

# arTeMiDe ver.1

Alexey A. Vladimirov

June 7, 2017

User manual for **arTeMiDe** package, which evaluated TMDs and related cross-sections.

This work is licensed under the Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International License. To view a copy of this license, visit <http://creativecommons.org/licenses/by-nc-sa/4.0/> or send a letter to Creative Commons, PO Box 1866, Mountain View, CA 94042, USA.

If you use the **arTeMiDe**, please, quote [1].

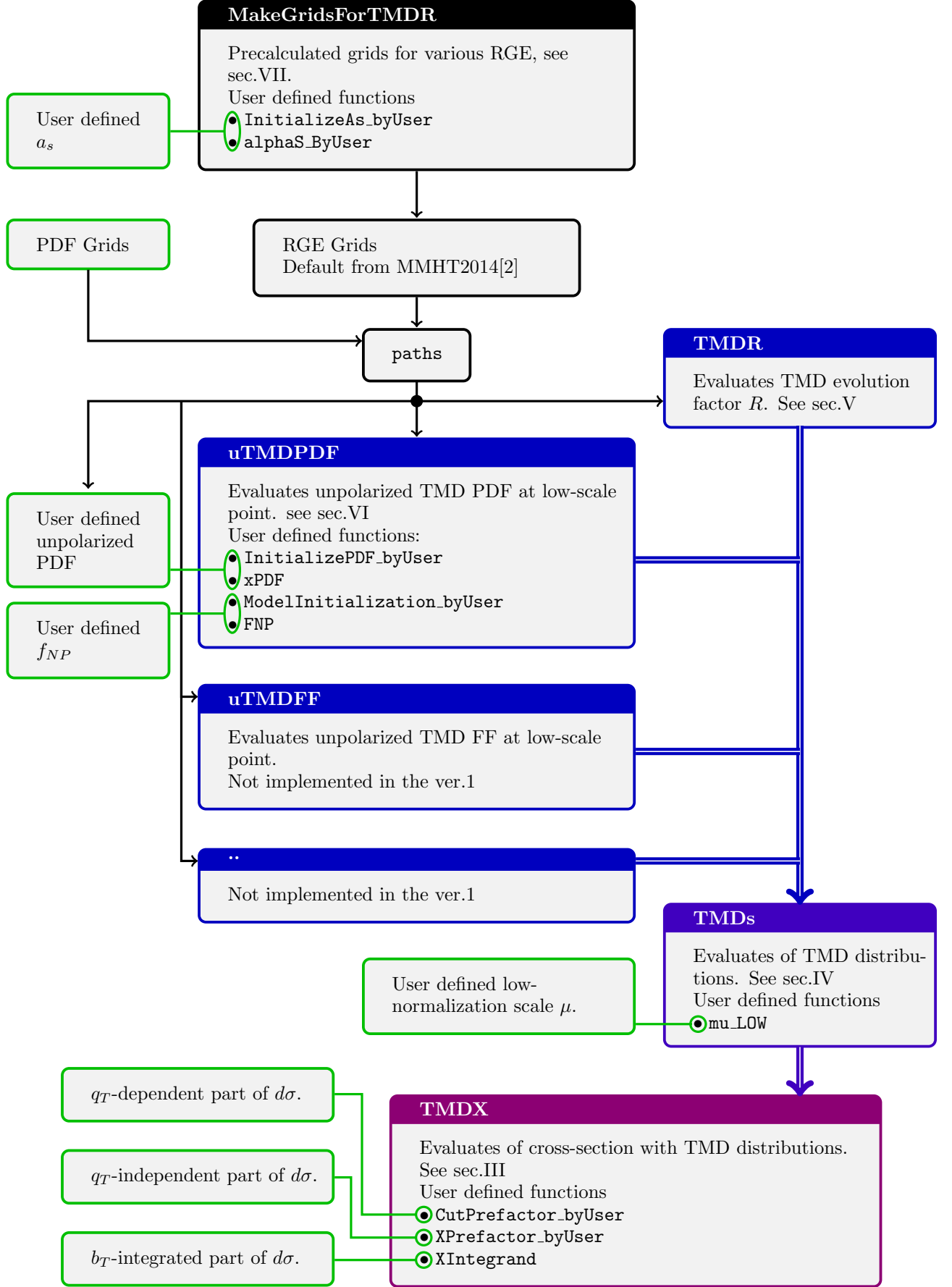
If you find mistakes, have suggestions, or have questions write to:

Alexey Vladimirov: *Alexey.Vladimirov@physik.uni-regensburg.de* or *vladimirov.aleksey@gmail.com*

## I. VERSION HISTORY

**Ver.1** Release: uTMDPDF, TMDR, TMDs and TMDX modules. Only Drell-Yan-like cross-sections.

## II. GENERAL STRUCTURE AND USER INPUT OF ARTEMIDE



### A. Functions to check and update

1. Build the grids for coupling constants, if necessary. See section VII. The default ones are evaluated from MMHT2014 [2].
2. Prepare the PDF initialization and reading PDF routine in the end of `uTMDPDF.f90` code. The default one uses MMHT2014 main set[2].
3. Update the file `paths` with paths to new grids, with accordance to the used order.
4. Set the non-perturbative function  $f_{NP}$  and its initialization in the end of the file `uTMDPDF.f90`.
5. Set the value of low-normalization scale  $\mu_i(b)$  in the end of the file `TMDs.f90`.

### III. TMDX MODULE

**Warning:** In the current version **arTeMiDe** evaluates only the unpolarized Drell-Yan-like processes. Therefore, the user interface is somewhat restricted to it. For SIDIS and other processes, as well as, on the polarized processes, it will be updated with inclusion of **uTMDFF** module.

The module **TMDX** joins the lower modules and performs the evaluation of the cross-section. In the current version (version 1) it is restricted to the following form

$$d\sigma(q_T) = \text{prefactor} \times \text{cuts} \times \int_0^\infty \frac{bdb}{2} J_0(bq_T) X(b), \quad (3.1)$$

where *prefactor* any  $q_T$  independent factor, *cuts* is any  $q_T$  dependent factor (typically it includes cuts),  $X(b)$  is the expression to integrate. Example, for the Drell-Yan process one has for  $d\sigma/dq_T$

$$\begin{aligned} \text{prefactor} &= \frac{4\pi}{9sQ^2} |C_V(Q, \mu_H)|^2 \\ \text{cuts} &= 1 + \frac{q_T^2}{2Q^2} \\ X(b) &= \sum_f |e_f|^2 F_f(x_A, b; \mu_H, \zeta_A) F_{\bar{f}}(x_B, b; \mu_H, \zeta_B). \end{aligned}$$

The expressions for these factors can be setup by user within the functions **Xprefactor\_byUser**, **cutPrefactor\_byUser**, **XIntegrand**, which are located in the end of the code **TMDX.f90**. Several examples of these factors can be found in the code.

In the definition of these function one can use the following variables and functions (list to be updated)

- s\_global** the variable  $s$  set by **TMDX.Xsetup**
- Q\_global** the variable  $Q$  set by **TMDX.Xsetup**
- y\_global** the variable  $y$  set by **TMDX.Xsetup**
- HardCoefficientDY()** the function which evaluate  $|C_V|^2$  for the Drell-Yan process.
- xA\_global**  $= Qe^y/\sqrt{s}$
- xB\_global**  $= Qe^{-y}/\sqrt{s}$
- **muHard\_global**  $= Q$
- **zetaA\_global**  $= Q^2$
- **zetaB\_global**  $= Q^2$

The variables marked by • are the part of the hard-factorization scale definition. They should be used as hard-factorization scales of TMDs. This variables also used to define the theoretical errors.

