Lecture Notes

PHYS260

Honor Physics II

Electrics, Magnetism and
A Brief Peek into Quantum Physics

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SEPTEMBER 11, 2025

Contents

A Electrics and Magnetism

Part A

ELECTRICS AND MAGNETISM

Chapter 0

Math Symbols Demo

0.1 Scalars, Vectors, Matrices and Tensors

$$\boldsymbol{L} = \mathcal{I}\boldsymbol{\omega} = \mathbf{I}\boldsymbol{\omega} = \begin{pmatrix} I_{\mathrm{ii}} & I_{\mathrm{ij}} & I_{\mathrm{ik}} \\ I_{\mathrm{ji}} & I_{\mathrm{jj}} & I_{\mathrm{jk}} \\ I_{\mathrm{ki}} & I_{\mathrm{kj}} & I_{\mathrm{kk}} \end{pmatrix} \begin{pmatrix} \omega_{\mathrm{i}} \\ \omega_{\mathrm{j}} \\ \omega_{\mathrm{k}} \end{pmatrix}$$

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} \int \frac{I \cdot \mathrm{d}\boldsymbol{l} \times (\boldsymbol{r} - \boldsymbol{r}')}{\|\boldsymbol{r} - \boldsymbol{r}'\|^3}$$

0.2 Math Operators

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \in \mathrm{Mat}(2 \times 2; \mathbb{R})$$

0.3 Normed Vectors and Canonical Basis

$$\boldsymbol{v} = x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}} + z\hat{\boldsymbol{k}}$$

0.4 Differential d's

$$\mathrm{d}U = \delta Q + \delta W$$

0.5 Units

$$R_{\mathrm{equiv}} = 2.71\,\Omega$$

Chapter 1

Electric Charge and Electric Fields

1.0 The Fundamental Forces

Last semester in PHYS160, we explored the fundamental principles governing the motion of objects in the macroscopic physical world. From the origin of $F = ma$, we used mathematical tools to derive the likes of potential energy, momentum conservation, rigid body motion, Kepler's laws, etc. We also explored the same ideas from the alternate perspective of Lagrangian mechanics, which takes the same form in any generalized coordinate systems.

Much of our focus was on gravitational forces, which is just one piece of a larger puzzle in physics. As we venture into new territory this semester, it's essential to broaden our perspective to include the full spectrum of interactions that shape the universe.

At the heart of modern physics lie the four fundamental forces, which are the basic interactions responsible for all phenomena we observe.

Definition 1 (Fundamental Forces).
Fundamental forces are the interactions in nature that cannot be reduced to more basic interactions. There are four fundamental forces known to exist:

Interaction	Strength	Range
Gravity	10^{-38}	Long
Electromagnetism	10^{-2}	Long
Weak	10^{-6}	Short (10^{-18})
Strong	1	Short (10^{-15})

Remark.
Every single force that we've encountered during the last semester can be either classified into one of the four fundamental forces, or decomposed into some of them.

For example, gravity is solely gravity; the force acted upon a table when someone pushes it is, at its essence, electromagnetic force. However, we still don't know the exact composition of certain really common forces, like friction.

The simplicity we see in highly abstract models are often approximations of emergent behaviors of microscopic particles, and this emergent behavior can sometimes hinder our understanding to their origins.

In the first part of the course, we will be focusing on electromagnetic force, the force acting between electrically charged particles.

1.1 Electric Charge

Similar to how most other physical quantities are discovered, electric charge was first observed as a qualitative phenomenon in 600 B.C. Back then, the Greeks called amber 'elektron', from which the word electric is derived.

Property 1 (Triboelectric Effect)**.**
Electric charge can be transferred from one object to another via contact or sliding with each other.

- Rubbing a plastic/amber rod with fur causes the rod to be *negatively charged*;
- Rubbing a glass rod with silk causes the rod to be *positively charged*.

Electric charge is one of the fundamental physical properties of matter. Like mass, we can measure the amount of electric charge that an object has, but we cannot exactly present it in a well-defined manner.

Notation 1 (Electric Charge)**.**
Electric charge is usually denoted by Q or q.
Electric charge is measured in *Coulombs* in the SI unit, or C in short.

Remark.
In circuitry, time-dependent quantities are usually represented by lowercase symbols, while uppercase letters more often represent invariants.

Instead of trying to actually define what charge is, we take an alternate route, and begin from understanding how charged objects actually behave.

Property 2 (Rules of Attraction and Repulsion)**.**

- Charges of the same kind repel each other;
- Charges of the opposite kind attract each other;
- Charged objects attract neutral objects.

Despite the exotic behaviors, we can't actually tell the difference between a charged rod and a neutrally charged one just by looking at their appearance. This might implies that the triboelectric effect acts on a microscopic level. This becomes clear once we inspect the structure of an atom.

At the center of an atom lies its nucleus, which accounts for more than 99.9% of the atom's total mass. It is consisted of positively charged protons and neutrally charged neutrons. The nucleus is enclosed by the negatively charged electron cloud.

