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The Spectral Elemet Method enanched by the Domain Reduction Method (DRM)

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1 The DRM

Large scale numerical wave propagation from the earthquake source to a specific site location implies great computational effort, especially if the problem is 3D and the propagation path has strong geological or topographical irregularities. The Domain Reduction Method (Bielak et al. 2003, Yoshimura et al., 2003, Faccioli et al., 2005, Bielak, 2005) is a very effective approach to overcome this problem. The main advantage of the procedure is the possibility of *substructuring* the original problem (figure 1.1) into two numerical simpler sub-models, with different scale dimensions, to be solved in two different steps (see the above-mentioned references for details of implementation).

STEP I

In this phase the simulation of the earthquake source and propagation path effects is performed, using an *auxiliary model* that includes both the source and the surrounding half space (the *external domain* conveniently described according to local crustal properties). This model, from which the structure of interest has been removed and replaced by the same material as the surrounding soil (figure 1.1a), may be characterized by a scale of thousand of meters.

STEP II

Subsequently, a model which encompasses the region of interest is generated. In this phase only a region with *reduced* spatial dimensions (*internal domain* in figure 1.1b) is modelled with the desired accuracy, focusing on the structure and the surrounding soil, but not the seismic source and most of the propagation path from the source to the site. The typical dimensions of the elements for the spatial discretization may range from few meters to fractions of meter. The dynamic input of the model is a set of effective nodal forces evaluated on the basis of the ground displacements calculated at the first step and applied within a strip of elements named *effective boundary* (orange boundary in figure 1.1). These forces are equivalent to and replace the original seismic forces used in the first step to reproduce the seismic source.

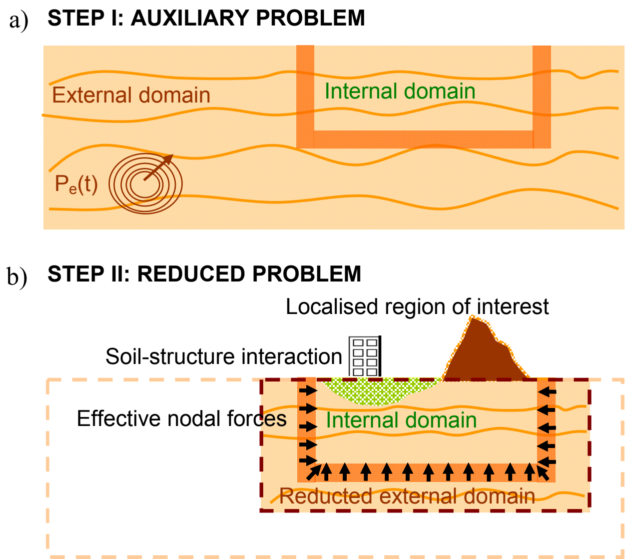

Fig. 1.1 – Summary of the two-step DRM. a) auxiliary problem including the seismic source and the background geological model; b) reduced model including the localized geological feature of interest. The seismic source is hire replaced by equivalent nodal forces applied at the boundary of effective nodes (dark orange boundary strip)

As a consequence, the main advantage of the DRM is the possibility of coupling solutions obtained with different methods.

The SE code GeoELSE has been improved in order to use the DRM method as an internal set of routines and processes. The effective nodal forces, equivalent to the original seismic source, may now be calculated from free field displacements time histories coming from different wave propagation methods: the SE as well, the semi-analytical method of Hisada and Bielak (2003) (named Hisada code in the following) or any other analytical or empirical method for plane wave propagation.

In the following two examples are shown where GeoELSE is used coupled with the Hisada code (section 2) or by itself (section 3).

2 The case of Düzce, Turkey, solved coupling GeoELSE with the Hisada's method

From a computational point of view, the main problem of coupling two different methods is the post processing of the great number of free field time histories (step I) at the LGL nodes of the effective boundary, especially in high polynomial degree analyses. For this purpose, only the free field displacement time histories at the corner nodes of the spectral elements (big points in figure 2.1) are calculated in step I, while the seismic response at the remaining LGL nodes (small points in figure 2.1) is calculated using an *ad hoc* interpolation procedure.

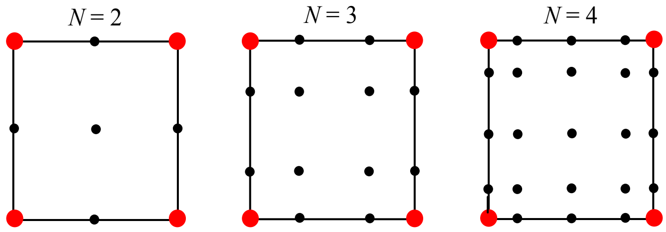

Fig. 2.1 – Legendre-Gauss-Lobatto (LGL) nodes within spectral elements for different spectral degree *N:* the big points indicate the corner nodes

The phases for the application of the DRM method coupling the Hisada approach with GeoELSE is herein described taking as an example the case of Düzce (illustrated in Scandella, 2007 and Scandella et al., 2007). As shown in figure 2.2, the semi-analytical approach of Hisada & Bielak (2003) has been used to solve the 3D auxiliary problem which includes the source and the wave propagation in the half-space, simulating the wave propagation field induced by an extended seismic source (step I). The Spectral Element Method (SEM) has been used to simulate the 2D wave propagation in the region of interest (step II).

The procedure requires 8 steps, as shown in figure 2.3:

1. Generation of the SE mesh of the reduced model for step II (e.g. by **CUBIT**);
2. Extrapolation of the effective boundary nodes from the SE mesh: list of the element corner nodes and list of the remaining LGL nodes (with **GeoELSE**, option SDRM 3);
3. Conversion of the corner nodes coordinates from the 2D reference system to 3D geographical coordinates for the analytical code of Hisada (with **trasf_for_else**);
4. DRM step I: **Hisada** run;
5. Integration of the velocity time histories calculated by the Hisada code at the corner nodes of the effective boundary elements (with **integration_vel_Hisada**)
6. Evaluation of the displacement time histories at the remaining LGL nodes of the effective boundary by interpolation (**interpolation**);
7. Conversion of the 3D displacement time histories at the effective boundary nodes in the 2D compatible format for GeoELSE (with **disp_for_else**);
8. DRM step II: 2D SE analysis by **GeoELSE**.

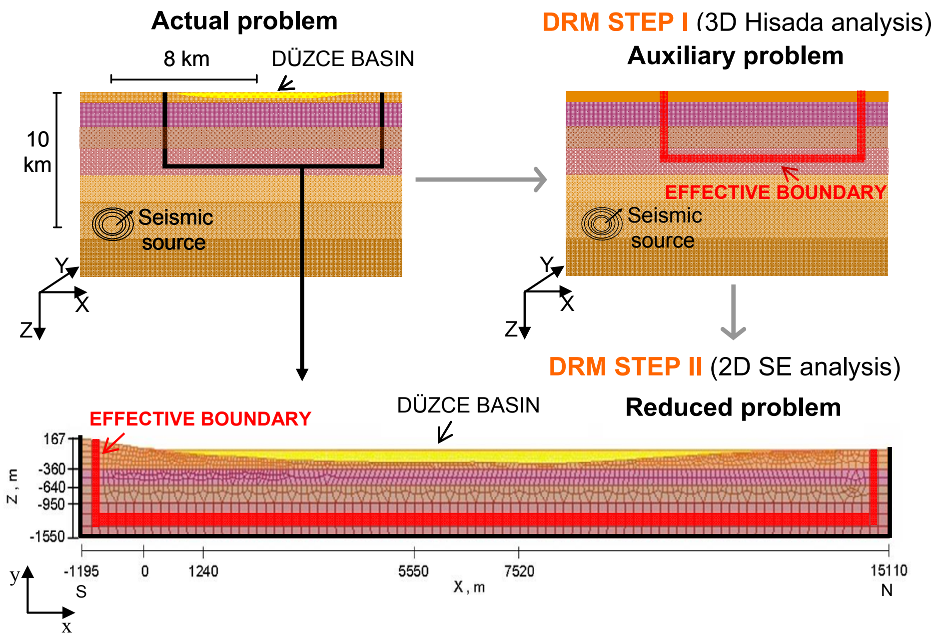

Fig. 2.2 – Scheme of the DRM procedure applied to the case of the Düzce urban area. The problem is subdivided into two simpler ones analyzed in different steps. Step I: 3D analysis of the source and the wave propagation in a layered half-space using the semi-analytical method of Hisada. Step II: 2D wave propagation in the Düzce basin by means of the Spectral Element Method

