

TDT4136 - Assignment 2

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Task 1 - Models and Entailment in Propositional Logic

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The tables above show all the possible worlds. The red square tells us that there is a wumpus occupying that square. The blue square means a pit is located on that square. And purple means that both a wumpus and a pit is on that square. The four first rows matches $\alpha_2 =$ “There is no pit in [2,2].” and the third column matches $\alpha_3 =$ “There is a wumpus in [1,3]”. The **KB** is true for the table in row 2, column 3.

2) Exercise 7.4

a) $False \models True$

False entails True, this is correct

b) $True \models False$

False

c) $(A \wedge B) \models (A \Leftrightarrow B)$

d) $(A \Leftrightarrow B) \models (A \vee B)$

e) $(A \Leftrightarrow B) \models (\neg A \Leftrightarrow B)$

f) $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B \vee C) \vee (B \vee C \vee (D \rightarrow E))$

g) $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B) \wedge (\neg D \vee E)$

h) $(A \vee B) \wedge \neg(A \rightarrow B)$

i) $(A \vee B) \rightarrow C \models \neg(A \rightarrow B)$

j) $(C \vee (\neg A \wedge \neg B)) \equiv ((A \rightarrow C) \wedge (B \rightarrow C))$

k) $(A \Leftrightarrow B) \wedge (\neg A \vee B)$

l) $(A \Leftrightarrow B) \Leftrightarrow C$ has the same number of models as $(A \Leftrightarrow B)$ for any fixed set of proposition symbols that includes A, B, C.

3) Exercise 7.7

a) $B \vee C$

There are 3 models in which this sentence is true

b) $\neg A \vee \neg B \vee \neg C \vee \neg D$

Using a truth table i have found the number of models to be 15.

b) $(A \rightarrow B) \vee A \vee \neg B \vee C \vee D$

This sentence is always false and has no models.

4)

5)

a) $A_1 \vee A_{73}$

$3/4 + 2^{100}$

b) $A_7 \vee (A_{19} \wedge A_{33})$

$5/8 * 2^{100}$

c) $A_{11} \rightarrow A_{22}$

$3/4 + 2^{100}$

Task 2 - Resolution in Propositional Logic

1) - Convert each of the following sentences to Conjunctive Normal Form (CNF)

a) $A \wedge B \wedge C$

Already in CNF

a) $A \vee B \vee C$

Already in CNF

a) $A \rightarrow (B \vee C)$

$\neg A \vee (A \vee B)$

2) - Consider the following Knowledge Base (KB):

- $(A \vee \neg B) \rightarrow \neg C$
- $(D \wedge E) \rightarrow C$
- $A \wedge D$

Use resolution to show that $KB \models \neg E$

The first step is to convert the knowledge base into Conjunctive Normal Form.

- $(\neg A \vee \neg C) \wedge (\neg C \vee E)$
- $C \vee \neg D \vee \neg E$
- $A \wedge D$