1 Expansion of the Sudakov form factor

In this section I work out the expansion of the quark Sudakov form factor to $\mathcal{O}(\alpha_s^2)$. The Sudakov form factor is independent of the process and thus the same expansion can be used for example for Drell-Yan and SIDIS.[1] One crucial point is that the expansion will be done in the so-called ζ-prescription introduced in Ref. [1].

The Sudakov form factor we are considering here is essentially the square of the quark evolution factor deriving from the solution of the Collins-Soper (CS) equations. As is well known, the CS equations govern the evolution of TMD distributions in two independent factorisation scales μ and ζ. Usually, μ and ζ are related to each other by assuming that $\mu = \sqrt{\zeta}$. This particular choice reduces the degrees of freedom of the problem. On the other hand, as observed in Ref. [1], this choice does not help cure large logarithms that appear in the matching functions. The mutual independence of μ and ζ can be exploited to guarantee that such large logarithms are reabsorbed in the definition of the scale ζ as a function of μ (or viceversa). This is the basic observation at the base of the ζ-prescription that achieves this goal by assuming $\zeta \equiv \zeta(\mu)$ and requiring that the total derivative of TMD distribution F with respect to μ is zero:

$$\mu^2 \frac{dF\left(x, b; \mu, \zeta(\mu)\right)}{d\mu^2} = 0. \tag{1}$$

This naturally leads to a differential equation in $\zeta(\mu)$ involving the CS anomalous dimensions and $\Gamma_{\rm cusp}$ that can be solved order by order in α_s. The exact form of the function $\zeta(\mu)$ has to be taken into account when expanding the Sudakov form factor.

The exact form of the Sudakov form factor, that we remind to be the squared evolution factor deriving from the solution of the CS equations and the evolves a pair of quark TMD distributions in b space from the scales (μ_i, ζ_i) to (μ_f, ζ_f), is:

$$\left[R(\mu_i, \zeta_i \rightarrow \mu_f, \zeta_f; b)\right]^2 = \exp\left[\int_{\mu_i^2}^{\mu_f^2} \frac{d\mu^2}{\mu^2}\left(-\gamma_V + \Gamma_{\rm cusp} \ln\left(\frac{\mu^2}{\zeta_f}\right)\right) - 2\mathcal{D} \ln\left(\frac{\zeta_f}{\zeta_i}\right)\right]. \tag{2}$$

where the following perturbative expansion hold:

$$\gamma_V = \sum_{n=1} a_s^n \gamma_V^{(n)} \qquad \Gamma_{\rm cusp} = \sum_{n=1} a_s^n \Gamma_{\rm cusp}^{(n)}\,, \tag{3}$$

and:

$$\mathcal{D} = \sum_{n=1} a_s^n \sum_{k=0}^{n} d^{(n,k)} L^k\,, \tag{4}$$

where I have defined:

$$a_s(\mu) = \frac{\alpha_s(\mu)}{4\pi} \quad \text{and} \quad L \equiv \ln\left(\frac{b^2\mu_i^2}{4e^{-2\gamma_E}}\right)\,. \tag{5}$$

The perturbative coefficients, $\gamma_V^{(n)}$, $\Gamma_{\rm cusp}^{(n)}$, and $d^{(n,k)}$ are know up to the order necessary to implement NNLL evolution.[2] The final scales μ_f and ζ_f are usually taken to be equal to $\mu_f = \sqrt{\zeta} = \kappa Q$, where Q is, for example the virtuality of the exchanged photon in DIS and the invariant mass of the lepton pair in Drell-Yan. In addition, using the ζ-prescription, we set:

$$\zeta_i \equiv \zeta(\mu_i) = \mu_i^2 \exp\left(\sum_{n=0} a_s^n \sum_{k=0}^{n+1} \ell^{(n,k)} L^k\right) \tag{6}$$

where the coefficients $\ell^{(n,k)}$ are constants.

Before expanding the exponential we need to write its argument as a polynomial in α_s (or better a_s) computed in Q. These terms will in turn multiply powers of $\ln(Q)$. In view of the integral over

[1] Note that for gluon-initiated processes the expression will be different.

[2] In fact, with the only exception of $\Gamma_{\rm cusp}^{(4)}$, it would be possible to implement evolution up to N^{3}LL.

the impact parameter b needed to obtain cross sections differential in the transverse momentum q_T, we need to assign μ_i a dependence on b. Despite this dependence is arbitrary and needs only be dimensionally correct , the most natural choice is:

$$\mu_i = \frac{C_0}{b}\,, \quad \text{with} \quad C_0 = 2e^{-\gamma_E}\,. \tag{7}$$

This choice is such that L defined in Eq. (5) vanishes so that:

$$\mathcal{D} = \sum_{n=1} a_s^n d^{(n,0)} \tag{8}$$

and:

$$\zeta(\mu_i) = \frac{C_0^2}{b^2} \exp\left(\sum_{n=0} a_s^n \ell^{(n,0)}\right)\,. \tag{9}$$

This way the Sudakov form factor in Eq. (2) reads:

$$\begin{aligned}
\left[R(Q,b)\right]^2 &= \exp\Bigg\{ -\sum_{n=1}^{2}\int_{C_0^2/b^2}^{Q^2} \frac{d\mu^2}{\mu^2} a_s^n(\mu)\left[\gamma_V^{(n)} + \Gamma_{\rm cusp}^{(n)} \ln\left(\frac{Q^2}{\mu^2}\right)\right] \\
&+ 2\sum_{n=1}^{2} a_s^n\left(\frac{C_0}{b}\right) d^{(n,0)}\left[-\ln\left(\frac{b^2Q^2}{C_0^2}\right) + \ell^{(0,0)} + a_s\left(\frac{C_0}{b}\right)\ell^{(1,0)}\right]\Bigg\}\,,
\end{aligned} \tag{10}$$

where we limited the contributions in the exponential to $\mathcal{O}(a_s^2)$ that is the order we need. Now, using the RGE:

$$\mu^2 \frac{da_s}{d\mu^2} = -\beta_0 a_s^2(\mu)\,, \tag{11}$$

whose solution is:

