Lecture 14: The IO Model & External Sorting

Today's Lecture

1. The Buffer
2. External Merge Sort

1. The Buffer

Transition to Mechanisms

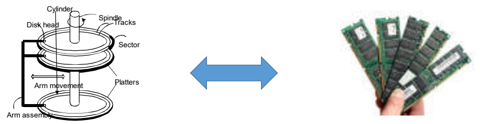
1. So you can **understand** what the database is doing!
   1. Understand the CS challenges of a database and how to use it.
   2. Understand how to optimize a query

2. Many **mechanisms** have become **stand-alone systems**
   - **Indexing** to Key-value stores
   - Embedded join processing
   - SQL-like languages take some aspect of what we discuss (PIG, Hive)

What you will learn about in this section

1. RECAP: Storage and memory model
2. Buffer primer

High-level: Disk vs. Main Memory



Disk:

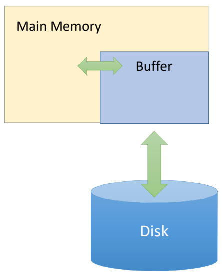
- ***Slow:*** Sequential *block* access
  - Read a blocks (not byte) at a time, so sequential access is cheaper than random
  - **Disk read / writes are expensive!**
- ***Durable:*** We will assume that once on disk, data is safe!
- ***Cheap***

Random Access Memory (RAM) or Main Memory:

- ***Fast:*** Random access, byte addressable
  - ~10x faster for sequential access
  - ~100,000x faster for random access!
- ***Volatile:*** Data can be lost if e.g. crash occurs, power goes out, etc!
- ***Expensive:*** For $100, get 16GB of RAM vs. 2TB of disk!

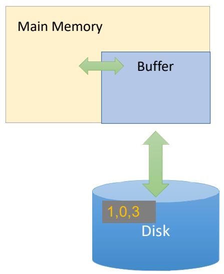
The Buffer

- A **buffer** is a region of physical memory used to store *temporary data*
  - *In this lecture:* a region in main memory used to store **intermediate data between disk and processes**
- *Key idea:* Reading / writing to disk is slow- need to cache data!



The (Simplified) Buffer

- In this class: We'll consider a buffer located in **main memory** that operates over **pages** and **files**:
  - **Read(page):** Read page from disk -> buffer *if not already in buffer*



The (Simplified) Buffer

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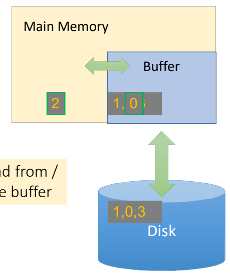
  - **Read(page):** Read page from disk -> buffer *if not already in buffer*

Processes can then read from / write to the page in the buffer



The (Simplified) Buffer

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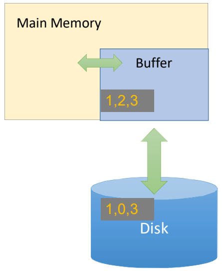
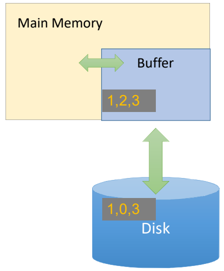
  - **Flush(page):** Evict page from buffer & write to disk



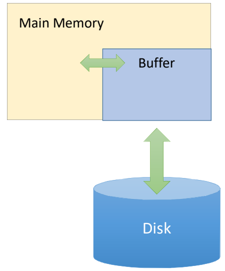
The (Simplified) Buffer

- In this class: We'll consider a buffer located in **main memory** that operates over **pages** and **files**:
  - **Read(page):** Read page from disk -> buffer *if not already in buffer*
  - **Flush(page):** Evict page from buffer & write to disk
  - **Release(page):** Evict page from buffer *without* writing to disk



Managing Disk: The DBMS Buffer

- Database maintains its own buffer
  - Why? The OS already does this…
  - DB knows more about access patterns.
    - Watch for how this shows up! (cf. *Sequential Flooding*)
  - Recovery and logging require ability to **flush** to disk.

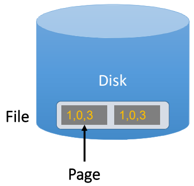


The Buffer Manager

- A **buffer manager** handles supporting operations for the buffer:
  - Primarily, handles & executes the "replacement policy"
    - i.e. finds a page in buffer to flush/release if buffer is full and a new page needs to be read in
  - DBMSs typically implement their own buffer management routines

A Simplified Filesystem Model

- For us, a **page** is a ***fixed-sized array*** of memory
  - Think: One or more disk blocks
  - Interface:
    - write to an entry (called a **slot**) or set to "None"
  - DBMS also needs to handle variable length fields
    - Page layout is important for good hardware utilization as well
- And a **file** is a *variable-length list* of pages
  - Interface: create / open / close; next_page(); etc.



2. External Merge & Sort

What you will learn about in this section

1. External Merge- Basics
2. External Merge- Extensions
3. External Sort

External Merge

Challenge: Merging Big Files with Small Memory

How do we *efficiently* merge two sorted files when both are much larger than our main memory buffer?

