11 Appendix A

The chosen aircraft is a typical large four-jet engine commercial transport aircraft, the Boeing 747-200. All relevant data are provided in the table below. All of the stability derivatives are dimensionless and referenced to the stability axes.

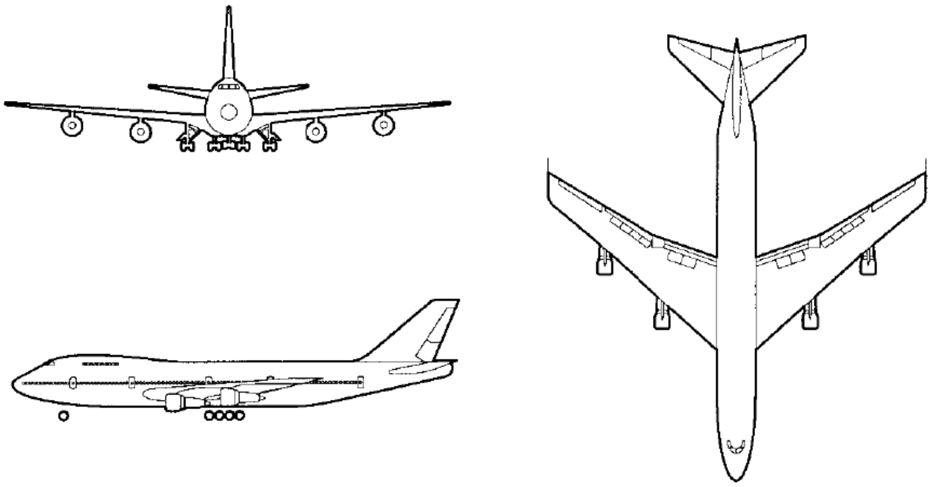

Figure 67: Boeing 747-200

Table 4: Reference Geometry & Mass Data

$S[m^2]$	510,96
$\bar{c}[m]$	8,32
$b[m]$	59,74
$m[kg]$	288773,23
$I_{xx}[kg \cdot m^2]$	24675886,69
$I_{yy}[kg \cdot m^2]$	44877574,145
$I_{zz}[kg \cdot m^2]$	67384152,115
$I_{xz}[kg \cdot m^2]$	1315143,4115

Table 5: Flight Condition Data & Steady State Coefficients

$h[m]$	6096
$Mach$	0,650
$(V_{TAS})_s[m/s]$	205,13
$\bar{q}[N/m^2]$	13888
$C.G.$	0,25
$(\alpha)_s[rad.]$	0,043633
C_{L_1}	0,40
C_{D_1}	0,0250
$C_{T_{x_1}}$	0,0250

Table 6: Longitudinal Coefficients and Stability Derivatrives

C_{D_0}	C_{D_u}	C_{D_α}	$C_{T_{x_0}}$	C_{L_0}	C_{L_u}	C_{L_α}	$C_{L_{\dot{\alpha}}}$	C_{L_q}	C_{m_0}	C_{m_u}	C_{m_α}	$C_{m_{\dot{\alpha}}}$	C_{m_q}
0,0164	0	0,20	-0,055	0,21	0,13	4,4	7,0	6,6	0	0,013	-1,0	-4,0	-20,5

Table 7: Lateral-Directional Stability Derivatives

C_{l_β}	C_{l_p}	C_{l_r}	C_{y_β}	C_{y_p}	C_{y_r}	C_{n_β}	$C_{n_{T_\beta}}$	C_{n_p}	C_{n_r}
0,0164	0	0,20	-0,055	0,21	0,13	4,4	7,0	6,6	0

Table 8: Control and Hinge Moment Derivatives

$C_{D_{\delta_e}}$	$C_{L_{\delta_e}}$	$C_{m_{\delta_e}}$	$C_{D_{i_h}}$	$C_{L_{i_h}}$	$C_{m_{i_h}}$	$C_{l_{\delta_a}}$	$C_{l_{\delta_r}}$	$C_{y_{\delta_a}}$	$C_{y_{\delta_r}}$	$C_{n_{\delta_a}}$	$C_{n_{\delta_r}}$
0	0,32	-1,30	0	0,70	-2,7	0,013	0,008	0	0,120	0,0018	-0,100

12 Appendix B

Each dimensional derivative represents either the linear or angular acceleration imparted to the aircraft as a result of a unit change in it's associated motion or control variable.

