# Structural Optimization with X-FEM in Architecture

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**Abstract.** *In this article the author describe how X-FEM capability could be used for three dimensional shape optimization in architecture. We will begin by a quick Review of structural optimization technique. Followed by a description of X-FEM principle and which changes are needed for applying it to shape optimization. Then we introduce some constrains that are took into account in this optimization process. We will finish by computing two numerical example: a classical cantilever and an architectural column.*

## 1 Introduction

Structural optimization has for purpose of using the material at their best. Classical objectives are (i) reducing the quantity of raw material in the hope that will also reduce the global cost of the structure or (ii) reducing the total thickness of structure thus reducing the visual impact of the structure or providing more room for services. For both of this reasons structural optimization could improve building quality, in particular we believe that the design of "free form" could be significantly improved by using structural optimization at early stage of the design process.

Nowadays, structure and geometric optimization are mainly driven by computer application. We could distinguish optimization method by looking how the geometry is handled by the optimization process.

Historically structures, in particular truss are described by "bars" linking two nodes each bar has a section and a material. The geometry is then described by a set of parameter thus any classical optimization method could handle the problem. However using a set of parameter in order to describe the geometry will reduce the field of possible geometry.

With the finite element analysis method the structure is described as a mesh (node and bar are in fact mesh composed only off 1D element). This method has been successfully used for performing shape finding for textile structure with method like dynamic relaxation. In this process nodes move mainly perpendicularly to the surface. However for other problem like in-plan cantilever the mesh evolution is so dramatic that, for maintaining the mesh quality, re-meshing is needed during the optimization process.

In order to avoid re-meshing the geometry could be described as a fix mesh with a density of material varying between zero and one. This description is called homogenization in [Allaire et al. 1997] or Bidirectional structural optimization

(BESO) in [Yang et al. 1999] or topology optimization. This last denomination emphasizes the ability of this geometric description to change the topology. However, this method has the drawback of introducing a new kind of material that does not exist (density 0.5 is not available unless we can approximate it with some kind of expensive laminated material). Therefore a post production strategy is needed in order to have real material.

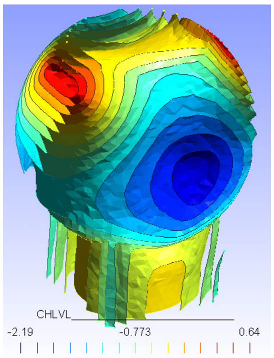


Figure 1: Example of level set field; only iso-layers are plotted, therefore the iso-layer zero is the actual boundary of the shape.

Another approach of the same idea is the description of the geometry on a fixed mesh but with only a density of 0 or 1 the discontinuity of this description is handled by using a test on a continuous function. This function is close to the signed distance of the shape border and called level-set function.

## 2 Optimization algorithm

Most of the optimization algorithms are aimed at finite set of parameter (like genetic algorithm, Gauss-Newtown, BFGS …), however in continuous shape optimization case we should find an algorithm capable of dealing with a varying or a very important number of parameter algorithm and multiple constraints. In addition we can compute the shape gradient but not the "Hessienne matrix", the problem have significant time cost for shape displacement and evaluation and relatively low time cost for computing the shape gradient.

Such perfect algorithm does not exist; we chose to work with a "Conjugate Gradient" methods with penalized constrains. This algorithm as the advantages to converge relatively quickly but does not satisfy all the constraint at the end of the process.

The optimization steps are the following:

1. Choose an initial geometry $\Phi_0$
2. Until the defined number of step has been reached do:
   a. Evaluate the value of the objective and penalized constraint for the current shape $\phi_k$

b. Compute the shape gradient $V_k$ for the objective and penalized constraint for the current shape $\phi_k$
c. $\beta_k$=max[0; ($V^T_k$. $V_k$- $V_{k-1}$)/ ($V^T_{k-1}$ .$V_{k-1}$)]
d. Compute the conjugate direction $V_{CG,k}=V_k+\beta_k V_{CG,k-1}$
e. Along the conjugate direction determine the best step $\alpha_k$
f. Compute the new shape: $\phi_{k+1}= \phi_k+\alpha_k V_{CG,k}$

The shape gradient should be computed for the objective function and constraint functions. This will be developed in chapter 4

# 3 Level set and X-FEM

## 3.1 Level set framework

This section recalls the framework of the level set method as proposed by [S. Osher et James A Sethian 1988]. The application of the level set method to structural optimization was pioneering in [M. Y. Wang, X. Wang, et Guo 2003], [Allaire et François Jouve 2005], [S. J. Osher et Santosa 2001], [J.A. Sethian et Wiegmann 2000].