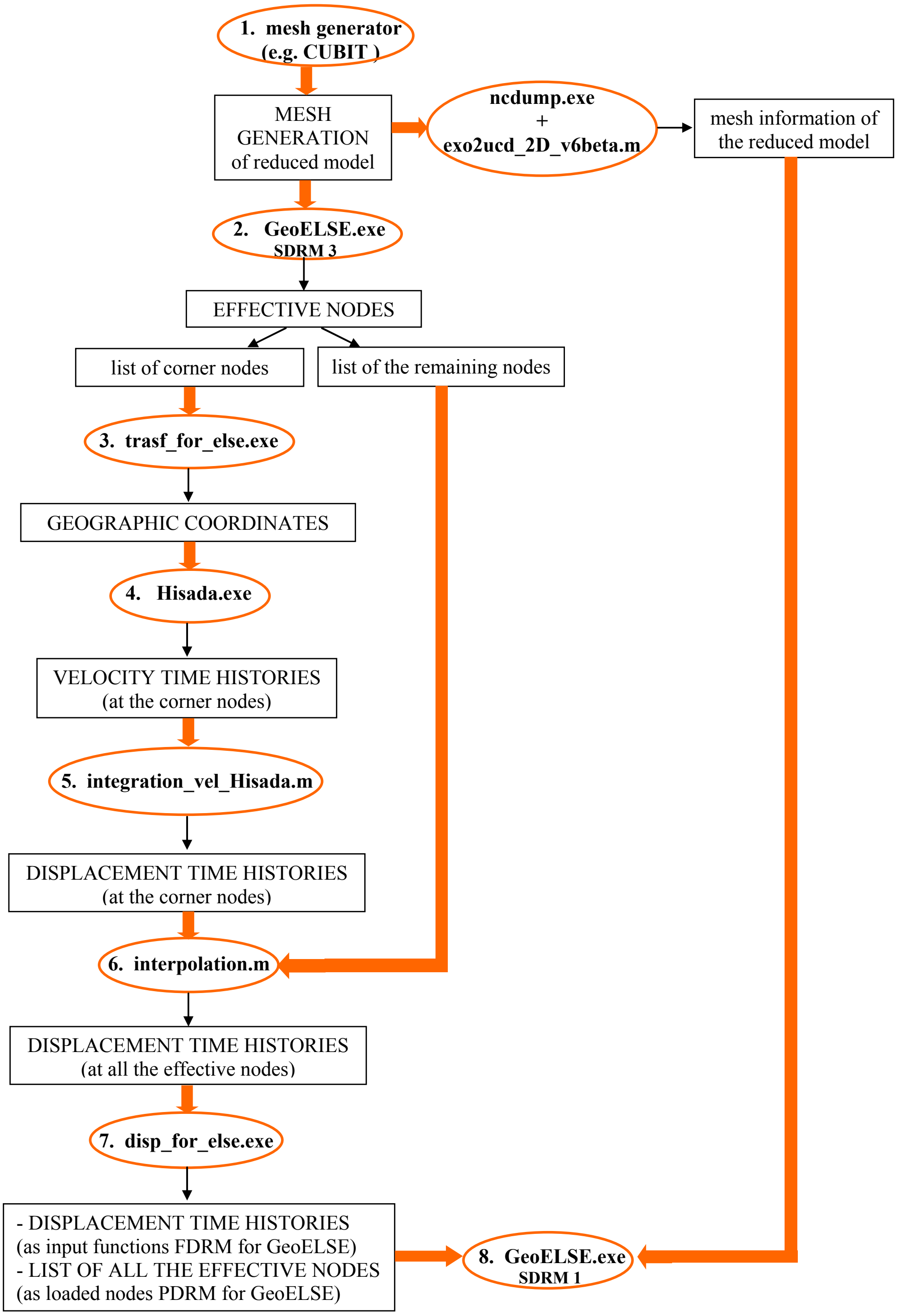

Fig. 2.3 – Main steps for the application of the DRM coupling GeoELSE with Hisada method

2.1 Mesh generation (e.g. with CUBIT)

While unstructured triad/tetrahedral (2D/3D) meshes can be achieved quite easily with commercial or non commercial software, the creation of a non structured SE quadrilateral/hexahedral mesh is still recognized as a challenging problem.

Thanks to the flexibility of the GeoELSE code in terms of mesh input, the decomposition into non overlapping quadrilateral/hexahedral elements was successfully achieved using the commercial software **CUBIT**.A clear description of the meshing strategy adopted to strictly honour the given geometry can be found in Stupazzini (2006), while other very promising strategy to tackle this challenging problem are illustrated in Casarotti et al. (2007).

The SE mesh of the 2D NS cross-section in the Düzce basin is shown in figure 2.2.

2.2 Extrapolation of the effective boundary nodes (with GeoELSE)

Node coordinates of the effective boundary elements in the SE mesh of the reduced model are extrapolated with **GeoELSE**. In particular, the list of the corner nodes (***lista_cornerLGL_coor.txt***) is required separately from the list of the remaining LGL nodes (***lista_otherLGL_coor.txt***), the former containing the nodes where in the DRM step I the free field displacement is calculated, the latter listing the nodes where the seismic response is evaluated by interpolation. These lists are obtained running the SE GeoELSE code.

The input files for GeoELSE are:

1. ***nomefile.inp*** (***duzce_SN_drm_II_Else.inp***) with the grid information;
2. ***nomefile.mat*** (***duzce_drm.mat***) with the MDRM, BDRM and SDRM options (figure 2.4);

mat	n°	type	rho	lambda	mu	gamma
MATE	1	2	1800	7.6050E+08	1.9013E+08	5.2360E-02
MATE	2	2	2000	9.4500E+08	4.0500E+08	3.1416E-02
MATE	3	2	2200	3.7161E+09	4.0244E+09	1.5708E-02
MATE	4	2	2250	1.0157E+10	1.0742E+10	1.0472E-02
MATE	5	2	2300	1.1397E+10	1.2702E+10	7.8540E-03
MATE	6	2	2300	1.5134E+10	1.6767E+10	7.8540E-03
MATE	7	2	2200	3.7161E+09	4.0244E+09	1.5708E-02
MATE	8	2	2250	1.0157E+10	1.0742E+10	1.0472E-02
MATE	9	2	2300	1.1397E+10	1.2702E+10	7.8540E-03
MATE	10	2	2300	1.5134E+10	1.6767E+10	7.8540E-03
MATE	11	2	2200	3.7161E+09	4.0244E+09	1.5708E-02
MATE	12	2	2250	1.0157E+10	1.0742E+10	1.0472E-02
MATE	13	2	2300	1.1397E+10	1.2702E+10	7.8540E-03
MATE	14	2	2300	1.5134E+10	1.6767E+10	7.8540E-03

ABSO 15

MDRM 7…
MDRM 8…
MDRM 9…
MDRM 10.. } blocks of effective boundary

BDRM 16 } block delimiting the internal boundary

SDRM.. 3. } option to compute the lists of nodes

Fig. 2.4 – Example of ***nomefile.mat (duzce_drm.mat)*** to compute the lists of the corner nodes (*lista_cornerLGL_coor.txt*) and of the remaining LGL nodes (*lista_otherLGL_coor.txt*) of the effective elements, and the lists of the node id in GeoELSE (*lista_cornerLGL.txt, lista_otherLGL.txt*)

MDRM 7 means that the material n° **7**, whose properties are listed before (green rectangle), is a block of elements of the effective boundary.

BDRM 16 means that the block n° **16** is the block of lines delimiting the internal boundary of the effective elements.

SDRM 3 is the option to be used for the computation of the required lists of nodes.

