# SOLOW SWAN MODEL HOMEWORK

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**Group J**

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1. Unrestricted estimation on the replication dataset of the human capital augmented Solow model, with y_17 as the dependent variable and s, δ_n_g and s_h as independent variables.

In [63]:
```
# Run regression on the non-oil countries sample
print('\t Non-oil countries')
mrw_h_rep_results = smf.ols('y_17 ~ s + δ_n_g + s_h', data=mrw_h_rep).fit()
print(mrw_h_rep_results.summary())
```

```
         Non-oil countries
                            OLS Regression Results
==============================================================================
Dep. Variable:                   y_17   R-squared:                       0.702
Model:                            OLS   Adj. R-squared:                  0.693
Method:                 Least Squares   F-statistic:                     71.61
Date:                Sun, 12 Dec 2021   Prob (F-statistic):           7.21e-24
Time:                        14:03:05   Log-Likelihood:                -95.617
No. Observations:                  95   AIC:                             199.2
Df Residuals:                      91   BIC:                             209.4
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept      4.2195      1.539      2.743      0.007       1.163       7.276
s              0.9181      0.235      3.915      0.000       0.452       1.384
δ_n_g         -1.9200      0.574     -3.343      0.001      -3.061      -0.779
s_h            1.1928      0.157      7.607      0.000       0.881       1.504
==============================================================================
Omnibus:                       38.592   Durbin-Watson:                   1.649
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              121.508
Skew:                          -1.340   Prob(JB):                     4.12e-27
Kurtosis:                       7.849   Cond. No.                         88.9
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

**Interpretation:**

As we can see here, the GDP per worker for the year 2017 is positively correlated with savings s and investment rate in human capital s_h and negatively with the number of 'δ*n*g', it is obvious that GDP per worker increases when savings s and investment rate in human capital s_h increases. If there is an increase in depreciation rate, increase in population growth then the GDP reduces.

The value of R-squared is 0.70, it means that the model can explain around 70% of total variation of GDP per capita across country

2. A restricted estimation on the replication dataset of the human capital augmented Solow model, with y_17 as the dependent variable, and appropriate restrictions as independent variables.

```
# Homework Task 2

mrw_rep['restricted'] = mrw_rep['s'] - mrw_rep['δ_n_g']
mrw_rep_res_1 = smf.ols('y_17 ~ restricted ', data = mrw_rep).fit()
α_β_res_1 = (mrw_rep_res_1.params[1]/(1+mrw_rep_res_1.params[1]))
print('only using restricted α=', mrw_rep_res_1.params[1] * (1-α_β_res_1))


mrw_rep['restricted'] = mrw_rep['s'] - mrw_rep['δ_n_g']
mrw_rep['restricted_h'] = mrw_rep['s_h'] - mrw_rep['δ_n_g']
print('\n\n\nHOMEWORK TASK 2')
mrw_rep_results_restricted = smf.ols('y_17 ~ restricted + restricted_h ', data = mrw_rep).fit()
print(mrw_rep_results_restricted.summary())

α_β_old = ((mrw_h_results_restricted.params[1]+mrw_h_results_restricted.params[2])/
(1+mrw_h_results_restricted.params[1]+mrw_h_results_restricted.params[2]))
α_β_new = ((mrw_rep_results_restricted.params[1]+mrw_rep_results_restricted.params[2])/
(1+mrw_rep_results_restricted.params[1]+mrw_rep_results_restricted.params[2]))

print('old α =', mrw_h_results_restricted.params[1] * (1-α_β_old))
print('old β =', mrw_h_results_restricted.params[2] * (1-α_β_old))
print('Implied α =', mrw_rep_results_restricted.params[1] * (1-α_β_new))
print('Implied β =', mrw_rep_results_restricted.params[2] * (1-α_β_new))
print('Test of restriction p-value =', mrw_rep_results.compare_f_test(mrw_rep_results_restricted)[1])
```