Consider $D \subset \mathbb{R}^3$ a bounded domain in which all admissible shapes Ω are included. The domain D will be meshed once and for all. We parameterize the boundary of Ω by means of a level set function ϕ, defined in D by

$$\begin{cases} \Phi(x) = 0 & \Leftrightarrow \quad x \in \partial\Omega \cap D \\ \Phi(x) < 0 & \leftrightarrow \quad x \in \Omega \\ \Phi(x) > 0 & <> \quad x \subset (D \setminus \Omega) \end{cases}$$

During the optimization process, the shape Ω(t) is going to evolve according to a fictitious time parameter $t \in \mathbb{R}$ which corresponds to descent stepping. The evolution of the level set function is governed by the following Hamilton-Jacobi transport equation:

$$\frac{\partial \Phi}{\partial t} + V|\bigtriangledown \Phi| = 0 \; in \; D$$

where V (t; x) is the normal velocity of the shape's boundary. This equation behaves better if ∇ ϕ is almost constant around the border ∂Ω. In practice the level set function ϕ is close to the signed distance to the border ∂Ω.

The main advantage of this method is its natural capacity fill or create hole in the shape. This is possible because, the boundary of the shape is not stored in the mesh but in the level set function in which creating a hole is simply setting some value to be positive.

For the computation of the shape’s behavior under different loading, finite element outside of the shape will be filled with a weak phase mimicking void but avoiding the singularity of the stiffness matrix. The drawback is that second order behavior like buckling or natural frequency could be altered by this weak phase, in which can be produced fictious eigenmodes see [Allaire et François Jouve 2005] [Bendsoe et Sigmund 2003]

### 3.2 X-Fem method

The X-Fem principle is to use local re-meshing, splitting finite element precisely on the boundary of a shape. Furthermore computed field could accept discontinuity when crossing the boundary. This results in computational independent volumes. Main industrial computation of this technology is aimed to crack propagation see [Geniaut, Massin, et Moës 2007].

Our development is based on the open-source finite elements software Code Aster owned by EDF. Code Aster is an “in house”, general purpose, finite elements analysis software designed for nuclear power plant structural analysis. Which mean that its first purpose is reliability and secondary purpose is to be able to perform specific computation, which are needed to asses a nuclear power plant safety. It worth noticing that EDF decided to make this software and it’s documentation open-source. The code is mainly FORTRAN with some C++ part and the user interface is PYTHON based. It is designed to work under Linux on local or remote computer.

The X-Fem module was designed to handle crack propagation in steel material. The crack is described by two intricate level-set. One for the localization of the crack plane and the other for the localization of the crack tip see [Colombo et Massin 2011] for complete explanation.

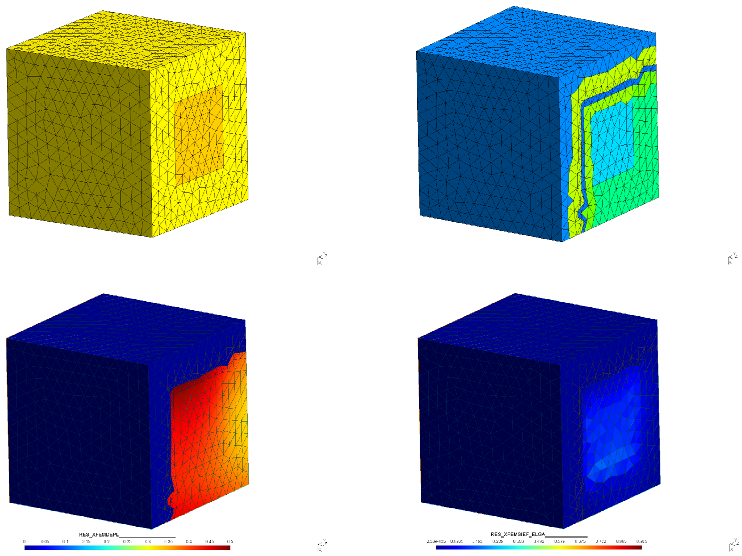

Figure 2: Example of X-Fem analysis done with Code Aster, from top left to bottom right: (i) Initial mesh, (ii) mesh with local remeshing, (iii) discontinuous field of displacement, (iv) discontinuous stress field.

