

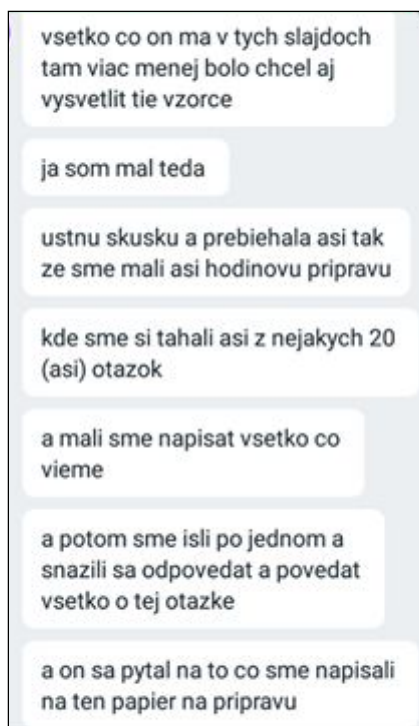
# Všeobecné info

## PODMIENKY ABSOLVOVANIA A SPÔSOB HODNOTENIA

- Projekty - **3x7 = 21 bodov)**
- Cvičenia - **10 bodov.**
- Skúška - **aspoň 7 bodov.**
- **Celkové hodnotenie:** A (50-46), B (45-41), C (40-36), D (35-31), E (30-**26**), Fx (25-0).

### Info od týpka z matfyzu

*\* My máme písomnú skúšku*



1. spoj. percepciu so sly

a) schéma

b) vstupno-výstupný vzťah

c) odvodenie učiac pravidla (delta) po kódech chyby

d) bias

2. BP

a) podstaty alg.

b) od čoho priamo závisí zmena váhy  $w_{ij}$  (vzťah)

c) vyloženie alg a ich účel

3. Autoasociat. (GI)

a) podstaty (slovné, matematické, graf.)

b) ktorú úlohu má Gram-Schmidt ortog.

4. FCA

a) schéma

b) OJO a vychytávanie

c) čo veľmi posúdiť o vzhľadu po konvergencii v GHA (lineárne neuróny)

d) ako je zapracované fungovanie pri viacerých neur. vo vrstve

5. SOM

a) alg. tréningu

b) dôvod zmeny skál

c) čo SOM aproximuje

6. RBF

a) schéma a funkcie

b) RBF vs MLP učenie a zobrazovanie neur.

c) k-meus ako a prečo

d) RLS v RBF?

7. SRN

a) schéma, aké rovnice

b) aké interné reprezentácie

c) ~~ako~~ aké alg. na tréningu

d) architekt. bias

8. Stoch. vs

a) ako PLS

b) koncepty

c) vyhod. stoch. vs. determin.

d) princíp (optimalizácia) v odvodzovaní učiac. pravidla (BP, LBN)

# Otázky na záverečnú skúšku - Neurónové siete

- Otázky som našiel na <http://dai.fmph.uniba.sk/courses/NN/ns-otazky.pdf>
- Vôbec to nemusia byť takéto otázky ale podobajú sa na to čo sme preberali

## 1. Stručná história konekcionizmu, vlastnosti biologického neurónu, model neurónu s prahovou logikou, implementácia Booleových funkcií. Paradigmy učenia a typy úloh pre NS.

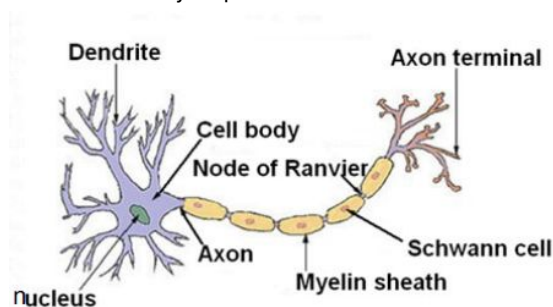
**Connectionism** – theory of information processing, inspired by biology (the brain). It is based on Artificial Neural Networks (ANNs).