External Merge Algorithm

- **Input**: 2 sorted lists of length M and N
- **Output:** 1 sorted list of length M + N
- **Required:** At least 3 Buffer Pages
- **IOs**: 2(M+N)

Key (Simple) Idea

To find an element that is no larger than all elements in two lists, one only needs to compare minimum elements from each list.

If:

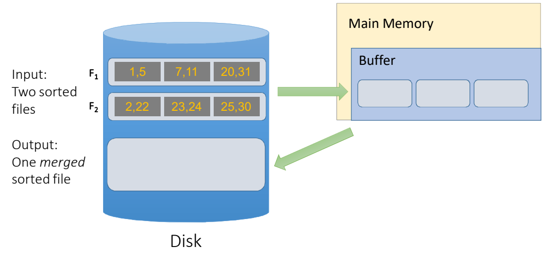
$$A_1 \le A_2 \le \cdots \le A_N$$
$$B_1 \le B_2 \le \cdots \le B_M$$

Then:

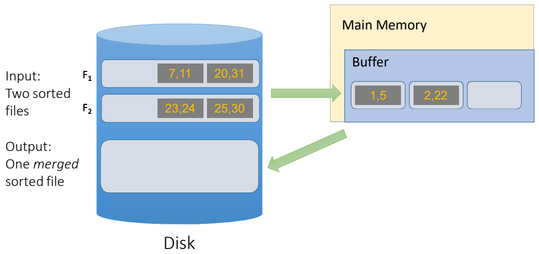
$$Min(A_1, B_1) \le A_i$$
$$Min(A_1, B_1) \le B_j$$

for i=1….N and j=1….M

External Merge Algorithm



External Merge Algorithm



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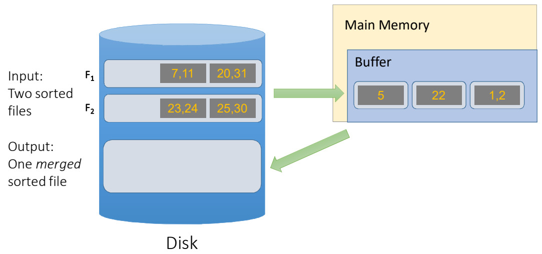

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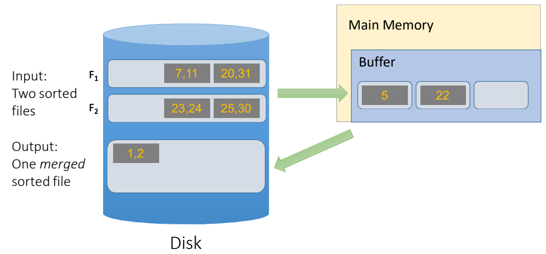

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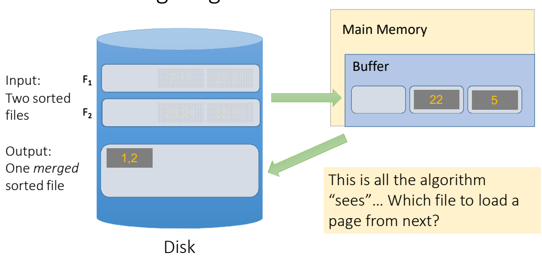

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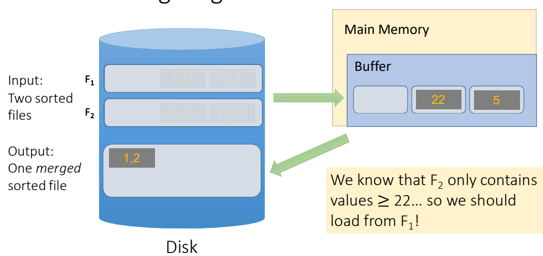

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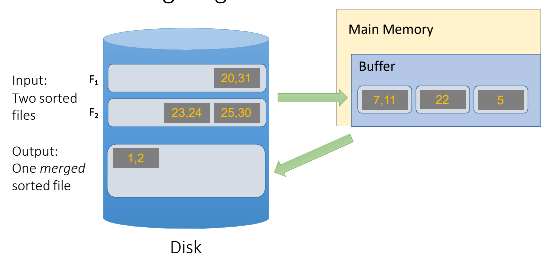
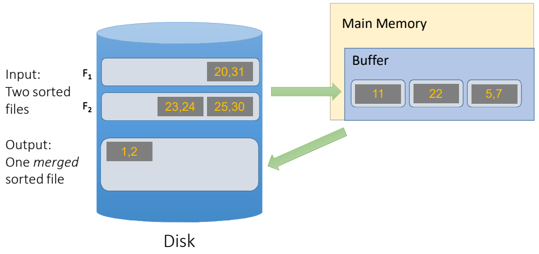