$$a_s(\mu) = \frac{a_s(Q)}{1 - a_s(Q)\beta_0 \ln(\mu^2/Q^2)} \simeq a_s(Q)\left[1 + a_s(Q)\beta_0\ln(Q^2/\mu^2) + \mathcal{O}(a_s^2)\right]\,, \tag{12}$$

we write every instance of a_s appearing in Eq. (10) in terms of $a_s(Q)$ and finally retain only terms up to $a_s^2(Q)$:

$$\begin{aligned}
\left[R(Q,b)\right]^2 &= \exp\Bigg\{ -a_s(Q)\int_{C_0^2/b^2}^{Q^2} \frac{d\mu^2}{\mu^2}\left[\gamma_V^{(1)} + \Gamma_{\rm cusp}^{(1)}\ln\left(\frac{Q^2}{\mu^2}\right)\right] \\
&- a_s^2(Q)\int_{C_0^2/b^2}^{Q^2} \frac{d\mu^2}{\mu^2}\left[\gamma_V^{(2)} + \left(\Gamma_{\rm cusp}^{(2)} + \gamma_V^{(1)}\beta_0\right)\ln\left(\frac{Q^2}{\mu^2}\right) + \Gamma_{\rm cusp}^{(1)}\beta_0\ln^2\left(\frac{Q^2}{\mu^2}\right)\right] \\
&+ 2a_s(Q)\left[d^{(1,0)}\ell^{(0,0)} - d^{(1,0)}\ln\left(\frac{b^2Q^2}{C_0^2}\right)\right] \\
&+ 2a_s^2(Q)\left[d^{(2,0)}\ell^{(0,0)} + d^{(1,0)}\ell^{(1,0)} + \left(-d^{(2,0)} + d^{(1,0)}\beta_0\ell^{(0,0)}\right)\ln\left(\frac{b^2Q^2}{C_0^2}\right) - d^{(1,0)}\beta_0\ln^2\left(\frac{b^2Q^2}{C_0^2}\right)\right]\Bigg\}\,.
\end{aligned} \tag{13}$$

The final step before carrying out the expansion is that of resolving the integrals:

$$\int_{C_0^2/b^2}^{Q^2}\frac{d\mu^2}{\mu^2}\ln^k\left(\frac{Q^2}{\mu^2}\right) = \int_{\ln(C_0^2/b^2)}^{\ln Q^2} d\ln\mu^2 \ln^k\left(\frac{Q^2}{\mu^2}\right) = \int_0^{\ln(b^2Q^2/C_0^2)} dx\, x^k = \frac{1}{k+1}\ln^{k+1}\left(\frac{b^2Q^2}{C_0^2}\right)\,. \tag{14}$$

If I define:

$$\mathcal{L} \equiv \ln\left(\frac{b^2Q^2}{C_0^2}\right)\,, \tag{15}$$

the Sudakov form fact in Eq. 13 reads:

$$
\begin{aligned}
[R(Q,\mathcal{L})]^2 &= \exp\left\{a_s(Q)\left[2d^{(1,0)}\ell^{(0,0)}-\left(\gamma_V^{(1)}+2d^{(1,0)}\right)\mathcal{L}-\frac{1}{2}\Gamma_{\rm cusp}^{(1)}\mathcal{L}^2\right]\right.\\
&+ a_s^2(Q)\left[2d^{(2,0)}\ell^{(0,0)}+2d^{(1,0)}\ell^{(1,0)}+\left(-2d^{(2,0)}+2d^{(1,0)}\beta_0\ell^{(0,0)}-\gamma_V^{(2)}\right)\mathcal{L}\right.\\
&- \left.\left.\frac{1}{2}\left(4d^{(1,0)}\beta_0+\Gamma_{\rm cusp}^{(2)}+\gamma_V^{(1)}\beta_0\right)\mathcal{L}^2-\frac{1}{3}\Gamma_{\rm cusp}^{(1)}\beta_0\mathcal{L}^3\right]\right\},
\end{aligned}
\tag{16}
$$

that can be conveniently written as:

$$
[R(Q,\mathcal{L})]^2=\exp\left\{\sum_{n=1}^{2}a_s^n(Q)\sum_{k=0}^{n+1}S^{(n,k)}\mathcal{L}^k\right\},
\tag{17}
$$

with:

$$
\begin{aligned}
&S^{(1,0)}=2d^{(1,0)}\ell^{(0,0)}=0\,,\quad S^{(1,1)}=-\left(\gamma_V^{(1)}+2d^{(1,0)}\right)=6C_F\,,\quad S^{(1,2)}=-\frac{1}{2}\Gamma_{\rm cusp}^{(1)}=-2C_F\,,\\
&S^{(2,0)}=2d^{(2,0)}\ell^{(0,0)}+2d^{(1,0)}\ell^{(1,0)}\,,\quad S^{(2,1)}=\left(-2d^{(2,0)}+2d^{(1,0)}\beta_0\ell^{(0,0)}-\gamma_V^{(2)}\right),\\
&S^{(2,2)}=-\frac{1}{2}\left(4d^{(1,0)}\beta_0+\Gamma_{\rm cusp}^{(2)}+\gamma_V^{(1)}\beta_0\right),\quad S^{(2,3)}=-\frac{1}{3}\Gamma_{\rm cusp}^{(1)}\beta_0\,.
\end{aligned}
\tag{18}
$$

The values of the coefficients of the anomalous dimensions and beta function can be read from Appendix D of Ref. [2]. The coefficients of the expansion of $\zeta(\mu)$ are instead reported in Eq. (2.29) of Ref. [1]. For the $\mathcal{O}(a_s)$ coefficients I reported the explicit values. This will help check the result against those in the literature.