It has two goals:

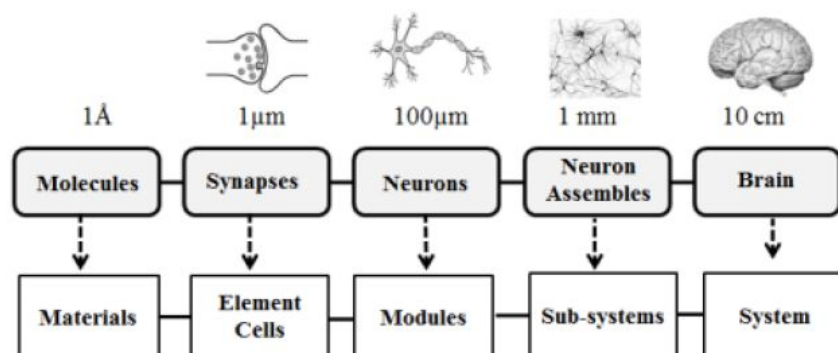
- **theoretical foundations of cognitive science** (modeling of cognitive processes)
  - contrasting with symbolic approaches
  - features: parallelism, robustness, learning from experience,...
- **applications in practical problems**
  - tasks: pattern recognition, classification, associative memory, time series prediction, dimensionality reduction, data visualization, ...

### Štruktúra biologického neurónu

V mozgu -  $\sim 10^{11}$  neurónov a  $\sim 10^{15}$  synáps



## Structural organization of levels in the brain

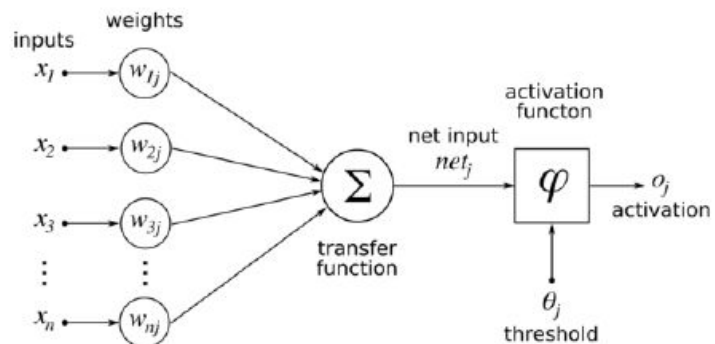


## Typical artificial neuron model

1. receives signals from other neurons (or sensors)
2. processes (integrates) incoming signals
3. sends the processed signal to other neurons (or muscles)

### Deterministic model

$$o = f(\sum_i w_i x_i - \theta)$$



**Stochastic model**  $P(o=1) = 1/(1+\exp(-net/T))$

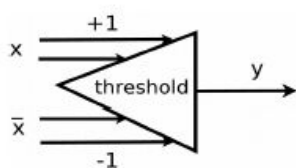
### History of classical connectionism

- Aristoteles (400 BC) – introduced concepts of memory, and connectionism
- Spencer (1855) – separated psychology from philosophy, postulated that “neural states affect psychological states”, knowledge is in connections.
- James (1890) – model of associative memory, “law of neural habit”
- Thorndike (1932) – distinguished sub-symbolic view on neural associations, formulated two laws of adaptation: “law of effect” and “law of exercise” (currently known as reinforcement in operant conditioning).
- McCulloch & Pitts (1943) – neural networks with threshold units
- Minsky (1967) extended their results to comprehensible form, and put them in the context of (formal) automata theory and theory of computation.

### First neural network

McCulloch & Pitts (1943) – neural networks with threshold logic units

- Threshold Logic unit:



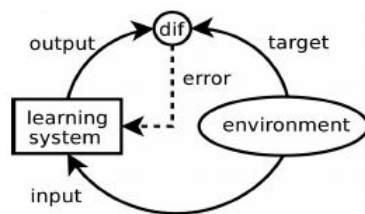
$$Y = 1, \text{ if } \text{sum}(x) - \text{sum}(\text{non } x) \geq \text{threshold}$$

$$0, \text{ otherwise.}$$

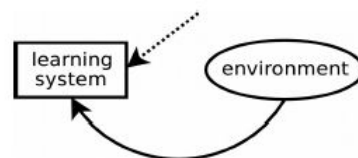
- can simulate any linearly separable Boolean function:
- Fixed weights, positive and negated inputs
- Theorem: Any Boolean function  $f: \{0,1\}^n \rightarrow \{0,1\}$  can be simulated by a two-layer NN with logical units.