3. ***else2_input.d*** with the information on input files and the spectral degree (figure 2.5).

```
-----------------------------------------------------------------------------------------------------
DEGREE 4

GRIDFILE   duzce_SN_DRM_II_Else
MATFILE    duzce_DRM
OUTFILE    duzce_DRM_out
-----------------------------------------------------------------------------------------------------
```

Fig. 2.5 – Example of ***else2_input.d*** for GeoLESE, with the information on the input files and the spectral degree

Output files at this stage are:

- ***lista_cornerLGL_coor.txt*** with the 2D coordinates x, y of the corner nodes of the effective elements, as shown in figure 2.6;
- ***lista_otherLGL_coor.txt*** with the 2D coordinates x, y of the LGL nodes (without the corner nodes) of the effective elements (figure 2.6);
- ***lista_cornerLGL.txt*** with the id of the corner nodes of the effective elements;
- ***lista_otherLGL.txt*** with the id of the LGL nodes (without the corner nodes) of the effective elements.

```
-----------------------------------------------------
14743.0000        -21.0000
14743.0000       -134.0000
14897.0000       -134.2133
14897.0000        -21.3200
14743.0000       -247.0000
14743.0000       -360.0000
               :
               :
               :
14743.0000      -1350.0000
14743.0000      -1150.0000
14897.0000      -1350.0000
14897.0000      -1150.0000
-----------------------------------------------------
```

x coordinate y coordinate

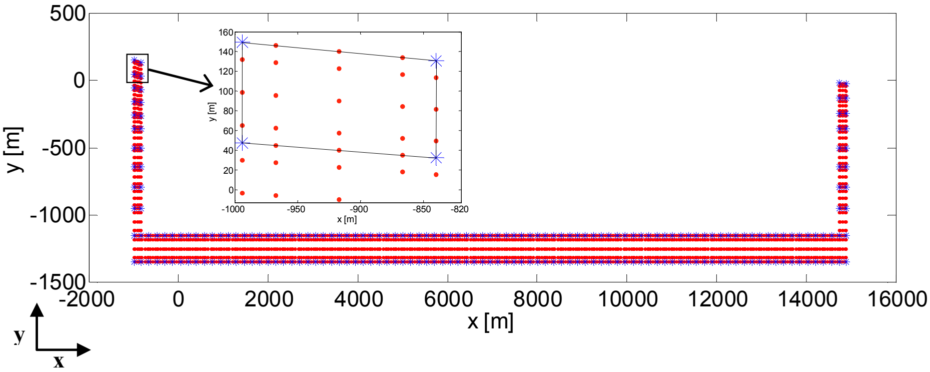

Fig. 2.6 – File ***lista_cornerLGL_coor.txt*** (referring to the Düzce case) with the list of the 2D x, y coordinates of the corner nodes of the effective boundary elements in the mesh used by GeoELSE. The figure below shows the nodes listed in the *lista_cornerLGL_coor.txt* (blue stars) and in *lista_otherLGL_coor.txt* (red points) which are the corner nodes of the effective boundary elements and the remaining LGL nodes without duplicates, respectively

2.3 From the 2D reference system of the grid to the 3D geographical reference system (with trasf_for_else)

The conversion of the 2D coordinates of the effective corner nodes into the 3D geographic coordinates (**X**, **Y**, **Z**) required by the Hisada code is performed with the **trasf_for_else** code.

In figure 2.7 the 2D reference coordinate systems of the grid is shown together with the 3D geographic one used by the Hisada code. The 3D geographic system follows the Aki & Richard (1880) conventions for the definition of the fault parameters. The origin **O** of the **X, Y** axes may be arbitrary. A reference point **o** on the 2D grid has to be chosen for the conversion of the coordinates (figure 2.7).

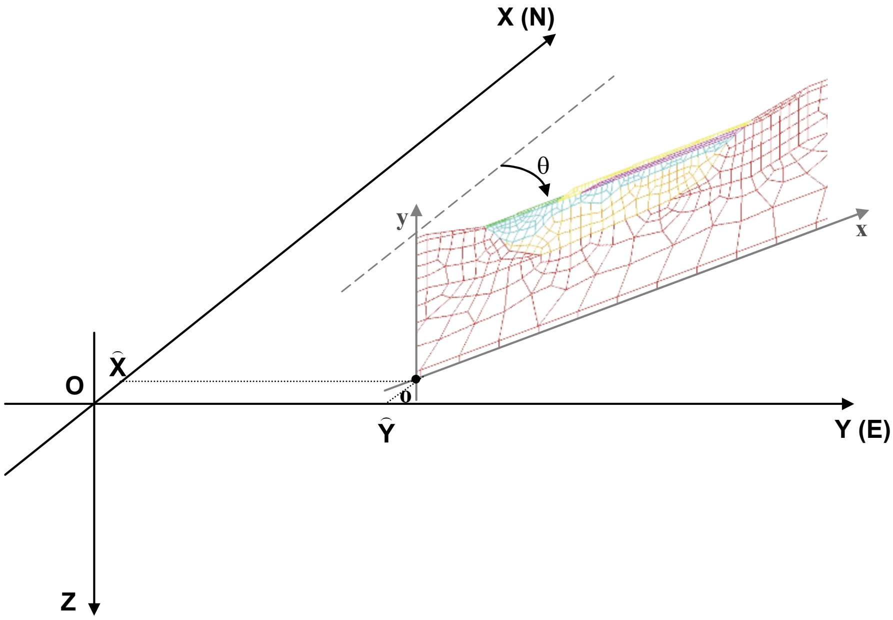

Fig. 2.7 – 2D reference coordinate systems of the grid with respect to the 3D geographic one. **O** is the arbitrary origin point of the **X**, **Y** axes. **o** is a reference point on the 2D grid, with geographic coordinates $\hat{X}$, $\hat{Y}$

Input information for **trasf_for_else** program is:

1. geographic coordinates ($\hat{X},\hat{Y}$) of the "reference" point of the 2D grid used (e.g.-3020, -12000 for Düzce);
2. x coordinate of the same "reference" point of the 2D grid (-2220 for Düzce);
3. rotation angle θ in degree of the 2D cross section, positive clockwise with respect to the North (0 for Düzce);
4. a file with the list of the coordinates of the corner nodes of the effective boundary elements: ***lista_cornerLGL_coor_Else.txt*** (following figure).

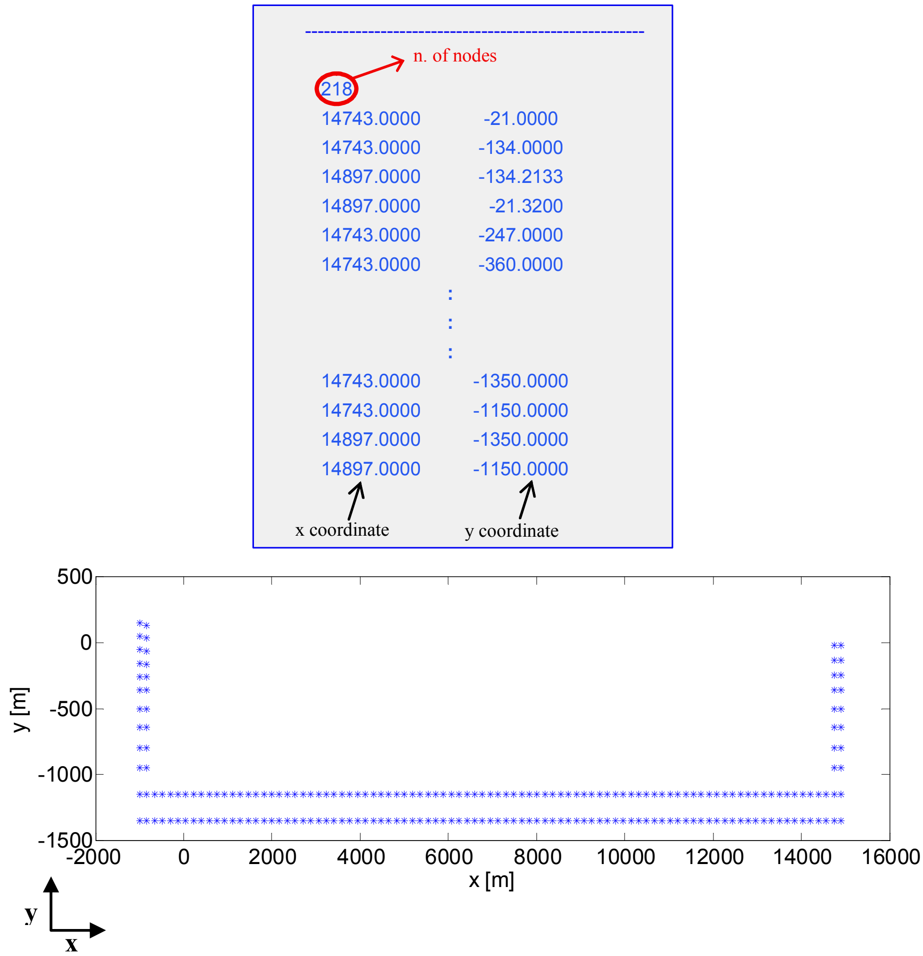

Fig. 2.8 – File ***lista_cornerLGL_coor_Else.txt*** (referring to the Düzce case) with the list of the 2D coordinates of the corner nodes of the effective boundary elements in the mesh for GeoELSE, (same as *lista_cornerLGL_coor.txt*, with first the number of the nodes)

Output file is:

– ***lista_cornerLGL_coor_Hisada.txt***, with the 3D geographic coordinates X, Y, Z of the corner nodes of the effective boundary elements for the Hisada code, as shown in figure 2.9:

13942.63	-12000.00	170.50
13942.63	-12000.00	283.50
14096.63	-12000.00	283.71
14096.63	-12000.00	170.82
13942.63	-12000.00	396.50
13942.63	-12000.00	509.50
	:	
	:	
	:	
13942.63	-12000.00	1499.50
13942.63	-12000.00	1299.50
14096.63	-12000.00	1499.50
14096.63	-12000.00	1299.50
↗	↑	↖
X coordinate	Y coordinate	Z coordinate

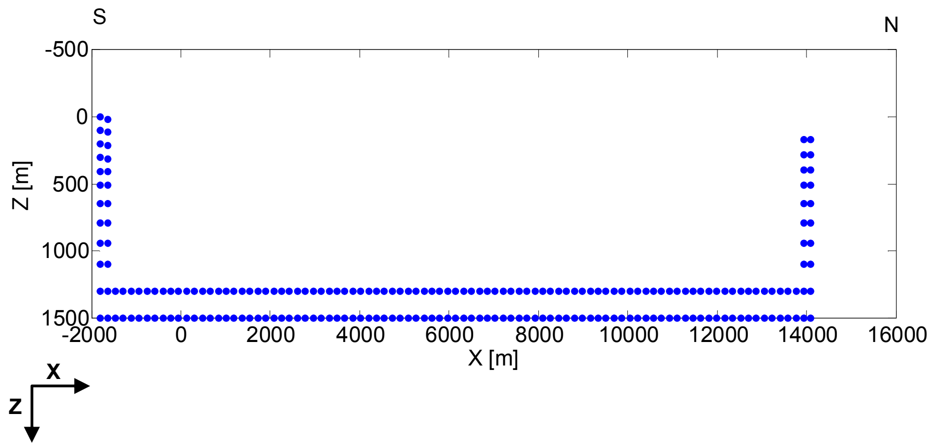

Fig. 2.9 – File ***lista_cornerLGL_coor_Hisada.txt*** (referring to the Düzce case) with the list of the 3D geographic coordinates of the corner nodes of the effective boundary elements in the layered soil model used by the Hisada method, in the reference system used by Hisada. In this case, the Düzce cross-section is aligned along the North direction **X** (θ = 0)

2.4 DRM step I: 3D Hisada run

Referring to the Hisada manual *grftl12s.manual.doc* (2006) for a thorough description of the use of the Hisada code, and to Scandella (2007) for the 3D Düzce simulated fault and the layered soil properties adopted, the input file ***grfit12s.in*** is shown hereafter.

Hisada code calculates at the required nodes the velocity time histories (***lista_cornerLGL_coor_Hisada.txt***), whose components X, Y and Z are saved in the files ***x_dat, y_dat*** and ***z_dat*** respectively.

```
-------------------------------------------------------------------------------------------------------------------
* Data for Dt, Duration, and Period * * 1999 Duzce, Turkey Earthquake *
 0.1500      512 : Delta Time (sec), and Number of Time (must be Power of 2)
 0.2000    0.000 : Minimum Period (sec), and Imaginary Omega for Phinney
  * MEDIUM DATA *
      7          : NL (NUMBER OF LAYERS)     t/m3,m/s,    ,m/s,    ,    m
2.20 2312.5  200.0 0.0  1352.5 100.0 0.0   490.0 :DNS, VP, Qp, VS, Qs, Thick
2.25 3750.0  250.0 0.0  2185.0 150.0 0.0   280.0 :DNS, VP, Qp, VS, Qs, Thick
2.30 4000.0  350.0 0.0  2350.0 200.0 0.0   310.0 :DNS, VP, Qp, VS, Qs, Thick
2.30 4600.0  350.0 0.0  2700.0 200.0 0.0   600.0 :DNS, VP, Qp, VS, Qs, Thick
2.40 5050.0  350.0 0.0  3000.0 200.0 0.0   820.0 :DNS, VP, Qp, VS, Qs, Thick
2.50 5500.0  500.0 0.0  3300.0 300.0 0.0  2000.0 :DNS, VP, Qp, VS, Qs, Thick
2.70 6300.0  800.0 0.0  3700.0 500.0 0.0     0.0 :DNS, VP, Qp, Vs, Qs, Thick
* FAULT DATA :
   16500.0   16500.0 : Total Length (STR) and Width (DIP) (m)
   5    5    6  : Number of Sub-Faults (NSTR,NDIP),Gauss Points per Wavelength
     0.5     1 : Time Delay(sec), Source Type(0:Ramp; 1:Smoothed; 2:exp)
 -3020.369 -8017.266 10000.000 : Location of Hypocenter (X, Y and Z: m)
    0.0      0.0    14954.079 : Location of Fault Origin (X, Y and Z: m)
    265.0      65.0 : Strike, Dip (deg)
   2800.0 : Rupture Velocity (m/s)
      1     0.0 : Number of Time Windows and Time Interval (sec)
    1.4     1.4 : Half-Rise time 1 & 2 for 1st Time Window (sec)
* dislocations (m) on sub-faults up to 6 time windows
  5.60     }
  .....    }  NSTR x NDIP slip values
* rake angles (degree) on sub-faults for 6 time windows
-168.0     }
........   }  NSTR x NDIP rake values
* Static Wavenumber Integ. Data for Greenfield's quadrature *
      2.0 : The first corner w*k on real axis (ex. 2.0)
       16 : Initial Num. of Intg. Points for Adaptive Newton-Cotes Quadrature
     10.0 : The second corner w*k on imag. axis (ex. 10.0)
       16 : Initial Num. of Intg. Points for Adaptive Newton-Cotes Quadrature
* Dynamic Wavenumber Integ. Data for Simpson's and Filon's quadratures *
      200 : Number of Integration Points from 0 to om/Ryleigh(min)
       50 : Number of Integration Points from om/Ryl(min) to om/c(final)
     10.0 : Factor for c(final): c(final)=Ryl(min)/Factor (Use 10, usually)
* CHANGE OF SIGNS OF IMAGINARY PARTS OF FINAL RESULTS (FOR FFT) *
        1 : CHANGE SIGN (3D1), NOT CHANGE SIGN (3D0)20
* OBSERVATION POINT DATA: System Origin *
      218 : NUMBER OF OP
  13942.63 -12000.00    170.50     }
  .......................................  }  <-> file lista_cornerLGL_coor_Hisada.txt
  14096.63 -12000.00   1299.50     }
-------------------------------------------------------------------------------------------------------------------
```

Fig. 2.10 – Input file ***grfit12s.in*** for Hisada code, simulating the 1999 Düzce earthquake

2.5 Integration of the velocity time histories calculated by the Hisada code (by integration_vel_Hisada.m)

The computation of the displacement time histories at the effective corner nodes is performed with the code **integration_vel_Hisada.m**. This is a matlab program which calculates at each run the displacement time histories in one direction (**X**, **Y**, **Z**), saved in a column output file. Three runs, corresponding to the 3D spatial coordinates, are required using as input the output file from the Hisada code:

1^{st} run Input: ***x_dat*** Output: ***x_dat_disp.txt***;

2^{nd} run Input: ***y_dat*** Output: ***y_dat_disp.txt***;

3^{rd} run Input: ***z_dat*** Output: ***z_dat_disp.txt***.

Moreover, the program requires to specify the time step used in the Hisada code (dt), the number of time steps (npun_eff) and, if necessary, a scaling factor (scala), a base line correction option (lba) and a filter option (ifiltro), whose parameters have to be specified in the "*parameter definition"* section of the program.

2.6 Interpolation of the displacement time histories (with "interpolation.m")

The displacement time histories at all the LGL nodes of the effective boundary, except the SE corner nodes, are evaluated by a suitable interpolation procedure using the matlab program "**interpolation.m**". The Biharmonic Spline algorithm (Sandwell, 1987) has been adopted, providing accurate estimates of the displacement wavefield, which is slightly affected by interpolation errors and distortions especially at the edges of the computation domain. This algorithm, implemented in Matlab in the function *griddata, method v4,* is based on Green functions of the biharmonic operator for the minimum curvature interpolation of irregularly spaced data points. The interpolation surface in 2D is a linear combination of Green functions centered at each data point. Their amplitude is adjusted so that the interpolating surface passes through the points. This technique corresponds physically to forcing an elastic beam or sheet to match the data points. The interpolating surface satisfies the biharmonic equation, and therefore has minimum curvature.

The program requires as input files:

1. ***lista_cornerLGL_coor.txt***: list of the corner nodes of the effective boundary elements, where the displacement time histories are known;
2. ***lista_otherLGL_coor.txt***: list of the remaining LGL nodes of the effective boundary elements, where the displacement time histories are unknown;
3. ***x_dat_disp.txt, y_dat_disp.txt, z_dat_disp.txt*** files with the known, computed displacement components at the corner nodes of the effective boundary elements;
4. time step (dt) used in the Hisada code.

