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CSE 351
HW #2

1-) $H(s) = (2s+3)/(s^2+5s+6)$

a) if the input $f(t) = e^{-3t}u(t)$

$$y(s) = H(s).F(s) = \frac{(2s+3)}{(s+3)(s^2+5s+6)} = \frac{2s+3}{(s+2)(s+3)^2}$$

$$= \frac{k}{s+2} + \frac{a_0}{(s+3)^2} + \frac{a_1}{s+3}$$

$$k = \left.\frac{2s+3}{(s+3)^2}\right|_{s=-2} \Rightarrow -1$$

$$a_0 = \left.\frac{2s+3}{s+2}\right|_{s=-3} \Rightarrow 3$$

$$y(s) = \frac{-1}{s+2} + \frac{3}{(s+3)^2} + \frac{a_1}{s+3} = \frac{2s+3}{(s+2)(s+3)^2}$$

$$0 + a_1 - 1 = 0 \Rightarrow a_1 = 1$$

$$y(s) = \frac{-1}{s+2} + \frac{3}{(s+3)^2} + \frac{1}{s+3}$$

$$y(t) = \left[-e^{-2t} + (1+3t)e^{-3t}\right]u(t)$$

b) $y(s) = \left(\dfrac{2s+3}{s^2+5s+6}\right) F(s)$

$$(s^2+5s+6)\, y(s) = (2s+3)\, F(s)$$

$$(D^2+5D+6)\, y(t) = (2D+3)\, f(t)$$

$$\frac{d^2y}{dt^2} + \frac{5dy}{dt} + 6y(t) = \frac{2df}{dt} + 3f(t)$$

c) $H(s) = \dfrac{s+2}{s(s+1)^2} = \dfrac{k}{s} + \dfrac{a_0}{(s+1)^2} + \dfrac{a_1}{s+1}$

$\underline{\underline{k=2}}$, $\underline{\underline{a_0 = -1}}$ (using partial fraction expansion method);

$$H(s) = \frac{s+2}{s(s+1)^2} = \frac{2}{s} - \frac{1}{(s+1)^2} + \frac{a_1}{s+1}$$

→ multiply both sides by s and let $s \to \infty$

$$0 = 2 + 0 + a_1 \rightarrow a_1 = -2$$

$$H(s) = \frac{2}{s} - \frac{1}{(s+1)^2} - \frac{2}{s+1}$$

$$\underline{\underline{h(t) = \left[2 - (2+t)\, e^{-t}\right] u(t)}}$$

(2) The system equation in delay form is obtained as follows;

$$2y[k] - 3y[k-1] + y[k-2] = 4f[k] - 3f[k-1]$$

Using the initial conditions and delay property,

$$y[k] \Longleftrightarrow Y[z]$$

$$y[k-1] \Longleftrightarrow \frac{1}{z} Y[z]$$

$$y(k-2) \Longleftrightarrow \frac{1}{z^2} Y[z] + 1$$

$$2Y[z] - \frac{3}{z} Y[z] + \frac{1}{z^2} Y[z] + 1 = \frac{4z-3}{z-0{,}25}$$

$$\left[2 - \frac{3}{z} + \frac{1}{z^2}\right] Y[z] = -1 + \frac{4z-3}{z-0{,}25} = \frac{3z-2{,}75}{z-0{,}25}$$

$$\frac{Y(z)}{z} = \frac{z(3z-2{,}75)}{(2z^2-3z+1)(z-0{,}25)} = \frac{z(3z-2{,}75)}{2(z-0{,}5)(z-1)(z-0{,}25)}$$

$$\frac{Y(z)}{z} = \frac{5/2}{z-1/2} + \frac{1/3}{z-1} - \frac{4/3}{z-0{,}25}$$

$$y(k) = \left[\frac{1}{3} + \frac{5}{2}(2)^{-k} - \frac{1}{3}(4)^{-k}\right] u[k]$$

③ $$\frac{H[z]}{z} = \frac{-5z+22}{(z+1)(z-2)^2} = \frac{3}{z+1} + \frac{k}{z-2} + \frac{4}{(z-2)^2}$$

multiply both sides by z and let $z \to \infty$

$$0 = 3 + k + 0 \rightarrow k = -3$$

$$H[z] = 3\frac{z}{z+1} - 3\frac{z}{z-2} + 4\frac{z}{(z-2)^2}$$

$$H[z] \Longleftrightarrow h[k]$$

$$h[k] = [3(-1)^k - 3(2)^k + 2k(2)^k]\,u[k]$$