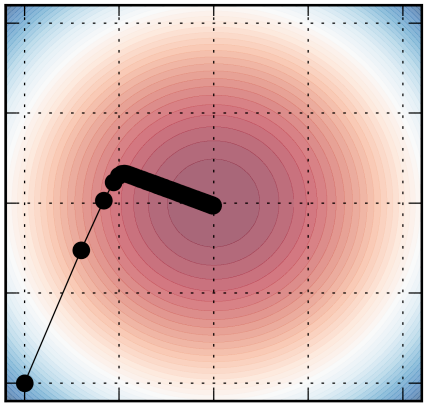
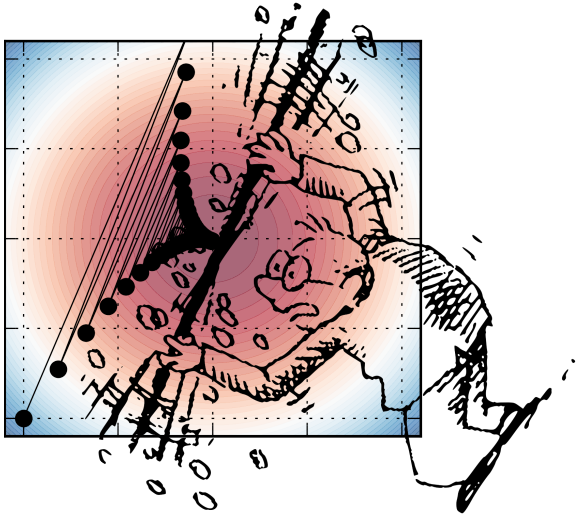
34th ALERT Doctoral School 2023

Machine Learning in Geomechanics

ALERT Geomaterials

Introduction to regression methods





Filippo Masi

The University of Sydney

THE UNIVERSITY OF SYDNEY

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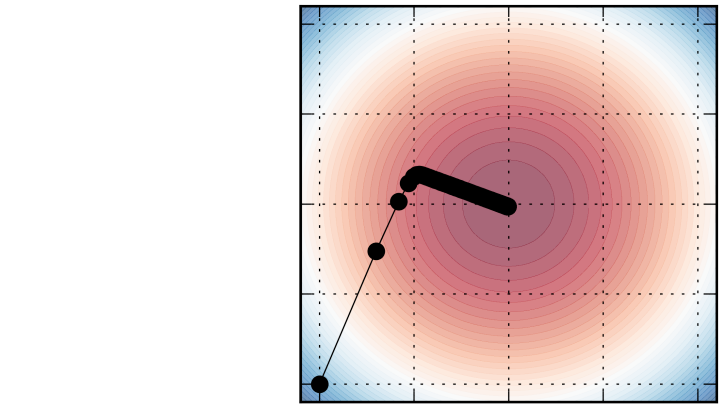

