1 — Translations

In this chapter, we develop a interpretation of $\lambda\mu$ in CDC. We prove some properties of this interpretation, including *soundness*. We concatenate this interpretation with van Bakel's interpretation of λ^{try} in $\lambda\mu$. This concatenation yields an interpretation of λ^{try} in CDC. This will then be used as a basis for the implementation of λ^{try} in Haskell.

1.1 Interpreting λ^{try} in $\lambda\mu$

Steffen van Bakel describes the interpretation of λ^{try} to $\lambda\mu$:

$$
\begin{array}{rcl}
\llbracket x \rrbracket & \triangleq & x \\
\llbracket \lambda x.M \rrbracket & \triangleq & \lambda x.\llbracket M \rrbracket \\
\llbracket MN \rrbracket & \triangleq & \llbracket M \rrbracket \llbracket N \rrbracket \\
\llbracket \text{try } M;\ \overline{\text{catch n}_i(x) = M_i;} & \text{catch m}(x) = L \rrbracket & \\
 & \triangleq & \\
 & (\lambda c_m.\mu\text{m}.[\text{m}]\llbracket \text{try } M;\ \overline{\text{catch n}_i(x) = M_i} \rrbracket)(\lambda x.\llbracket L \rrbracket) & \\
\llbracket \text{try } M;\ \text{catch m}(x) = L \rrbracket & \triangleq & (\lambda c_m.\mu\text{m}.[\text{m}]\llbracket M \rrbracket)(\lambda x.\llbracket L \rrbracket) \\
\llbracket \text{throw n}(M) \rrbracket & \triangleq & \lambda \circ .[\text{n}]c_n \llbracket M \rrbracket
\end{array}
$$

throw n(M) terms are modelled using $\lambda\mu$-abstractions of non-occurring names. This has the effect of removing all terms it is applied to:

$$(\mu \circ .M)NOP \rightarrow (\mu \circ .M)OP \rightarrow (\mu \circ .M)P \rightarrow \mu \circ .M$$

The contents of the $\lambda\mu$-abstraction calls c_n. This λ-variable is bound by the translation of **try** terms. This binding means that the exception handlers, represented by $\lambda x.\llbracket L \rrbracket$, are in scope for the reduction of the body of the try M.

1.2 Interpreting $\lambda\mu$ in CDC

The translation of $\lambda\mu$-terms into CDC assumes that there is a single global prompt P_0 . It also assumes that this prompt has already been pushed onto the stack. This means that the translation of a full $\lambda\mu$-program M in CDC is:

Definition 1.2.1 (INITIALIZATION OF STACK FOR RUNNING M IN CDC)

$$(\lambda \text{P}_0 . \text{PP} \quad \text{P}_0 \quad [\![M]\!]) \text{ NP}$$

This creates a new prompt P_0 which is in scope for all subterms of M. It also prepares the stack by pushing P_0 immediately. With the abstract machine prepared, the interpretation of $\lambda\mu$ terms into CDC proceeds as follows:

Definition 1.2.2 (INTERPRETATION OF $\lambda\mu$ INTO CDC)

$$\begin{array}{lcl}
[\![x]\!] & \triangleq & x \\
[\![\lambda x.M]\!] & \triangleq & \lambda x.[\![M]\!] \\
[\![MN]\!] & \triangleq & [\![M]\!][\![N]\!] \\
[\![\mu\alpha.M]\!] & \triangleq & \text{WSC} \quad \text{P}_0 \quad \lambda\alpha.\text{PP} \quad \text{P}_0 \quad [\![M]\!] \\
[\![[\beta]M]\!] & \triangleq & \text{PSC} \quad \beta \ [\![M]\!]
\end{array}$$

To implement $\lambda\mu$-abstractions, we capture the subcontinuation until the last occurrence of P_0 on the stack. This subcontinuation is bound to α which ensures the subcontinuation is distributed to all occurrences of α in M. P_0 is then pushed back onto the stack before the evaluation of M.

