right)t}{2 m}}\right) g\, m\, \sqrt{k\left(-c^2 + 4 k m\right)} - \right.\right.$$

$$2\, k^{3/2}\, m\, v1 + 2\, e^{\frac{i\sqrt{-c^2 + 4 k m}\, t}{m}} k^{3/2}\, m\, v1 + i\, \sqrt{k^3\left(-c^2 + 4 k m\right)}\, x1 +$$

$$\left.\left.i\, e^{\frac{i\sqrt{-c^2 + 4 k m}\, t}{m}} \sqrt{k^3\left(-c^2 + 4 k m\right)}\, x1 + c\left(-1 + e^{\frac{i\sqrt{-c^2 + 4 k m}\, t}{m}}\right)\sqrt{k}\ (g\, m + k\, x1)\right), \text{FontSize} \to 34\right]\right]$$

$$\frac{1}{2\,k^{3/2}\,\sqrt{c^2-4\,k\,m}}\,e^{-\frac{t\left(c+i\,\sqrt{4\,k\,m-c^2}\right)}{2\,m}}$$

$$\left(c\,\sqrt{k}\,\left(-1+e^{\frac{i\,t\,\sqrt{4\,k\,m-c^2}}{m}}\right)(g\,m+k\,\text{x1})+\right.$$

$$i\,g\,m\,\sqrt{k\left(4\,k\,m-c^2\right)}$$

$$\left(e^{\frac{i\,t\,\sqrt{4\,k\,m-c^2}}{m}}-2\,e^{\frac{t\left(c+i\,\sqrt{4\,k\,m-c^2}\right)}{2\,m}}+1\right)+$$

$$2\,k^{3/2}\,m\,\text{v1}\,e^{\frac{i\,t\,\sqrt{4\,k\,m-c^2}}{m}}+$$

$$i\,\text{x1}\,\sqrt{k^3\left(4\,k\,m-c^2\right)}\,e^{\frac{i\,t\,\sqrt{4\,k\,m-c^2}}{m}}+$$

$$\left.i\,\text{x1}\,\sqrt{k^3\left(4\,k\,m-c^2\right)}-2\,k^{3/2}\,m\,\text{v1}\right)$$

```
FullSimplify[DSolve[
  {(x''[t] + 2 * zeta * omega0 * x'[t] + omega0^2 * x[t] == -g) /. subs /. {c → Sqrt[4 * k * m]},
   x'[0] == v1, x[0] == x1}, x[t], t],
 {Element[t, Reals], Element[v1, Reals], Element[x1, Reals], Element[m, Reals],
  Element[g, Reals], Element[k, Reals], m > 0, g > 0, k > 0}]
```

$$\left\{\left\{x[t]\rightarrow\frac{e^{-\sqrt{\frac{k}{m}}\,t}\left(g\left(m-e^{\sqrt{\frac{k}{m}}\,t}\,m+\sqrt{k\,m}\,t\right)+k\left(\text{x1}+t\left(\text{v1}+\sqrt{\frac{k}{m}}\,\text{x1}\right)\right)\right)}{k}\right\}\right\}$$

```
TraditionalForm[
```

$$\text{Style}\left[\frac{e^{-\sqrt{\frac{k}{m}}\,t}\left(g\left(m-e^{\sqrt{\frac{k}{m}}\,t}\,m+\sqrt{k\,m}\,t\right)+k\left(x1+t\left(v1+\sqrt{\frac{k}{m}}\,x1\right)\right)\right)}{k}, \text{FontSize} \to 34\right]\right]$$

$$\frac{1}{k}\,e^{t\left(-\sqrt{\frac{k}{m}}\right)}\left(g\left(m\left(-e^{t\,\sqrt{\frac{k}{m}}}\right)+t\,\sqrt{k\,m}+m\right)+\right.$$

$$\left.k\left(t\left(x1\,\sqrt{\frac{k}{m}}+v1\right)+x1\right)\right)$$