Table 9: Longitudinal Dimensional Stability Derivatives

$$X_u = \frac{-\bar{q}S(C_{D_u}+2C_{D_1})}{m(V_0)_e} \qquad Z_u = \frac{-\bar{q}S(C_{L_u}+2C_{L_1})}{m(V_0)_e} \qquad Z_{\delta_e} = \frac{-\bar{q}SC_{L_{\delta_e}}}{m}$$

$$X_{T_u} = \frac{\bar{q}S(C_{T_{x_u}}+2C_{T_{x_1}})}{m(V_0)_e} \qquad Z_\alpha = \frac{-\bar{q}S(C_{L_\alpha}+C_{D_1})}{m} \qquad M_u = \frac{\bar{q}S\bar{c}(C_{m_u}+2C_{m_1})}{I_{yy}(V_0)_e}$$

$$X_\alpha = \frac{-\bar{q}S(C_{D_\alpha}-C_{L_1})}{m} \qquad Z_{\dot{\alpha}} = \frac{-\bar{q}S\bar{c}C_{L_{\dot{\alpha}}}}{2m(V_0)_e} \qquad M_{T_u} = \frac{\bar{q}S\bar{c}(C_{m_{T_u}}+2C_{m_{T_1}})}{I_{yy}(V_0)_e}$$

$$X_{\delta_e} = \frac{-\bar{q}SC_{D_{\delta_e}}}{m} \qquad Z_q = \frac{-\bar{q}S\bar{c}C_{L_q}}{2m(V_0)_e} \qquad M_\alpha = \frac{\bar{q}S\bar{c}C_{m_\alpha}}{I_{yy}}$$

$$M_{\dot{\alpha}} = \frac{\bar{q}S\bar{c}^2C_{m_{\dot{\alpha}}}}{2I_{yy}(V_0)_e} \qquad M_q = \frac{\bar{q}S\bar{c}^2C_{m_q}}{2I_{yy}(V_0)_e} \qquad M_{T_\alpha} = \frac{\bar{q}S\bar{c}C_{m_{T_\alpha}}}{I_{yy}}$$

$$M_{\delta_e} = \frac{\bar{q}S\bar{c}C_{m_{\delta_e}}}{I_{yy}}$$

Table 10: Lateral-Directional Dimensional Stability Derivatives

$$Y_\beta = \frac{\bar{q}SC_{y_\beta}}{m} \qquad L_\beta = \frac{\bar{q}SbC_{l_\beta}}{I_{xx}} \qquad N_\beta = \frac{\bar{q}SbC_{n_\beta}}{I_{zz}}$$

$$Y_p = \frac{\bar{q}SbC_{y_p}}{2m(V_0)_e} \qquad L_p = \frac{\bar{q}Sb^2C_{l_p}}{2I_{xx}(V_0)_e} \qquad N_{T_\beta} = \frac{\bar{q}SbC_{n_{T_\beta}}}{I_{zz}}$$

$$Y_r = \frac{\bar{q}SbC_{y_r}}{2m(V_0)_e} \qquad L_r = \frac{\bar{q}Sb^2C_{l_r}}{2I_{xx}(V_0)_e} \qquad N_p = \frac{\bar{q}Sb^2C_{n_p}}{2I_{zz}(V_0)_e}$$

$$Y_{\delta_a} = \frac{\bar{q}SC_{y_{\delta_a}}}{m} \qquad L_{\delta_a} = \frac{\bar{q}SbC_{l_{\delta_a}}}{I_{xx}} \qquad N_r = \frac{\bar{q}Sb^2C_{n_r}}{2I_{zz}(V_0)_e}$$