To implement named-terms, the subcontinuation β is pushed into the stack before evaluating M. This means the reduct of M will be returned to this subcontinuation. In effect, this reduces M and passes the result to β.

1.2.1 Notation

To carry out proofs, the full state of the abstract machine is displayed:

$$M \quad D \quad E$$

where each column corresponds to one component of the original abstract machine. Our translations only use a single prompt so we omit the final column of the abstract machine (used for representing the global prompt counter).

When an empty context is in a sequence, it has no effect on the machine: a sequence $D : \Box : D'$ is extensionally equivalent to $D : D'$. For this reason, we omit empty contexts from sequences. For example in the case of the following reduction

$$\text{PSC} \ \beta \ M \quad \Box \quad \text{P}_0 \quad \rightarrow_{CDC} \quad M \quad \Box \quad \beta : \Box : \text{P}_0$$

we will instead write

$$\text{PSC} \ \beta \ M \quad \Box \quad \text{P}_0 \quad \rightarrow_{CDC} \quad M \quad \Box \quad \beta : \text{P}_0$$

1.2.2 Additional Translations

We alter μ-reduction to consume multiple variables. The application of a μ abstraction to multiple variables will consume them all at once:

Definition 1.2.3 μ-REDUCTION TO CONSUME MULTIPLE VARIABLES

$$(\mu\alpha.[\beta]M)\overline{N} \quad \rightarrow_\mu \quad \mu\alpha.[\beta](M[[\alpha]M'\overline{N}/[\alpha]M'])$$

This does not change the behaviour of μ-reduction but condenses the reduction steps. The entire applicative context is consumed. Therefore the remaining μ abstraction will point α to $\Box$. This means that all labelled subterms $[\alpha]M'$ will be translated to PSC $\Box$ $[\![M']\!]$. CDC reduces PSC $\Box$ $[\![M']\!]$ to $[\![M']\!]$. This reduction means we can discard the α labels after consuming the entire context. Given this, we can define the following translation for multiple-variable consumption:

Definition 1.2.4 TRANSLATION OF MULTIPLE VARIABLE CONSUMPTION TO CDC

$$[\![M[[\alpha]M'\overline{N}/[\alpha]M']]\!] \quad \triangleq \quad [\![M]\!][\Box[\![\overline{N}]\!]/\alpha]$$

With this translation, the proof that $[\![\cdot]\!]$ respects $\rightarrow_\mu$ is easy.

1.2.3 Properties

Theorem 1.2.5 (SOUNDNESS OF $[\![\cdot]\!]$)

$$M \rightarrow_\mu N \Rightarrow \exists P.\ [\![M]\!] \rightarrow^{nf} P \wedge [\![N]\!] \rightarrow^{nf} P$$

Proof. By induction on the definition of $\rightarrow_\mu$

$$\begin{array}{llll}
\multicolumn{3}{l}{\mu\alpha.[\alpha]M \rightarrow M :} & (\alpha \notin fv(M)) \\
 & [\![\mu\alpha.[\alpha]M]\!] & & \\
\triangleq & \text{WSC P}_0\ \lambda\alpha.\text{PP P}_0\ (\text{PSC}\ \alpha[\![M]\!]) & D & \text{P}_0 \\
\rightarrow_{CDC} & (\lambda\alpha.\text{PP P}_0\ (\text{PSC}\ \alpha[\![M]\!]))(D) & \Box & [] \\
\rightarrow_{CDC} & (\text{PP P}_0\ (\text{PSC}\ \alpha\ [\![M]\!]))[D/\alpha] & \Box & [] \\
\rightarrow_{CDC} & \text{PP P}_0\ (\text{PSC}\ \alpha\ [\![M]\!])[D/\alpha] & \Box & [] \\
\rightarrow_{CDC} & (\text{PSC}\ \alpha\ [\![M]\!])[D/\alpha] & \Box & \text{P}_0 \\
\rightarrow_{CDC} & \text{PSC}\ D\ [\![M]\!][D/\alpha] & \Box & \text{P}_0 \\
\rightarrow_{CDC} & [\![M]\!][\Box/\alpha] & \Box & D:\Box:\text{P}_0 \\
\rightarrow_{CDC} & [\![M]\!] & \Box & D:\Box:\text{P}_0 \\
\rightarrow_{CDC} & [\![M]\!] & D & \text{P}_0 \\
\triangleq & M & D & \text{P}_0
\end{array}$$