The program saves the output files:

- ***x_disp_all.txt, y_disp_all.txt, z_disp_all.txt*** with the interpolated 3D time histories at all the LGL nodes. The displacements at the corner nodes of the effective elements are followed by the interpolated displacements at the other LGL nodes, in the same order as in the *lista_cornerLGL_coor.txt* and *lista_otherLGL_coor.txt* lists respectively;
- ***time.txt*** with the time steps.

2.7 Transformation of the 3D displacement wave field in the 2D input for GeoELSE (with disp_for_Else)

The 3D displacement time history components at all the effective LGL nodes (***x_disp_all.txt, y_disp_all.txt, z_disp_all.txt***) are used to compute the 2D effective input by the code **disp_for_Else**.

The program requires as input files:

1. ***x_disp_all.txt, y_disp_all.txt, z_disp_all.txt*** with the interpolated displacement time histories in the three spatial directions X, Y, Z at all the LGL nodes;

2. ***time.txt*** with the time steps;
3. ***lista_cornerLGL_coor.txt***;
4. ***lista_otherLGL_coor.txt***;
5. the rotation angle θ of the 2D cross-section with respect to the North (clockwise), for the transformation of the 3D displacement components (X=NS, Y=EW, Z=UD), in the 2D reference system of the cross-section.

The program gives as output files:

– ***fdrm.dat*** with the 2D displacement time histories at all the LGL nodes of the effective boundary, as shown in figure 2.11. The keyword **FDRM** indicates the displacement time histories that will be used in GeoELSE second step to calculate the effective loading forces (applied as seismic input). The time histories are saved in the same order as the LGL nodes are listed in the file ***lista_all_coor.txt***.

```
                  type of function
n. of effective node  for GeoELSE
                                  n. of time steps
                                   t          ux          uy
  ---------------------------------------------------------------------------------------
  FDRM    1    50  1025  0.000000   0.000000   0.000000   0.7500028E-01   0.5927678E-05 -
0.7944061E-05  0.1500006   0.1134535E-04 -0.1609106E-04  0.2250008   0.1644928E-04 -0.2456832E-
04   0.3000011        0.1799566E-04  -0.3283838E-04   0.3750014       0.1888873E-04  -0.3997566E-04
0.4500017      0.2007082E-04 -0.4738449E-04   0.5250020   0.1823481E-04 -0.5447005E-04   0.6000023
0.1520370E-04 -0.5978754E-04   0.6750025 0.1150010E-04 -0.6530623E-04   0.7500028 0.7065618E-05 -
0.7047548E-04    0.8250031         0.5278567E-05  -0.7228690E-04    0.9000034       0.6742111E-05  -
0.7135958E-04  0.9750037  0.1531507E-04 ..........................................................
  ..........................................................................................
  76.57529   -0.2970805E-04   0.2026278E-04   76.65029   -0.1844230E-04  0.1396632E-04   76.72529
-0.8624359E-05   0.7120525E-05   76.80000   0.000000       0.000000
.........................................................................................
  FDRM   2165   50   1025    0.000000   0.000000    0.000000    0.7500028E-01   0.4622232E-05 -
0.8647306E-05  0.1500006    0.8820752E-05 -0.1560602E-04  0.2250008   0.1300696E-04 -0.2354649E-
04    0.3000011        0.1524834E-04  -0.3121556E-04    0.3750014     0.1709878E-04  -0.3688239E-04
0.4500017     0.1867863E-04 -0.4378103E-04   0.5250020    0.1828890E-04 -0.5043816E-04   0.6000023
0.1706981E-04 -0.5500718E-04  0.6750025    0.1530161E-04 -0.6023167E-04  0.7500028    0.1391945E-
04  -0.6502244E-04   0.8250031       0.1455521E-04 -0.6707234E-04   0.9000034       0.1790917E-04 -
0.6679076E-04  0.9750037   0.2860002E-04 ........................................................
  ..........................................................................................
  76.57529       -0.1853167E-04   0.2325002E-04     76.65029       -0.1206831E-04   0.1526366E-04
76.72529    -0.6157185E-05  0.8226303E-05   76.80000      0.000000      0.000000
  ---------------------------------------------------------------------------------------
```

Fig. 2.11 – File ***fdrm.dat*** with the displacement time histories at all the LGL nodes of the effective boundary, following the node order in the list ***lista_all_coor.txt*** saved by the program "**disp_for_Else.exe**". FDRM is the option for the input function in the second step of the DRM

FDRM 1 50 1025 0.000000 0.000000 0.000000 0.7500028E-01….

defines the displacement time history components associated to the node n. **1.** **50** is the type of function recognised by GeoELSE, with the time histories at **1025** time steps listed as follows: t_1, u_{x1}, u_{y1}, t_2, u_{x2}, u_{y2}, t_3, u_{x3}, u_{y3}, ….., t_{1025}, bu_{x1025}, u_{y1025}. A list with all the displacement time histories associated to the LGL nodes of the effective boundary has to be specified (2165 for the Düzce case).

– ***lista_all_coor.txt*** with the coordinates of all the LGL effective nodes, as shown in figure 2.12. In particular, the corner nodes of the effective elements (*lista_cornerLGL_coor.txt*) are followed by the other LGL effective nodes (*lista_otherLGL_coor.txt*). The keyword **PDRM** indicates the loading points in the DRM second step.

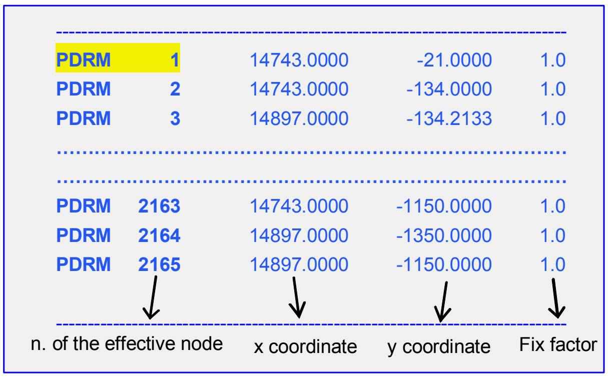

Fig. 2.12 – File ***lista_all_coor.txt*** (saved by the program "**disp_for_Else.exe**") with the list of the coordinates of all the LGL nodes of the effective boundary in the 2D reference system of the mesh used by GeoELSE. **PDRM** stands for point (LGL node) of the DRM effective boundary where an effective force is applied in the second step of the DRM

PDRM 1 14743.00 -21.000 1.0 means that the node with coordinates x=14743.0 and y=-21.0 is a node of the DRM effective boundary where an effective force is applied as input in the second step. **1** is the id associated to the node. 1.0 is a fixed coefficient. A list with all the LGL nodes of the effective boundary is saved (2165 nodes for the Düzce case).

2.8 DRM step II: SE analysis with GeoELSE

In the second step of the DRM the free field displacements evaluated before (***fdrm.dat***) are read as input functions and used to calculate the effective nodal forces (seismic input equivalent to the original seismic source). The required input files are:

1. ***nomefile.inp*** (***duzce_SN_drm_II_Else.inp***) with the grid information;
2. ***nomefile.mat*** (***duzce_drm.mat***) with options MDRM, BDRM, SDRM PDRM and FDRM, as shown in figure 2.13. The PDRM and FDRM lists of the files lista_all_coor.txt and fdrm.dat, previously evaluated, are added at the end of the file.

```
---------------------------------------------------------------------------------------------------------
 mat     n°  type   rho          lambda          mu            gamma
MATE     1    2    1800       7.6050E+08     1.9013E+08     5.2360E-02
MATE     2    2    2000       9.4500E+08     4.0500E+08     3.1416E-02
MATE     3    2    2200       3.7161E+09     4.0244E+09     1.5708E-02
MATE     4    2    2250       1.0157E+10     1.0742E+10     1.0472E-02
MATE     5    2    2300       1.1397E+10     1.2702E+10     7.8540E-03
MATE     6    2    2300       1.5134E+10     1.6767E+10     7.8540E-03
MATE     7    2    2200       3.7161E+09     4.0244E+09     1.5708E-02
MATE     8    2    2250       1.0157E+10     1.0742E+10     1.0472E-02
MATE     9    2    2300       1.1397E+10     1.2702E+10     7.8540E-03
MATE    10    2    2300       1.5134E+10     1.6767E+10     7.8540E-03
MATE    11    2    2200       3.7161E+09     4.0244E+09     1.5708E-02
MATE    12    2    2250       1.0157E+10     1.0742E+10     1.0472E-02
MATE    13    2    2300       1.1397E+10     1.2702E+10     7.8540E-03
MATE    14    2    2300       1.5134E+10     1.6767E+10     7.8540E-03

ABSO 15

MDRM   7..
MDRM   8..
MDRM   9..
MDRM  10..

BDRM 16

SDRM  2..

PDRM       1        14743.0000        -21.0000      1.0
.............................................................
PDRM    2165        14897.0000      -1150.0000      1.0
```
↔ file ***lista_all_coor.txt***

```
FDRM       1  50  1025    0.000000  0.000000  0.000000  0.7500028E-01  0.5927678E-05..
.........................................................................................
FDRM    2165  50  1025  0.000000  0.000000  0.000000  0.7500028E-01  0.4622232E-05..
------------------------------------------------------------------------------------------
```
↔ file ***fdrm.dat***

Fig. 2.13 – Example of ***nomefile.mat (duzce_drm.mat)*** for DRM step II with GeoELSE. The files *lista_all_coor.txt* and *fdrm.dat* are copied at the end of the file of figure 2.4 and the option SDRM is changed in "2" (DRM step 2)

SDRM 2 means that the second step of the DRM is performed.