#### A. Initialization

Prior the usage module is to be initialized (once per run) by  
call **TMDX.Initialize(orderTMD, orderH)**  
here:

**orderTMD** declaration of order a unpolarized TMDPDF. It can be 'NLL', 'NLO', 'NNLL' or 'NNLO'. This is a complex declaration, which implies particular orders for coefficient functions, PDFs, anomalous dimension, etc. For detailed definition see [1].

**orderH** declaration of the order of hard-coefficient to be used.

### B. Setting up the cross-section

Prior to evaluation of cross-section set up the kinematics and the definition of the cross-section, by

`call TMDX.XSetup(s,Q,y,process)`

where `s` is the Mandelstam variable  $s$ ,  $Q$  is hard virtuality  $Q$ ,  $y$  is the rapidity parameter  $y$  and `process` is the number of process to be used (it is used in the functions `Xprefactor_byUser`, and `XIntegrand` only).

All non-perturbative parameters are defined in the TMDs and lower-level modules. For user convenience there is a subroutine, which passes the values of parameters to TMDs. It is

`call TMDX.SetNPPParameters(bmax,gK,lambda)`

where `bmax` and `gK` are `real*8`, `lambda` is `real*8(1:number of parameters)`. For details see IV B.

### C. Cross-section evaluation

After the parameters of cross-section are set up, the values of the cross-section at different  $q_T$  can be obtained by

`call CalculateXsection(X,qt)`

where `X` is `real*8(1,N)` variable where cross-section will be stored, `qt` is `real*8(1,N)` is the list of values of  $q_T$ 's at which the `X` is to be calculated.

This command has following extensions

`CalculateXsection_Yint(X,qt,yMin,yMax) =  $\int_{y_{\min}}^{y_{\max}} dy d\sigma(q_T)$`  here `yMin,yMax` are `real*8`.

`CalculateXsection_YintComplete(X,qt) =  $\int_{-y_0}^{y_0} dy d\sigma(q_T)$`  where  $y_0$  is maximum allowed  $y$ , i.e.  $y_0 = \ln(s/Q^2)/2$ .

`CalculateXsection_Qint_Yint(X,qt,QMin,QMax,yMin,yMax) =  $\int_{Q_{\min}}^{Q_{\max}} 2Q dQ \int_{y_{\min}}^{y_{\max}} dy d\sigma(q_T)$`  here `QMin,QMax` are `real*8`.

`CalculateXsection_PTint_Yint(X,qtMin,qtMax,yMin,yMax) =  $\int_{q_{T\min}}^{q_{T\max}} 2q_T dq_T \int_{y_{\min}}^{y_{\max}} dy d\sigma(q_T)$`  here `qtMin,qtMax` are `real*8(1:N)`

`CalculateXsection_PTint_YintComplete(X,qtMin,qtMax) =  $\int_{q_{T\min}}^{q_{T\max}} 2q_T dq_T \int_{-y_0}^{y_0} dy d\sigma(q_T)$`

`CalculateXsection_PTint_Qint_Yint(X,qtMin,qtMax,QMin,QMax,yMin,yMax) =  $\int_{q_{T\min}}^{q_{T\max}} 2q_T dq_T \int_{Q_{\min}}^{Q_{\max}} 2Q dQ \int_{-y_0}^{y_0} dy d\sigma(q_T)$`

Take care that every next function is heavier to evaluate then the previous one.

**Important:** The evaluation of integrand over cross-sections is made by Simpson method with fixed numbers of sections. This is done for speeding up the calculation. It is expected that the function (i.e. cross-section) does not dramatically changed within the range of integration. These procedures will be updated in the future versions. Currently, do not use the too wide range of integrations!

**Note:** If the quadrature does not converge after 64 Ogata nodes, which is a lot, it implies some blowing up integrand. It can happen if e.g. TMD distribution rise at  $b \rightarrow \infty$  (if e.g.  $g_K < 0$ , which should not happen). In this case, some generic very large number will be returned. It will be returned for any further evaluations of cross-sections, until `TMDX.SetNPPParameters` called.