$(\mu\alpha.[\alpha]M)\overline{N} \to \mu\alpha.[\alpha]M([\alpha]M'\overline{N}/[\alpha]M')\overline{N}$:

$$
\begin{array}{llll}
 & \llbracket(\mu\alpha.[\alpha]M)\overline{N}\rrbracket & & \\
\triangleq & \llbracket(\mu\alpha.[\alpha]M)\rrbracket\llbracket\overline{N}\rrbracket & & \\
\triangleq & (\textsc{wsc}\ \mathrm{P}_0\ \lambda\alpha.\textsc{pp}\ \mathrm{P}_0\ (\textsc{psc}\ \alpha\ \llbracket M\rrbracket))\llbracket\overline{N}\rrbracket & \square & \mathrm{P}_0 \\
\to_{CDC} & \textsc{wsc}\ \mathrm{P}_0\ \lambda\alpha.\textsc{pp}\ \mathrm{P}_0\ (\textsc{psc}\ \alpha\ \llbracket M\rrbracket) & \square\llbracket\overline{N}\rrbracket & \mathrm{P}_0 \\
\to_{CDC} & \lambda\alpha.\textsc{pp}\ \mathrm{P}_0\ (\textsc{psc}\ \alpha\ \llbracket M\rrbracket)(\square\llbracket\overline{N}\rrbracket) & \square & [] \\
\to_{CDC} & (\textsc{pp}\ \mathrm{P}_0\ (\textsc{psc}\ \alpha\ \llbracket M\rrbracket))[\square\llbracket\overline{N}\rrbracket/\alpha] & \square & [] \\
\to_{CDC} & \textsc{pp}\ \mathrm{P}_0\ (\textsc{psc}\ \alpha\ \llbracket M\rrbracket)[\square\llbracket\overline{N}\rrbracket/\alpha] & \square & [] \\
\to_{CDC} & \textsc{pp}\ \mathrm{P}_0\ (\textsc{psc}\ \square\llbracket\overline{N}\rrbracket\ \llbracket M\rrbracket[\square\llbracket\overline{N}\rrbracket/\alpha]) & \square & [] \\
\to_{CDC} & \textsc{psc}\ \square\llbracket\overline{N}\rrbracket\ \llbracket M\rrbracket[\square\llbracket\overline{N}\rrbracket/\alpha] & \square & \mathrm{P}_0 \\
\to_{CDC} & \llbracket M\rrbracket[\square\llbracket\overline{N}\rrbracket/\alpha] & \square & \square\llbracket\overline{N}\rrbracket : \square : \mathrm{P}_0 \\
\to_{CDC} & \llbracket M\rrbracket[\square\llbracket\overline{N}\rrbracket/\alpha] & \square\llbracket\overline{N}\rrbracket & \mathrm{P}_0
\end{array}
$$

This final state is P. Now we must prove $\llbracket N\rrbracket \to^{nf} P$:

$$
\begin{array}{llll}
 & \llbracket\mu\alpha.[\alpha]M([\alpha]M'\overline{N}/[\alpha]M')\overline{N}\rrbracket & & \\
\triangleq & \textsc{wsc}\ \mathrm{P}_0\ \lambda\alpha.\textsc{pp}\ \mathrm{P}_0\ (\textsc{psc}\ \alpha\ \llbracket M[[\alpha]M'\overline{N}/[\alpha]M']\overline{N}\rrbracket) & \square & \mathrm{P}_0 \\
\to_{CDC} & \lambda\alpha.\textsc{pp}\ \mathrm{P}_0\ (\textsc{psc}\ \alpha\ \llbracket M[[\alpha]M'\overline{N}/[\alpha]M']\overline{N}\rrbracket)(\square) & \square & [] \\
\to_{CDC} & (\textsc{pp}\ \mathrm{P}_0\ (\textsc{psc}\ \alpha\ \llbracket M[[\alpha]M'\overline{N}/[\alpha]M']\overline{N}\rrbracket))[\square/\alpha] & \square & [] \\
\to_{CDC} & \textsc{pp}\ \mathrm{P}_0\ (\textsc{psc}\ \alpha\ \llbracket M[[\alpha]M'\overline{N}/[\alpha]M']\overline{N}\rrbracket)[\square/\alpha] & \square & [] \\
\to_{CDC} & (\textsc{psc}\ \alpha\ \llbracket M[[\alpha]M'\overline{N}/[\alpha]M']\overline{N}\rrbracket)[\square/\alpha] & \square & \mathrm{P}_0 \\
\to_{CDC} & \textsc{psc}\ \square\ \llbracket M[[\alpha]M'\overline{N}/[\alpha]M']\overline{N}\rrbracket[\square/\alpha] & \square & \mathrm{P}_0 \\
\to_{CDC} & \llbracket M[[\alpha]M'\overline{N}/[\alpha]M']\overline{N}\rrbracket[\square/\alpha] & \square & \mathrm{P}_0 \\
\to_{CDC} & \llbracket M[[\alpha]M'\overline{N}/[\alpha]M']\rrbracket[\square/\alpha]\ \llbracket\overline{N}\rrbracket & \square & \mathrm{P}_0 \\
\to_{CDC} & \llbracket M[[\alpha]M'\overline{N}/[\alpha]M']\rrbracket[\square/\alpha] & \square\llbracket\overline{N}\rrbracket & \mathrm{P}_0 \\
\to_{CDC} & \llbracket M\rrbracket[\square\overline{N}/\alpha][\square/\alpha] & \square\llbracket\overline{N}\rrbracket & \mathrm{P}_0 \\
\to_{CDC} & \llbracket M\rrbracket[\square\overline{N}/\alpha] & \square\llbracket\overline{N}\rrbracket & \mathrm{P}_0
\end{array}
$$

Remark 1.2.6 α cannot occur in $\overline{N}$ because it has been pulled in from outside the μ-abstraction. Neither can it occur in $\llbracket M\rrbracket[\square\overline{N}/\alpha]$ because all occurrences have been substituted for $\square\llbracket\overline{N}\rrbracket$.

$(\mu\alpha.[\beta]M)\overline{N} \to \mu\alpha.[\beta](M[[\alpha]M'N/[\alpha]M'])$:

$$
\begin{array}{llll}
 & \llbracket(\mu\alpha.[\beta]M)\overline{N}\rrbracket & & \\
\triangleq & \llbracket(\mu\alpha.[\beta]M)\rrbracket\llbracket\overline{N}\rrbracket & & \\
\triangleq & (\textsc{wsc}\ \mathrm{P}_0\ \lambda\alpha.\textsc{pp}\ \mathrm{P}_0\ (\textsc{psc}\ \beta\ \llbracket M\rrbracket))\llbracket\overline{N}\rrbracket & \square & \mathrm{P}_0 \\
\to_{CDC} & (\textsc{wsc}\ \mathrm{P}_0\ \lambda\alpha.\textsc{pp}\ \mathrm{P}_0\ (\textsc{psc}\ \beta\ \llbracket M\rrbracket)) & \square\llbracket\overline{N}\rrbracket & \mathrm{P}_0 \\
\to_{CDC} & (\lambda\alpha.\textsc{pp}\ \mathrm{P}_0\ (\textsc{psc}\ \beta\ \llbracket M\rrbracket))(\square\llbracket\overline{N}\rrbracket) & \square & [] \\
\to_{CDC} & (\textsc{pp}\ \mathrm{P}_0\ (\textsc{psc}\ \beta\ \llbracket M\rrbracket))[\square\llbracket\overline{N}\rrbracket/\alpha] & \square & [] \\
\to_{CDC} & \textsc{pp}\ \mathrm{P}_0\ (\textsc{psc}\ \beta\ \llbracket M\rrbracket)[\square\llbracket\overline{N}\rrbracket/\alpha] & \square & [] \\
\to_{CDC} & (\textsc{psc}\ \beta\ \llbracket M\rrbracket)[\square\llbracket\overline{N}\rrbracket/\alpha] & \square & \mathrm{P}_0 \\
\to_{CDC} & \textsc{psc}\ \beta\ \llbracket M\rrbracket[\square\llbracket\overline{N}\rrbracket/\alpha] & \square & \mathrm{P}_0 \\
\to_{CDC} & \llbracket M\rrbracket[\square\llbracket\overline{N}\rrbracket/\alpha] & \square & \beta : [] : \mathrm{P}_0
\end{array}
$$