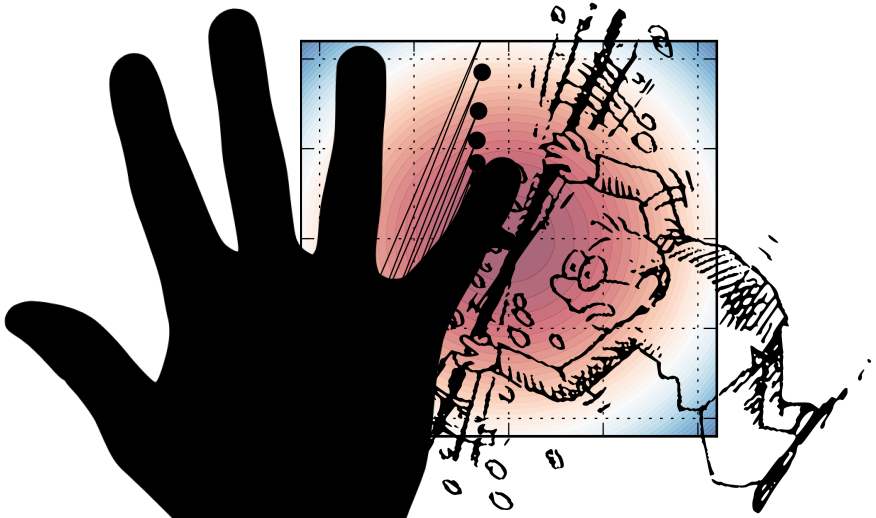

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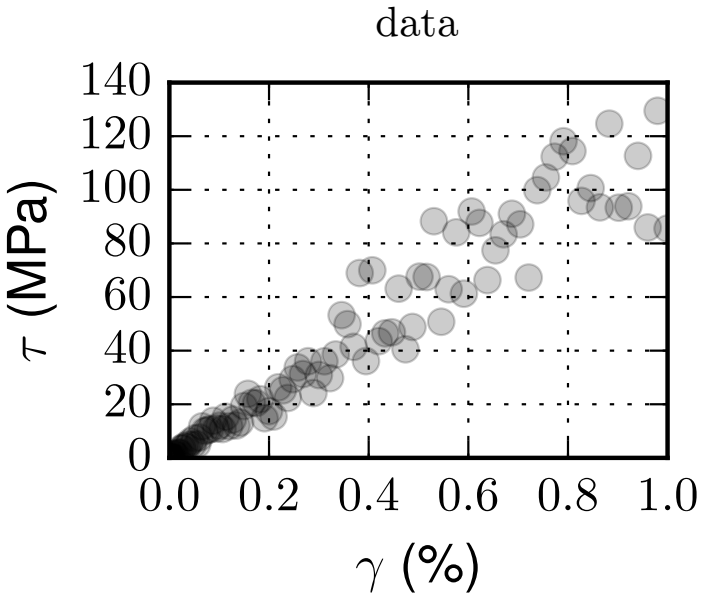
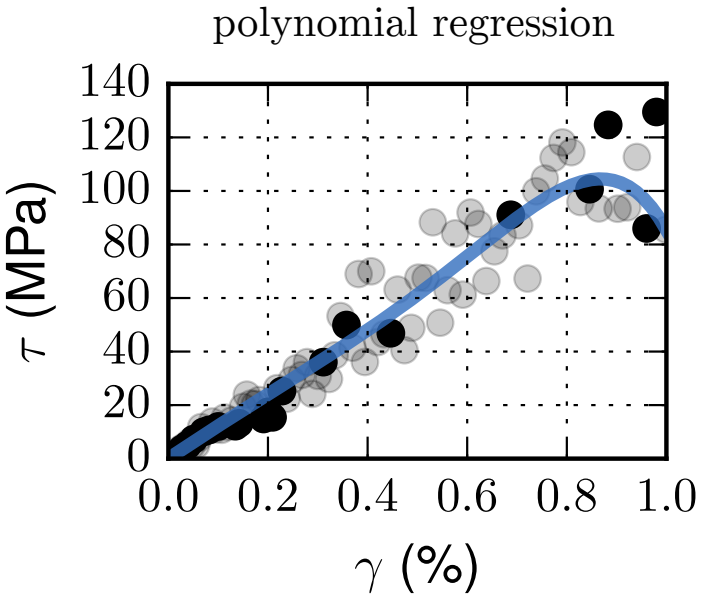
Regression methods / Exercises

https://qrco.de/regr

Exercise 1

Perform regression to obtain an interpretable (hyper-elastic) constitutive model that best fits stress-strain data





[TODO-1] Is the above polynomial model satisfactory?

Hint: Check the predictions of the model for larger deformations (extrapolation).

[TODO-2] Can you find a more accurate (yet interpretable) constitutive model?

Hint: Yes, you can!