### D. Theoretical uncertainties

There are three perturbative scales in the definition of the cross-section and TMD distributions,  $c_{1,2,3}$ . For precise definition check [1]. The theoretical uncertainty resulted from the truncation of the perturbative series is estimated by variation of  $c_{1,2,3} \in [0.5, 2]$ .

The subroutines `CalculateXsection` have overloaded versions with two extra variables following the cross-section variable `X`

`CalculateXsection...(X,Xmin,Xmax,qt,...)` where `Xmin(Xmax)` is `real*8(1,N)` which filled by the lowest (highest) value of cross-section obtained from the variation of constants  $c_{1,2,3}$ .

Note, that during evaluation the overloaded function calculates cross-section 7 times, and therefore, 7 times slower then the initial version.

## E. Example

```

program example
!to include the module
use TMDX
use LeptonCutDY
implicit none
real*8,dimension(1:4)::xSec,pt_list,xSecMin,xSecMax
pt_list=(/5d0,10d0,15d0,20d0/)
!initialize all at NNLO
call TMDX.Initialize('NNLO','NNLO')
! state  $b_{\max} = 1$ ,  $g_k = 0$ ,  $\lambda_1 = 0.18$  and  $\lambda_2 = 0.0024$ .
call TMDX.SetNPPParameters(1d0,0d0,(/0.180d0,0.0024d0/))
! let  $Q = 91\text{GeV}$  and  $\sqrt{s} = 8\text{TeV}$ , mid-rapidity, process=2.
call TMDX.XSetUp(8000d0**2,91d0,0d0,2)
! ATLAS cuts:  $p_T > 20$ ,  $-2.4 < \eta < 2.4$ 
call SetCuts(.true.,20d0,-2.4d0,2.4d0)
! Evaluate cross-section
call CalculateXsection(xSec,pt_list)
write(*,*) xSec
! Evaluate cross-section and theoretical variations
call CalculateXsection(xSec,xSecMin,xSecMax,pt_list)
end program example

```

Compile e.g. by line

```
f95 src/*.f*
```

## IV. TMDs MODULE

The module **TMDs** joins the lower modules and performs the evaluation of various TMD distributions in the  $\zeta$ -prescription (in version 1 only unpolarized TMDPDF is accessible). Generally a TMD distribution is given by the expression

$$F_f(x, b; \mu, \zeta) = R_f[b, (\mu, \zeta) \rightarrow (\mu_i, \zeta_{\mu_i})] \tilde{F}_f(x, b; \mu_i, f_{NP}), \quad (4.1)$$

where  $R$  is the TMD evolution kernel,  $\tilde{F}$  is a TMD distribution at low scale. It depends on  $f_{NP}$  which is defined by user for each TMD distribution, see secVI C. It also uses the external PDFs which should be provided by user, see VI B. Note, that **TMDs** initializes the lower modules automatically. Therefore, no special initializations should be done.

**Note:** this module (and consequently the manual section) will be seriously in the next versions.

### A. Initialization

Prior the usage module is to be initialized (once per run) by  
`call TMDs_Initialize(order)`  
 here:

**order** declaration of order a unpolarized TMDPDF. It can be 'NLL', 'NLO', 'NNLL' or 'NNLO'. This is a complex declaration, which implies particular orders for coefficient functions, PDFs, anomalous dimension, etc. For detailed definition see [1].

### B. Definition of non-perturbative parameters, and low-scale

The low scale  $\mu_i$  is stated in the function `mu_LOW(bt)` which can be found in the end of `TMDs.f90` code. Modify it if needed.

There are multiple possibilities to include the non-perturbative parameters. The parameters are distributed between different modules. In the current version (ver 1.0, which includes only uTMDPDF module), there are following non-parameters

$b_{\max}$  The parameter defines the low scale  $\mu_i$ .

$g_K$  The parameter parametrizes the non-perturbative part of TMD evolution.