Currently buckling analysis cannot be performed with an X-Fem model. Therefore optimization of structure susceptible to buckle will not be performed. In architecture this includes most of slender steel structure like column of wind bracing beam.

As the source code was available we modified the software to be able to:

- Initiate a level-set function with any scalar field in Figure 2 (i) the initialization is done by setting all nodes in a square volume to -1 and all the other to +1 leading to a boundary going exactly through the middle of the tetrahedron.

- Create specific area labeling according to the finite element position result can be see in Figure 2 (ii).

- Keep some specified area inside the level set, by setting to negative value these area.

- Redefine the propagation step for a transport equation of only one level-set.

# 4 Control of the optimization process

It is very important to control the result of the optimization. This can be achieve by objective and constrains function but it could also be controlled by the shape propagation step.

## 4.1 Physical space and design space

Design space and physical space are defined as follow. Physical space gathers the physical property that should be set in order to solve the objective function or constrains function. This will describe the behavior of the geometry in the “real” world. It could be load cases and usual boundary condition. Design space gathers all the property that determines how the geometry can change during the optimization process for example the domain mesh geometry.

## 4.2 Physical space

We implemented two objective functions: shape volume and total energy under external load (also called compliance). [Allaire et François Jouve 2005] derived shape gradient for natural frequency and buckling coefficient, however second order resolution are not available in the XFEM software we used. They also propose to approach maximum stress and maximum displacement within the shape by the differentiable function $x^3$.

## 4.3 Design space

The easiest way to control the shape is of course by the mesh shape. As the level set is defined on the mesh’s node, the curvature and precision of the resulting shape will be largely influenced by the mesh’s tetrahedron size.

We implemented a smoothing algorithm that reduces to a specified magnitude the curvature of the shape at each optimization step. We implemented a filling algorithm which guaranties that selected volume will always filled by material.

# 5 Numerical result

## 5.1 Cantilever

The cantilever is a classical study for structural shape optimization. Benchmark study is available for 2d plate optimization as well as 3d optimization with the BESO method see the work done by [Borrvall et Petersson 2001] or [Young et al. 1999].

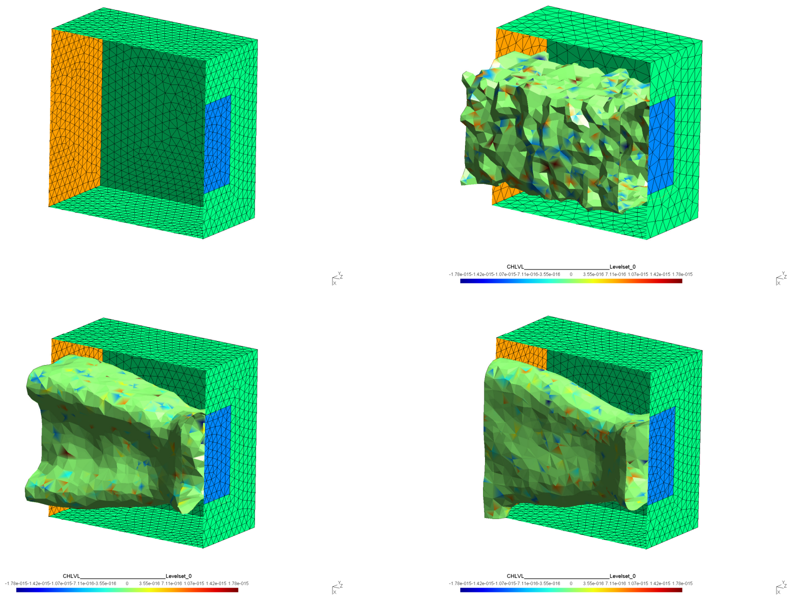

Figure 3: Optimization result on a canteliver problem done with Code Aster. From top left to bottom rigth: (i) meshed volume, (ii) initial filled volume, (iii) solution case A, (iv) solution case B

In our study the meshed volume is a cube. The physical conditions are the following: on the left face the blue square is loaded downward, the opposite face (in orange) is fixed. Regarding the design space, a volume around the loaded area (blue square) should be included in the result shape; this for avoiding instability, in addition a smoothing step is performed at every shape evolution.

The purpose of the optimization is to reduce the volume while keeping the same compliance under an external load.