[TODO-3] Compute the effective stiffness G (in GPa)

$$G \equiv \frac{d\tau}{d\gamma}(\gamma)$$

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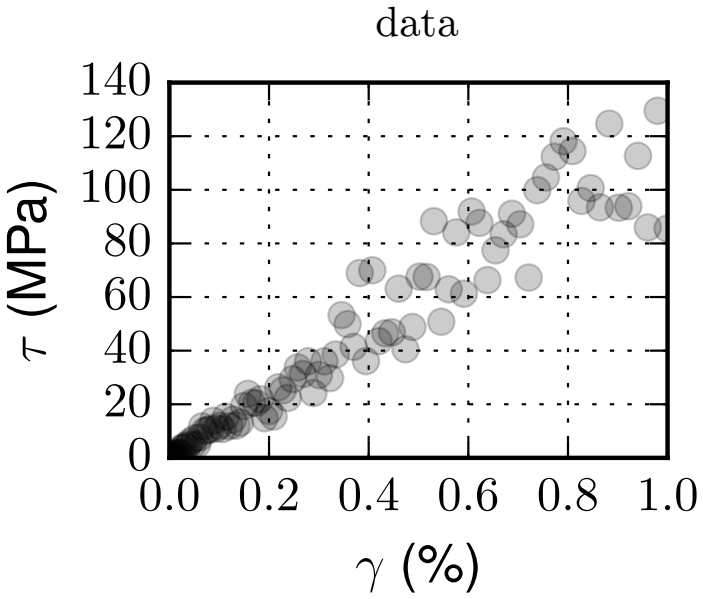

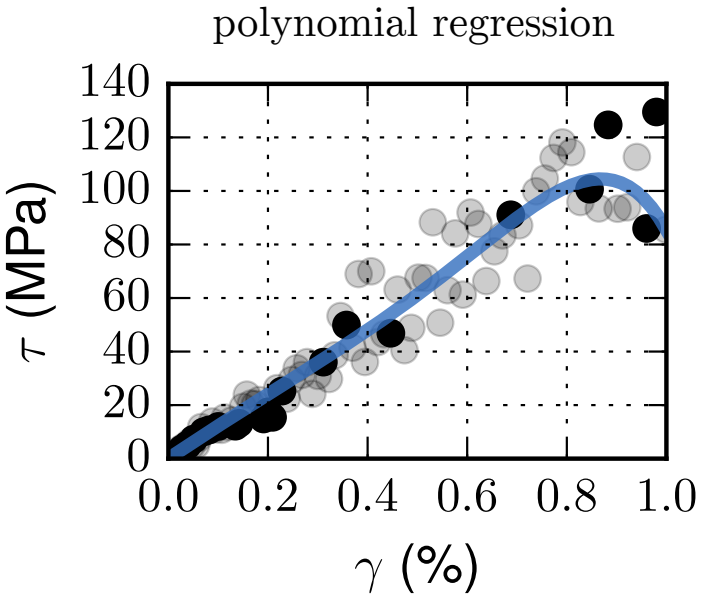

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WINNER!

$$G \equiv \frac{d\tau}{d\gamma}(\gamma)$$

Exercise 1 - Solution

Perform regression to obtain an interpretable (hyper-elastic) constitutive model that best fits stress-strain data

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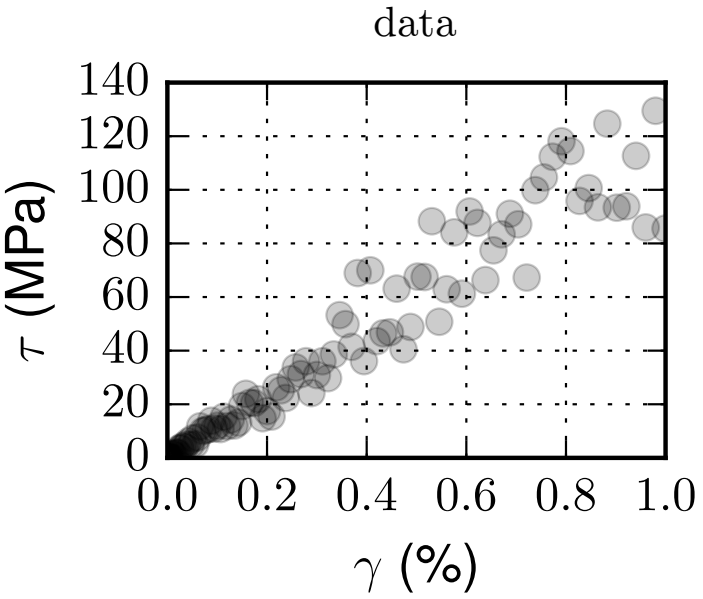