## Learning paradigms in NN

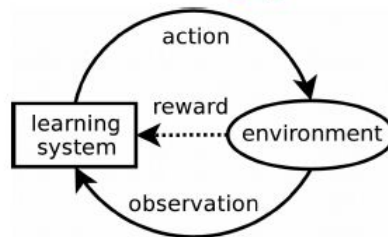
### supervised (with teacher)



### unsupervised (self-organized)



### reinforcement learning (partial feedback)



### Learning tasks

- Pattern association (auto-, hetero-)
- Pattern classification (within pattern recognition)
- Feature extraction (within PR or independently)
- Data compression
- Function approximation
- Control
- Filtering
- Prediction
- Signal generation (with recurrent networks)
- ...

2. Binárny perceptrón: pojem učenia s učiteľom, učiace pravidlo, algoritmus tréovania, deliaca nadrovina, klasifikácia vzorov, lineárna separovateľnosť, náčrt dôkazu konvergenzie, definícia a príklad.

## Summary of perceptron algorithm

*Given:* training data: input-target  $\{x, d\}$  pairs, unipolar perceptron

*Initialization:* randomize weights, set learning rate,  $E = 0$ .

*Training:*

1. choose input  $x$ , compute output  $y$
2. evaluate error function  $e(t) = \frac{1}{2} (d - y)^2$ ,  $E = E + e(t)$
3. adjust weights using delta rule (if  $e(t) > 0$ )
4. if all patterns used, then goto 5, else go to 1
5. if  $E = 0$  (all patterns in the set classified correctly), then end  
else shuffle inputs,  $E = 0$ , go to 1



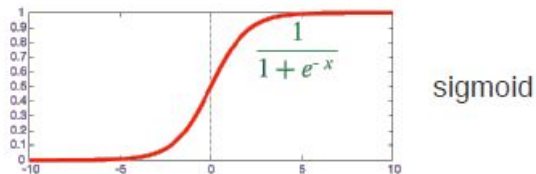
3. Spojitý perceptrón: Rôzne aktivačné funkcie perceptrónu, chybové funkcie a spôsob minimalizácie, učiace pravidlá, algoritmus trénovania perceptrónu. Súvis s Bayesovským klasifikátorom.

## Continuous perceptron

- **Nonlinear unit** with activation function:  $y = f(net) = 1 / (1 + e^{-net})$

Has nice properties:

- boundedness
- monotonicity
- differentiability



- Quadratic error function:  $E(\mathbf{w}) = 1/2 \sum_p (d^{(p)} - y^{(p)})^2$  [ $p \sim$  patterns]
- (unconstrained) minimization of the error function: necessary conditions  $e(\mathbf{w}^*) \leq e(\mathbf{w})$  and  $\nabla e(\mathbf{w}^*) = 0$ , gradient operator

$\nabla = [\partial/\partial w_1, \partial/\partial w_2, \dots]^T$ . Minimizing  $E(\mathbf{w})$  leads to

- (stochastic, online) **gradient descent learning**:

$$w_j(t+1) = w_j(t) + \alpha (d^{(p)} - y^{(p)}) f'(net) x_j = w_j(t) + \alpha \delta^{(p)} x_j$$

- (alternative) batch learning: (update after each epoch)

$$w_j(t+1) = w_j(t) + \alpha \sum_p \delta^{(p)} x_j^{(p)}$$

4. Viacvrstvé dopredné neurónové siete: architektúra a aktivačné vzorce, odvodenie metódy učenia pomocou spätného šírenia chýb (BP) pre dvojvrstvovú doprednú NS, modifikácie BP, typy úloh pre použitie doprednej NS.