Eq. (17) can be easily expanded as up to order a_s^2 as:

$$
\begin{aligned}
[R(Q,\mathcal{L})]^2 &= 1+a_s(Q)\sum_{k=0}^{2}S^{(1,k)}\mathcal{L}^k+a_s^2(Q)\left[\sum_{k=0}^{3}S^{(2,k)}\mathcal{L}^k+\frac{1}{2}\left(\sum_{k=0}^{2}S^{(1,k)}\mathcal{L}^k\right)^2\right]+\mathcal{O}(a_s^3)\\
&= 1+a_s(Q)\sum_{k=0}^{2}S^{(1,k)}\mathcal{L}^k+a_s^2(Q)\sum_{k=0}^{4}\widetilde{S}^{(2,k)}\mathcal{L}^k+\mathcal{O}(a_s^3)\\
&\equiv 1+a_s(Q)R^{(1)}+a_s^2(Q)R^{(2)}+\mathcal{O}(a_s^3)\,,
\end{aligned}
\tag{19}
$$

with:

$$
\begin{aligned}
\widetilde{S}^{(2,0)}&=S^{(2,0)}+\frac{1}{2}\left[S^{(1,0)}\right]^2,\\
\widetilde{S}^{(2,1)}&=S^{(2,1)}+S^{(1,0)}S^{(1,1)}\\
\widetilde{S}^{(2,2)}&=S^{(2,2)}+\frac{1}{2}\left[S^{(1,1)}\right]^2+S^{(1,2)}S^{(1,0)}\,,\\
\widetilde{S}^{(2,3)}&=S^{(2,3)}+S^{(1,1)}S^{(1,2)}\\
\widetilde{S}^{(2,4)}&=\frac{1}{2}\left[S^{(1,2)}\right]^2.
\end{aligned}
\tag{20}
$$

Now that the expansion of the Sudakov form factor is done to $\mathcal{O}(a_s^2)$, it can be combined to the rest of the perturbative quantities entering the computation of the cross section. In the following, we will concentrate on SIDIS that involves both TMD PDFs F and TMD FFs D. Specifically, in b space the SIDIS cross section is a combination of terms having the following structure:

$$
\begin{aligned}
B_{ij} &= H(Q)\, F_i(x,b;Q)\, D_j(z,b;Q) = H(Q)\ [R(Q,\mathcal{L})]^2\ F_i\left(x,b;\frac{C_0}{b}\right)\ D_j\left(z,b;\frac{C_0}{b}\right) \\
&= H(Q)\ [R(Q,\mathcal{L})]^2 \left[\sum_k \mathcal{C}_{ik}(x,\mathcal{L}) \underset{x}{\otimes} f_k\,(x;Q)\right] \left[\sum_l \mathbb{C}_{jl}(z,\mathcal{L}) \underset{z}{\otimes} d_l\,(z;Q)\right] \\
&\equiv \sum_{kl} \hat{B}_{ij,kl}(x,z;b) \underset{x}{\otimes} f_k \underset{z}{\otimes} d_l\,,
\end{aligned}
\tag{21}
$$

where f_k and d_l are the (non-perturbative) collinear PDFs and FFs, respectively. The other terms, including R^2, are perturbatively computable as:

$$
\begin{aligned}
& H(Q) = 1 + a_s(Q)H^{(1)} + a_s^2(Q)H^{(2)} + \mathcal{O}(a_s^3)\,, \\
& \mathcal{C}_{ik}(x,\mathcal{L}) = \delta_{ik}\delta(1-x) + a_s(Q)\mathcal{C}_{ik}^{(1)}(x,\mathcal{L}) + a_s^2(Q)\mathcal{C}_{ik}^{(2)}(x,\mathcal{L}) + \mathcal{O}(a_s^3)\,, \\
& \mathbb{C}_{jl}(z,\mathcal{L}) = \delta_{jl}\delta(1-z) + a_s(Q)\mathbb{C}_{jl}^{(1)}(z,\mathcal{L}) + a_s^2(Q)\mathbb{C}_{jl}^{(2)}(z,\mathcal{L}) + \mathcal{O}(a_s^3)\,.
\end{aligned}
\tag{22}
$$

I now need to put everything together and truncate to order a_s^2 in such a way that:

$$
\hat{B}_{ij,kl}(x,z;b) = \hat{B}_{ij,kl}^{(0)}(x,z;b) + a_s(Q)\hat{B}_{ij,kl}^{(1)}(x,z;b) + a_s^2(Q)\hat{B}_{ij,kl}^{(2)}(x,z;b) + \mathcal{O}(a_s^3)\,,
\tag{23}
$$

with:

$$
\begin{aligned}
\hat{B}_{ij,kl}^{(0)} &= \delta_{ik}\delta_{jl}\delta(1-x)\delta(1-z)\,, \\
\hat{B}_{ij,kl}^{(1)} &= (H^{(1)} + R^{(1)})\delta_{ik}\delta_{jl}\delta(1-x)\delta(1-z) \\
&+ \mathcal{C}_{ik}^{(1)}(x,\mathcal{L})\delta_{jl}\delta(1-z) + \delta_{ik}\delta(1-x)\mathbb{C}_{jl}^{(1)}(z,\mathcal{L}) \\
\hat{B}_{ij,kl}^{(2)} &= (H^{(2)} + H^{(1)}R^{(1)} + R^{(2)})\delta_{ik}\delta_{jl}\delta(1-x)\delta(1-z) \\
&+ (H^{(1)} + R^{(1)})\left[\mathcal{C}_{ik}^{(1)}(x,\mathcal{L})\delta_{jl}\delta(1-z) + \delta_{ik}\delta(1-x)\mathbb{C}_{jl}^{(1)}(z,\mathcal{L})\right] \\
&+ \mathcal{C}_{ik}^{(2)}(x,\mathcal{L})\delta_{jl}\delta(1-z) + \mathcal{C}_{ik}^{(1)}(x,\mathcal{L})\mathbb{C}_{jl}^{(1)}(z,\mathcal{L}) + \delta_{ik}\delta(1-x)\mathbb{C}_{jl}^{(2)}(z,\mathcal{L})
\end{aligned}
\tag{24}
$$