$$Y_{\delta_r} = \frac{\bar{q}SC_{y_{\delta_r}}}{m} \qquad L_{\delta_r} = \frac{\bar{q}SbC_{l_{\delta_r}}}{I_{xx}} \qquad N_{\delta_a} = \frac{\bar{q}SbC_{n_{\delta_a}}}{I_{zz}}$$

$$N_{\delta_r} = \frac{\bar{q}SbC_{n_{\delta_r}}}{I_{zz}}$$

13 Appendix C

This section concerns the algebraic derivation of the state space form along with its limitation. Knowledge of the translational, rotational and kinematic equations set out in Chapter 1 is required. In perturbed state flight, all motion variables are defined relative to a known steady state flight condition. The derivation was largely taken from [1]. Let us make the following substitutions referring to steady–state and perturbed variables. Substituting into the general translational equations leads to

$$
\begin{array}{ccc}
U = U_s + u & V = V_s + v & W = W_s + w \\
P = P_s + p & Q = Q_s + q & R = R_s + r \\
\Phi = \phi_s + \phi & \Theta = \theta_s + \theta & \Psi = \psi_s + \psi \\
X_A = (X_A)_s + x_A & Y_A = (Y_A)_s + y_A & Z_A = (Z_A)_s + z_A \\
X_T = (X_T)_s + x_T & Y_T = (Y_T)_s + y_T & Z_T = (Z_T)_s + z_T \\
\mathcal{L}_A = (\mathcal{L}_A)_s + l_A & \mathcal{M}_A = (\mathcal{M}_A)_s + m_A & \mathcal{N}_A = (\mathcal{N}_A)_s + n_A \\
\mathcal{L}_T = (\mathcal{L}_T)_s + l_T & \mathcal{M}_T = (\mathcal{M}_T)_s + m_T & \mathcal{N}_T = (\mathcal{N}_T)_s + n_T
\end{array}
$$

$$
\begin{aligned}
\dot{u} &= \frac{1}{m}((X_A)_s + x_A + (X_T)_s + x_T) - (W_s + w)(Q_s + q) + (V_s + v)(R_s + r) - g\sin(\theta_s + \theta), \\
\dot{v} &= \frac{1}{m}((Y_A)_s + y_A + (Y_T)_s + y_T) - (U_s + u)(R_s + r) + (W_s + w)(P_s + p) + g\cos(\theta_s + \theta)\sin(\phi_s + \phi), \\
\dot{w} &= \frac{1}{m}((Z_A)_s + z_A + (Z_T)_s + z_T) - (V_s + v)(P_s + p) + (U_s + u)(Q_s + q) + g\cos(\theta_s + \theta)\cos(\phi_s + \phi).
\end{aligned}
$$

The same applies to to rotational equations and kinematic equations.

$$
\begin{aligned}
\dot{p} =& \frac{1}{I_{xx}I_{zz} - I_{xz}^2}[I_{zz}((\mathcal{L}_A)_s + l_A + (\mathcal{L}_T)_s + l_T - (Q_s + q)(R_s + r)(I_{zz} - I_{yy}) + I_{xz}(P_s + p)(Q_s + q))+ \\
& I_{xz}((\mathcal{N}_A)_s + n_A + (\mathcal{N}_T)_s + n_T - (P_s + p)(Q_s + q)(I_{yy} - I_{xx}) - I_{xz}(Q_s + q)(R_s + r))], \\
\dot{q} =& \frac{1}{I_{yy}}\left[(\mathcal{M}_A)_s + m_A + (\mathcal{M}_T)_s + m_T - (R_s + r)(P_s + p)(I_{xx} - I_{zz}) - I_{xz}((P_s + p)^2 - (R_s + r)^2)\right], \\
\dot{r} =& \frac{1}{I_{xx}I_{zz} - I_{xz}^2}[I_{xz}((\mathcal{L}_A)_s + l_A + (\mathcal{L}_T)_s + l_T - (R_s + r)(Q_s + q)(I_{zz} - I_{yy}) + I_{xz}(P_s + p)(Q_s + q))+ \\
& I_{xx}((\mathcal{N}_A)_s + n_A + (\mathcal{N}_T)_s + n_T - (P_s + p)(Q_s + q)(I_{yy} - I_{xx}) - I_{xz}(Q_s + q)(R_s + r))].
\end{aligned}
$$