External Merge Algorithm



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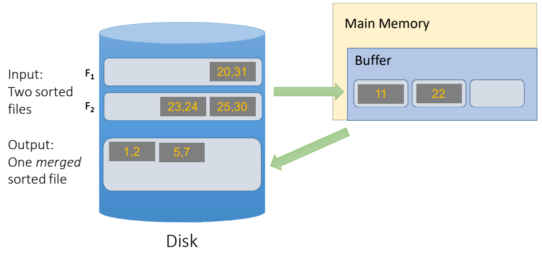
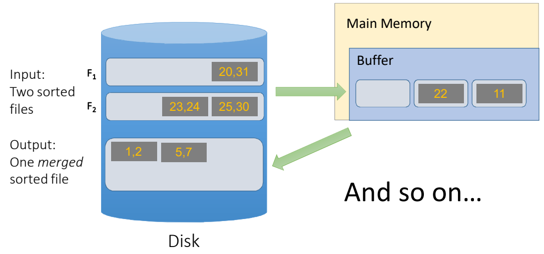


External Merge Algorithm



And so on…

We can merge lists of **arbitrary length** with *only* 3 buffer pages.

If lists of size M and N, then
Cost: 2(M+N) IOs
Each page is read once, written once

With B+1 buffer pages, can merge B lists. How?

B+ Trees:
An IO-Aware Index Structure

"If you don't find it in the index, look very carefully through the entire catalog"

- Sears, Roebuck and Co., Consumers Guide, 1897

Today's Lecture

1. *External Merge Sort & Sorting Optimizations*
2. Indexes: Motivations & Basics
3. B+ Trees

1. External Merge Sort

What you will learn about in this section

1. External merge sort
2. External merge sort on larger files
3. Optimizations for sorting

Recap: External Merge Algorithm

- Suppose we want to merge two **sorted** files both much larger than main memory (i.e. the buffer)
- We can use the **external merge algorithm** to merge files of ***arbitrary length*** in **2*(N+M) IO** operations with only **3 buffer pages**!

Our first example of an "IO aware" algorithm / cost model

External Merge Sort

Why are Sort Algorithms Important?

- Data requested from DB in sorted order is **extremely common**
  - e.g., find students in increasing GPA order
- **Why not just use quicksort in main memory??**
  - What about if we need to sort 1TB of data with 1GB of RAM…

A classic problem in computer science!

More reasons to sort…

- Sorting useful for eliminating *duplicate copies* in a collection of records (Why?)
- Sorting is first step in *bulk loading* B+ tree index. *Coming up…*
- *Sort-merge* join algorithm involves sorting *Next lecture*

Do people care?

http://sortbenchmark.org

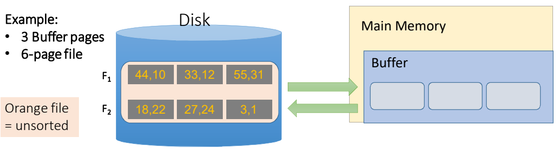
Sort benchmark bears his name

So how do we sort big files?

1. Split into chunks small enough to **sort in memory *("runs")***
2. **Merge** pairs (or groups) of runs ***using the external merge algorithm***
3. **Keep merging** the resulting runs ***(each time = a "pass")*** until left with one sorted file!

External Merge Sort Algorithm

Example:
- 3 Buffer pages
- 6-page file



1. Split into chunks small enough to **sort in memory**

External Merge Sort Algorithm

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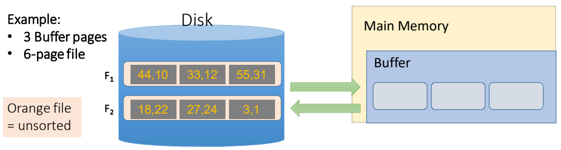

1. Split into chunks small enough to **sort in memory**

External Merge Sort Algorithm

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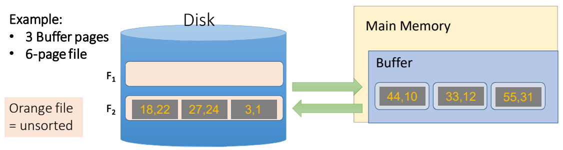
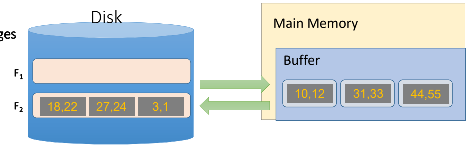


1. Split into chunks small enough to **sort in memory**

External Merge Sort Algorithm

Example:
- 3 Buffer pages
- 6-page file

Orange file = unsorted

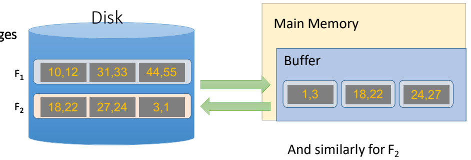


1. Split into chunks small enough to **sort in memory**

External Merge Sort Algorithm

Example:
- 3 Buffer pages
- 6-page file

Each sorted file is a called a *run*

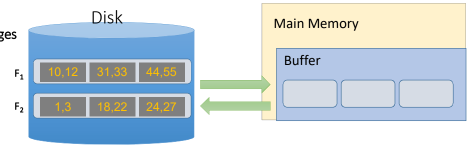


And similarly for F_2

1. Split into chunks small enough to **sort in memory**

External Merge Sort Algorithm

Example:
- 3 Buffer pages
- 6-page file



2. Now just run the **external merge** algorithm & we're done!

Calculating IO Cost

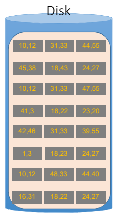
For 3 buffer pages, 6 page file:

1. Split into **<u>two 3-page files</u>** and **sort in memory**
   1. **= 1 R + 1 W for each file = 2*(3 + 3) = 12 IO operations**

2. **Merge** each pair of sorted chunks ***using the external merge algorithm***
   1. **= 2*(3 + 3) = 12 IO operations**

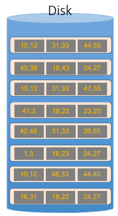
3. **Total cost = 24 IO**

Running External Merge Sort on Larger Files



Assume we still only have 3 buffer pages *(Buffer not pictured)*

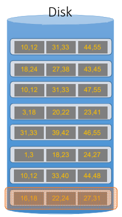
Running External Merge Sort on Larger Files



1. Split into files small enough to sort in buffer…

Assume we still only have 3 buffer pages *(Buffer not pictured)*

Running External Merge Sort on Larger Files

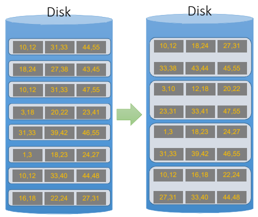


1. Split into files small enough to sort in buffer… and sort

Assume we still only have 3 buffer pages *(Buffer not pictured)*

Call each of these sorted files a *run*

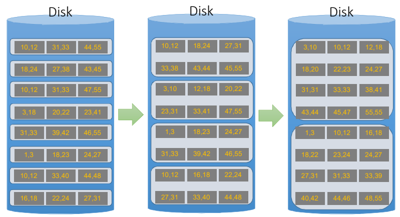
Running External Merge Sort on Larger Files



Assume we still only have 3 buffer pages *(Buffer not pictured)*

2. Now merge pairs of (sorted) files… **the resulting files will be sorted!**

Running External Merge Sort on Larger Files

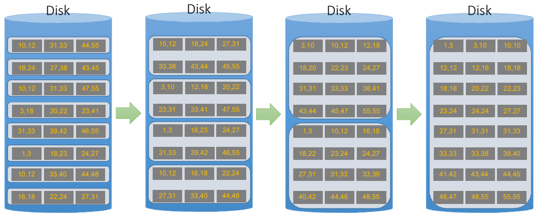


Assume we still only have 3 buffer pages *(Buffer not pictured)*

3. And repeat…

Call each of these steps a ***pass***

Running External Merge Sort on Larger Files

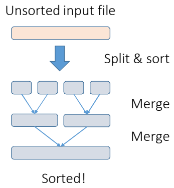


4. And repeat!

Simplified 3-page Buffer Version

Assume for simplicity that we split an N-page file into N single-page ***runs*** and sort these; then:

- First pass: Merge **N/2 *pairs* of runs** each of length **1 page**
- Second pass: Merge **N/4 *pairs* of runs** each of length **2 pages**
- In general, for **N** pages, we do $\lceil \log_2 N \rceil$ passes
  - +1 for the initial split & sort
- Each pass involves reading in & writing out all the pages = ***2N IO***



→ 2N*($\lceil \log_2 N \rceil$+1) total IO cost!

Using B+1 buffer pages to reduce # of passes

Suppose we have B+1 buffer pages now; we can:

1. **Increase length of initial runs**. Sort B+1 at a time!

At the beginning, we can split the N pages into runs of length B+1 and sort these in memory

IO Cost:

$2N(\lceil \log_2 N \rceil + 1)$ ➡ $2N(\lceil \log_2 \frac{N}{B+1} \rceil + 1)$

Starting with runs of length 1 | Starting with runs of length ***B+1***

Using B+1 buffer pages to reduce # of passes

Suppose we have B+1 buffer pages now; we can:

2. Perform a B-way merge.

On each pass, we can merge groups of ***B*** runs at a time (vs. merging pairs of runs)!