## Multi-layer perceptrons

- Generalization of simple perceptrons
- Features:
  - contains hidden-layer(s)
  - neurons have non-linear differentiable activation function
  - full connectivity b/w layers
- (supervised) error “back-propagation” learning algorithm introduced
- originated after 1985: Rumelhart & McClelland: *Parallel distributed processing* (described earlier by Werbos, 1974, 1982)
- theoretical analysis difficult
- response to earlier critique of perceptrons (Minsky & Papert, 1969)

## Two-layer perceptron

- Inputs  $x$ , weights  $w$ ,  $v$ , outputs  $y$
- Nonlinear activation function  $f$
- Unit activation:

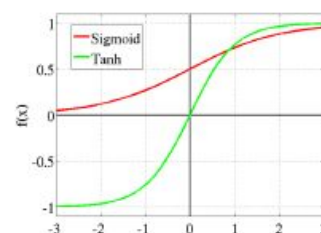
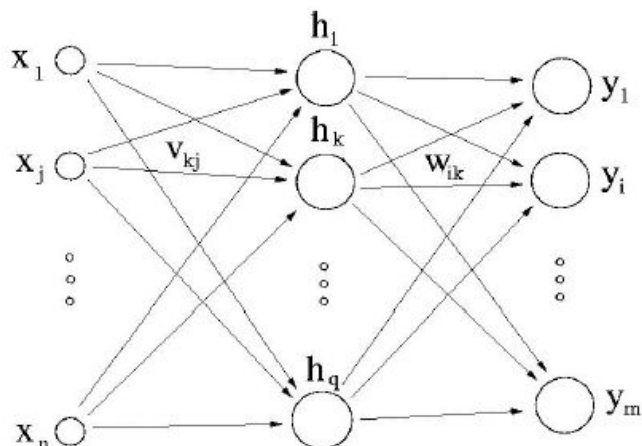
$$h_k = f\left(\sum_{j=1}^{n+1} v_{kj} x_j\right)$$

$$y_i = f\left(\sum_{k=1}^{q+1} w_{ik} h_k\right)$$

- Bias input:  $x_{n+1} = h_{q+1} = -1$
- Activation function examples:

$$f(net) = \sigma(net) = \frac{1}{1 + e^{-net}}$$

$$f(net) = \tanh(net) = \frac{e^{net} - e^{-net}}{e^{net} + e^{-net}} = \frac{2}{1 + e^{-2net}} - 1$$





## Summary of back-propagation algorithm

*Given:* training data: input-target  $\{\mathbf{x}^{(p)}, \mathbf{d}^{(p)}\}$  pairs

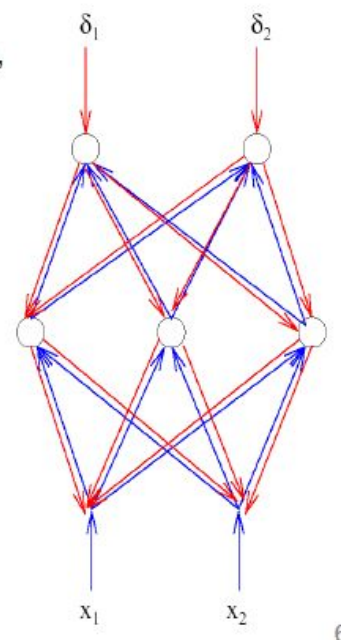
*Initialization:* randomize weights, set learning parameters

*Training:*

1. choose input  $\mathbf{x}^{(p)}$ , compute outputs  $\mathbf{y}^{(p)}$  (**forward pass**),
2. evaluate chosen error function  $e(t)$ ,  $E \leftarrow E + e(t)$
3. compute  $\delta_i, \delta_k$  (**backward pass**)
4. adjust weights  $\Delta w_{ik}$  and  $\Delta v_{kj}$
5. if all patterns used, then goto 6, else go to 1
6. if stopping\_criterion is met, then end  
else permute inputs and go to 1

No well-defined stopping criteria exist for BP, neither can it be shown in general to converge. Suggestions:

- when change in  $E_{epoch}$  is sufficiently small (<1%)
- when generalization performance is adequate



# Sequential and batch modes of training

## Sequential mode

- on line (example-by-example), stochastic
- able to track small changes in training data
- easier to implement, requires less local storage
- difficult to establish theoretical conditions for convergence