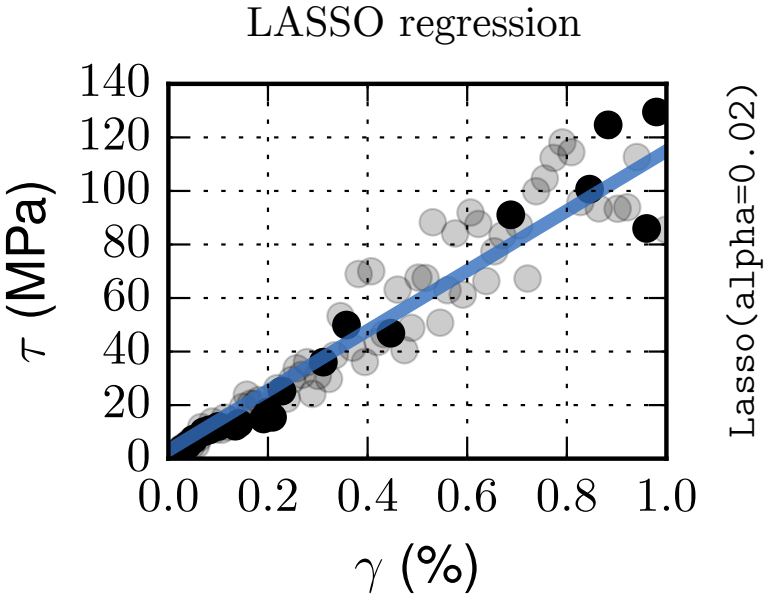

Constitutive model: $\bar{\tau} = -0.0009 + 0.95\bar{\gamma}$

Effective stiffness: $$G \equiv \frac{d\tau}{d\gamma}(\gamma) = \frac{d\tau}{d\bar{\tau}}\frac{d\bar{\tau}}{d\bar{\gamma}}\frac{d\bar{\gamma}}{d\gamma} = \frac{\alpha_\tau}{\alpha_\gamma}\frac{d\bar{\tau}}{d\bar{\gamma}} = 11.187608 \text{ GPa}$$

$$\bar{\tau} = \frac{\tau - \beta_\tau}{\alpha_\tau}, \quad \bar{\gamma} = \frac{\gamma - \beta_\gamma}{\alpha_\gamma}$$

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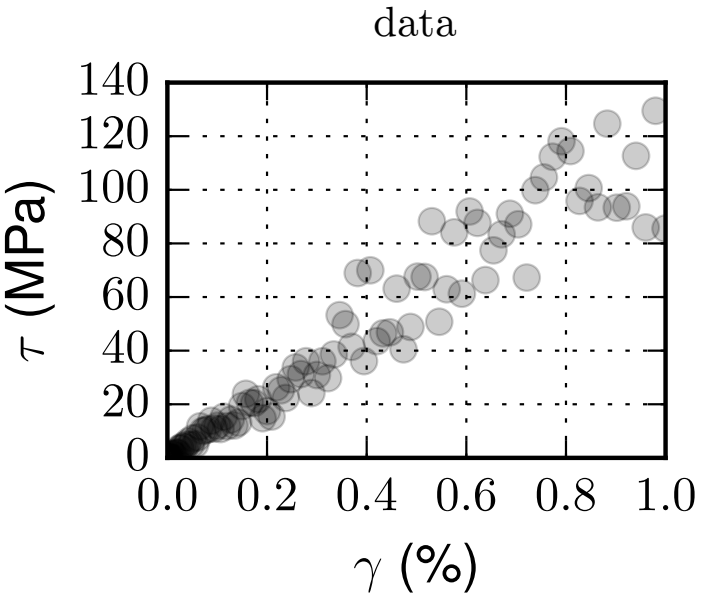
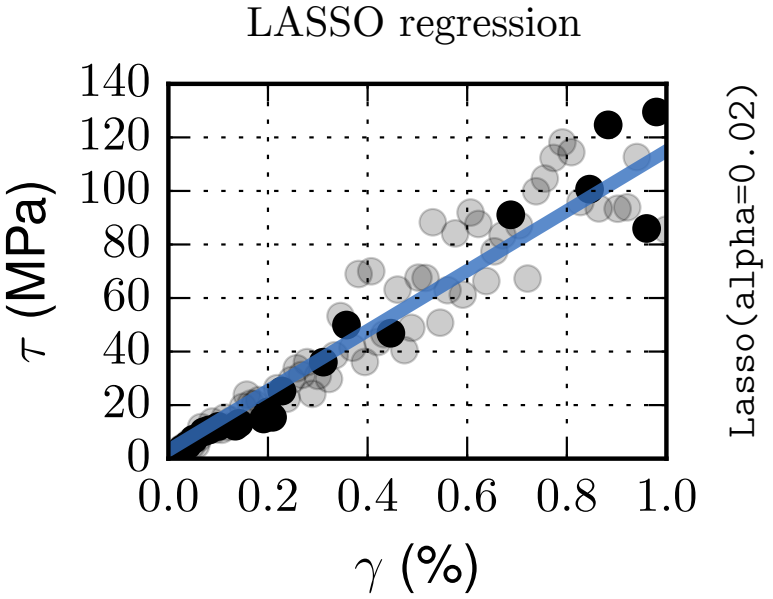
Perform regression to obtain an interpretable (hyper-elastic) constitutive model that best fits stress-strain data