IO Cost:

$2N(\lceil \log_2 N \rceil + 1)$ ➡ $2N(\lceil \log_2 \frac{N}{B+1} \rceil + 1)$ ➡ $2N(\lceil \log_B \frac{N}{B+1} \rceil + 1)$

Starting with runs of length 1 | Starting with runs of length ***B+1*** | Performing ***B***-way merges

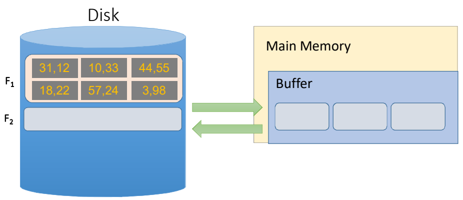
Repacking

Repacking for even longer initial runs

- With B+1 buffer pages, we can now start with ***B+1-length initial runs*** (and use ***B-way merges***) to get $2N(\lceil \log_B \frac{N}{B+1} \rceil + 1)$ IO cost…
- Can we reduce this cost more by getting even longer initial runs?
- Use **repacking**- produce longer initial runs by "merging" in buffer as we sort at initial stage

Repacking Example: 3 page buffer

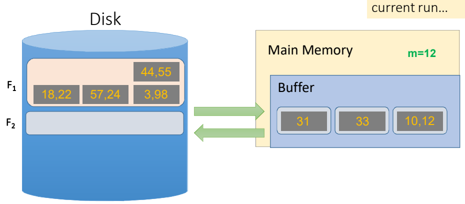
- Start with unsorted single input file, and load 2 pages



Repacking Example: 3 page buffer

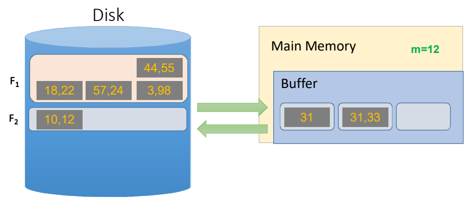
- Take the minimum two values, and put in output page

Also keep track of max (last) value in current run…



Repacking Example: 3 page buffer

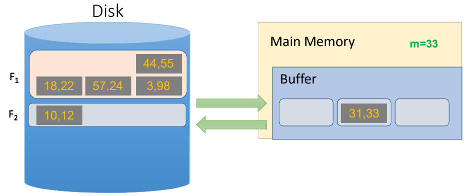
- Next, ***repack***



Lecture 15 > Section 1 > Optimizations for sorting

Repacking Example: 3 page buffer

- Next, ***repack***, then load another page and continue!



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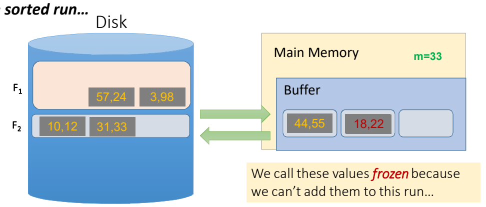
Repacking Example: 3 page buffer

- Now, however, ***the smallest values are less than the largest (last) in the sorted run…***



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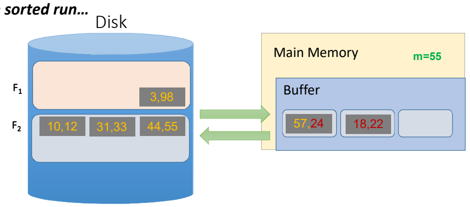

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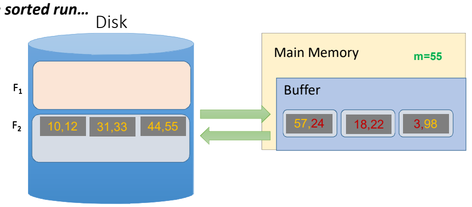

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Repacking Example: 3 page buffer

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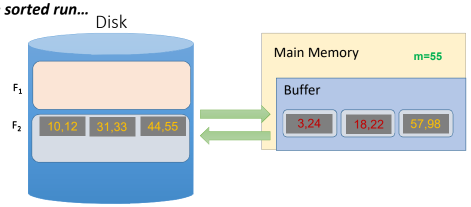

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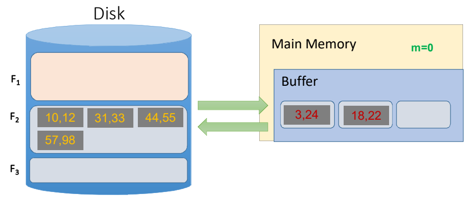
Repacking Example: 3 page buffer

- Once ***all buffer pages have a frozen value,*** or input file is empty, start new run with the frozen values



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Repacking Example: 3 page buffer

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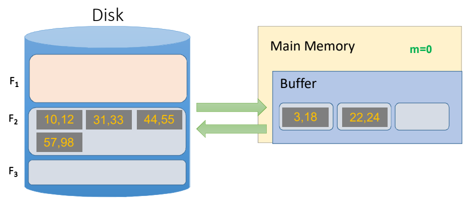


Lecture 15 > Section 1 > Optimizations for sorting

Repacking

- Note that, for buffer with B+1 pages:
  - If input file is sorted → nothing is frozen → we get **a single** run!
  - If input file is reverse sorted (worst case) → everything is frozen → we get runs of length **B+1**
- In general, with repacking we do **no worse** than without it!
- What if the file is already sorted?
- Engineer's approximation: runs will have **~2(B+1)** length

$$\sim 2N\left(\left\lceil \log_B \frac{N}{2(B+1)} \right\rceil + 1\right)$$

Lecture 15 > Section 1 > SUMMARY

Summary

- Basics of IO and buffer management.
  - See notebook for more fun! (Learn about *sequential flooding*)
- We introduced the IO cost model using **sorting**.
  - Saw how to do merges with few IOs,
  - Works better than main-memory sort algorithms.
- Described a few optimizations for sorting