3. ***else2_input.d*** with the information about the input file to be read, the spectral degree, the output files, the time step and the list of the required receivers, as shown in figure 2.14.

```
-----------------------------------------------------------------------------------------------------
DEGREE 4

GRIDFILE   duzce_SN_DRM_II_Else
MATFILE    duzce_DRM
OUTFILE    duzce_DRM_out

OPTIOUT    1 1 1 0 1 0 0 2

TIMESTEP     0.1e-4
STOPTIME     40
TMONITOR     1000

MONITOR  3180.000000   -21.320000     }
MONITOR  3180.000000   -25.0          }  list of requested
MONITOR  3180.000000   -30.0          }  receivers
.........................................................
-----------------------------------------------------------------------------------------------------
```

Fig. 2.14 – Example of ***else2_input.d*** file for DRM step II with GeoELSE

OPTIOUT x x x x x x x x specifies what has to be saved. The former 6 values can be **1** or **0** depending on the fact that the related field has to be saved or not. The 6 fields relate to the ground motion in terms of: 1) displacement, 2) velocity, 3) acceleration, 4) stress, 5) strain and 6) rotation respectively. If a field is **1**, for each **MONITOR** of the required receiver list, an output file is saved with the appropriate, relative extension:

1. DISPLACEMENT	***monitorxxxx.d***	**| Δt | u_x | u_y |;**
2. VELOCITY	***monitorxxxx.v***	**| Δt | v_x | v_y |;**
3. ACCELERATION	***monitorxxxx.a***	**| Δt | a_x | a_y |;**
4. STRESS	***monitorxxxx.s***	**| Δt | σ_{xx} | σ_{yy} | σ_{xy} | σ_{zz} |;**
5. STRAIN	***monitorxxxx.e***	**| Δt | ε_{xx} | ε_{yy} | ε_{xy} |;**
6. ROTATION	***monitorxxxx.w***	**| Δt | ω_{xy} |.**

xxxx is the id number that identifies the receiver in the list. If a field is **0** the relative output files are not saved. In the case of Düzce in figure 2.14 stresses and rotations are not required; therefore the files ***monitorxxxx.s*** and ***monitorxxxx.w*** are not created by the code.

The seventh number is for animation options:

0 no animation files required;

1 file outputs for GID program are saved: ***sperem.res, sperem.dat***, with the information for all the nodes of the grid at all the time steps;

2 file outputs for MTV are saved:

- ***file_out_xxx_D.mtv*** ($\sqrt{u_x^2 + u_y^2}$)
- ***file_out_xxx_X.mtv*** (u_x)
- ***file_out_xxx_Y.mtv*** (u_y)
- ***file_out _xxx_SXX.mtv*** (σ_{xx})
- ***file_out _xxx_SYY.mtv*** (σ_{yy})

- ***file_out _xxx_SXY.mtv*** (σ_{xy})
- ***file_out _xxx_SZZ.mtv*** (σ_{zz})
- ***file_out _xxx_SPR.mtv*** $(1/3(\sigma_{xx}+\sigma_{yy}+\sigma_{zz}))$
- ***file_out_xxx_SVM.mtv***
  $(\sigma_{xx}^2+\sigma_{yy}^2+\sigma_{zz}^2-\sigma_{zz}\sigma_{zz}-\sigma_{xx}\sigma_{zz}-\sigma_{xx}\sigma_{yy}+3\sigma_{xy}\sigma_{yx})$

If **1** or **2** is chosen ***.bin*** files are also written, one for each time step, with the information for all the nodes of the grid:

- ***file_outE_xx_xxx.bin*** (displacement)
- ***file_outV_xx_xxx.bin*** (velocity)
- ***file_outA_xx_xxx.bin*** (acceleration)

The last number specifies which results are to be saved for animations: **1** displacements, velocities and accelerations; **2** displacements, velocities and accelerations, stress and strains. In the case of Düzce in figure 2.14 the animation options are not used (**0 2**).

3 The case of the Acquasanta Bridge, Italy, solved using GeoELSE for both DRM steps

The SE GEoELSE code may be used for both DRM steps. In this way, the free field seismic response at all the LGL nodes of the effective boundary elements is calculated in the DRM first step and used as input in the DRM second step. An interfacing program is required to post-process the output data from the first step in a readable format for the second one.

The procedure is illustrated referring to the 2D case of the Acquasanta Bridge, situated in the NW Italy (Scandella, 2007, Stupazzini et al., 2006). This is a soil-structure interaction study on the Genoa-Ovada railway.

The procedure requires 4 steps, as shown in figure 3.1:

1. Generation of the SE meshes for the auxiliary model (step I) and for the reduced one (step II);
2. DRM step I: SE analysis with GeoELSE;
3. Post process of the displacement time histories evaluated in the first step;
4. DRM step II: SE analysis with GeoELSE.

The procedure is the same for the 3D case. To this purpose the available sequential 3D code has some limits for what concerns the processing of the seismic response at the nodes. To overcome these limitations the introduction of the DRM in the parallel kernel is under study.