$\{\lambda_i\}$  The set of parameters which define the non-perturbative function  $f_{NP}$ , for unpolarized TMD PDF.

To set particular values of these parameters use

`call TMDs_SetNPPParameters(bmax,gK,lambda)`

where `bmax` and `gK` are `real*8`, `lambda` is `real*8(1:number of parameters)`.

### C. Evaluating unpolarized TMD PDFs

The expression for unpolarized TMD PDF is obtained by the functions

`uTMDPDF_3(x,b,mu,zeta)`

where

`x` (`real*8`) Bjorken- $x$  ( $0 < x < 1$ )

`b` (`real*8`) Transverse distance ( $b > 0$ ) in GeV

`mu` (`real*8`) The scale  $\mu_f$  in GeV. Typically,  $\mu_f = Q$ .

`zeta` (`real*8`) The scale  $\zeta_f$  in  $\text{GeV}^2$ . Typically,  $\zeta_f = Q^2$ .

This function return the vector `real*8(-3:3)` for  $\bar{s}, \bar{u}, \bar{d}, ?, d, u, s$ . Gluon contribution is undefined, but taken into account in the mixing contribution. The mixing with  $c$  and  $b$  quarks is not included.

The following additional functions evaluate the TMD PDFs of different flavours simultaneously.

`uTMDPDF_30(x,b,mu,zeta)` returns (real\*8) array(-3:3) for  $\bar{s}, \bar{u}, \bar{d}, g, d, u, s$ . The mixing with  $c$  and  $b$  quarks is not included. It is a bit ( $\sim 5 - 15\%$  depending on order) slower then the previous command. If gluons are not needed use previous.

`uTMDPDF_5(x,b,mu,zeta)` returns (real\*8) array(-5:5) for  $\bar{b}, \bar{c}, \bar{s}, \bar{u}, \bar{d}, ?, d, u, s, c, b$ . Gluon contribution is undefined, but taken into account in the mixing contribution.

`uTMDPDF_50(x,b,mu,zeta)` returns (real\*8) array(-5:5) for  $\bar{b}, \bar{c}, \bar{s}, \bar{u}, \bar{d}, g, d, u, s, c, b$ . It is a bit ( $\sim 5 - 15\%$  depending on order) slower then the previous command. If gluons are not needed use previous.

#### D. Example

```
program example
!to include the module
use TMDs
implicit none
!initialize all at NNLO
call TMDs_Initialize('NNLO')
! state  $b_{\max} = 1$ ,  $g_k = 0$ ,  $\lambda_1 = 0.18$  and  $\lambda_2 = 0.0024$ .
call TMDs_SetNPPParameters(1d0,0d0,(/0.18d0,0.0024d0/))
!obtain TMD PDF for all flavours at  $x = 0.1$   $b = 1\text{GeV}$ ,  $\mu = 91\text{GeV}$  and  $\zeta = 91^2\text{GeV}^2$ 
write(*,*) uTMDPDF_50(0.1d0,1d0,91d0,91d0**2)
end program example
```

Compile e.g. by line  
f95 src/\*.f\*



## V. TMDR MODULE

The module **TMDR** performs the evaluation of the TMD evolution kernel in the  $(\mu, \zeta)$ -plane from the point  $(\mu_f, \zeta_f)$  to  $(\mu_i, \zeta_i)$ . It is given by the following expression

$$R_f[\mathbf{b}; (\mu_f, \zeta_f) \rightarrow (\mu_i, \zeta_i); \mu_0] = \exp \left[ \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma_F(\mu, \zeta_f) - \int_{\mu_0}^{\mu_i} \frac{d\mu}{\mu} \Gamma(\mu) \ln \left( \frac{\zeta_f}{\zeta_i} \right) \right] \left( \frac{\zeta_f}{\zeta_i} \right)^{-\mathcal{D}_{\text{perp}}^f(\mu_0, \mathbf{b}) - g_K b^2}. \quad (5.1)$$

where  $f$  is the flavour of parton,  $\gamma_F$  is the ultraviolet TMD anomalous dimension and  $\mathcal{D}$  is the rapidity anomalous dimension. The following entries defines this expression

- The coefficient functions can be of NLO, NNLO, NNNLO.
- The order of coupling constant  $a_s$  used in the integrals is defined by user.
- By definition  $g_K = 0$ , but can be switched on.