B+ Trees: An IO-Aware Index Structure

What you will learn about in this section

1. Indexes: Motivation
2. Indexes: Basics
3. ACTIVITY: Creating indexes

Index Motivation

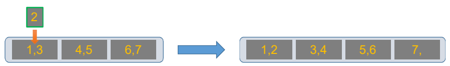
Person(name, age)

- Suppose we want to search for people of a specific age
- ***First idea:*** Sort the records by age… we know how to do this fast!
- How many IO operations to search over ***N sorted*** records?
  - Simple scan: ***O(N)***
  - Binary search: $O(\log_2 N)$

Could we get even cheaper search? E.g. go from $\log_2 N$ $\rightarrow \log_{200} N$?

Index Motivation

- What about if we want to **insert** a new person, but keep the list sorted?



- We would have to potentially shift ***N*** records, requiring up to **~ 2*N/P** IO operations (where P = # of records per page)!
  - We could leave some "slack" in the pages…

Could we get faster insertions?

Index Motivation

- What about if we want to be able to search quickly along multiple attributes (e.g. not just age)?
  - We could keep multiple copies of the records, each sorted by one attribute set… this would take a lot of space

Can we get fast search over multiple attribute (sets) without taking too much space?

We'll create separate data structures called *indexes* to address all these points

Further Motivation for Indexes: NoSQL!

- NoSQL engines are (basically) ***just indexes!***
  - A lot more is left to the user in NoSQL… one of the primary remaining functions of the DBMS is still to provide index over the data records, for the reasons we just saw!
  - Sometimes use B+ Trees (covered next), sometimes hash indexes (not covered here)

Indexes are critical across all DBMS types

Indexes: High-level

- An *index* on a file speeds up selections on the *search key fields* for the index.
  - Search key properties
    - Any subset of fields
    - is **not** the same as *key of a relation*
- *Example:*

Product(name, maker, price)

On which attributes would you build indexes?

More precisely

- An *index* is a **data structure** mapping search keys to sets of rows in a database table
  - Provides efficient lookup & retrieval by search key value- usually much faster than searching through all the rows of the database table
- An index can store the full rows it points to (*primary index*) or pointers to those rows (*secondary index*)
  - We'll mainly consider secondary indexes

Operations on an Index

- Search: Quickly find all records which meet some *condition on the search key attributes*
  - More sophisticated variants as well. Why?
- Insert / Remove entries
  - Bulk Load / Delete. Why?

Indexing is one the most important features provided by a database for performance

Conceptual Example

What if we want to return all books published after 1867? The above table might be very expensive to search over row-by-row…

Russian_Novels

BID	Title	Author	Published	Full_text
001	*War and Peace*	Tolstoy	1869	…
002	*Crime and Punishment*	Dostoyevsky	1866	…
003	*Anna Karenina*	Tolstoy	1877	…

```
SELECT *
FROM Russian_Novels
WHERE Published > 1867
```


Conceptual Example

By_Yr_Index

Published	BID
1866	002
1869	***001***
1877	***003***

Russian_Novels

BID	Title	Author	Published	Full_text
001	*War and Peace*	Tolstoy	1869	…
002	*Crime and Punishment*	Dostoyevsky	1866	…
003	*Anna Karenina*	Tolstoy	1877	…

Maintain an index for this, and search over that!

Why might just keeping the table sorted by year not be good enough?

Conceptual Example

By_Yr_Index

Published	BID
1866	002
1869	001
1877	003

Russian_Novels

BID	Title	Author	Published	Full_text
001	*War and Peace*	Tolstoy	1869	…
002	*Crime and Punishment*	Dostoyevsky	1866	…
003	*Anna Karenina*	Tolstoy	1877	…

By_Author_Title_Index

Author	Title	BID
Dostoyevsky	Crime and Punishment	002
Tolstoy	Anna Karenina	003
Tolstoy	War and Peace	001

Can have multiple indexes to support multiple search keys

Indexes shown here as tables, but in reality we will use more efficient data structures…

Covering Indexes

By_Yr_Index

Published	BID
1866	002
1869	001
1877	003

We say that an index is <u>covering</u> *for a specific query* if the index contains all the needed attributes- ***meaning the query can be answered using the index alone!***

The "needed" attributes are the union of those in the SELECT and WHERE clauses…

Example:

```
SELECT Published, BID
FROM Russian_Novels
WHERE Published > 1867
```


High-level Categories of Index Types

- B-Trees *(covered next)*
  - Very good for range queries, sorted data
  - Some old databases only implemented B-Trees
  - *We will look at a variant called **B+ Trees***
- Hash Tables *(not covered)*
  - There are variants of this basic structure to deal with IO
  - Called ***linear*** or ***extendible hashing***- IO aware!