## Batch mode

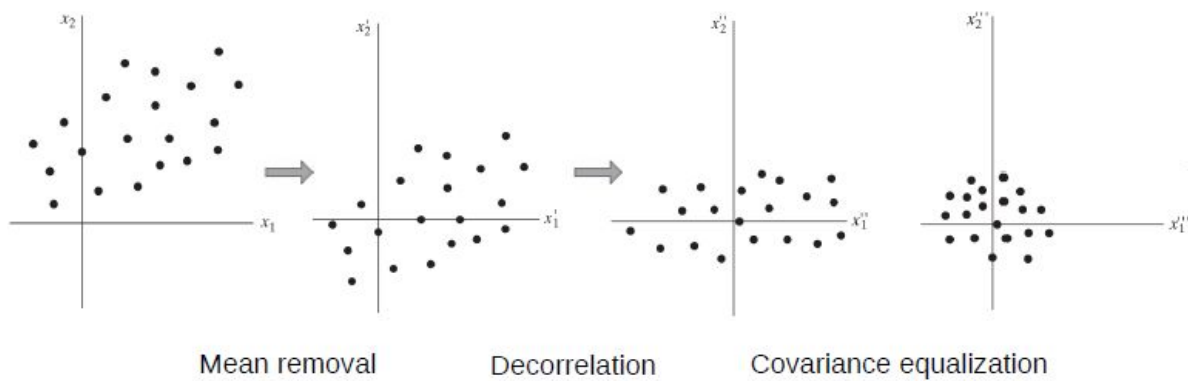
- adaptation performed at the end of each epoch, deterministic
- provides an accurate estimate of gradient vector, statistical inference
- parallelization possible

$$E_{av} = 1/(2N) \sum_{p=0}^N (\mathbf{d}^{(p)} - \mathbf{y}^{(p)})^2$$
$$\Delta w_{ik} \propto -\partial E_{av}(t)/\partial w_{ik}$$
$$\Delta v_{kj} \propto -\partial E_{av}(t)/\partial v_{kj}$$

- **typy úloh pre použitie doprednej NS**
  - Klasifikacia
  - Regresia

5. Viacvrstvová dopredná NS ako univerzálny aproximátor funkcií (teorém), trénovacia a testovacia množina, generalizácia, preučenie, skoré zastavenie učenia, selekcia modelu, validácia modelu. Hlboké učenie NS.

## Normalization of inputs



## MLP as a universal approximator

*Theorem:* Let's have  $A_{\text{train}} = \{\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(p)}, \dots, \mathbf{x}^{(N)}\}$ ,  $\mathbf{x}^{(p)} \in \mathbb{R}^n$ . For  $\epsilon > 0$  and arbitrary continuous function  $F: \mathbb{R}^n \rightarrow (0,1)$  defined on discrete set  $A_{\text{train}}$  there exists such a function  $G$ :

$$G(\mathbf{x}^{(p)}) = f\left(\sum_{k=1}^{q+1} w_k f\left(\sum_{j=1}^{n+1} v_{kj} x_j^{(p)}\right)\right)$$

where parameters  $w_k, v_{kj} \in \mathbb{R}$  and  $f(z) = \mathbb{R} \rightarrow (0,1)$  is a continuous and monotone-increasing function satisfying  $f(-\infty) = 0$  and  $f(\infty) = 1$ , such that:

$$\sum_p |F(\mathbf{x}^{(p)}) - G(\mathbf{x}^{(p)})| < \epsilon.$$

We say that  $G$  approximates  $F$  on  $A_{\text{train}}$  with accuracy  $\epsilon$ .

$G$  can be interpreted as a **2-layer feedforward NN with 1 output neuron**.

- it is an existence theorem
- **curse of dimensionality** – sparsity problem, how to get a dense sample for large  $n$  and complex  $F$

Hecht-Nielsen (1987), Hornik, Stinchcombe & White (1989)