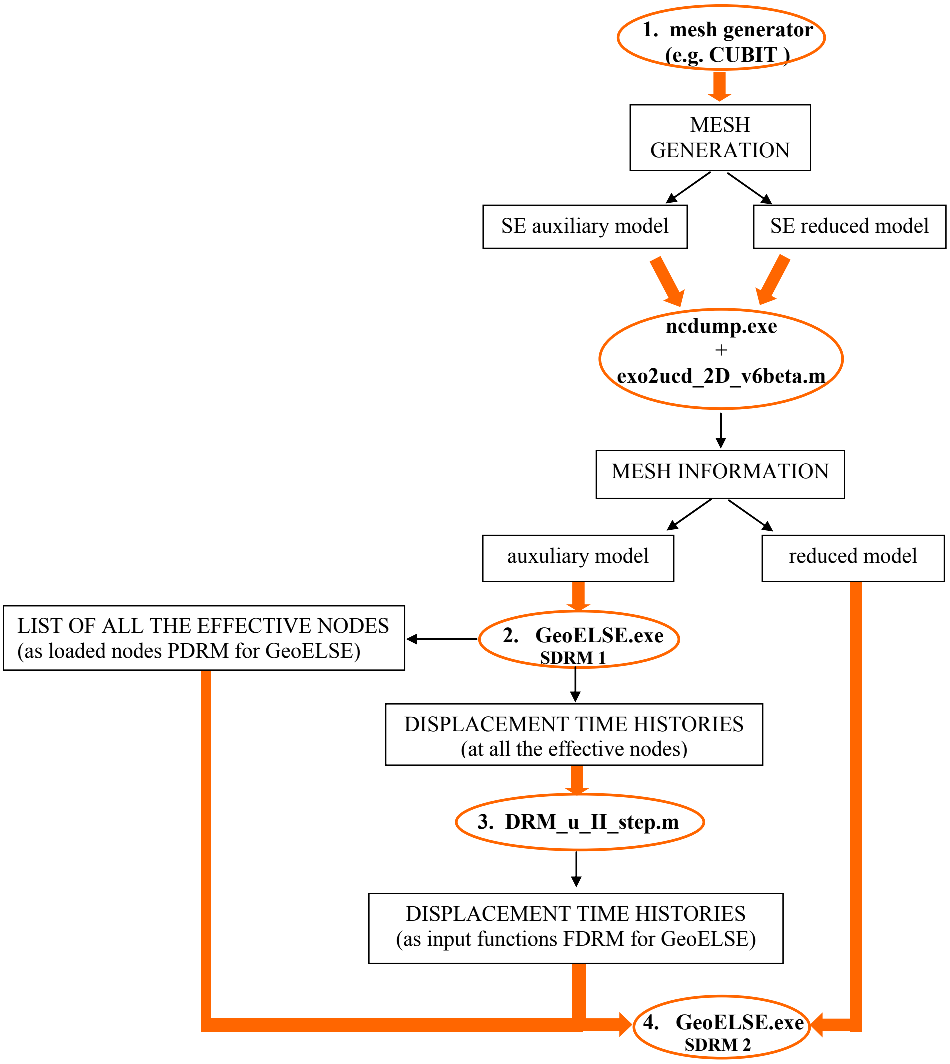

Fig. 3.1 – Steps for the application of the DRM using GeoELSE for both steps

3.1 Mesh generation (with CUBIT)

In the DRM step I the seismic source (fault) is included, the structure of interest is removed, and the soil properties of the internal domain of interest, zoomed in figure 3.2, are replaced by the ones of the surrounding layered half space. As a consequence, the numerical grid of the internal domain has fewer elements than it would have been in a classical analysis, not using the DRM. The superficial alluvium deposit (shown in light yellow in figure 3.3) is in fact not modelled here, and the filling material at the abutments of the bridge is removed with the bridge itself.

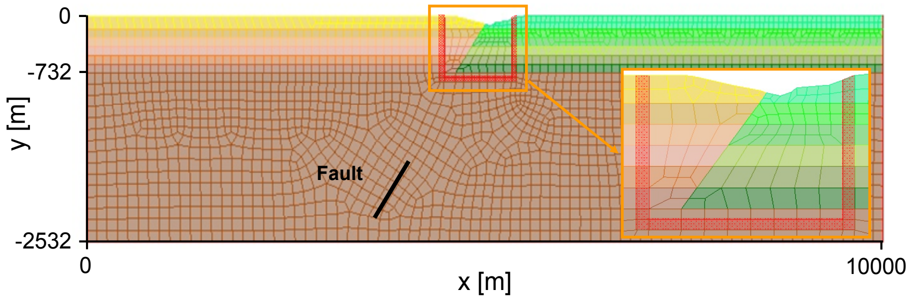

Fig. 3.2 – 2D SE model of the auxiliary problem for the DRM step I analysis of the Acquasanta bridge (Genoa, Italy) including the seismic source and the topography. The dark red strip represents the effective boundary where free field displacements are calculated

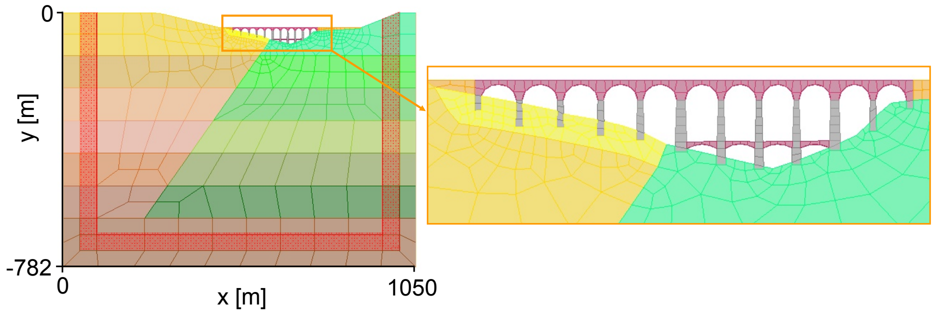

Fig. 3.3 – 2D SE model of the reduced problem for the DRM step II analysis of the Acquasanta bridge (Genoa, Italy) including the viaduct. The dark red strip represents the effective boundary where the equivalent nodal forces are applied as input motion

Note that, to achieve accurate results, the effective boundary has to include *all* the layers of the auxiliary model, including the bedrock, so that the only source of error may be the efficiency of the absorbing boundary. In some cases, due to the negligible influence of the deeper layers, only the uppermost layers may be included in the internal domain, as in the Düzce case, reasonably neglecting some contributions of the scattered field in the second step.

3.2 DRM step I: SE analysis with GeoELSE

As previously mentioned, GeoELSE requires three input files:

1. ***nomefile.inp*** (***acquasanta2D_1st.inp***);
2. ***nomefile.mat*** (***acquasanta2D_1st.mat***) with the MDRM, BDRM and SDRM options, as shown in figure 3.4;
3. ***else2_input.d*** with the information about the input file to be read, the spectral degree, the output files (see section 2.8), the time step and the list of the required receivers, as shown in figure 3.5;

```
-------------------------------------------------------------------------
MATERIALE DENTRO AL DRM
 BLOCK 7 - calcescito sano (superficie)  Q = 150 (fpeak = 1Hz)
MATE   1   2    2500  9.0000E+08       3.6000E+09       2.0944E-02
 BLOCK 8 - calcescito 2 -  Q = 150 (fpeak = 1Hz)
MATE   2   2    2550  1.4331E+09       3.8058E+09       2.0944E-02
 BLOCK 9 - calcescito 3 -  Q = 150 (fpeak = 1Hz)
MATE   3  2    2600   2.0177E+09       4.0193E+09       2.0944E-02
 BLOCK 10 - calcescito 4 -  Q = 150 (fpeak = 1Hz)
MATE  4    2    2650   2.6554E+09       4.2406E+09       2.0944E-02
 BLOCK 11 - calcescito 5 -  Q = 150 (fpeak = 1Hz)
MATE   5   2    2700  3.3482E+09       4.4699E+09       2.0944E-02
 BLOCK 12 - calcescito 6 -  Q = 150 (fpeak = 1Hz)
MATE   6   2    2750  4.0979E+09       4.7073E+09       2.0944E-02
-------------------------------------------------------------------------
CONTORNO DEL DRM
 BLOCK 13 - calcescito sano (superficie)  Q = 150 (fpeak = 1Hz)
MATE   7    2    2500 9.0000E+08       3.6000E+09       2.0944E-02
 BLOCK 14 - calcescito 2 -  Q = 150 (fpeak = 1Hz)
MATE   8    2    2550 1.4331E+09       3.8058E+09       2.0944E-02
 BLOCK 15 - calcescito 3 -  Q = 150 (fpeak = 1Hz)
MATE   9    2    2600 2.0177E+09       4.0193E+09       2.0944E-02
 BLOCK 16 - calcescito 4 -  Q = 150 (fpeak = 1Hz)
MATE   10    2   2650 2.6554E+09       4.2406E+09       2.0944E-02
 BLOCK 17 - calcescito 5 -  Q = 150 (fpeak = 1Hz)
MATE   11    2    2700 3.3482E+09       4.4699E+09       2.0944E-02
 BLOCK 18 - calcescito 6 -  Q = 150 (fpeak = 1Hz)
MATE   12   2   2750  4.0979E+09       4.7073E+09       2.0944E-02
-------------------------------------------------------------------------
MATERIALE DENTRO AL DRM
 BLOCK 19 - Serpentini (Giurassico) (superficie)  Q = 125 (fpeak = 1Hz)
MATE   13   2    2400           9.6852E+08       9.6774E+08       2.5133E-02
 BLOCK 20 - Serpentini 2
MATE   14   2    2466.666667 1.3875E+09       1.3906E+09       2.4322E-02
 BLOCK 21 - Serpentini 3
MATE   15  2    2533.333333  1.8944E+09       1.9028E+09       2.3562E-02
 BLOCK 22 - Serpentini 4
MATE   16   2    2600           2.4944E+09       2.5098E+09       2.2848E-02
 BLOCK 23 - Serpentini 5
MATE   17   2    2666.666667 3.1929E+09       3.2169E+09       2.2176E-02
 BLOCK 24 - Serpentini 6
MATE   18  2    2733.333333  3.9950E+09       4.0295E+09       2.1542E-02
```

```
--------------------------------------------------------------------------------------------------------------
MATERIALE DENTRO AL DRM
 BLOCK 25 - Serpentini (Giurassico) (superficie)  Q = 125 (fpeak = 1Hz)
MATE   19   2    2400          9.6852E+08      9.6774E+08      2.5133E-02
 BLOCK 26 - Serpentini 2
MATE   20   2    2466.666667 1.3875E+09      1.3906E+09      2.4322E-02
 BLOCK 27 - Serpentini 3
MATE   21   2    2533.333333 1.8944E+09      1.9028E+09      2.3562E-02
 BLOCK 28 - Serpentini 4
MATE   22   2    2600          2.4944E+09      2.5098E+09      2.2848E-02
 BLOCK 29 - Serpentini 5
MATE   23   2    2666.666667 3.1929E+09      3.2169E+09      2.2176E-02
 BLOCK 30 - Serpentini 6
MATE   24   2    2733.333333 3.9950E+09      4.0295E+09      2.1542E-02
 ----------------------------------------------------------------------
  BLOCK 31 - Serpentini = Calcescisti DENTRO  DRM
MATE   25   2    2800 4.9062E+09      4.9529E+09      2.0944E-02
 ----------------------------------------------------------------------
  BLOCK 31 - Serpentini = Calcescisti BORDO DRM
MATE   26   2    2800 4.9062E+09      4.9529E+09      2.0944E-02
 ----------------------------------------------------------------------
 CALCSCISTI FUORI DAL DRM
 BLOCK 33 - calcescito sano (superficie)  Q = 150 (fpeak = 1Hz)
MATE   27   2    2500          9.0000E+08      3.6000E+09      2.0944E-02
 BLOCK 34 - calcescito 2 -  Q = 150 (fpeak = 1Hz)
MATE   28   2    2550          1.4331E+09      3.8058E+09      2.0944E-02
 BLOCK 35 - calcescito 3 -  Q = 150 (fpeak = 1Hz)
MATE   29   2    2600 2.0177E+09      4.0193E+09      2.0944E-02
 BLOCK 36 - calcescito 4 -  Q = 150 (fpeak = 1Hz)
MATE   30   2    2650 2.6554E+09      4.2406E+09      2.0944E-02
 BLOCK 37 - calcescito 5 -  Q = 150 (fpeak = 1Hz)
MATE   31   2    2700          3.3482E+09      4.4699E+09      2.0944E-02
 BLOCK 38 - calcescito 6 -  Q = 150 (fpeak = 1Hz)
MATE   32   2    2750  4.0979E+09      4.7073E+09      2.0944E-02
SERPENTINI FUORI DAL DRM
 BLOCK 39 - Serpentini (Giurassico) (superficie)  Q = 125 (fpeak = 1Hz)
MATE   33   2    2400          9.6852E+08      9.6774E+08      2.5133E-02
 BLOCK 40 - Serpentini 2
MATE   34   2    2466.666667 1.3875E+09      1.3906E+09      2.4322E-02
 BLOCK 41 - Serpentini 3
MATE   35   2    2533.333333  1.8944E+09      1.9028E+09      2.3562E-02
 BLOCK 42 - Serpentini 4
MATE   36   2    2600          2.4944E+09      2.5098E+09      2.2848E-02
 BLOCK 43 - Serpentini 5
MATE   37   2    2666.666667 3.1929E+09      3.2169E+09      2.2176E-02
 BLOCK 44 - Serpentini 6
MATE   38   2    2733.333333  3.9950E+09      4.0295E+09      2.1542E-02
--------------------------------------------------------------------------------------------------------------
```

```
----------------------------------------------------------------------------------------------------
  BLOCK 45 - Serpentini = Calcescisti FUORI DAL  DRM
MATE   39   2    2800 4.9062E+09       4.9529E+09       2.0944E-02
-----------------------------------------------------------------------------------------------------
ABSO 41

SDRM  1

MDRM    7
MDRM    8
MDRM    9
MDRM   10
MDRM   11
MDRM   12
MDRM   19
MDRM   20
MDRM   21
MDRM   22
MDRM   23
MDRM   24
MDRM   26

BDRM   40

SISM  9    1    -1356.16  -2193.32  -1356.16  -2193.32  -792.57  -1367.26  0.5635  0.8260
-0.8260  0.5635 1.0e+12  1064
     fsism  tagsism    xc        yc          x1          y1          x2           y2         s1          s2
    n1        n2      Amp     Vel_rup
  NODO 1757 -> centro    -       NODO 1753 -> Punto 1     -    NODO 1755 -> Punto 2

FUNC  9  31 0.3 .50 1

FMAX 10
```