### A. Initialization

Prior the usage module is to be initialized (once per run). By

`call TMDR.Initialize(oA,oCUSP,oAD,oZETA)`

here:

`oA` declaration of order for  $a_s$ . It can be 'LO', 'NLO' or 'NNLO'. It affects the grid for  $a_s$  which would be used. The path to grid is set in the file `paths`.

`oCUSP` declaration of order for cusp-anomalous dimension (typically one order higher then the rest anomalous dimensions). It can be 'NLO', 'NNLO' or 'NNNLO'.

`oAD` declaration of order for (non-cusp) part of  $\gamma_F$  and  $\mathbf{D}_{\text{pert}}$ . It can be 'NLO', 'NNLO' or 'NNNLO'.

`oZETA` declaration of order for  $\zeta_\mu$ , which is used in  $\zeta$ -prescription. It can be 'NLL', 'NLO', 'NNLO' or 'NNLL' (see definition in [1]).

### B. Definition of $g_K$

The value of the non-perturbative parameter  $g_K$  is 0 default. It can be changed by

`call TMDR.SetgK(gK)`

### C. Evaluating TMD evolution kernel

The expression for TMD evolution kernel is given by the function

`TMDR.R(b,zetaf,muf,zetai,mui,mu0,f)`

where

`b` (real\*8) Transverse distance ( $b > 0$ ) in GeV

`zetaf,muf` (real\*8) hard-factorization scales ( $\zeta_f, \mu_f$ ) in GeV. Typically,  $=(Q^2, Q)$

`zetai,mui` (real\*8) low-factorization scales ( $\zeta_i, \mu_i$ ) in GeV.

`mu0` (real\*8) The scale of perturbative definition of rapidity anomalous dimension  $\mathcal{D}$   $\mu_0$  in GeV.

`f` (integer) parton flavor. 0 for gluon,  $\neq 0$  for quarks.

Typically, one does not need the function  $R$  in the full glory. Therefore, this function has following derivatives, which are optimized with accordance to set of scales.

`TMDR.full.zetaP(bT,zetaf,muf,mui,mu0,f)` same as `TMD.R` but with  $\zeta_i = \zeta_\mu$ .

`TMDR.zetaP(bT,zetaf,muf,mui,f)` same as `TMD.R` but with  $\zeta_i = \zeta_\mu$  and  $\mu_0 = \mu_i$ . This is the most often case.

## D. Example

```

program example
!to include the module
use TMDR
implicit none
!initialize all at NNLO,  $\Gamma_{cusp}$  at NNNLO
call TMDR.Initialize('NNLO','NNNLO','NNLO','NNLO')
!state  $g_K = 0.01$ 
call TMDR.SetgK(0.001d0)
!obtain TMD evolution kernel for quark in  $\zeta$ -prescription from  $(\mu_f, \zeta_f) = (91, 91^2)$  to  $\mu_i = 5\text{GeV}$ , at  $b = 0.1\text{GeV}$ 
write(*,*) TMDR.zetaP(0.1d0,91d0**2,91d0,5d0,1)
end program example

```

Compile e.g. by line (if compile separately take care to include the module `readAs.f90`)  
**f95 src/TMDR.f90**

## VI. UTMDPDF MODULE

The module `uTMDPDF` performs the evaluation of the unpolarized TMD PDF at low scale  $\mu_i$  in  $\zeta$ -prescription. It is given by the following integral

$$F_f(x, b; \mu_i) = \int_x^1 \frac{dz}{z} C_{f \leftarrow f'}(z, b, \mu_i) f_{f'}\left(\frac{z}{x}, \mu_i\right) f_{NP}(z, b, \{\lambda\}), \quad (6.1)$$

where  $f_f(x, \mu)$  is PDF of flavor  $f$ ,  $C$  is the coefficient function in  $\zeta$ -prescription,  $f_{NP}$  is the non-perturbative function. The following entries defines this expression