## Generalization

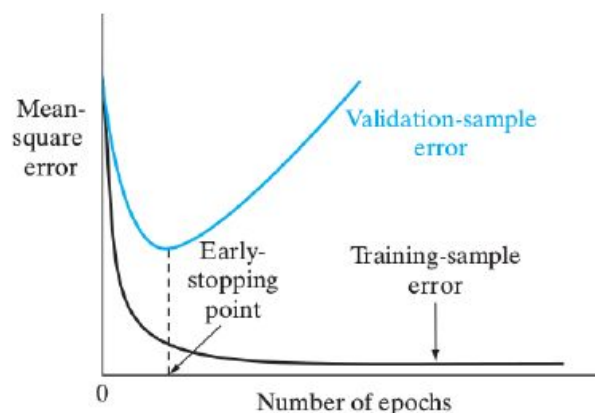
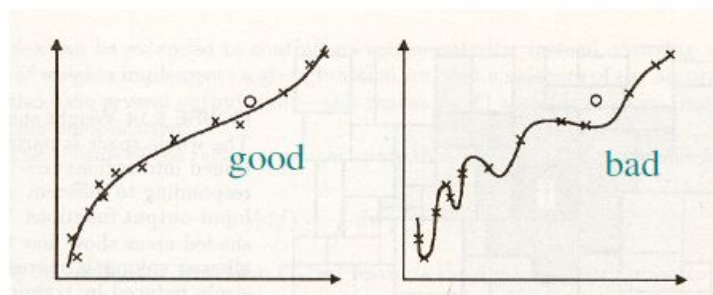
Data set:

$$A = A_{\text{estim}} \cup A_{\text{val}} \cup A_{\text{test}}$$

- Validation set is used for model selection.
- Generalization (= testing set performance) is important in using ANNs.

Generalization is influenced by:

- size of  $A_{\text{estim}}$  and its representativeness
- architecture of NN
- complexity of the problem



**Cross validacia** - typicky na 10 - 20 % trénovacej množiny

### Early stopping

- “BP algorithm is considered to have converged when
  - ... the Euclidean norm of the gradient vector reaches a sufficiently small gradient threshold.” (Kramer and Sangiovanni-Vincentelli, 1989)
  - ... the absolute rate of change in the average squared error per epoch is sufficiently small.”

**6. Lineárne modely NS:** vzťah pre riešenie systému lin. rovníc v jednovrstvovej sieti, pojem pseudoinverzie matice (Moore-Penrose), autoasociatívna pamäť: lineárny obal, princíp funkcie modelu, detektor novosti.

## Linear NN models

Input vector:  $\mathbf{x} = [x_1, x_2, \dots, x_n]^T$

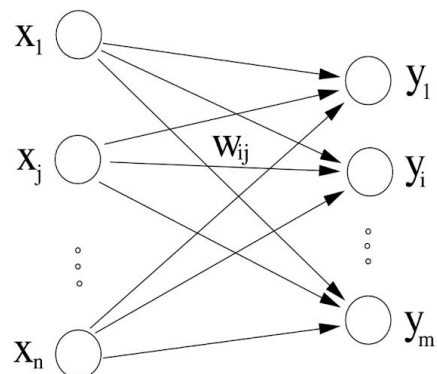
Output vector:  $\mathbf{y} = [y_1, y_2, \dots, y_m]^T$

Weight matrix:  $\mathbf{W} \sim \text{type } [m \times n]$

Linear transformation  $\varphi : \mathbb{R}^n \rightarrow \mathbb{R}^m, \mathbf{y} = \mathbf{W}\mathbf{x}$

☹ ignores saturation property of neurons

☺ allows to find analytic solutions using linear algebra.



(Kohonen, 1970;  
Anderson, 1972;  
Cooper, 1973)

- **Adding layers in a linear NN does not appear reasonable** (since no complexity is added).
- But: It allows nonlinear learning dynamics in linear deep networks (Saxe, 2015).



## Auto-associator case

Let's consider  $N < n$  and the **autoassociative case**:  $\mathbf{y}^{(p)} = \mathbf{x}^{(p)}$ ,  $m = n$

Model is supposed to remember  $N$  **prototypes**  $[\mathbf{x}^{(1)} \mathbf{x}^{(2)} \dots \mathbf{x}^{(N)}]$ .

*Goal*: train on prototypes and then submit a corrupted version of a prototype. Model should be able to reconstruct it.

Since  $\mathbf{Y} = \mathbf{X}$ ,  $\mathbf{W} = \mathbf{X}\mathbf{X}^+$ . How to interpret  $\mathbf{W}$ ?

In special case, which is too restrictive ( $N = n$ , linearly independent inputs), we would have a trivial solution  $\mathbf{W} = \mathbf{I}$  (identity).

How about a general case?

