Real Options Valuation: A Dynamic Programming Approach

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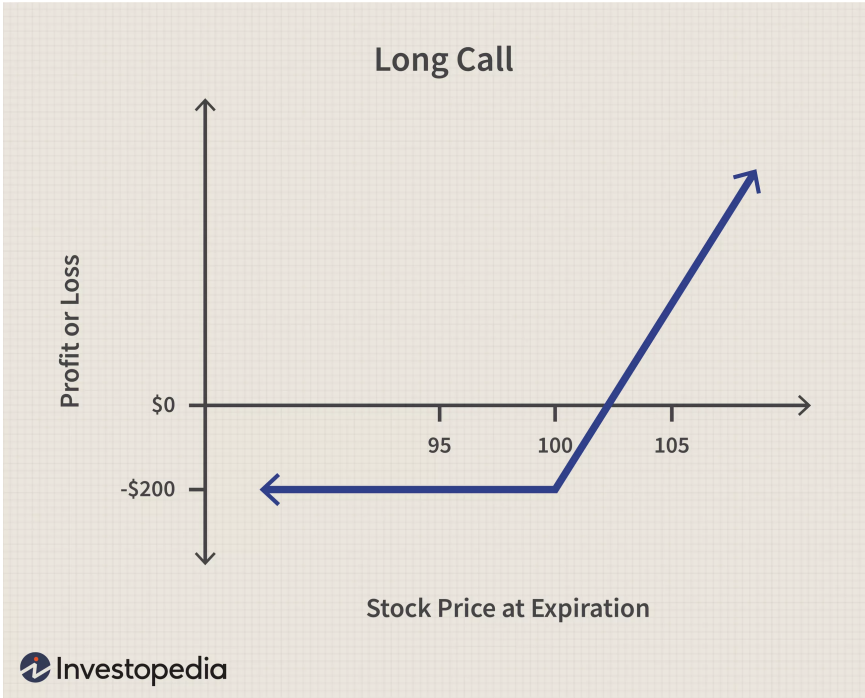

Figure: Call Option [?]

Introduction

Detailed

- Three classes of ROA authors (BSM model analogy)
- SDT framework $>$ frameworks used in economy
- Formulation of ROA problem in SDT framework

Table 7.1. Analysis of the Option to Invest in a Project when Construction Takes One Period

$X(i,n)$	0	1	2	3	4	5
0	100.00	125.00	156.25	195.31	244.14	305.18
1		80.00	100.00	125.00	156.25	195.31
2			64.00	80.00	100.00	125.00 ← Step 1: Construct binomial tree for the state variable
3				51.20	64.00	80.00
4					40.96	51.20
5						32.77

$V_b(i,n)$	0	1	2	3	4	5
0	1.88	7.82	31.93	65.41	107.26	0.00
1		0.30	1.43	6.76	31.93	0.00 ← Step 2: Fill in final column using equation (7.1)
2			0.00	0.00	0.00	0.00
3				0.00	0.00	0.00
4					0.00	0.00 ← Step 3: Fill in remaining columns using equation (7.2)
5						0.00

Figure: Economical framework example [**?**]

ROA inspiration

Graeme Guthrie

- Replicating portfolio (efficient markets)
- Risk neutral probabilities
- Backward induction
- Probability of up and down move [**?**]:

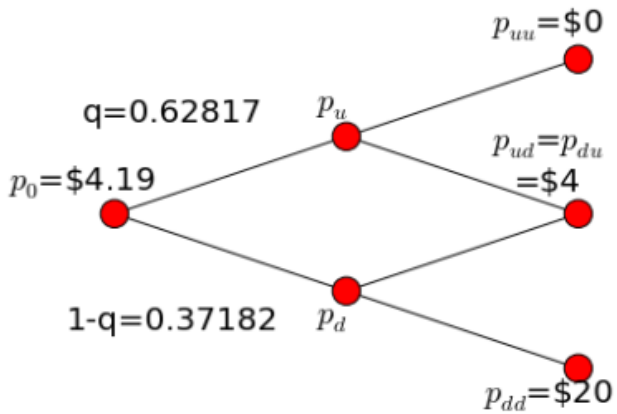

Figure: Binomial model

$$\pi_u = \frac{ZR_f - X_d}{X_u - X_d}, \pi_d = \frac{X_u - ZR_f}{X_u - X_d} \quad (1)$$

ROA limitations

General

- Limited number of uncertainty sources - one (efficient market)
- Simple distributions - binomial model
- Limited by computational complexity for higher-dimensional problems
- Complicated scaling in types and scale of real options

SDT Improvements

General

- Allows for multiple sources of uncertainty - seamless integration
- Allows continuous distributions, theoretically of any type
- Computational complexity tools - ADP
- Real options (actions) easily scaled with action set

Furthermore:

- Allows for simple integration of Bayesian learning
- Allows for complex creation of prior probability densities

SDT Improvements

Preserving economical truths

- Time value of money - discounting factors
- Risk aversion of investors - utility theory
- Risk-neutral probabilities - Bayesian priors

SDT Improvements

ADP

- To be done

Valuation of gas power plant

General Idea

- Power generating company
- In the next 5 years, possibility to build 200MW or 400MW gas unit for 65/130M EUR
- Prices of gas, power, CO2 allowances are 24EUR, 9EUR, 40EUR per MWh
- Government policy favoring renewables can rise -¿ higher volatility of prices.
- Lifespan of gas power plant is set to 25 years, loan possible with 3% interest rate
- Power plant is selling its power as monthly contracts

Valuation of gas power plant

Details

Considered real options:

- Wait for more favorable market prices - Timing option
- Build 200MW gas power plant. Then possibility to increase to 400MW - Scaling option
- Sell the power plant - abandonment option
- Mothball the plant. It is not able to produce, but the fixed costs are lowered
- Ability to not run the power plant (paying fixed costs)- switching option

Sources of uncertainty:

- Price of gas, power and CO2 - lognormal process with different volatilities
- Government policy for renewables - discrete with positive mean

Results

- To be computed

Summary

- Item
- Item
- Item
- Item
- Possible directions of future research:
  - Item
  - Item
  - Item

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