- The coefficient function can be of LO, NLO, NNLO.
- The order of coupling constant  $a_s$  and PDF  $f(x)$  are defined by used they can be LO, NLO, NNLO.
- Optionally, the renormalon contribution can be added. In this case the coefficient function takes the form

$$C_{f \leftarrow f'}(z, b, \mu_i) \rightarrow C_{f \leftarrow f'}(z, b, \mu_i) + \lambda_2 b^2 \delta_{ff'} C_{f \leftarrow q}^{\text{ren}}(z, b, \mu_i). \quad (6.2)$$

### A. Initialization

Prior the usage module is to be initialized (once per run). By

`call uTMDPDF_Initialize(oC,oA,oPDF)`

here:

`oC` declaration of order for coefficient function. It can be 'LO', 'NLO' or 'NNLO'.

`oA` declaration of order for  $a_s$ . It can be 'LO', 'NLO' or 'NNLO'. It affects the grid for  $a_s$  which would be used. The path to grid is set in the file `paths`.

`oPDF` declaration of order for PDF. It can be 'LO', 'NLO' or 'NNLO'. It affects the grid PDF which would be used. The path to grid is set in the file `paths`. The initialization for PDF set is made in the subroutine `InitializePDF_byUser` in the end of the module. Update this subroutine if needed.

### B. Definition of PDFs

The PDFs are defined by user. The initialization routine `InitializePDF_byUser` is called during the initialization stage. It refers to the grid file under the path defined in `paths`-file. The function for evaluation of a PDF `xPDF` is given in the end of module. Modify it if needed.

### C. Definition of non-perturbative part, $f_{NP}$ and parameters

To modify the definition of the non-perturbative part two function should be modified. Both located in the end of the file `uTMDPDF.f90`.

The first is the subroutine `ModelInitialization_byUser()`. It contains definition of constants `includeRenomalon` and `LambdaNPLength`.

`includeRenomalon` (logical) statement to include the renormalon correction into coefficient function. It is `.true.` or `.false.`. If renormalon correction is included the constant  $\lambda_2$  would be used as prefactor for it. See eqn.(6.2).

`LambdaNPLength` (integer) number of non-perturbative constants  $\lambda$  to be used.

The second, user should provide the definition of the non-perturbative function in `FNP`. It uses the parameters  $\lambda_{1,2,\dots}$  which are given by `(real*8(1:LambdaNPLength))` variable `lambdaNP`.

To set the values for array `lambdaNP` use

`call uTMDPDF_SetLambdaNP((/lambda1, lambda2,.../))`

Note, that the constant  $\lambda_2$  is reserved for the renormalon-contribution prefactor (if included).

### D. Evaluating unpolarized TMD PDFs

The expression for unpolarized TMD PDF is given by the function

`uTMDPDF_lowScale(f,x,b,mu)`

where

`f` (integer) parton flavor. 0 for gluon,  $i$  for quarks ( $\{1,2,3,4,5\}=\{d,u,s,c,b\}$ ),  $-i$  for anti-quarks.

`x` (real\*8) Bjorken- $x$  ( $0 < x < 1$ )

`b` (real\*8) Transverse distance ( $b > 0$ ) in GeV

`mu` (real\*8) The scale  $\mu_i$  in GeV. It should be bigger than 0.5GeV. we suggest to take it larger than 1 GeV, since the variation of scales can occasionally make it smaller than 0.5 GeV.

**Warning:** we do not recommend use this function, because it is slower and numerically less accurate than the following functions.

The following additional functions evaluate the TMD PDFs of different flavours simultaneously.