Computation result: the number of node in the model is around 4000, there are 32000 tetrahedra, the computation took around 80 steps in 2 hours on a 2005 laptop. Mesh grid size is a key point for a quick convergence. In fact reducing the minimal length by a factor q will increase the computation time by more than $q^3$

due to increasing number of step for the evolution of the level-set (stability issue of the transport equation). The computation time could be reduced by stat of the art computer or “could computing”, but will no disappear.

Results of the computation are show in the bottom of figure 3. In case A the acceptable radius of curvature has been reduced by comparison of case B. In both case we see that the optimization process led to an “I” section beam, which is a well know optimum for this problem.

### 5.2 Support of monumental column

This example is closer to the real-world study. A steel roof is supported by a series of “quadripode” each “quadripode” is composed of four steel columns supported by a unique concrete support.

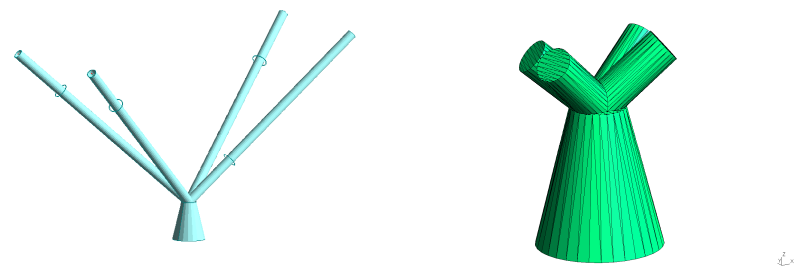

Figure 4: Global view of a "quadripode"; zoom on the concrete base the height the base is around 3 meters.

Physical space: load from the roof have been computed according to the global analytical model. Three load cases have been applied to the calculation model. The shape is fixed at the bottom.

As previously, regarding the design space: a volume around the loaded area (circle on top) is included in the result shape, this for avoiding instability. And a smoothing step is performed at every shape evolution. The purpose of the optimization is to reduce the volume while keeping the same compliance under the external load.

The computation takes around 120 steps in 6 hours for approximately the same mesh size. The difference of computation time could be attributed to the increase number of load case (from 1 to 3).

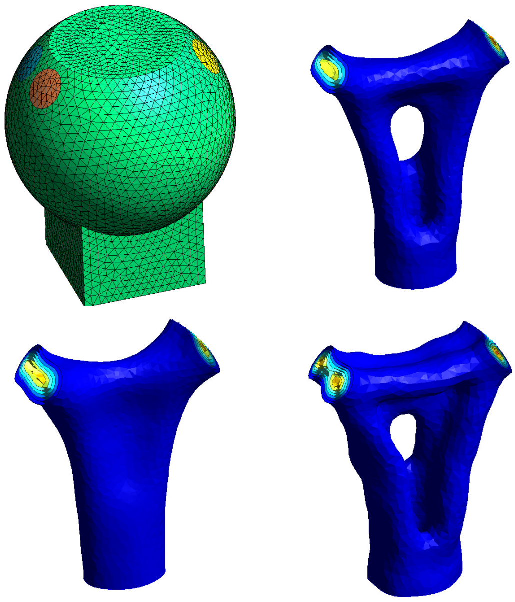

Figure 5: Shape optimization results for the «quadripode», on the top left the global mesh, the other view are solution with different smoothing parameters

Results of the computation are show in the bottom of figure 4. The result shape work as follow, a link between loaded areas will equilibrate horizontal forces and the remaining loads are taking down to the ground. We can see small asymmetry at proximity to loaded area due to the asymmetry of loading. Result shapes are not intuitive as they could not easily be guessed from the initial problem in particular the variation according to the acceptable radius of curvature. Structurally the central part carries the shear forces and plays the role of a web.

Graphically result shapes strongly express the load involved in the structure. In this project the source of the load --the roof-- is visible at the same times than the columns. The dialogue between these elements linked by the “cold” steel column, create a very specific kind of organic atmosphere.

## 6 Conclusion and future work

The method described in this article has been successful to handle solid design like column foot or connection. This method is promising for environmental performance due to the reduction of material quantity and the creation of a specific atmosphere. We saw that 3D problem create new issue regarding computation time and for very precise analysis specific solution are needed. Theoretically X-FEM could better handle buckling or frequency constraint than classical level-set method.

Development in this area will be needed to handle slender structure like steel column.

## Acknowledgements

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