Definition 2 (Elementary Charge)**.**
e, the elementary charge, is a fundamental physical constant, defined as the electric charge

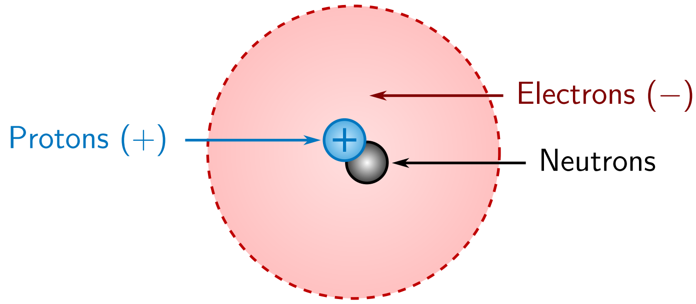


Figure 1.1: The structure of an atom

carried by a single proton, or the negative of the electric charge carried by a single electron.

$$e = 1.60217663 \times 10^{-19}\ [\mathrm{C}]$$

Further experimentation gives us the two following essential properties of electric charge.

Property 3 (Charge Conservation).
The algebraic sum of electric charge in an isolated system is conserved.

Property 4 (Charge Quantization).
The charge of any object is an integer multiple of the elementary charge e, i.e.

$$Q = n \cdot e, \quad n \in \mathbb{Z}.$$

1.2 Conductors and Insulators

It is commonly seen that solid objects allow the flow of charge in a certain uniform direction. Solid materials are classified with accordance to their abilities to conduct this directed flow of charge into the following categories:

Definition 3.

Superconductor
: A perfect conductor. Charge can move in superconductors without any resistance.

Conductor
: Charge can move freely in conductors, but with a certain resistance. The energy dissipated by this resistance is transformed into a byproduct – heat.

Semiconductor
: Becomes conductor upon meeting certain requirements. Semiconductors are often used in the field of logic design.

Insulator
Has no mobile charges.

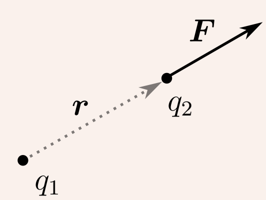
Remark.
The idea of perfect conductors and perfect insulators are only ideal models. In reality, materials lie somewhere in between the two ends of this spectrum.

1.3 Induced Charge*

1.4 Coulomb's Law*

Law 1 (Coulomb's Law)**.**
The magnitude of the force between two charged particles at rest follows the inverse square law, and the direction of the force always points from one charged particle to the other, abiding the rules of attraction and repulsion:

$$\begin{aligned}\boldsymbol{F} &= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{\|\boldsymbol{r}\|^2}\hat{\boldsymbol{r}} \\ &= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{\|\boldsymbol{r}\|^3}\boldsymbol{r},\end{aligned}$$



where q_1, q_2 are the electric charge carried by the two particles, and r the distance between them.

Remark.
Coulomb is a large quantity! If we have $q_1 = q_2 = 1\,\mathrm{C}$, $r = 1\,\mathrm{m}$, then the Coulomb force between the two charges would be approximately $9 \times 10^9\,\mathrm{N}$!

Example.
If we compare the Coulomb force and the gravitational force between two α-particles, we would get:

$$\frac{\|\boldsymbol{F}_{\text{elec}}\|}{\|\boldsymbol{F}_{\text{grav}}\|} = \frac{q_\alpha^2}{4\pi\varepsilon_0 \cdot G m_\alpha^2} \approx 3.31 \times 10^{35},$$

i.e. the Coulomb force is tens of orders of magnitude larger than the gravitational force!

1.5 Electric Field*

A setup consisting of several charged particles at rest would induce a Coulomb force to another test particle in space. Let's first recapitulate some fundamental results on potential force fields from mathematics and the discussion of last semester:

Definition 4 (Potential Field in $\mathbb{R}^3$)**.**
Let $\Omega \subset \mathbb{R}^3$ be an open set. A vector field $\boldsymbol{F} : \Omega \to \mathbb{R}^3$ is a potential field if there exist a differentiable potential function $U : \Omega \to \mathbb{R}$ such that

$$\boldsymbol{F}(x) = -\nabla U(x), \quad \forall x \in \Omega.$$

Definition 5 (Simply Connected Set)**.**
Loosely speaking, a set $\Omega \subset \mathbb{R}^3$ is said to be simply connected if

- Ω is pathwise connected and
- Every closed curve in Ω can be contracted to a simple point within Ω.

Other than explicitly integrating in order to find the potential function U, there is a handy criteria to help identify if a vector field on a simply connected set is a potential field.

Theorem 1 (Criteria for Potential Field)**.**
Let $\Omega \subset \mathbb{R}^3$ be a simply connected open set and suppose that $\boldsymbol{F} : \Omega \to \mathbb{R}^3$ is continuously differentiable. Then

$$\nabla \times \boldsymbol{F} = -\nabla \times \nabla U = 0 \quad \Longrightarrow \quad \boldsymbol{F} \text{ is a potential field.}$$

Definition 6 (Electric Field)**.**
The electric field that the charged particle Q induces at the position occupied by q is

$$\boldsymbol{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{\|\boldsymbol{r}\|^2}\hat{\boldsymbol{r}},$$

in which $\boldsymbol{r}$ is the displacement of q with respect to Q.

Proposition 1.
The electric field induced by a point charge is spherically symmetrical.