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CSE 351 #1

① linearity condition,

i) $f_1(t) \rightarrow \square \rightarrow y_1(t)$

$f_2(t) \rightarrow \square \rightarrow y_2(t)$

ii) $k_1 f_1(t) + k_2 f_2(t) \rightarrow \square \rightarrow k_1 . y_1(t) + k_2 y_2(t)$

a) $$\frac{dy(t)}{dt} + 2y(t) = f^2(t)$$

i) $$\frac{dy_1}{dt} + 2y_1(t) = f_1^2(t) \rightarrow \frac{dk_1 y_1(t)}{dt} + 2k_1 y_1(t) = k_1 f_1^2(t)$$

$$\frac{dy_2}{dt} + 2y_2(t) = f_2^2(t) \rightarrow \frac{dk_2 y_2(t)}{dt} + 2k_2 y_2(t) = k_2 f_2^2(t)$$

$$\frac{d}{dt}\underbrace{\left[k_1 y_1(t) + k_2 y_2(t)\right]}_{y(t)} + 2\underbrace{\left[k_1 y_1(t) + k_2 y_2(k)\right]}_{y(t)} = \underbrace{k_1 f_1^2(t) + k_2 f_2^2(t)}_{f_i(t)}$$

$$\boxed{f_i(t) = k_1 f_1^2(t) + k_2 f_2^2(t)}$$

ii) $$\frac{d}{dt}\Big[\underbrace{k_1 y_1(t) + k_2 y_2(t)}_{y(t)}\Big] + 2\Big[\underbrace{k_1 y_1(t) + k_2 y_2(t)}_{y(t)}\Big] = \underbrace{[k_1 f_1(t) + k_2 f_2]^2}_{f_{ii}(t)}$$

Since $f_i(t) \neq f_{ii}(t)$, this system is non-linear.

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b) $$\frac{dy(t)}{dt} + 3ty(t) = t^2 f(t)$$

i) $$\frac{dy_1(t)}{dt} + 3ty_1(t) = t^2 f_1(t) \rightarrow \frac{dk_1 y_1(t)}{dt} + 3tk_1 y_1(t) = t^2 k_1 f_1(t)$$

$$\frac{dy_2(t)}{dt} + 3ty_2(t) = t^2 f_2(t) \rightarrow \frac{dk_2 y_2(t)}{dt} + 3tk_2 y_2(t) = t^2 k_2 f_2(t)$$

$$\frac{d}{dt}\Big[\underbrace{k_1 y_1(t) + k_2 y_2(t)}_{y(t)}\Big] + 3t\Big[\underbrace{k_1 y_1(t) + k_2 y_2(t)}_{y(t)}\Big] = t^2\Big[\underbrace{k_1 f_1(t) + k_2 f_2(t)}_{f_i(t)}\Big]$$

ii) $$\frac{d}{dt}\Big[\underbrace{k_1 y_1(t) + k_2 y_2(t)}_{y(t)}\Big] + 3t\Big[\underbrace{k_1 y_1(t) + k_2 y_2(t)}_{y(t)}\Big] = t^2\Big[\underbrace{k_1 f_1(t) + k_2 f_2(t)}_{f_{ii}(t)}\Big]$$

Since $f_i(t) = f_{ii}(t)$, this system is linear.

2)

a) Characteristic Equation : $(\lambda^2+5\lambda+6)=0$

$$(\lambda+2)(\lambda+3)=0$$

$$\lambda_1=-2 \quad \lambda_2=-3$$

$$y_0(t)=c_1e^{\lambda_1 t}+c_2e^{\lambda_2 t}=c_1e^{-2t}+c_2e^{-3t}$$

$$y_0'(t)=-2c_1e^{-2t}-3c_2e^{-3t}$$

set $t=0$ using $y_0(0)=2$, $y_0(0)=-1$

$$\left.\begin{array}{l} c_1+c_2=2 \\ -2c_1-3c_2=-1 \end{array}\right\} \begin{array}{l} c_1=5 \\ c_2=-3 \end{array}$$

$$\boxed{y_0(t)=5e^{-2t}-3e^{-3t}}$$

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b) $f(t)=u(t) \rightarrow \boxed{h(t)=e^{-t}u(t)} \rightarrow y(t)=?$

$$y(t)=h(t)*f(t)$$
$$=e^{-t}u(t)*u(t)$$
$$=(1-e^{-t})\,u(t)$$

3)

a) $y(k+1) + 2y(k) = f(k) \rightarrow (E+2)\, y(k) = f(k)$

$$h(k) = \frac{b_0}{a_0}\delta(k) + y_0(k)\, u(k)$$

Characteristic Equation $(\gamma+2) = 0 \Rightarrow \gamma = -2$

$$y_0(k) = c.(\gamma)^k = c(-2)^k$$

Also, $a_0 = 2$, $b_0 = 1$

Therefore

$$h(k) = \frac{1}{2}\delta(k) + c(-2)^k$$

Use iterative solution to determine c.

$$(E+2)\, h(k) = \delta(k)$$

$$h(k+1) + 2h(k) = \delta(k)$$

Set $k=-1$ and substitute $h(-1) = \delta(-1) = 0$

$$h(0) = 0$$

For $k=0$ and use $h(0) = 0$

$$0 = \frac{1}{2} + c(-2)^0 \Rightarrow c = -\frac{1}{2}$$

As a result,

$$\boxed{h(k) = \frac{1}{2}\delta(k) - \frac{1}{2}(-2)^k u(k)}$$

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b) $h(k) = (-2)^k u(k)$, $f(k) = e^{-k} u(k) \rightarrow y(k) = f(k) * h(k)$

$f_1(k) = \gamma^k u(k)$, $f_2(k) = \gamma_2 u(k)$, $\gamma_1 \neq \gamma_2$

$$f_1(k) * f_2(k) = \left[\frac{\gamma_1^{k+1} - \gamma_2^{k+1}}{\gamma_1 - \gamma_2} \right] u(k)$$

$$y(k) = e^{-k} u(k) * (-2)^k u(k)$$

$$= \left(\frac{1}{e}\right)^k u(k) * (-2)^k . u(k)$$

$$= \left[\frac{(1/e)^{k+1} - (-2)^{k+1}}{(1/e) + 2} \right] u(k)$$

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