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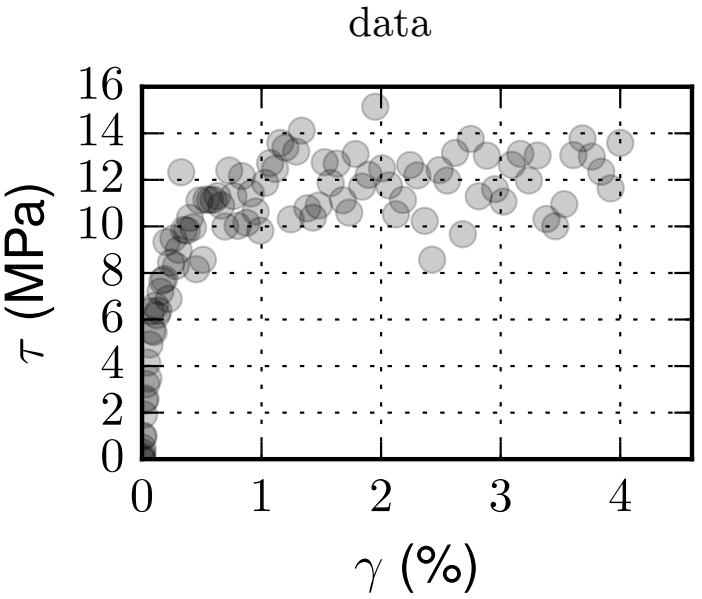
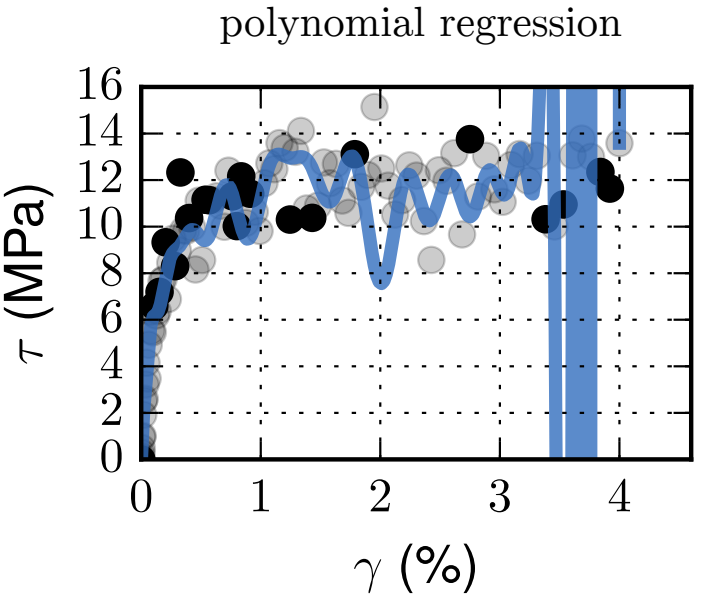
$$G^{\text{truth}} = 12.3456789 \text{ GPa} \qquad \bar{\tau} = \frac{\tau - \beta_\tau}{\alpha_\tau}, \quad \bar{\gamma} = \frac{\gamma - \beta_\gamma}{\alpha_\gamma}$$

