

## Air Quality Model Performance Metric Definitions

Common Variables:

- $N, n$  count of samples (model-obs pairs)
- $i$  index for individual observation or model estimate pair
- $e_i$  model estimated or predicted value of a variable at time or location  $i$
- $\bar{e}$  mean value of the model estimated values of a variable over a time or space
- $e'$  the deviation of the model estimated value of a variable from the mean value
- $o_i$  observed or expected value of a variable at location/time  $i$ .
- $\bar{o}$  mean of the observed values of a variable over a time or space.
- $o'$  the deviation of the observed value of a variable from the mean value

### Standard Error and Bias Metrics (units of obs.)

The bias (BIAS) in predicting a variable can be given as:

$$BIAS = 1/N \sum_{i=1}^n (e_i - o_i)$$

Other names & Abbrev: Mean Error (ME), Mean Bias (MB)

The mean absolute error (MAE) in predicting a variable can be given as:

$$MAE = 1/N \sum_{i=1}^n |e_i - o_i|$$

Other names & Abbrev: MAE, mean absolute gross error

The root mean square error (RMSE) in predicting a variable can be given as:

$$RMSE = \sqrt{1/N \sum_{i=1}^n (e_i - o_i)^2}$$

## Systematic vs Unsystematic Error (units of obs.)

Systematic RMSE ( $RMSE_s$ ) in predicting a variable can be given as:

$$RMSE_s = \sqrt{1/N \sum_{i=1}^n (C - o_i)^2}$$

Unsystematic RMSE ( $RMSE_u$ ) in predicting a variable can be given as:

$$RMSE_u = \sqrt{1/N \sum_{i=1}^n (C - e_i)^2}$$

$$C = a + b e_i$$

Where: **a** and **b** are the least squares regression coefficients of  **$o_i$**  (observed) and  **$e_i$**  (model estimated) value.

## Normalization Metrics (unitless)

Normalized Mean Bias:

$$NMB = \frac{\sum_{i=1}^n (e_i - o_i)}{\sum_{i=1}^n o_i}$$

Other names & Abbrev: Normalized bias

Normalized Mean Error:

$$NME = \frac{\sum_{i=1}^n |e_i - o_i|}{\sum_{i=1}^n o_i}$$

Other names & Abbrev: Normalized error

## Fractional Metrics (unitless)

Fractional Bias:

$$FB = \frac{1}{n} \left[ \frac{\sum_{i=1}^n (e_i - o_i)}{\sum_{i=1}^n \frac{(e_i + o_i)}{2}} \right]$$

Other names & Abbrev: Mean Fractional Bias (MFB)

Fractional Error:

$$FE = \frac{1}{n} \left[ \frac{\sum_{i=1}^n |e_i - o_i|}{\sum_{i=1}^n \frac{(e_i + o_i)}{2}} \right]$$

Other names & Abbrev: Mean Fractional Error (MFE)

## Correlation Metrics (unitless)

The anomaly correlation (AC) coefficient used in AMET in predicting a variable is based on uncentered approach and can be given as:

$$AC = \frac{1/N \sum_{i=1}^n e' o'}{\sqrt{1/N (\sum_{i=1}^n (e')^2 * 1/N \sum_{i=1}^n (o')^2)}}$$

where:

$$e' = e_i - \bar{e}$$

$$o' = o_i - \bar{o}.$$

The index of agreement (IOA) between the predictions and the observations of a variable can be given as:

$$IOA = 1 - \frac{\sum_{i=1}^n (e_i - o_i)^2}{\sum_{i=1}^n (|e_i - \bar{o}| + |o_i - \bar{o}|)^2}$$

The Pearson correlation coefficient (r) between the predictions and the observations of a variable can be given as:

$$r = \frac{\sum_{i=1}^n (e_i - \bar{e}) (o_i - \bar{o})}{\sqrt{\sum_{i=1}^n (e_i - \bar{e})^2 \sum_{i=1}^n (o_i - \bar{o})^2}}$$

Other names & Abbrev: COR, CORR, R

## Variability Measures

The standard deviation (SDEV) of a variable predicted by a model (or observation or other like model-obs difference) can be given as:

$$SDEV = \sqrt{1/N \sum_{i=1}^n (e_i - \bar{e})^2}$$

Other names & Abbrev: SD, StDev,  $\sigma$

Coefficient of Variation (standard deviation normalized by the average of the data):

$$COV = \frac{SDEV}{\bar{e}}$$