Despite we derived the expansion of the resummed cross section up to $\mathcal{O}(a_s^2)$, these are not yet the final formulas. The reason is that, in order some of the terms of the expansion above do not depend on the impact parameter b. This means that these terms, upon Fourier transform needed to obtain the expression in the transverse moment q_T, will give rise to terms proportional to $\delta(q_T)$. In view of the matching procedure, since the fixed-order expression we are going to match to does not include any $\delta(q_T)$ terms, we need to make sure that they are not subtracted.Therefore, we need to identify and remove any term that does not depend on b. In fact, by construction, this is equivalent to leave only terms proportional to a power of $\mathcal{L}$. In order to do so, we start observing that the hard factor H does not contain any $\mathcal{L}$. In addition, we have to pay attention to the coefficients $R^{(n)}$ because they are polynomials in $\mathcal{L}$ but also include a constant term ($\mathcal{L}^0$). Finally, the ζ-prescription, Eq. (1), provides us with a simple recipe to compute the logarithmic terms of the matching functions $\mathcal{C}_{ik}^{(n)}$ and $\mathbb{C}_{jl}^{(n)}$. Specifically, the condition of independence of the TMD PDFs and FFs from the factorisation scale μ is such that TMDs behave like physical observable (*e.g.* deep-inelastic-scattering or single-inclusive-annihilation structure functions) and thus obey the standard scale variation rules derived, for example, in Eq. (2.17) of Ref. [3]. More in particular, one finds that the matching function coefficients s have the usual logarithmic structure:

$$\mathcal{C}_{ij}^{(n)}(x,\mathcal{L}) = \sum_{k=0}^{n} \mathcal{C}_{ij}^{(n,k)}(x)\mathcal{L}^k \quad \text{and} \quad \mathbb{C}_{ij}^{(n)}(x,\mathcal{L}) = \sum_{k=0}^{n} \mathbb{C}_{ij}^{(n,k)}(x)\mathcal{L}^k\,, \tag{25}$$

where the non-logarithmic terms $\mathcal{C}_{ij}^{(n,0)}$ and $\mathbb{C}_{ij}^{(n,0)}$ have to be computed explicitly while the other terms proportional to a positive power of $\mathcal{L}$ can be expressed in terms of the non-logarithmic term of the previous orders and of the coefficients of the DGLAP splitting functions and of the QCD β function:

$$\begin{aligned}
\mathcal{C}_{ij}^{(1,1)}(x) &= -\mathcal{P}_{ij}^{(1)}(x)\,,\\
\mathcal{C}_{ij}^{(2,1)}(x) &= -\left(\mathcal{P}_{ij}^{(2)}(x) + \mathcal{C}_{ik}^{(1,0)}(x)\otimes\mathcal{P}_{kj}^{(1)}(x) - \beta_0\mathcal{C}_{ij}^{(1,0)}(x)\right)\,,\\
\mathcal{C}_{ij}^{(2,2)}(x) &= \frac{1}{2}\left(\mathcal{P}_{ik}^{(1)}(x)\otimes\mathcal{P}_{kj}^{(1)}(x) - \beta_0\mathcal{P}_{ij}^{(1)}(x)\right)\,,
\end{aligned} \tag{26}$$

and:

$$\begin{aligned}
\mathbb{C}_{ij}^{(1,1)}(x) &= -\mathbb{P}_{ij}^{(1)}(x)\,,\\
\mathbb{C}_{ij}^{(2,1)}(x) &= -\left(\mathbb{P}_{ij}^{(2)}(x) + \mathbb{C}_{ik}^{(1,0)}(x)\otimes\mathbb{P}_{kj}^{(1)}(x) - \beta_0\mathbb{C}_{ij}^{(1,0)}(x)\right)\,,\\
\mathbb{C}_{ij}^{(2,2)}(x) &= \frac{1}{2}\left(\mathbb{P}_{ik}^{(1)}(x)\otimes\mathbb{P}_{kj}^{(1)}(x) - \beta_0\mathbb{P}_{ij}^{(1)}(x)\right)\,,
\end{aligned} \tag{27}$$

where $\mathcal{P}_{ij}^{(n)}$ and $\mathbb{P}_{ij}^{(n)}$ are the coefficients of the a_s^n terms of the space- and time-like splitting functions, respectively. With this information at hand, knowing the logarithmic expansion of the coefficients $R^{(n)}$, and keeping in mind that the hard coefficients $H^{(n)}$ do not contain any logarithms, we can organize the coefficients inEq. (24) in terms of powers of $\mathcal{L}$. Specifically, we find that:

$$\hat{B}_{ij,kl}^{(n)} = \sum_{p=0}^{2n} \hat{B}_{ij,kl}^{(n,p)}\mathcal{L}^p\,, \tag{28}$$

with the $\mathcal{O}(1)$ coefficient being:

$$\hat{B}_{ij,kl}^{(0,0)} = \delta_{ik}\delta_{kl}\delta(1-x)\delta(1-z)\,, \tag{29}$$

the $\mathcal{O}(a_s)$ coefficients being:

$$\begin{aligned}
\hat{B}_{ij,kl}^{(1,0)} &= (H^{(1)} + S^{(1,0)})\delta_{ik}\delta_{jl}\delta(1-x)\delta(1-z) + \mathcal{C}_{ik}^{(1,0)}(x)\delta_{jl}\delta(1-z) + \delta_{ik}\delta(1-x)\mathbb{C}_{jl}^{(1,0)}(z)\,,\\
\hat{B}_{ij,kl}^{(1,1)} &= S^{(1,1)}\delta_{ik}\delta_{jl}\delta(1-x)\delta(1-z) + \mathcal{C}_{ik}^{(1,1)}(x)\delta_{jl}\delta(1-z) + \delta_{ik}\delta(1-x)\mathbb{C}_{jl}^{(1,1)}(z)\,,\\
\hat{B}_{ij,kl}^{(1,2)} &= S^{(1,2)}\delta_{ik}\delta_{jl}\delta(1-x)\delta(1-z)
\end{aligned} \tag{30}$$

and the $\mathcal{O}(a_s^2)$ coefficients being:

$$
\begin{aligned}
\hat{B}^{(2,0)}_{ij,kl} &= (H^{(2)} + H^{(1)} S^{(1,0)} + \widetilde{S}^{(2,0)})\delta_{ik}\delta_{jl}\delta(1-x)\delta(1-z)\,, \\
&+ (H^{(1)} + S^{(1,0)})\left[\mathcal{C}^{(1,0)}_{ik}(x)\delta_{jl}\delta(1-z) + \delta_{ik}\delta(1-x)\mathbb{C}^{(1,0)}_{jl}(z)\right] \\
&+ \mathcal{C}^{(2,0)}_{ik}(x)\delta_{jl}\delta(1-z) + \mathcal{C}^{(1,0)}_{ik}(x)\mathbb{C}^{(1,0)}_{jl}(z) + \delta_{ik}\delta(1-x)\mathbb{C}^{(2,0)}_{jl}(z) \\
\hat{B}^{(2,1)}_{ij,kl} &= (H^{(1)} S^{(1,1)} + \widetilde{S}^{(2,1)})\delta_{ik}\delta_{jl}\delta(1-x)\delta(1-z) \\
&+ (H^{(1)} + S^{(1,0)})\left[\mathcal{C}^{(1,1)}_{ik}(x)\delta_{jl}\delta(1-z) + \delta_{ik}\delta(1-x)\mathbb{C}^{(1,1)}_{jl}(z)\right] \\
&+ S^{(1,1)}\left[\mathcal{C}^{(1,0)}_{ik}(x)\delta_{jl}\delta(1-z) + \delta_{ik}\delta(1-x)\mathbb{C}^{(1,0)}_{jl}(z)\right] \\
&+ \mathcal{C}^{(2,1)}_{ik}(x)\delta_{jl}\delta(1-z) + \mathcal{C}^{(1,1)}_{ik}(x)\mathbb{C}^{(1,0)}_{jl}(z) + \mathcal{C}^{(1,0)}_{ik}(x)\mathbb{C}^{(1,1)}_{jl}(z) + \delta_{ik}\delta(1-x)\mathbb{C}^{(2,1)}_{jl}(z) \\
\hat{B}^{(2,2)}_{ij,kl} &= (H^{(1)} S^{(1,2)} + \widetilde{S}^{(2,2)})\delta_{ik}\delta_{jl}\delta(1-x)\delta(1-z) \\
&+ \widetilde{S}^{(1,2)}\left[\mathcal{C}^{(1,0)}_{ik}(x)\delta_{jl}\delta(1-z) + \delta_{ik}\delta(1-x)\mathbb{C}^{(1,0)}_{jl}(z)\right] \\
&+ S^{(1,1)}\left[\mathcal{C}^{(1,1)}_{ik}(x)\delta_{jl}\delta(1-z) + \delta_{ik}\delta(1-x)\mathbb{C}^{(1,1)}_{jl}(z)\right] \\
&+ \mathcal{C}^{(2,2)}_{ik}(x)\delta_{jl}\delta(1-z) + \mathcal{C}^{(1,1)}_{ik}(x)\mathbb{C}^{(1,1)}_{jl}(z) + \delta_{ik}\delta(1-x)\mathbb{C}^{(2,2)}_{jl}(z) \\
\hat{B}^{(2,3)}_{ij,kl} &= \widetilde{S}^{(2,3)}\delta_{ik}\delta_{jl}\delta(1-x)\delta(1-z) + S^{(1,2)}\left[\mathcal{C}^{(1,1)}_{ik}(x)\delta_{jl}\delta(1-z) + \delta_{ik}\delta(1-x)\mathbb{C}^{(1,1)}_{jl}(z)\right] \\
\hat{B}^{(2,4)}_{ij,kl} &= \widetilde{S}^{(2,4)}\delta_{ik}\delta_{jl}\delta(1-x)\delta(1-z)
\end{aligned}
\tag{31}
$$

In order to obtain a differential cross section in q_T, we need to take the Fourier transform of $\hat{B}_{ij,kl}$, that is:

$$
\hat{B}_{ij,kl}(x,z;q_T) \equiv \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q}_T} \hat{B}_{ij,kl}(x,z;b) = \sum_{n=0}^{2} a_s^n(Q) \sum_{p=0}^{2n} \hat{B}^{(n,p)}_{ij,kl}(x,z) I_p(q_T)\,, \tag{32}
$$

where I have defined:

$$
I_p(q_T) = \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q}_T} \mathcal{L}^p = \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q}_T} \ln^p\left(\frac{b^2 Q^2}{C_0^2}\right) = \frac{1}{2}\int_0^\infty db\, b J_0(bq_T) \ln^p\left(\frac{b^2Q^2}{C_0^2}\right). \tag{33}
$$

Results for I_p have been computed up to $p = 4$ in Eq. (136) of Appendix B of Ref. [4]. Specifically, and including the trivial tranform with $p = 0$, they read:

$$
\begin{aligned}
I_0(q_T) &= \delta(q_T)\,,\\
I_1(q_T) &= -\frac{1}{q_T^2}\,,\\
I_2(q_T) &= -\frac{2}{q_T^2}\ln\left(\frac{Q^2}{q_T^2}\right)\,,\\
I_3(q_T) &= -\frac{3}{q_T^2}\ln^2\left(\frac{Q^2}{q_T^2}\right)\,,\\
I_4(q_T) &= -\frac{4}{q_T^2}\left[\ln^3\left(\frac{Q^2}{q_T^2}\right) - 4\zeta_3\right]\,.
\end{aligned}
\tag{34}
$$

As clear from the transforms above, all terms with $p = 0$ will be proportional to $\delta(q_T)$. We do not need to consider these terms because analogous terms are not included in the fixed-order calculation and thus does not need to be subtracted. Therefore, we write:

$$
\hat{B}_{ij,kl}(x,z;q_T) = \sum_{n=1}^{2} a_s^n(Q) \sum_{p=1}^{2n} \hat{B}_{ij,kl}^{(n,p)}(x,z) I_p(q_T) + \left(\sum_{n=0}^{2} a_s^n(Q)\hat{B}_{ij,kl}^{(n,0)}(x,z)\right)\delta(q_T) + \mathcal{O}(a_s^3)\,, \tag{35}
$$

and we are not going to consider the term proportional to $\delta(q_T)$, even though all terms have been derived above.

As clear from Eq. (35), removing all terms proportional to $\delta(q_T)$ also means removing the full $\mathcal{O}(1)$ terms such that leading-order term is now $\mathcal{O}(a_s)$.