$$
\begin{array}{llll}
 & \llbracket \mu\alpha.[\beta](M[[\alpha]M'N/[\alpha]M']) \rrbracket & & \\
\triangleq & \textsc{wsc}\ \textsc{p}_0\ \lambda\alpha.\textsc{pp}\ \textsc{p}_0\ (\textsc{psc}\ \beta\llbracket M[[\alpha]M'N/[\alpha]M'] \rrbracket) & \square & \textsc{p}_0 \\
\rightarrow_{CDC} & (\lambda\alpha.\textsc{pp}\ \textsc{p}_0\ (\textsc{psc}\ \beta\llbracket M[[\alpha]M'N/[\alpha]M'] \rrbracket))(\square) & \square & [] \\
\rightarrow_{CDC} & (\textsc{pp}\ \textsc{p}_0\ (\textsc{psc}\ \beta\llbracket M[[\alpha]M'N/[\alpha]M'] \rrbracket))[\square/\alpha] & \square & [] \\
\rightarrow_{CDC} & \textsc{pp}\ \textsc{p}_0\ (\textsc{psc}\ \beta\llbracket M[[\alpha]M'N/[\alpha]M'] \rrbracket)[\square/\alpha] & \square & [] \\
\rightarrow_{CDC} & \textsc{psc}\ \beta\llbracket M[[\alpha]M'N/[\alpha]M'] \rrbracket[\square/\alpha] & \square & \textsc{p}_0 \\
\rightarrow_{CDC} & \llbracket M[[\alpha]M'N/[\alpha]M'] \rrbracket[\square/\alpha] & \square & \beta : [] : \textsc{p}_0 \\
\rightarrow_{CDC} & M[\square\llbracket N \rrbracket/\alpha][\square/\alpha] & \square & \beta : [] : \textsc{p}_0 \\
\rightarrow_{CDC} & M[\square\llbracket N \rrbracket/\alpha] & \square & \beta : [] : \textsc{p}_0
\end{array}
$$

$\mu\alpha.[\beta]\mu\gamma.[\delta]M \rightarrow \mu\alpha[\delta]M[\beta/\gamma]$: $\qquad (\gamma \neq \delta)$