$$
\begin{aligned}
\dot{\phi} &= (P_s + p) + (Q_s + q)\sin(\phi_s + \phi)\operatorname{tg}(\theta_s + \theta) + (R_s + r)\cos(\phi_s + \phi)\operatorname{tg}(\theta_s + \theta), \\
\dot{\theta} &= (Q_s + q)\cos(\phi_s + \phi) - (R_s + r)\sin(\phi_s + \phi), \\
\dot{\psi} &= (Q_s + q)\frac{\sin(\phi_s + \phi)}{\cos(\theta_s + \theta)} + (R_s + r)\frac{\cos(\phi_s + \phi)}{\cos(\theta_s + \theta)}.
\end{aligned}
$$

Several assumptions will now be made for the simplification of the equations and their manipulation. The first simplification will be performed using small angle approximations. For angle $\varphi \approx 0$, we can write $\cos\varphi \approx 1$, $\sin\varphi \approx \varphi$ and $\operatorname{tg}\varphi \approx \varphi$. This simplification holds quite well for angles up to 15 degrees. We can now represent the simplified trigonometric identities in the following table.

$$
\sin(\varphi_s + \varphi) \approx \sin\varphi_s + \varphi\cos\varphi_s,
$$

$$
\begin{aligned}
\cos(\varphi_s + \varphi) &\approx \cos\varphi_s - \varphi \sin\varphi_s, \\
\operatorname{tg}(\varphi_s + \varphi) &\approx \frac{\operatorname{tg}\varphi_s + \varphi}{1 - \operatorname{tg}\varphi_s \varphi}.
\end{aligned}
$$

The second assumption made will be the elimination of terms representing steady–state motion, which is assumed to be satisfied as the motion is defined relative to a steady–state condition. Finally, the assumption of small perturbation will be made to eliminate the products and cross products of the perturbed variables, which are considered to be negligible. The remaining linear terms form a linearized set of equations of motion.
Translational equations:

$$
\begin{aligned}
m\left(\dot{u} - V_s r - R_s v + W_s q\right) &= -mg\theta\cos\theta_s + x_A + x_T, \\
m\left(\dot{v} + U_s r + R_s u - W_s p - P_s w\right) &= -mg\theta\sin\phi_s\sin\theta_s + mg\phi\cos\phi_s\cos\theta_s + y_A + y_T, \\
m\left(\dot{w} - U_s q - Q_s u + V_s p + P_s v\right) &= -mg\theta\cos\phi_s\sin\theta_s - mg\phi\sin\phi_s\cos\theta_s + z_A + z_T,
\end{aligned}
$$

Rotational equations:

$$
\begin{aligned}
I_{xx}\dot{p} - I_{xz}\dot{r} - I_{xz}\left(P_s q + Q_s p\right) + \left(I_{zz} - I_{yy}\right)\left(R_s q + Q_s r\right) &= l_A + l_T, \\
I_{yy}\dot{q} + \left(I_{xx} - I_{zz}\right)\left(P_s r + R_s p\right) + I_{xz}\left(2P_s p + 2R_s r\right) &= m_A + m_T, \\
I_{zz}\dot{r} - I_{xz}\dot{p} + \left(I_{yy} - I_{xx}\right)\left(P_s q + Q_s p\right) + I_{xz}\left(Q_s r + R_s q\right) &= n_A + n_T.
\end{aligned}
$$