In order to validate the results above, it is opportune to compare the $\mathcal{O}(a_s)$ expressions to those present in the literature. To this end, we write explicitly the expression for $\hat{B}_{ij,kl}^{(1)}$ in q_T space without the $\delta(q_T)$ term:

$$
\begin{aligned}
\hat{B}_{ij,kl}^{(1)}(x,z;q_T) &= -\hat{B}_{ij,kl}^{(1,1)}(x,z)\mathcal{L}\frac{1}{q_T^2} - \hat{B}_{ij,kl}^{(1,2)}(x,z)\frac{2}{q_T^2}\ln\left(\frac{Q^2}{q_T^2}\right)\\
&= \frac{1}{q_T^2}\left[4C_F\left(\ln\left(\frac{Q^2}{q_T^2}\right) - \frac{3}{2}\right)\delta_{ik}\delta_{jl}\delta(1-x)\delta(1-z)\right.\\
&+ \left.\mathcal{P}_{ik}^{(1)}(x)\delta_{jl}\delta(1-z) + \delta_{ik}\delta(1-x)\mathbb{P}_{jl}^{(1)}(z)\right]\,,
\end{aligned}
\tag{36}
$$

so that:

$$
\begin{aligned}
B_{ij}(x,z;q_T) &= a_s(Q)\sum_{kl}\hat{B}_{ij,kl}^{(1)} \underset{x}{\otimes} f_k(x,Q) \underset{z}{\otimes} d_l(z,Q) + \mathcal{O}(a_s^2)\\
&= a_s(Q)\frac{1}{q_T^2}\left[4C_F\left(\ln\left(\frac{Q^2}{q_T^2}\right) - \frac{3}{2}\right) f_i(x,Q)d_j(z,Q)\right.\\
&+ \left.\left(\sum_k \mathcal{P}_{ik}^{(1)}(x) \underset{x}{\otimes} f_k(x,Q)\right) d_j(z,Q) + f_i(x,Q)\left(\sum_l \mathbb{P}_{jl}^{(1)}(z) \underset{z}{\otimes} d_l(z,Q)\right)\right] + \mathcal{O}(a_s^2)\,.
\end{aligned}
\tag{37}
$$

This result, up to som pre-factor and to selecting the right flavour combinations,[3] nicely agrees with that of, *e.g.*, Refs. [5, 7, 6].

[3] We have purposely left the flavour indices i and j unspecified because they do not necessarily run over the quark flavours u, d, etc. In fact, it turns out to be useful to adopt a different basis (usual referred to as evolution basis) that simplifies the structure of the products between, for example, PDFs/FFs and splitting functions.

In order to check that the matching is actually removing the double counting terms, it is instructive to derive Eq. (37) extracting the asymptote from the fixed-order computation at $\mathcal{O}(a_s)$. We take the expressions for the coefficient functions from Eqs. (106)-(109) of Appendix B of Ref. [7] or from Eqs. (4.6)-(4.20) of Ref. [8].Referring to the second reference, some simplifications apply. First, we consider cross sections with unpolarised projectiles ($\lambda_e = 0$) over unpolarised targets ($S_\perp^\mu = 0$) and integrated over the azimuthal angles ϕ_H and ϕ_S. By doing so and after a simple manipulation, the cross section simplifies greatly and can be written in terms of structure functions as:

$$\frac{d\sigma}{dxdydzdq_T^2} = \frac{2\pi\alpha^2}{xyQ^2}\left[Y_+ F_{UU,T} + 2(1-y)F_{UU,L}\right] = \frac{2\pi\alpha^2}{xyQ^2} Y_+ \left[F_{UU,2} - \frac{y^2}{Y_+} F_{UU,L}\right], \tag{38}$$

with:

$$Y_+ \equiv 1 + (1-y)^2\,, \tag{39}$$

and where we have defined the structure function:

$$F_{UU,2} \equiv F_{UU,T} + F_{UU,L}\,. \tag{40}$$

Notice that, as compared to Ref. [8], we have factored out from the structure functions a further factor $1/(\pi z^2)^4$ so that they factorize as:

$$\begin{aligned} F_{UU,S} \quad = \quad & a_s \frac{x}{Q^2} \sum_i e_i^2 \int_x^1 \frac{d\bar{x}}{\bar{x}} \int_z^1 \frac{d\bar{z}}{\bar{z}} \delta\left(\frac{q_T^2}{Q^2} - \frac{(1-\bar{x})(1-\bar{z})}{\bar{x}\bar{z}}\right) \left[\hat{B}_{qq}^{S,\mathrm{FO}}(\bar{x},\bar{z},q_T) f_i\left(\frac{x}{\bar{x}}\right) d_i\left(\frac{z}{\bar{z}}\right)\right. \\ & + \left. \hat{B}_{qg}^{S,\mathrm{FO}}(\bar{x},\bar{z},q_T) f_g\left(\frac{x}{\bar{x}}\right) d_i\left(\frac{z}{\bar{z}}\right) + \hat{B}_{gq}^{S,\mathrm{FO}}(\bar{x},\bar{z},q_T) f_i\left(\frac{x}{\bar{x}}\right) d_g\left(\frac{z}{\bar{z}}\right)\right] + \mathcal{O}(a_s^2)\,. \end{aligned} \tag{41}$$

with $S = 2, L$ and where the sum over i runs over the active quark and antiquark flavours and e_i is the electric charge of the i-th flavour. The expression for the coefficient functions are:

$$\begin{aligned} \hat{B}_{qq}^{2,\mathrm{FO}}(x,z,q_T) &= 2C_F\left[(1-x)(1-z) + 4xz + \frac{1+x^2z^2}{xz}\frac{Q^2}{q_T^2}\right], \\ \hat{B}_{qq}^{L,\mathrm{FO}}(x,z,q_T) &= 8C_F xz\,, \\ \hat{B}_{qg}^{2,\mathrm{FO}}(x,z,q_T) &= 2C_F\left[(1-x)z + 4x(1-z) + \frac{1+x^2(1-z)^2}{xz}\frac{1-z}{z}\frac{Q^2}{q_T^2}\right], \\ \hat{B}_{qg}^{L,\mathrm{FO}}(x,z,q_T) &= 8C_F x(1-z)\,, \\ \hat{B}_{gq}^{2,\mathrm{FO}}(x,z,q_T) &= 2T_R\left[[x^2+(1-x)^2][z^2+(1-z)^2]\frac{1-x}{xz^2}\frac{Q^2}{q_T^2} + 8x(1-x)\right], \\ \hat{B}_{gq}^{L,\mathrm{FO}}(x,z,q_T) &= 16T_R x(1-x)\,. \end{aligned} \tag{42}$$