The data structures we present here are "IO aware"

Real difference between structures: costs of ops *determines which index you pick and why*

Demo on Index

1. B+ Trees

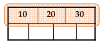
What you will learn about in this section

1. B+ Trees: Basics
2. B+ Trees: Design & Cost
3. Clustered Indexes

B+ Trees

- Search trees
  - B does not mean binary!
- Idea in B Trees:
  - make 1 node = 1 physical page
  - Balanced, height adjusted tree (not the B either)
- Idea in B+ Trees:
  - Make leaves into a linked list (for range queries)

B+ Tree Basics

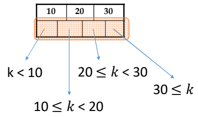


Parameter ***d*** = the degree

Each *non-leaf ("interior")* ***node*** has ≥ d and ≤ 2d ***keys****

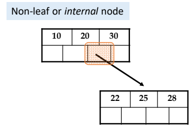
except for root node, which can have between **1 and 2d keys*

B+ Tree Basics



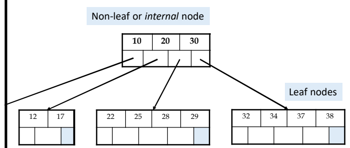
The *n* keys in a node define *n+1* ranges

B+ Tree Basics



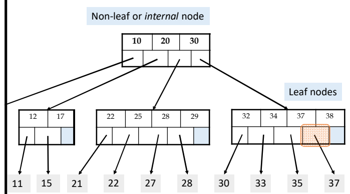
For each range, in a *non-leaf* node, there is a **pointer** to another node with keys in that range

B+ Tree Basics



Leaf nodes also have between *d* and *2d* keys, and are different in that:

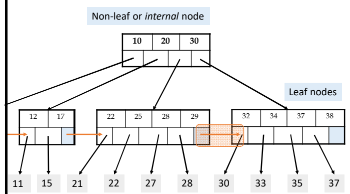
B+ Tree Basics



Leaf nodes also have between *d* and *2d* keys, and are different in that:

Their key slots contain pointers to data records

B+ Tree Basics

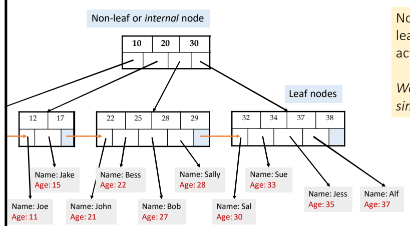


Leaf nodes also have between *d* and *2d* keys, and are different in that:

Their key slots contain pointers to data records

They contain a pointer to the next leaf node as well, ***for faster sequential traversal***

B+ Tree Basics



Note that the pointers at the leaf level will be to the actual data records (rows).

We might truncate these for simpler display (as before)…

Some finer points of B+ Trees

Searching a B+ Tree

- For exact key values:
  - Start at the root
  - Proceed down, to the leaf

```
SELECT name
FROM   people
WHERE  age = 25
```

- For range queries:
  - As above
  - *Then sequential traversal*

```
SELECT name
FROM   people
WHERE  20 <= age
 AND  age <= 30
```

B+ Tree Exact Search Animation

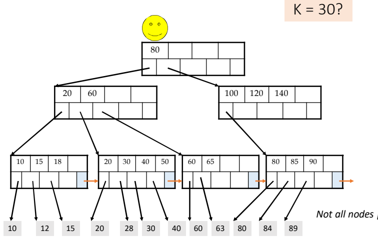
K = 30?

30 < 80

30 in [20,60)

30 in [30,40)

To the data!



Not all nodes pictured

Lecture 14 > Section 1 > B+ Tree design & cost

B+ Tree Range Search Animation

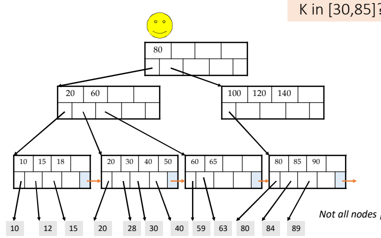
K in [30,85]?

30 < 80

30 in [20,60)

30 in [30,40)

To the data!



Not all nodes pictured

Lecture 14 > Section 1 > B+ Tree design & cost

B+ Tree Design

- How large is ***d***?
- Example:
  - Key size = 4 bytes
  - Pointer size = 8 bytes
  - Block size = 4096 bytes
- We want each *node* to fit on a single *block/page*
  - 2d x 4 + (2d+1) x 8 <= 4096 → ***d <= 170***

NB: Oracle allows 64K = 2^16 byte blocks
→ d <= 2730

Lecture 14 > Section 1 > B+ Tree design & cost

B+ Tree: High Fanout = Smaller & Lower IO

- As compared to e.g. binary search trees, B+ Trees have **high *fanout*** (***between d+1 and 2d+1***)
- This means that the **depth of the tree is small** → getting to any element requires very few IO operations!
  - Also can often store most or all of the B+ Tree in main memory!
- A TiB = 2^{40} Bytes. What is the height of a B+ Tree (with fill-factor = 1) that indexes it (with 64K pages)?
  - $(2*2730 + 1)^h = 2^{40}$ → ***h = 4***

The <u>fanout</u> is defined as the number of pointers to child nodes coming out of a node

Note that fanout is dynamic- we'll often assume it's constant just to come up with approximate eqns!