Exercise 2

Perform regression to obtain an interpretable (nonlinear hypo-elastic) constitutive model that best fits stress-strain data





[TODO-1] Is the above polynomial model satisfactory?

Hint: Check the predictions of the model for larger deformations (extrapolation).

[TODO-2] Can you find a more accurate (yet interpretable) constitutive model?

[TODO-3] Compute the expression of the effective stiffness G.

$$G \equiv \frac{d\tau}{d\gamma}(\gamma)$$

Exercise 2 - Solution

Perform regression to obtain an interpretable (nonlinear hypo-elastic) constitutive model that best fits stress-strain data

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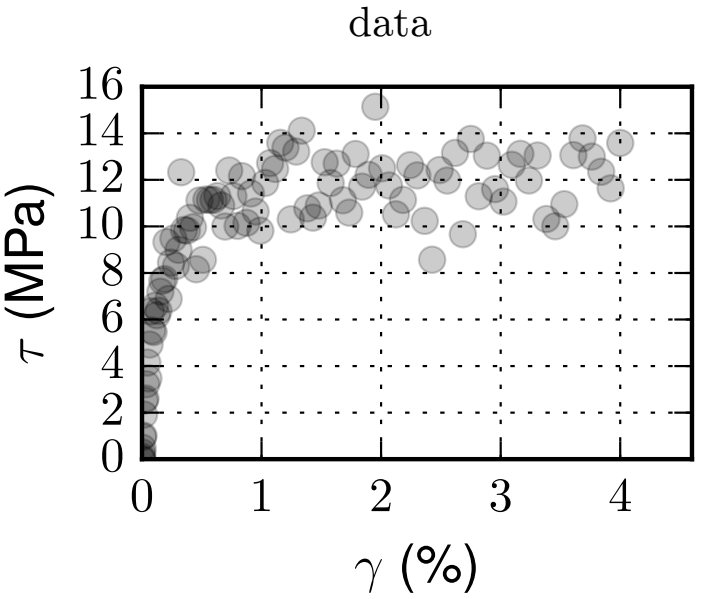

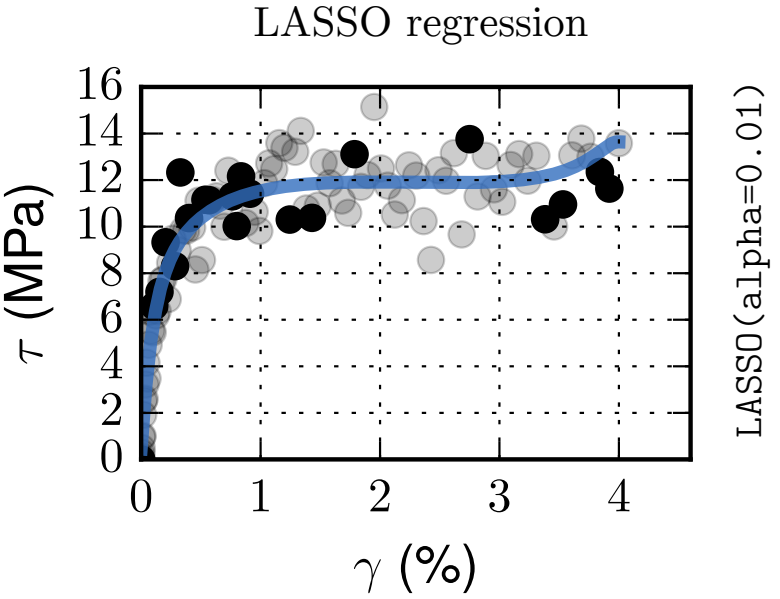

Constitutive model: $$\bar{\tau} = 0.48 + 0.61\bar{\gamma}^5 - 0.2\bar{\gamma}^8 - 0.1\bar{\gamma}^{10}$$