These expressions are enough to compute the SIDIS cross section at $\mathcal{O}(a_s)$ in the region $q_T \lesssim Q$. In order to match Eq. (37) one has to take the limit $q_T \ll Q$ and retain in the coefficient functions only the terms enhanced as Q^2/q_T^2. This automatically means that $F_{UU,L}$ does not contribute in the $q_T \leftarrow 0$ limit:

$$F_{UU,L} \underset{q_T \ll Q}{\longrightarrow} 0\,. \tag{43}$$

[4]Factor z^2 is the consequence of the fact that we are writing the cross section differential in q_T^2 that is the transverse momentum of the exchanged photon while in Ref. [8] the cross section is differentila in p_T^2 that is the transverse momentum of the of the outgoing hadrons. Since $p_T = zq_T$, the factor z^2 cancels.

Another crucial observation is that the δ-function in Eq. (41) can be expanded as follows:

$$\delta\left(\frac{q_T^2}{Q^2}-\frac{(1-x)(1-z)}{xz}\right)=\ln\left(\frac{Q^2}{q_T^2}\right)\delta(1-x)\delta(1-z)+\frac{x\delta(1-z)}{(1-x)_+}+\frac{z\delta(1-x)}{(1-z)_+}+\mathcal{O}\left(\frac{Q^2}{q_T^2}\ln\left(\frac{Q^2}{q_T^2}\right)\right), \tag{44}$$

so that:

$$\begin{aligned}F_{UU,2} \quad \underset{q_T\ll Q}{\longrightarrow} \quad & a_s\frac{x}{q_T^2}\sum_i e_i^2\int_x^1\frac{d\bar{x}}{\bar{x}}\int_z^1\frac{d\bar{z}}{\bar{z}}\Bigg[\hat{B}_{qq}^{2,\text{asy}}(\bar{x},\bar{z},q_T)f_i\left(\frac{x}{\bar{x}}\right)d_i\left(\frac{z}{\bar{z}}\right)\\ + \quad & \hat{B}_{qg}^{2,\text{asy}}(\bar{x},\bar{z},q_T)f_g\left(\frac{x}{\bar{x}}\right)d_i\left(\frac{z}{\bar{z}}\right)+\hat{B}_{gq}^{2,\text{asy}}(\bar{x},\bar{z},q_T)f_i\left(\frac{x}{\bar{x}}\right)d_g\left(\frac{z}{\bar{z}}\right)\Bigg]+\mathcal{O}(a_s^2)\,.\end{aligned} \tag{45}$$

with:

$$\begin{aligned}\hat{B}_{qq}^{2,\text{asy}}(x,z,q_T) &= 2C_F\left[2\ln\left(\frac{Q^2}{q_T^2}\right)+\frac{1+x^2}{(1-x)_+}\delta(1-z)+\delta(1-x)\frac{1+z^2}{(1-z)_+}\right]\\ &= 2C_F\left[2\ln\left(\frac{Q^2}{q_T^2}\right)-3\right]\delta(1-x)\delta(1-z)+\mathcal{P}_{qq}^{(1)}(x)\delta(1-z)+\delta(1-x)\mathbb{P}_{qq}^{(1)}(z)\,,\\ \hat{B}_{qg}^{2,\text{asy}}(x,z,q_T) &= 2C_F\left[\frac{1+(1-z)^2}{z}\right]=\delta(1-x)\mathbb{P}_{gq}^{(1)}(z)\,,\\ \hat{B}_{gq}^{2,\text{asy}}(x,z,q_T) &= 2T_R\left[x^2+(1-x)^2\right]=\mathcal{P}_{gq}^{(1)}(x)\delta(1-z)\,.\end{aligned} \tag{46}$$

It is thus easy to see that we can rewrite Eq. (45) as:

$$\begin{aligned}F_{UU,2} \quad \underset{q_T\ll Q}{\longrightarrow} \quad & a_s\frac{x}{z^2q_T^2}\sum_i e_i^2\Bigg[4C_F\left(\ln\left(\frac{Q^2}{q_T^2}\right)-\frac{3}{2}\right)f_i\left(x\right)d_i\left(z\right)\\ + \quad & \left(\sum_{k=q,g}\mathcal{P}_{qk}^{(1)}(x)\otimes f_k\left(x\right)\right)d_i\left(z\right)+f_i\left(x\right)\left(\sum_{k=q,g}\mathbb{P}_{qk}^{(1)}(z)\otimes d_k\left(z\right)\right)\Bigg]+\mathcal{O}(a_s^2)\,.\end{aligned} \tag{47}$$

Eq. (47) agrees with Eq. (37). This confirms that this term removes the double-counting terms when doing the matching.

In order to provide a version of Eq. (41) that can be readily implemented, we need to perform one of the integrals making use of the δ-function. We integrate over $\bar{x}$ so that we write:

$$\delta\left(\frac{q_T^2}{Q^2}-\frac{(1-\bar{x})(1-\bar{z})}{\bar{x}\bar{z}}\right)=\frac{\bar{z}\bar{x}_0^2}{1-\bar{z}}\delta(\bar{x}-\bar{x}_0)\,, \tag{48}$$

with:

$$\bar{x}_0=\frac{1-\bar{z}}{1-\bar{z}\left(1-\frac{q_T^2}{Q^2}\right)}\,. \tag{49}$$