Kinematic equations:

$$
\begin{aligned}
p &= \dot{\phi} - \dot{\psi}_s\theta\cos\theta_s - \dot{\psi}\sin\theta_s, \\
q &= -\dot{\theta}_s\phi\sin\phi_s + \dot{\theta}\cos\phi_s + \dot{\psi}_s\phi\cos\theta_s\cos\phi_s - \dot{\psi}_s\theta\sin\theta_s\sin\phi_s + \dot{\psi}\cos\theta_s\sin\phi_s, \\
r &= -\dot{\psi}_s\phi\cos\theta_s\sin\phi_s - \dot{\psi}_s\theta\sin\theta_s\cos\phi_s + \dot{\psi}\cos\theta_s\cos\phi_s - \dot{\theta}_s\phi\cos\phi_s - \dot{\theta}\sin\phi_s.
\end{aligned}
$$

Now, for most steady–state conditions data measured for the aircraft, wings–level steady–state straight flight is usually assumed so that $V_s = \phi_s = P_s = Q_s = R_s = \dot{\phi}_s = \dot{\theta}_s = \dot{\psi}_s = 0$. The equations simplify to Translational equations:

$$
\begin{aligned}
m\left(\dot{u} + W_s q\right) &= -mg\theta\cos\theta_s + x_A + x_T, \\
m\left(\dot{v} + U_s r - W_s p\right) &= mg\phi\cos\theta_s + y_A + y_T, \\
m\left(\dot{w} - U_s q\right) &= -mg\theta\sin\theta_s + z_A + z_T,
\end{aligned}
$$

Rotational equations:

$$
\begin{aligned}
I_{xx}\dot{p} - I_{xz}\dot{r} &= l_A + l_T, \\
I_{yy}\dot{q} &= m_A + m_T, \\
I_{zz}\dot{r} - I_{xz}\dot{p} &= n_A + n_T.
\end{aligned}
$$

Kinematic equations:

$$
\begin{aligned}
p &= \dot{\phi} - \dot{\psi}\sin\theta_s, \\
q &= \dot{\theta},
\end{aligned}
$$

$$r = \dot{\psi} \cos \theta_s .$$

Using dimensional stability derivatives from Appendix B, we can now represent the perturbed forces and moments for both the longitudinal and lateral directional equations of motion and substitute the state variable $w = (V_{TAS})_s \alpha$ and $U_s = (V_{TAS})_s$. We also assume $W_s = 0$ so that the set of equations decouple completely. Note that the mass of the aircraft m is incorporated into the stability derivatives.

$$\begin{aligned}
\dot{u} &= -g\theta \cos \theta_s + X_u u + X_{T_u} u + X_\alpha \alpha + X_{\delta_e} \delta_e, \\
V_{TAS}\dot{\alpha} - (V_{TAS})_s q &= -g\theta \sin \theta_s + Z_u u + Z_\alpha \alpha + Z_{\dot{\alpha}} \dot{\alpha} + Z_q q + Z_{\delta_e} \delta_e, \\
I_{yy}\dot{q} &= M_u u + M_{T_u} u + M_\alpha \alpha + M_{T_\alpha} \alpha + M_{\dot{\alpha}} \dot{\alpha} + M_q q + M_{\delta_e} \delta_e, \\
q &= \dot{\theta}.
\end{aligned}$$

From there, the matrix format presented in the text can be easily derived. For the lateral directional equations, using substitutions $v = V_{TAS}\beta$

$$\begin{aligned}
\dot{\beta} (V_{TAS})_s + (V_{TAS})_s r &= g\phi \cos \theta_s + Y_\beta \beta + Y_p p + Y_r r + Y_{\delta_a} \delta_a + Y_{\delta_r} \delta_r, \\
I_{xx}\dot{p} - I_{xz}\dot{r} &= L_\beta \beta + L_p p + L_r r + L_{\delta_a} \delta_a + L_{\delta_r} \delta_r, \\
I_{zz}\dot{r} - I_{xz}\dot{p} &= N_\beta \beta + N_p p + N_r r + N_{\delta_a} \delta_a + N_{\delta_r} \delta_r, \\
p &= \dot{\phi} - \dot{\psi} \sin \theta_s, \\
r &= \dot{\psi} \cos \theta_s .
\end{aligned}$$