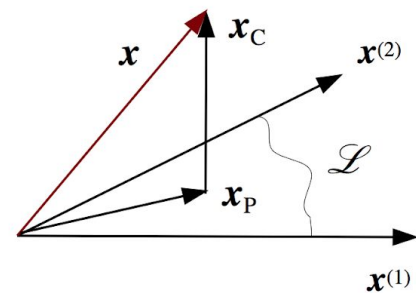
Training set  $\mathbf{A}_{\text{train}} = \{\mathbf{x}^{(p)}, p = 1, 2, \dots, N\}$  forms the linear manifold  $\mathcal{L}$ .

NN considers every departure  $\mathbf{x}$  from  $\mathcal{L}$  as added noise that needs to be filtered out by projecting  $\mathbf{x}$  to  $\mathcal{L}$ :

We need to show that output  $\mathbf{W}\mathbf{x} = \mathbf{X}\mathbf{X}^+\mathbf{x} = \mathbf{x}_p$  (filtered version of  $\mathbf{x}$ ), i.e. that operator  $\mathbf{W} = \mathbf{X}\mathbf{X}^+$  makes an **orthogonal projection** to  $\mathcal{L}$ .

Alternatively, the NN model with operator  $\mathbf{W} = \mathbf{I} - \mathbf{X}\mathbf{X}^+$  is called **novelty detector**, where  $\mathbf{W}\mathbf{x} = \mathbf{x}_c \in \mathcal{L}^\perp$ .

Now assume: you learned  $N$  patterns, and want to add  $(N+1)$ -st pattern.  
How to change  $\mathbf{W}$  efficiently?



**7. Lineárne modely NS: účel Grammovho-Schmidtovho ortogonalizačného procesu, GI model. Pamäť korelačnej matice ako autoasociatívna pamäť, vzťah pre výpočet váh, presluch, porovnanie s GI.**

## Summary

- Linear models were studied during connectionist depression in the 1970s
- Single layer models as auto-associative memories
- Analytic solutions possible
- General inverse model – noise filtering by projection to linear manifold (of the training data)
- GI – as novelty detector
- Correlation Matrix Memory – Hebbian-based learning, subject to cross-talk
- GI better in general, for sufficiently dissimilar inputs both models are comparable

**8. Samoorganizácia v NS, základné princípy, pojem učenia bez učiteľa, typy úloh použitia, Ojovo pravidlo učenia pre jeden lineárny neurón, vysvetlenie konvergenencie.**

**9. Metóda hlavných komponentov pomocou algoritmu GHA a APEX, architektúra modelu, vzťah pre adaptáciu váh, pojem vlastných vektorov a vlastných čísel, redukcia dimenzie, aplikácia na kompresiu obrazu.**

**10.SOM model: algoritmus, parametre, základné koncepty, vlastnosti, príklad použitia.**

**11. RBF model: aktivačné vzorce, báзовé funkcie, príznakový priestor, problém interpolácie, tréning modelu, aproximačné vlastnosti RBF siete, princíp algoritmu RLS.**

**12. NS na spracovanie sekvenčných dát: reprezentácia času, typy úloh pre rekurentné NS. Modely s časovým oknom do minulosti, výhody a nedostatky, príklad použitia.**

**13. Rekurentné NS: princíp trénovania pomocou algoritmu BPTT a RTRL. Teoretické vlastnosti RNS.**

**14. Elmanova sieť: interné reprezentácie pri symbolovej dynamike, Markovovské správanie, architekturná predispozícia.**

**15. Sieť s echo stavmi (ESN): architektúra, inicializácia, trénovanie modelu, vplyv parametrov na vlastnosti rezervoára, echo vlastnosť, pamäťová kapacita.**

**16. Hopfieldov model NS: deterministická dynamika, energia systému, relaxácia, typy atraktorov, autoasociatívna pamäť – nastavenie váh, princíp výpočtu kapacity pamäte.**

**17. Nelineárne dynamické systémy: stavový portrét, dynamika, typy atraktorov. Stochastický Hopfieldov model NS: parameter inverznej teploty, princíp odstránenia falošných atraktorov.**

**18. Hlboké učenie: základné koncepty, spôsoby pred/trénovania trénovania hlbokých sietí (DN), diskriminatívny a generatívny prístup, koncept konvolúcie, autoenkóder, GAN model, príklady úspešného použitia DN.**