$$
\begin{array}{llll}
 & \llbracket \mu\alpha.[\beta]\mu\gamma.[\delta]M \rrbracket & & \\
\triangleq & \textsc{wsc}\ \textsc{p}_0\ \lambda\alpha.\textsc{pp}\ \textsc{p}_0\ (\textsc{psc}\ \beta\ \llbracket \mu\gamma.[\delta]M \rrbracket) & \square & \textsc{p}_0 \\
\rightarrow_{CDC} & (\lambda\alpha.\textsc{pp}\ \textsc{p}_0\ (\textsc{psc}\ \beta\ \llbracket \mu\gamma.[\delta]M \rrbracket))(\square) & \square & [] \\
\rightarrow_{CDC} & (\textsc{pp}\ \textsc{p}_0\ (\textsc{psc}\ \beta\ \llbracket \mu\gamma.[\delta]M \rrbracket))[\square/\alpha] & \square & [] \\
\rightarrow_{CDC} & \textsc{pp}\ \textsc{p}_0\ (\textsc{psc}\ \beta\ \llbracket \mu\gamma.[\delta]M \rrbracket)[\square/\alpha] & \square & [] \\
\rightarrow_{CDC} & (\textsc{psc}\ \beta\ \llbracket \mu\gamma.[\delta]M \rrbracket)[\square/\alpha] & \square & \textsc{p}_0 \\
\rightarrow_{CDC} & \textsc{psc}\ \beta\ \llbracket \mu\gamma.[\delta]M \rrbracket[\square/\alpha] & \square & \textsc{p}_0 \\
\rightarrow_{CDC} & \llbracket \mu\gamma.[\delta]M \rrbracket[\square/\alpha] & \square & \beta : [] : \textsc{p}_0 \\
\triangleq & (\textsc{wsc}\ \textsc{p}_0\ \lambda\gamma.\textsc{pp}\ \textsc{p}_0\ (\textsc{psc}\ \delta\llbracket M \rrbracket))[\square/\alpha] & \square & \beta : [] : \textsc{p}_0 \\
\rightarrow_{CDC} & \textsc{wsc}\ \textsc{p}_0\ \lambda\gamma.\textsc{pp}\ \textsc{p}_0\ (\textsc{psc}\ \delta\llbracket M \rrbracket)[\square/\alpha] & \square & \beta : [] : \textsc{p}_0 \\
\rightarrow_{CDC} & (\lambda\gamma.\textsc{pp}\ \textsc{p}_0\ (\textsc{psc}\ \delta\llbracket M \rrbracket)[\square/\alpha])(\beta) & \square & [] \\
\rightarrow_{CDC} & (\textsc{pp}\ \textsc{p}_0\ (\textsc{psc}\ \delta\llbracket M \rrbracket)[\square/\alpha])[\beta/\gamma] & \square & [] \\
\rightarrow_{CDC} & \textsc{pp}\ \textsc{p}_0\ (\textsc{psc}\ \delta\llbracket M \rrbracket)[\square/\alpha][\beta/\gamma] & \square & [] \\
\rightarrow_{CDC} & (\textsc{psc}\ \delta\llbracket M \rrbracket)[\square/\alpha][\beta/\gamma] & \square & \textsc{p}_0 \\
\rightarrow_{CDC} & \textsc{psc}\ \delta\llbracket M \rrbracket[\square/\alpha][\beta/\gamma] & \square & \textsc{p}_0 \\
\rightarrow_{CDC} & \llbracket M \rrbracket[\square/\alpha][\beta/\gamma] & \square & \delta : [] : \textsc{p}_0
\end{array}
$$

$$
\begin{array}{llll}
 & \llbracket \mu\alpha[\delta]M[\beta/\gamma] \rrbracket & & \\
\triangleq & \textsc{wsc}\ \textsc{p}_0\ \lambda\alpha.\textsc{pp}\ \textsc{p}_0\ (\textsc{psc}\ \delta\ \llbracket M \rrbracket[\beta/\gamma]) & \square & \textsc{p}_0 \\
\rightarrow_{CDC} & (\lambda\alpha.\textsc{pp}\ \textsc{p}_0\ (\textsc{psc}\ \delta\ \llbracket M \rrbracket[\beta/\gamma]))(\square) & \square & [] \\
\rightarrow_{CDC} & (\textsc{pp}\ \textsc{p}_0\ (\textsc{psc}\ \delta\ \llbracket M \rrbracket[\beta/\gamma]))[\square/\alpha] & \square & [] \\
\rightarrow_{CDC} & \textsc{pp}\ \textsc{p}_0\ (\textsc{psc}\ \delta\ \llbracket M \rrbracket[\beta/\gamma])[\square/\alpha] & \square & [] \\
\rightarrow_{CDC} & (\textsc{psc}\ \delta\ \llbracket M \rrbracket[\beta/\gamma])[\square/\alpha] & \square & \textsc{p}_0 \\
\rightarrow_{CDC} & \textsc{psc}\ \delta\ \llbracket M \rrbracket[\beta/\gamma][\square/\alpha] & \square & \textsc{p}_0 \\
\rightarrow_{CDC} & \llbracket M \rrbracket[\beta/\gamma][\square/\alpha] & \square & \delta : [] : \textsc{p}_0
\end{array}
$$

Remark 1.2.7 The terms $\llbracket M \rrbracket[\beta/\gamma][\square/\alpha]$ and $\llbracket M \rrbracket[\square/\alpha][\beta/\gamma]$ are denotationally equivalent if $\alpha \neq \beta \neq \gamma$.