The known universe contains ~10^{80} particles… what is the height of a B+ Tree that indexes these?

Lecture 14 > Section 1 > B+ Tree design & cost

B+ Trees in Practice

- Typical order: d=100. Typical fill-factor: 67%.
  - average fanout = 133
- Typical capacities:
  - Height 4: 133^4 = 312,900,700 records
  - Height 3: 133^3 = 2,352,637 records
- Top levels of tree sit *in the buffer pool*:
  - Level 1 = 1 page = 8 Kbytes
  - Level 2 = 133 pages = 1 Mbyte
  - Level 3 = 17,689 pages = 133 MBytes

<u>Fill-factor</u> is the percent of available slots in the B+ Tree that are filled; is usually < 1 to leave slack for (quicker) insertions

Typically, only pay for one IO!

Lecture 14 > Section 1 > B+ Tree design & cost

Simple Cost Model for Search

- Let:
  - ***f*** = fanout, which is **in [d+1, 2d+1]** ***(we'll assume it's constant for our cost model…)***
  - ***N*** = the total number of *pages* we need to index
  - ***F*** = fill-factor (usually ~= 2/3)
- Our B+ Tree needs to have room to index ***N / F*** pages!
  - We have the fill factor in order to leave some open slots for faster insertions
- What height (*h*) does our B+ Tree need to be?
  - h=1 → Just the root node- room to index f pages
  - h=2 → f leaf nodes- room to index f^2 pages
  - h=3 → f^2 leaf nodes- room to index f^3 pages
  - …
  - h → f^{h-1} leaf nodes- room to index f^h pages!

→ We need a B+ Tree of height h = $\left\lceil \log_f \frac{N}{F} \right\rceil$!

Lecture 14 > Section 1 > B+ Tree design & cost

Simple Cost Model for Search

- Note that if we have ***B*** available buffer pages, by the same logic:
  - We can store $\boldsymbol{L_B}$ levels of the B+ Tree in memory
  - where $\boldsymbol{L_B}$ ***is the number of levels such that the sum of all the levels' nodes fit in the buffer:***
    - $B \geq 1 + f + \cdots + f^{L_B - 1} = \sum_{l=0}^{L_B - 1} f^l$
- In summary: to do exact search:
  - We read in one page per level of the tree
  - However, levels that we can fit in buffer are free!
  - Finally we read in the actual record

IO Cost: $\left\lceil \log_f \frac{N}{F} \right\rceil - L_B + 1$

where $B \geq \sum_{l=0}^{L_B - 1} f^l$

Lecture 14 > Section 1 > B+ Tree design & cost

Simple Cost Model for Search

- To do range search, we just follow the horizontal pointers
- The IO cost is that of loading additional leaf nodes we need to access + the IO cost of loading each ***page*** of the results- we phrase this as "Cost(OUT)"

IO Cost: $\left\lceil \log_f \frac{N}{F} \right\rceil - L_B + Cost(OUT)$

where $B \geq \sum_{l=0}^{L_B - 1} f^l$

Lecture 14 > Section 1 > B+ Tree design & cost

Fast Insertions & Self-Balancing

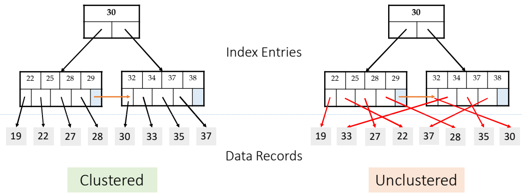
- We won't go into specifics of B+ Tree insertion algorithm, but has several attractive qualities:
  - **~ Same cost as exact search**
  - ***Self-balancing:*** B+ Tree remains **balanced** (with respect to height) even after insert

B+ Trees also (relatively) fast for single insertions!
However, can become bottleneck if many insertions (if fill-factor slack is used up…)

Clustered Indexes

An index is ***clustered*** if the underlying data is ordered in the same way as the index's data entries.

Clustered vs. Unclustered Index



Clustered vs. Unclustered Index

- Recall that for a disk with block access, **sequential IO is much faster than random IO**
- For exact search, no difference between clustered / unclustered
- For range search over R values: difference between **1 random IO + R sequential IO**, and **R random IO**:
  - A random IO costs ~ 10ms (sequential much much faster)
  - For R = 100,000 records- **difference between ~10ms and ~17min!**

Summary

- We covered an algorithm + some optimizations for sorting larger-than-memory files efficiently
  - An ***IO aware*** algorithm!
- We create **indexes** over tables in order to support ***fast (exact and range) search*** and ***insertion*** over ***multiple search keys***
- **B+ Trees** are one index data structure which support very fast exact and range search & insertion via ***high fanout***
  - ***Clustered vs. unclustered*** makes a big difference for range queries too