Effective stiffness: $$G \equiv \frac{d\tau}{d\gamma}(\gamma) = \frac{d\tau}{d\bar{\tau}}\frac{d\bar{\tau}}{d\bar{\gamma}}\frac{d\bar{\gamma}}{d\gamma} = \frac{\alpha_\tau}{\alpha_\gamma}\frac{d\bar{\tau}}{d\bar{\gamma}}$$

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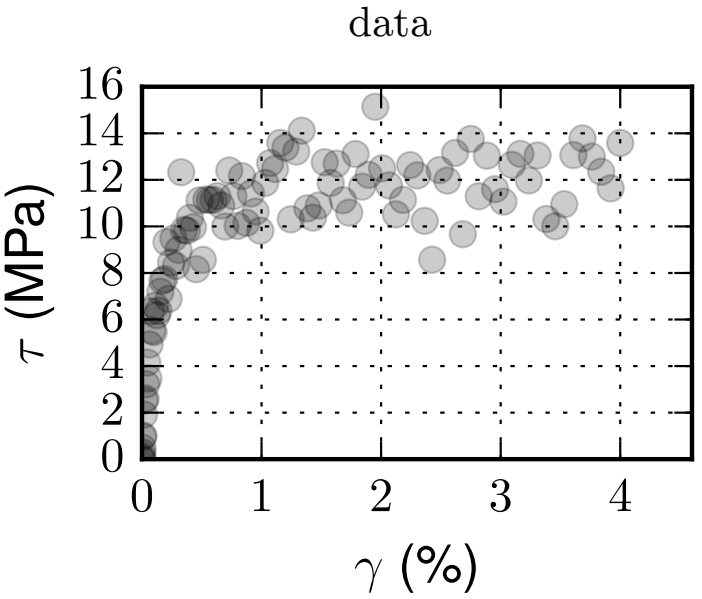
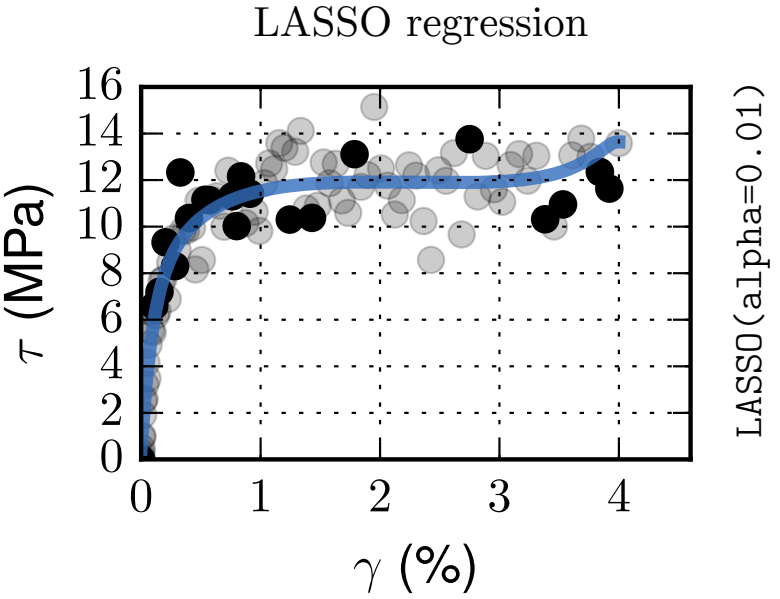
Perform regression to obtain an interpretable (nonlinear hypo-elastic) constitutive model that best fits stress-strain data





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$$\tau = \frac{\beta G \gamma}{\beta + |\gamma|}, \quad \beta = 0.001,\ G = 12.345\ \text{GPa}$$

Quite a failure!