```
only using restricted α= 0.6777674774077055



HOMEWORK TASK 2
                            OLS Regression Results
==============================================================================
Dep. Variable:                   y_17   R-squared:                       0.702
Model:                            OLS   Adj. R-squared:                  0.696
Method:                 Least Squares   F-statistic:                     108.5
Date:                Sun, 12 Dec 2021   Prob (F-statistic):           6.34e-25
Time:                        16:27:23   Log-Likelihood:                -95.658
No. Observations:                  95   AIC:                             197.3
Df Residuals:                      92   BIC:                             205.0
Df Model:                           2
Covariance Type:            nonrobust
================================================================================
                   coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------
Intercept        3.8147      0.537      7.106      0.000       2.749       4.881
restricted       0.8987      0.223      4.029      0.000       0.456       1.342
restricted_h     1.1753      0.143      8.212      0.000       0.891       1.460
==============================================================================
Omnibus:                       37.896   Durbin-Watson:                   1.668
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              116.973
Skew:                          -1.321   Prob(JB):                     3.98e-26
Kurtosis:                       7.751   Cond. No.                         39.1
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
old α = 0.3082141483520783
old β = 0.27431268216163596
Implied α = 0.2923657451901177
Implied β = 0.3823287533440458
Test of restriction p-value = 0.7793650591954614
```

**Interpretation:**

When we use the restricted estimation using the Solow Swan model, the value of alpha approximates to 0.6. But the value of alpha is expected between 0.3-0.4. Hence, when using firms profit maximization the value of alpha is reduced to 0.29~0.3.

Change in the value of alpha:
Old value of alpha: 0.308
New value of implied alpha: 0.29.
There is a decrement of 6.8% of the value of alpha. There is however a problem: implied level of α is well below 0.3, whereas economists usually think that α ∈ [0.3, 0.4].

Change in the value of beta:
Old value of beta: 0.274
New value of implied beta: 0.382
There is a decrement of 28% of the value of beta.

3. A convergence estimation on the replication dataset of the human capital augmented Solow model, with y_17_85 as the dependent variable and y_85, s, δ_n_g and s_h as independent variables.

```
print('\t HOMEWORK TASK 3')

mrw_rep['y_17_85'] = mrw_rep['y_17'] - mrw_rep['y_85']

mrw_rep_results = smf.ols('y_17_85 ~ y_85 + s + δ_n_g + s_h', data=mrw_rep).fit()
print(mrw_rep_results.summary())
print('')
print('Implied λ =', np.log(1+mrw_results_O.params[1])/(-32))
```

```
         HOMEWORK TASK 3
                            OLS Regression Results
==============================================================================
Dep. Variable:                y_17_85   R-squared:                       0.200
Model:                            OLS   Adj. R-squared:                  0.165
Method:                 Least Squares   F-statistic:                     5.639
Date:                Sun, 12 Dec 2021   Prob (F-statistic):           0.000427
Time:                        17:47:35   Log-Likelihood:                -81.212
No. Observations:                  95   AIC:                             172.4
Df Residuals:                      90   BIC:                             185.2
Df Model:                           4
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept      0.7581      1.464      0.518      0.606      -2.150       3.666
y_85          -0.4142      0.104     -3.993      0.000      -0.620      -0.208
s              0.4658      0.218      2.138      0.035       0.033       0.899
δ_n_g         -1.3327      0.507     -2.629      0.010      -2.340      -0.325
s_h            0.5061      0.182      2.780      0.007       0.144       0.868
==============================================================================
Omnibus:                       88.768   Durbin-Watson:                   1.595
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             1176.200
Skew:                          -2.820   Prob(JB):                    3.90e-256
Kurtosis:                      19.289   Cond. No.                         261.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Implied λ = 0.015843235571682643
```

```
# Convergence plots

par = mrw_rep_results.params

# plt.scatter(mrw_rep['y_85'], 100*((mrw_rep['Y17']/mrw_rep['Y85'])**(1/32)-1))
plt.scatter(mrw_rep['y_85'], 100*mrw_rep['y_17_85']/32)

plt.title('Unconditional')
plt.xlabel('Log output per working age adult: 1985')
plt.ylabel('Growth rate: 1985-2017')

plt.show()

###
plt.scatter(mrw_rep['y_85'],
            100/25*(mrw_rep['y_17_85']
                    -par[2]*(mrw_rep['s']-mrw_rep['s'].mean())
                    -par[3]*(mrw_rep['δ_n_g']-mrw_rep['δ_n_g'].mean())
                    -par[4]*(mrw_rep['s_h']-mrw_rep['s_h'].mean())))

plt.title('Conditional on saving, population growth and human capital')
plt.xlabel('Log output per working age adult: 1985')
plt.ylabel('Growth rate: 1985-2017')

plt.show()
```