$\mu\alpha.[\beta]\mu\gamma.[\gamma]M \to \mu\alpha[\beta]M[\beta/\gamma]$:

$$
\begin{array}{llll}
 & \llbracket \mu\alpha.[\beta]\mu\gamma.[\gamma]M \rrbracket & & \\
\triangleq & \textsc{wsc}\ \textsc{p}_0\ \lambda\alpha.\textsc{pp}\ \textsc{p}_0\ (\textsc{psc}\ \beta\ \llbracket \mu\gamma.[\gamma]M \rrbracket) & \square & \textsc{p}_0 \\
\to_{CDC} & (\lambda\alpha.\textsc{pp}\ \textsc{p}_0\ (\textsc{psc}\ \beta\ \llbracket \mu\gamma.[\gamma]M \rrbracket))(\square) & \square & [] \\
\to_{CDC} & (\textsc{pp}\ \textsc{p}_0\ (\textsc{psc}\ \beta\ \llbracket \mu\gamma.[\gamma]M \rrbracket))[\square/\alpha] & \square & [] \\
\to_{CDC} & \textsc{pp}\ \textsc{p}_0\ (\textsc{psc}\ \beta\ \llbracket \mu\gamma.[\gamma]M \rrbracket)[\square/\alpha] & \square & [] \\
\to_{CDC} & (\textsc{psc}\ \beta\ \llbracket \mu\gamma.[\gamma]M \rrbracket)[\square/\alpha] & \square & \textsc{p}_0 \\
\to_{CDC} & \textsc{psc}\ \beta\ \llbracket \mu\gamma.[\gamma]M \rrbracket[\square/\alpha] & \square & \textsc{p}_0 \\
\to_{CDC} & \llbracket \mu\gamma.[\gamma]M \rrbracket[\square/\alpha] & \square & \beta : [] : \textsc{p}_0 \\
\triangleq & (\textsc{wsc}\ \textsc{p}_0\ \lambda\gamma.\textsc{pp}\ \textsc{p}_0\ (\textsc{psc}\ \gamma\llbracket M \rrbracket))[\square/\alpha] & \square & \beta : [] : \textsc{p}_0 \\
\to_{CDC} & \textsc{wsc}\ \textsc{p}_0\ \lambda\gamma.\textsc{pp}\ \textsc{p}_0\ (\textsc{psc}\ \gamma\llbracket M \rrbracket)[\square/\alpha] & \square & \beta : [] : \textsc{p}_0 \\
\to_{CDC} & (\lambda\gamma.\textsc{pp}\ \textsc{p}_0\ (\textsc{psc}\ \gamma\llbracket M \rrbracket)[\square/\alpha])(\beta) & \square & [] \\
\to_{CDC} & (\textsc{pp}\ \textsc{p}_0\ (\textsc{psc}\ \gamma\llbracket M \rrbracket)[\square/\alpha])[\beta/\gamma] & \square & [] \\
\to_{CDC} & \textsc{pp}\ \textsc{p}_0\ (\textsc{psc}\ \gamma\llbracket M \rrbracket)[\square/\alpha][\beta/\gamma] & \square & [] \\
\to_{CDC} & (\textsc{psc}\ \gamma\llbracket M \rrbracket)[\square/\alpha][\beta/\gamma] & \square & \textsc{p}_0 \\
\to_{CDC} & \textsc{psc}\ \beta\llbracket M \rrbracket[\square/\alpha][\beta/\gamma] & \square & \textsc{p}_0 \\
\to_{CDC} & \llbracket M \rrbracket[\square/\alpha][\beta/\gamma] & \square & \beta : [] : \textsc{p}_0 \\
 & & & \\
 & \llbracket \mu\alpha[\beta]M[\beta/\gamma] \rrbracket & & \\
\triangleq & \textsc{wsc}\ \textsc{p}_0\ \lambda\alpha.\textsc{pp}\ \textsc{p}_0\ (\textsc{psc}\ \beta\ \llbracket M \rrbracket[\beta/\gamma]) & \square & \textsc{p}_0 \\
\to_{CDC} & (\lambda\alpha.\textsc{pp}\ \textsc{p}_0\ (\textsc{psc}\ \beta\ \llbracket M \rrbracket[\beta/\gamma]))(\square) & \square & [] \\
\to_{CDC} & (\textsc{pp}\ \textsc{p}_0\ (\textsc{psc}\ \beta\ \llbracket M \rrbracket[\beta/\gamma]))[\square/\alpha] & \square & [] \\
\to_{CDC} & \textsc{pp}\ \textsc{p}_0\ (\textsc{psc}\ \beta\ \llbracket M \rrbracket[\beta/\gamma])[\square/\alpha] & \square & [] \\
\to_{CDC} & (\textsc{psc}\ \beta\ \llbracket M \rrbracket[\beta/\gamma])[\square/\alpha] & \square & \textsc{p}_0 \\
\to_{CDC} & \textsc{psc}\ \beta\ \llbracket M \rrbracket[\beta/\gamma][\square/\alpha] & \square & \textsc{p}_0 \\
\to_{CDC} & \llbracket M \rrbracket[\beta/\gamma][\square/\alpha] & \square & \beta : [] : \textsc{p}_0
\end{array}
$$