This allows us to write:

$$\begin{aligned}F_{UU,S} \quad = \quad & a_s\frac{x}{Q^2}\sum_i e_i^2\int_z^{z_{\max}}\frac{d\bar{z}}{1-\bar{z}}\bar{x}_0\Bigg[\hat{B}_{qq}^{S,\text{FO}}(\bar{x}_0,\bar{z},q_T)f_i\left(\frac{x}{\bar{x}_0}\right)d_i\left(\frac{z}{\bar{z}}\right)\\ + \quad & \hat{B}_{qg}^{S,\text{FO}}(\bar{x}_0,\bar{z},q_T)f_g\left(\frac{x}{\bar{x}_0}\right)d_i\left(\frac{z}{\bar{z}}\right)+\hat{B}_{gq}^{S,\text{FO}}(\bar{x}_0,\bar{z},q_T)f_i\left(\frac{x}{\bar{x}_0}\right)d_g\left(\frac{z}{\bar{z}}\right)\Bigg]+\mathcal{O}(a_s^2)\,,\end{aligned} \tag{50}$$

with:

$$z_{\max}=\frac{1-x}{1-x\left(1-\frac{q_T^2}{Q^2}\right)}\,. \tag{51}$$

Now I would like to rewrite the cross section above in such a way that it matches that at $\mathcal{O}(a_s)$ of Ref. [9]. That would allow us to confidently use the $\mathcal{O}(a_s^2)$ calculation presented right in that reference for the matching to the resummed calculation. This is made a little tricky by the different notation used in Ref. [9] and from the fact that in that paper the cross section is differential in a different set of variables. Specifically, we would like it to be differential in x, y, z, and q_T^2 while in Ref. [9] it is differential in x, Q^2, η, and p_T^2. Eq. (13) of Ref. [9], can be translated into our notation by noticing that:

$$\frac{d\sigma}{dxdQ^2dp_T^2d\eta} = \frac{x}{zQ^2}\sum_{i,j}\int_z^{z_{\max}}\frac{d\bar{z}}{1-\bar{z}}f_i\left(\frac{x}{\bar{x}_0}\right)d_j\left(\frac{z}{\bar{z}}\right)\frac{d\sigma_{ij}^{(1)}}{dxdQ^2dp_T^2d\eta} + \mathcal{O}(a_s^2) \tag{52}$$

where we have exploited the δ-functions in Eqs. (18)-(20) to get rid of the integral over z.[5]

The $\mathcal{O}(a_s)$ partonic cross sections in Eqs. (18)-(20) of Ref. [9], setting $\varepsilon = 0$, can be written as:

$$\frac{d\sigma_{ij}^{(1)}}{dxdQ^2dp_T^2d\eta} = \frac{2\pi\alpha^2 a_s e_q^2\bar{x}_0}{xQ^4}Y_+\left[\underbrace{\left(F_{UU,M}^{ij}(\bar{x}_0,\bar{z}) + \frac{3}{2}F_{UU,L}^{ij}(\bar{x}_0,\bar{z})\right)}_{F_{UU,2}^{ij}} - \frac{y^2}{Y_+}F_{UU,L}^{ij}(\bar{x}_0,\bar{z})\right]. \tag{53}$$

One can verify that $F_{UU,2}^{qq}(\bar{x}_0,\bar{z})$, $F_{UU,L}^{qq}(\bar{x}_0,\bar{z})$, $F_{UU,M}^{qg}(\bar{x}_0,\bar{z})$, $F_{UU,L}^{qg}(\bar{x}_0,\bar{z})$, $F_{UU,M}^{gq}(\bar{x}_0,\bar{z})$, and $F_{UU,L}^{gq}(\bar{x}_0,\bar{z})$ exactly correspond to $\hat{B}_{qq}^{2,\mathrm{FO}}(\bar{x}_0,\bar{z},q_T)$, $\hat{B}_{qq}^{L,\mathrm{FO}}(\bar{x}_0,\bar{z},q_T)$, $\hat{B}_{qg}^{M,\mathrm{FO}}(\bar{x}_0,\bar{z},q_T)$, $\hat{B}_{qg}^{L,\mathrm{FO}}(\bar{x}_0,\bar{z},q_T)$, $\hat{B}_{gq}^{M,\mathrm{FO}}(\bar{x}_0,\bar{z},q_T)$, and $\hat{B}_{gq}^{L,\mathrm{FO}}(\bar{x}_0,\bar{z},q_T)$ of Eq. (42). It is crucial to notice that the correspondence holds only if $\bar{x}_0$ defined in Eq. (49) as a function of $\bar{z}$ is used. Putting all pieces together, Eq. (52) can be recast as:

$$\frac{d\sigma}{dxdQ^2dp_T^2d\eta} = \frac{2\pi\alpha^2}{zxQ^4}Y_+\left[F_{UU,2} - \frac{y^2}{Y_+}F_{UU,L}\right], \tag{54}$$

with:

$$\begin{aligned} F_{UU,S} &= a_s\frac{x}{Q^2}\sum_i e_i^2\int_z^{z_{\max}}\frac{d\bar{z}}{1-\bar{z}}\bar{x}_0\left[F_{UU,L}^{qq}(\bar{x}_0,\bar{z})f_i\left(\frac{x}{\bar{x}_0}\right)d_i\left(\frac{z}{\bar{z}}\right)\right.\\ &+ \left.F_{UU,L}^{qg}(\bar{x}_0,\bar{z})f_g\left(\frac{x}{\bar{x}_0}\right)d_i\left(\frac{z}{\bar{z}}\right) + F_{UU,L}^{gq}(\bar{x}_0,\bar{z})f_i\left(\frac{x}{\bar{x}_0}\right)d_g\left(\frac{z}{\bar{z}}\right)\right] + \mathcal{O}(a_s^2). \end{aligned} \tag{55}$$

Therefore the structure of the observables is exactly the same. What is left to work out is the Jacobian to express the cross section as differential in the same variable as in Eq. (50). What we need is to know how the variables Q^2, p_T, and η are related to y, z, and q_T. The relevant relations are:

$$\left.\begin{aligned} Q^2 &= xyS \\ p_T^2 &= z^2q_T^2 \\ \eta &= \frac{1}{2}\ln\left(\frac{S}{q_T^2}\right) \end{aligned}\right.\quad\Longrightarrow\quad dQ^2dp_T^2d\eta = \frac{zQ^2}{y}dydzdq_T^2. \tag{56}$$

so that:

$$\frac{d\sigma}{dxdydzdq_T^2} = \frac{Q^2z}{y}\frac{d\sigma}{dxdQ^2dp_T^2d\eta} = \frac{2\pi\alpha^2}{xyQ^2}Y_+\left[F_{UU,2} - \frac{y^2}{Y_+}F_{UU,L}\right], \tag{57}$$

exactly like in Eq. (38).

[5]Notice that the z variable of Ref. [9] does not coincide with our definition.

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