Fig. 3.4 – Example of ***nomefile.mat (acquasanta2D_1st.mat)*** to run the DRM first step with GeoELSE

SDRM 1 means that the first step of the DRM is performed.

MDRM 7… means that the material n° **7**, whose properties are listed before (green rectangle), is a block of elements of the effective boundary.

BDRM 40 means that the block n° **40**, is the block of lines which delimits the internal boundary of the effective strip.

In this case the seismic source is modelled as a kinematic source with a seismic moment tensor density (**SISM 9**) specified by function n. 31 (**FUNC 9 31**).

```
-----------------------------------------------------------------------------------------------------
DEGREE 4
GRIDFILE  acquasanta2D_1st
MATFILE   acquasanta2D_1st
OUTFILE   acquasanta2D_1st_out

TIMESTEP 5.5e-004
STOPTIME 10.1
TMONITOR 18

OPTIOUT    1 0 0 0 0 0    1 2

MONITOR     0.000000    0.000000
MONITOR    28.911494    0.000000      list of requested
MONITOR    52.690983    0.000004      receivers
......................................................
-----------------------------------------------------------------------------------------------------
```

Fig. 3.5 – Example of ***else2_input.d*** file to run the DRM step I with GeoELSE

During the step I the program saves:

- ***monitorDRMxxxx.d*** files, one for each LGL node of the DRM effective boundary, with 3 columns: **| Δt | u_x | u_y |;**
- ***nomefile_out (acquasanta2D_1st_out)*** with the connectivity of the DRM effective boundary elements and the list of coordinates of the LGL nodes in the effective boundary, as shown in figure 3.6.

  Note that, in this case, the corner nodes of the effective boundary elements are not listed separately from the other LGL effective nodes, but in a single list.

```
------------------------------------------------------------------------------------------------------------------------
# Number of DRM domain elements =    27
# DRM domain elements connectivity
     40   55  3127  3128  3129    12  3130  3136  3137  3138  2586  3131  3139  3140  3141  2585
3132  3142  3143  3144  2584    54  3133  3134  3135    11
     41   56  3145  3146  3147    13  3148  3151  3152  3153  2598  3149  3154  3155  3156  2597
3150  3157  3158  3159  2596    55  3127  3128  3129    12
     42   57  3160  3161  3162    32  3163  3166  3167  3168  2809  3164  3169  3170  3171  2810
3165  3172  3173  3174  2811    56  3145  3146  3147    13
     43   58  3175  3176  3177    38  3178  3181  3182  3183  2904  3179  3184  3185  3186  2903
3180  3187  3188
.......................................................................................................................
    134   158  4570  4571  4572   159  4558  4576  4577  4578  4573  4559  4579  4580  4581  4574
4560  4582  4583  4584  4575   147  4417  4418  4419   148
    135   159  4585  4586  4587   160  4573  4594  4595  4596  4591  4574  4597  4598  4599  4592
4575  4600  4601  4602  4593   148  4588  4589  4590   161
    136   148  4588  4589  4590   161  4420  4606  4607  4608  4603  4421  4609  4610  4611  4604
4422  4612  4613  4614  4605   125  4294  4295  4296   139
# Number of DRM boundary lines =    25
# Number of DRM element nodes =   444
# Number of DRM internal boundary nodes =   101
# Number of DRM total nodes =    545
# DRM domain nodes
PDRM      1     -402.9841      45.1144      1.0000
PDRM      2     -402.9841     -20.8477      1.0000
PDRM      3     -402.9841     -86.8098      1.0000      list of the coordinates
.......................................................      of the LGL nodes in
PDRM    543      472.5530    -595.4434      1.0000      the effective boundary
PDRM    544      488.9193    -595.4434      1.0000
PDRM    545      497.5530    -595.4434      1.0000
------------------------------------------------------------------------------------------------------------------------

n. of the LGL node   x coordinate   y coordinate   Fix factor
```

Fig. 3.6 – Example of ***file_out (acquasanta2D_1st_out)*** file, output of the DRM step I by GeoELSE, with the list of the LGL nodes of the effective boundary (**PDRM**) required as loading nodes in second step

3.3 Post process of the displacement time histories evaluated in the first step

The files ***monitorDRMxxxx.d,*** with the displacement time histories at the effective LGL nodes, are converted in the single file ***fdrm.dat*** (see figure 2.11) with the matlab program **DRM_u_II_step.m**. The program reads the files ***monitorDRMxxxx.d*** and requires as parameter the number of the effective LGL nodes.

3.4 DRM step II: SE analysis by GeoELSE

The input and output files are the same described in section 2.8. The only difference is that in the file ***nomefile.mat (acquasanta2D_2st.mat)*** the list of all the effective node coordinates (indicated by the keyword PDRM), loaded points in the DRM second step, is taken from the file ***acquasanta2D_1st_out*** (figure 3.6). The files are not reported for brevity.

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