$\square$

Theorem 1.2.8 (Completeness of $\llbracket\cdot\rrbracket$)

$$\llbracket M \rrbracket \to^{nf} \llbracket N \rrbracket \Rightarrow \exists Q.M \to^* N \wedge \llbracket N \rrbracket \to^{nf} Q$$

Proof. To come... $\square$

1.3 Interpreting λ^{try} in CDC

By appending the interpretation of λ^{try} in $\lambda\mu$ with the interpretation of $\lambda\mu$ in CDC, we get a translation from λ^{try} to CDC:

Definition 1.3.1 $\quad$ TRANSLATION OF λ^{try} INTO CDC

$$
\begin{array}{rcl}
[\![x]\!] & \triangleq & x \\
[\![\lambda x.M]\!] & \triangleq & \lambda x.[\![M]\!] \\
[\![MN]\!] & \triangleq & [\![M]\!][\![N]\!] \\
[\![\mathrm{throw\ n}(M)]\!] & \triangleq & \textsc{wsc}\ \ \textsc{p}_0\ \ \lambda\circ.\textsc{pp}\ \ \textsc{p}_0\ \ (\textsc{psc}\ \ n\ (c_n\ [\![M]\!])) \\
[\![\mathrm{try}\ M;\ \mathrm{catch\ n}(x) = L]\!] & \triangleq & (\lambda c_n.\textsc{wsc}\ \ \textsc{p}_0\ \ \lambda n.\textsc{pp}\ \ \textsc{p}_0\ \ (\textsc{psc}\ \ n\ [\![M]\!]))(\lambda x.[\![L]\!]) \\
\multicolumn{3}{l}{[\![\mathrm{try}\ M;\ \overline{\mathrm{catch\ n}_i(x) = M_i};\ \mathrm{catch\ m}(x) = L]\!]} \\
 & \triangleq & \\
\multicolumn{3}{r}{(\lambda c_m.\textsc{wsc}\ \ \textsc{p}_0\ \ \lambda m.\textsc{pp}\ \ \textsc{p}_0\ \ (\textsc{psc}\ \ m\ [\![\mathrm{try}\ M;\ \overline{\mathrm{catch\ n}_i(x) = M_i}]\!]))(\lambda x.[\![L]\!])}
\end{array}
$$