Air Quality Model Performance Metric Definitions

Common Variables:

N,n = count of samples (model-obs pairs)
i = index for individual observation or model estimate pair
e_i: model estimated or predicted value or a variable at time or location i.
$\bar{e}$: mean value of the model estimated values of a variable over a time or space.
e': the deviation of the model estimated value of a variable from its mean value at time or location i.
o_i: observed or expected value or a variable at location/time i.
$\bar{o}$: mean value of the observed values of a variable over a time or space.
o': the deviation of the observed value of a variable from its mean value

Standard Error and Bias Metrics (units of obs.)

The bias (BIAS) in predicting a variable can be given as:

$$BIAS = 1/N \sum_{i=1}^{n} (e_i - o_i)$$

Other names & Abbrev: Mean Error (ME), Mean Bias (MB)

The mean absolute error (MAE) in predicting a variable can be given as:

$$MAE = 1/N \sum_{i=1}^{n} |e_i - o_i|$$

Other names & Abbrev: MAE, mean absolute gross error

The root mean square error (RMSE) in predicting a variable can be given as:

$$RMSE = \sqrt{1/N \sum_{i=1}^{n} (e_i - o_i)^2}$$

Systematic vs Unsystematic Error (units of obs.)

Systematic RMSE ($RMSE_s$) in predicting a variable can be given as:

$$RMSE_s = \sqrt{1/N \sum_{i=1}^{n} (C - o_i)^2}$$

Unsystematic RMSE ($RMSE_u$) in predicting a variable can be given as:

$$RMSE_u = \sqrt{1/N \sum_{i=1}^{n} (C - e_i)^2}$$

$$C = a + b\, e_i$$

Where: **a** and **b** are the least squares regression coefficients of $\mathbf{o_i}$ (observed) and $\mathbf{e_i}$ (model estimated) value.

Normalization Metrics (unitless)

Normalized Mean Bias:

$$NMB = \frac{\sum_{i=1}^{n}(e_i - o_i)}{\sum_{i=1}^{n} o_i}$$

Other names & Abbrev: Normalized bias

Normalized Mean Error:

$$NME = \frac{\sum_{i=1}^{n}|e_i - o_i|}{\sum_{i=1}^{n} o_i}$$

Other names & Abbrev: Normalized error

Fractional Metrics (unitless)

Fractional Bias:

$$FB = \frac{1}{n}\left[\frac{\sum_{i=1}^{n}(e_i - o_i)}{\sum_{i=1}^{n}\frac{(e_i + o_i)}{2}}\right]$$

Other names & Abbrev: Mean Fractional Bias (MFB)

Fractional Error:

$$FE = \frac{1}{n}\left[\frac{\sum_{i=1}^{n}|e_i - o_i|}{\sum_{i=1}^{n}\frac{(e_i + o_i)}{2}}\right]$$

Other names & Abbrev: Mean Fractional Error (MFE)

Correlation Metrics (unitless)

The anomaly correlation (AC) coefficient used in AMET in predicting a variable is based on uncentered approach and can be given as:

$$AC = \frac{1/N\sum_{i=1}^{n} e\prime o\prime}{\sqrt{1/N\left(\sum_{i=1}^{n}(e\prime)^2 * 1/N\sum_{i=1}^{n}(o\prime)^2\right)}}$$

where:
e’ = $e_i - \bar{e}$
o’ = $o_i - \bar{o}$.

The index of agreement (IOA) between the predictions and the observations of a variable can be given as:

$$IOA = 1 - \frac{\sum_{i=1}^{n}(e_i - o_i)^2}{\sum_{i=1}^{n}(|e_i - \bar{o}| + |o_i - \bar{o}|)^2}$$

The Pearson correlation coefficient (r) between the predictions and the observations of a variable can be given as:

$$r = \frac{\sum_{i=1}^{n}(e_i - \overline{e})(o_i - \overline{o})}{\sqrt{\sum_{i=1}^{n}(e_i - \overline{e})^2 \sum_{i=1}^{n}(o_i - \overline{o})^2}}$$

Other names & Abbrev: COR, CORR, R

Variability Measures

The standard deviation (SDEV) of a variable predicted by a model (or observation or other like model-obs difference) can be given as:

$$SDEV = \sqrt{1/N \sum_{i=1}^{n} (e_i - \bar{e})^2}$$

Other names & Abbrev: SD, StDev, σ

Coefficient of Variation (standard deviation normalized by the average of the data):

$$COV = \frac{SDEV}{\bar{e}}$$