`uTMDPDF_lowScale3(x,b,mu)` returns (real\*8) array(-3:3) for  $\bar{s}, \bar{u}, \bar{d}, ?, d, u, s$ . Gluon contribution is undefined, but taken into account in the mixing contribution. The mixing with  $c$  and  $b$  quarks is not included.

`uTMDPDF_lowScale30(x,b,mu)` returns (real\*8) array(-3:3) for  $\bar{s}, \bar{u}, \bar{d}, g, d, u, s$ . The mixing with  $c$  and  $b$  quarks is not included. It is a bit ( $\sim 5 - 15\%$  depending on order) slower than the previous command. If gluons are not needed use previous.

`uTMDPDF_lowScale5(x,b,mu)` returns (real\*8) array(-5:5) for  $\bar{b}, \bar{c}, \bar{s}, \bar{u}, \bar{d}, ?, d, u, s, c, b$ . Gluon contribution is undefined, but taken into account in the mixing contribution.

`uTMDPDF_lowScale50(x,b,mu)` returns (real\*8) array(-5:5) for  $\bar{b}, \bar{c}, \bar{s}, \bar{u}, \bar{d}, g, d, u, s, c, b$ . It is a bit ( $\sim 5 - 15\%$  depending on order) slower than the previous command. If gluons are not needed use previous.

These commands have essentially less number of calls for `xPDF` and, therefore, significantly, faster than `uTMDPDF_lowScale`. We recommend to use these functions.

### E. Example

```
program example
!to include the module
use uTMDPDF
implicit none
!initialize all at NNLO
call uTMDPDF_Initialize('NNLO','NNLO','NNLO')
!state  $\lambda_1 = 0.18$  and  $\lambda_2 = 0.0024$ .
!It is assumed includeRenomalon=.true. and lambdaNPLenght=2
call uTMDPDF_SetLambdaNP((/0.180d0,0.0024d0/))
!obtain TMD PDF for all flavours at  $x = 0.1$   $b = 1\text{GeV}$  and  $\mu_i = 5\text{GeV}$ 
write(*,*) uTMDPDF_lowScale50(0.1d0,1d0,5d0)
end program example
```

Compile e.g. by line (if compile separately take care to include the module `readAs.f90`)

`f95 src/*.f*`

## VII. PREPARING GRIDS FOR TMDR

To improve the evaluation timing TMDR uses the precalculated grids of the renormalization group integrals such as

$$\int_{\mu_r}^{\mu} \frac{d\mu'}{\mu'} a_s^n(\mu'), \quad \int_{\mu_r}^{\mu} \frac{d\mu'}{\mu'} a_s^n(\mu') \ln(\mu), \quad \int_{\mu_r}^{\mu} \frac{d\mu'}{\mu'} \Gamma_{cusp}(a_s) \ln \mu', \quad \text{etc.} \quad (7.1)$$

Here,  $\mu_r = 1\text{GeV}$  is the common reference point. Grids can be prepared by evaluation of the program `MakeGridsForTMDR.f90` in the directory `/Precalculate` (note, that default version already contains necessary grids made on the MMHT2014 values of  $a_s$ ).

As an input for the program user should prepare functions `InitializeAs_byUser` and `alphaS_ByUser`. Both are located in the module `UserDefinedAlphaS.f90`. The subroutine `InitializeAs_byUser` runs initialization routine, if needed. The function `alphaS_ByUser` gives access to the values of  $\alpha_s(\mu) = g^2(\mu)/(4\pi)$ .

As a result of evaluation program create the file `As_grid.dat`. Rename it conveniently at put to the directory `/Grids`. Update the file `paths` with new names.

---

[1] I. Scimemi and A. Vladimirov, arXiv:1706.01473 [hep-ph].

[2] L. A. Harland-Lang, A. D. Martin, P. Motylinski and R. S. Thorne, Eur. Phys. J. C **75** (2015) no.5, 204 doi:10.1140/epjc/s10052-015-3397-6 [arXiv:1412.3989 [hep-ph]].